NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2129

METHOD OF CALCULATING THE LATERAL MOTIONS OF AIRCRAFT BASED ON THE LAPLACE TRANSFORM

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Washington
July 1950

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SUMMARY

The lateral motions of aircraft are obtained by means of the Laplace transform which gives solutions expressed in terms of elementary functions for the free and forced motions. These equations permit the calculation of the free motion of an aircraft following any initial condition or the forced motion following the application of constant external forces and moments. These forced motions can be used to obtain by means of Duhamel's integral the response to any arbitrary forcing function. All the classical stability concepts can be deduced from these same solution equations largely by inspection. These equations for the lateral motion are applied to the calculation of the lateral stability of a specific airplane and to the calculation of certain of its free and forced motions.

INTRODUCTION

The lateral motions of aircraft are represented by three simultaneous differential equations which are generally assumed to be linear. The fundamental problem of lateral dynamics involves the solution of these differential equations in terms of the aerodynamic and mass parameters of the airplane. The solutions can then be used to obtain numerically the motion of the airplane as a function of time.

The recent application of the Laplace transform to the solution of systems of linear differential equations permits a more general analysis of the problem of airplane motion than that of reference 1, which is based upon Heaviside's operational calculus. Heaviside's operational calculus permits a calculation of the forced motion, which is the motion following the application of external forces and moments. The Laplace transform permits these same calculations and also permits the direct calculation of the free motion, which is the motion following finite initial values of the variables and their first derivatives in the absence of externally applied forces and moments. This calculation cannot be made by use of Heaviside's operational calculus. The Laplace transform solutions, which include both the free and forced motions, may be written in a closed form from which all the classical stability concepts can be deduced largely by inspection. The form of the equations of motions of the airplane is independent of such aerodynamic parameters as Reynolds number and Mach number, and these parameters enter the equations only as they effect the values of the aerodynamic constants or stability derivatives appearing in the equations. The values of the stability derivatives must be obtained by actual measurements during physical tests or from aerodynamic theory before motion calculations can be attempted.

Investigations of some of the possibilities of applying the Laplace transform to the study of aircraft motion have been reported in references 2 and 3, and in two British reports, one by K. Mitchell, the other by J. Watham and E. Priestley. The British papers do not give final equations in a form suitable for calculation purposes. The analysis of reference 2 closely parallels that of the present paper until the point of taking the inverse Laplace transform is reached. At this point, reference 2 indicates that the inverse Laplace transform can be taken either by means of the relatively simple partial-fraction expansion (used in the present paper) or the more complicated inversion theorem of the Laplace transform. Neither approach in reference 2 is carried to the point of final equations containing only elementary functions and in a form particularly suited for computation. A solution similar to that of the present paper is indicated in reference 3. Only the form of the analysis is shown in reference 3, however, and all the details necessary for practical applications have not been carried out.

The present paper presents an analysis based on the representation of the lateral motion of an aircraft by differential equations. The results of the analysis are solutions in closed form expressing the free and forced motions in terms of elementary functions. These equations permit the calculation of the free motion of an aircraft following any initial condition or the forced motion following the application of constant external forces and moments. These forced motions can be used to obtain, by means of Duhamel's integral, the response to any arbitrary forcing function as shown in references 4 and 5. The solutions are readily adaptable to calculation by digital-type calculating machines and the calculation is an arithmetical process requiring no knowledge of the theory of the Laplace transform. The solution equations of motion have been applied on an automatic calculating machine to the calculation of the lateral stability of a specific airplane and to the calculation of certain free and forced motions as illustrative examples.

COEFFICIENTS AND SYMBOLS

C_L	trim lift coefficient $(W \cos \gamma / qS)$
C_l	rolling-moment coefficient (L/qSb)
C_n	yawing-moment coefficient (N/qSb)
C_Y	lateral-force coefficient (Y/qS)
W	airplane weight, pounds
L	rolling moment
M	pitching moment
N	yawing moment
Y	lateral force
H_a	aileron hinge moment
H_e	elevator hinge moment
H_r	rudder hinge moment
q	dynamic pressure $(\rho V^2/2)$
S	wing area, square feet
b	wing span, feet
γ	inclination of flight path to horizontal (positive in climb), degrees
α	angle of attack, degrees
θ	angle of pitch, degrees
ρ	mass density of air, slugs per cubic foot
V	free-stream velocity, feet per second
m	airplane mass, slugs (W/g)
g	acceleration due to gravity, feet per second per second

s_b	nondimensional time (tV/b)
t	time, seconds
$D_b = \frac{d}{ds_b}$	
η	inclination of principal longitudinal axis of inertia (positive for axis above flight path at nose), degrees
μ_b	airplane relative-density factor $(m/\rho Sb)$
ϕ	angle of bank, radians $\left(\int_0^t p\ dt\right)$
ψ	angle of yaw or azimuth, radians $\left(\int_0^t r\ dt\right)$
p	rolling velocity about stability X-axis, radians per second
r	yawing velocity about stability Z-axis, radians per second
β	angle of sideslip, radians
C_{l_c}	rolling-moment coefficient of forcing-function couple in roll
C_{n_c}	yawing-moment coefficient of forcing-function couple in yaw
C_{Y_c}	lateral-force coefficient of lateral forcing function
P,P'	periods of oscillatory modes, seconds
$T_{1/2},T_{1/2}'$	times to damp to half-amplitude of oscillatory modes, seconds
$N_{1/2},N_{1/2}'$	cycles to damp to half-amplitude of oscillatory modes
δ_a	aileron deflection, degrees
δ_r	rudder deflection, degrees

δ_e	elevator deflection, degrees
K_X	nondimensional radius of gyration about stability X-axis $\left(\sqrt{K_{X_0}{}^2\cos^2\eta + K_{Z_0}{}^2\sin^2\eta}\right)$
K_Z	nondimensional radius of gyration about stability Z-axis $\left(\sqrt{K_{Z_0}{}^2\cos^2\eta + K_{X_0}{}^2\sin^2\eta}\right)$
K_{XZ}	nondimensional product of inertia between stability X- and Z-axes $\left(\left(K_{Z_0}{}^2 - K_{X_0}{}^2\right)\sin\eta\cos\eta\right)$
K_{X_0}	nondimensional radius of gyration about principal X-axis $\left(k_{X_0}/b\right)$
K_{Z_0}	nondimensional radius of gyration about principal Z-axis $\left(k_{Z_0}/b\right)$
k_{X_0}	radius of gyration about principal X-axis, feet
k_{Z_0}	radius of gyration about principal Z-axis, feet
σ	Laplace transform of s_b
Δ	stability quartic
$\lambda_1,\lambda_2,\lambda_3,\lambda_4$	roots of $\Delta = 0$
p_σ, q_σ	polynomials in σ
$q_\sigma' = \frac{dq_\sigma}{d\sigma}$	
λ_n	roots of $q_\sigma = 0$
R_1	real part of λ_3 and λ_4 when λ_3 and λ_4 are complex conjugates
I_1	imaginary part of λ_3 and λ_4 when λ_3 and λ_4 are complex conjugates
R_1'	real part of λ_1 and λ_2 when λ_1 and λ_2 are complex conjugates

I_1'	imaginary part of λ_1 and λ_2 when λ_1 and λ_2 are complex conjugates
A,B,C,D,E	coefficients of stability quartic
R	Routh's discriminant
A_1,A_2,A_3,A_4,A_5,A_6	amplitude coefficients for ϕ
B_1,B_2,B_3,B_4,B_5,B_6	amplitude coefficients for ψ
C_1,C_2,C_3,C_4,C_5	amplitude coefficients for β
R_A	real part of A_3 and A_4 when λ_3 and λ_4 are complex conjugates
I_A	imaginary part of A_3 and A_4 when λ_3 and λ_4 are complex conjugates
R_B	real part of B_3 and B_4 when λ_3 and λ_4 are complex conjugates
I_B	imaginary part of B_3 and B_4 when λ_3 and λ_4 are complex conjugates
R_C	real part of C_3 and C_4 when λ_3 and λ_4 are complex conjugates
I_C	imaginary part of C_3 and C_4 when λ_3 and λ_4 are complex conjugates
R_A'	real part of A_1 and A_2 when λ_1 and λ_2 are complex conjugates
I_A'	imaginary part of A_1 and A_2 when λ_1 and λ_2 are complex conjugates
R_B'	real part of B_1 and B_2 when λ_1 and λ_2 are complex conjugates
I_B'	imaginary part of B_1 and B_2 when λ_1 and λ_2 are complex conjugates

R_C' real part of C_1 and C_2 when λ_1 and λ_2 are complex conjugates

I_C' imaginary part of C_1 and C_2 when λ_1 and λ_2 are complex conjugates

K_A amplitude coefficient for ϕ oscillation corresponding to complex conjugate roots λ_3 and λ_4 $\left(2\sqrt{R_A^2 + I_A^2}\right)$

K_B amplitude coefficient for ψ oscillation corresponding to complex conjugate roots λ_3 and λ_4 $\left(2\sqrt{R_B^2 + I_B^2}\right)$

K_C amplitude coefficient for β oscillation corresponding to complex conjugate roots λ_3 and λ_4 $\left(2\sqrt{R_C^2 + I_C^2}\right)$

K_A' amplitude coefficient for ϕ oscillation corresponding to complex conjugate roots λ_1 and λ_2 $\left(2\sqrt{R_A'^2 + I_A'^2}\right)$

K_B' amplitude coefficient for ψ oscillation corresponding to complex conjugate roots λ_1 and λ_2 $\left(2\sqrt{R_B'^2 + I_B'^2}\right)$

K_C' amplitude coefficient for β oscillation corresponding to complex conjugate roots λ_1 and λ_2 $\left(2\sqrt{R_C'^2 + I_C'^2}\right)$

ω_A phase angle for ϕ oscillation corresponding to conjugate complex roots λ_3 and λ_4, radians $\left(\tan^{-1} \frac{I_A}{R_A}\right)$

ω_B phase angle for ψ oscillation corresponding to conjugate complex roots λ_3 and λ_4, radians $\left(\tan^{-1} \frac{I_B}{R_B}\right)$

ω_C phase angle for β oscillation corresponding to conjugate complex roots λ_3 and λ_4, radians $\left(\tan^{-1} \frac{I_C}{R_C}\right)$

ω_A' phase angle for ϕ oscillation corresponding to conjugate complex roots λ_1 and λ_2, radians $\left(\tan^{-1} \frac{I_A'}{R_A'}\right)$

ω_B' phase angle for ψ oscillation corresponding to conjugate complex roots λ_1 and λ_2, radians $\left(\tan^{-1} \frac{I_B'}{R_B'}\right)$

ω_C' phase angle for β oscillation corresponding to conjugate complex roots λ_1 and λ_2, radians $\left(\tan^{-1} \frac{I_C'}{R_C'}\right)$

$$C_{l_r} = \frac{\partial C_l}{\partial\left(\frac{rb}{2V}\right)}$$

$$C_{l_p} = \frac{\partial C_l}{\partial\left(\frac{pb}{2V}\right)}$$

$$C_{l_\beta} = \frac{\partial C_l}{\partial\beta}$$

$$C_{n_r} = \frac{\partial C_n}{\partial\left(\frac{rb}{2V}\right)}$$

$$C_{n_p} = \frac{\partial C_n}{\partial\left(\frac{pb}{2V}\right)}$$

$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta}$$

$$C_{Y_r} = \frac{\partial C_Y}{\partial\left(\frac{rb}{2V}\right)}$$

$$C_{Y_p} = \frac{\partial C_Y}{\partial\left(\frac{pb}{2V}\right)}$$

$$C_{Y_\beta} = \frac{\partial C_Y}{\partial \beta}$$

a_0,a_1,a_2,a_3,a_4,a_5 coefficients appearing in numerator terms of amplitude coefficients for ϕ

b_0,b_1,b_2,b_3,b_4,b_5 coefficients appearing in numerator terms of amplitude coefficients for ψ

c_0,c_1,c_2,c_3,c_4 coefficients appearing in numerator terms of amplitude coefficients for β

Subscripts:

0 initial value

σ transformed variable

ANALYSIS

The linear equations of motion, referred to the axis system shown in figure 1 and representing the lateral motion of an airplane are

$$\left.\begin{aligned}
&2\mu_b K_X{}^2 D_b{}^2\phi - \frac{1}{2}C_{l_p} D_b\phi + 2\mu_b K_{XZ} D_b{}^2\psi - \frac{1}{2}C_{l_r} D_b\psi - C_{l_\beta}\beta - C_{l_c} = 0 \\
&2\mu_b K_{XZ} D_b{}^2\phi - \frac{1}{2}C_{n_p} D_b\phi + 2\mu_b K_Z{}^2 D_b{}^2\psi - \frac{1}{2}C_{n_r} D_b\psi - C_{n_\beta}\beta - C_{n_c} = 0 \\
&-\frac{1}{2}C_{Y_p} D_b\phi - C_L\phi + 2\mu_b D_b\psi - C_L \tan\gamma\psi - \frac{1}{2}C_{Y_r} D_b\psi - C_{Y_\beta}\beta + \\
&\qquad 2\mu_b D_b\beta - C_{Y_c} = 0
\end{aligned}\right\} \quad (1)$$

The terms C_{l_c}, C_{n_c}, and C_{Y_c} are forcing functions which represent disturbances imposed upon the state of motion of the airplane by control movement or atmospheric turbulence. These terms, in general, are arbitrary functions of time, but for the purpose of this analysis, they are considered to be constants applied at zero time. After a solution has been obtained in terms of constant forcing quantities this solution can be used to obtain a new solution for an arbitrary forcing function by Duhamel's integral as explained in references 4 and 5.

Transformation of Equations

When the Laplace transform is applied (reference 6, p. 8), the transformed equations become after multiplying through by σ

$$\left.\begin{aligned}
&\left(2\mu_b K_X{}^2\sigma^3 - \frac{1}{2}C_{l_p}\sigma^2\right)\phi_\sigma + \left(2\mu_b K_{XZ}\sigma^3 - \frac{1}{2}C_{l_r}\sigma^2\right)\psi_\sigma + \left(-C_{l_\beta}\sigma\right)\beta_\sigma = r_1 \\
&r_1 = \left(2\mu_b K_X{}^2\sigma^2 - \frac{1}{2}C_{l_p}\sigma\right)\phi_0 + \left(2\mu_b K_{XZ}\sigma^2 - \frac{1}{2}C_{l_r}\sigma\right)\psi_0 + \\
&\qquad \left(2\mu_b K_X{}^2\sigma\right)\left(D_b\phi\right)_0 + \left(2\mu_b K_{XZ}\sigma\right)\left(D_b\psi\right)_0 + C_{l_c}
\end{aligned}\right\} \quad (2a)$$

$$\left.\begin{aligned}&\left(2\mu_b K_{XZ}\sigma^3 - \tfrac{1}{2}C_{n_p}\sigma^2\right)\phi_\sigma + \left(2\mu_b K_Z{}^2\sigma^3 - \tfrac{1}{2}C_{n_r}\sigma^2\right)\psi_\sigma + \left(-C_{n_\beta}\sigma\right)\beta_\sigma = r_2\\&r_2 = \left(2\mu_b K_{XZ}\sigma^2 - \tfrac{1}{2}C_{n_p}\sigma\right)\phi_0 + \left(2\mu_b K_Z{}^2\sigma^2 - \tfrac{1}{2}C_{n_r}\sigma\right)\psi_0 +\\&\qquad\left(2\mu_b K_{XZ}\sigma\right)\left(D_b\phi\right)_0 + \left(2\mu_b K_Z{}^2\sigma\right)\left(D_b\psi\right)_0 + C_{n_c}\end{aligned}\right\} \quad (2b)$$

$$\left.\begin{aligned}&\left(-\tfrac{1}{2}C_{Y_p}\sigma^2 - C_L\sigma\right)\phi_\sigma + \left(2\mu_b\sigma^2 - \tfrac{1}{2}C_{Y_r}\sigma^2 - C_L \tan\gamma\sigma\right)\psi_\sigma +\\&\qquad\left(2\mu_b\sigma^2 - C_{Y_\beta}\sigma\right)\beta_\sigma = r_3\\&r_3 = \left(-\tfrac{1}{2}C_{Y_p}\sigma\right)\phi_0 + \left(2\mu_b\sigma - \tfrac{1}{2}C_{Y_r}\sigma\right)\psi_0 + \left(2\mu_b\sigma\right)\beta_0 + C_{Y_c}\end{aligned}\right\} \quad (2c)$$

Solution of Transformed Equations

After equations (2) are solved by determinants, the expression for ϕ_σ is

$$\phi_\sigma = \frac{a_0\sigma^5 + a_1\sigma^4 + a_2\sigma^3 + a_3\sigma^2 + a_4\sigma + a_5}{\sigma^2\Delta} \quad (3)$$

where

$$\Delta = A\sigma^4 + B\sigma^3 + C\sigma^2 + D\sigma + E \quad (4)$$

and the constants are given by

$$a_0 = \phi_0\left(8\mu_b{}^3\begin{vmatrix}K_X{}^2 & K_{XZ}\\ K_{XZ} & K_Z{}^2\end{vmatrix}\right)$$

$$a_1 = \phi_0\left(2\mu_b{}^2\begin{vmatrix}K_X{}^2 & K_{XZ} & C_{l_r}\\ K_{XZ} & K_Z{}^2 & C_{n_r}\\ 0 & 1 & -2C_{Y_\beta}\end{vmatrix} - 2\mu_b{}^2\begin{vmatrix}C_{l_p} & K_{XZ}\\ C_{n_p} & K_Z{}^2\end{vmatrix}\right) + (D_b\phi)_0\left(8\mu_b{}^3\begin{vmatrix}K_X{}^2 & K_{XZ}\\ K_{XZ} & K_Z{}^2\end{vmatrix}\right)$$

$$a_2 = \phi_0\left(\mu_b\begin{vmatrix}C_{l_\beta} & C_{l_p} & K_{XZ}\\ C_{n_\beta} & C_{n_p} & K_Z{}^2\\ C_{Y_\beta} & C_{Y_p} & 0\end{vmatrix} + \mu_b\begin{vmatrix}C_{l_\beta} & K_X{}^2 & C_{l_r}\\ C_{n_\beta} & K_{XZ} & C_{n_r}\\ C_{Y_\beta} & 0 & C_{Y_r}\end{vmatrix} - 4\mu_b{}^2\begin{vmatrix}C_{l_\beta} & K_X{}^2\\ C_{n_\beta} & K_{XZ}\end{vmatrix} + \right.$$

$$\left.\frac{1}{2}\mu_b\begin{vmatrix}C_{l_p} & C_{l_r}\\ C_{n_p} & C_{n_r}\end{vmatrix}\right) + (D_b\phi)_0\left(2\mu_b{}^2\begin{vmatrix}K_X{}^2 & K_{XZ} & C_{l_r}\\ K_{XZ} & K_Z{}^2 & C_{n_r}\\ 0 & 1 & -2C_{Y_\beta}\end{vmatrix}\right) +$$

$$(D_b\psi)_0\left(2\mu_b{}^2\begin{vmatrix}C_{l_r} & K_{XZ}\\ C_{n_r} & K_Z{}^2\end{vmatrix}\right) + \beta_0\left(4\mu_b{}^2\begin{vmatrix}C_{l_\beta} & K_{XZ}\\ C_{n_\beta} & K_Z{}^2\end{vmatrix}\right) + 4\mu_b{}^2\begin{vmatrix}C_{l_c} & K_{XZ}\\ C_{n_c} & K_Z{}^2\end{vmatrix}$$

$$a_3 = \phi_0\left(\mu_b\begin{vmatrix}C_{l_\beta} & C_{l_p} & K_X^2 \\ C_{n_\beta} & C_{n_p} & K_{XZ} \\ 0 & -2\tan\gamma C_L & 1\end{vmatrix} - \frac{1}{4}\begin{vmatrix}C_{l_\beta} & C_{l_p} & C_{l_r} \\ C_{n_\beta} & C_{n_p} & C_{n_r} \\ C_{Y_\beta} & C_{Y_p} & C_{Y_r}\end{vmatrix}\right) +$$

$$(D_b\phi)_0\left(\mu_b\begin{vmatrix}C_{l_\beta} & K_X^2 & C_{l_r} \\ C_{n_\beta} & K_{XZ} & C_{n_r} \\ C_{Y_\beta} & 0 & C_{Y_r}\end{vmatrix} - 4\mu_b^2\begin{vmatrix}C_{l_\beta} & K_X^2 \\ C_{n_\beta} & K_{XZ}\end{vmatrix}\right) +$$

$$\psi_0\left(2\mu_b\tan\gamma C_L\begin{vmatrix}C_{l_\beta} & K_{XZ} \\ C_{n_\beta} & K_Z^2\end{vmatrix}\right) + (D_b\psi)_0\left(\mu_b\begin{vmatrix}C_{l_\beta} & K_{XZ} & C_{l_r} \\ C_{n_\beta} & K_Z^2 & C_{n_r} \\ C_{Y_\beta} & 0 & C_{Y_r}\end{vmatrix} -\right.$$

$$\left.4\mu_b^2\begin{vmatrix}C_{l_\beta} & K_{XZ} \\ C_{n_\beta} & K_Z^2\end{vmatrix}\right) + \beta_0\left(-\mu_b\begin{vmatrix}C_{l_\beta} & C_{l_r} \\ C_{n_\beta} & C_{n_r}\end{vmatrix}\right) + \left(2\mu_b\begin{vmatrix}C_{l_\beta} & K_{XZ} & C_{l_c} \\ C_{n_\beta} & K_Z^2 & C_{n_c} \\ C_{Y_\beta} & 0 & C_{Y_c}\end{vmatrix} +\right.$$

$$\left.\mu_b\begin{vmatrix}C_{l_r} & C_{l_c} \\ C_{n_r} & C_{n_c}\end{vmatrix}\right)$$

$$a_4 = \phi_0\left(-\frac{1}{2}\tan\gamma C_L\begin{vmatrix}C_{l_\beta} & C_{l_p}\\ C_{n_\beta} & C_{n_p}\end{vmatrix}\right) + (D_b\phi)_0\left(2\mu_b\tan\gamma C_L\begin{vmatrix}C_{l_\beta} & K_X^2\\ C_{n_\beta} & K_{XZ}\end{vmatrix}\right) +$$

$$\psi_0\left(-\frac{1}{2}\tan\gamma C_L\begin{vmatrix}C_{l_\beta} & C_{l_r}\\ C_{n_\beta} & C_{n_r}\end{vmatrix}\right) + (D_b\psi)_0\left(2\mu_b\tan\gamma C_L\begin{vmatrix}C_{l_\beta} & K_{XZ}\\ C_{n_p} & K_Z^2\end{vmatrix}\right) +$$

$$\left(\frac{1}{2}\begin{vmatrix}C_{l_\beta} & C_{l_c} & C_{l_r}\\ C_{n_\beta} & C_{n_c} & C_{n_r}\\ C_{Y_\beta} & C_{Y_c} & C_{Y_r}\end{vmatrix} - 2\mu_b\begin{vmatrix}C_{l_\beta} & C_{l_c}\\ C_{n_\beta} & C_{n_c}\end{vmatrix}\right)$$

$$a_5 = \tan\gamma C_L\begin{vmatrix}C_{l_\beta} & C_{l_c}\\ C_{n_\beta} & C_{n_c}\end{vmatrix}$$

$$A = 8\mu_b^3\begin{vmatrix}K_X^2 & K_{XZ}\\ K_{XZ} & K_Z^2\end{vmatrix}$$

$$B = 2\mu_b^2\begin{vmatrix}K_X^2 & K_{XZ} & C_{l_r}\\ K_{XZ} & K_Z^2 & C_{n_r}\\ 0 & 1 & -2C_{Y_\beta}\end{vmatrix} - 2\mu_b^2\begin{vmatrix}C_{l_p} & K_{XZ}\\ C_{n_p} & K_Z^2\end{vmatrix}$$

$$C = \mu_b\begin{vmatrix}C_{l_\beta} & C_{l_p} & C_{l_r}\\ 0 & K_{XZ} & K_Z^2\\ C_{Y_\beta} & C_{Y_p} & C_{Y_r}\end{vmatrix} + \mu_b\begin{vmatrix}0 & K_X^2 & K_{XZ}\\ C_{n_\beta} & C_{n_p} & C_{n_r}\\ C_{Y_\beta} & C_{Y_p} & C_{Y_r}\end{vmatrix} + \frac{1}{2}\mu_b\begin{vmatrix}C_{l_p} & C_{l_r}\\ C_{n_p} & C_{n_r}\end{vmatrix} - 4\mu_b^2\begin{vmatrix}C_{l_\beta} & K_X^2\\ C_{n_p} & K_{XZ}\end{vmatrix}$$

$$D = 2\mu_b C_L \begin{vmatrix} C_{l_\beta} & K_X{}^2 & K_{XZ} \\ C_{n_\beta} & K_{XZ} & K_Z{}^2 \\ 0 & 1 & \tan\gamma \end{vmatrix} - \frac{1}{4}\begin{vmatrix} C_{l_\beta} & C_{l_p} & C_{l_r} \\ C_{n_\beta} & C_{n_p} & C_{n_r} \\ C_{Y_\beta} & C_{Y_p} & C_{Y_r} \end{vmatrix} + \mu_b \begin{vmatrix} C_{l_\beta} & C_{l_p} \\ C_{n_\beta} & C_{n_p} \end{vmatrix}$$

$$E = -\frac{1}{2}C_L \begin{vmatrix} C_{l_\beta} & C_{l_p} & C_{l_r} \\ C_{n_\beta} & C_{n_p} & C_{n_r} \\ 0 & 1 & \tan\gamma \end{vmatrix}$$

The expression for ψ_σ is

$$\psi_\sigma = \frac{b_0\sigma^5 + b_1\sigma^4 + b_2\sigma^3 + b_3\sigma^2 + b_4\sigma + b_5}{\sigma^2\Delta} \tag{5}$$

where the constants are given by

$$b_0 = \psi_0\left(8\mu_b{}^3 \begin{vmatrix} K_X{}^2 & K_{XZ} \\ K_{XZ} & K_Z{}^2 \end{vmatrix}\right)$$

$$b_1 = \psi_0\left(2\mu_b{}^2 \begin{vmatrix} K_X{}^2 & K_{XZ} & C_{l_r} \\ K_{XZ} & K_Z{}^2 & C_{n_r} \\ 0 & 1 & -2C_{Y_\beta} \end{vmatrix} - 2\mu_b{}^2 \begin{vmatrix} C_{l_p} & K_{XZ} \\ C_{n_p} & K_Z{}^2 \end{vmatrix}\right) + (D_b\psi)_0\left(8\mu_b{}^3 \begin{vmatrix} K_X{}^2 & K_{XZ} \\ K_{XZ} & K_Z{}^2 \end{vmatrix}\right)$$

$$b_2 = (D_b\phi)_0\left(-2\mu_b^2\begin{vmatrix}C_{l_p} & K_X^2\\ C_{n_p} & K_{XZ}\end{vmatrix}\right) + \psi_0\left(\mu_b\begin{vmatrix}0 & K_X^2 & K_{XZ}\\ C_{n_\beta} & C_{n_p} & C_{n_r}\\ C_{Y_\beta} & C_{Y_p} & C_{Y_r}\end{vmatrix} + \mu_b\begin{vmatrix}C_{l_\beta} & C_{l_p} & C_{l_r}\\ 0 & K_{XZ} & K_Z^2\\ C_{Y_\beta} & C_{Y_p} & C_{Y_r}\end{vmatrix} -\right.$$

$$\left.4\mu_b^2\begin{vmatrix}C_{l_\beta} & K_X^2\\ C_{n_\beta} & K_{XZ}\end{vmatrix} + \frac{1}{2}\mu_b\begin{vmatrix}C_{l_p} & C_{l_r}\\ C_{n_p} & C_{n_r}\end{vmatrix}\right) + (D_b\psi)_0\left(-2\mu_b^2\begin{vmatrix}K_X^2 & C_{l_p} & K_{XZ}\\ K_{XZ} & C_{n_p} & K_Z^2\\ 1 & -2C_{Y_\beta} & 0\end{vmatrix}\right) +$$

$$\beta_0\left(-4\mu_b^2\begin{vmatrix}C_{l_\beta} & K_X^2\\ C_{n_\beta} & K_{XZ}\end{vmatrix}\right) + \left(-4\mu_b^2\begin{vmatrix}C_{l_c} & K_X^2\\ C_{n_c} & K_{XZ}\end{vmatrix}\right)$$

$$b_3 = \phi_0\left(-2\mu_b C_L\begin{vmatrix}C_{l_\beta} & K_X^2\\ C_{n_\beta} & K_{XZ}\end{vmatrix}\right) + (D_b\phi)_0\left(\mu_b\begin{vmatrix}C_{l_\beta} & C_{l_p} & K_X^2\\ C_{n_\beta} & C_{n_p} & K_{XZ}\\ C_{Y_\beta} & C_{Y_p} & 0\end{vmatrix}\right) +$$

$$\psi_0\left(\mu_b\begin{vmatrix}C_{l_\beta} & C_{l_p} & K_{XZ}\\ C_{n_\beta} & C_{n_p} & K_Z^2\\ 0 & 2C_L & 1\end{vmatrix} - \frac{1}{4}\begin{vmatrix}C_{l_\beta} & C_{l_p} & C_{l_r}\\ C_{n_\beta} & C_{n_p} & C_{n_r}\\ C_{Y_\beta} & C_{Y_p} & C_{Y_r}\end{vmatrix}\right) + (D_b\psi)_0\left(\mu_b\begin{vmatrix}C_{l_\beta} & C_{l_p} & K_{XZ}\\ C_{n_\beta} & C_{n_p} & K_Z^2\\ C_{Y_\beta} & C_{Y_p} & 0\end{vmatrix}\right) +$$

$$\beta_0\left(\mu_b\begin{vmatrix}C_{l_\beta} & C_{l_p}\\ C_{n_\beta} & C_{n_p}\end{vmatrix}\right) + \left(2\mu_b\begin{vmatrix}C_{l_\beta} & C_{l_c} & K_X^2\\ C_{n_\beta} & C_{n_c} & K_{XZ}\\ C_{Y_\beta} & C_{Y_c} & 0\end{vmatrix} + \mu_b\begin{vmatrix}C_{l_c} & C_{l_p}\\ C_{n_c} & C_{n_p}\end{vmatrix}\right)$$

$$b_4 = \phi_0\left(\frac{1}{2}C_L\begin{vmatrix}C_{l_\beta} & C_{l_p}\\ C_{n_\beta} & C_{n_p}\end{vmatrix}\right) + (D_b\phi)_0\left(-2\mu_b C_L\begin{vmatrix}C_{l_\beta} & K_X{}^2\\ C_{n_\beta} & K_{XZ}\end{vmatrix}\right) + \psi_0\left(\frac{1}{2}C_L\begin{vmatrix}C_{l_\beta} & C_{l_r}\\ C_{n_\beta} & C_{n_r}\end{vmatrix}\right) +$$

$$(D_b\psi)_0\left(-2\mu_b C_L\begin{vmatrix}C_{l_\beta} & K_{XZ}\\ C_{n_\beta} & K_Z{}^2\end{vmatrix}\right) + \frac{1}{2}\begin{vmatrix}C_{l_\beta} & C_{l_p} & C_{l_c}\\ C_{n_\beta} & C_{n_p} & C_{n_c}\\ C_{Y_\beta} & C_{Y_p} & C_{Y_c}\end{vmatrix}$$

$$b_5 = -C_L\begin{vmatrix}C_{l_\beta} & C_{l_c}\\ C_{n_\beta} & C_{n_c}\end{vmatrix}$$

The expression for β_σ is

$$\beta_\sigma = \frac{c_0\sigma^4 + c_1\sigma^3 + c_2\sigma^2 + c_3\sigma + c_4}{\sigma\Delta} \tag{6}$$

where the constants are given by

$$c_0 = \beta_0\left(8\mu_b{}^3\begin{vmatrix}K_X{}^2 & K_{XZ}\\ K_{XZ} & K_Z{}^2\end{vmatrix}\right)$$

$$c_1 = \phi_0\left(4\mu_b{}^2C_L\begin{vmatrix}K_X{}^2 & K_{XZ}\\ K_{XZ} & K_Z{}^2\end{vmatrix}\right) + (D_b\phi)_0\left(2\mu_b{}^2C_{Y_p}\begin{vmatrix}K_X{}^2 & K_{XZ}\\ K_{XZ} & K_Z{}^2\end{vmatrix}\right) +$$

$$\psi_0\left(4\mu_b{}^2\tan\gamma\; C_L\begin{vmatrix}K_X{}^2 & K_{XZ}\\ K_{XZ} & K_Z{}^2\end{vmatrix}\right) + (D_b\psi)_0\left(2\mu_b{}^2\left(C_{Y_r} - 4\mu_b\right)\begin{vmatrix}K_X{}^2 & K_{XZ}\\ K_{XZ} & K_Z{}^2\end{vmatrix}\right) +$$

$$\beta_0\left(2\mu_b{}^2\begin{vmatrix}C_{l_r} & K_X{}^2\\ C_{n_r} & K_{XZ}\end{vmatrix} - 2\mu_b{}^2\begin{vmatrix}C_{l_p} & K_{XZ}\\ C_{n_p} & K_Z{}^2\end{vmatrix}\right) + 4\mu_b{}^2C_{Y_c}\begin{vmatrix}K_X{}^2 & K_{XZ}\\ K_{XZ} & K_Z{}^2\end{vmatrix}$$

$$c_2 = \phi_0\left(\mu_bC_L\begin{vmatrix}C_{l_r} & K_X{}^2\\ C_{n_r} & K_{XZ}\end{vmatrix} - \mu_bC_L\begin{vmatrix}C_{l_p} & K_{XZ}\\ C_{n_p} & K_Z{}^2\end{vmatrix}\right) + (D_b\phi)_0\left(\frac{1}{2}\mu_b\begin{vmatrix}K_X{}^2 & C_{l_p} & C_{l_r}\\ K_{XZ} & C_{n_p} & C_{n_r}\\ 0 & C_{Y_p} & C_{Y_r}\end{vmatrix} -\right.$$

$$\left.2\mu_b{}^2\begin{vmatrix}K_X{}^2 & C_{l_p} & K_{XZ}\\ K_{XZ} & C_{n_p} & K_Z{}^2\\ 0 & 2C_L & 1\end{vmatrix}\right) + \psi_0\left(\mu_b\tan\gamma C_L\begin{vmatrix}C_{l_r} & K_X{}^2\\ C_{n_r} & K_{XZ}\end{vmatrix} -\right.$$

$$\left.\mu_b\tan\gamma C_L\begin{vmatrix}C_{l_p} & K_{XZ}\\ C_{n_p} & K_Z{}^2\end{vmatrix}\right) + (D_b\psi)_0\left(\frac{1}{2}\mu_b\begin{vmatrix}K_{XZ} & C_{l_p} & C_{l_r}\\ K_Z{}^2 & C_{n_p} & C_{n_r}\\ 0 & C_{Y_p} & C_{Y_r}\end{vmatrix} +\right.$$

$$\left.2\mu_b{}^2\begin{vmatrix}K_X{}^2 & C_{l_p} & K_{XZ}\\ K_{XZ} & C_{n_p} & K_Z{}^2\\ 1 & -2\tan\gamma C_L & 0\end{vmatrix}\right) + \beta_0\left(\frac{1}{2}\mu_b\begin{vmatrix}C_{l_p} & C_{l_r}\\ C_{n_p} & C_{n_r}\end{vmatrix}\right) +$$

$$\left(-\mu_b\begin{vmatrix}0 & K_X{}^2 & K_{XZ}\\ C_{n_c} & C_{n_p} & C_{n_r}\\ C_{Y_c} & C_{Y_p} & C_{Y_r}\end{vmatrix} - \mu_b\begin{vmatrix}C_{l_c} & C_{l_p} & C_{l_r}\\ 0 & K_{XZ} & K_Z{}^2\\ C_{Y_c} & C_{Y_p} & C_{Y_r}\end{vmatrix} - 4\mu_b{}^2\begin{vmatrix}K_X{}^2 & C_{l_c}\\ K_{XZ} & C_{n_c}\end{vmatrix}\right)$$

$$c_3 = \phi_0\left(\frac{1}{4}C_L\begin{vmatrix}C_{l_p} & C_{l_r}\\ C_{n_p} & C_{n_r}\end{vmatrix}\right) + (D_b\phi)_0\left(\mu_bC_L\begin{vmatrix}K_X^2 & C_{l_p} & C_{l_r}\\ K_{XZ} & C_{n_p} & C_{n_r}\\ 0 & 1 & \tan\gamma\end{vmatrix}\right) +$$

$$\psi_0\left(\frac{1}{4}\tan\gamma C_L\begin{vmatrix}C_{l_p} & C_{l_r}\\ C_{n_p} & C_{n_r}\end{vmatrix}\right) + (D_b\psi)_0\left(\mu_bC_L\begin{vmatrix}K_{XZ} & C_{l_p} & C_{l_r}\\ K_Z^2 & C_{n_p} & C_{n_r}\\ 0 & 1 & \tan\gamma\end{vmatrix}\right) +$$

$$\left(\frac{1}{4}\begin{vmatrix}C_{l_c} & C_{l_p} & C_{l_r}\\ C_{n_c} & C_{n_p} & C_{n_r}\\ C_{Y_c} & C_{Y_p} & C_{Y_r}\end{vmatrix} - 2\mu_bC_L\begin{vmatrix}C_{l_c} & K_X^2 & K_{XZ}\\ C_{n_c} & K_{XZ} & K_Z^2\\ 0 & 1 & \tan\gamma\end{vmatrix} + \mu_b\begin{vmatrix}C_{l_p} & C_{l_c}\\ C_{n_p} & C_{n_c}\end{vmatrix}\right)$$

$$c_4 = \frac{1}{2}C_L\begin{vmatrix}C_{l_c} & C_{l_p} & C_{l_r}\\ C_{n_c} & C_{n_p} & C_{n_r}\\ 0 & 1 & \tan\gamma\end{vmatrix}.$$

All the determinants given in this paper are expanded in the appendix.

In order to obtain the actual variables ϕ, ψ, and β from the transformed variables an inverse LaPlace transformation must be applied to ϕ_σ, ψ_σ, and β_σ. The expressions for ϕ_σ, ψ_σ, and β_σ are of the form p_σ/q_σ where p_σ and q_σ are polynomials, the degree of q_σ being higher than that of p_σ. Reference 6, page 45, indicates that the inverse transform of a function of this type is (in terms of the variables used herein)

$$L^{-1}\left(\frac{p_\sigma}{q_\sigma}\right) = \sum_{n=1}^{m}\frac{p_\sigma(\lambda_n)}{q_\sigma'(\lambda_n)}\,e^{\lambda_n s_b}$$

This equation assumes all the roots λ_n of $q_\sigma = 0$ to be distinct. All roots of $q_\sigma = 0$ are distinct for β_σ; however, for ϕ_σ and ψ_σ, $q_\sigma = 0$ has double zero roots. (See equations (3), (5), and (6).) The terms in the equations for ϕ and ψ resulting from the two zero roots of $q_\sigma = 0$ are given according to reference 6, page 49, by

$$\frac{d\Omega}{d\sigma}(0) + \Omega(0)s_b$$

where

$$\Omega = \frac{p_\sigma}{q_\sigma}\sigma^2$$

The analysis takes three forms depending upon the character of the nonzero roots of $q_\sigma = 0$ which are the same as the roots of $\Delta = 0$. Four real roots, two real roots plus a pair of conjugate complex roots, or two pairs of conjugate complex roots may exist.

Four Real Roots

The inverse Laplace transform of ϕ_σ is

$$\phi = A_1e^{\lambda_1 s_b} + A_2e^{\lambda_2 s_b} + A_3e^{\lambda_3 s_b} + A_4e^{\lambda_4 s_b} + A_5 s_b + A_6 \tag{7}$$

where the constants are

$$A_1 = \frac{a_0\lambda_1{}^5 + a_1\lambda_1{}^4 + a_2\lambda_1{}^3 + a_3\lambda_1{}^2 + a_4\lambda_1 + a_5}{6A\lambda_1{}^5 + 5B\lambda_1{}^4 + 4C\lambda_1{}^3 + 3D\lambda_1{}^2 + 2E\lambda_1}$$

$$A_2 = \frac{a_0\lambda_2{}^5 + a_1\lambda_2{}^4 + a_2\lambda_2{}^3 + a_3\lambda_2{}^2 + a_4\lambda_2 + a_5}{6A\lambda_2{}^5 + 5B\lambda_2{}^4 + 4C\lambda_2{}^3 + 3D\lambda_2{}^2 + 2E\lambda_2}$$

$$A_3 = \frac{a_0\lambda_3{}^5 + a_1\lambda_3{}^4 + a_2\lambda_3{}^3 + a_3\lambda_3{}^2 + a_4\lambda_3 + a_5}{6A\lambda_3{}^5 + 5B\lambda_3{}^4 + 4C\lambda_3{}^3 + 3D\lambda_3{}^2 + 2E\lambda_3}$$

$$A_4 = \frac{a_0\lambda_4^5 + a_1\lambda_4^4 + a_2\lambda_4^3 + a_3\lambda_4^2 + a_4\lambda_4 + a_5}{6A\lambda_4^5 + 5B\lambda_4^4 + 4C\lambda_4^3 + 3D\lambda_4^2 + 2E\lambda_4}$$

$$A_5 = \frac{a_5}{E}$$

$$A_6 = \frac{1}{E}\left(a_4 - a_5\frac{D}{E}\right)$$

The inverse Laplace transform of ψ_σ is

$$\psi = B_1e^{\lambda_1 s_b} + B_2e^{\lambda_2 s_b} + B_3e^{\lambda_3 s_b} + B_4e^{\lambda_4 s_b} + B_5 s_b + B_6 \tag{8}$$

where the constants are

$$B_1 = \frac{b_0\lambda_1^5 + b_1\lambda_1^4 + b_2\lambda_1^3 + b_3\lambda_1^2 + b_4\lambda_1 + b_5}{6A\lambda_1^5 + 5B\lambda_1^4 + 4C\lambda_1^3 + 3D\lambda_1^2 + 2E\lambda_1}$$

$$B_2 = \frac{b_0\lambda_2^5 + b_1\lambda_2^4 + b_2\lambda_2^3 + b_3\lambda_2^2 + b_4\lambda_2 + b_5}{6A\lambda_2^5 + 5B\lambda_2^4 + 4C\lambda_2^3 + 3D\lambda_2^2 + 2E\lambda_2}$$

$$B_3 = \frac{b_0\lambda_3^5 + b_1\lambda_3^4 + b_2\lambda_3^3 + b_3\lambda_3^2 + b_4\lambda_3 + b_5}{6A\lambda_3^5 + 5B\lambda_3^4 + 4C\lambda_3^3 + 3D\lambda_3^2 + 2E\lambda_3}$$

$$B_4 = \frac{b_0\lambda_4^5 + b_1\lambda_4^4 + b_2\lambda_4^3 + b_3\lambda_4^2 + b_4\lambda_4 + b_5}{6A\lambda_4^5 + 5B\lambda_4^4 + 4C\lambda_4^3 + 3D\lambda_4^2 + 2E\lambda_4}$$

$$B_5 = \frac{b_5}{E}$$

$$B_6 = \frac{1}{E}\left(b_4 - b_5\frac{D}{E}\right)$$

The inverse Laplace transform of β_σ is

$$\beta = C_1e^{\lambda_1 s_b} + C_2e^{\lambda_2 s_b} + C_3e^{\lambda_3 s_b} + C_4e^{\lambda_4 s_b} + C_5 \tag{9}$$

where the constants are

$$C_1 = \frac{c_0\lambda_1^5 + c_1\lambda_1^4 + c_2\lambda_1^3 + c_3\lambda_1^2 + c_4\lambda_1}{6A\lambda_1^5 + 5B\lambda_1^4 + 4C\lambda_1^3 + 3D\lambda_1^2 + 2E\lambda_1}$$

$$C_2 = \frac{c_0\lambda_2^5 + c_1\lambda_2^4 + c_2\lambda_2^3 + c_3\lambda_2^2 + c_4\lambda_2}{6A\lambda_2^5 + 5B\lambda_2^4 + 4C\lambda_2^3 + 3D\lambda_2^2 + 2E\lambda_2}$$

$$C_3 = \frac{c_0\lambda_3^5 + c_1\lambda_3^4 + c_2\lambda_3^3 + c_3\lambda_3^2 + c_4\lambda_3}{6A\lambda_3^5 + 5B\lambda_3^4 + 4C\lambda_3^3 + 3D\lambda_3^2 + 2E\lambda_3}$$

$$C_4 = \frac{c_0\lambda_4^5 + c_1\lambda_4^4 + c_2\lambda_4^3 + c_3\lambda_4^2 + c_4\lambda_4}{6A\lambda_4^5 + 5B\lambda_4^4 + 4C\lambda_4^3 + 3D\lambda_4^2 + 2E\lambda_4}$$

$$C_5 = \frac{c_4}{E}$$

The quantity $D_b\phi$ can be obtained from equation (7) by differentiation as

$$D_b\phi = A_1'e^{\lambda_1 s_b} + A_2'e^{\lambda_2 s_b} + A_3'e^{\lambda_3 s_b} + A_4'e^{\lambda_4 s_b} + A_5' \qquad (10)$$

where the constants are

$$A_1' = \lambda_1 A_1$$

$$A_2' = \lambda_2 A_2$$

$$A_3' = \lambda_3 A_3$$

$$A_4' = \lambda_4 A_4$$

$$A_5' = A_5$$

The quantity $D_b\psi$ can be obtained from equation (8) by differentiation as

$$D_b\psi = B_1'e^{\lambda_1 s_b} + B_2'e^{\lambda_2 s_b} + B_3'e^{\lambda_3 s_b} + B_4'e^{\lambda_4 s_b} + B_5' \qquad (11)$$

where the constants are

$$B_1' = \lambda_1 B_1$$

$$B_2' = \lambda_2 B_2$$

$$B_3' = \lambda_3 B_3$$

$$B_4' = \lambda_4 B_4$$

$$B_5' = B_5$$

The quantity $D_b\beta$ can be obtained from equation (9) by differentiation as

$$D_b\beta = C_1'e^{\lambda_1 s_b} + C_2'e^{\lambda_2 s_b} + C_3'e^{\lambda_3 s_b} + C_4'e^{\lambda_4 s_b} \tag{12}$$

where the constants are

$$C_1' = \lambda_1 C_1$$

$$C_2' = \lambda_2 C_2$$

$$C_3' = \lambda_3 C_3$$

$$C_4' = \lambda_4 C_4$$

Collecting the equations of motion (equations (7) to (12)) for the case of four real roots gives

$$\left.\begin{aligned}
\phi &= A_1e^{\lambda_1 s_b} + A_2e^{\lambda_2 s_b} + A_3e^{\lambda_3 s_b} + A_4e^{\lambda_4 s_b} + A_5 s_b + A_6 \\
\psi &= B_1e^{\lambda_1 s_b} + B_2e^{\lambda_2 s_b} + B_3e^{\lambda_3 s_b} + B_4e^{\lambda_4 s_b} + B_5 s_b + B_6 \\
\beta &= C_1e^{\lambda_1 s_b} + C_2e^{\lambda_2 s_b} + C_3e^{\lambda_3 s_b} + C_4e^{\lambda_4 s_b} + C_5 \\
D_b\phi &= A_1'e^{\lambda_1 s_b} + A_2'e^{\lambda_2 s_b} + A_3'e^{\lambda_3 s_b} + A_4'e^{\lambda_4 s_b} + A_5' \\
D_b\psi &= B_1'e^{\lambda_1 s_b} + B_2'e^{\lambda_2 s_b} + B_3'e^{\lambda_3 s_b} + B_4'e^{\lambda_4 s_b} + B_5' \\
D_b\beta &= C_1'e^{\lambda_1 s_b} + C_2'e^{\lambda_2 s_b} + C_3'e^{\lambda_3 s_b} + C_4'e^{\lambda_4 s_b}
\end{aligned}\right\} \tag{13}$$

Two Real Roots and a Pair of Conjugate Complex Roots

If a pair of conjugate complex roots λ_3 and λ_4 exists, the coefficients of the terms of ϕ, ψ, and β (equation (13)) corresponding to λ_3 and λ_4 are conjugate complex. The complex number $\lambda_3{}^k$ can be written

$$\lambda_3{}^k = R_k + I_k i$$

Thus,

$$\lambda_3 = R_1 + I_1 i$$

$$\lambda_3^2 = R_2 + I_2 i$$

$$\lambda_3^3 = R_3 + I_3 i$$

$$\lambda_3^4 = R_4 + I_4 i$$

$$\lambda_3^5 = R_5 + I_5 i$$

The coefficients of the terms of ϕ resulting from the complex roots are

$$A_3 = \frac{a_0R_5 + a_1R_4 + a_2R_3 + a_3R_2 + a_4R_1 + a_5 + \left(a_0I_5 + a_1I_4 + a_2I_3 + a_3I_2 + a_4I_1\right)i}{6AR_5 + 5BR_4 + 4CR_3 + 3DR_2 + 2ER_1 + \left(6AI_5 + 5BI_4 + 4CI_3 + 3DI_2 + 2EI_1\right)i}$$

or after rationalizing

$$A_3 = R_A + I_A i$$

and

$$A_4 = R_A - I_A i$$

Similarly, the coefficients of the terms of ψ resulting from the complex roots are

$$B_3 = R_B + I_B i$$

and

$$B_4 = R_B - I_B i$$

and the coefficients of the terms of β resulting from the complex roots are

$$C_3 = R_C + I_C i$$

and

$$C_4 = R_C - I_C i$$

The terms of ϕ corresponding to the conjugate complex roots are

$$A_3e^{\lambda_3 s_b} + A_4e^{\lambda_4 s_b} = K_A e^{R_1 s_b} \cos(I_1 s_b + \omega_A)$$

where

$$K_A = 2\sqrt{R_A{}^2 + I_A{}^2}$$

and

$$\omega_A = \tan^{-1} \frac{I_A}{R_A}$$

Similarly, these terms for ψ are

$$B_3e^{\lambda_3 s_b} + B_4e^{\lambda_4 s_b} = K_B e^{R_1 s_b} \cos(I_1 s_b + \omega_B)$$

where

$$K_B = 2\sqrt{R_B{}^2 + I_B{}^2}$$

and

$$\omega_B = \tan^{-1} \frac{I_B}{R_B}$$

and for β are

$$C_3e^{\lambda_3 s_b} + C_4e^{\lambda_4 s_b} = K_C e^{R_1 s_b} \cos(I_1 s_b + \omega_C)$$

where

$$K_C = 2\sqrt{R_C{}^2 + I_C{}^2}$$

and

$$\omega_C = \tan^{-1} \frac{I_C}{R_C}$$

The final equations corresponding to two real and two conjugate complex roots are

$$\left.\begin{aligned}
\phi &= A_1e^{\lambda_1 s_b} + A_2e^{\lambda_2 s_b} + K_Ae^{R_1 s_b}\cos\left(I_1 s_b + \omega_A\right) + A_5 s_b + A_6 \\
\psi &= B_1e^{\lambda_1 s_b} + B_2e^{\lambda_2 s_b} + K_Be^{R_1 s_b}\cos\left(I_1 s_b + \omega_B\right) + B_5 s_b + B_6 \\
\beta &= C_1e^{\lambda_1 s_b} + C_2e^{\lambda_2 s_b} + K_Ce^{R_1 s_b}\cos\left(I_1 s_b + \omega_C\right) + C_5 \\
D_b\phi &= A_1{}'e^{\lambda_1 s_b} + A_2{}'e^{\lambda_2 s_b} + K_A\sqrt{R_1^{\,2} + I_1^{\,2}}\,e^{R_1 s_b}\cos\left(I_1 s_b + \omega_A + \right. \\
&\qquad \left.\tan^{-1}\frac{I_1}{R_1}\right) + A_5 \\
D_b\psi &= B_1{}'e^{\lambda_1 s_b} + B_2{}'e^{\lambda_2 s_b} + K_B\sqrt{R_1^{\,2} + I_1^{\,2}}\,e^{R_1 s_b}\cos\left(I_1 s_b + \omega_B + \right. \\
&\qquad \left.\tan^{-1}\frac{I_1}{R_1}\right) + B_5 \\
D_b\beta &= C_1{}'e^{\lambda_1 s_b} + C_2{}'e^{\lambda_2 s_b} + K_C\sqrt{R_1^{\,2} + I_1^{\,2}}\,e^{R_1 s_b}\cos\left(I_1 s_b + \omega_C + \right. \\
&\qquad \left.\tan^{-1}\frac{I_1}{R_1}\right)
\end{aligned}\right\} \quad (1$$

Two Pairs of Conjugate Complex Roots

If two pairs of conjugate complex roots, λ_1, λ_2, and λ_3, λ_4 exist, another cosine term is introduced into equations (14) in place of the exponentials so that

$$\left.\begin{aligned}
\phi &= K_A' e^{R_1' s_b} \cos\left(I_1' s_b + \omega_A'\right) + K_A e^{R_1 s_b} \cos\left(I_1 s_b + \omega_A\right) + A_5 s_b + A_6 \\
\psi &= K_B' e^{R_1' s_b} \cos\left(I_1' s_b + \omega_B'\right) + K_B e^{R_1 s_b} \cos\left(I_1 s_b + \omega_B\right) + B_5 s_b + B_6 \\
\beta &= K_C' e^{R_1' s_b} \cos\left(I_1' s_b + \omega_C'\right) + K_C e^{R_1 s_b} \cos\left(I_1 s_b + \omega_C\right) + C_5 \\
D_b\phi &= K_A' \sqrt{R_1'^2 + I_1'^2}\, e^{R_1' s_b} \cos\left(I_1' s_b + \omega_A' + \tan^{-1} \frac{I_1'}{R_1'}\right) + \\
&\quad K_A \sqrt{R_1^2 + I_1^2}\, e^{R_1 s_b} \cos\left(I_1 s_b + \omega_A + \tan^{-1} \frac{I_1}{R_1}\right) + A_5 \\
D_b\psi &= K_B' \sqrt{R_1'^2 + I_1'^2}\, e^{R_1' s_b} \cos\left(I_1' s_b + \omega_B' + \tan^{-1} \frac{I_1'}{R_1'}\right) + \\
&\quad K_B \sqrt{R_1^2 + I_1^2}\, e^{R_1 s_b} \cos\left(I_1 s_b + \omega_B + \tan^{-1} \frac{I_1}{R_1}\right) + B_5 \\
D_b\beta &= K_C' \sqrt{R_1'^2 + I_1'^2}\, e^{R_1' s_b} \cos\left(I_1' s_b + \omega_C' + \tan^{-1} \frac{I_1'}{R_1'}\right) + \\
&\quad K_C \sqrt{R_1^2 + I_1^2}\, e^{R_1 s_b} \cos\left(I_1 s_b + \omega_C + \tan^{-1} \frac{I_1}{R_1}\right)
\end{aligned}\right\} \quad (15)$$

DISCUSSION

The lateral motions of aircraft have been obtained by means of the Laplace transform. This analysis resulted in equations from which the free lateral motion of an aircraft can be calculated for any initial condition, or the forced motion can be calculated for any constant lateral force or moment applied at zero time. In general, the lateral forces and moments applied to the airplane by control movement or atmospheric turbulence are not constant but are arbitrary functions of time. After a solution has been obtained in terms of constant disturbing forces and moments, however, the solution for the arbitrary forces and moments can be obtained by Duhamel's integral as explained in references 4 and 5.

The nature of the motion indicated by equations (13) to (15) depends upon the form of the roots of the polynomial

$$\Delta = 0$$

which is commonly referred to as the stability quartic. The roots of the quartic can take three forms - first, all four roots real; second, two real roots and a pair of conjugate complex roots; and third, two pairs of conjugate complex roots. In the case of the lateral motions of airplanes the first form almost never occurs; the second form is very common; and the third form occurs under rather rare conditions. The actual motions indicated by equations (13) to (15) can be seen to be composed, in general, of the sums of terms which are the amplitude coefficients (the A's, B's, C's, and K's of equations (13) to (15)) modulated by exponential and cosine factors.

All the classical stability concepts can be obtained from equations (13) to (15). Because stability is concerned only with the free motion (motion due to initial conditions) the forcing or disturbing quantities C_{l_c}, C_{n_c}, and C_{Y_c} can be set equal to zero so that the amplitude coefficients A_5, B_5, and C_5 vanish. The variation of the amplitude of the motion with time, which determines the stability, is now dependent entirely upon the damping coefficients $e^{\lambda_1 s_b}$, $e^{\lambda_2 s_b}$, $e^{\lambda_3 s_b}$, $e^{\lambda_4 s_b}$, $e^{R_1 s_b}$, and $e^{R_1' s_b}$. The motion diminishes with time (stable) if λ_1, λ_2, λ_3, λ_4, R_1, and R_1' are all negative. Thus, these criteria for stability are that all real roots of $\Delta = 0$ and the real parts of all complex roots be negative. These criteria have been expressed in reference 7 in terms of the signs of the coefficients of the quartic $\Delta = 0$ and the sign of Routh's discriminant which is written as

$$R = BCD - AD^2 - EB^2 \tag{16}$$

These criteria in the present case can be expressed as follows: The necessary and sufficient conditions for the real roots and the real parts of the complex roots to be negative are that every coefficient of the quartic and R should be positive.

If the motions contain oscillations, the periods of the oscillations in seconds are from equations (14) and (15)

$$\left.\begin{aligned} P &= \frac{2\pi b}{I_1 V} \\ P' &= \frac{2\pi b}{I_1' V} \end{aligned}\right\} \qquad (17)$$

and the times to damp to half-amplitude in seconds are

$$\left.\begin{aligned} T_{1/2} &= -\frac{b \log_e 2}{R_1 V} \\ T_{1/2}' &= -\frac{b \log_e 2}{R_1' V} \end{aligned}\right\} \qquad (18)$$

and the cycles to damp to half-amplitude are

$$\left.\begin{aligned} N_{1/2} &= \frac{T_{1/2}}{P} = \frac{I_1 \log_e 2}{2\pi R_1} \\ N_{1/2}' &= \frac{T_{1/2}'}{P'} = \frac{I_1' \log_e 2}{2\pi R_1'} \end{aligned}\right\} \qquad (19)$$

APPLICATION

The equations for the motion of an aircraft resulting from the analysis were used to calculate illustrative examples of certain free and forced motions of an experimental swept-wing airplane, a three-view drawing of which is shown in figure 2. The calculations were made by use of the Bell Telephone Laboratories X-66744 relay computer available at the Langley Laboratory. The calculations were based upon stability derivatives measured on a model of the experimental airplane in the

Langley stability tunnel and presented in reference 8. Motions were calculated for true airspeeds of 140 and 200 miles per hour under standard conditions. The stability derivatives, other related aerodynamic quantities, and the mass characteristics of the experimental airplane as used in the calculations are shown in table I. The coefficients of the stability quartic (equation (4)) and value of Routh's discriminant (equation (16)) were calculated as

C_L	True airspeed (mph)	A	B	C	D	E	R
0.693	140	26.19791	10.18804	3.021074	0.6312249	0.00235618	5.8
.340	200	26.20030	9.818377	2.504971	.4623735	.00014875	8.7

The positive signs of all these quantities indicate complete stability of the lateral motion for both airspeeds. The coefficients of the quartic are such as to give two real roots and a pair of conjugate complex roots which are

C_L	True airspeed (mph)	λ_1	λ_2	$R_1 \pm iI_1$
0.693	140	-0.2802853	-0.003603100	-0.05249952 ± 0.28590791
.340	200	-.2649690	-.0003222716	-.05472583 ± 0.25197541

Discussions of methods of obtaining the roots of the quartic can be found in references 9 to 12.

The first or second powers of σ multiplied by Δ in the denominators of equations (3), (5), and (6) introduce one or two zero roots, respectively, in addition to the roots given in the preceding table. These zero roots lead to the terms containing the amplitude coefficients A_5, A_6, B_5, B_6, and C_5 of equations (14). For the experimental airplane, the motion can be thought of as composed of three modes - the oscillatory mode resulting from the pair of conjugate complex roots, the rolling-subsidence mode resulting from the large negative real root λ_1, and the spiral mode resulting from the small negative real root and the zero roots. Stability of the free spiral motion is indicated by the negative sign of the small real root λ_2.

The period, time to damp to half-amplitude, and cycles to damp to half-amplitude of the oscillatory motion were calculated by use of equations (17) to (19) from the imaginary and real parts of the conjugate complex roots as

C_L	True airspeed (mph)	P (sec)	$T_{1/2}$ (sec)	$N_{1/2}$
0.693	140	3.60	2.16	0.60
.340	200	2.86	1.45	.51

The motions calculated fell into two categories which may be termed free and forced motions.

Free Motions

Free motions are those which exist following an initial condition and in the absence of any forcing function. The five possible initial conditions are ϕ_0, ψ_0, β_0, r_0, and p_0. Every free motion that the airplane is capable of executing can be obtained by superposition of the motions following these initial conditions taken separately. Figures 3 to 6 show the calculated free motion following the initial conditions ϕ_0, β_0, r_0, and p_0 for the experimental airplane in level flight according to equations (14). No airplane response to ψ_0 occurs when the angle of climb is zero. Table II gives the values of the amplitude coefficients (see equations (14)) corresponding to the motions of figures 3 to 6. These figures show the total airplane motion and show the separate contributions to the motion by the rolling-subsidence and spiral modes when the motion resulting from these modes is appreciable. Figures 3 to 6 indicate that in the case of the experimental airplane the initial condition β_0 predominately excites the oscillatory mode of motion, r_0 excites both the oscillatory and spiral modes, and ϕ_0 and p_0 predominately excite the spiral mode. The rolling-subsidence mode appears for a very short period of time in the initial phases of any motion involving appreciable rolling velocity. The principal effects upon the motions of figures 3 to 6 of increasing the airspeed from 140 to 200 miles per hour is to reduce the period as well as the time and cycles to damp to half-amplitude.

Forced Motions

Forced motions are those which exist during the action of forcing functions upon the airplane. Any forced motion of an airplane can be built up by proper superposition (Duhamel's integral, reference 4) of the motions following the application at zero time of constant values of the forcing functions C_{l_c}, C_{n_c}, and C_{Y_c}. Figures 7 and 8 show the calculated response according to equations (14) of the experimental airplane to the constant value 0.02 for C_{l_c} and C_{n_c} applied at zero time. The response to a value of 0.02 for C_{Y_c} was also calculated but during a time period of 8 seconds was negligible compared with responses resulting from C_{l_c} and C_{n_c}. For the experimental airplane, the value $C_{l_c} = 0.02$ corresponds to a total aileron deflection of 21.0°, the value of $C_{n_c} = 0.02$ corresponds to a rudder deflection of 13.7°, and the value $C_{Y_c} = 0.02$ corresponds to a rudder deflection of 7.5°. The response to C_{l_c} is predominately in the spiral mode of motion; whereas the response to C_{n_c} is predominately in the spiral and oscillatory modes.

A large number of forced motions calculated for the experimental airplane corresponding to various flight conditions are presented in reference 8. These motions were built up by superposition of motions such as those of figures 7 and 8 following the application of the constant forcing functions C_{l_c} and C_{n_c}. A large number of comparisons are made in reference 8 between calculations and flight tests for a large variety of flight conditions. The agreement between calculated and flight motions is good and indicates the practicability of analyzing the dynamic lateral flying qualities of aircraft by use of the theory of lateral dynamics such as that herein developed if experimentally determined values of the aerodynamic and mass parameters of the airplane are available. Reference 8 also indicates rather insignificant effects upon the calculated motions that result from a consideration of slight nonlinearities which occur in certain aerodynamic parameters of the experimental airplane.

CONCLUDING REMARKS

The lateral motions of aircraft were determined by means of the Laplace transform which gave solutions expressed in terms of elementary functions for the free and forced motions. These equations permit the calculation of the free motion of an aircraft following any initial condition or the forced motion following the application of constant external forces and moments. These forced motions can be used to obtain the response to any arbitrary forcing function by means of Duhamel's integral. All the classical stability concepts can be deduced from these same solutions largely by inspection. These equations for the lateral motion were applied to the calculation of the lateral stability of a specific airplane and to the calculation of certain of its free and forced motions.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., April 3, 1950

APPENDIX

EXPANSION OF THE COEFFICIENT DETERMINANTS

The stability quartic coefficients are

$$A = 8\mu_b{}^3\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right)$$

$$B = 2\mu_b{}^2\left[2C_{Y_\beta}\left(K_{XZ}{}^2 - K_X{}^2K_Z{}^2\right) + K_{XZ}\left(C_{l_r} + C_{n_p}\right) - K_X{}^2C_{n_r} - K_Z{}^2C_{l_p}\right]$$

$$C = 4\mu_b{}^2\left(K_X{}^2C_{n_\beta} - K_{XZ}C_{l_\beta}\right) + \mu_bK_X{}^2\left(C_{n_r}C_{Y_\beta} - C_{n_\beta}C_{Y_r}\right) + \mu_bK_Z{}^2\left(C_{l_p}C_{Y_\beta} - C_{l_\beta}C_{Y_p}\right) + \mu_bK_{XZ}\left(C_{l_\beta}C_{Y_r} - C_{l_r}C_{Y_\beta} + C_{n_\beta}C_{Y_p} - C_{n_p}C_{Y_\beta}\right) + \frac{1}{2}\mu_b\left(C_{l_p}C_{n_r} - C_{l_r}C_{n_p}\right)$$

$$D = 2\mu_bC_L\left[\tan\gamma\left(K_{XZ}C_{l_\beta} - K_X{}^2C_{n_\beta}\right) + \left(K_{XZ}C_{n_\beta} - K_Z{}^2C_{l_\beta}\right)\right] + \mu_b\left(C_{l_\beta}C_{n_p} - C_{l_p}C_{n_\beta}\right) + \frac{1}{4}\left(C_{l_r}C_{n_p}C_{Y_\beta} + C_{l_p}C_{n_\beta}C_{Y_r} + C_{l_\beta}C_{n_r}C_{Y_p} - C_{l_\beta}C_{n_p}C_{Y_r} - C_{l_p}C_{n_r}C_{Y_\beta} - C_{l_r}C_{n_\beta}C_{Y_p}\right)$$

$$E = \frac{1}{2}C_L\left[\tan\gamma\left(C_{l_p}C_{n_\beta} - C_{l_\beta}C_{n_p}\right) + C_{l_\beta}C_{n_r} - C_{l_r}C_{n_\beta}\right]$$

The coefficients appearing in the numerator of amplitude coefficients for ϕ are

$$a_0 = \phi_0\left[8\mu_b{}^3\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right)\right]$$

$$a_1 = \phi_0\left[2\mu_b{}^2K_{XZ}\left(C_{l_r} + C_{n_p}\right) + 4\mu_b{}^2C_{Y_\beta}\left(K_{XZ}{}^2 - K_X{}^2K_Z{}^2\right) - 2\mu_b{}^2\left(K_X{}^2C_{n_r} + K_Z{}^2C_{l_p}\right)\right] + \left(D_b\phi\right)_0\left[8\mu_b{}^3\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right)\right]$$

$$a_2 = \phi_0\left[\mu_bK_X{}^2\left(C_{n_r}C_{Y_\beta} - C_{n_\beta}C_{Y_r}\right) + 4\mu_b{}^2\left(K_X{}^2C_{n_\beta} - K_{XZ}C_{l_\beta}\right) + \mu_bK_Z{}^2\left(C_{l_p}C_{Y_\beta} - C_{l_\beta}C_{Y_p}\right) + \mu_bK_{XZ}\left(C_{n_\beta}C_{Y_p} - C_{n_p}C_{Y_\beta} + C_{l_\beta}C_{Y_r} - C_{l_r}C_{Y_\beta}\right) + \frac{1}{2}\mu_b\left(C_{l_p}C_{n_r} - C_{l_r}C_{n_p}\right)\right] + \left(D_b\phi\right)_0\left[4\mu_b{}^2C_{Y_\beta}\left(K_{XZ}{}^2 - K_X{}^2K_Z{}^2\right) + 2\mu_b{}^2\left(K_{XZ}C_{l_r} - K_X{}^2C_{n_r}\right)\right] + \left(D_b\psi\right)_0\left[2\mu_b{}^2\left(K_Z{}^2C_{l_r} - K_{XZ}C_{n_r}\right)\right] + \beta_0\left[4\mu_b{}^2\left(K_Z{}^2C_{l_\beta} - K_{XZ}C_{n_\beta}\right)\right] + 4\mu_b{}^2\left(K_Z{}^2C_{l_c} - K_{XZ}C_{n_c}\right)$$

$$a_3 = \phi_0\left[2\mu_b \tan\gamma C_L\left(K_{XZ}C_{l_\beta} - K_X^2C_{n_\beta}\right) + \mu_b\left(C_{l_\beta}C_{n_p} - C_{l_p}C_{n_\beta}\right) + \right.$$

$$\frac{1}{4}\left(C_{l_r}C_{n_p}C_{Y_\beta} + C_{l_p}C_{n_\beta}C_{Y_r} + C_{l_\beta}C_{n_r}C_{Y_p} - C_{l_\beta}C_{n_p}C_{Y_r} - C_{l_p}C_{n_r}C_{Y_\beta} - \right.$$

$$\left.\left.C_{l_r}C_{n_\beta}C_{Y_p}\right)\right] + \left(D_b\phi\right)_0\left[4\mu_b^2\left(K_X^2C_{n_\beta} - K_{XZ}C_{l_\beta}\right) + \mu_bK_X^2\left(C_{n_r}C_{Y_\beta} - \right.\right.$$

$$\left.\left.C_{n_\beta}C_{Y_r}\right) + \mu_bK_{XZ}\left(C_{l_\beta}C_{Y_r} - C_{l_r}C_{Y_\beta}\right)\right] + \psi_0\left[2\mu_b \tan\gamma C_L\left(K_Z^2C_{l_\beta} - \right.\right.$$

$$\left.\left.K_{XZ}C_{n_\beta}\right)\right] + \left(D_b\psi\right)_0\left[\mu_bK_Z^2\left(C_{l_\beta}C_{Y_r} - C_{l_r}C_{Y_\beta}\right) + 4\mu_b^2\left(K_{XZ}C_{n_\beta} - K_Z^2C_{l_\beta}\right) + \right.$$

$$\left.\mu_bK_{XZ}\left(C_{n_r}C_{Y_\beta} - C_{n_\beta}C_{Y_r}\right)\right] + \beta_0\left[\mu_b\left(C_{l_r}C_{n_\beta} - C_{l_\beta}C_{n_r}\right)\right] +$$

$$2\mu_bK_Z^2\left(C_{l_\beta}C_{Y_c} - C_{l_c}C_{Y_\beta}\right) + 2\mu_bK_{XZ}\left(C_{n_c}C_{Y_\beta} - C_{n_\beta}C_{Y_c}\right) +$$

$$\mu_b\left(C_{l_r}C_{n_c} - C_{l_c}C_{n_r}\right)$$

$$a_4 = \phi_0\left[\frac{1}{2} \tan \gamma C_L \left(C_{l_p}C_{n_\beta} - C_{l_\beta}C_{n_p}\right)\right] + \left(D_b\phi\right)_0\left[2\mu_b \tan \gamma C_L \left(K_{XZ}C_{l_\beta} - K_X{}^2C_{n_\beta}\right)\right] + \psi_0\left[\frac{1}{2} \tan \gamma \; C_L\left(C_{l_r}C_{n_\beta} - C_{l_\beta}C_{n_r}\right)\right] +$$

$$\left(D_b\psi\right)_0\left[2\mu_b \tan \gamma C_L \left(K_Z{}^2C_{l_\beta} - K_{XZ}C_{n_\beta}\right)\right] + \left[2\mu_b\left(C_{l_c}C_{n_\beta} - C_{l_\beta}C_{n_c}\right) + \frac{1}{2}C_{l_c}\left(C_{n_r}C_{Y_\beta} - C_{n_\beta}C_{Y_r}\right) + \frac{1}{2} C_{n_c}\left(C_{l_\beta}C_{Y_r} - C_{l_r}C_{Y_\beta}\right) + \frac{1}{2} C_{Y_c}\left(C_{l_r}C_{n_\beta} - C_{l_\beta}C_{n_r}\right)\right]$$

$$a_5 = \tan \gamma C_L \left(C_{l_\beta}C_{n_c} - C_{l_c}C_{n_\beta}\right)$$

The coefficients appearing in the numerator of amplitude coefficients for ψ are

$$b_0 = \psi_0\left[8\mu_b{}^3\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right)\right]$$

$$b_1 = \psi_0\left[2\mu_b{}^2K_{XZ}\left(C_{n_p} + C_{l_r}\right) - 4\mu_b{}^2C_{Y_\beta}\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right) - 2\mu_b{}^2\left(K_Z{}^2C_{l_p} + K_X{}^2C_{n_r}\right)\right] + \left(D_b\psi\right)_0\left[8\mu_b{}^3\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right)\right]$$

$$b_2 = \left(D_b\phi\right)_0\left[2\mu_b{}^2\left(K_X{}^2C_{n_p} - K_{XZ}C_{l_p}\right)\right] + \psi_0\left[\mu_bK_X{}^2\left(C_{n_r}C_{Y_\beta} - C_{n_\beta}C_{Y_r}\right) + \right.$$

$$\mu_bK_Z{}^2\left(C_{l_p}C_{Y_\beta} - C_{l_\beta}C_{Y_p}\right) + \mu_bK_{XZ}\left(C_{n_\beta}C_{Y_p} - C_{n_p}C_{Y_\beta} + C_{l_\beta}C_{Y_r} - C_{l_r}C_{Y_\beta}\right) +$$

$$\left. 4\mu_b{}^2\left(K_X{}^2C_{n_\beta} - K_{XZ}C_{l_\beta}\right) + \frac{1}{2}\mu_b\left(C_{l_p}C_{n_r} - C_{l_r}C_{n_p}\right)\right] +$$

$$\left(D_b\psi\right)_0\left[2\mu_b{}^2\left(K_{XZ}C_{n_p} - K_Z{}^2C_{l_p}\right) - 4\mu_b{}^2C_{Y_\beta}\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right)\right] +$$

$$\beta_0\left[4\mu_b{}^2\left(K_X{}^2C_{n_\beta} - K_{XZ}C_{l_\beta}\right)\right] + 4\mu_b{}^2\left(K_X{}^2C_{n_c} - K_{XZ}C_{l_c}\right)$$

$$b_3 = \phi_0\left[2\mu_bC_L\left(K_X{}^2C_{n_\beta} - K_{XZ}C_{l_\beta}\right)\right] + \left(D_b\phi\right)_0\left[\mu_bK_X{}^2\left(C_{n_\beta}C_{Y_p} - C_{n_p}C_{Y_\beta}\right) + \right.$$

$$\left.\mu_bK_{XZ}\left(C_{l_p}C_{Y_\beta} - C_{l_\beta}C_{Y_p}\right)\right] + \psi_0\left[\mu_b\left(C_{l_\beta}C_{n_p} - C_{l_p}C_{n_\beta}\right) - 2\mu_bC_L\left(K_Z{}^2C_{l_\beta} - \right.\right.$$

$$\left. K_{XZ}C_{n_\beta}\right) + \frac{1}{4}\left(C_{l_r}C_{n_p}C_{Y_\beta} + C_{l_p}C_{n_\beta}C_{Y_r} + C_{l_\beta}C_{n_r}C_{Y_p} - C_{l_\beta}C_{n_p}C_{Y_r} - \right.$$

$$\left.\left. C_{l_p}C_{n_r}C_{Y_\beta} - C_{l_r}C_{n_\beta}C_{Y_p}\right)\right] + \left(D_b\psi\right)_0\left[\mu_bK_Z{}^2\left(C_{l_p}C_{Y_\beta} - C_{l_\beta}C_{Y_p}\right) + \right.$$

$$\left.\mu_bK_{XZ}\left(C_{n_\beta}C_{Y_p} - C_{n_p}C_{Y_\beta}\right)\right] + \beta_0\left[\mu_b\left(C_{l_\beta}C_{n_p} - C_{l_p}C_{n_\beta}\right)\right] +$$

$$\mu_b\left(C_{l_c}C_{n_p} - C_{l_p}C_{n_c}\right) + 2\mu_bK_{XZ}\left(C_{l_c}C_{Y_\beta} - C_{l_\beta}C_{Y_c}\right) +$$

$$2\mu_bK_X{}^2\left(C_{n_\beta}C_{Y_c} - C_{n_c}C_{Y_\beta}\right)$$

$$b_4 = \phi_0\left[\frac{1}{2}C_L\left(C_{l_\beta}C_{n_p} - C_{l_p}C_{n_\beta}\right)\right] + \left(D_b\phi\right)_0\left[2\mu_b C_L\left(K_X{}^2C_{n_\beta} - K_{XZ}C_{l_\beta}\right)\right] +$$

$$\psi_0\left[\frac{1}{2}C_L\left(C_{l_\beta}C_{n_r} - C_{l_r}C_{n_\beta}\right)\right] + \left(D_b\psi\right)_0\left[2\mu_b C_L\left(K_{XZ}C_{n_\beta} - K_Z{}^2C_{l_\beta}\right)\right] +$$

$$\frac{1}{2}C_{l_c}\left(C_{n_\beta}C_{Y_p} - C_{n_p}C_{Y_\beta}\right) + \frac{1}{2}C_{n_c}\left(C_{l_p}C_{Y_\beta} - C_{l_\beta}C_{Y_p}\right) +$$

$$\frac{1}{2}C_{Y_c}\left(C_{l_\beta}C_{n_p} - C_{l_p}C_{n_\beta}\right)$$

$$b_5 = C_L\left(C_{l_c}C_{n_\beta} - C_{l_\beta}C_{n_c}\right)$$

The coefficients appearing in the numerator of amplitude coefficients for β are

$$c_0 = \beta_0\left[8\mu_b{}^3\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right)\right]$$

$$c_1 = \phi_0\left[4\mu_b{}^2C_L\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right)\right] + \left(D_b\phi\right)_0\left[2\mu_b{}^2C_{Y_p}\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right)\right] +$$

$$\psi_0\left[4\mu_b{}^2 \tan\gamma C_L\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right)\right] + \left(D_b\psi\right)_0\left[2\mu_b{}^2\left(C_{Y_r} - 4\mu_b\right)\left(K_X{}^2K_Z{}^2 -\right.\right.$$

$$\left.\left.K_{XZ}{}^2\right)\right] + \beta_0\left[2\mu_b{}^2\left(K_{XZ}C_{l_r} - K_X{}^2C_{n_r}\right) + 2\mu_b{}^2\left(K_{XZ}C_{n_p} - K_Z{}^2C_{l_p}\right)\right] +$$

$$4\mu_b{}^2C_{Y_c}\left(K_X{}^2K_Z{}^2 - K_{XZ}{}^2\right)$$

$$c_2 = \phi_0\left[\mu_b C_L\left(K_{XZ}C_{l_r} - K_X{}^2 C_{n_r}\right) + \mu_b C_L\left(K_{XZ}C_{n_p} - K_Z{}^2 C_{l_p}\right)\right] +$$

$$\left(D_b\phi\right)_0\left[\frac{1}{2}\mu_b K_X{}^2\left(C_{n_p}C_{Y_r} - C_{n_r}C_{Y_p}\right) + 2\mu_b{}^2 K_X{}^2\left(2K_Z{}^2 C_L - C_{n_p}\right) +\right.$$

$$\left.2\mu_b{}^2 K_{XZ}\left(C_{l_p} - 2K_{XZ}C_L\right) + \frac{1}{2}\mu_b K_{XZ}\left(C_{l_r}C_{Y_p} - C_{l_p}C_{Y_r}\right)\right] +$$

$$\psi_0\left[\mu_b \tan\gamma C_L\left(K_{XZ}C_{l_r} - C_{n_r}K_X{}^2\right) + \mu_b \tan\gamma C_L\left(K_{XZ}C_{n_p} - K_Z{}^2 C_{l_p}\right)\right] +$$

$$\left(D_b\psi\right)_0\left[\frac{1}{2}\mu_b K_{XZ}\left(C_{n_p}C_{Y_r} - C_{n_r}C_{Y_p}\right) + \frac{1}{2}\mu_b K_Z{}^2\left(C_{l_r}C_{Y_p} - C_{l_p}C_{Y_r}\right) +\right.$$

$$\left.4\mu_b{}^2 \tan\gamma C_L\left(K_X{}^2 K_Z{}^2 - K_{XZ}{}^2\right) + 2\mu_b{}^2\left(K_Z{}^2 C_{l_p} - K_{XZ}C_{n_p}\right)\right] +$$

$$\beta_0\left[\frac{1}{2}\mu_b\left(C_{l_p}C_{n_r} - C_{l_r}C_{n_p}\right)\right] + 4\mu_b{}^2\left(K_{XZ}C_{l_c} - K_X{}^2 C_{n_c}\right) + \mu_b K_Z{}^2\left(C_{l_c}C_{Y_p} -\right.$$

$$\left.C_{l_p}C_{Y_c}\right) + \mu_b K_X{}^2\left(C_{n_c}C_{Y_r} - C_{n_r}C_{Y_c}\right) + \mu_b K_{XZ}\left(C_{n_p}C_{Y_c} - C_{n_c}C_{Y_p}\right) +$$

$$\mu_b K_{XZ}\left(C_{l_r}C_{Y_c} - C_{l_c}C_{Y_r}\right)$$

$$c_3 = \phi_0\left[\frac{1}{4}C_L\left(C_{l_p}C_{n_r} - C_{l_r}C_{n_p}\right)\right] + \left(D_b\phi\right)_0\left[\mu_b C_L \tan\gamma\left(K_X{}^2 C_{n_p} - K_{XZ}C_{l_p}\right) + \right.$$

$$\left.\mu_b C_L\left(K_{XZ}C_{l_r} - K_X{}^2 C_{n_r}\right)\right] + \psi_0\left[\frac{1}{4}\tan\gamma C_L\left(C_{l_p}C_{n_r} - C_{l_r}C_{n_p}\right)\right] +$$

$$\left(D_b\psi\right)_0\left[\mu_b C_L\left(K_Z{}^2 C_{l_r} - K_{XZ}C_{n_r}\right) + \mu_b \tan\gamma C_L\left(K_{XZ}C_{n_p} - K_Z{}^2 C_{l_p}\right)\right] +$$

$$\mu_b\left(C_{l_p}C_{n_c} - C_{l_c}C_{n_p}\right) + 2\mu_b C_L\left(K_Z{}^2 C_{l_c} - K_{XZ}C_{n_c}\right) +$$

$$2\mu_b \tan\gamma C_L\left(K_X{}^2 C_{n_c} - K_{XZ}C_{l_c}\right) + \frac{1}{4}\left(C_{l_c}C_{n_p}C_{Y_r} + C_{l_p}C_{n_r}C_{Y_c} + \right.$$

$$\left.C_{l_r}C_{n_c}C_{Y_p} - C_{l_r}C_{n_p}C_{Y_c} - C_{l_p}C_{n_c}C_{Y_r} - C_{l_c}C_{n_r}C_{Y_p}\right)$$

$$c_4 = \frac{1}{2}C_L\left(C_{l_r}C_{n_c} - C_{l_c}C_{n_r}\right) + \frac{1}{2}\tan\gamma C_L\left(C_{l_c}C_{n_p} - C_{l_p}C_{n_c}\right)$$

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TABLE I

AERODYNAMIC AND MASS CHARACTERISTICS OF AN

EXPERIMENTAL SWEPT-WING AIRPLANE

^{1}V	140	200
W	8700	8700
S	250	250
m	270.2	270.2
ρ	0.00238	0.00238
γ	0	0
b	33.6	33.6
C_L	0.693	0.340
α	10.6	4.8
η	11.05	5.25
μ_b	13.51	13.51
V/b	6.111	8.730
$K_X{}^2$	0.02329	0.02219
$K_Z{}^2$	0.05932	0.06042
K_{XZ}	0.007316	0.003544
C_{l_β}	-0.0659	-0.0275
C_{n_β}	0.100	0.0975
C_{Y_β}	-0.739	-0.722
C_{l_p}	-0.325	-0.31
C_{n_p}	-0.1	-0.06
C_{Y_p}	0.44	0.3
C_{l_r}	0.12	0.07
C_{n_r}	-0.280	-0.280
C_{Y_r}	0.36	0.40

[1]Miles-per-hour units.

TABLE II

AMPLITUDE COEFFICIENTS OF FREE AND FORCED MOTIONS OF EXPERIMENTAL SWEPT-WING AIRPLANE

(a) V = 140 miles per hour.

Initial conditions and forcing functions	Variables	Rolling-subsidence mode		Oscillatory mode		Spiral mode					
	ϕ	A_1	0.04073926	K_A	0.05404332	A_2	0.4374647	A_5	0	A_6	0
	ψ	B_1	-.00222650	K_B	.04009448	B_2	-3.038911	B_5	0	B_6	3.029296
ϕ_0	β	C_1	-.00131258	K_C	.04330260	C_2	.01392006	C_5	0	--	------------
	p	A_1'	-.06978088	$K_A\sqrt{R_1^2+I_1^2}$	.09600416	A_2'	-.00963249	A_5'	0	--	------------
	r	B_1'	.00381366	$K_B\sqrt{R_1^2+I_1^2}$	.07122481	B_2'	.06691349	B_5'	0	--	------------
	ϕ	A_1	-.01808626	K_A	.2450096	A_2	-.02458282	A_5	0	A_6	0
	ψ	B_1	.00973284	K_B	.18177064	B_2	.17076788	B_5	0	B_6	0
β_0	β	C_1	.00573756	K_C	.19631484	C_2	.00078222	C_5	0	--	------------
	p	A_1'	.30503482	$K_A\sqrt{R_1^2+I_1^2}$	.43524085	A_2	.00054129	A_5'	0	--	------------
	r	B_1'	-.01667089	$K_B\sqrt{R_1^2+I_1^2}$	.3229020	B_2'	-.00376012	B_5'	0	--	------------
	ϕ	A_1	-.27132714	K_A	.02412880	A_2	28175964	A_5	0	A_6	0
	ψ	B_1	.01482880	K_B	.01790107	B_2	-1.9572863	B_5	0	B_6	1.9260299
p_0	β	C_1	.00874177	K_C	.01933340	C_2	.00896556	C_5	0	--	------------
	p	A_1'	.46473072	$K_A\sqrt{R_1^2+I_1^2}$	.04286193	A_2'	-.00620389	A_5'	0	--	------------
	r	B_1'	-.02539885	$K_B\sqrt{R_1^2+I_1^2}$	.03179911	B_2'	.04309623	B_5'	0	--	------------
	ϕ	A_1	-.21275154	K_A	.35205361	A_2	.46209967	A_5	0	A_6	0
	ψ	B_1	.01162773	K_B	.26118594	B_2	-3.2100413	B_5	0	B_6	3.1796150
r_0	β	C_1	.00685435	K_C	.28208436	C_2	.01470395	C_5	0	--	------------
	p	A_1'	.36440206	$K_A\sqrt{R_1^2+I_1^2}$	.62538141	A_2'	-.01017468	A_5'	0	--	------------
	r	B_1'	-.01991605	$K_B\sqrt{R_1^2+I_1^2}$	.46396553	B_2'	.07067985	B_5'	0	--	------------
	ϕ	A_1	.3534235	K_A	.07815380	A_2	-25.21345	A_5	0	A_6	24.93682
	ψ	B_1	-.01931556	K_B	.05798158	B_2	175.1489	B_5	.6199628	B_6	-175.1797
C_{l_c}	β	C_1	-.01138685	K_C	.06262090	C_2	-.8022885	C_5	.8679479	--	------------
	p	A_1'	-.60536104	$K_A\sqrt{R_1^2+I_1^2}$	.13883429	A_2'	.55517272	A_5'	0	--	------------
	r	B_1'	.03308464	$K_B\sqrt{R_1^2+I_1^2}$	.1029990	B_2'	-3.8565875	B_5'	3.7886547	--	------------
	ϕ	A_1	.07219731	K_A	.1935925	A_2	-16.45365	A_5	0	A_6	16.22009
	ψ	B_1	-.00394581	K_B	.1436248	B_2	114.2976	B_5	.4085555	B_6	-114.1513
C_{n_c}	β	C_1	-.00232607	K_C	.1551168	C_2	-.5235526	C_5	.3719777	--	------------
	p	A_1'	-.12366306	$K_A\sqrt{R_1^2+I_1^2}$	.34390240	A_2'	.36229131	A_5'	0	--	------------
	r	B_1'	.00675858	$K_B\sqrt{R_1^2+I_1^2}$	.25513879	B_2'	-2.5167086	B_5'	2.4967235	--	------------
	ϕ	A_1	.00235150	K_A	.00311940	A_2	.02525049	A_5	0	A_6	-.02886004
	ψ	B_1	-.00012851	K_B	.00231425	B_2	-.1754060	B_5	0	B_6	.1748510
C_{Y_c}	β	C_1	-.00007576	K_C	.00249943	C_2	.00080347	C_5	0	--	------------
	p	A_1'	-.00402776	$K_A\sqrt{R_1^2+I_1^2}$	.00554138	A_2'	-.00055599	A_5'	0	--	------------
	r	B_1'	.00022013	$K_B\sqrt{R_1^2+I_1^2}$	.00411110	B_2'	.00386225	B_5'	0	--	------------

TABLE II

AMPLITUDE COEFFICIENTS OF FREE AND FORCED MOTIONS OF EXPERIMENTAL SWEPT-WING AIRPLANE - Concluded

(b) V = 200 miles per hour.

Initial conditions and forcing functions	Variables	Rolling-subsidence mode		Oscillatory mode		Spiral mode					
	ϕ	A_1	0.01197011	K_A	0.01640600	A_2	0.4807800	A_5	0	A_6	0
	ψ	B_1	-.00027474	K_B	.02347936	B_2	-18.21875	B_5	0	B_6	18.21429
ϕ_0	β	C_1	-.0003213	K_C	.02426556	C_2	.0083529	C_5	0	--	------------
	p	A_1'	-.02769022	$K_A\sqrt{R_1^2+I_1^2}$	.03693084	A_2'	-.00135277	A_5'	0	--	------------
	r	B_1'	.00049674	$K_B\sqrt{R_1^2+I_1^2}$	.05285335	B_2'	.05126683	B_5'	0	--	------------
	ϕ	A_1	-.10082626	K_A	.13447276	A_2	-.00492530	A_5	0	A_6	0
	ψ	B_1	.00180874	K_B	.19245072	B_2	.18665748	B_5	0	B_6	0
β_0	β	C_1	.00270588	K_C	.19889500	C_2	-.00008558	C_5	0	--	------------
	p	A_1'	.23323073	$K_A\sqrt{R_1^2+I_1^2}$	.30270574	A_2'	.00001386	A_5'	0	--	------------
	r	B_1'	-.00418398	$K_B\sqrt{R_1^2+I_1^2}$	.43321690	B_2'	-.00052525	B_5'	0	--	------------
	ϕ	A_1	-.2093808	K_A	.00718425	A_2	.2112909	A_5	0	A_6	0
	ψ	B_1	.00375619	K_B	.01028182	B_2	-8.006691	B_5	0	B_6	7.995698
p_0	β	C_1	.00561934	K_C	.01062610	C_2	.00367088	C_5	0	--	------------
	p	A_1'	.48433861	$K_A\sqrt{R_1^2+I_1^2}$	.01617213	A_2'	-.00059456	A_5'	0	--	------------
	r	B_1'	-.00868879	$K_B\sqrt{R_1^2+I_1^2}$	.02314494	B_2'	.02253050	B_5'	0	--	------------
	ϕ	A_1	-.07670895	K_A	.1518581	A_2	.1869734	A_5	0	A_6	0
	ψ	B_1	.00137619	K_B	.2173312	B_2	-7.085196	B_5	0	B_6	7.097827
r_0	β	C_1	.00205853	K_C	.2246084	C_2	.00324842	C_5	0	--	------------
	p	A_1'	.17744277	$K_A\sqrt{R_1^2+I_1^2}$	.34184086	A_2'	-.00052614	A_5'	0	--	------------
	r	B_1'	-.00318338	$K_B\sqrt{R_1^2+I_1^2}$	.48922468	B_2'	.01993745	B_5'	0	--	------------
	ϕ	A_1	.4547069	K_A	.03147098	A_2	-365.6037	A_5	0	A_6	365.1805
	ψ	B_1	-.00815719	K_B	.04503932	B_2	13855.46	B_5	4.457143	B_6	-13855.50
C_{l_c}	β	C_1	-.01220331	K_C	.04654752	C_2	-6.351295	C_5	6.400000	--	------------
	p	A_1'	-1.0518225	$K_A\sqrt{R_1^2+I_1^2}$	.07084298	A_2'	1.0286136	A_5'	0	--	------------
	r	B_1'	.01886912	$K_B\sqrt{R_1^2+I_1^2}$	.10138614	B_2'	-38.981861	B_5'	38.911304	--	------------
	ϕ	A_1	.03526760	K_A	.1276446	A_2	-102.7051	A_5	0	A_6	102.5640
	ψ	B_1	-.00063272	K_B	.1826785	B_2	3892.267	B_5	1.257143	B_6	-3892.093
C_{n_c}	β	C_1	-.00094653	K_C	.1887956	C_2	-1.784201	C_5	1.600000	--	------------
	p	A_1'	-.08158060	$K_A\sqrt{R_1^2+I_1^2}$	.28733535	A_2'	.28895732	A_5'	0	--	------------
	r	B_1'	.00146360	$K_B\sqrt{R_1^2+I_1^2}$	.41121949	B_2'	-10.950758	B_5'	10.974984	--	------------
	ϕ	A_1	.00140829	K_A	.00193011	A_2	.05655717	A_5	0	A_6	-.05882356
	ψ	B_1	-.00002526	K_B	.00276228	B_2	-2.143383	B_5	0	B_6	2.142857
C_{Y_c}	β	C_1	-.00003779	K_C	.00285477	C_2	.00098269	C_5	0	--	------------
	p	A_1'	-.00325766	$K_A\sqrt{R_1^2+I_1^2}$	.00434479	A_2'	-.00015915	A_5'	0	--	------------
	r	B_1'	.00005844	$K_B\sqrt{R_1^2+I_1^2}$	.00621801	B_2'	.00603139	B_5'	0	--	------------

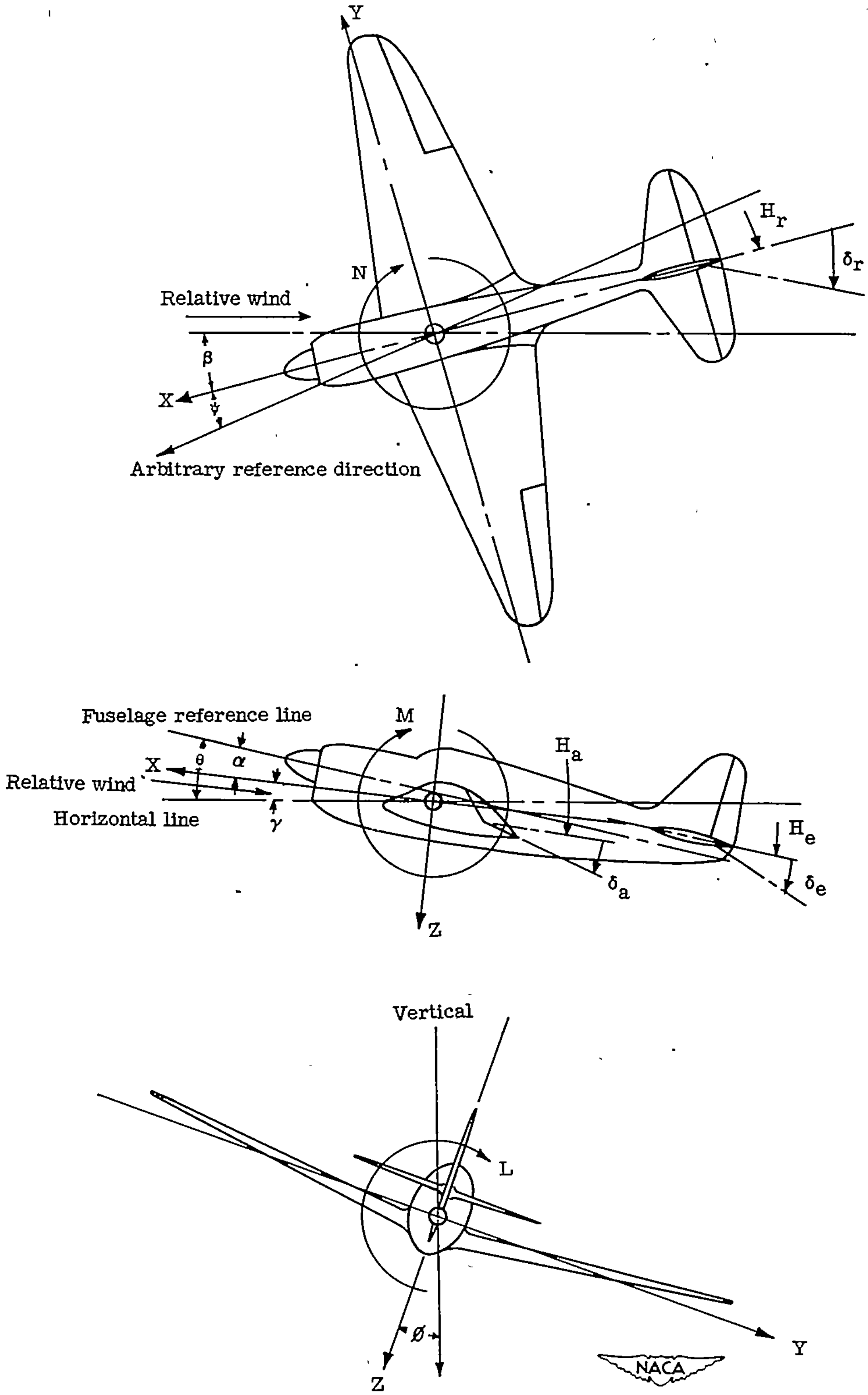

Figure 1.– Stability axis system. Positive values of forces, moments, and angles are indicated by arrows.

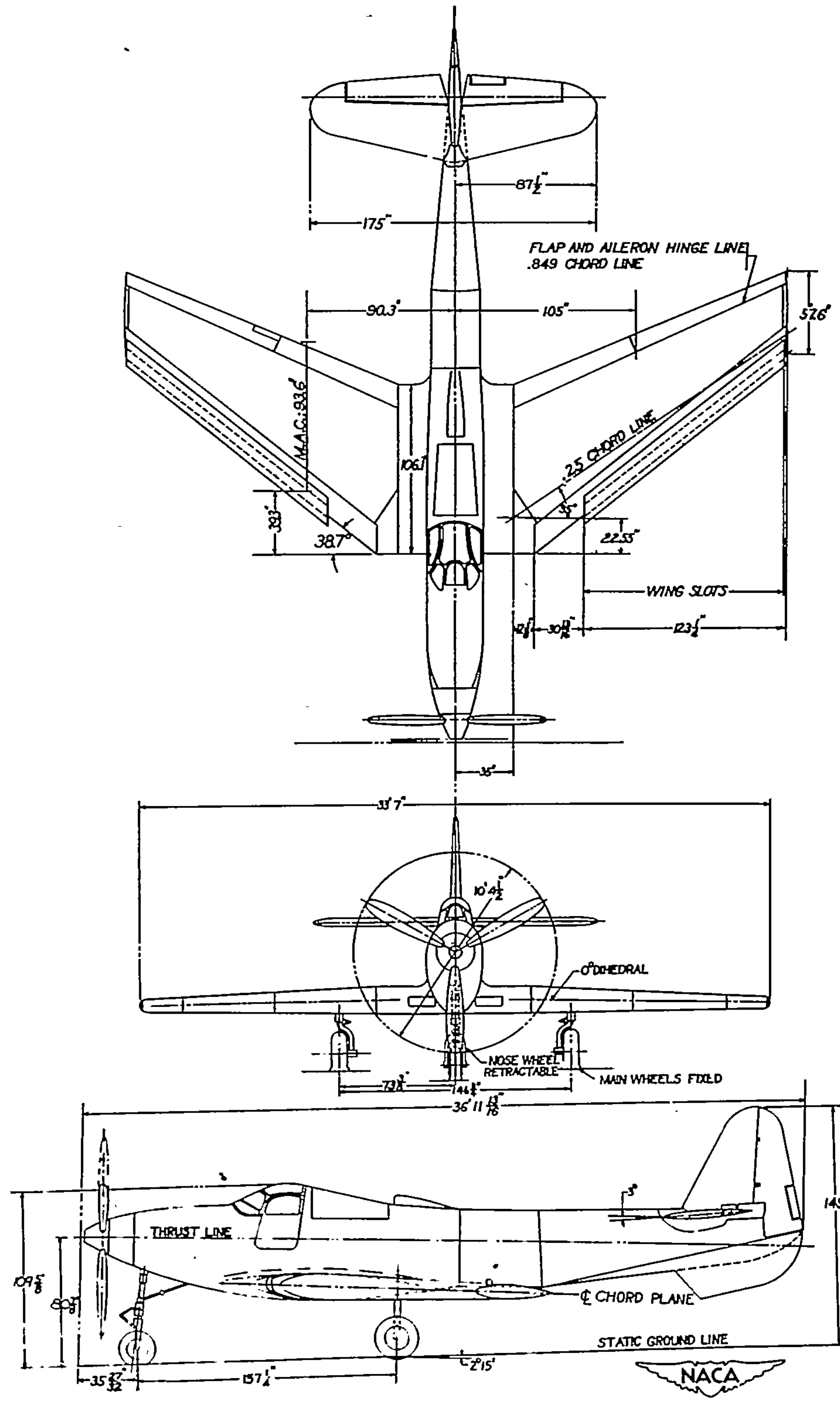

Figure 2.- Three-view drawing of experimental swept-wing airplane.

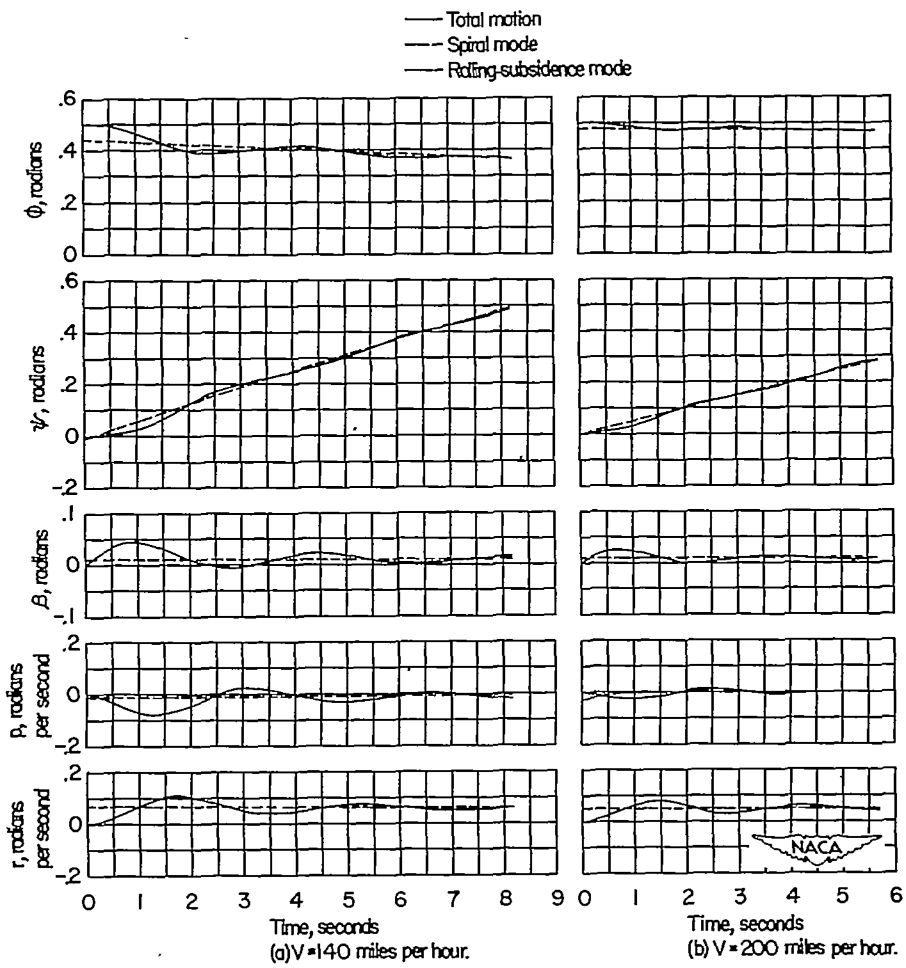

Figure 3.— Response of experimental swept-wing airplane to an initial angle of bank. $\phi_0 = 0.5$ radian; $\gamma = 0^{\circ}$.

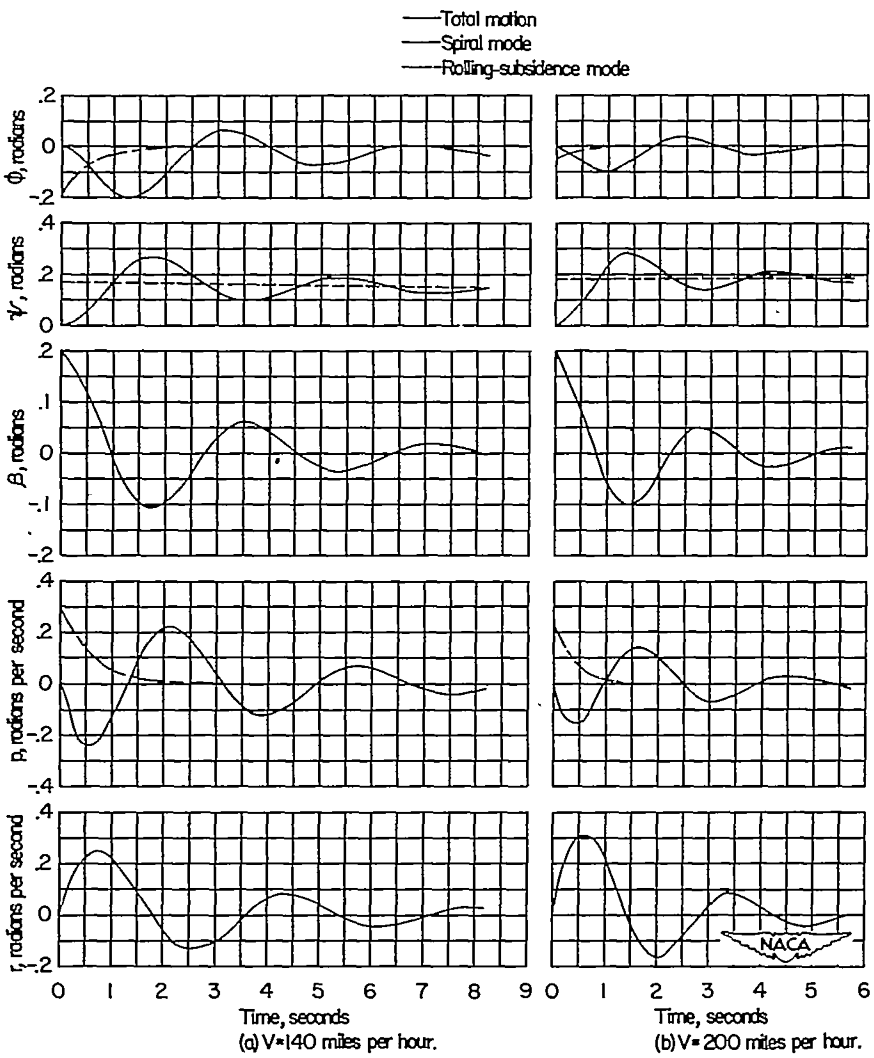

Figure 4.– Response of experimental swept-wing airplane to an initial sideslip angle. $\beta_0 = 0.2$ radian; $\gamma = 0^{\circ}$.

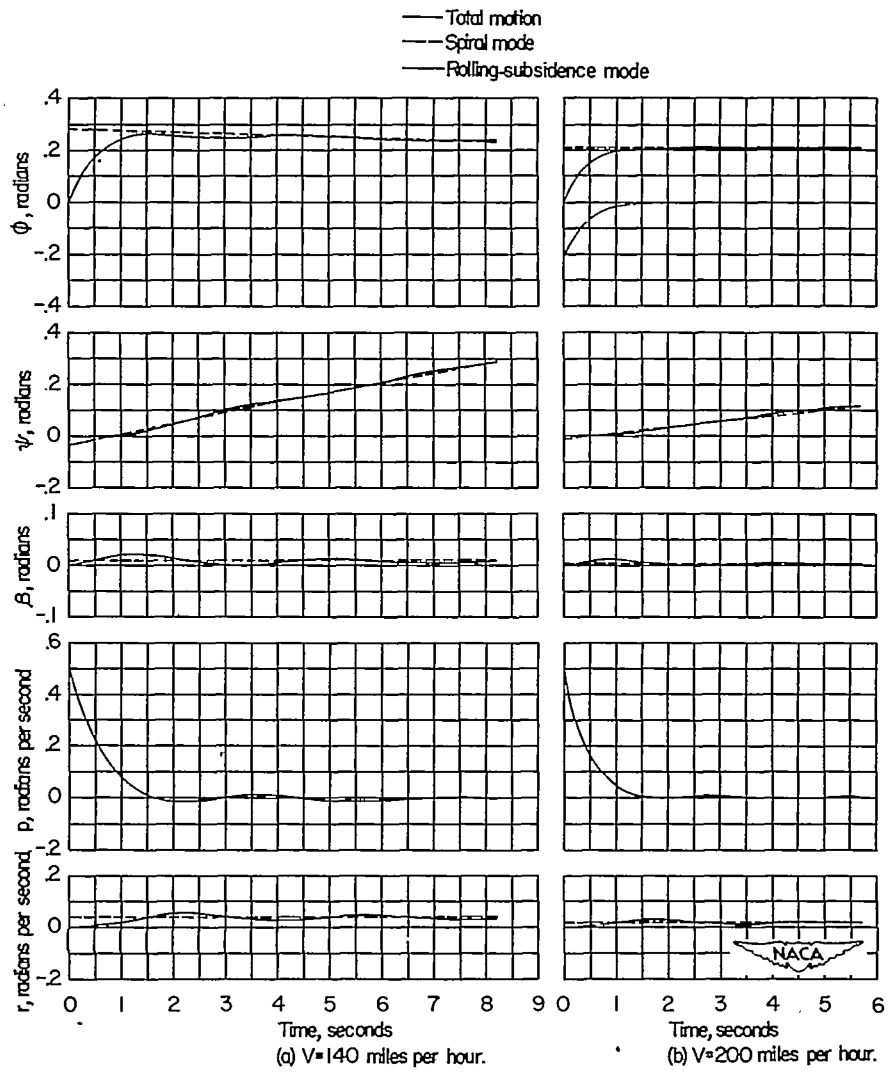

Figure 5.– Response of experimental swept-wing airplane to an initial rolling velocity. $p_0 = 0.5$ radian per second; $\gamma = 0^{\circ}$.

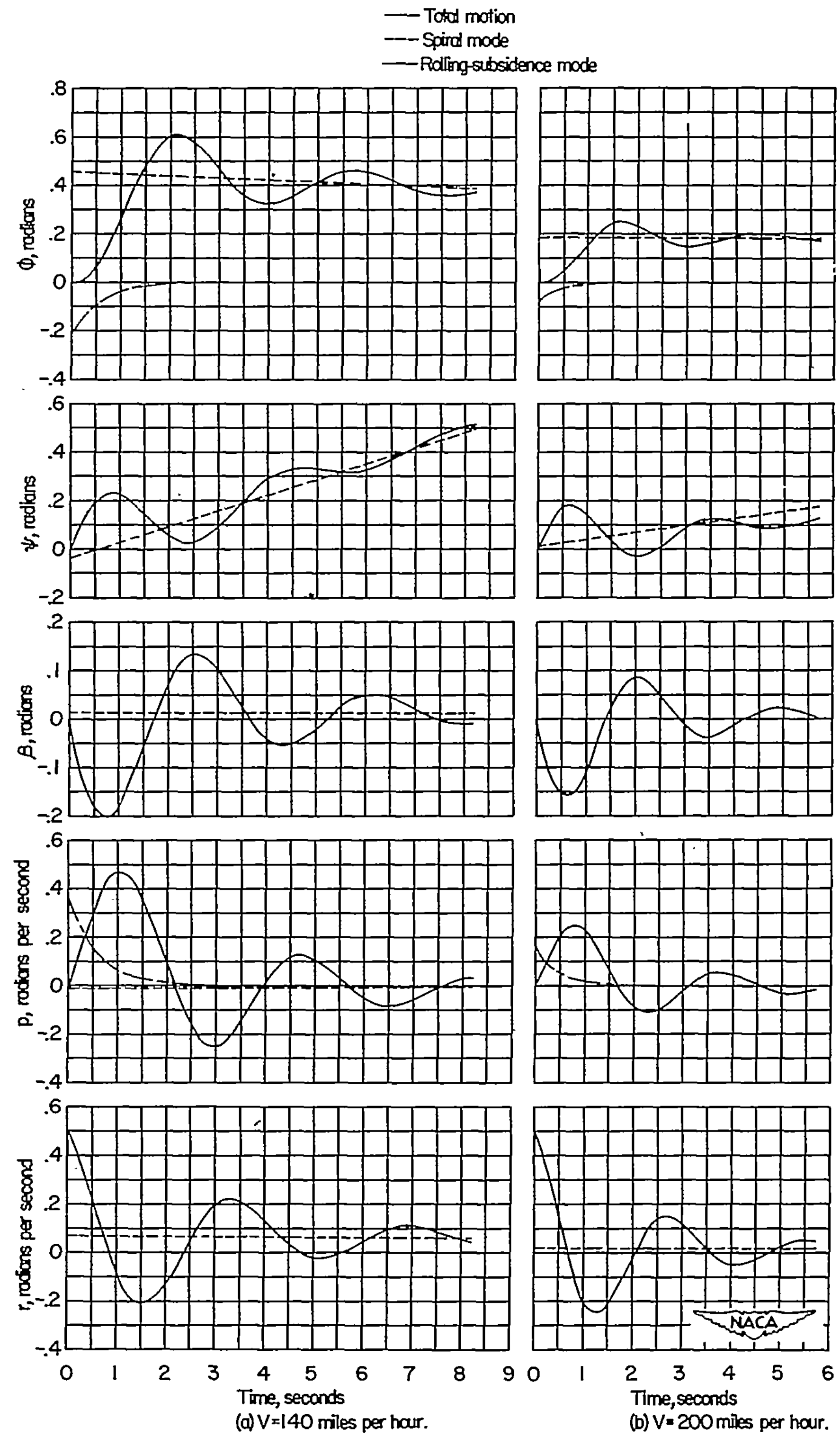

Figure 6.– Response of experimental swept-wing airplane to an initial yawing velocity. $r_0 = 0.5$ radian per second; $\gamma = 0^{\circ}$.

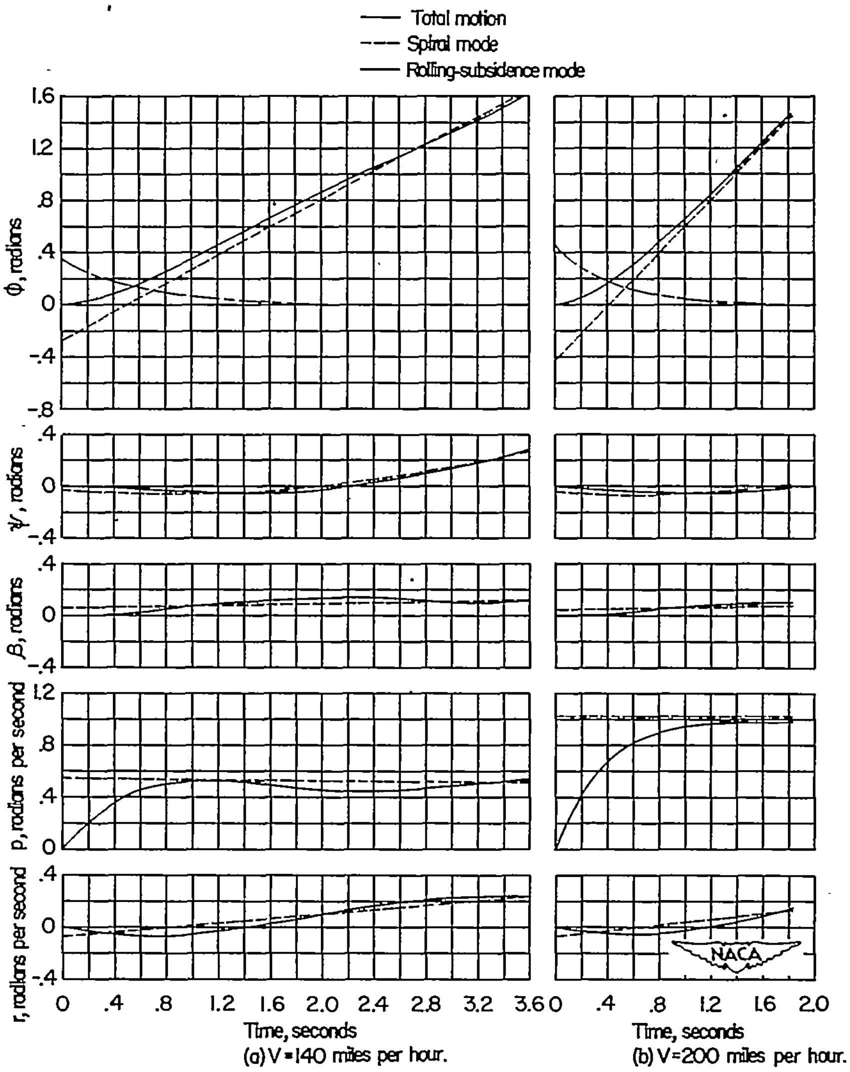

Figure 7.— Response of experimental swept-wing airplane to the application at zero time of a constant rolling-moment coefficient. $C_{l_c} = 0.02$; $\gamma = 0^{\circ}$.

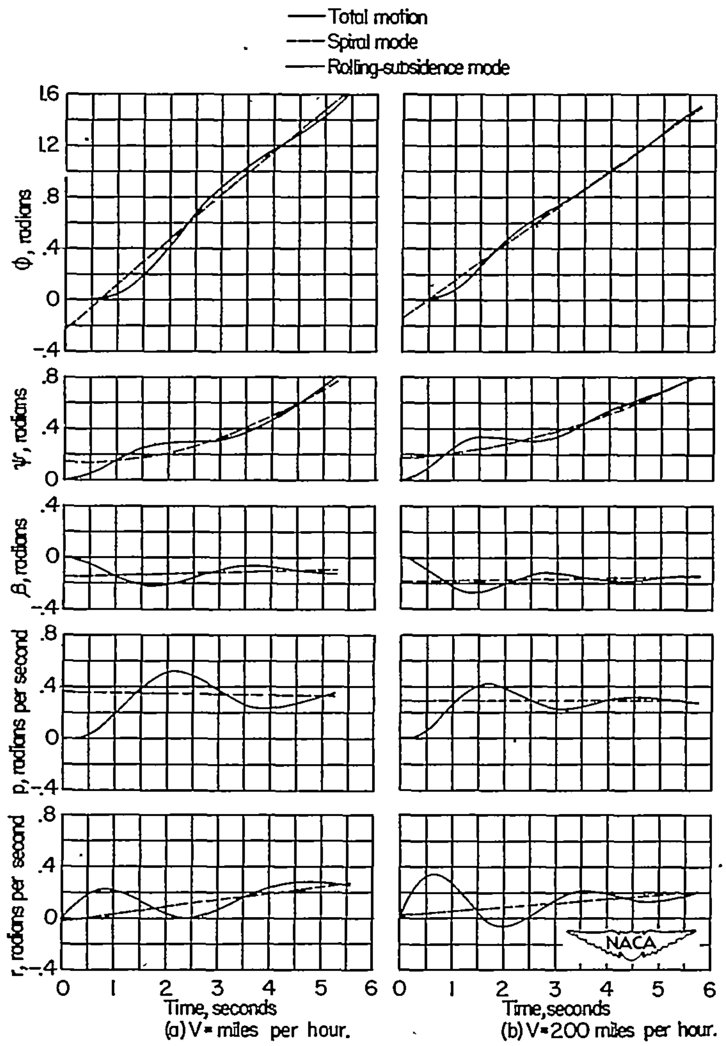

Figure 8.— Response of experimental swept-wing airplane to the applicatior at zero time of a constant yawing-moment coefficient. $C_{n_c} = 0.02$; $\gamma = 0^{\circ}$.

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