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TECHNICAL NOTE 2125

PRANDTL-MEYER FLOW FOR A DIATOMIC GAS OF VARIABLE SPECIFIC HEAT

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SUMMARY

Tables and charts, which give the results of an analysis that accounts for variation in specific heats of a nonviscous compressible fluid (diatomic gas) during the Prandtl-Meyer flow process (commonly called the flow around a corner), are presented. Comparison is made to the constant-specific-heat solution with the ratio of specific heats $\gamma = 1.4$ and with this ratio corresponding to the total fluid temperature. The comparison showed that variation in specific heats appreciably affected the magnitude of some of the parameters pertinent to this flow but that a close approximation to the variable-specific heat solution could be obtained by the use of a constant value of the ratio of specific heats corresponding to the total fluid temperature.

INTRODUCTION

In the supersonic flow of a fluid from a high- to a relatively low-pressure region, the resultant expansion is accomplished through the medium of expansion waves, which simultaneously turn and diverge the fluid streamlines. Supersonic flow of a fluid around a sharp convex corner of a wall represents one such expansion process. Understanding of this expansion process is of fundamental importance in some supersonic-flow problems inasmuch as the solution for this particular process can be used to describe other expansion processes, such as expansion along the curved boundary of a two-dimensional supersonic wing or supersonic nozzle, or expansion in a free-jet stream such as would exist in underexpanding supersonic nozzles or in the clearance space between turbine nozzles and turbine blades.

The solution of the corner flow of a nonviscous compressible fluid given by Prandtl and Meyer is based on the assumption that the ratio of specific heats remains constant during the expansion. When high fluid temperatures and high expansion pressure ratios are simultaneously involved, however, the variation of specific heats during the process may become appreciable.

In a study made at the NACA Lewis laboratory, the Prandtl-Meyer solution has been extended to the case of a diatomic gas having variable specific heats. The effect of specific-heat lag on the flow process is assumed negligible. The results are presented in convenient tabular and chart form. The flow variables presented are streamline angle, ray angle, Mach angle, local Mach number, pressure ratio, temperature ratio, ratio of local velocity to velocity at a Mach number of unity, and the ratio of the density-velocity product to its value at a Mach number of unity. The range of variables investigated corresponds to streamline angles from 0° to 50°.

SYMBOLS

The following symbols are used in this report:

C_p specific heat at constant pressure

E_i internal energy per unit mass of gas

E_v vibrational energy per unit mass of gas

H total enthalpy per unit mass of gas

k constant resulting from method of removing singularity

M Mach number

M_o initial Mach number

P total pressure

p static pressure

R gas constant

ΔS^o change in dilute-phase entropy

Δs change in total entropy

T absolute total temperature

t absolute static temperature

u velocity in direction of radius vector

V vector velocity having components u and v

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v velocity in direction perpendicular to radius vector

α streamline angle ($\alpha = 0$ for $M = 1$), degrees

α_o streamline angle resulting from hypothetical expansion from $M = 1$ to initial Mach number, degrees

β Mach angle, degrees

γ ratio of specific heats

θ characteristic temperature of molecular vibration

ρ density of gas

τ θ/t

ϕ ray angle ($\phi = 0$ for $M = 1$), degrees

ϕ_o ray angle resulting from hypothetical expansion from $M = 1$ to the initial Mach number, degrees

Superscript:

$*$ conditions in gas when $M = 1$

METHOD OF ANALYSIS

A brief description of the method of analysis of the flow of a nonviscous compressible fluid of variable specific heat around a corner is presented; the details are given in the appendix.

The trace of a streamline in flowing around a corner boundary is schematically shown in figure 1. A gas initially flowing along a wall at a Mach number $M \geqslant 1$ approaches the corner at which an isentropic expansion occurs. The expansion is accomplished through the medium of expansion waves, which emanate from the corner. As a result of the expansion, a streamline will be turned through an angle $\alpha - \alpha_o$ at a Mach line OM, where α_o is the streamline angle resulting from a hypothetical expansion of the uniform stream from a Mach number of unity to the Mach number M_o and α is the streamline angle for an expansion from a Mach number of unity to any

Mach number. The fluid continues to expand until the static pressure in the stream is equal to the ambient pressure or until the flow is parallel to the wall. Associated with the deflection angle $\alpha - \alpha_o$ are the ray angle $\phi - \phi_o$ and the Mach angle β. 1284

The solution of this flow was obtained by simultaneously solving the equations of motion, (equations (A1) and (A2) in appendix), the continuity equation (equation (A3) in appendix), and the energy equation. At sufficiently low temperatures, the energy equation needs to include only the internal energy due to rotation and translation of the molecules, and the kinetic energy and flow energy of the gas stream. As the temperature of a diatomic gas is increased, the energy of vibration of the molecules becomes significant and thus should be included in the energy equation. It is this energy contribution that is reflected as a variation of the specific heats.

By assuming the rotational energy to be that of a rigid rotator, the vibrational energy of a diatomic gas in which the molecules oscillate harmonically was obtained from the kinetic theory of gases (reference 1) by

$$E_v = Rt\left(\frac{\tau}{e^{\tau} - 1}\right) \qquad (1)$$

The internal energy of the gas, including the vibration energy in addition to the translational and rotational energy, was then given by

$$E_i = Rt\left(\frac{5}{2} + \frac{\tau}{e^{\tau} - 1}\right) \qquad (2)$$

and the energy equation for a conservative system thus became

$$H = \frac{V^2}{2} + \left(\frac{7}{2} + \frac{\tau}{e^{\tau} - 1}\right) Rt = \text{constant} \qquad (3)$$

Equation (3), together with the equations of motion and the continuity equation were simultaneously solved to obtain a single differential equation relating the ray angle to the static gas temperature. This equation was integrated by numerical means and from the results other parameters pertinent to the corner flow solution were obtained.

RESULTS OF ANALYSIS

The results of the analysis of the flow of a nonviscous compressible diatomic gas of variable specific heat around a corner for two different ratios of total fluid temperature to characteristic temperature of molecular vibration T/θ are shown in table I. Shown in the table as functions of the streamline angle α (fig. 1) are the ray angle ϕ, the Mach angle β, the ratio of static pressure to total pressure p/P, the local Mach number M, the ratio of fluid velocity to its value at a Mach number of unity V/V^*, the ratio of static temperature to total temperature t/T, and the mass-flow ratio $\rho V/\rho^* V^*$. The ratio $\rho V/\rho^* V^*$ is the ratio of the product of the density and fluid velocity to the value of this product at a Mach number of unity.

In figure 2 the calculated results, given in table I, are compared with results obtained by the conventional constant-specific-heat solution where $\gamma = 1.4$, as obtained from reference 2. These data are tabulated in table II. The error that results from use of the conventional constant-specific-heat computations (fig. 2) can be appreciable at high temperatures. The value $\gamma = 1.4$, however, is known to be incorrect at these temperatures. Use of a constant value of γ corresponding to the total temperature of the fluid considerably reduces the error. This result is shown in figure 3, where the percentage error resulting from use of $\gamma = 1.303$ (which corresponds to $T/\theta = 0.6$ (see equation (A26) of appendix)) is plotted against streamline angle α for each of the flow parameters considered. (Percentage error, divided by 100, is the value of a parameter computed for $\gamma = 1.303$ minus the value of the parameter computed for a variable ratio of specific heat all divided by the value of the parameter computed for the γ variable.) For a streamline angle of 40°, the pressure ratio and the temperature ratio are in error by approximately 3 percent; the remaining flow parameters are in error by 0.5 percent or less (fig. 3). Smaller errors in all parameters would be involved for values of T/θ less than the value of 0.6, which was used in the comparison of figure 3.

In order to use the results of the variable-specific-heat analysis in a practical problem, the characteristic temperature of gas vibration must be known. This temperature is a constant only for a single diatomic gas. For air and lean exhaust-gas mixtures, however, the variation of θ with exhaust-gas temperature is so small that an average value of θ may be chosen that will closely fit experimental specific-heat data in the region of interest. Values of θ (obtained by substituting experimental specific-heat data in equation (A26) in appendix) recommended for use in the tables and charts for air, for products of combustion with 400 percent of theoretical air, and for products of combustion with 200 percent of theoretical air are 5450° R, 4500° R and 3900° R, respectively.

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SUMMARY OF RESULTS

The results of calculation of the flow of a nonviscous compressible diatomic gas around a corner showed that the effect of variation in specific heat on the magnitude of parameters pertinent to this flow is appreciable at high temperatures and high expansion pressure ratios. An analysis showed that in the absence of the variable-specific-heat tabulations presented very close approximations to the true value of the parameters can be calculated when a constant specific-heat value corresponding to the total fluid temperature is assumed.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, November 15, 1949.

APPENDIX - DETAILS OF ANALYSIS

For the steady nonviscous flow of a compressible fluid around a corner, all derivatives with respect to the radius vector are zero. The equations of motion and the continuity equation are thus (fig. 1):

$$\frac{du}{d\phi} - v = 0 \tag{A1}$$

$$v\frac{dv}{d\phi} + uv = -\frac{1}{\rho}\frac{dp}{d\phi} \tag{A2}$$

$$\rho u + \frac{d}{d\phi}(\rho v) = 0 \tag{A3}$$

By assuming the rotational energy of a diatomic gas to be that for a rigid rotator and the vibrational energy to be that for a harmonic oscillator, the vibrational energy is

$$E_v = Rt\left(\frac{\tau}{e^{\tau} - 1}\right) \tag{1}$$

With this relation, the internal energy is then given by

$$E_i = Rt\left(\frac{5}{2} + \frac{\tau}{e^{\tau} - 1}\right) \tag{2}$$

and the total enthalpy by

$$H = \frac{u^2 + v^2}{2} + E_i + Rt = \text{constant} \tag{A4}$$

Substitution of equation (A1), that is, $v = du/d\phi$, into equations (A2), (A3), and (A4) results in the following equations:

$$\frac{du}{d\phi}\left(\frac{d^2u}{d\phi^2}\right) + u\frac{du}{d\phi} = -\frac{1}{\rho}\frac{dp}{d\phi} \tag{A5}$$

$$\rho u + \rho\frac{d^2u}{d\phi^2} + \frac{du}{d\phi}\left(\frac{d\rho}{d\phi}\right) = 0 \tag{A6}$$

$$u^2 + \left(\frac{du}{d\phi}\right)^2 + 2(E_i + Rt) = 2H \tag{A7}$$

Because for a perfect gas $p = \rho Rt$, equation (A5) may be expanded in the form

$$\frac{du}{d\phi}\left(\frac{d^2u}{d\phi^2}\right) + u\frac{du}{d\phi} = - Rt\left(\frac{d\ \log_e\rho}{d\phi} + \frac{d\ \log_e t}{d\phi}\right) \tag{A8}$$

From equation (A6)

$$\frac{d\ \log_e\rho}{d\phi} = -\left(\frac{u + \frac{d^2u}{d\phi^2}}{\frac{du}{d\phi}}\right) \tag{A9}$$

Hence equation (A8) becomes

$$\left[\left(\frac{du}{d\phi}\right)^2 - Rt\right]\left(u + \frac{d^2u}{d\phi^2}\right) + R\left(\frac{du}{d\phi}\right)\left(\frac{dt}{d\phi}\right) = 0 \tag{A10}$$

From equation (A7) after differentiating with respect to ϕ

$$u + \frac{d^2u}{d\phi^2} = -\left[\frac{\frac{dE_i}{dt}\left(\frac{dt}{d\phi}\right) + R\frac{dt}{d\phi}}{\frac{du}{d\phi}}\right] \tag{A11}$$

From equations (A10) and (A11),

$$\left[\left(\frac{du}{d\phi}\right)^2 - Rt\right]\left(\frac{dE_i}{dt} + R\right) - R\left(\frac{du}{d\phi}\right)^2 = 0 \tag{A12}$$

and thus

$$\frac{du}{d\phi} = \sqrt{\frac{Rt}{dE_i/dt}\left(\frac{dE_i}{dt} + R\right)} \tag{A13}$$

By substituting equation (A13) in equation (A7)

$$u = \sqrt{2(H - E_i - Rt) - \frac{Rt}{dE_i/dt}\left(\frac{dE_i}{dt} + R\right)} \tag{A14}$$

The variable u and its derivatives may now be substituted into equation (A10) so as to give a single equation involving t and ϕ. This equation is of the form

$$-\tau^2\left(\frac{d\phi}{d\tau}\right) = \frac{A-B}{D} \tag{A15}$$

where

$$A = \frac{e^\tau}{4}\left(\frac{2\tau}{e^\tau - 1}\right)^2\left(\tau - 2 + \frac{2\tau}{e^\tau - 1}\right) \tag{A16}$$

$$B = 2\left[\frac{7}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2}\right]\left[\frac{6}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2}\right]\left[\frac{5}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2}\right] \tag{A17}$$

$$D = 2\left[\frac{5}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2}\right]\sqrt{\frac{2}{\tau}\left(\frac{H}{R\theta} - \frac{7}{2\tau} - \frac{1}{e^\tau - 1}\right)\left[\frac{5}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2}\right]\left[\frac{7}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2}\right] - \frac{1}{\tau^2}\left[\frac{7}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2}\right]^2} \tag{A18}$$

When $\tau = \tau^*$, which for a given θ corresponds to the value of the static temperature in the gas when the local Mach number is unity, the denominator term D is zero. This singularity was investigated and found to be a pole of order minus one half. It was removed by assuming that

$$\frac{d\phi}{d\tau} = \frac{k}{\sqrt{\tau - \tau^*}} + X(\tau) = \frac{1}{\tau^2}\left(\frac{B}{D} - \frac{A}{D}\right) \tag{A19}$$

where $X(\tau)$ is everywhere finite and has finite derivatives and $X(\tau^*) = 0$. The constant k was computed by evaluating, for each value of $\frac{H}{R\theta}$, the expression

$$k = \left(\frac{B - A}{\tau^2}\right) \lim_{\tau \to \tau^*} \frac{\sqrt{\tau - \tau^*}}{D} \tag{A20}$$

In the integration of equation (A19), seven point formulas obtained from reference 3 were used everywhere except near the region of the singularity. Near the singularity, the function X was approximated by the form

$$\frac{X}{\sqrt{\tau - \tau^*}} = C_1\ (\tau - \tau^*) + C_2\ (\tau - \tau^*)^2 + C_3\ (\tau - \tau^*)^3 + C_4\ (\tau - \tau^*)^4 + C_5\ (\tau - \tau^*)^5 \tag{A21}$$

where C is a constant.

The total enthalpy was computed from the total gas temperature by the relation

$$\frac{H}{R\theta} = \frac{7}{2}\left(\frac{T}{\theta}\right) + \frac{1}{e^{\theta/T} - 1} \tag{A22}$$

For an isentropic flow

$$\Delta s = \int_{t^*}^{t} \frac{C_p}{t}\, dt - R \log_e \frac{p}{p^*} = \Delta S^o - R \log_e \frac{p}{p^*} = 0 \tag{A23}$$

where

$$\frac{\Delta S^o}{R} = \frac{\tau}{e^{\tau} - 1} - \frac{\tau^*}{e^{\tau^*} - 1} - \log_e \frac{\left(\frac{e^{\tau} - 1}{e^{\tau}}\right)}{\left(\frac{e^{\tau^*} - 1}{e^{\tau^*}}\right)} + \frac{7}{2} \log_e \frac{\tau^*}{\tau} \tag{A24}$$

The ratio of static pressure to initial static pressure at a Mach number of unity was obtained from the relation

$$\frac{p}{p^*} = e^{\frac{\Delta S^o}{R}} \tag{A25}$$

The specific heat at constant pressure was given by

$$\frac{C_p}{R} = \frac{\gamma}{\gamma-1} = \frac{1}{R}\frac{d}{dt}\left(E_i' + Rt\right) = \frac{7}{2} + \frac{\tau^2 e^{\tau}}{\left(e^{\tau}-1\right)^2} \tag{A26}$$

After the pressure, the temperature, and the specific heat, are obtained other variables pertinent to the flow can be determined from fundamental thermodynamic relations.

REFERENCES

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2. Ferri, Antonio: Elements of Aerodynamics of Supersonic Flows. The Macmillan Co., 1949, pp. 399-400.

3. Bickley, W. G.: Formulae for Numerical Integration. The Mathematical Gazette, vol. XXIII, no. 256, Oct. 1939, pp. 352-359.

TABLE I - CORNER-FLOW PARAMETERS WITH VARIABLE SPECIFIC HEAT

(a) $T/\theta = 0.3$.

α (deg)	∅ (deg)	β (deg)	p/P	M	V/V*	t/T	ρV/ρ*V*
0	0	90.00	0.540	1.000	1.000	0.851	1.000
1	23.21	67.79	.491	1.080	1.067	.830	.995
2	29.73	62.27	.462	1.130	1.108	.817	.987
3	34.50	58.50	.437	1.173	1.142	.805	.978
4	38.44	55.56	.415	1.213	1.174	.794	.966
5	41.88	53.12	.394	1.250	1.202	.783	.954
6	44.98	51.02	.376	1.286	1.230	.773	.941
7	47.84	49.16	.358	1.322	1.256	.763	.927
8	50.48	47.52	.341	1.356	1.280	.754	.913
9	52.97	46.03	.325	1.390	1.304	.744	.898
10	55.35	44.65	.310	1.423	1.328	.734	.882
11	57.61	43.39	.296	1.456	1.350	.725	.867
12	59.79	42.21	.282	1.488	1.372	.716	.851
13	61.90	41.10	.269	1.521	1.394	.707	.834
14	63.93	40.07	.256	1.554	1.415	.698	.817
15	65.91	39.09	.244	1.586	1.435	.688	.800
16	67.84	38.16	.232	1.618	1.455	.679	.783
17	69.72	37.28	.221	1.651	1.475	.670	.766
18	71.56	36.44	.210	1.684	1.495	.661	.748
19	73.36	35.64	.200	1.716	1.514	.652	.730
20	75.13	34.87	.190	1.749	1.533	.643	.712
21	76.86	34.14	.180	1.782	1.551	.634	.694
22	78.57	33.43	.171	1.815	1.569	.625	.677
23	80.25	32.75	.162	1.849	1.587	.616	.659
24	81.91	32.09	.154	1.882	1.605	.607	.641
25	83.55	31.45	.146	1.916	1.622	.598	.623
26	85.16	30.84	.138	1.951	1.640	.589	.605
27	86.76	30.24	.131	1.985	1.656	.580	.588
28	88.33	29.67	.124	2.020	1.673	.571	.570
29	89.89	29.11	.117	2.056	1.690	.562	.553
30	91.44	28.56	.110	2.092	1.706	.553	.536
31	92.97	28.03	.104	2.128	1.722	.544	.518
32	94.48	27.52	.098	2.164	1.738	.536	.501
33	95.99	27.01	.092	2.202	1.754	.527	.485
34	97.48	26.52	.087	2.239	1.769	.518	.468
35	98.96	26.04	.082	2.278	1.785	.509	.452
36	100.42	25.58	.077	2.316	1.800	.500	.436
37	101.88	25.12	.072	2.356	1.815	.492	.420
38	103.33	24.67	.068	2.396	1.829	.483	.404
39	104.77	24.23	.064	2.436	1.844	.474	.390
40	106.19	23.81	.060	2.478	1.858	.466	.375
41	107.62	23.38	.056	2.520	1.873	.457	.360
42	109.03	22.97	.052	2.562	1.887	.448	.345
43	110.43	22.57	.049	2.606	1.900	.440	.332
44	111.83	22.17	.045	2.650	1.914	.431	.317
45	113.22	21.78	.042	2.695	1.928	.423	.304
46	114.60	21.40	.039	2.741	1.941	.414	.291
47	115.98	21.02	.037	2.787	1.954	.406	.278
48	117.35	20.65	.034	2.835	1.967	.397	.266
49	118.71	20.29	.032	2.884	1.980	.389	.253
50	120.07	19.93	.029	2.933	1.993	.381	.242

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TABLE I - CORNER-FLOW PARAMETERS WITH VARIABLE SPECIFIC HEAT - Concluded

(b) $T/\theta = 0.6$.

α (deg)	ϕ (deg)	β (deg)	p/P	M	V/V^*	t/T	$\rho V/\rho^* V^*$
0	0	90.00	0.546	1.000	1.000	0.867	1.000
1	23.09	67.91	.498	1.079	1.068	.848	.995
2	29.57	62.43	.469	1.128	1.109	.836	.987
3	34.32	58.68	.445	1.171	1.144	.826	.978
4	38.24	55.76	.422	1.210	1.176	.816	.966
5	41.68	53.32	.402	1.248	1.205	.806	.954
6	45.03	50.97	.384	1.283	1.232	.798	.941
7	47.65	49.35	.367	1.316	1.258	.789	.928
8	50.47	47.53	.350	1.350	1.284	.780	.913
9	53.03	45.97	.334	1.384	1.308	.772	.898
10	55.26	44.74	.319	1.416	1.331	.763	.883
11	57.29	43.71	.305	1.447	1.353	.755	.868
12	59.50	42.50	.291	1.479	1.376	.747	.852
13	61.60	41.40	.278	1.510	1.398	.739	.835
14	63.59	40.41	.266	1.541	1.419	.730	.819
15	65.51	39.49	.254	1.572	1.440	.722	.802
16	67.42	38.58	.242	1.604	1.460	.714	.785
17	69.29	37.71	.231	1.634	1.480	.706	.768
18	71.11	36.89	.220	1.666	1.500	.698	.750
19	72.89	36.11	.210	1.697	1.520	.690	.733
20	74.64	35.36	.200	1.728	1.539	.682	.715
21	76.35	34.65	.190	1.759	1.558	.674	.698
22	78.04	33.96	.181	1.790	1.576	.666	.680
23	79.71	33.29	.172	1.822	1.595	.658	.662
24	81.35	32.65	.164	1.853	1.613	.650	.645
25	82.96	32.04	.156	1.885	1.631	.641	.628
26	84.56	31.44	.148	1.917	1.649	.633	.610
27	86.14	30.86	.140	1.949	1.666	.625	.592
28	87.70	30.30	.133	1.982	1.683	.617	.575
29	89.24	29.76	.126	2.014	1.700	.609	.558
30	90.76	29.24	.119	2.047	1.717	.601	.542
31	92.27	28.73	.113	2.081	1.734	.593	.524
32	93.77	28.23	.107	2.114	1.751	.585	.508
33	95.25	27.75	.101	2.148	1.767	.577	.492
34	96.72	27.28	.096	2.182	1.783	.569	.475
35	98.18	26.82	.090	2.217	1.799	.561	.459
36	99.63	26.37	.085	2.252	1.815	.552	.444
37	101.07	25.93	.080	2.287	1.830	.544	.428
38	102.50	25.50	.076	2.322	1.846	.536	.413
39	103.91	25.09	.071	2.359	1.861	.528	.398
40	105.32	24.68	.067	2.395	1.876	.520	.383
41	106.72	24.28	.063	2.432	1.891	.512	.369
42	108.11	23.89	.059	2.470	1.906	.504	.355
43	109.50	23.50	.055	2.508	1.921	.496	.341
44	110.88	23.12	.052	2.546	1.935	.488	.328
45	112.25	22.75	.049	2.586	1.950	.480	.314
46	113.61	22.39	.046	2.625	1.964	.472	.301
47	114.97	22.03	.043	2.666	1.979	.463	.289
48	116.32	21.68	.040	2.707	1.992	.455	.276
49	117.66	21.34	.037	2.748	2.005	.447	.264
50	119.00	21.00	.035	2.791	2.019	.439	.252

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TABLE II - CORNER-FLOW PARAMETERS FOR γ = 1.4

α (deg)	ϕ (deg)	β (deg)	p/P	M	V/V*	t/T	$\rho V/\rho^* V^*$
0	0	90.00	0.528	1.000	1.000	0.833	1.000
1	23.38	67.62	.479	1.081	1.066	.810	.995
2	30.02	61.98	.450	1.133	1.107	.796	.986
3	34.78	58.22	.425	1.176	1.140	.783	.977
4	38.85	55.15	.402	1.218	1.172	.771	.965
5	42.35	52.65	.382	1.258	1.201	.760	.953
6	45.38	50.62	.364	1.294	1.227	.749	.940
7	48.25	48.75	.346	1.330	1.252	.739	.926
8	50.90	47.10	.330	1.365	1.276	.729	.912
9	53.42	45.58	.314	1.400	1.300	.718	.897
10	55.83	44.17	.299	1.435	1.323	.708	.881
11	58.10	42.90	.285	1.469	1.345	.698	.865
12	60.30	41.70	.271	1.503	1.367	.689	.849
13	62.40	40.60	.258	1.537	1.387	.679	.832
14	64.45	39.55	.246	1.571	1.408	.670	.815
15	66.45	38.55	.234	1.605	1.428	.660	.798
16	68.38	37.62	.222	1.638	1.448	.651	.780
17	70.27	36.73	.211	1.672	1.467	.641	.762
18	72.12	35.88	.201	1.706	1.486	.632	.744
19	73.93	35.07	.190	1.741	1.505	.623	.726
20	75.73	34.27	.181	1.775	1.523	.613	.708
21	77.47	33.53	.172	1.810	1.541	.604	.690
22	79.18	32.82	.162	1.845	1.559	.595	.672
23	80.87	32.13	.154	1.880	1.576	.586	.654
24	82.52	31.48	.146	1.915	1.593	.577	.635
25	84.17	30.83	.138	1.950	1.610	.568	.617
26	85.77	30.23	.131	1.986	1.627	.559	.600
27	87.38	29.62	.123	2.024	1.644	.550	.581
28	88.97	29.03	.116	2.060	1.660	.541	.563
29	90.52	28.48	.110	2.097	1.676	.532	.546
30	92.03	27.97	.104	2.132	1.690	.524	.530
31	93.60	27.40	.098	2.173	1.707	.514	.511
32	95.10	26.90	.092	2.210	1.722	.506	.494
33	96.60	26.40	.087	2.248	1.737	.497	.478
34	98.08	25.92	.081	2.288	1.752	.488	.461
35	99.58	25.42	.076	2.329	1.767	.480	.444
36	101.03	24.97	.072	2.369	1.781	.471	.428
37	102.50	24.50	.067	2.411	1.796	.462	.412
38	103.93	24.07	.063	2.452	1.810	.454	.397
39	105.38	23.62	.059	2.495	1.824	.446	.381
40	106.80	23.20	.055	2.537	1.838	.437	.366
41	108.20	22.80	.052	2.581	1.851	.429	.352
42	109.62	22.38	.048	2.626	1.865	.420	.337
43	111.02	21.98	.045	2.671	1.878	.412	.323
44	112.40	21.60	.042	2.718	1.892	.404	.309
45	113.78	21.22	.039	2.765	1.905	.395	.295
46	115.17	20.83	.036	2.812	1.917	.387	.282
47	116.53	20.47	.034	2.860	1.930	.379	.270
48	117.92	20.08	.031	2.911	1.943	.371	.257
49	119.27	19.73	.029	2.961	1.955	.363	.245
50	120.62	19.38	.027	3.012	1.967	.355	.234

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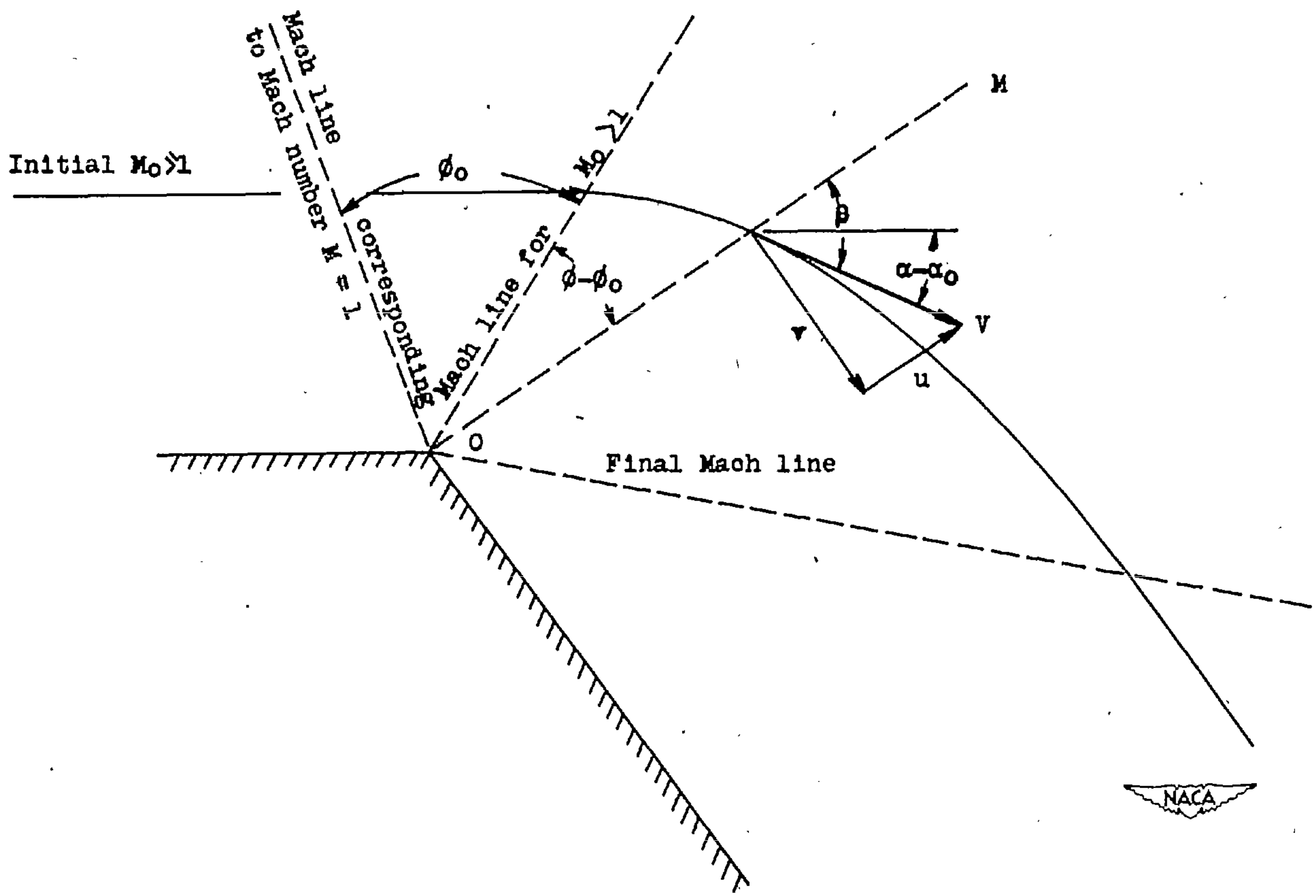
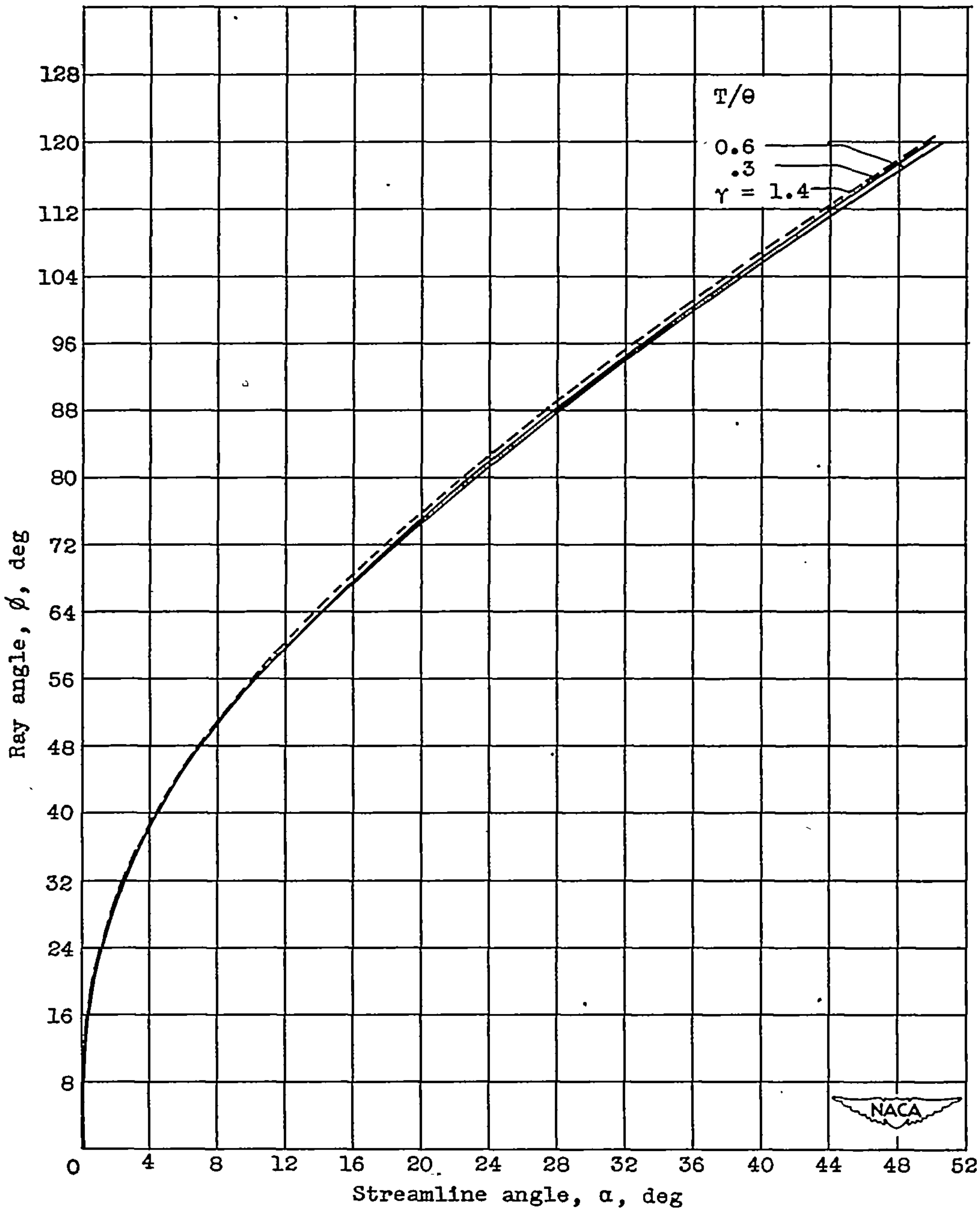
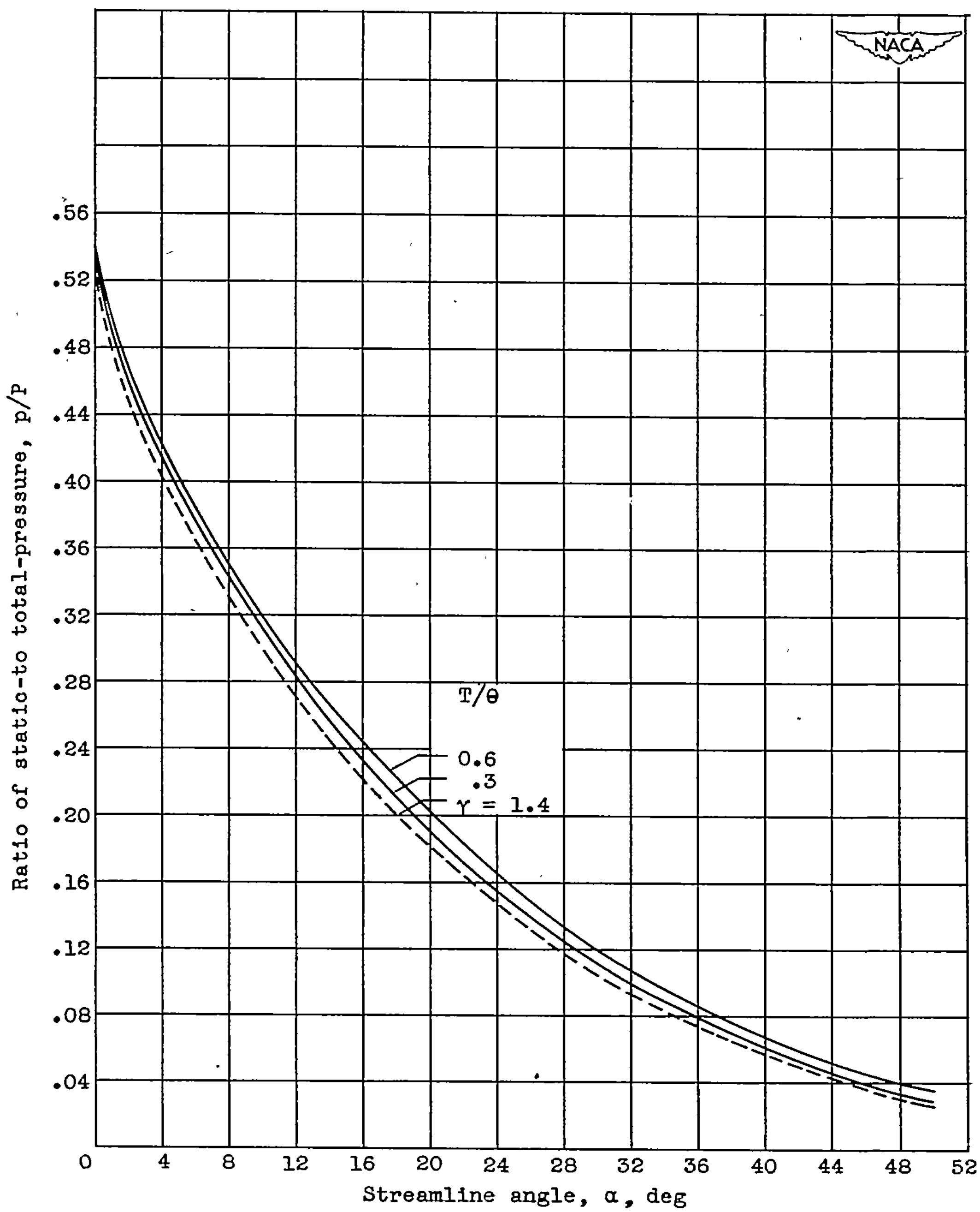


Figure 1. - Schematic representation of supersonic flow around a corner.



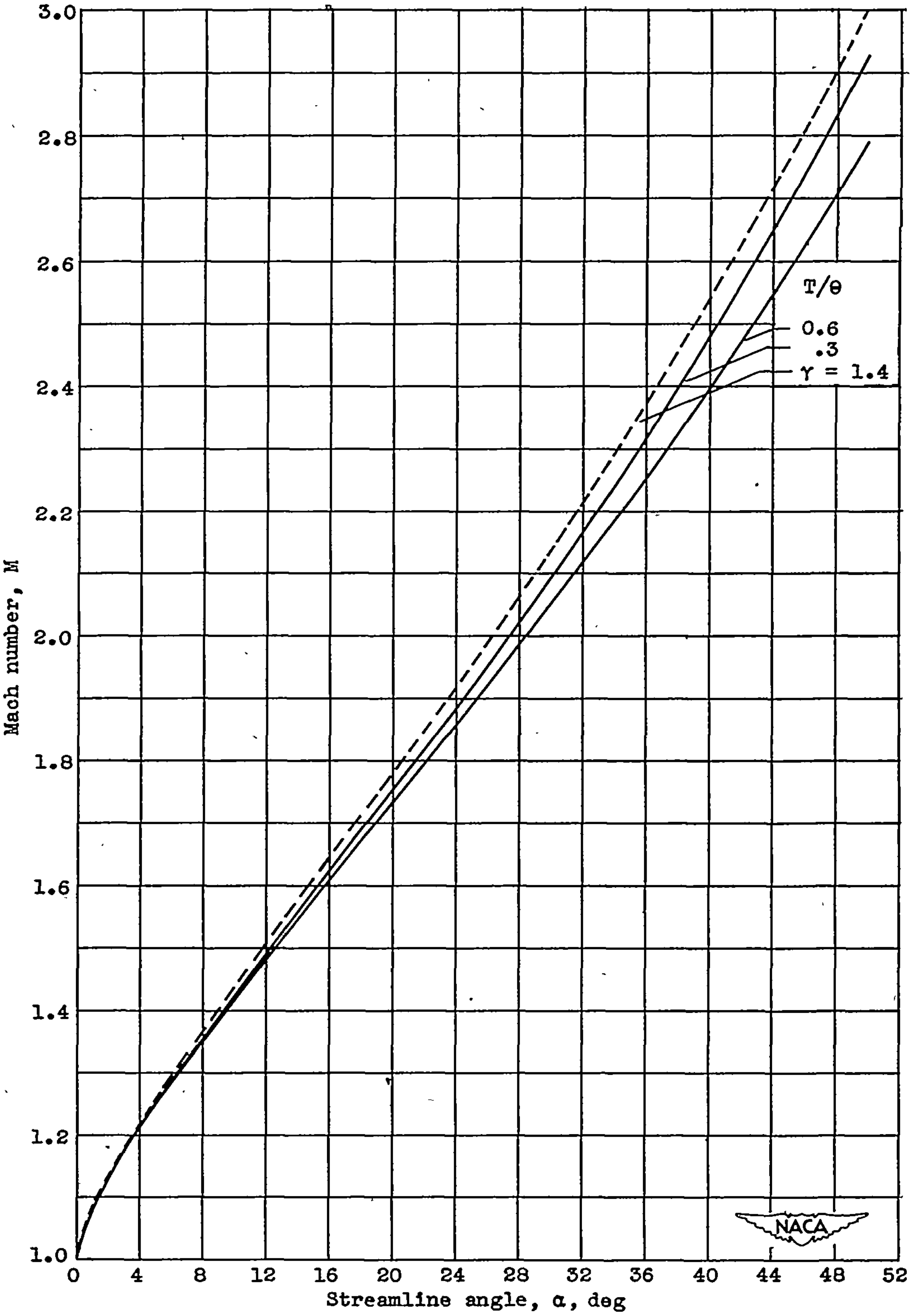
(a) Ray angle ϕ.

Figure 2. - Variation of corner-flow parameters with streamline angle. (Data for curve of $\gamma = 1.4$ is taken from reference 2.)



(b) Ratio of static to total pressure p/P.

Figure 2. - Continued. Variation of corner-flow parameters with streamline angle. (Data for curve of $\gamma = 1.4$ is taken from reference 2.)



(c) Mach number M.

Figure 2. - Continued. Variation of corner-flow parameters with streamline angle. (Data for curve of $\gamma = 1.4$ is taken from reference 2.)

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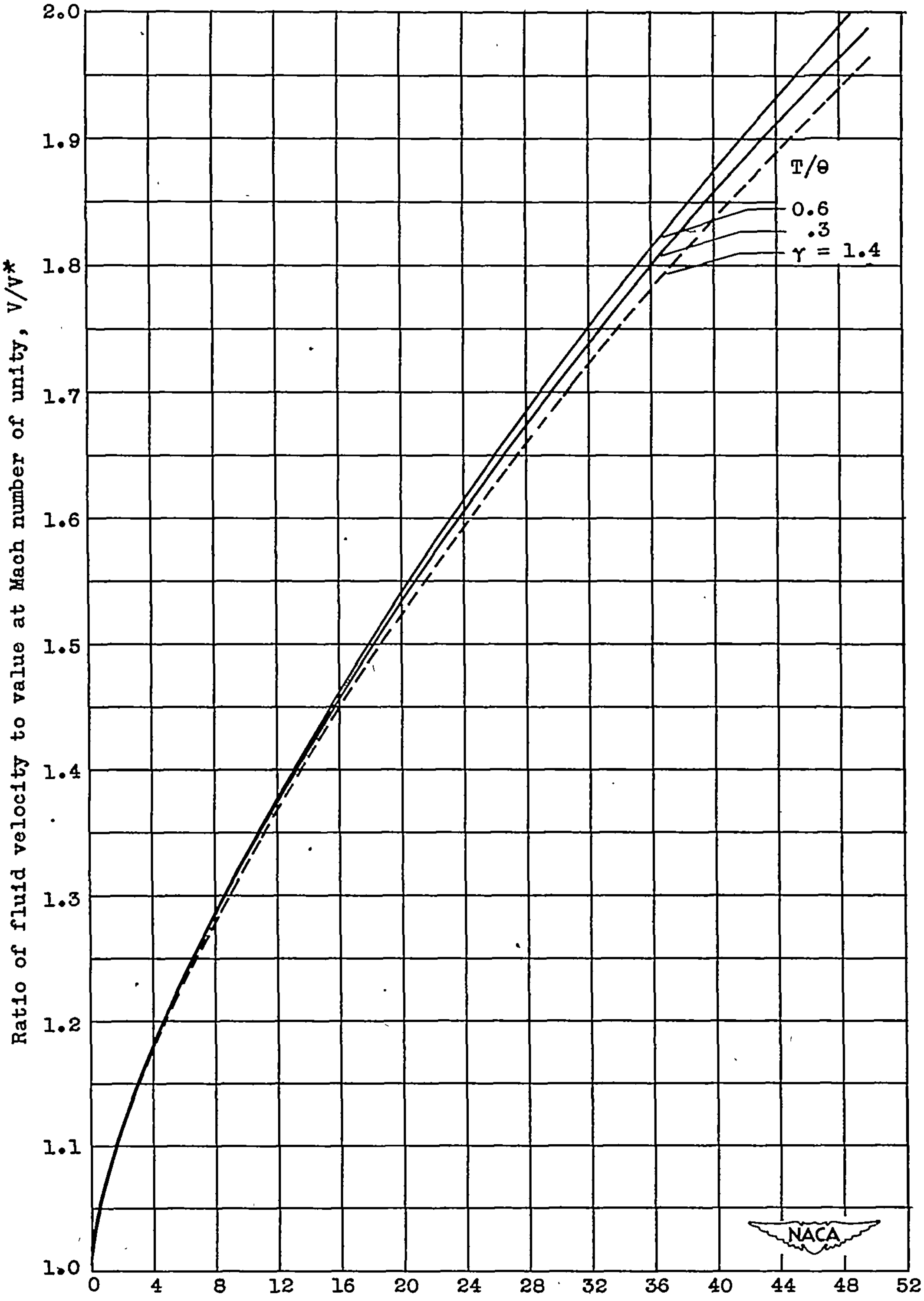

(d) Ratio of fluid velocity to initial value at Mach number of unity V/V*.

Figure 2. - Continued. Variation of corner-flow parameters with streamline angle. (Data for curve of $\gamma = 1.4$ is taken from reference 2.)

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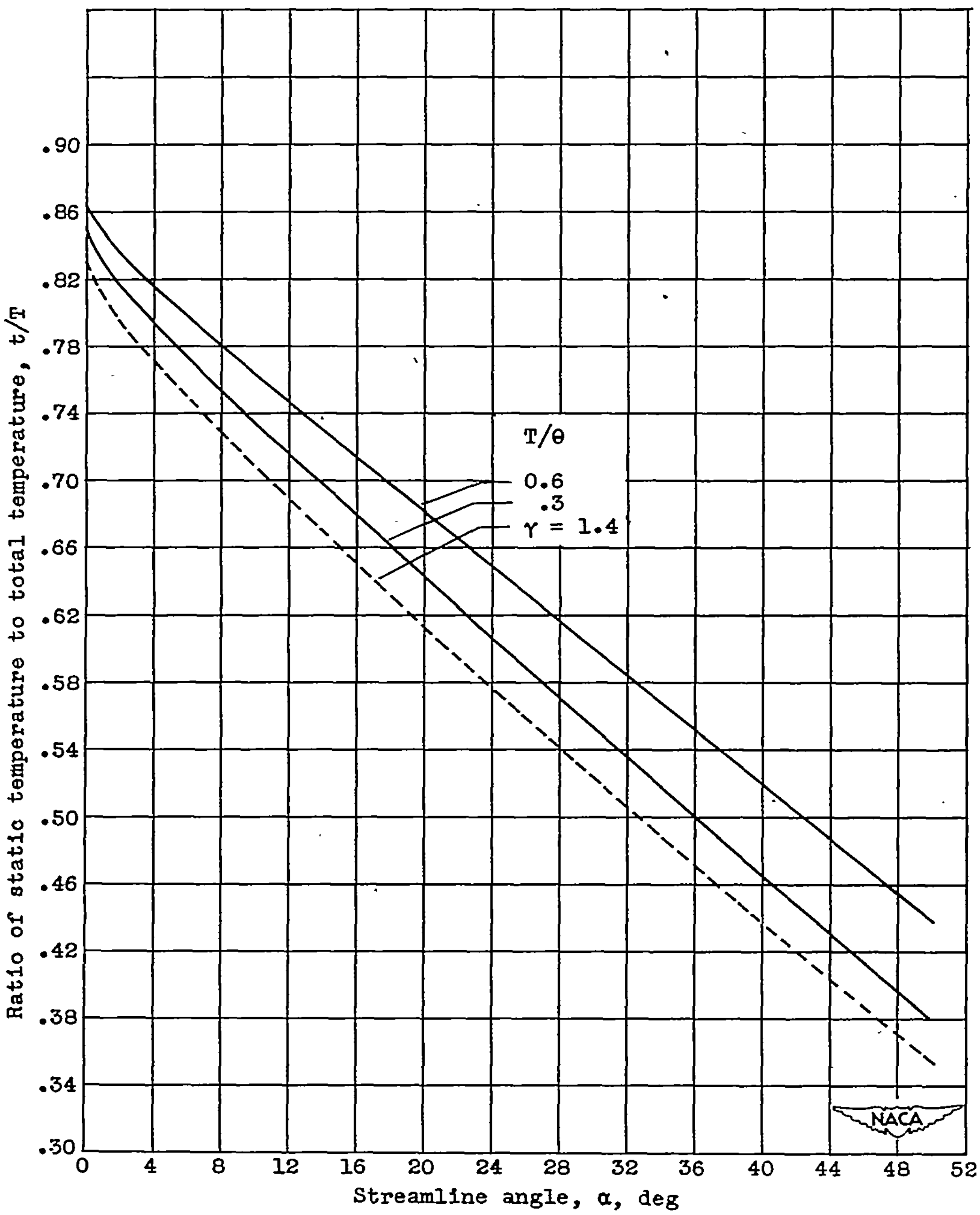
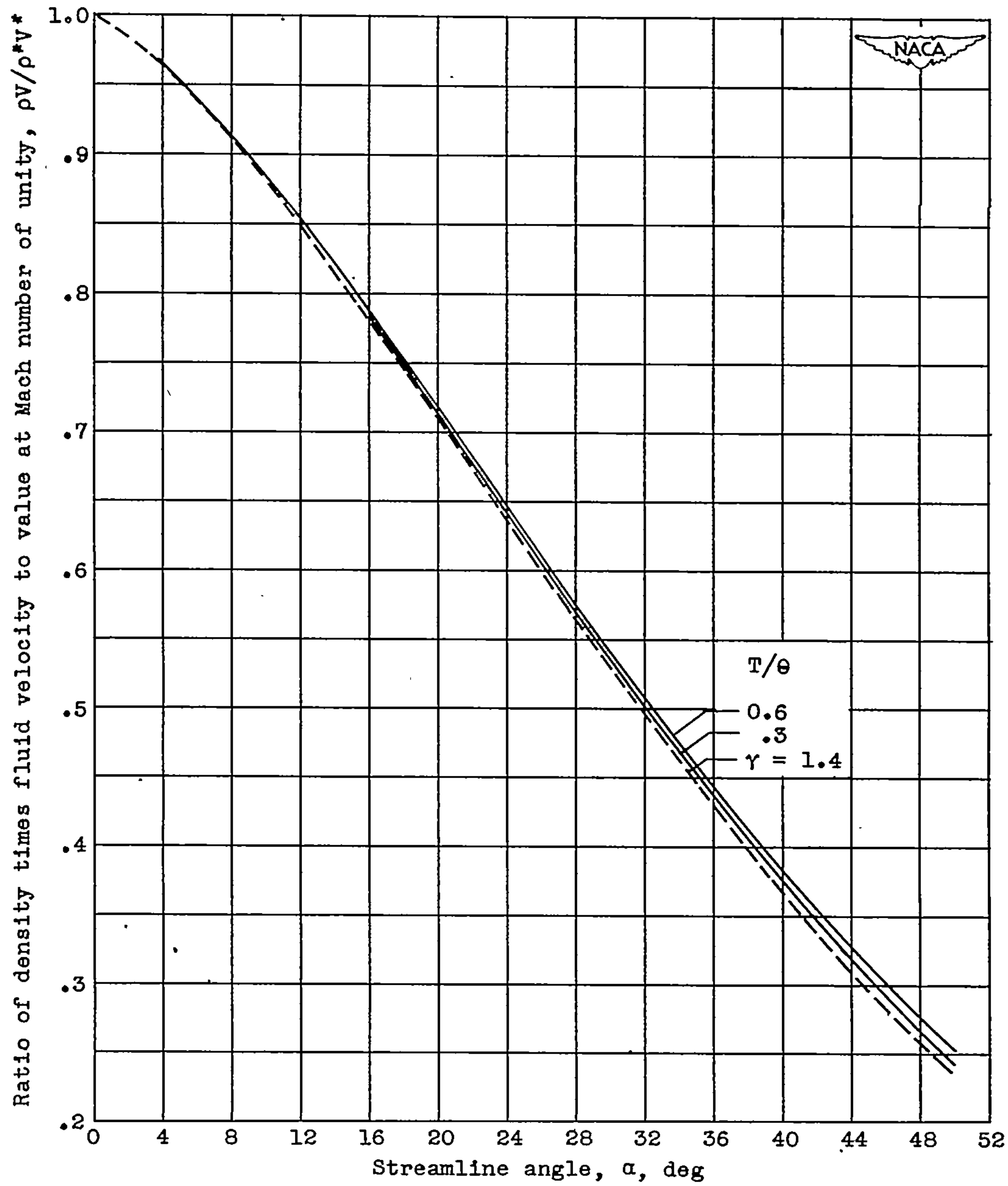


(e) Ratio of static temperature to total temperature t/T.

Figure 2. - Continued. Variation of corner-flow parameters with streamline angle. (Data for curve of $\gamma = 1.4$ is taken from reference 2.)



(f) Ratio of density times fluid velocity to value at Mach number of unity $\rho V/\rho^* V^*$.

Figure 2. - Concluded. Variation of corner-flow parameters with streamline angle. (Data for curve of $\gamma = 1.4$ is taken from reference 2.)

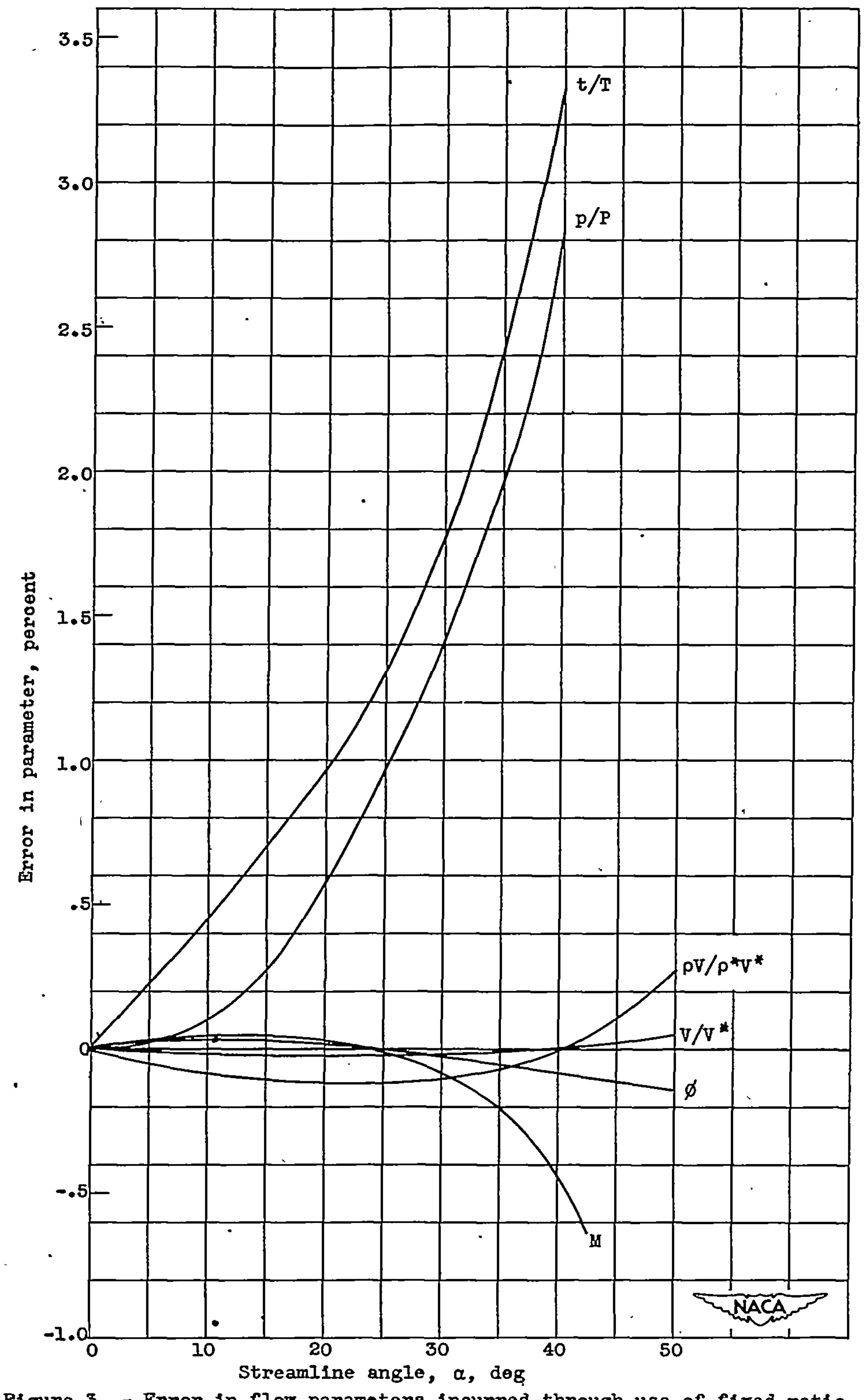


Figure 3. - Error in flow parameters incurred through use of fixed ratio of specific heats ($\gamma = 1.303$) evaluated at total temperature of fluid ($T/\theta = 0.6$).