NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2071

CHARTS OF THERMODYNAMIC PROPERTIES OF AIR AND

COMBUSTION PRODUCTS FROM 300° TO 3500° R

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SUMMARY

Thermodynamic data of gas properties are necessary in order to compute the performance of gas-turbine engines from a knowledge of design conditions, to determine from measured gas conditions the performance of a component of such an engine, and to design the engine components.

Convenient nomographic charts that permit the determination of the enthalpy change for various processes are presented. In order to simplify calculation, enthalpy and a function ϕ, defined as $\int \frac{c_p}{T} dT$ (where c_p is the specific heat at constant pressure and T is the temperature), are presented for mixtures of air, water vapor, and the products of complete combustion of hydrocarbon fuels with air. The charts cover a range of temperature from 300° to 3500° R, hydrogen-carbon ratios from 0.10 to 0.20, and fuel-air ratios from zero to stoichiometric. The effects of pressure and dissociation are considered negligible.

The charts have the following characteristics:

(1) The data are presented for temperatures as high as those for which the dissociation effects are small.

(2) The scales of the two types of nomograph were selected to permit quick reading of the values of enthalpy or ϕ within ±2.5° R, or easy computation of enthalpy or ϕ within ±0.5° R.

(3) The units in which the data are presented are those commonly used in cycle analyses so that the data may be utilized without converting between two systems of units.

Methods are presented for the application of these data to the solution of common thermodynamic problems.

INTRODUCTION

Thermodynamic data of gas properties are necessary in order to compute the performance of gas-turbine engines from a knowledge of the design conditions or to determine from measured gas conditions the performance of a component of such an engine. These data are also necessary for the design of engine components.

Most gas-turbine power cycles are constructed from the following five processes:

(1) Compression of air

(2) Constant-pressure combustion of hydrocarbon fuel with air

(3) Reheating combustion

(4) Expansion of combustion products

(5) Heat exchange

The presentation of thermodynamic data should be in a form that permits simple, quick, and accurate computations to be made for determining the change in state and the energy requirements for each of these five processes; the data should cover the range of pressures, temperatures, and gas compositions required. The large amount of data already available attests to the importance of this problem. (See references 1 to 10.)

Specialized computational techniques for the analysis of part of a power cycle are described in references 1 to 7. Each of references 8 to 10 supplies data for the analysis of all five of these processes but is limited either by inconvenience of use, range of data presented, or presentation of data for only one or two hydrogen-carbon ratios.

Convenient nomographic charts that permit the determination of enthalpy changes for various processes were developed at the NACA Lewis laboratory and are presented herein. In the charts, which were constructed principally from the data of reference 11, enthalpy and a function ϕ, defined as $\int \frac{c_p}{T} \, dT$ (where c_p is the specific heat at constant pressure and T is temperature) are given for mixtures of air, water vapor, and the products of complete combustion of hydrocarbon fuel with air. The temperature range is from 600° to 3500° R for hydrogen-carbon ratios from 0.10 to 0.20 and fuel-air

ratios from zero to stoichiometric; data from reference 2 were used
to extend the chart of air properties from 600° to 300° R. The data
for water vapor (reference 12) range from 500° to 3500° R. The fol-
lowing variables and units are employed: temperature in degrees
Rankine, enthalpy in Btu per pound, ϕ in Btu per pound degrees
Rankine, fuel-air ratio, and hydrogen-carbon ratio.

In the construction of these charts, any effects of pressure
or dissociation were neglected. The effect of pressure on the
thermodynamic properties of air, as presented in reference 13, has
been examined in reference 10 with the conclusion "that the char-
acteristics of the isentropic . . . are suitable for precise work
even to pressures in the hundreds of pounds per square inch." The
effects of dissociation are considered in reference 7 and are found
to be negligible below temperatures of 3000° to 3200° R. These
considerations indicate that low-pressure specific heats obtained
by spectroscopic means are reliable without correction for disso-
ciation or compressibility at conditions normally encountered in
gas-turbine engines. The data presented should, however, be used
with caution above 3000° R. The effects of pressure and dissoci-
ation at high temperatures are presented in references 5, 14, and
15.

The charts presented herein have the following characteristics:

(1) The data are presented for temperatures as high as those
for which dissociation effects are small.

(2) The scales of the two types of nomograph were selected to
permit quick reading of the values of enthalpy or ϕ within
±2.5° R, or easy computation of enthalpy or ϕ within ±0.5° R.

(3) The units in which the data are presented are those com-
monly used in cycle analyses so that the data may be utilized with-
out converting between two systems of units.

The use of the charts is explained and methods are described
for employing the charts to solve several thermodynamic problems.
A series of numerical examples is presented. The methods used to
obtain the charts are presented in the appendix.

SYMBOLS

The following symbols are used in this report:

a,b,c,d   gases

c_p specific heat at constant pressure, $\text{Btu}/(\text{lb-}^\circ\text{R})$

f   fuel-air ratio

$\bar{H}$ lower heating value of fuel at 600° R, $\text{Btu}/(\text{lb})$

H/C   hydrogen-carbon ratio

h   enthalpy, $\text{Btu}/(\text{lb})$

J   mechanical equivalent of heat, 778.156 $(\text{ft-lb})/\text{Btu}$

m   specific humidity, (lb water/lb air)

n   number of hydrogen atoms in fuel for each carbon atom

p   pressure, (lb/sq ft)

R   gas constant, $(\text{ft-lb/lb-}^\circ\text{R})$

T   temperature, $(^\circ\text{R})$

v   specific volume, (cu ft/lb)

w   weight, (lb)

X   number of moles of fuel

η_b burner efficiency

η_c compressor efficiency

η_t turbine efficiency

$$\phi \equiv \int \frac{c_p}{T}\, dT, \quad \text{Btu}/(\text{lb-}^\circ\text{R})$$

$$\psi_h \equiv \frac{(h - h_0)(1 + f)}{f}, \quad \text{Btu}/(\text{lb})$$

$$\psi_\phi \equiv \frac{(\phi - \phi_0)(1 + f)}{f}, \quad \text{Btu}/(\text{lb-}^\circ\text{R})$$

Subscripts:

a,b,c,d	gases
C	carbon
f	fuel
i	ideal
H	hydrogen
l	liquid
m	water vapor-air mixture
n	number of hydrogen atoms in fuel for each atom of carbon
t	turbine
w	water
∞	small stage
0	dry air, zero fuel-air ratio
1	initial state
2	final state

Superscripts:

'	first trial or approximation
''	second trial or approximation
'''	third trial or approximation

READING THE CHARTS

Thermodynamic properties of dry air, combustion gases, and water vapor are plotted in charts I, II to IV, and V, respectively. Two types of chart of gas properties are included: type 1 includes charts I and V; and type 2 includes charts II to IV.

Type 1 - Charts I and V

The relation among three variables, any one of which may be an independent variable with the remaining two variables dependent upon the first, is described in charts I and V. Chart I shows the relation among ϕ, enthalpy h, and temperature T for dry air, and chart V for water vapor. The reading of these charts is illustrated by figure 1. At a temperature of 410° R this enlarged view of chart I indicates that

$$h_0 = 2.39 \text{ Btu/lb}$$

$$\phi_0 = 0.006 \text{ Btu/lb-}^\circ\text{R}$$

Chart I may easily be read with a precision of $\pm 0.5^\circ$ R and chart V, $\pm 2.5^\circ$ R.

Type 2 - Charts II to IV

The relation among four variables, any two of which may be independent variables with the remaining two variables dependent upon the first two, is presented in charts II to IV. Chart II, for which the temperature scale is divided into 5° intervals, shows the relation among ϕ, enthalpy h, fuel-air ratio f, and temperature T for gases resulting from the complete combustion of fuel with dry air for a fuel having a hydrogen-carbon ratio of 0.167; this chart may be used for hydrogen-carbon ratios close to 0.167. Charts III and IV present the relation among ψ_h, ψ_ϕ, the hydrogen-carbon ratio H/C, and temperature T for gases resulting from the complete combustion of hydrocarbon fuels with dry air, and are used in conjunction with chart I. The reading of these charts is illustrated in figure 2, which is an enlarged view of chart II. At a temperature of 1025° R and a fuel-air ratio of 0.04, the enthalpy is determined by drawing a line of constant temperature between two points at 1025° R and fuel-air ratios of 0 and 0.0678; at the point at which this line intersects the 0.04 fuel-air ratio line, the enthalpy may be read as 156.6 Btu per pound. The inverse problem occurs if f and either h or ϕ are known. In such a case, the point f, h (or f, ϕ) is located on chart II and a constant-temperature line is drawn through this point; the temperature may then be read from the scales at the side of the chart. Charts III and IV are read in a similar fashion; the values of ψ_h and ψ_ϕ may be used with fuel-air ratio f, and h_0 and ϕ_0 (from chart I) to determine h and ϕ by substitution in the following equations: (For derivation, see the appendix.)

$$h = h_O + \psi_h \left(\frac{f}{1 + f} \right) \tag{35}$$

$$\phi = \phi_O + \psi_\phi \left(\frac{f}{1 + f} \right) \tag{36}$$

In order to simplify these computations, the relation between f and $f/(1 + f)$ is shown in chart VI.

If the fuel-air ratio f, hydrogen-carbon ratio H/C, and either ϕ or enthalpy h are known, the temperature may be determined by using charts II to IV as follows: For the specified fuel-air ratio and enthalpy, for example, the temperature may be estimated from chart II. With this temperature and the hydrogen-carbon ratio, the value of ψ_h may be read from chart III or IV. The value of h_O may then be determined from equation (35) and, by using this h_O value, the temperature read from chart I. For results within ±0.5° R, not more than two such approximations are required.

For a hydrogen-carbon ratio of 0.167, the same data are obtainable from chart II, or from a combination of charts I and IV and a short computation. The computation of h or ϕ by means of charts I and IV produces results that, although within ±0.5° R, are obtained more slowly than results from chart II, which are within ±2.5° R. Charts III and IV should therefore be used only when the results from chart II are insufficiently accurate.

If chart II is used for computations, two inaccuracies may be introduced: (1) The data may be read less precisely than with charts III and IV; and (2) the gas properties vary with hydrogen-carbon ratio. Chart II is correct only for a hydrogen-carbon ratio of 0.167. Changes in enthalpy for given pressure ratios are shown in reference 7 (appendix D and fig. 9) to be relatively insensitive to changes in hydrogen-carbon ratio. Changes in enthalpy between given temperatures are more sensitive to variations in hydrogen-carbon ratio.

APPLICATION OF CHARTS

Methods for applying the charts to the solution of several problems involving compression, combustion and reheating, expansion, and heat exchange are described. A subsequent section presents numerical examples.

Isentropic Expansion or Compression

For an isentropic process

$$J \, dh = v \, dp \tag{1}$$

For a perfect gas

$$pv = RT \tag{2}$$

$$dh = c_p \, dT \tag{3}$$

By combining equations (1), (2), and (3)

$$\frac{R}{J} \log_e \left(\frac{p_2}{p_1} \right) = \int_{T_1}^{T_2} \frac{c_p}{T} \, dT \equiv \Delta\phi_i \tag{4}$$

Thus, for an isentropic process the change in ϕ may be determined from the pressure ratio. The corresponding temperature and enthalpy changes may be determined from $\Delta\phi$ and the charts; for example, known quantities are T_1, ϕ_1, h_1, and p_2/p_1. From equation (4) and the relation

$$\phi_2 = \phi_1 + \Delta\phi$$

the value of ϕ_2 is calculated. From the chart, T_2 and h_2 at ϕ_2 are determined; Δh can then be found.

Expansion or Compression with Constant Small-Stage Efficiency

The small-stage, or polytropic, efficiency for an expansion in a turbine is defined as

$$\eta_{t,\infty} \equiv \frac{J \, c_p \, dT}{v \, dp} \tag{5}$$

By combining equations (2) and (5) and the definition of ϕ

$$d\phi = \frac{c_p}{T} \, dT = \eta_{t,\infty} \frac{R}{J} \frac{dp}{p}$$

For constant small-stage efficiency,

$$\Delta\phi = \eta_{t,\infty} \frac{R}{J} \log_e \left(\frac{p_2}{p_1} \right) \qquad (6)$$

The small-stage efficiency for compression is defined as

$$\eta_{c,\infty} \equiv \frac{v \; dp}{J \, c_p \; dT}$$

For constant small-stage efficiency, this expression reduces to

$$\Delta\phi = \frac{R}{J \, \eta_{c,\infty}} \log_e \left(\frac{p_2}{p_1} \right) \qquad (7)$$

For either a compressor or a turbine, the change in enthalpy Δh may be determined from $\Delta\phi$, the initial state, and the charts.

Steady-Flow Combustion with No Useful Work

For the combustion of fuel with air,

$$\eta_b \, f \, \overline{H} = (1 + f) \, h_2 - h_{0,1} - f \, h_f \qquad (8)$$

This expression may be revised to read

$$f = \frac{h_{0,2} - h_{0,1}}{\eta_b \, \overline{H} - h_{0,2} - \psi_{h,2} + h_f} \qquad (9)$$

where

$$\psi_{h,2} = \frac{(h_2 - h_{0,2})(1 + f)}{f}$$

Equation (12) in reference 3 gives the following relation between the lower heating value $\overline{H}$ and the hydrogen-carbon ratio:

$$\overline{H} = 15,935 + 15,800 \; (H/C) \; Btu/lb \qquad (10)$$

Although the datum for the gas charts is 600° R, lower heating values may be used even though based on reference temperatures different from 600° R. The change in the lower heating value is less than

0.2 Btu per pound of fuel for each 1° R change in reference temperature (reference 3); the datum state for the fuel may therefore be taken as liquid at 600° R with negligible error. If the fuel is injected in a state other than as a liquid at 600° R, the initial enthalpy of the fuel h_f must be determined. For most liquid hydrocarbon fuels, the specific heat may be assumed equal to 0.5 Btu/(lb)($^\circ$R).

For the combustion of fuel with the products of a previous combustion with the same fuel (commonly called reheating)

$$\eta_b \, \overline{H} \, (f_2 - f_1) = (1 + f_2) \, h_2 - (1 + f_1) \, h_1 - (f_2 - f_1) \, h_f$$

$$(11)$$

Solving for f_2

$$f_2 = \frac{\eta_b \, \overline{H} \, f_1 + h_{0,2} - (1 + f_1) \, h_1 + f_1 \, h_f}{\eta_b \, \overline{H} - h_{0,2} - \Psi_{h,2} + h_f} \qquad (12)$$

Or, solving for $f_2 - f_1$

$$f_2 - f_1 = \frac{(1 + f_1)(h_{0,2} - h_{0,1}) + f_1 \, (\Psi_{h,2} - \Psi_{h,1})}{\eta_b \, \overline{H} - h_{0,2} - \Psi_{h,2} + h_f} \qquad (13)$$

Mixtures of Gases

Consider the mixture of two or more gases, each of which is the product of a previous combustion with a hydrocarbon fuel. Let these gases be designated a, b, c, and d. Then the enthalpy h of the resulting uniform mixture is

$$h = \frac{w_a \, h_a + w_b \, h_b + \cdots}{w_a + w_b + \cdots}$$

and

$$\phi = \frac{w_a \, \phi_a + w_b \, \phi_b + \cdots}{w_a + w_b + \cdots}$$

$$\left. \right\} \qquad (14)$$

The weight of air $w_{O,a}$ and the weight of fuel $w_{f,a}$ required to produce gas a are

$$w_{O,a} = \frac{w_a}{1 + f_a}$$

$$w_{f,a} = \frac{f_a \, w_a}{1 + f_a}$$

For the mixture of a, b, c, and d, the weights of air w_O and fuel w_f are

$$w_O = \frac{w_a}{1 + f_a} + \frac{w_b}{1 + f_b} + \cdots$$

$$w_f = \frac{f_a \, w_a}{1 + f_a} + \frac{f_b \, w_b}{1 + f_b} + \cdots$$

and the corresponding fuel-air ratio f is

$$f = \frac{\dfrac{f_a \, w_a}{1 + f_a} + \dfrac{f_b \, w_b}{1 + f_b} + \cdots}{\dfrac{w_a}{1 + f_a} + \dfrac{w_b}{1 + f_b} + \cdots} \tag{15}$$

For gas a, the weights of carbon $w_{C,a}$ and hydrogen $w_{H,a}$ are

$$w_{C,a} = \frac{f_a \, w_a}{(1 + f_a) \left[1 + (H/C)_a \right]}$$

$$w_{H,a} = \frac{f_a \, w_a \, (H/C)_a}{(1 + f_a) \left[1 + (H/C)_a \right]}$$

For the mixture,

$$\left(\frac{H}{C}\right) = \frac{\left(\dfrac{f_a}{1+f_a}\right)\left[\dfrac{(H/C)_a}{1+(H/C)_a}\right]w_a + \left(\dfrac{f_b}{1+f_b}\right)\left[\dfrac{(H/C)_b}{1+(H/C)_b}\right]w_b + \cdots}{\left(\dfrac{f_a}{1+f_a}\right)\left[\dfrac{w_a}{1+(H/C)_a}\right] + \left(\dfrac{f_b}{1+f_b}\right)\left[\dfrac{w_b}{1+(H/C)_b}\right] + \cdots}$$

(16)

For a mixture of any two gases, equation (14) may be modified to read

$$\left.\begin{aligned} h &= h_a + \frac{w_b}{w_a+w_b}\,(h_b - h_a) \\[2ex] \phi &= \phi_a + \frac{w_b}{w_a+w_b}\,(\phi_b - \phi_a) \end{aligned}\right\}$$

(17)

If one of the constituents of the mixture of several gases is water that was injected as a liquid into the gas stream, supplementary information concerning the initial enthalpy of the water is required in order to compute an energy balance. In reference 16, in which the datum is the state of saturated liquid water at 32° F, the enthalpy of water vapor at 1 pound per square inch and 140° F is given as 1123.3 Btu per pound. From this information, the enthalpy of the liquid water above the datum of vapor at 600° R and zero pressure may be stated as

$$h_{w,l} = -1123 + (T - 492)$$

(18)

by assuming that the specific heat of liquid water is 1.00 Btu/(lb)($^\circ$R); at temperatures as high as 700° R, this assumption produces an error of less than 0.5 Btu per pound in equation (18).

Heat Exchange

Consider the heat exchange between two gases a and b, which are at different temperatures. Then from the conservation of energy,

$$w_a\,(h_{a,1} - h_{a,2}) = w_b\,(h_{b,2} - h_{b,1})$$

(19)

NUMERICAL EXAMPLES

The following six examples demonstrate the use of the charts and some of the problems to which the charts are applicable. The problems are stated and solved with the source of information (equations and chart number) listed in a column at the right of each page.

1. Calculation of h and ϕ

The simplest problem occurs when values of temperature, hydrogen-carbon ratio, and fuel-air ratio are given as the independent variables, and the values of h and ϕ are desired. The process of calculation is direct.

Given:

$$T = 2000^{\circ}\ R$$

$$H/C = 0.160$$

$$f = 0.02$$

To find: h and ϕ

Chart II is constructed only for a hydrogen-carbon ratio of 0.167, but can be used in this case because the assigned H/C is close to 0.167.

As in figure 2, connect the temperature value at f = 0 to the same temperature value at f = 0.0678 and read the h or ϕ value at the desired f value.

h = 420 Btu/(lb mixture) at T and f   (chart II(d))

ϕ = 0.4120 Btu/(lb mixture)($^{\circ}$R) at T and f   (chart II(d))

For more precise values of h and ϕ, use charts I and IV and the correct value of H/C.

h_0 = 409.6 Btu/(lb air) at T   (chart I(e))

ϕ_0 = 0.4035 Btu/(lb air)($^{\circ}$R) at T   (chart I(e))

ψ_h = 515 at T and H/C   (chart IV(b))

$$\Psi_\phi = 0.4150 \text{ at } T \text{ and } H/C \qquad \text{(chart IV(b))}$$

$$f/(1 + f) = 0.01961 \text{ at } f \qquad \text{(chart VI(a))}$$

Therefore

$$h = 409.6 + 0.01961 \times 515 \qquad (35)$$

$$= 419.7 \text{ Btu/(lb mixture)}$$

$$\phi = 0.4035 + 0.01961 \times 0.4150 \qquad (36)$$

$$= 0.4117 \text{ Btu/(lb mixture)}(^\circ R)$$

2. Calculation of Temperature

The inverse situation of example 1 occurs when values of h
or ϕ, hydrogen-carbon ratio H/C, and fuel-air ratio f are given
as independent variables and values of temperature T are desired.
When h or ϕ values are given for H/C values near 0.167, the
solution is read directly from chart II. If the H/C values are
remote from 0.167, chart II is used to estimate the temperature and
charts I and III or IV are used to find the temperature by a suc-
cessive approximation method.

Given:

$$\phi = 0.4000$$

$$H/C = 0.145$$

$$f = 0.03$$

To find: T

The temperature is first estimated from chart II.

Locate ϕ = 0.4000 at f = 0.03   (chart II(d))

Extend line through this point so that the temperature values
at f = 0 and f = 0.0678 are identical and read the temperature
as

$$T = 1895^\circ R$$

. This temperature is the first approximation and should be rounded off to the nearest 5°, that is,

$$T' = 1895° \text{ R}$$

$$\psi_\phi' = 0.360 \text{ at } T' \text{ and } H/C \qquad \text{(chart III(a))}$$

$$f/(1 + f) = 0.02912 \text{ at } f \qquad \text{(chart VI(a))}$$

Now ϕ_0' can be obtained at T'

$$\phi = \phi_0' + \psi_\phi' \left[f/(1 + f) \right] \qquad (36)$$

Thus

$$\phi_0' = \phi - \psi_\phi' \left[f/(1 + f) \right]$$

$$\phi_0' = 0.4000 - 0.360 \times 0.02912$$

$$= 0.3895 \text{ Btu}/(\text{lb air})(°\text{R})$$

The second approximate temperature can now be found from chart I at ϕ_0'.

$$T'' = 1901° \text{ R at } \phi_0' \qquad \text{(chart I(e))}$$

This value, however, is within the accuracy of the charts and can be considered the actual value.

3. Compression Process

Either adiabatic or small-stage efficiency may be used with the chart values in determining the actual Δh for a process. In demonstrating the solution of the compression problem, the adiabatic efficiency η_c will be given; enthalpy change Δh, final temperature T_2, and the corresponding small-stage efficiency $\eta_{c,\infty}$ will be determined. The solution will be divided into two parts.

Adiabatic compression: - Compress dry air.

Given:

$$\frac{p_2}{p_1} = 4$$

$T_1 = 518.4^\circ$ R

$\eta_c = 0.90$ adiabatic compressor efficiency

To find: Δh and T_2 for the process.

In initial state

$h_{0,1} = 28.40$ Btu/(lb air) at T_1 \hfill (chart I(a))

$\phi_{0,1} = 0.0622$ Btu/(lb air)($^\circ$R) at T_1 \hfill (chart I(a))

The ideal $\Delta\phi$ is now determined.

$$\Delta\phi_{0,i} = \frac{R_0}{J} \log_e\left(\frac{p_2}{p_1}\right) \tag{4}$$

$$\Delta\phi_{0,i} = \frac{53.379}{778.156} \times \log_e 4 = 0.0950$$

$$\phi_{0,2,i} = \phi_{0,1} + \Delta\phi_{0,i}$$

$$= 0.1572 \text{ Btu/(lb air)}(^\circ R)$$

$T_{2,i} = 768.7^\circ$ R at $\phi_{0,2,i}$ \hfill (chart I(b))

$h_{0,2,i} = 88.8$ Btu/(lb air) at $T_{2,i}$ \hfill (chart I(b))

$$\Delta h = (h_{0,2,i} - h_{0,1})\frac{1}{\eta_c}$$

$$\Delta h = 60.4 \times \frac{1}{0.90} = 67.1 \text{ Btu/(lb air)}$$

Actual $h_{0,2} = h_{0,1} + \Delta h$

Thus,

$$h_{0,2} = 28.4 + 67.1 = 95.5 \text{ Btu/(lb air)}$$

and

$$T_2 = 796.8^\circ \text{ R}$$

\hfill (chart I(b))

at $h_{0,2}$.

Small-stage efficiency. - Find what $\eta_{c,\infty}$ will give same T_2 as η_c.

Given:

$$T_1 = 518.4^\circ \text{ R}$$

$$T_2 = 796.8^\circ \text{ R}$$

$$\frac{p_2}{p_1} = 4$$

To find: $\eta_{c,\infty}$

$$\phi_{0,2} = 0.1658 \text{ Btu/(lb air)}(^\circ\text{R}) \text{ at } T_2 \qquad \text{(chart (b))}$$

$$\Delta\phi_0 = \phi_{0,2} - \phi_{0,1} = 0.1658 - 0.0622 = 0.1036$$

$\Delta\phi_0$ is also given by equation (7).

$$\Delta\phi_0 = \frac{1}{\eta_{c,\infty}} \frac{R_0}{J} \log_e \frac{p_2}{p_1} \qquad (7)$$

For air

$$\Delta\phi = \Delta\phi_0$$

$$0.1036 = \frac{1}{\eta_{c,\infty}} \times \frac{53.379}{778.156} \log_e 4$$

and

$$\eta_{c,\infty} = 0.917$$

This value of $\eta_{c,\infty} = 0.917$ is equivalent to the $\eta_c = 0.90$ in this case. Because $\eta_{c,\infty}$ is more convenient to use, it will be used in the rest of the examples.

4. Combustion Process

Two variations of the combustion process occur. One is: given the temperatures before and after combustion, find the fuel-air ratio; the other is: given the temperature before combustion and the fuel-air ratio, find the temperature after combustion.

Find the fuel-air ratio. - This problem can be solved directly and requires no approximations.

Given:

$$T_1 = 900^\circ \text{ R}$$

$$T_2 = 2000^\circ \text{ R}$$

$$H/C = 0.189$$

$$\eta_b = 0.95$$

The temperature of liquid fuel is 500° R.

To find: f

$$f = \frac{h_{0,2} - h_{0,1}}{\eta_b \bar{H} - h_{0,2} - \psi_{h,2} + h_f} \tag{9}$$

$$h_{0,1} = 120.9 \text{ Btu/(lb air)} \qquad \qquad \text{(chart I(b))}$$

$$h_{0,2} = 409.6 \text{ Btu/(lb air)} \qquad \qquad \text{(chart I(e))}$$

$$\bar{H} = 15,935 + 15,800 \times 0.189 = 18,924 \text{ Btu/(lb fuel)} \qquad (10)$$

$$\psi_{h,2} = 583 \text{ at } T_2 \text{ and } H/C \qquad \qquad \text{(chart IV(b))}$$

$$h_f = 0.5 \times (500-600) = -50 \text{ Btu/(lb fuel)} \quad \text{(See Steady-Flow Combustion with No Useful Work.)}$$

$$f = \frac{409.6 - 120.9}{18,924 \times 0.95 - 409.6 - 583 - 50}$$

$$f = 0.01705 \text{ lb fuel/(lb air)}$$

Find the temperature after combustion. - This solution requires an approximate temperature from chart II and use of equation (8).

Given:

$$T_1 = 900^\circ \text{ R}$$

$$H/C = 0.189$$

$$\eta_b = 1.00$$

$$f = 0.02$$

The temperature of the liquid fuel is 600° R.

To find: T_2

$$\eta_b \, \overline{\overline{H}} \, f = (1 + f) \, h_2 - h_{0,1} - f \, h_f \qquad\qquad (8)$$

$$h_{0,1} = 120.9 \text{ Btu/(lb air)} \qquad\qquad \text{(chart I(b))}$$

$$\overline{\overline{H}} = 18{,}924 \text{ Btu/(lb fuel)} \qquad\qquad (10)$$

$$h_f = 0 \text{ at } 600^\circ \text{ R} \qquad\qquad \substack{\text{(See Mixtures} \\ \text{of Gases.)}}$$

$$h_2 = \frac{1.00 \times 18{,}924 \times 0.02 + 120.9 + 0}{1.02} \qquad\qquad (8)$$

$$= 489.6 \text{ Btu/(lb mixture)}$$

The approximate temperature is obtained from chart II

$$T_2' = 2240^\circ \text{ R at } f \text{ and } h_2 \qquad\qquad \text{(chart II(e))}$$

$$h_{0,2}' = h_2 - \psi_{h,2}' \left[f/(1 + f) \right] \qquad\qquad (35)$$

where

$$f/(1 + f) = 0.01961 \text{ at } f \qquad\qquad \text{(chart VI(a))}$$

$$\psi_{h,2}' = 712 \text{ at } T_2' \text{ and } H/C \qquad\qquad \text{(chart IV(b))}$$

$$h_{0,2}' = 489.6 - 712 \times 0.01961$$

$$= 475.64 \text{ Btu/(lb air)}$$

$$T_2'' = T_2 = 2236^\circ \text{ R at } h_{0,2}' \qquad \text{(chart I(e))}$$

This equality is true because a second trial using $T_2'' = 2236$ would result in the same answer.

5. Expansion Process

The expansion process involving a mixture of combustion products and air can be handled in two different ways for certain conditions. Chart II can be used when the H/C ratio is sufficiently close to 0.167. Charts I and III or IV give a more exact solution applicable to any H/C ratio from 0.100 to 0.200. This method, whereas more exact, is also more laborious.

Given:

$$T_1 = 3000^\circ \text{ R}$$

$$\Delta h = -200 \text{ Btu/(lb air)}$$

$$\eta_{t,\infty} = 0.90$$

$$f = 0.03$$

$$H/C = 0.190$$

To find: p_1/p_2

The problem will first be solved with chart II, although the H/C ratio is remote from 0.167.

$$h_1 = 727 \text{ Btu/(lb mixture)} \qquad \text{(chart II(h))}$$

$$\phi_1 = 0.5385 \text{ Btu/(lb mixture)}(^\circ\text{R}) \qquad \text{(chart II(h))}$$

Now h_2 is determined per pound of mixture from h_1 and Δh.

$$h_2 = h_1 + \frac{\Delta h}{1 + f}$$

$$h_2 = 727 - \frac{200}{1.03} = 532.82 \text{ Btu/(lb mixture)}$$

and

$$T_2 = 2360^\circ \text{ R at } h_2 \text{ and } f \qquad\qquad \text{(chart II(e))}$$

$$\phi_2 = 0.4650 \text{ Btu/(lb mixture)}(^\circ\text{R}) \text{ at } T_2 \text{ and } f \text{ (chart II(e))}$$

$$\Delta\phi = \phi_2 - \phi_1$$

$$\Delta\phi = 0.4650 - 0.5385 = -0.0735 \text{ Btu/(lb mixture)}(^\circ\text{R})$$

also

$$\Delta\phi = \eta_{t,\infty} \frac{R_t}{J} \log_e \left(\frac{p_2}{p_1}\right) \qquad\qquad\qquad (6)$$

where

$$R_t = \frac{53.379 + \dfrac{0.03 \times 1546.35 \times 0.19}{4.032 \times 1.19}}{1.03} \qquad \text{(table I)}$$

$$R_t = 53.61 \text{ ft-lb/(lb mixture)}(^\circ\text{R})$$

$$\log_e \left(\frac{p_2}{p_1}\right) = \frac{-0.0735 \times 778.156}{0.90 \times 53.61} = -1.1854$$

and

$$p_1/p_2 = 3.27$$

For the more exact solution, charts I and IV are used to account for H/C deviations from 0.167 and to obtain more precise results. As in example 2, chart II must be used to obtain a first approximation of the temperatures.

$$h_{0,1} = 695.8 \text{ Btu/(lb air) at } 3000^\circ \text{ R} \qquad \text{(chart I(g))}$$

$$\phi_{0,1} = 0.5194 \text{ Btu/(lb air)}(^\circ\text{R}) \text{ at } 3000^\circ \text{ R} \quad \text{(chart I(g))}$$

$$\psi_{h,1} = 1172 \text{ at } 3000^\circ \text{ R and } H/C = 0.190 \qquad (\text{chart IV(d)})$$

$$\psi_{\phi,1} = 0.709 \text{ at } 3000^\circ \text{ R and } H/C = 0.190 \qquad (\text{chart IV(d)})$$

$$f/(1 + f) = 0.2912 \text{ at } f \qquad\qquad\qquad (\text{chart VI(a)})$$

$$h_1 = 695.8 + 1172 \times 0.02912 = 729.93 \text{ Btu/(lb mixture)} \quad (35)$$

and

$$\phi_1 = 0.5194 + 0.709 \times 0.02912 = 0.5400 \qquad\qquad (36)$$

$$h_2 = h_1 + \frac{\Delta h}{1 + f}$$

$$h_2 = 729.93 - \frac{200}{1.03} = 535.76 \text{ Btu/(lb mixture)}$$

As in example 2, the first approximation of temperature is determined from chart II. Round off the first approximate temperatures to the nearest 5°.

$$T_2' = 2375^\circ \text{ R at } h_2 \text{ and } f \qquad (\text{chart II(e)})$$

$$\psi_{h,2}' = 790 \text{ at } T_2' \text{ and } H/C \qquad (\text{chart IV(b)})$$

From equation (35) $h_{0,2}'$ is determined.

$$h_2' = h_{0,2}' + \psi_{h,2}' \left[f/(1 + f) \right] \qquad\qquad (35)$$

$$h_{0,2}' = 535.76 - 790 \times 0.02912$$

$$= 512.76 \text{ Btu/(lb air)}$$

From the first approximation, T_2'' is within the accuracy of the charts and thus can be considered to be the actual T_2.

$$T_2'' = T_2 = 2367 \text{ at } h_{0,2}' \qquad\qquad (\text{chart I(f)})$$

$$\phi_{0,2} = 0.4509 \text{ Btu/(lb air)}(^\circ\text{R}) \text{ at } T_2 \qquad (\text{chart I(f)})$$

$$\psi_{\phi,2} = 0.5635 \text{ at } T_2 \text{ and } H/C \qquad (\text{chart IV(b)})$$

$$\phi_2 = 0.4509 + 0.5635 \times .02912 \qquad (36)$$

$$= 0.4673 \text{ Btu}/(\text{lb mixture})(^{\circ}R)$$

As previously shown,

$$\Delta\phi = \phi_2 - \phi_1 = \eta_{t,\infty} \frac{R_t}{J} \log_e \left(\frac{p_2}{p_1}\right) \qquad (6)$$

$$0.4673 - 0.5400 = 0.90 \times \frac{53.61}{778.156} \log_e \left(\frac{p_2}{p_1}\right)$$

or

$$\log_e \frac{p_2}{p_1} = -1.1725$$

$$\frac{p_1}{p_2} = 3.24$$

Thus with H/C ratio remote from $H/C = 0.167$ (as in this problem $H/C = 0.190$), the temperature determined from chart II differs from the actual temperature by 7° R and the calculated pressure ratio differs by 1 percent.

6. Compress Moist Air

Problems involving water vapor and air or gas mixture require the use of chart V in conjunction with chart I, chart II, or charts I, III, and IV. The properties of the individual components are found and then combined to determine the properties of the mixture. The reverse process, in which the temperature of the mixture is desired, requires a trial-and-error solution.

Given: Compress moist air

$$\frac{p_2}{p_1} = 4$$

$$T_1 = 550^{\circ} \text{ R}$$

$$\eta_{c,\infty} = 0.90$$

$m = 0.05$ (lb water)/(lb air)

To find: Δh and T_2 for process

For the state before compression, equation (14) may be modified to read

$$h_1 = \frac{h_{0,1} + m\, h_w}{1 + m}$$

$$\phi_1 = \frac{\phi_{0,1} + m\, \phi_w}{1 + m}$$

where

$h_{0,1} = 36.0$ Btu/(lb air) at T_1 (chart I(a))

$h_{w,1} = -22.0$ Btu/(lb water) at T_1 (chart V(a))

$$h_1 = \frac{36 + (0.05 \times -22.0)}{1.05} = 33.24 \text{ Btu/(lb mixture)}$$

also

$\phi_{0,1} = 0.0763$ Btu/(lb air)($^\circ$R) at T_1 (chart I(a))

$\phi_{w,1} = -0.038$ Btu/(lb water)($^\circ$R) at T_1 (chart V(a))

$$\phi_1 = \frac{0.0763 + (0.05 \times -0.038)}{1.05}$$

$$= 0.0709 \text{ Btu/(lb mixture)}(^\circ\text{R})$$

$R_w = 85.8$ ft-lb/(lb water)($^\circ$R)   (reference 17)

$$R_m = \frac{53.379 + 0.05 \times 85.8}{1.05} \qquad\qquad \text{(table I)}$$

$$= 54.923 \text{ ft-lb/(lb mixture)}(^\circ\text{R})$$

For process

$$\Delta\phi_m = \frac{1}{\eta_{c,\infty}} \left(\frac{R_m}{778.156}\right) \log_e\!\left(\frac{p_2}{p_1}\right) \tag{7}$$

$$\Delta\phi_m = \frac{1}{0.90} \times \frac{54.923}{778.156} \log_e 4 = 0.1087$$

$$\phi_2 = \phi_1 + \Delta\phi_m = 0.0709 + 0.1087$$

$$\phi_2 = 0.1796 \text{ Btu/(lb mixture)}(^\circ R)$$

An estimate of the final temperature must now be made. A close approximation is to consider ϕ_2 as the value for dry air and look up the equivalent temperature in chart I. The actual T_2 should be a bit higher because of the water vapor content; a slightly higher temperature approximation is therefore made.

$$T_2' \cong 850^\circ \text{ R} \qquad\qquad\qquad\qquad \text{(chart I(b))}$$

$$\phi_{w,2}' = 0.161 \text{ Btu/(lb water)}(^\circ R) \text{ at } T_2' \qquad \text{(chart V(a))}$$

$$\phi_{0,2}' = 0.1816 \text{ Btu/(lb air)}(^\circ R) \text{ at } T_2' \qquad \text{(chart I(b))}$$

Now from equation (17) $\phi_{0,2}''$ can be calculated.

$$\phi_{0,2}'' = \phi_2 - \frac{m}{1+m} (\phi_{w,2}' - \phi_{0,2}')$$

$$\phi_{0,2}'' = 0.1796 - \frac{0.05}{1.05} (0.161 - 0.1816)$$

$$= 0.1806 \text{ Btu/(lb air)}(^\circ R)$$

Make a final trial

$$T_2'' = 847^\circ \text{ R at } \phi_{0,2}'' \qquad\qquad\qquad \text{(chart I(b))}$$

$$\phi_{w,2}'' = 0.1508 \text{ Btu/(lb water)}(^\circ R) \text{ at } T_2'' \qquad \text{(chart V(a))}$$

$$\phi_{0,2}''' = 0.1796 - \frac{0.05}{1.05} (0.1508 - 0.1806) \qquad\qquad (17)$$

$$= 0.1810 \text{ Btu/(lb air)}(^\circ R)$$

$$T_2''' = 848^\circ \text{ R} = T_2 \text{ final} \qquad\qquad\qquad \text{(chart I(b))}$$

Now find enthalpy of mixture at T_2.

$h_{0,2} = 108.10$ Btu/(lb air) at T_2 (chart I(b))

$h_{w,2} = 113.0$ Btu/(lb water) at T_2 (chart V(a))

$$h_2 = \frac{108.1 + 0.05 \times 113}{1.05} = 108.33 \text{ Btu/(lb mixture)} \tag{14}$$

From h_1 and h_2, the work per pound of air for compression of the moist air is

$$\Delta h = (h_2 - h_1)(1 + m)$$

$$= (108.33 - 33.24)(1.05) = 78.85 \text{ Btu/(lb air)}$$

Lewis Flight Propulsion Laboratory,
    National Advisory Committee for Aeronautics,
        Cleveland, Ohio, August 30, 1949.

APPENDIX - DERIVATION OF CHARTS

Charts I to IV

The specific heats given in reference 11 were plotted as functions of temperature for the following gases: oxygen, nitrogen, carbon dioxide, and water. Empirical equations of the following form were fitted to the curves over subintervals of the temperature range from 600° to 3500° R:

$$c_p = a_0 + a_1 \, T + a_2 \, T^2 + a_3 \, T^3 \tag{20}$$

The values of c_p obtained from equation (20) agree with the data of reference 11 within 0.1 percent.

The Gibbs-Dalton law (reference 17) was employed to obtain the properties of various gas mixtures resulting from the complete combustion of hydrocarbon fuels with dry air. Air was assumed to have the following composition on a volume basis (reference 2, p. 57):

Oxygen	0.2099
Nitrogen	.7803
Argon	.0098
	1.0000

The following atomic weights were selected:

Oxygen	16.000
Nitrogen	14.008
Argon	39.950
Carbon	12.010
Hydrogen	1.008

The specific heat at constant pressure for argon was assumed to be 0.1243 Btu per pound °R, or, on a molal basis, 5/2 R.

By use of equation (20), the following two integrals were evaluated for temperatures from 600° to 3500° R, fuel-air ratios of zero and the stoichiometric values, and hydrogen-carbon ratios of 0.100, 0.150, 0.167, and 0.200.

$$h = \int_{600}^{T} c_p \, dT + 48.00 \tag{21}$$

$$\phi \equiv \int_{600}^{T} \frac{c_p}{T} \, dT + 0.09729 \qquad (22)$$

The values of h and ϕ were assigned equal to 48.00 Btu/(lb) and 0.09729 Btu/(lb)($^\circ$R), respectively, at 600° R in order that the values computed for a fuel-air ratio of zero would agree with the data of reference 2.

Enthalpy is used in the usual engineering sense in which the chemical energy is not included in the enthalpy. The result is that at the datum temperature of 600° R the enthalpy of the various gas mixtures considered is 48.00 Btu per pound, regardless of gas composition.

Chart I was constructed directly from the evaluation of equations (21) and (22). For charts II to IV, the method is described by which the computations for stoichiometric fuel-air ratios and four hydrogen-carbon ratios may be interpolated to permit determining h and ϕ for any leaner-than-stoichiometric fuel-air ratio and any hydrogen-carbon ratio from 0.1 to 0.2.

Changes in h and ϕ between specified temperature limits were determined from charts I to IV and from references 10 and 12; a comparison of these changes indicated that results from charts I to IV differ by not over 0.2 percent from reference 10 and less than 0.2 percent from reference 12.

Chart I. - In order to determine h_0 and ϕ_0 for dry air, equations (21) and (22) were evaluated for temperatures from 600° to 3500° R. The results are presented in chart I. The data of reference 2 were used to extend the range of this chart from 600° to 300° R.

Chart II. - Chart II was constructed in order to determine h and ϕ of mixtures of air and combustion products for an average jet-engine fuel (H/C = 0.167) and for fuel-air ratios of zero to stoichiometric.

The method of interpolation for fuel-air ratios between zero and stoichiometric was established as follows:

For the complete combustion of X moles of a hydrocarbon fuel (CH_n) with 1 pound of dry air

$$X \ CH_n + \frac{1}{28.9692} \ (0.2099 \ O_2 + 0.7803 \ N_2 + 0.0098 \ A)$$

$$\rightarrow \frac{1}{28.9692} \ (0.2099 \ O_2 + 0.7803 \ N_2 + 0.0098 \ A) +$$

$$X \left[CO_2 + \frac{n}{2} \ H_2O - \left(1 + \frac{n}{4}\right) O_2 \right] \tag{23}$$

Equation (23) may be rewritten as

$$X \ CH_n + \frac{1}{28.9692} \ (0.2099 \ O_2 + 0.7803 \ N_2 + 0.0098 \ A)$$

$$\rightarrow \frac{1}{28.9692} \ (0.2099 \ O_2 + 0.7803 \ N_2 + 0.0098 \ A) +$$

$$\frac{f}{12.01 \left[1 + (H/C)\right]} (CO_2 - O_2) + \frac{f \ (H/C)}{2.016 \left[1 + (H/C)\right]} (H_2O - 1/2 \ O_2) \tag{24}$$

where

$$\left(\frac{H}{C}\right) = \frac{1.008 \ n}{12.01}$$

$$f = 12.01(X) \left[1 + (H/C)\right]$$

or

$$X = \frac{f}{12.01 \ \left[1 + (H/C)\right]}$$

The products of combustion in equation (24) may be considered as a mixture of two gases a and b where

$$a = \frac{f}{1 + (H/C)} \left[\frac{(CO_2 - O_2)}{12.01} + \frac{(H/C)}{2.016} \ (H_2O - 1/2 \ O_2) \right]$$

$$b = \frac{1}{28.9692} \ (0.2099 \ O_2 + 0.7803 \ N_2 + 0.0098A) \tag{25}$$

For a mixture of two gases a and b

$$c_p = \frac{w_a}{w_a + w_b} \ (c_{p,a} - c_{p,b}) + c_{p,b} \tag{26}$$

For the mixture of gases in equation (24), equation (26) may be modified to read

$$c_p = \frac{f}{1+f}(c_{p,a} - c_{p,b}) + c_{p,b} \tag{27}$$

By combining equations (21) and (27), and equations (22) and (27) and differentiating the resulting expressions

$$\frac{\partial h}{\partial\left(\frac{f}{1+f}\right)} = \int (c_{p,a} - c_{p,b})\ dT \tag{28}$$

$$\frac{\partial\phi}{\partial\left(\frac{f}{1+f}\right)} = \int \frac{c_{p,a} - c_{p,b}}{T}\ dT \tag{29}$$

The integrals in the right-hand members of equations (28) and (29) are functions only of temperature. For a given temperature, h and ϕ therefore vary linearly with $f/(1+f)$. Thus, in chart II the h and ϕ scales are linear and the fuel-air ratio scale is linear in $f/(1+f)$. Linear interpolation at constant temperature between lines of constant f consequently yields the value of h or ϕ at the desired fuel-air ratio.

Charts III and IV. - Gas a of equation (25) may be considered as a mixture of gases c and d where

$$\left.\begin{aligned}c &= (CO_2 - O_2)/12.01 \\[2mm] d &= \frac{(H/C)}{2.016}\left(H_2O - \frac{1}{2}O_2\right)\end{aligned}\right\} \tag{30}$$

By combining equations (26) and (30)

$$c_{p,a} = \frac{1}{1+(H/C)}(c_{p,c} - c_{p,d}) + c_{p,d} \tag{31}$$

where

$$w_c = 1 \text{ pound} \quad \text{and} \quad w_d = H/C \text{ pound}$$

By combining equations (28) and (31) and differentiating

$$\frac{\partial^2 h}{\partial \left(\dfrac{f}{1+f}\right)\partial\left[\dfrac{1}{1+(H/C)}\right]} = \int (c_{p,c} - c_{p,d})\ dT \qquad (32)$$

Similarly

$$\frac{\partial^2 \phi}{\partial \left(\dfrac{f}{1+f}\right)\partial\left[\dfrac{1}{1+(H/C)}\right]} = \int \frac{c_{p,c} - c_{p,d}}{T}\ dT \qquad (33)$$

Equations (32) and (33) indicate that for a given temperature $\dfrac{\partial h}{\partial\left[f/(1+f)\right]}$ and $\dfrac{\partial \phi}{\partial\left[f/(1+f)\right]}$ vary linearly with $\dfrac{1}{1+(H/C)}$.
These equations suggest a convenient method of interpolation using tabular values of h_0, h_c, h_d, ϕ_0, ϕ_c, and ϕ_d and the following equations:

$$\left.\begin{aligned}
h &= \frac{f}{1+f}\left[\frac{h_c + (H/C)\ h_d}{1+(H/C)}\right] + \frac{h_0}{1+f}\\[2ex]
\phi &= \frac{f}{1+f}\left[\frac{\phi_c + (H/C)\ \phi_d}{1+(H/C)}\right] + \frac{\phi_0}{1+f}
\end{aligned}\right\} \qquad (34)$$

The results of evaluating equations (28) and (29) for hydrogen-carbon ratios of 0.100, 0.150, and 0.200, temperatures of 600° to 3500° R, and stoichiometric fuel-air ratios are presented in charts III and IV. The spacing of the lines of constant hydrogen-carbon ratio is linear in $\dfrac{1}{1+(H/C)}$ so that a linear interpolation between lines of constant H/C at constant temperature yields the value of ψ_h or ψ_ϕ at the desired hydrogen-carbon ratio and temperature.

The values of h and ϕ at a specified fuel-air ratio, hydrogen-carbon ratio, and temperature may be determined from charts I and III or IV and the following relations:

$$h = h_0 + \int_0^{\frac{f}{1+f}} \frac{\partial h}{\partial \left(\frac{f}{1+f} \right)} \, d \left(\frac{f}{1+f} \right)$$

and

$$\phi = \phi_0 + \int_0^{\frac{f}{1+f}} \frac{\partial \phi}{\partial \left(\frac{f}{1+f} \right)} \, d \left(\frac{f}{1+f} \right)$$

Because $\dfrac{\partial h}{\partial \left[f/(1+f) \right]}$ is constant at a given temperature and hydrogen-carbon ratio,

$$h = h_0 + \left[\frac{\partial h}{\partial \left(\frac{f}{1+f} \right)} \right]_{T,H/C} \left(\frac{f}{1+f} \right)$$

From this relation

$$\left[\frac{\partial h}{\partial \left(\frac{f}{1+f} \right)} \right]_{T,H/C} = \frac{(h - h_0)(1+f)}{f} \equiv \psi_h$$

Similarly

$$\left[\frac{\partial \phi}{\partial \left(\frac{f}{1+f} \right)} \right]_{T,H/C} = \frac{(\phi - \phi_0)(1+f)}{f} \equiv \psi_\phi$$

Finally

$$h = h_0 + \psi_h \left(\frac{f}{1+f} \right) \tag{35}$$

and

$$\phi = \phi_0 + \psi_\phi \left(\frac{f}{1+f} \right) \tag{36}$$

Chart V. - Chart V was prepared directly from the tabular data of reference 12 by fairing a smooth curve through the data.

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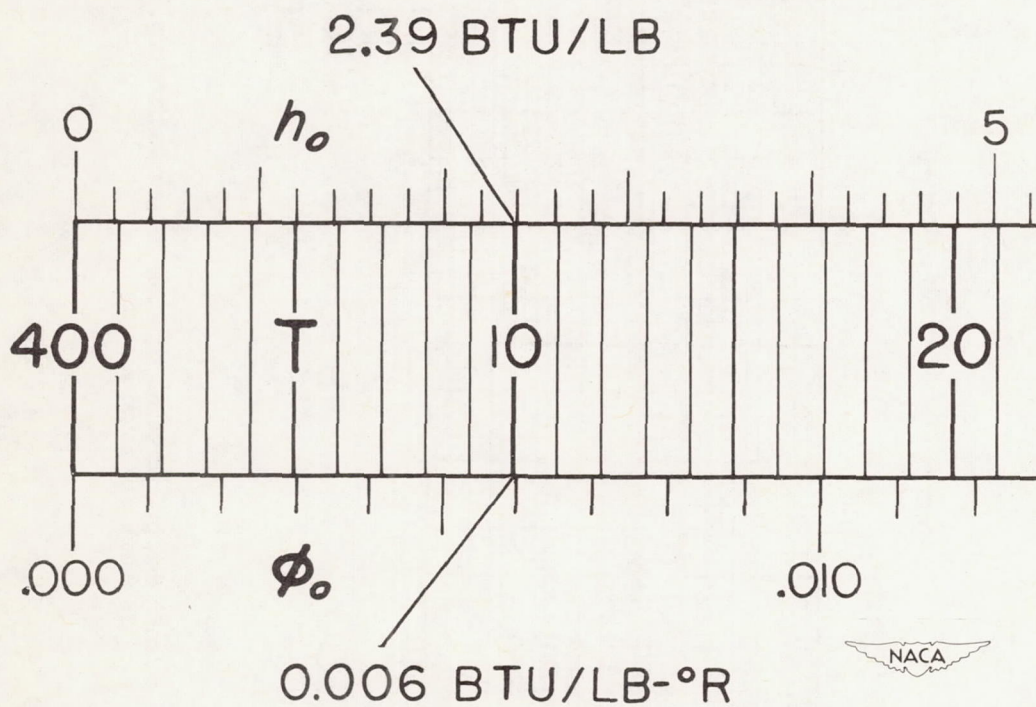
Figure 1. - Enlarged section of chart I showing use of charts of type I. Given temperature, 410° R; h_0, 2.39 Btu per pound air; ϕ_0, 0.006 Btu per pound air per °R.
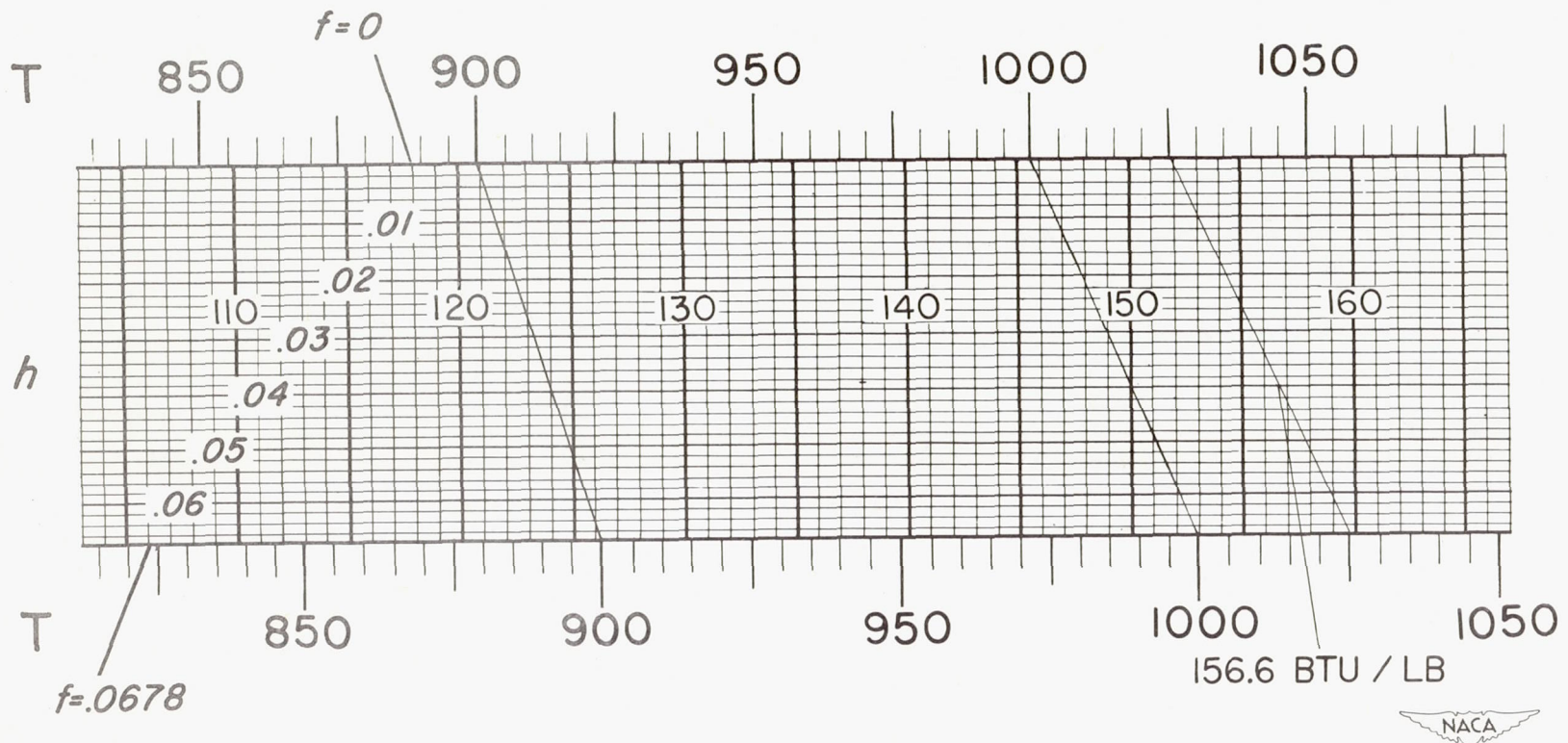
Figure 2. - Enlarged section of chart II showing use of charts of type II. Given temperature, 1025° R; f, 0.04; h, 156.6 Btu per pound mixture.

CHART INDEX

I   Dry-air values of ϕ_0, h_0, and T.
           Linear T scale

II   Combustion-gas values of ϕ, h, T, and f for hydrogen-
           carbon ratio of 0.167.
           Linear ϕ and h scales
           Hyperbolic f scale

III   Combustion-gas value of ψ_h, ψ_ϕ, and T for hydrogen-
           carbon ratios from 0.10 to 0.15.
           Linear ψ_h, ψ_ϕ scales
           Hyperbolic H/C scales

IV   Combustion-gas values of ψ_h, ψ_ϕ, and T for hydrogen-
           carbon ratios from 0.15 to 0.20.
           Linear ψ_h, ψ_ϕ scales
           Hyperbolic H/C scale

V   Water-vapor values of h_w, ϕ_w, and T.
           Linear T scale

VI   Variation of $\dfrac{f}{1+f}$ with fuel-air ratio f.
           Linear $\dfrac{f}{1+f}$ scale

TABLE I - USEFUL CONSTANTS AND EQUATIONS

universal gas constant, 1.9872 cals/(gm mole)(^{O}R) or
1546.35 ft-lb/(lb mole)(^{O}R)

J mechanical equivalent of heat, 778.156 ft-lb/Btu

R_O gas constant for air 53.379, ft-lb/(lb air)(^{O}R)

R_w gas constant for water vapor, 85.8 ft-lb/(lb water vapor)(^{O}R)

R_t gas constant for combustion products of fuel and dry air,

$$\frac{53.379 + \dfrac{f \times 1546.35 \ (H/C)}{4.032 \ (1 + H/C)}}{1 + f} \quad \text{(equation (21) reference 7)}$$

R_m gas constant for mixture of water plus air, $\dfrac{53.379 + m\ 85.8}{1 + m}$

g standard acceleration due to gravity, 32.174 ft/sec^2

0° R = $- 459.69^O$ F

$$\Delta\phi = \eta_{t,\infty} \frac{R_t}{J} \log_e \left(\frac{p_2}{p_1}\right), \text{ for expansion process} \tag{6}$$

$$\Delta\phi = \frac{R}{J \, \eta_{c,\infty}} \log_e \left(\frac{p_2}{p_1}\right), \text{ for compression process} \tag{7}$$

$$\phi_2 = \phi_1 + \Delta\phi$$

$$f = \frac{h_{0,2} - h_{0,1}}{\eta_b \overline{H} - h_{0,2} - \Psi_{h,2} + h_f} \tag{9}$$

$$\overline{H} = 15{,}935 + 15{,}800 \ (H/C) \text{ Btu/lb} \tag{10}$$

$$h = h_0 + \Psi_h \left(\frac{f}{1 + f}\right) \tag{35}$$

$$\phi = \phi_0 + \Psi_\phi \left(\frac{f}{1 + f}\right) \tag{36}$$

CHART I
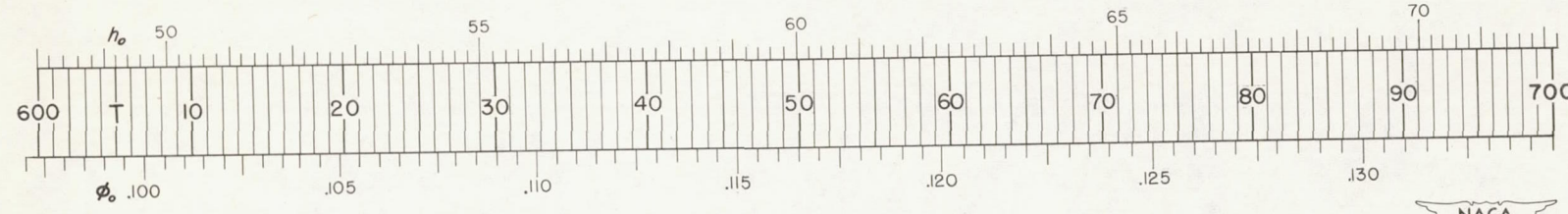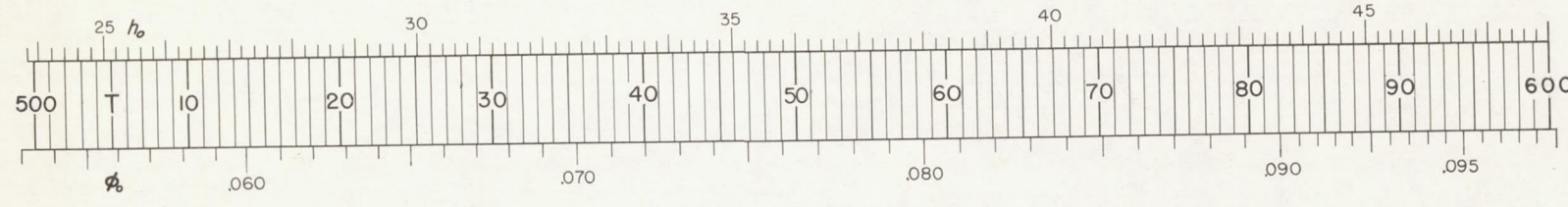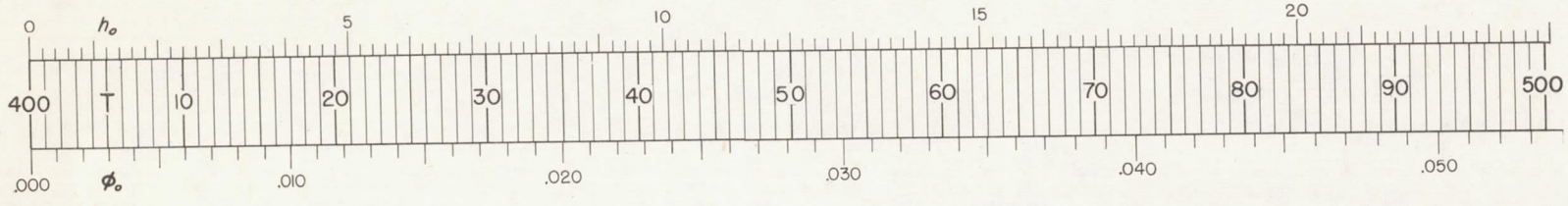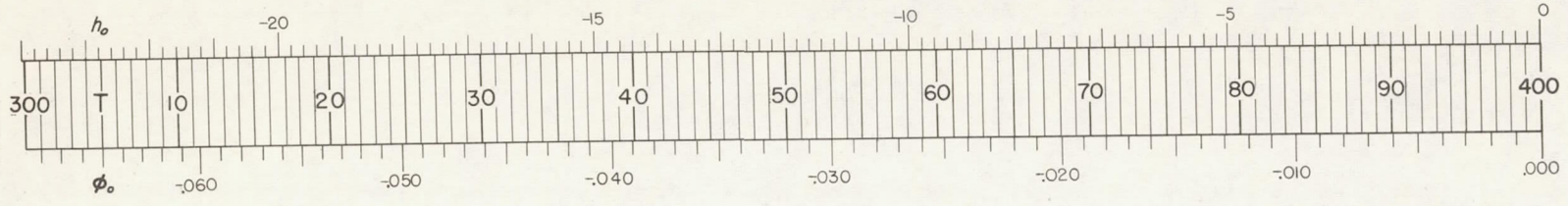
(a) Temperature range, 300° to 700° R.

Chart I. - Dry-air values of ϕ_0, h_0, and T.
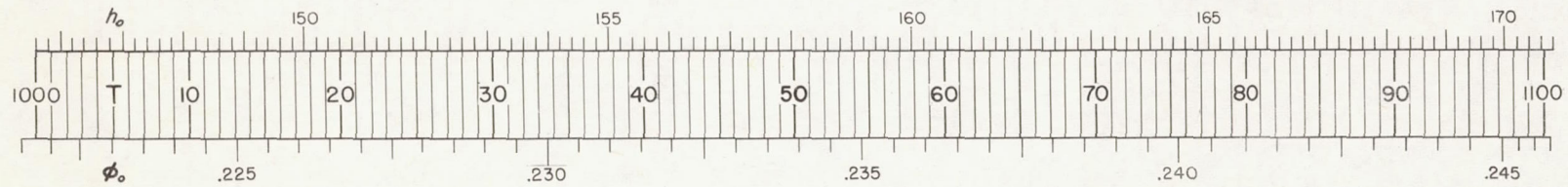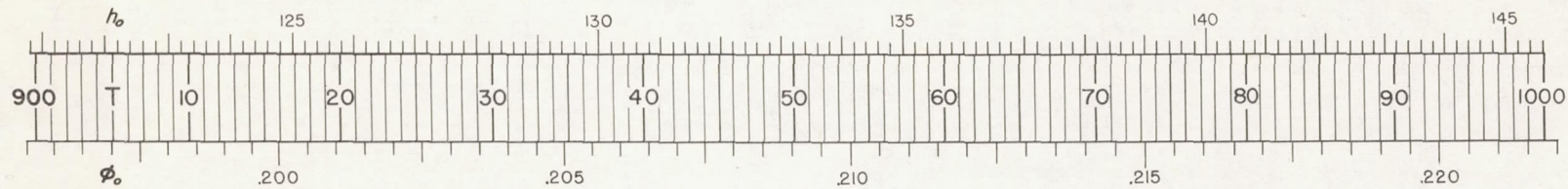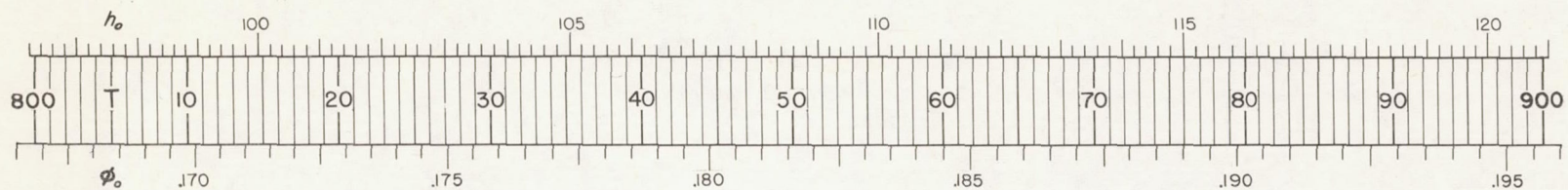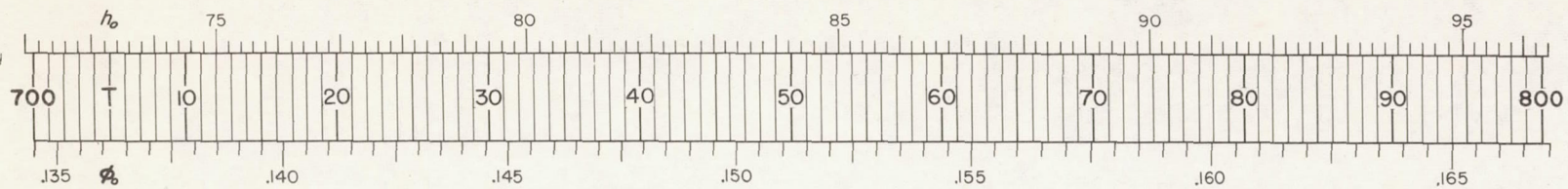
(b) Temperature range, 700° to 1100° R.

Chart I. - Continued. Dry-air values of ϕ_0, h_0, and T.
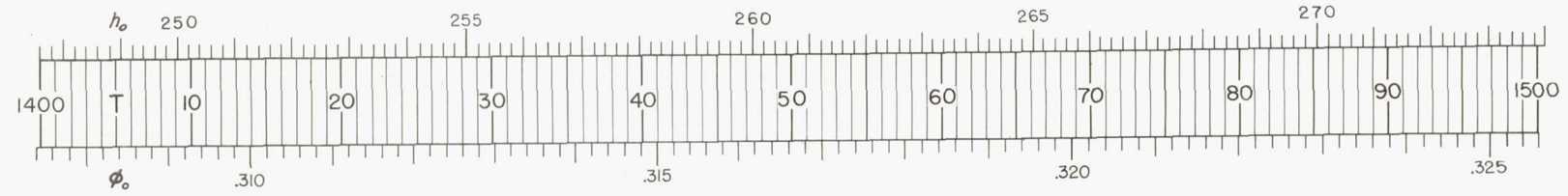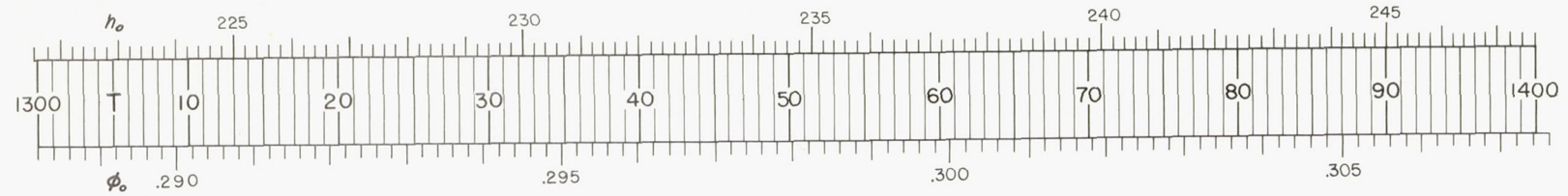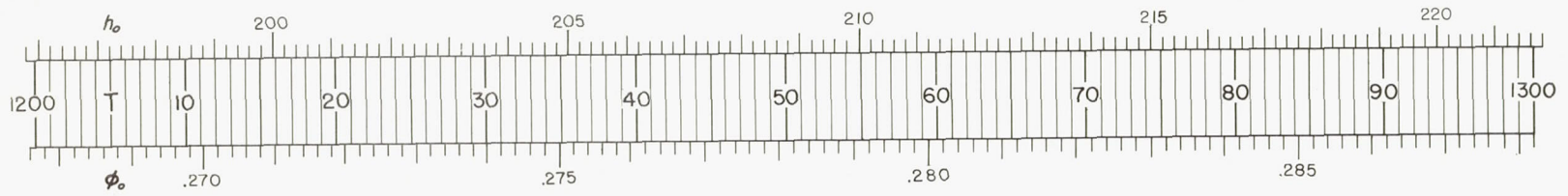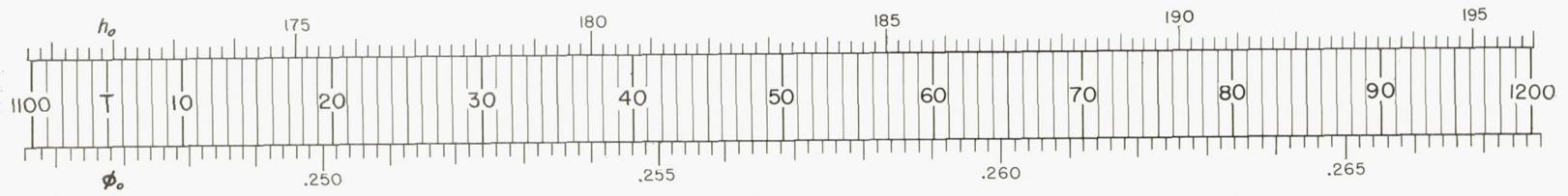
(c) Temperature range, 1100° to 1500° R.

Chart I. - Continued. Dry-air values of ϕ_0, h_0, and T.
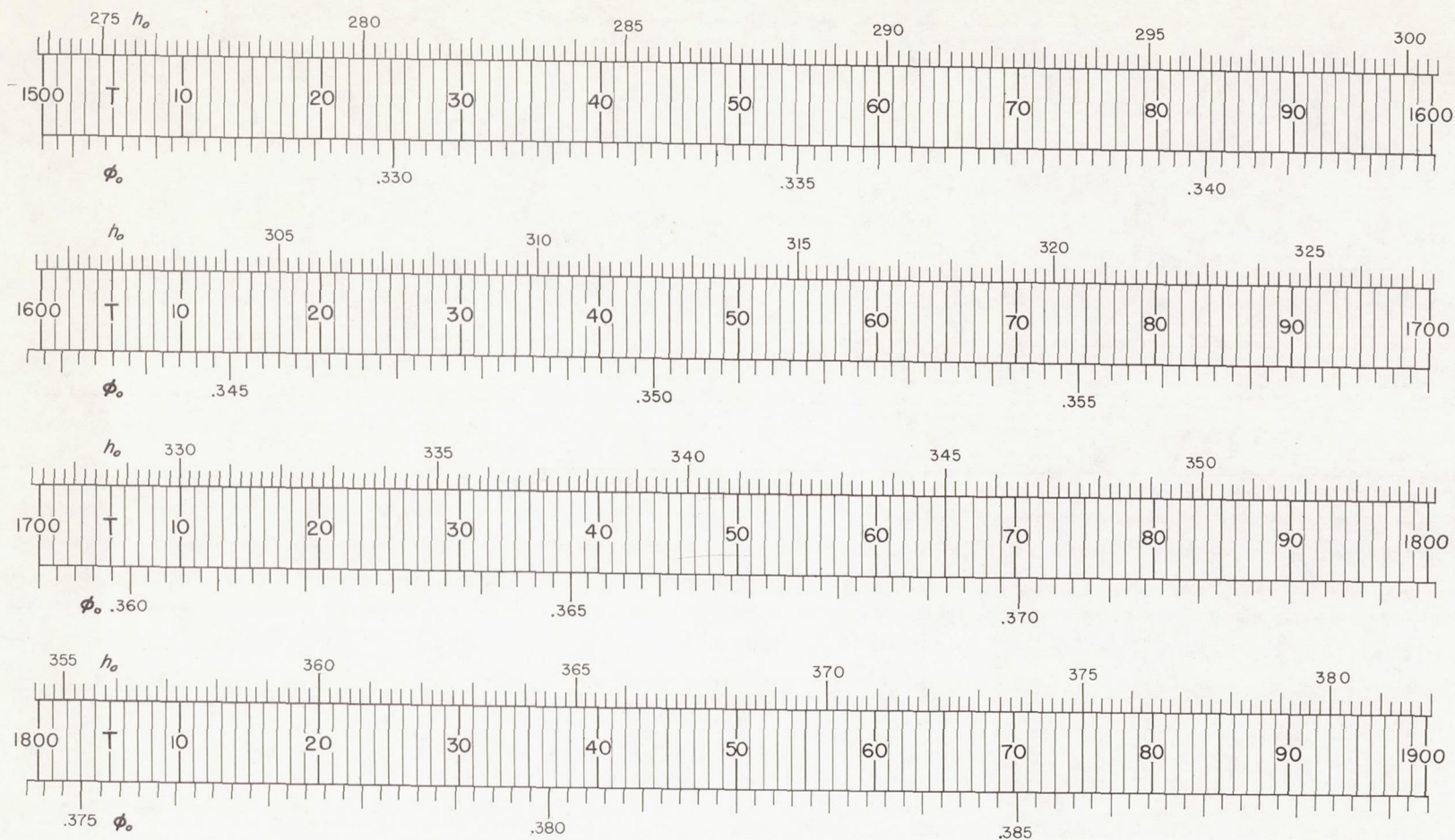
(d) Temperature range, 1500° to 1900° R.

Chart I. - Continued. Dry-air values of ϕ_0, h_0, and T.
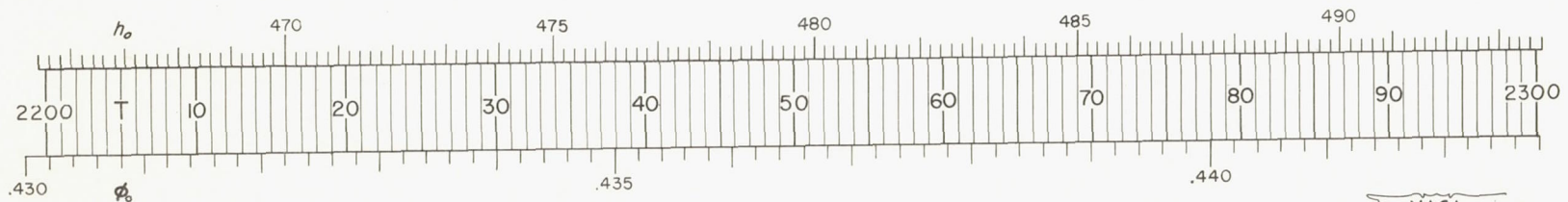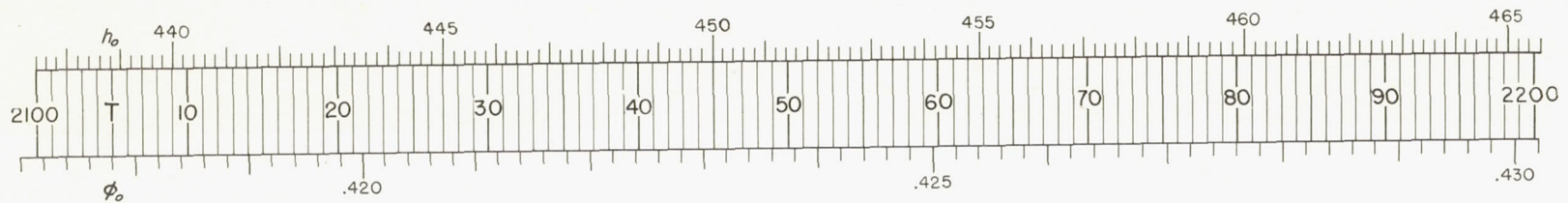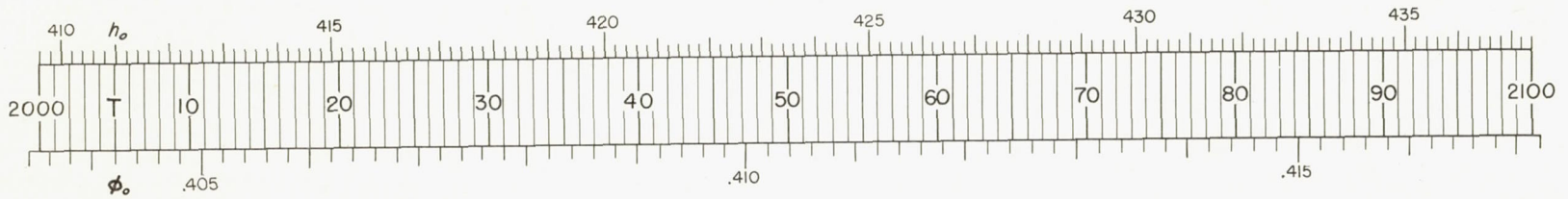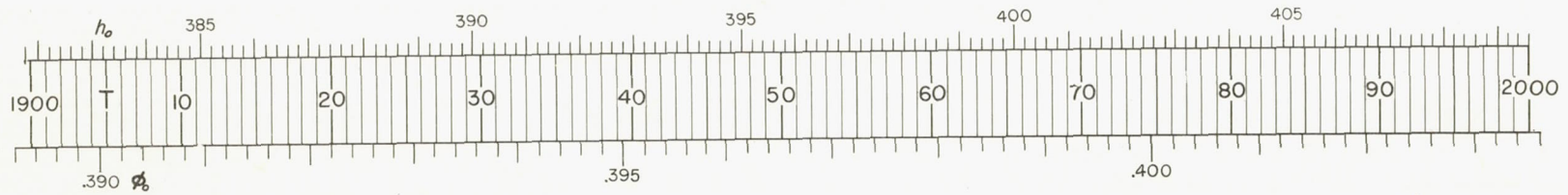
(e) Temperature range, 1900° to 2300° R.

Chart I. - Continued. Dry-air values of ϕ_0, h_0, and T.
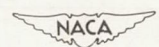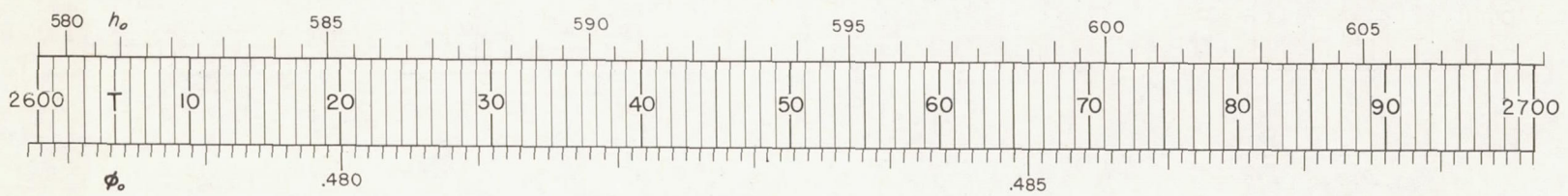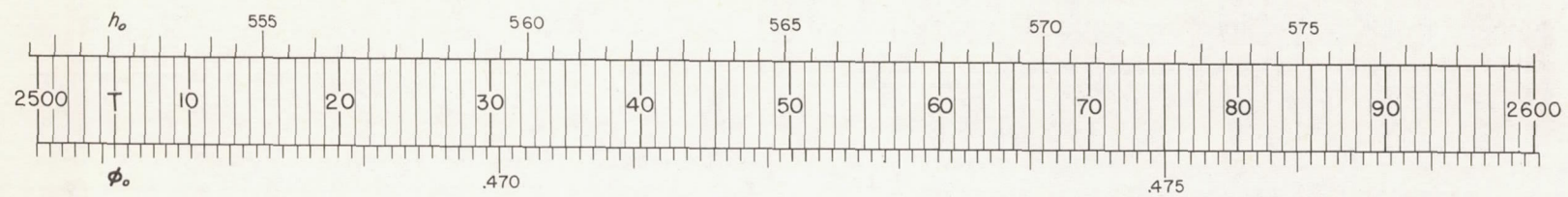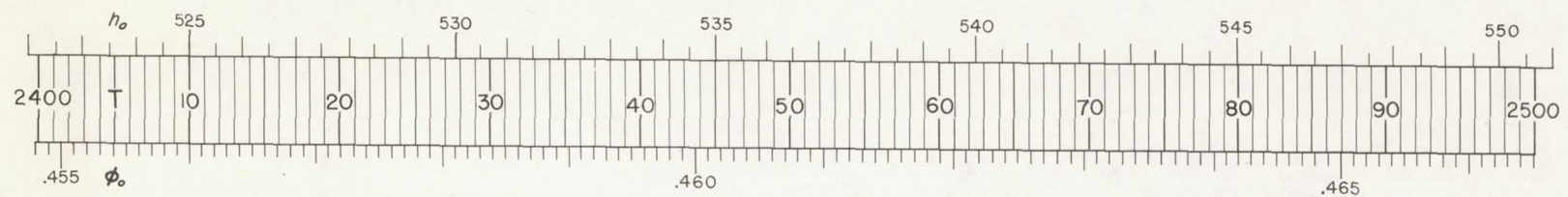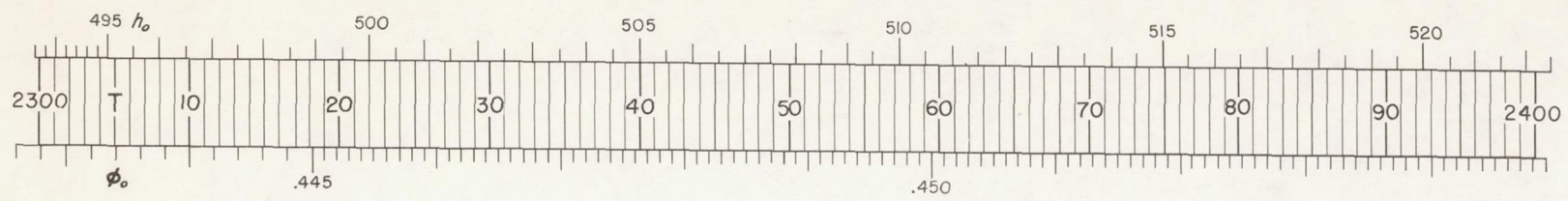
(f) Temperature range, 2300° to 2700° R.

Chart I. - Continued. Dry-air values of ϕ_0, h_0, and T.
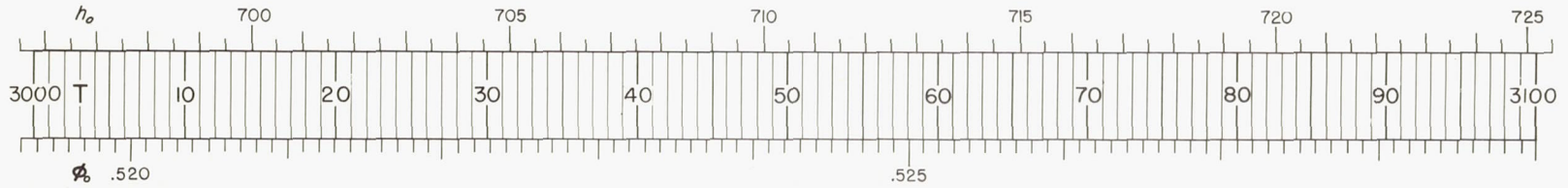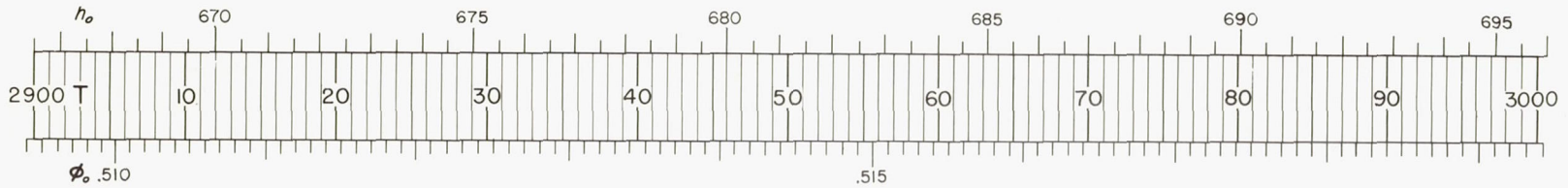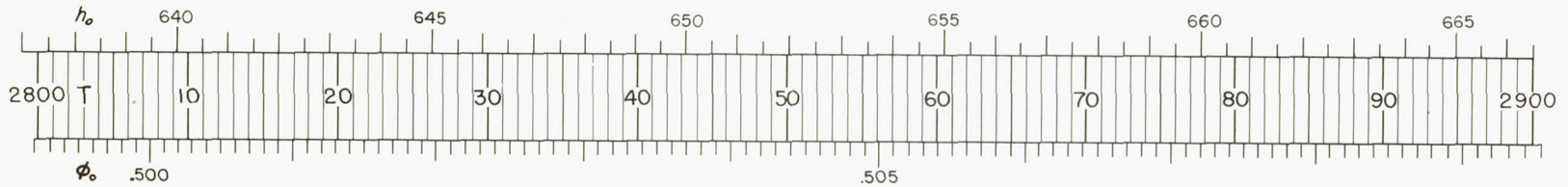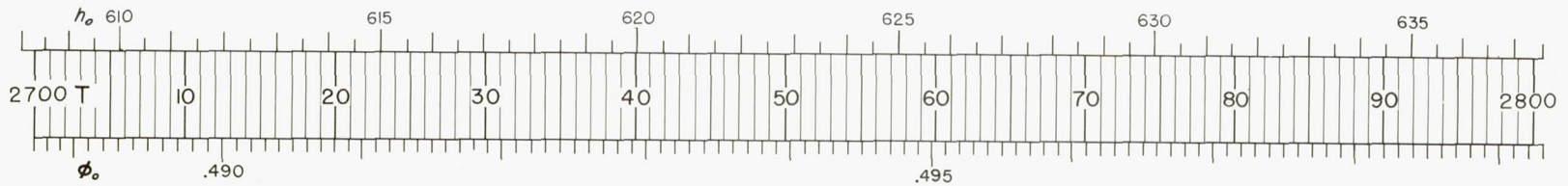
(g) Temperature range, 2700° to 3100° R.

Chart I. - Continued. Dry-air values of ϕ_0, h_0, and T.
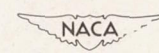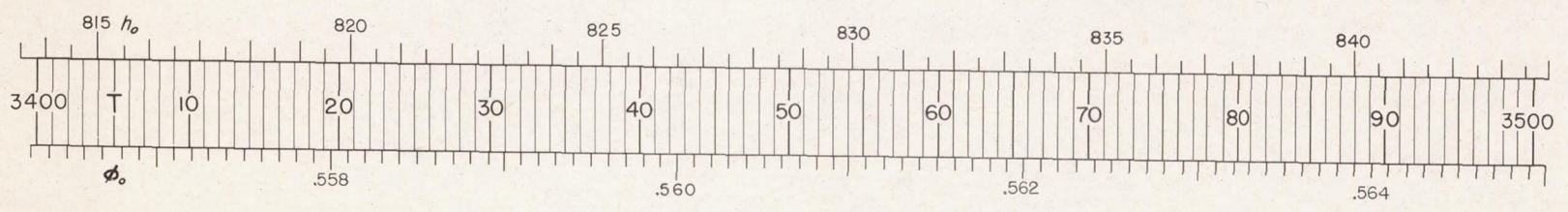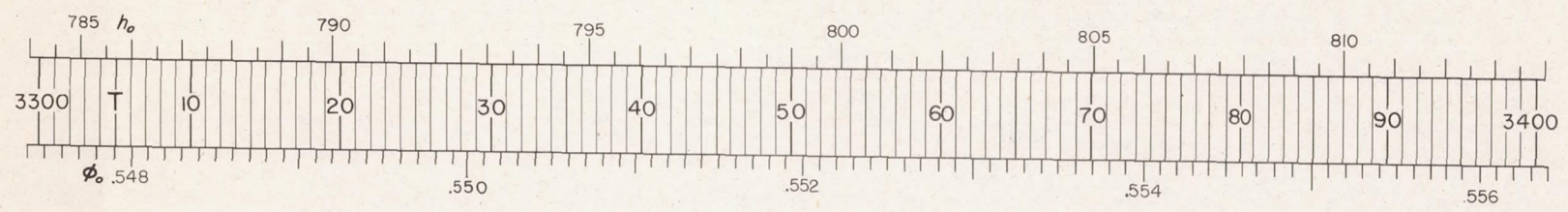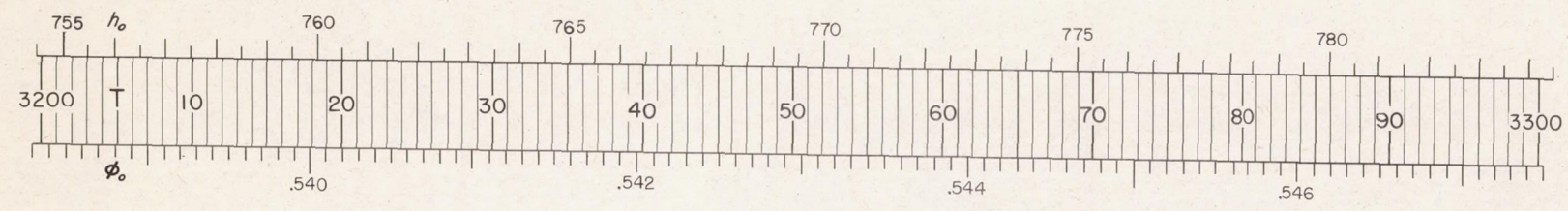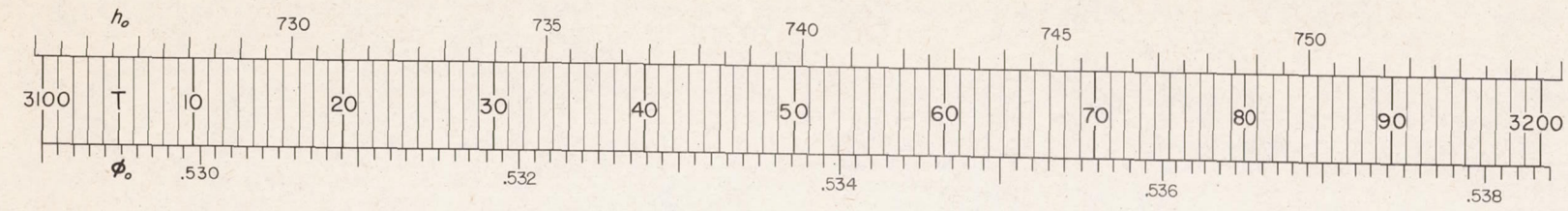
(h) Temperature range, 3100° to 3500° R.

Chart I. - Concluded. Dry-air values of ϕ_0, h_0, and T.
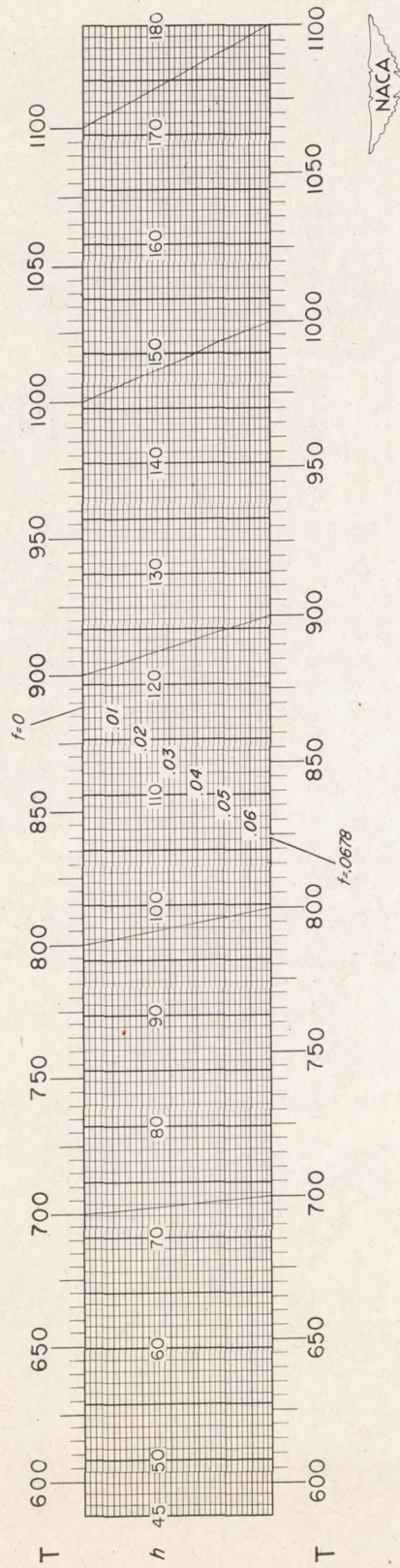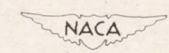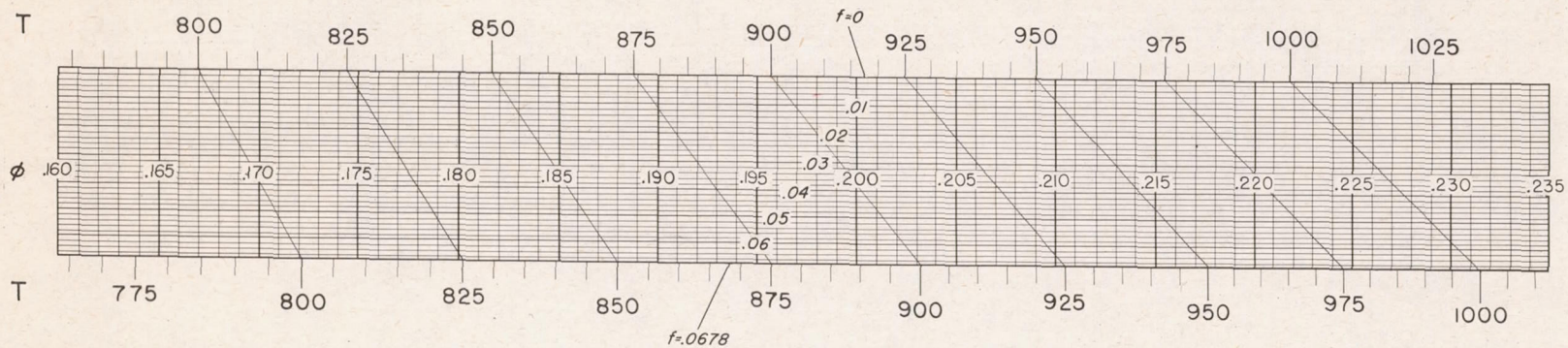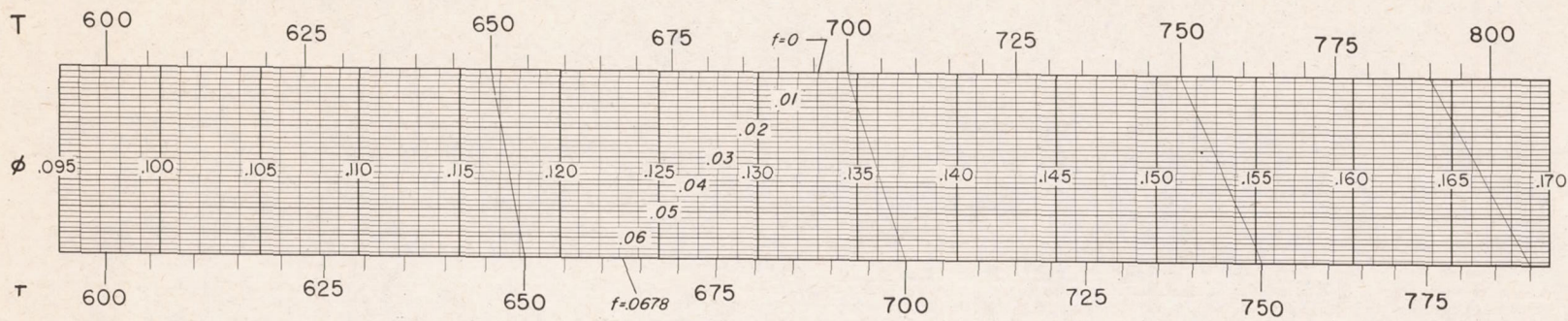
CHART II

(a) Temperature range, 600° to 1100° R.

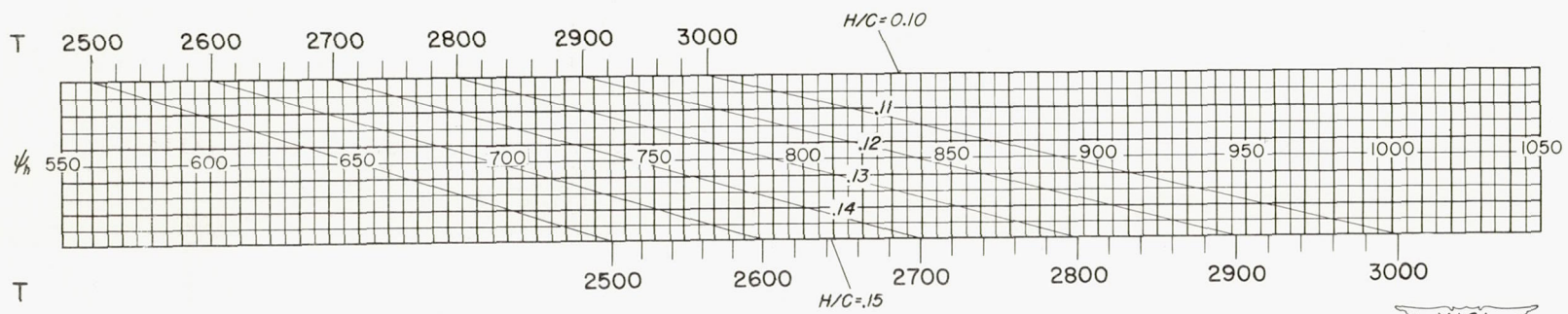Chart II. - Combustion-gas values of ϕ, h, T, and f for hydrogen-carbon ratio of 0.167.

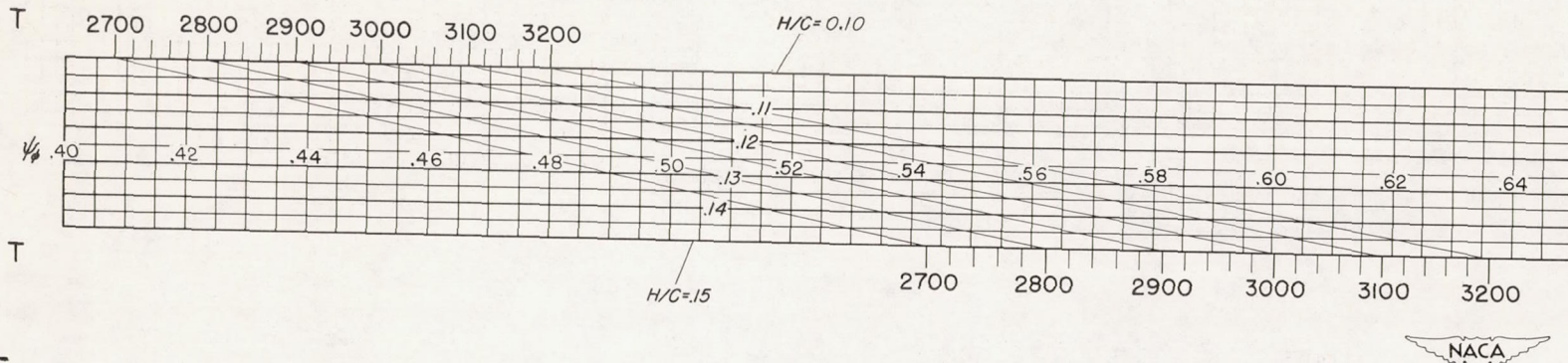(b) Temperature range, 1000° to 1500° R.

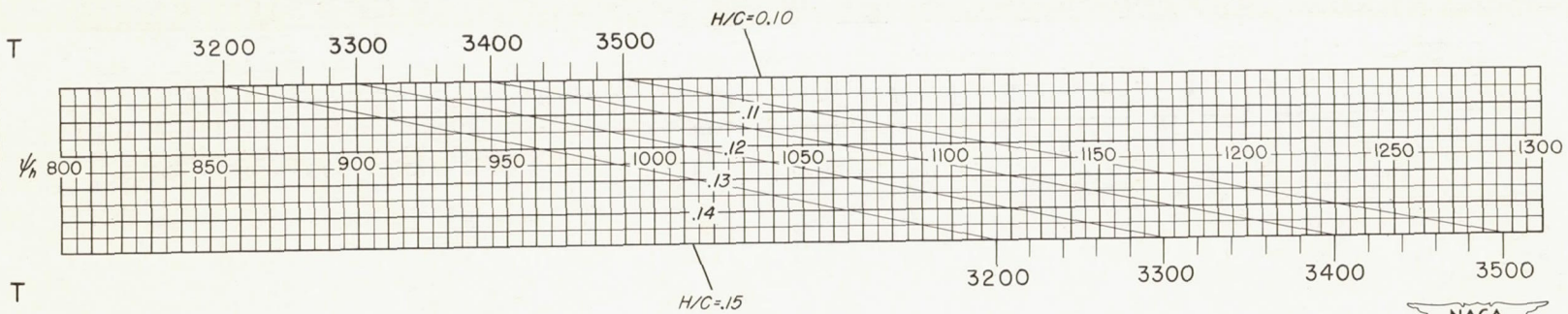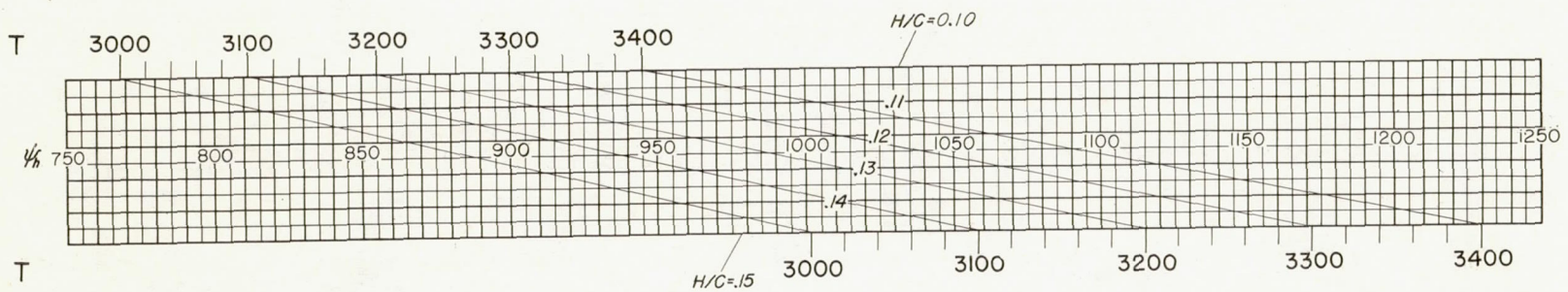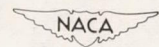Chart II. - Continued. Combustion-gas values of ∅, h, T, and f for hydrogen-carbon ratio of 0.167.
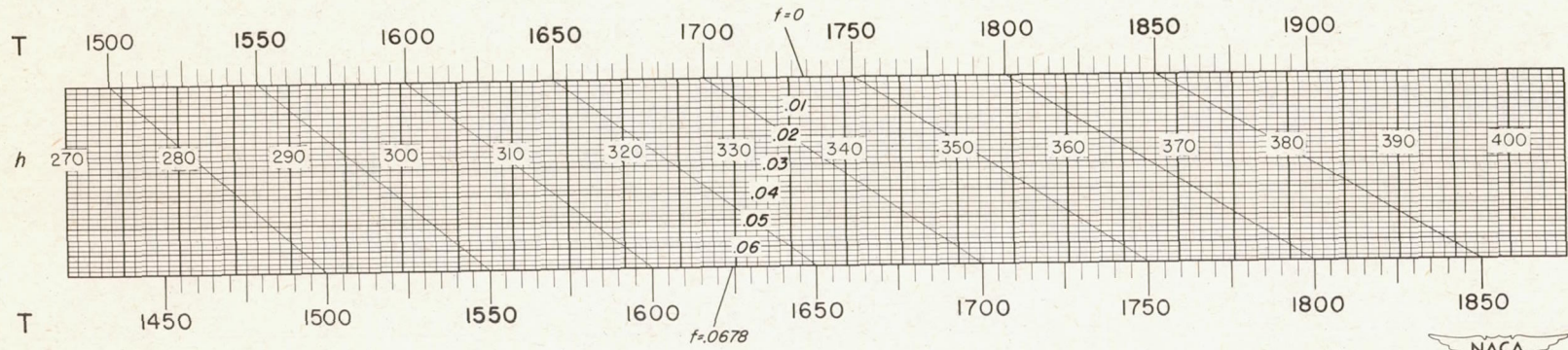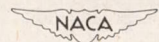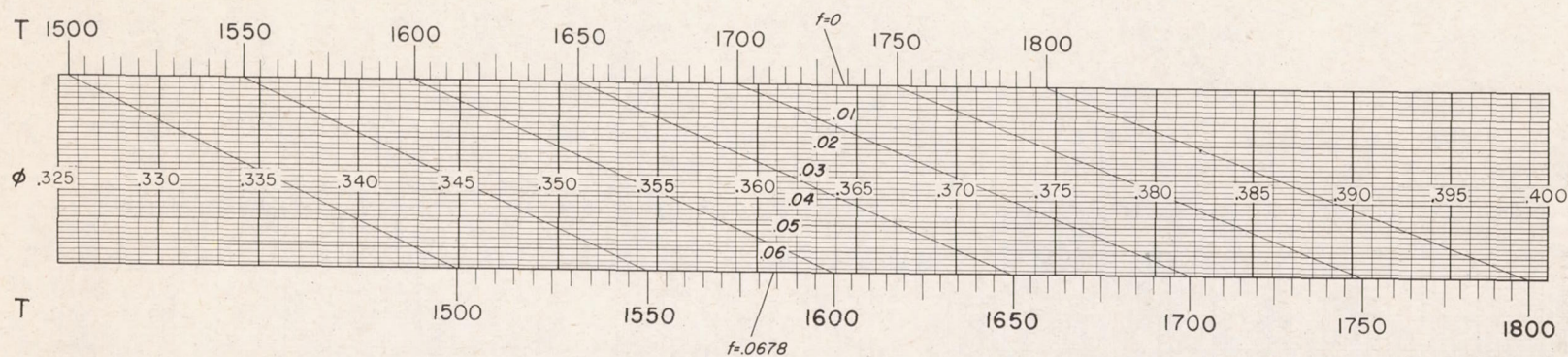
(c) Temperature range, 1500° to 1850° R.

Chart II. - Continued. Combustion-gas values of ϕ, h, T, and f for hydrogen-carbon ratio of 0.167.
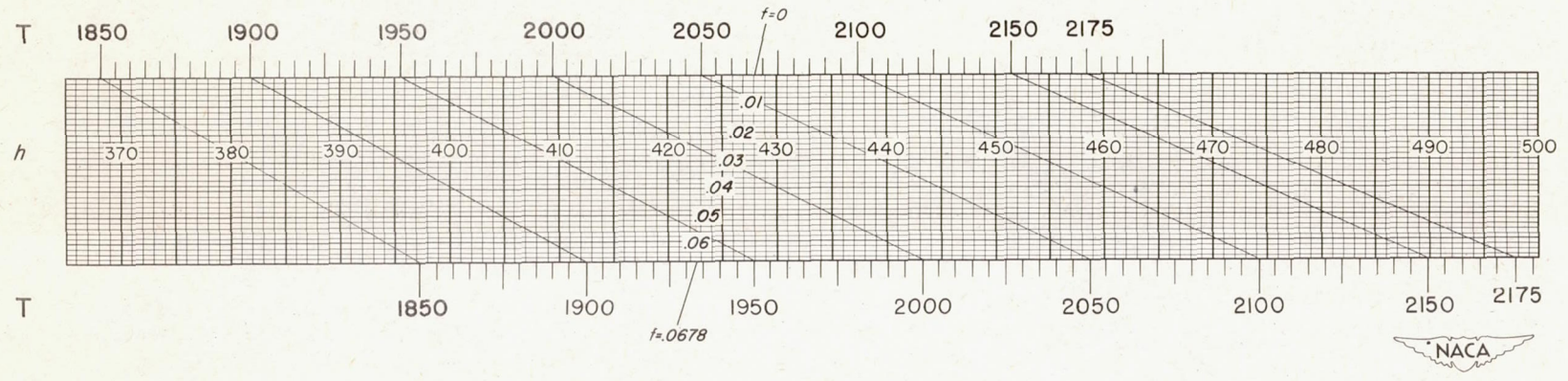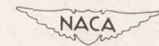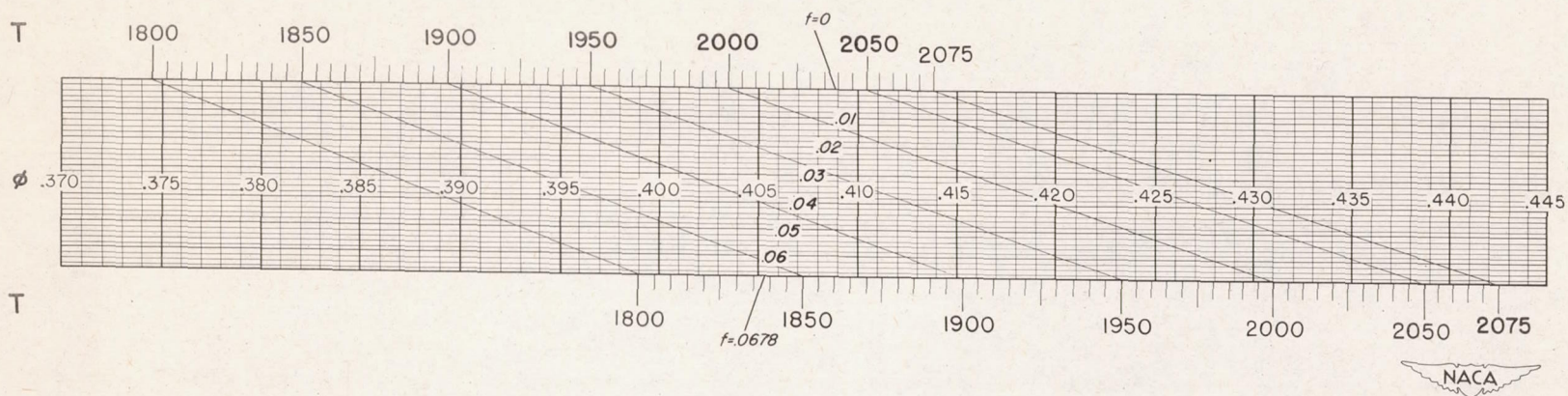
(d) Temperature range, 1800° to 2175° R.

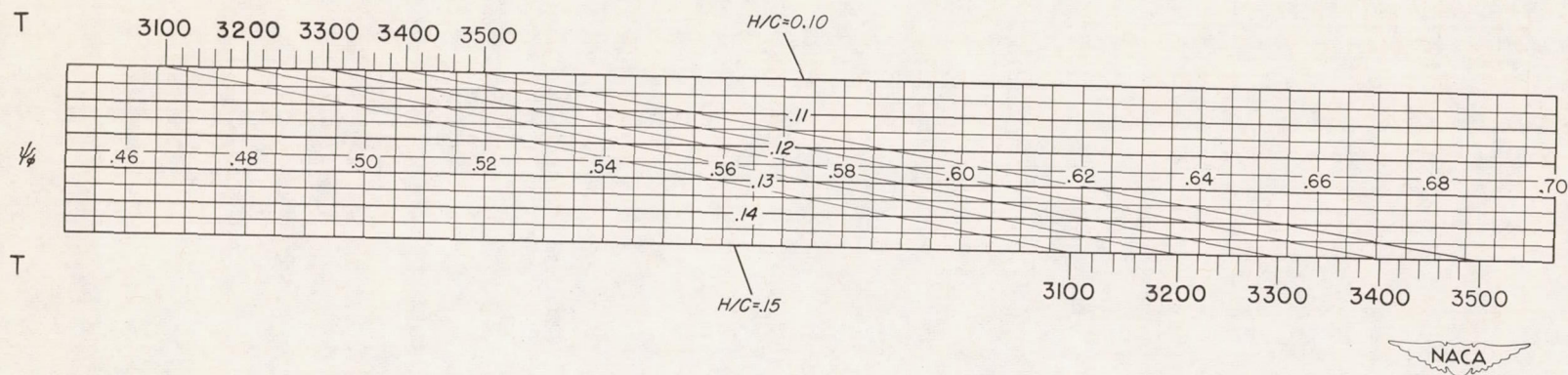Chart II. - Continued. Combustion-gas values of ϕ, h, T, and f for hydrogen-carbon ratio of 0.167.
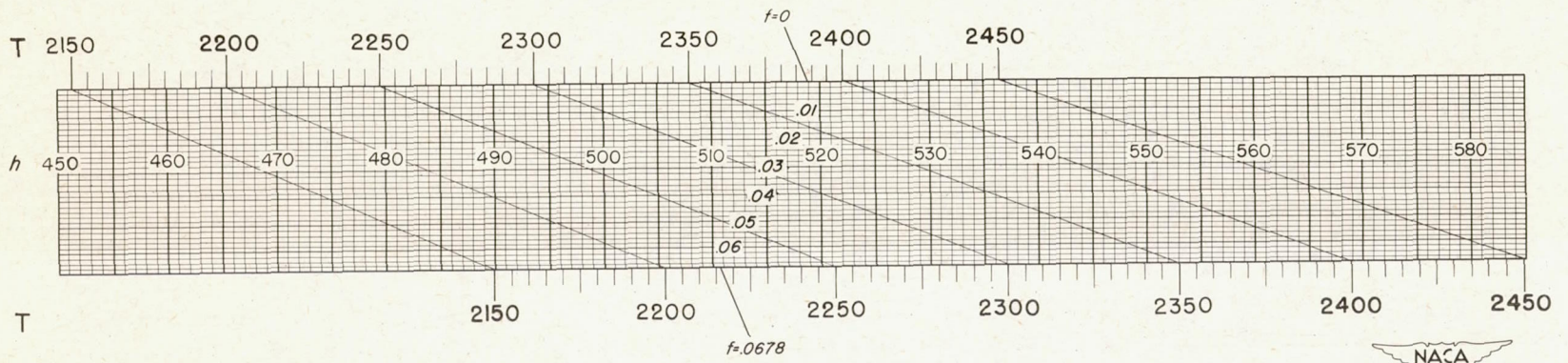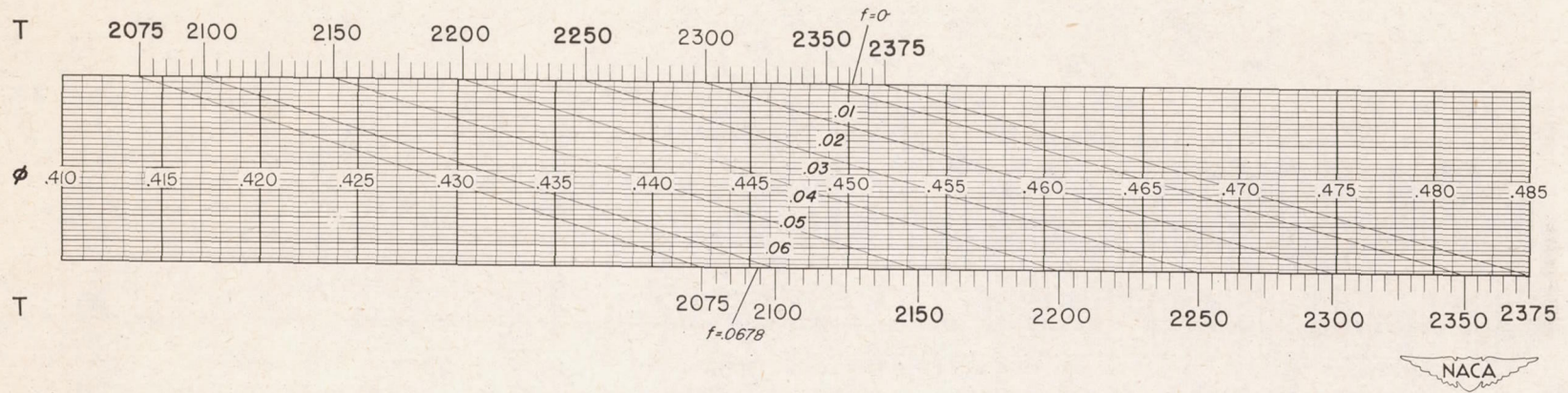
(e) Temperature range, 2075° to 2450° R.

Chart II. - Continued. Combustion-gas values of ϕ , h, T, and f for hydrogen-carbon ratio of 0.167.
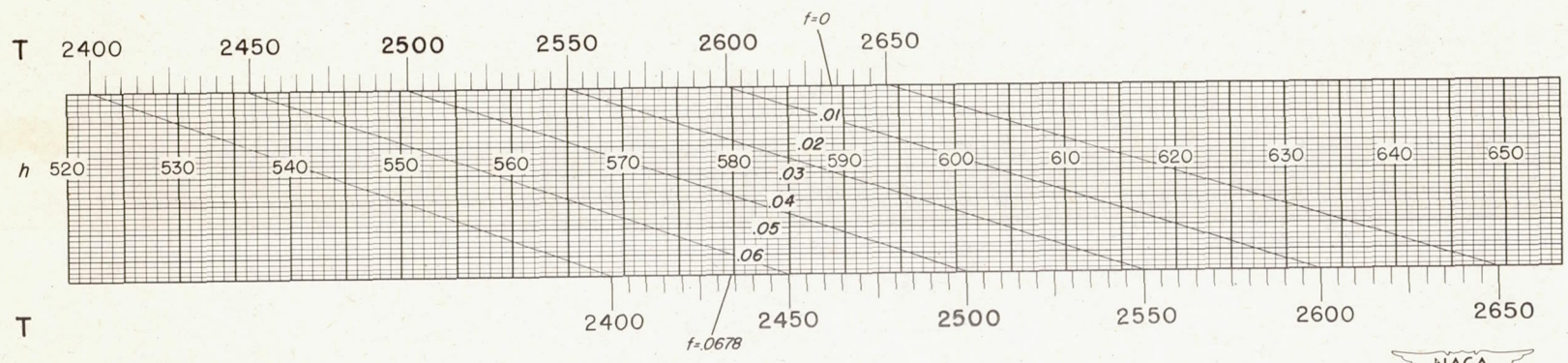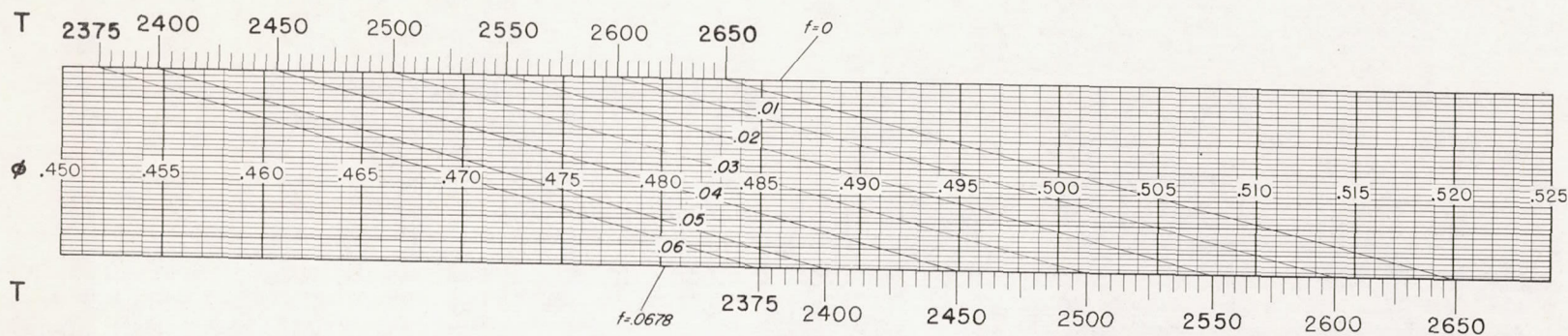
(f) Temperature range, 2375° to 2650° R.

Chart II. - Continued. Combustion-gas values of ϕ, h, T, and f for hydrogen-carbon ratio of 0.167.

(g) Temperature range, 2650° to 2950° R.

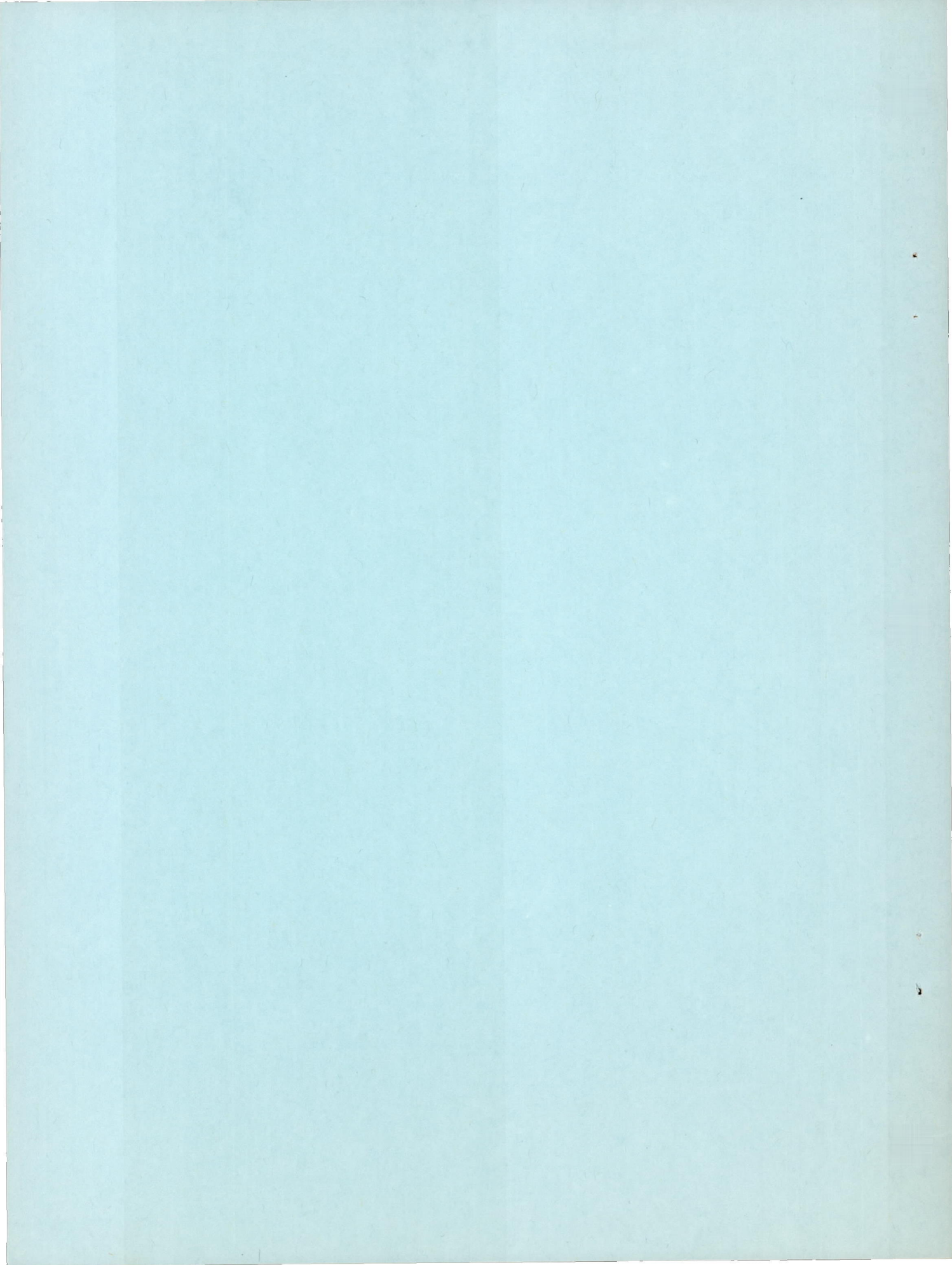Chart II. - Continued. Combustion-gas values of ϕ, h, T, and f for hydrogen-carbon ratio of 0.167.

(h) Temperature range, 2850° to 3240° R.

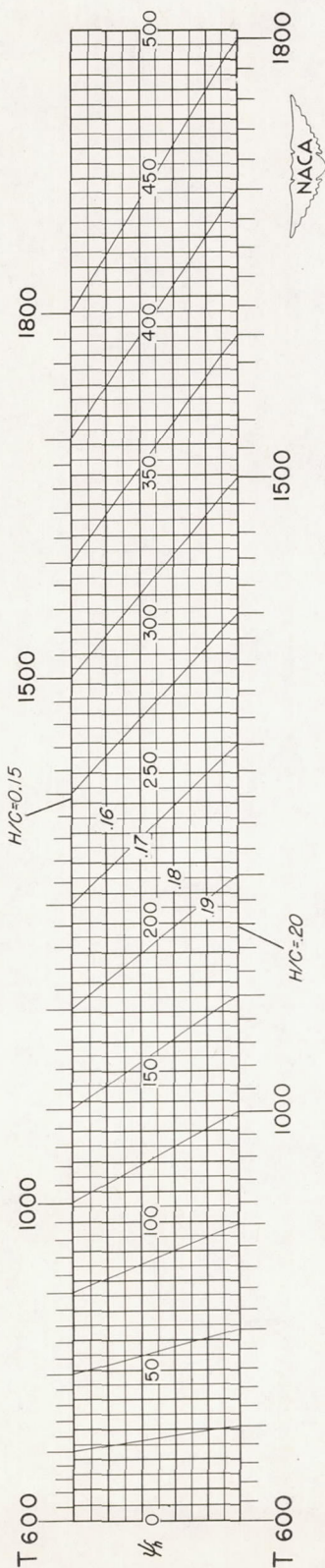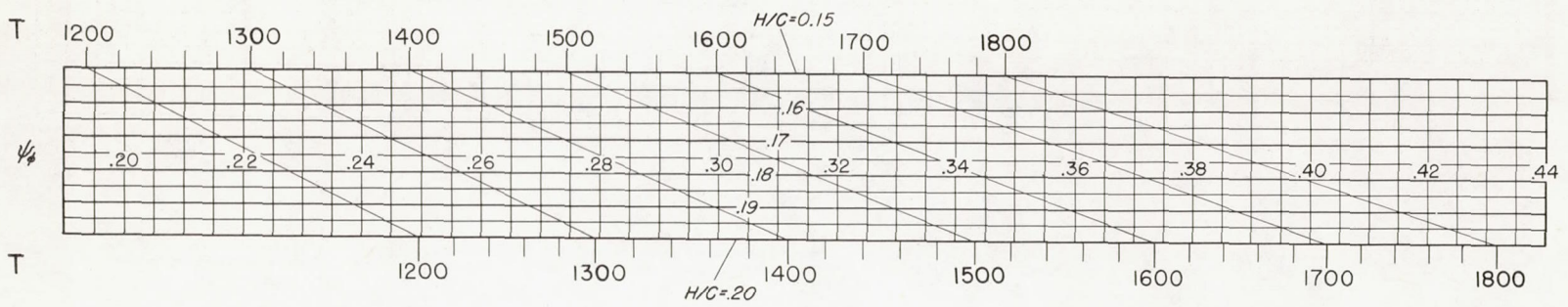Chart II. - Continued. Combustion-gas values of ϕ, h, T, and f for hydrogen-carbon ratio of 0.167.

(i) Temperature range, 3225° to 3500° R.

Chart II. - Concluded. Combustion-gas values of ϕ, h, T, and f for hydrogen-carbon ratio of 0.167.

CHART III
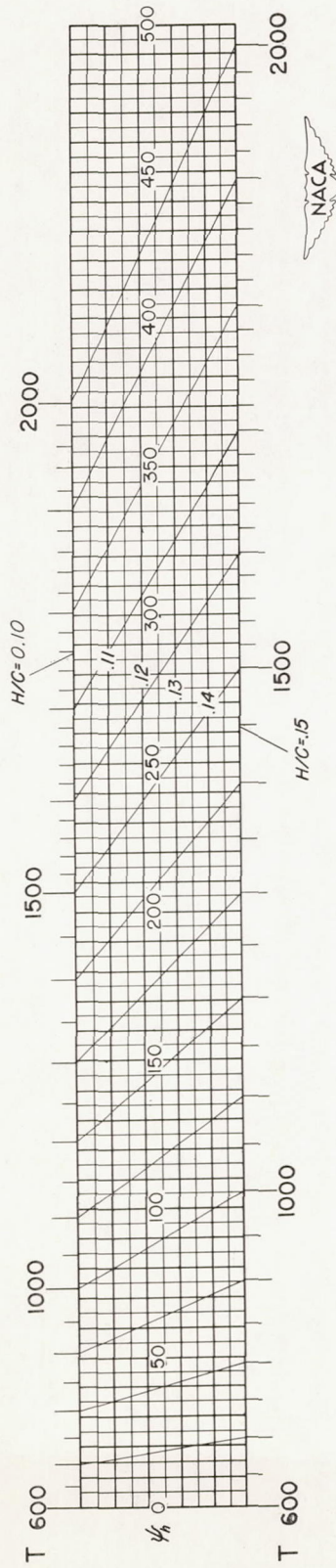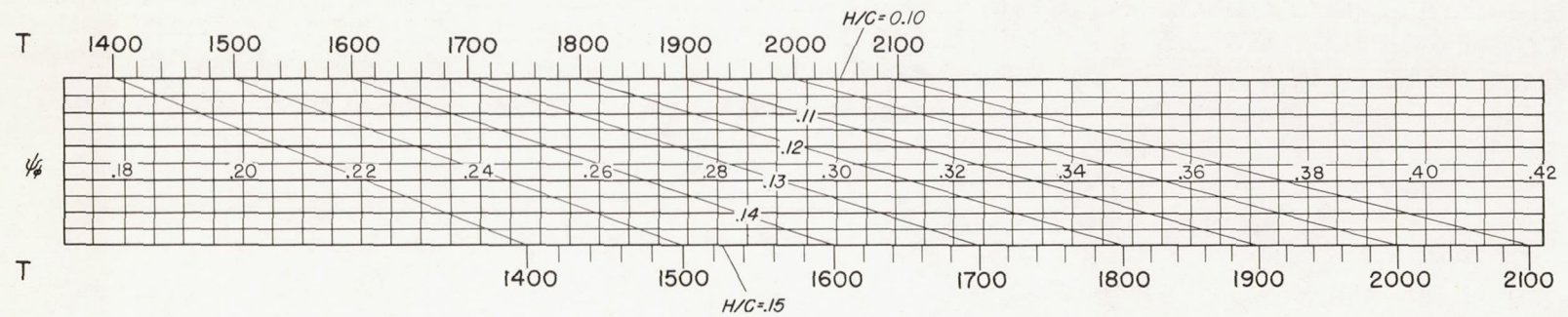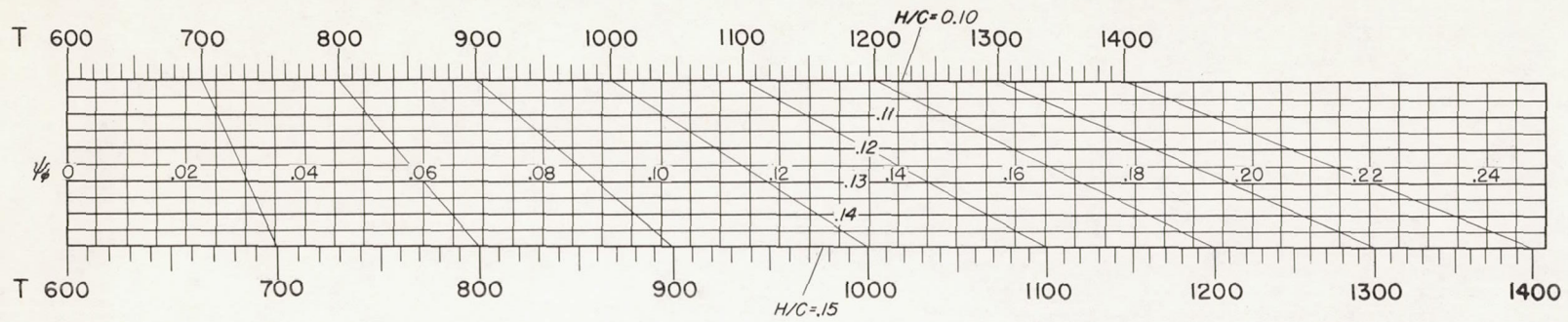
(a) Temperature range, 600° to 2100° R.

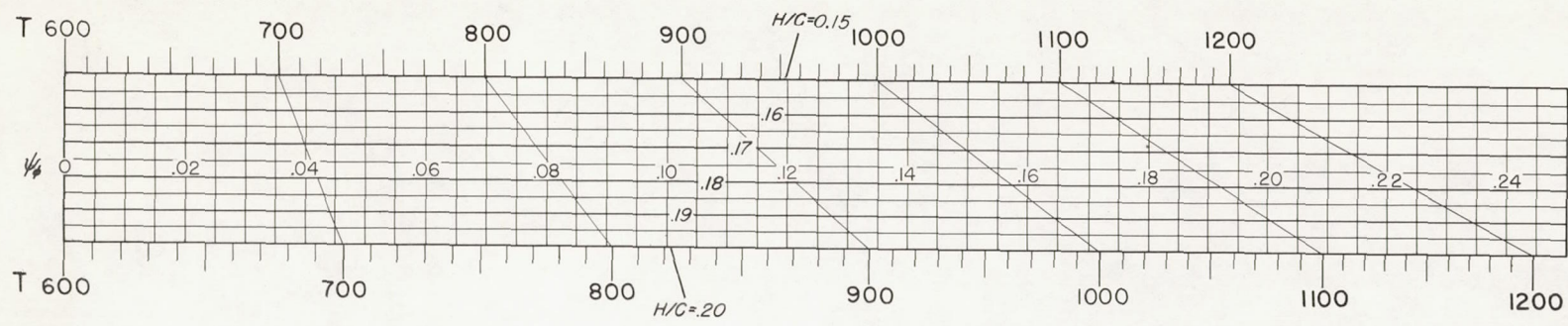Chart III. - Combustion-gas values of Ψ_h, Ψ_ϕ, and T for hydrogen-carbon ratios from 0.10 to 0.15.
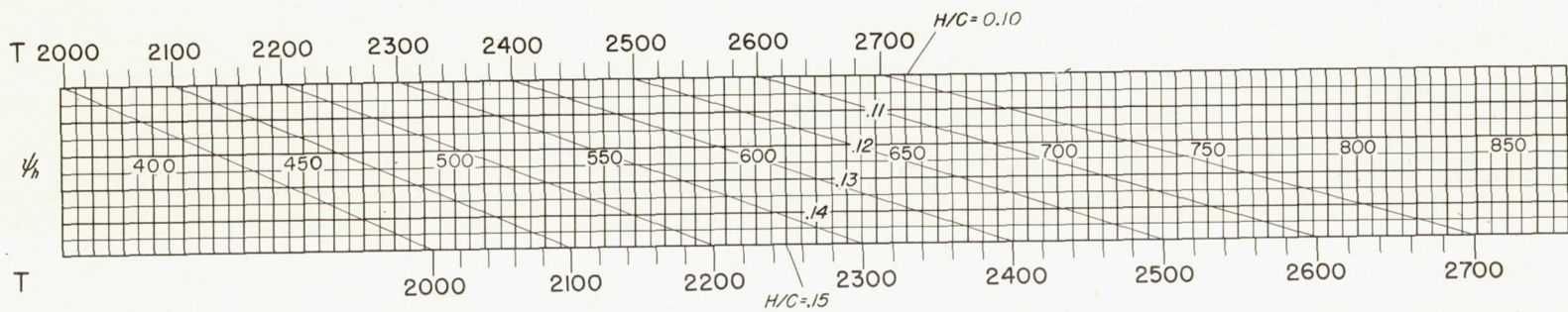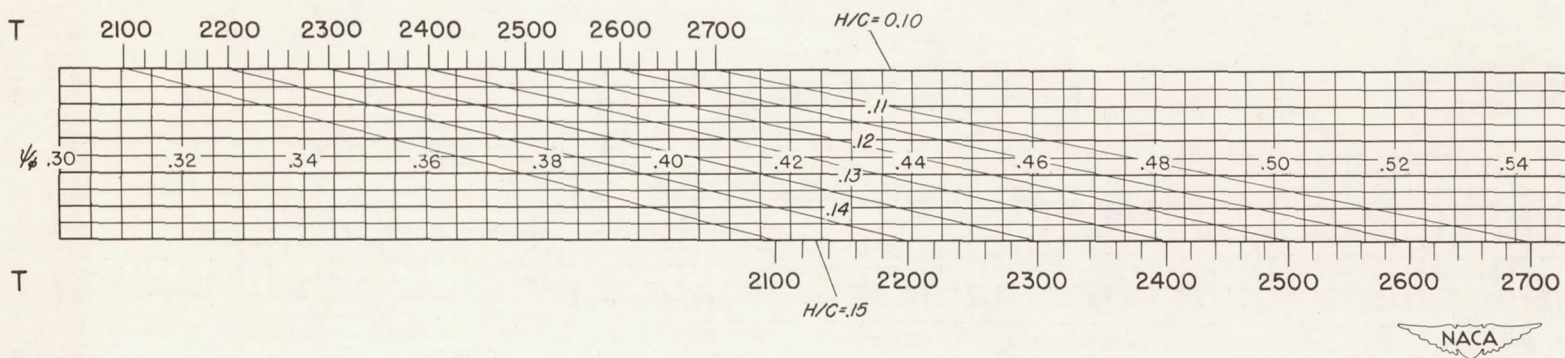
(b) Temperature range, 2000° to 2700° R.

Chart III. - Continued. Combustion-gas values of ψ_h, ψ_ϕ, and T for hydrogen-carbon ratios from 0.10 to 0.15.

T 2700 2800 2900 3000 3100 3200   H/C= 0.10

ψ/φ .40   .42   .44   .46   .48   .50   .52   .54   .56   .58   .60   .62   .64

.11
.12
.13
.14

T

H/C=.15   2700   2800   2900   3000   3100   3200

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(c) Temperature range, 2500° to 3200° R.

Chart III. - Continued. Combustion-gas values of ψ_h, ψ_ϕ, and T for hydrogen-carbon ratios from 0.10 to 0.15.

(d) Temperature range, 3000° to 3500° R.

Chart III. - Concluded. Combustion-gas values of ψ_h, ψ_ϕ, and T for hydrogen-carbon ratios from 0.10 to 0.15.

CHART IV

(a) Temperature range, 600° to 1800° R.

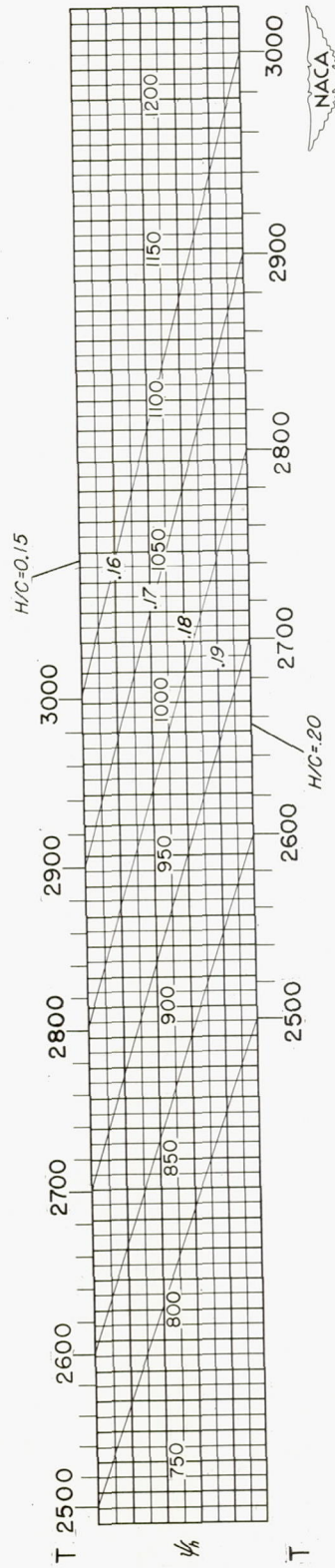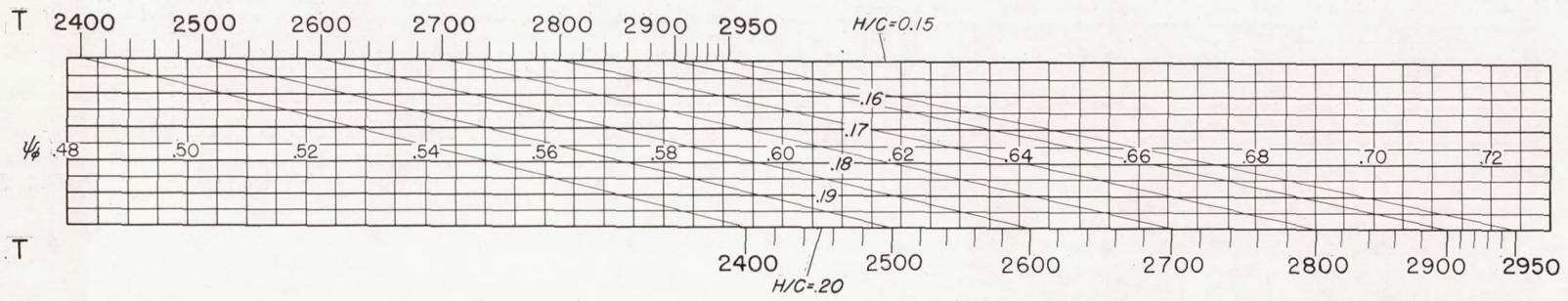Chart IV. - Combustion-gas values of ψ_h, ψ_ϕ, and T for hydrogen-carbon ratios of 0.15 to 0.20.
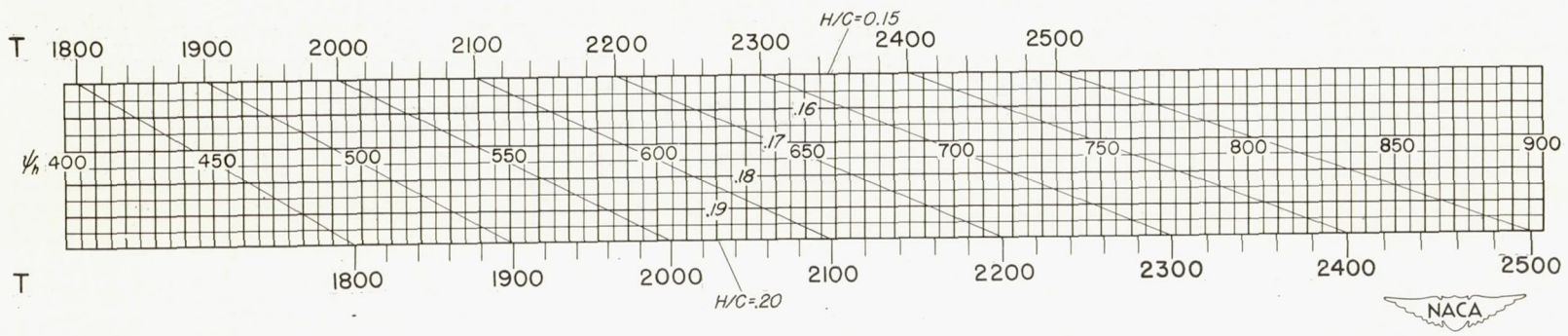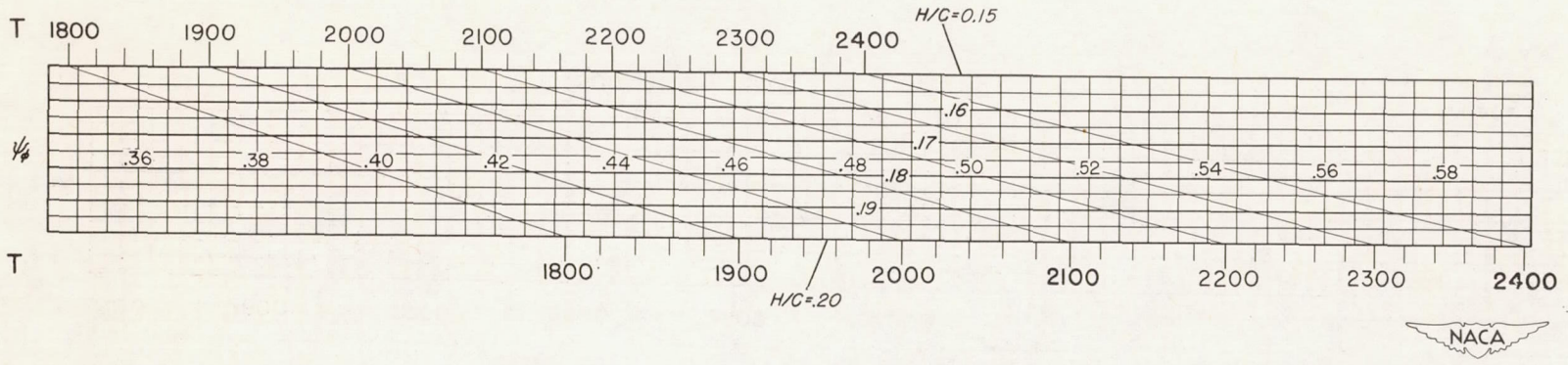
(b) Temperature range, 1800° to 2500° R.

Chart IV. - Continued. Combustion-gas values of ψ_h, ψ_ϕ, and T for hydrogen-carbon ratios of 0.15 to 0.20.
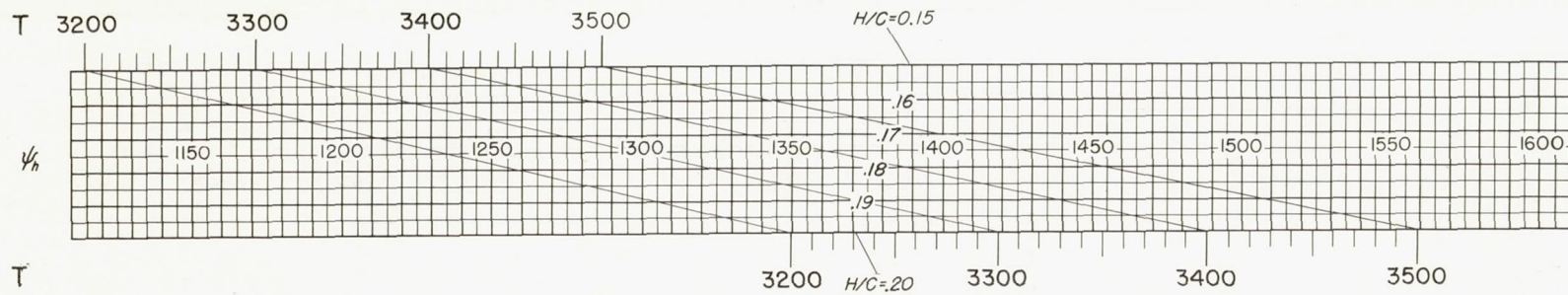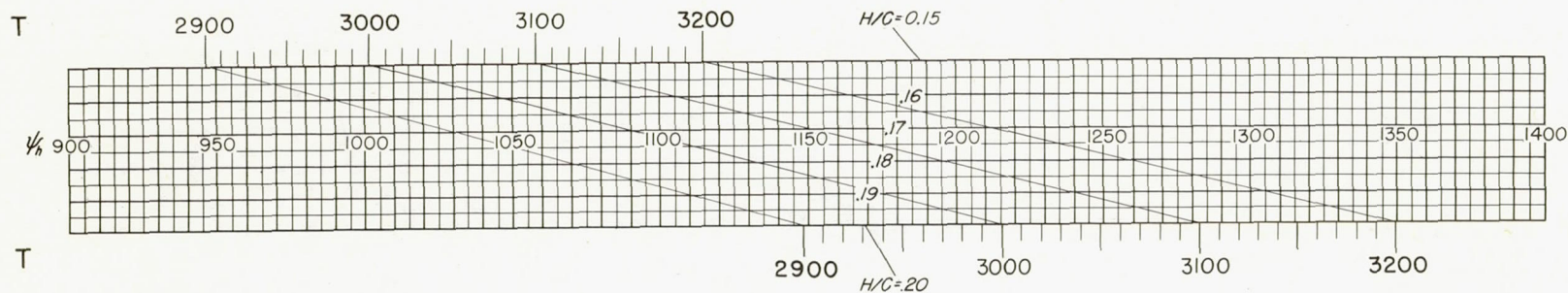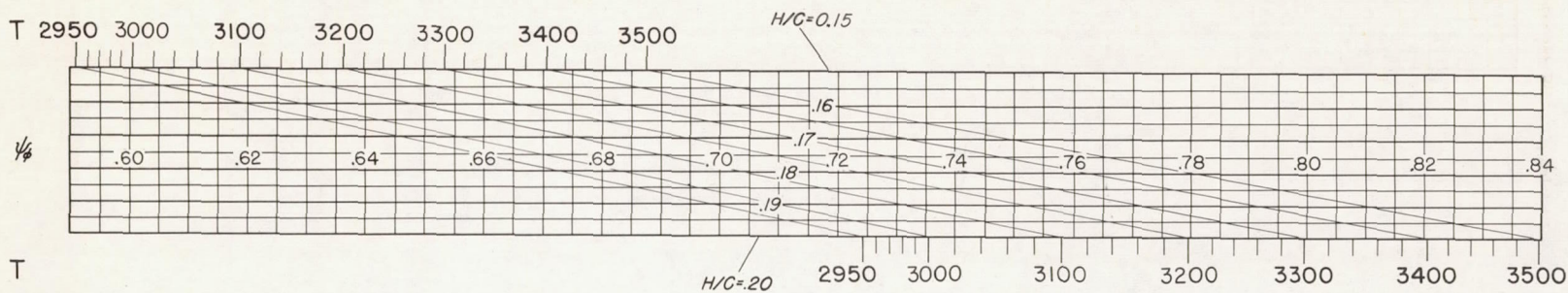
(c) Temperature range, 2400° to 3000° R.

Chart IV. - Continued. Combustion-gas values of Ψ_h, Ψ_ϕ, and T for hydrogen-carbon ratios of 0.15 to 0.20.
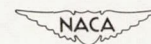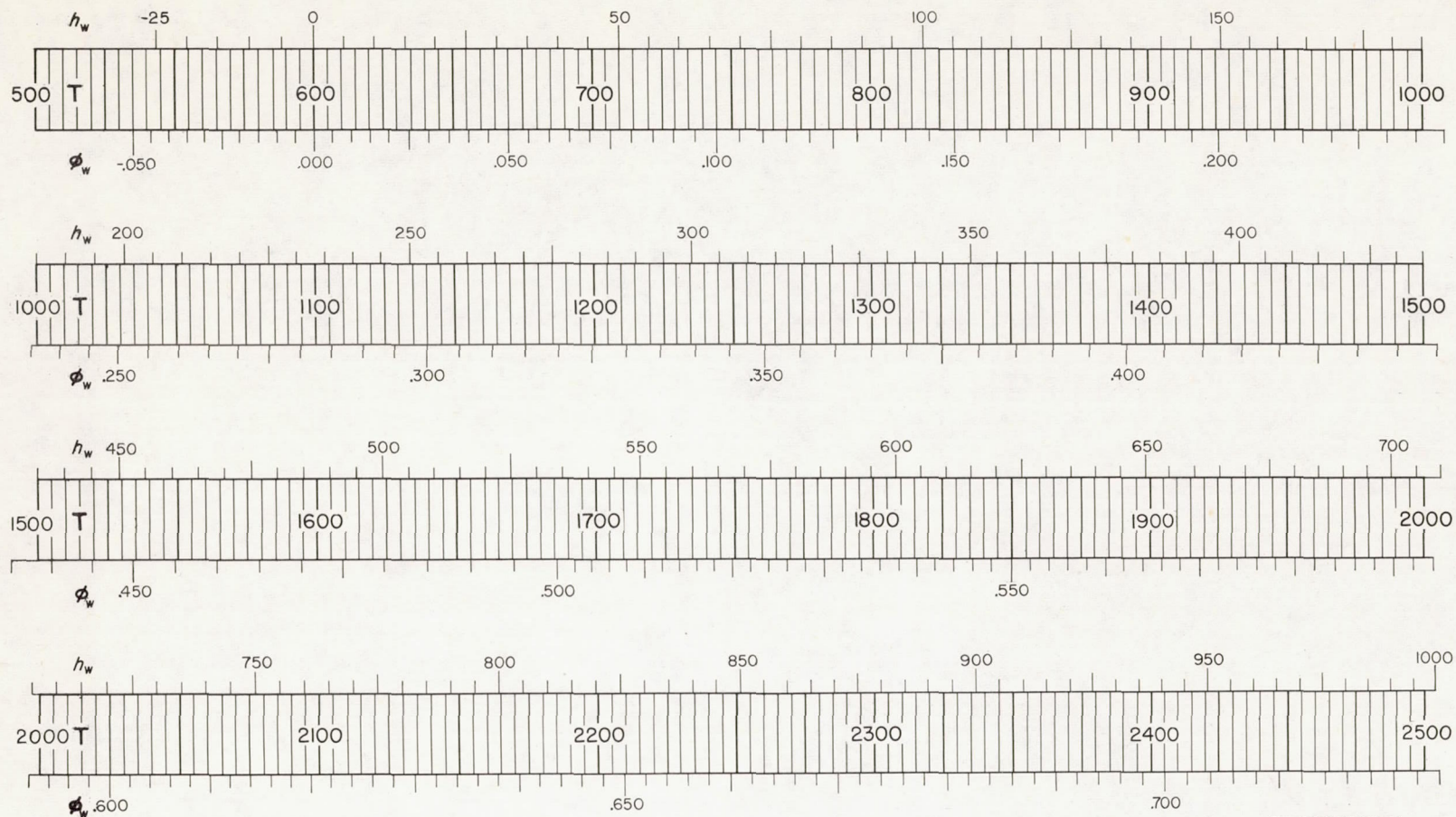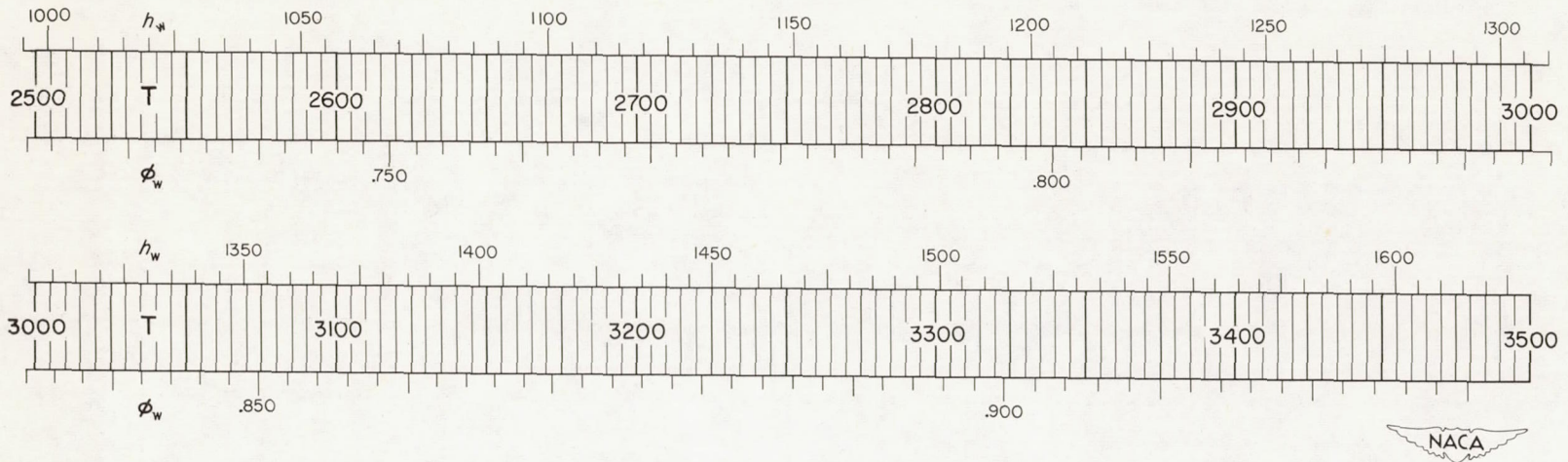
(d) Temperature range, 2900° to 3500° R.

Chart IV. - Concluded. Combustion-gas values of ψ_h, ψ_ϕ, and T for hydrogen-carbon ratios of 0.15 to 0.20.

CHART V

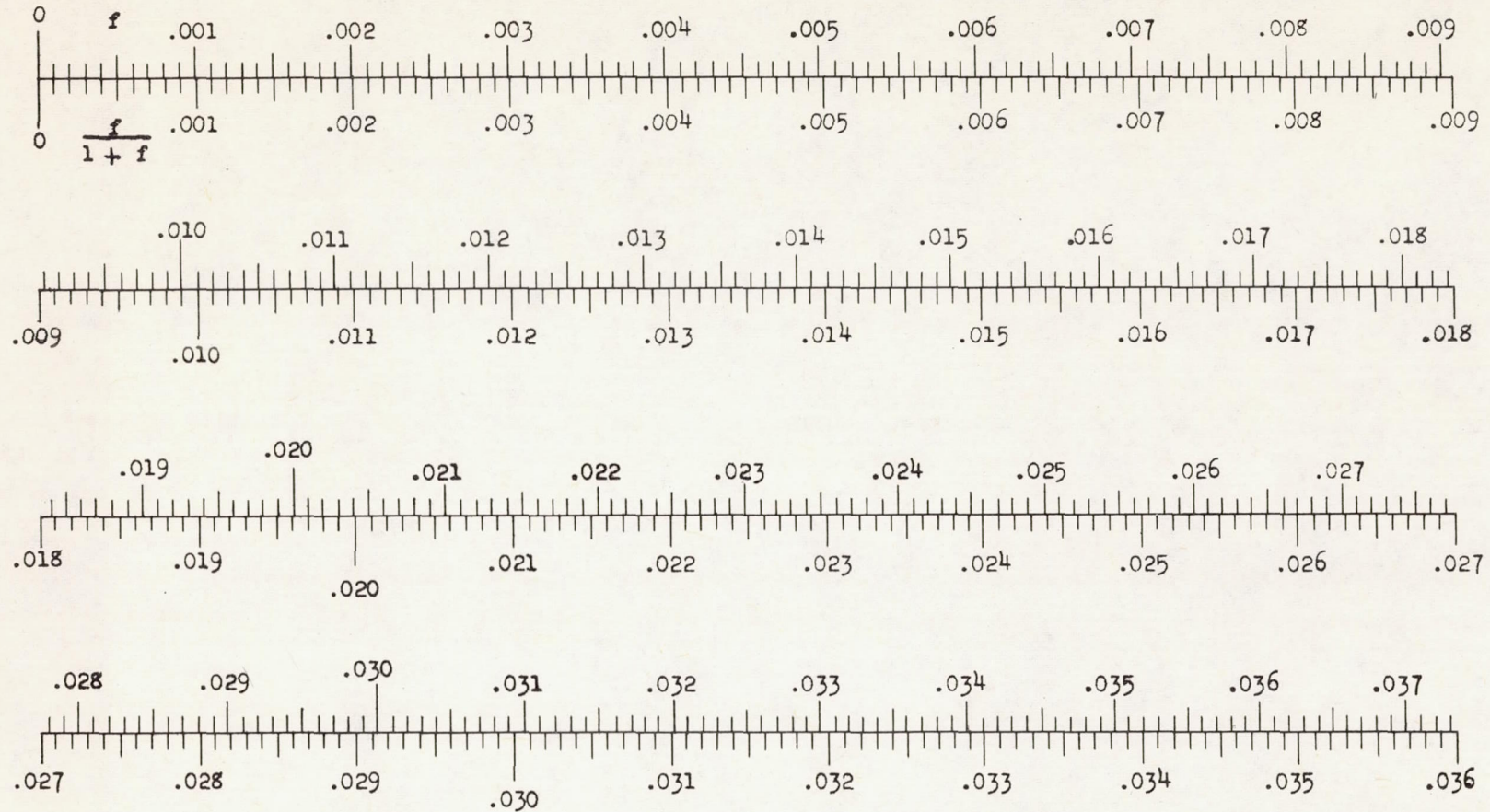(a) Temperature range, 500° to 2500° R.

Chart V. - Water-vapor values of h_w, ϕ_w, and T.

(b) Temperature range, 2500° to 3500° R.

Chart V. - Concluded. Water-vapor values of h_w, ϕ_w, and T.

CHART VI

f

.001 .002 .003 .004 .005 .006 .007 .008 .009

$\dfrac{f}{1+f}$

0   .001 .002 .003 .004 .005 .006 .007 .008 .009

.010 .011 .012 .013 .014 .015 .016 .017 .018

.009 .010 .011 .012 .013 .014 .015 .016 .017 .018

.020
.019 .021 .022 .023 .024 .025 .026 .027

.018 .019 .020 .021 .022 .023 .024 .025 .026 .027

.028 .029 .030 .031 .032 .033 .034 .035 .036 .037

.027 .028 .029 .030 .031 .032 .033 .034 .035 .036

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(a) Fuel-air ratio range, 0 to 0.03734.

Chart VI. - Variation of $\dfrac{f}{1+f}$ with fuel-air ratio f.
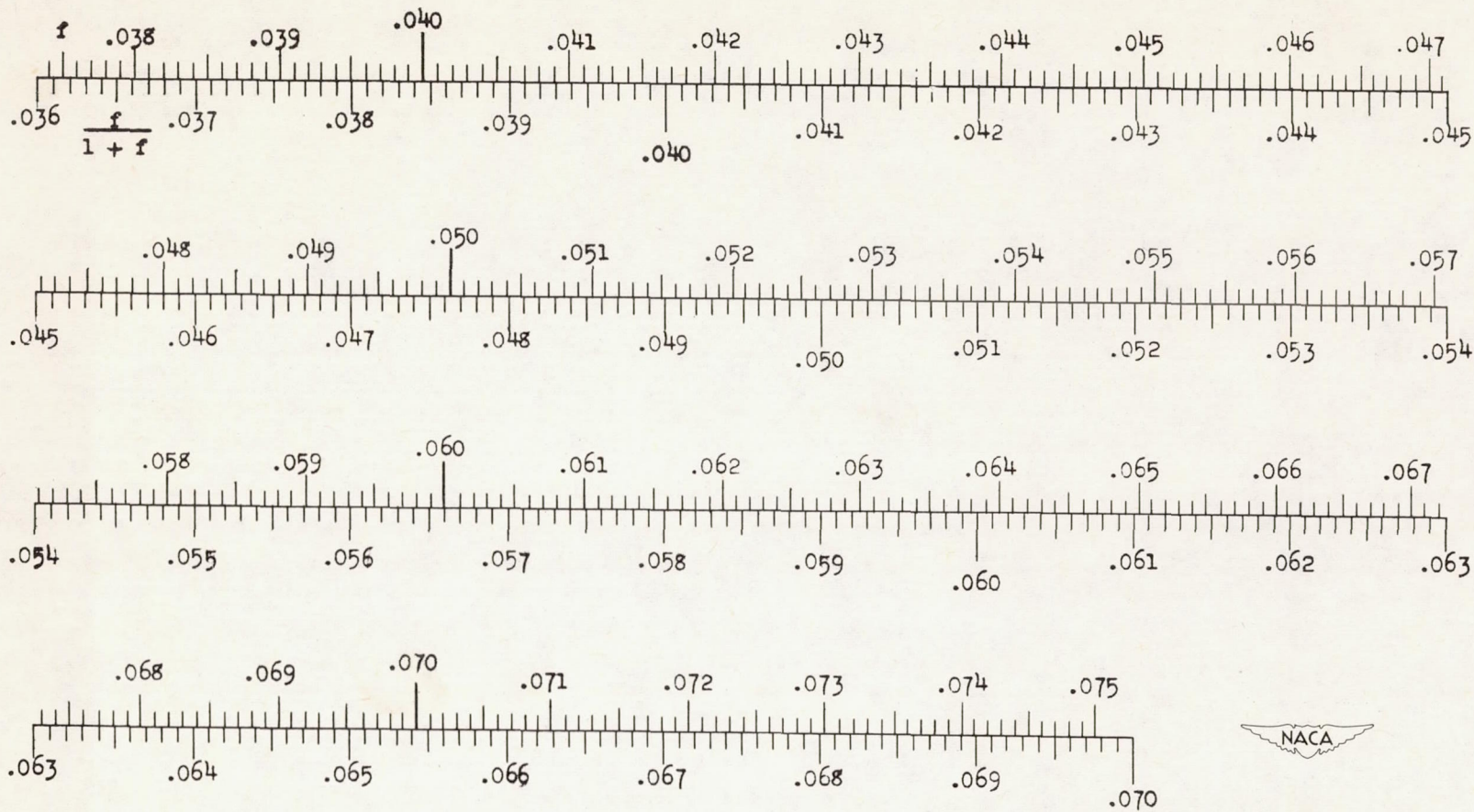
(b) Fuel-air ratio range, 0.03733 to 0.075.

Chart VI. - Concluded. Variation of $\frac{f}{1+f}$ with fuel-air ratio f.