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TECHNICAL NOTE 2182

ANALYSIS OF EFFECT OF VARIATIONS IN PRIMARY VARIABLES ON TIME CONSTANT AND TURBINE-INLET-TEMPERATURE OVERSHOOT OF TURBOJET ENGINE

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Washington
September 1950

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ANALYSIS OF EFFECT OF VARIATIONS IN PRIMARY VARIABLES ON TIME CONSTANT AND TURBINE-INLET-TEMPERATURE OVERSHOOT OF TURBOJET ENGINE

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SUMMARY

An analysis is presented of the effect of independent variations in the component efficiency characteristics, flight conditions, and engine size on the time constant and the turbine-inlet-temperature overshoot of a turbojet engine with a centrifugal compressor and a turbine with choked stator. The dynamic factors were calculated from thermodynamic equations of engine-component performance.

The engine time constant, corrected for engine size and altitude, increased 45 percent with a 10-percent decrease in compressor-tip Mach number. At constant Mach number, independent variations over a normal operating range of compressor efficiency, ram pressure ratio, and exhaust-to-turbine nozzle-area ratio varied the engine time constant 15 to 25 percent; the effect of turbine efficiency was negligible. The effect of independent variations in the rate of change of component efficiency during transient engine operation on the time constant was of significant magnitude and should be given consideration in generalizing the dynamic behavior of an engine. Turbine-inlet-temperature overshoot decreased with an increase in compressor Mach number except under conditions of high ram pressure ratio. The effect of independent variations in compressor and turbine efficiency and exhaust-to-turbine nozzle-area ratio on overshoot was small compared with the effect of ram pressure ratio. Contrary to the engine time constant, the turbine-inlet-temperature overshoot required no correction for engine size and altitude.

INTRODUCTION

The design of turbojet-engine control systems is intimately related to the dynamic characteristics of the engine and has advanced to the stage where theoretically correct control constants can be determined for fixed dynamic properties (references 1 and 2). In a turbojet engine, these control constants can generally be accurately evaluated for fixed engine operating conditions. Variations in dynamic characteristics resulting from changes in operating conditions or engine performance, however, necessitate compromises in the selection of the most suitable control constants. With respect to the control designer, a knowledge of the engine parameters affecting dynamic behavior and the magnitude of the variations that may be incurred would therefore facilitate the design of the engine control systems.

Some preliminary experimental investigations have been made of the variations in dynamic properties resulting from changes in operating conditions (references 3 and 4). Such investigations are useful in the design of control systems for the engines investigated. The results cannot be accurately extrapolated to similar engines of different size or performance, however, because the changes in dynamic behavior encountered during the investigations were the result of the combined effect of a number of engine variables on dynamic behavior. An evaluation of the primary variables affecting dynamic behavior and the independent effect each has on the behavior would be of value. It would provide basic information on the variations in dynamic characteristics that may be expected from performance variations and design changes between engines without the necessity of individually investigating each engine.

This investigation was therefore conducted at the NACA Lewis laboratory to determine the primary variables, such as component efficiency characteristics, flight conditions, and engine size, that affect dynamic characteristics of a turbojet engine and to evaluate the effect of independently varying these variables.

A simple turbojet engine with a centrifugal compressor and a turbine with choked stator was analyzed. Transfer functions for the engine were derived by combining the static relation between dependent and independent variables with the dynamic expression for the engine mechanical configuration. Two pertinent factors expressing dynamic characteristics were obtained from the transfer functions for this type engine: (1) the engine time constant, a characteristic time in the response of engine speed to a change in fuel flow, and (2) the turbine-inlet-temperature overshoot resulting from a step change in fuel flow. The expressions for these dynamic factors were expanded in terms of primary engine variables by the use of thermodynamic equations of engine performance and used to evaluate the effect of variations in primary variables on the time constant and turbine-inlet-temperature overshoot. The use of the thermodynamic equations presupposes that the engine processes are quasi-static. Such an assumption is shown to be valid for a turbojet engine in references 3 and 4.

METHOD OF ANALYSIS

The two dynamic characteristics considered herein, the engine time constant and the turbine-inlet-temperature overshoot, are related to the acceleration properties of the engine in response to a change in an operating condition. By theoretical analysis, assuming the engine to be a quasi-static system, an expression in terms of primary engine variables can be derived for both these characteristics. The dynamic factors are derived using the Laplace transform, a method generally applied to control analysis. The symbols used are defined in Appendix A.

Transfer Functions

If it is assumed that the engine is a first-order system and that the unbalanced torque between the compressor and the turbine is a function of engine speed and fuel flow, the significant terms in a linear expansion about a steady-state operating point (unbalanced torque equals zero) are given in the following equation:

$$\Delta Q = \left(\frac{\partial Q}{\partial N}\right)_{w_f} \Delta N + \left(\frac{\partial Q}{\partial w_f}\right)_N \Delta w_f \tag{1}$$

In the region around the steady-state operating condition, equation (1) may be considered as a differential equation in the same variables. If equation (1) is combined with Newton's second law of motion and the Laplace transform is applied with initial conditions equal to zero, the transfer function relating speed and fuel flow is given by

$$L(N) = r_f \frac{1}{1+\tau_f p} L(w_f) \tag{2}$$

where

$$r_f = -\left(\frac{\partial Q}{\partial w_f}\right)_N \bigg/ \left(\frac{\partial Q}{\partial N}\right)_{w_f}$$

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and

$$\tau_f = -\frac{I}{J}\bigg/\left(\frac{\partial Q}{\partial N}\right)_{w_f}$$

The factor τ_f is the engine time constant, a characteristic dynamic factor in the response of speed to the independent variable fuel flow.

The unbalanced torque can also be assumed a function of speed and turbine-inlet temperature. If this assumption is made, the following transfer function can be derived:

$$L(N) = r_T \frac{1}{1+\tau_T p} L(T_4) \qquad (3)$$

where

$$r_T = -\left(\frac{\partial Q}{\partial T_4}\right)_N \bigg/ \left(\frac{\partial Q}{\partial N}\right)_{T_4}$$

and

$$\tau_T = -\frac{I}{J}\bigg/\left(\frac{\partial Q}{\partial N}\right)_{T_4}$$

combining the two transfer functions, equations (2) and (3), and eliminating speed gives

$$L(T_4) = \frac{r_f}{r_T}\left[\frac{1+\tau_T p}{1+\tau_f p}\right] L(w_f) \qquad (4)$$

The dynamic relation between fuel flow and turbine-inlet temperature is represented by equation (4). If τ_T is larger than τ_f, the form of this equation is such that, for a step change in fuel flow, the turbine-inlet temperature is at a maximum value at the instant the step is made.

If τ_T is assumed larger than τ_f and the initial-value theorem (reference 2) is applied,

$$T_{4(max)} = \frac{r_f}{r_T} \frac{\tau_T}{\tau_f} w_f \tag{5}$$

The final value is obtained when time is infinite. Applying the final-value theorem to equation (4) (reference 2) gives

$$T_{4(final)} = \frac{r_f}{r_T} w_f \tag{6}$$

When equations (5) and (6) are combined, the following expression for turbine-inlet-temperature overshoot is obtained:

$$\frac{T_{4(max)} - T_{4(final)}}{T_{4(final)}} = \frac{\tau_T}{\tau_f} - 1 \tag{7}$$

From the definition of τ_f and τ_T, equation (7) can be written in the form

$$\frac{T_{4(max)} - T_{4(final)}}{T_{4(final)}} = \left(\frac{\partial Q}{\partial N}\right)_{w_f} \Bigg/ \left(\frac{\partial Q}{\partial N}\right)_{T_4} - 1 \tag{8}$$

The temperature overshoot given by equation (8) is for initial conditions equal to zero.

Thermodynamic Expressions

In order to evaluate the engine time constant (equation (2)) and the turbine-inlet-temperature overshoot (equation (8)), two engine characteristics, $\left(\frac{\partial Q}{\partial N}\right)_{w_f}$ and $\left(\frac{\partial Q}{\partial N}\right)_{T_4}$, the changes in torque with speed at constant fuel flow and at constant turbine-inlet temperature, respectively, must be known. These two factors can be obtained from thermodynamic equations of the engine components.

In the thermodynamic analysis of the engine, simplified assumptions of engine performance are made to facilitate the derivation. These assumptions include constant specific heats, burner pressure drop, fuel-air ratio, compressor slip factor, and a noncompressible-flow equation for the exhaust nozzle. These assumptions will result in some error in the absolute magnitude of the dynamic factors; they are, however, sufficiently accurate to calculate the relative changes in a dynamic factor resulting from variations in primary variables. For example, the noncompressible-flow equation results in a variation in the engine time constant that differs by only 5 percent from that obtained with a compressible-flow equation for a range of exhaust-nozzle conditions between choked and extremely small pressure ratios.

The following thermodynamic equations are used in this analysis:

(a) Relation of unbalanced engine torque, neglecting friction and accessory power, to other engine parameters:

$$NQ = w_g(H_4-H_5) - w_a(H_3-H_2) \tag{9}$$

(b) Compressor performance:

$$\frac{H_3-H_2}{H_2} = \frac{1}{\eta_c}\left[\left(\frac{P_3}{P_2}\right)^{0.283} - 1\right] \tag{10}$$

$$H_3-H_2 = \frac{1}{4gJ} D_c{}^2 \frac{\psi}{\eta_c} N^2 \tag{11}$$

(c) Burner performance:

$$w_gH_4 - w_aH_3 = \eta_b h w_f \tag{12}$$

(d) Turbine performance:

$$\frac{H_4-H_5}{H_4} = \eta_t\left[1 - \left(\frac{P_5}{P_4}\right)^{0.248}\right] \tag{13}$$

$$w_g = 0.274\,\frac{A_t P_4}{\sqrt{H_4}} \tag{14}$$

(e) Gas flow through exhaust nozzle:

$$w_g = A_e\sqrt{\frac{2g}{RT_5}\,p_0(P_5-p_0)} \tag{15}$$

In addition to these equations of component performance, the following assumptions are made:

$$w_g = 1.02\ w_a \tag{16}$$

$$P_4 = 0.95\ P_3 \tag{17}$$

$$\eta_c,\eta_t,\eta_b = f(N,w_f) \tag{18}$$

These equations are sufficient to derive a single equation for engine performance in terms of torque, speed, and fuel flow. For this analysis, torque is assumed to vary linearly with speed and fuel flow. An equation expressing this relation is obtained by linearizing the thermodynamic equations in terms of the variable parameters and then combining the linearized equations to obtain a single equation linearly relating changes in torque with changes in speed and fuel flow. The algebraic process of this derivation and the complete expression for this equation is given in appendix B, equation (B36). A simplified form of this equation is

$$\Delta Q = \frac{A_t D^2 \delta}{\sqrt{\theta}} \left\{ f_1 \left[M, \frac{P_2}{P_0}, \frac{A_t}{A_e}, \eta_c, \eta_t, \left(\frac{\frac{\partial \eta_c}{\eta_c}}{\frac{\partial N}{N}} \right)_{w_f}, \left(\frac{\frac{\partial \eta_t}{\eta_t}}{\frac{\partial N}{N}} \right)_{w_f}, \left(\frac{\frac{\partial \eta_b}{\eta_b}}{\frac{\partial N}{N}} \right)_{w_f} \right] \right\} \Delta N + f_2(v_1) \Delta w_f \tag{19}$$

where f_2 is a function of variables similar to those expressed for f_1. This equation expresses in terms of primary engine variables the relation assumed in equation (1). The coefficient of the speed term of equation (19) is equal to the change in torque with speed at constant fuel flow for small deviations from the steady-state operating conditions. The engine time constant is then equal to

$$\tau_f = -\frac{I}{J} \bigg/ \frac{A_t D^2 \delta}{\sqrt{\theta}} \left\{ f_1 \left[M, \frac{P_2}{P_0}, \frac{A_t}{A_e}, \eta_c, \eta_t, \left(\frac{\frac{\partial \eta_c}{\eta_c}}{\frac{\partial N}{N}} \right)_{w_f}, \left(\frac{\frac{\partial \eta_t}{\eta_t}}{\frac{\partial N}{N}} \right)_{w_f}, \left(\frac{\frac{\partial \eta_b}{\eta_b}}{\frac{\partial N}{N}} \right)_{w_f} \right] \right\} \tag{20}$$

From equation (20), the time constant is seen to be directly proportional to I and $\sqrt{\theta}$ and is inversely proportional to A_t, D^2, and δ. A corrected value of the time constant in terms of these factors representing engine size and altitude, may accordingly be presented in the form

$$\frac{\tau_f \delta}{\sqrt{\theta}}\left(\frac{A_t D^2}{I}\right)$$

An equation linearly relating torque with speed and turbine-inlet temperature rather than fuel flow can also be derived. For this case compressor, turbine, and combustion efficiencies are assumed constant because varying efficiencies would complicate the presentation of results to the extent that they would have no informative value. A simplified form of the complete expression derived in appendix B, equation (B37) is as follows:

$$\Delta Q = \frac{A_t D^2 \delta}{\sqrt{\theta}}\left[f_3\left(M, \frac{P_2}{p_0}, \frac{A_t}{A_e}, \eta_c, \eta_t\right)\right]\Delta N + f_4(V_2)\Delta T_4 \qquad (21)$$

The coefficient of the speed term in equation (21) is the change of torque with speed at constant turbine-inlet temperature. The turbine-inlet-temperature overshoot is then the ratio of the coefficients of the speed terms in equations (19) and (21), as defined by equation (8).

$$\frac{T_{4(max)} - T_{4(final)}}{T_{4(final)}} = \frac{f_5\left(M, \frac{P_2}{p_0}, \frac{A_t}{A_e}, \eta_c, \eta_t,\right)}{f_3\left(M, \frac{P_2}{p_0}, \frac{A_t}{A_e}, \eta_c, \eta_t\right)} - 1 \qquad (22)$$

The numerator of equation (22) differs from the coefficient of the speed term in equation (19) only in that the rate of changes in component efficiency during a transient change is neglected.

Independent Variations of Primary Variables

From equation (20), the corrected time constant is shown to be a function of the engine variables $M, \frac{P_2}{p_0}, \frac{A_t}{A_e}, \eta_c, \eta_t,$

$$\left(\frac{\partial\eta_c}{\eta_c}\bigg/\frac{\partial N}{N}\right)_{w_f}, \left(\frac{\partial\eta_t}{\eta_t}\bigg/\frac{\partial N}{N}\right)_{w_f}, \text{ and } \left(\frac{\partial\eta_b}{\eta_b}\bigg/\frac{\partial N}{N}\right)_{w_f}.$$

Steady-state engine performance is generally presented with respect to compressor-tip Mach number. In this analysis of the effect of variations in primary variables on the engine time constant, the effect will also be presented with respect to compressor-tip Mach number. The Mach number used is the ratio of the compressor-tip velocity to the velocity of sound at the compressor inlet.

The effect of independent variations in primary engine variables on the time constant were calculated with respect to reference engine conditions and for this purpose values of each of the primary engine variables were chosen. The assumed reference conditions are

$$\frac{P_2}{p_0} = 1.0 \qquad \left(\frac{\partial\eta_c}{\eta_c}\bigg/\frac{\partial N}{N}\right)_{w_f} = 0$$

$$\frac{A_t}{A_e} = 0.50 \qquad \left(\frac{\partial\eta_t}{\eta_t}\bigg/\frac{\partial N}{N}\right)_{w_f} = 0$$

$$\eta_c = 0.80 \qquad \left(\frac{\partial\eta_b}{b}\bigg/\frac{\partial N}{N}\right)_{w_f} = 0$$

$$\eta_t = 0.85$$

These reference conditions, with an assumption of compressor-tip Mach number, determine all other engine variables. The results of independently varying the primary variables are shown in figure 1, which presents the corrected engine time constant as a function of compressor-tip Mach number.

Turbine-inlet-temperature overshoot is a function of the variables M, P_2/p_0, A_t/A_e, η_c, and η_t when the effect of the rate of change of efficiency is neglected. With the assumption of the same reference conditions of the primary variables as were used for the engine time constant, the effect of independent variations in the primary variables on the relation between the turbine-inlet-temperature overshoot and compressor-tip Mach number was calculated. These effects are shown in figure 2.

DISCUSSION OF RESULTS

Engine Time Constant

The effect of altitude on the engine time constant is indicated from the corrected value of the time constant used in this analysis. As has been indicated by other investigators (for example, reference 3), the engine time constant is directly proportional to the square root of the ambient temperature and inversely proportional to the ambient pressure at a constant compressor-tip Mach number. These relations are, of course, only the independent effect of altitude; additional effects may be introduced if a change in altitude results in a change in other primary variables.

The corrected engine time constant also shows that at constant compressor-tip Mach number the engine time constant is directly proportional to the ratio I/A_tD^2. Reference 3 also shows that the engine time constant is directly proportional to moment of inertia; however, such a single proportionality is inadequate for a comparison of time constants between engines of different size. For geometrically similar engines, both A_t and D increase as I increases and therefore the direct proportionality with I is minimized.

In figure (1), which presents the engine time constant, the corrected value of time constant increases with a decrease in compressor-tip Mach number. For an engine of constant size and with the reference values chosen for this analysis, a 10-percent decrease in the actual speed will result in approximately a 45-percent increase in the time constant.

The independent effect of steady-state compressor efficiency on the relation between compressor-tip Mach number and time constant is shown in figure 1(a). At constant Mach number, a decrease of 0.05 in compressor efficiency results in approximately a 15-percent increase in corrected time constant, the increase being larger at higher Mach numbers. The effect of steady-state turbine efficiency on the time constant is less than that of compressor efficiency, as shown in figure 1(b). A decrease in turbine efficiency of 0.05 results in approximately a 3-percent decrease in time constant at low compressor Mach numbers with the effect approaching zero as Mach number increases.

The exhaust-to-turbine nozzle-area ratio has an appreciable effect on the relation between Mach number and time constant. In figure 1(c), the time constant is shown to increase approximately 20 percent for a 20-percent increase in area ratio at constant Mach number. The effect of ram pressure ratio is shown in figure 1(d). The engine time constant decreases approximately 25 percent for an increase in ram pressure ratio of 20 percent at constant compressor-tip Mach number.

The effect of the rate of change of compressor, turbine, and burner transient efficiency on the time constant is shown in figures 1(e), 1(f), and 1(g), respectively. The parameter representing this efficiency gradient $\left(\frac{\partial\eta}{\eta}\bigg/\frac{\partial N}{N}\right)_{w_f}$ is, for small changes, the ratio of the percentage change in efficiency to the percentage change in speed at constant fuel flow. The value of this ratio is dependent on the performance characteristics of the components. It is not related to the value of the efficiencies at a steady-state operating condition but expresses the efficiency in the region surrounding the steady-state point. Although the steady-state efficiency may be high, the efficiency may decrease rapidly in the vicinity of the steady-state point with attendant large values of the efficiency gradient factor for only perceptible changes in operating conditions. The value of this factor may even be larger than those presented herein if the engine is operated near the region of compressor surge, burner blow-out, or at rapidly decreasing turbine efficiencies.

When the efficiency gradient is positive, which represents a condition of increasing efficiency with speed at constant fuel flow, the time constant increases at a constant Mach number. This effect is shown in figures 1(e) to 1(g). For compressor efficiency the effect is less at a low compressor-tip Mach number, whereas for the turbine and burner efficiencies it is less at a high Mach number.

The dependence of the engine time constant on the transient efficiency characteristics leads to difficulty in generalizing the dynamic behavior of an engine. Although steady-state data may be adequately corrected for altitude and ram pressure, there is no assurance that the dynamic behavior can be corrected to the same degree of accuracy. Altitude and ram conditions may affect the dynamic behavior of a component in such a manner that the component efficiency at a steady-state operating condition will not be materially affected but the rate of change of efficiency for a transient condition will vary significantly.

Turbine-Inlet-Temperature Overshoot

The turbine-inlet-temperature overshoot calculated herein is a value not readily obtained in an actual engine. The magnitude of overshoot shown in figure 2 is only obtained for a perfect step change in energy input to the engine. Because of lags in the fuel system and in combustion, this perfect step change is nearly impossible to produce. The values obtained, however, are valuable for comparative purposes and can be used to show the effect of the primary variables.

Contrary to the engine time constant, turbine-inlet-temperature overshoot is unaffected by engine size or altitude. The curves in figure 2 therefore are applicable to any size engine and altitude operating condition.

The effect of independent variation in compressor and turbine efficiency on the relation between compressor-tip Mach number and temperature overshoot are shown in figures 2(a) and 2(b), respectively. These curves show that a decrease in efficiency of 0.05 results in approximately a 6-percent increase in the overshoot at a constant Mach number. The effect of independent variations in exhaust-to-turbine nozzle-area ratio is shown in figure 2(c). At high values of Mach number and small values of area ratio, the overshoot decreases as the area ratio decreases. At low values of Mach number, an opposite but much smaller effect is obtained.

Ram pressure ratio has the most dominant effect on overshoot, as shown in figure 2(d). At a ram pressure ratio of somewhat less than 1.2, overshoot is approximately constant. At a ram pressure ratio of 1.4 overshoot increases with Mach number, whereas at a ram pressure ratio of 1.0, the overshoot decreases with increasing Mach number. The over-all effect of increasing the ram pressure ratio is a decrease in the overshoot.

SUMMARY OF RESULTS

From an analysis of the effect of independent variations in primary variables on the dynamic characteristics of a turbojet engine with centrifugal compressor and choked turbine, the following results were obtained:

1. A time-constant expression corrected for engine size and altitude was expressed in the form

$$\frac{\tau_f \delta}{\sqrt{\theta}}\left(\frac{A_t D^2}{I}\right)$$

where

τ_f	engine time constant, characteristic dynamic factor in response of speed to fuel flow, (sec)
δ	ambient pressure divided by standard sea-level pressure
θ	ambient enthalpy divided by standard sea-level enthalpy
A_t	effective turbine-nozzle area, (sq ft)
D	compressor-tip diameter, (ft)
I	total polar moment of inertia, (ft-lb)(sec^2)

2. The engine time constant increased 45 percent for a decrease in compressor-tip Mach number of 10 percent.

3. At constant compressor-tip Mach number, independent variations over a normal operating range of the steady-state values of compressor efficiency, ram pressure ratio, and exhaust-to-turbine nozzle-area ratio each varied the engine time constant by 15 to 25 percent; the variation of turbine efficiency had a negligible effect on the time constant.

4. The effect of variations in the efficiency gradient of compressor, turbine, and burner was of significant magnitude and should be given consideration in generalizing the dynamic characteristics of an engine for flight conditions.

5. Turbine-inlet-temperature overshoot required no correction for engine size or altitude.

6. Turbine-inlet-temperature overshoot decreased with an increase in compressor-tip Mach number except under conditions of high ram pressure ratio.

7. The steady-state values of turbine and compressor efficiency and the exhaust-to-turbine nozzle-area ratio had a small effect on turbine-inlet-temperature overshoot as compared with the effect of ram pressure ratio.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, March 20, 1950.

APPENDIX A

SYMBOLS

The following symbols are used in this report:

A	effective nozzle area, sq ft
$C_1, C_2, \ldots, C_7$	dimensionless constants
D	compressor-tip diameter, ft
g	gravitational constant, 32.2 ft/sec^2
H	enthalpy, Btu/lb
h	heating value of fuel, Btu/lb
I	total polar moment of inertia, (ft-lb)(sec^2)
J	mechanical equivalent of heat, 778 ft-lb/Btu
L	Laplace transform
M	compressor-tip Mach number based on compressor-inlet conditions
N	engine speed, radians/sec
P	total pressure, lb/ft^2
p	complex number
p_0	ambient static pressure, lb/ft^2
Q	unbalanced torque, Btu
R	gas constant, ft-lb/(lb)($^\circ$R)
r_f	ratio of torque change with fuel flow to torque change with speed, radians/lb
r_T	ratio of torque change with turbine-inlet temperature to torque change with speed, radians/($^\circ$R)(sec)

T	total temperature, °R
v_1	expression in terms of primary engine variables, (Btu)(sec)/lb
v_2	expression in terms of primary engine variables, Btu/°R
w_a	engine air flow, lb/sec
w_f	engine fuel flow, lb/sec
w_g	engine exhaust-gas flow, lb/sec
Δ	incremental change in steady-state value of engine parameter
δ	ambient pressure divided by standard sea-level pressure
ϵ	dimensionless constant equal to $\frac{H_2}{H_2 + \frac{h\eta_b w_f}{w_a}}$
η	efficiency
θ	ambient enthalpy divided by standard sea-level enthalpy
τ_f	engine time constant, characteristic dynamic factor in response of speed to fuel flow, sec
τ_T	engine time constant, characteristic dynamic factor in response of speed to turbine-inlet temperature, sec
ϕ	dimensionless constant equal to $\frac{H_4}{H_2 + \frac{h\eta_b w_f}{w_a}}$
ψ	compressor pressure coefficient

Subscripts:

0	ambient
2	compressor inlet
3	burner inlet
4	turbine inlet
5	turbine outlet
b	burner
c	compressor
e	exhaust
max	maximum
t	turbine

APPENDIX B

DERIVATION OF ENGINE-TORQUE EQUATION

The following equations are assumed to express engine performance

$$QN = w_g(H_4-H_5) - w_a(H_3-H_2) \tag{B1}$$

$$\frac{H_3-H_2}{H_2} = \frac{1}{\eta_c}\left[\left(\frac{P_3}{P_2}\right)^{0.283} - 1\right] \tag{B2}$$

$$H_3-H_2 = \frac{1}{4gJ} \; D^2 \; \frac{\psi}{\eta_c} N^2 = 9.48 \times 10^{-6} D^2N^2 \tag{B3}$$

where $\psi/\eta_c = 0.95$.

$$w_gH_4 - w_aH_3 = \eta_b h w_f \tag{B4}$$

$$\frac{H_4-H_5}{H_4} = \eta_t\left[1 - \left(\frac{P_5}{P_4}\right)^{0.248}\right] \tag{B5}$$

$$w_g = 0.274 \frac{A_tP_4}{\sqrt{H_4}} \tag{B6}$$

where a ratio of specific heats equal to 1.33 is used.

$$w_g = A_e\sqrt{\frac{2gp_0}{RT_5}(P_5-p_0)} = 0.557 A_e\sqrt{\frac{p_0}{H_5}(P_5-p_0)} \tag{B7}$$

where a specific heat equal to 0.263 is used.

$$w_g = 1.02\ w_a \tag{B8}$$

$$P_4 = 0.95\ P_3 \tag{B9}$$

$$\eta_c,\ \eta_t,\ \eta_b = f(N,w_f) \tag{B10}$$

Combining equations (B1), (B4), and (B8) yields

$$QN = h\eta_b w_f - w_a(1.02\ H_5 - H_2) \tag{B11}$$

When equations (B5), (B6), and (B8) are combined

$$\frac{H_4 - H_5}{H_4} = \eta_t \left[1 - \left(\frac{0.274\ P_5 A_t}{1.02\ w_a\sqrt{H_4}}\right)^{0.248}\right] \tag{B12}$$

If equations (B3), (B4), and (B8) are combined

$$1.02\ H_4 - \left(9.48 \times 10^{-6}\ D^2N^2 + H_2\right) = \eta_b h \frac{w_f}{w_a} \tag{B13}$$

Combining equations (B2), (B3), (B6), (B8), and (B9) results in

$$\frac{9.48 \times 10^{-6}\ D^2N^2}{H_2} = \frac{1}{\eta_c}\left[\left(\frac{1.02\ w_a\sqrt{H_4}}{0.27 \times 0.96\ A_t P_2}\right)^{0.283} - 1\right] \tag{B14}$$

Equations (B7) and (B8) yield

$$1.02\ w_a = 0.563\ A_e\sqrt{\frac{p_0}{H_5}\left(P_5 - p_0\right)} \tag{B15}$$

In equations (B11) to (B15), the parameters assumed variable during a transient change in analyzing dynamic behavior are Q, N, w_f, w_a, H_4, H_5, P_4, P_5, η_c, η_t, and η_b. Differentiating these equations in terms of their variable parameters and assuming the coefficients of the differential equation to be constant for finite differences of the variables results in equations in which the parameters are linearly related. The linear form of these equations and their simplification is as follows:

From equation (B11)

$$\frac{Q}{h\eta_b w_f}\Delta N + \frac{N}{h\eta_b w_f}\Delta Q = \frac{\Delta w_f}{w_f} + \frac{\Delta\eta_b}{\eta_b} - \frac{1.02\ H_5 w_a}{h\eta_b w_f}\frac{\Delta H_5}{H_5} - \frac{w_a\ (1.02\ H_5 - H_2)}{h\eta_b w_f}\frac{\Delta w_a}{w_a}$$

The coefficients of this equation as well as the following linear equations are assumed constant and are defined at a steady-state operating condition. The unbalanced torque Q in the first term of this equation is then equal to zero. The difference between the initial and final value of unbalanced torque ΔQ is then equal to the unbalance torque during a transience.

At steady-state conditions, the following expressions can be obtained from equation (B11):

$$\frac{w_a\ (1.02\ H_5 - H_2)}{h\eta_b w_f} = 1$$

and

$$\frac{1.02\ H_5 w_a}{h\eta_b w_f} = \frac{H_2 + \dfrac{h\eta_b w_f}{w_a}}{\dfrac{h\eta_b w_f}{w_a}}$$

Making these substitutions in the coefficients of the linear equation gives

$$\frac{N}{h\eta_b w_f}\Delta Q = \frac{\Delta w_f}{w_f} + \frac{\Delta\eta_b}{\eta_b} - \frac{\Delta w_a}{w_a} - \left(\frac{H_2 + \dfrac{h\eta_b w_f}{w_a}}{\dfrac{h\eta_b w_f}{w_a}}\right)\frac{\Delta H_5}{H_5}$$

When the term

$$\epsilon = \frac{H_2}{H_2 + \frac{h\eta_b w_f}{w_a}}$$

is substituted, the final form of the linear equation is

$$\frac{N}{h\eta_b w_f}\Delta Q = \frac{\Delta w_f}{w_f} + \frac{\Delta\eta_b}{\eta_b} - \frac{\Delta w_a}{w_a} - \left(\frac{1}{1-\epsilon}\right)\frac{\Delta H_5}{H_5} \qquad \text{(B16)}$$

The linear form of equation (B12) is

$$\frac{\Delta H_5}{H_5} = \left[1 - \frac{1}{2}\,0.248\left(\frac{\eta_t^{-1}}{\frac{H_5}{H_4}} + 1\right)\right]\frac{\Delta H_4}{H_4} - \left(\frac{1}{\frac{H_5}{H_4}} - 1\right)\frac{\Delta\eta_t}{\eta_t} -$$

$$0.248\left(\frac{\eta_t^{-1}}{\frac{H_5}{H_4}} + 1\right)\frac{\Delta w_a}{w_a} + 0.248\left(\frac{\eta_t^{-1}}{\frac{H_5}{H_4}} + 1\right)\frac{\Delta P_5}{P_5}$$

From equation (B11) at steady state

$$\frac{H_5}{H_4} = \left(\frac{H_2 + \frac{\eta h w_f}{w_a}}{1.02\ H_4}\right)$$

If

$$\phi = \frac{1.02\ H_4}{H_2 + \frac{h\eta_b w_f}{w_a}}$$

then

$$\frac{H_5}{H_4} = \frac{1}{\phi}$$

Making this substitution in the linear equation gives

$$\frac{\Delta H_5}{H_5} = \left\{1 - \frac{1}{2}\,0.248\left[(\eta_t-1)\phi + 1\right]\right\}\frac{\Delta H_4}{H_4} - (\phi-1)\,\frac{\Delta \eta_t}{\eta_t} +$$

$$0.248\left[(\eta_t-1)\phi + 1\right]\frac{\Delta P_5}{P_5} - 0.248\left[(\eta_t-1)\phi + 1\right]\frac{\Delta w_a}{w_a} \qquad \text{(B17)}$$

The linear form of equation (B13) is

$$\frac{\Delta w_a}{w_a} = \frac{\Delta w_f}{w_f} + 2\left(\frac{1.02\;H_4 - H_2}{\frac{h\eta_b w_f}{w_a}} - 1\right)\frac{\Delta N}{N} + \frac{\Delta \eta_b}{\eta_b} - \frac{1.02\;H_4}{\frac{h\eta_b w_f}{w_a}}\,\frac{\Delta H_4}{H_4}$$

Substituting the expressions for ϕ and ϵ gives

$$\frac{\Delta w_a}{w_a} = \frac{\Delta w_f}{w_f} + \frac{\Delta \eta_b}{\eta_b} + 2\left(\frac{\phi-1}{1-\epsilon}\right)\frac{\Delta N}{N} - \frac{\phi}{1-\epsilon}\,\frac{\Delta H_4}{H_4} \qquad \text{(B18)}$$

The linear form of equation (B14) is

$$\frac{\Delta H_4}{H_4} = -2\,\frac{\Delta w_a}{w_a} - \frac{2}{0.283}\left(\frac{1}{1 + \frac{H_2}{9.48 \times 10^{-6}\, D^2N^2\eta_c}}\right)\frac{\Delta\eta_c}{\eta_c} +$$

$$\frac{4}{0.283}\left(\frac{1}{1 + \frac{H_2}{9.48 \times 10^{-6}\, D^2N^2\eta_c}}\right)\frac{\Delta N}{N}$$

Combining equations (B3) and (B4) gives

$$\frac{H_2}{9.48 \times 10^{-6}\, D^2N^2} = \frac{H_2}{H_3 - H_2} = \frac{H_2}{1.02\, H_4 - \frac{\eta_b h w_f}{w_a} - H_2} = \frac{\epsilon}{\phi - 1}$$

then

$$\frac{\Delta H_4}{H_4} = -2\,\frac{\Delta w_a}{w_a} - \left(\frac{\frac{2}{0.283}}{1 + \frac{\frac{\epsilon}{\eta_c}}{\phi - 1}}\right)\frac{\Delta\eta_c}{\eta_c} + \left(\frac{\frac{4}{0.283}}{1 + \frac{\frac{\epsilon}{\eta_c}}{\phi - 1}}\right)\frac{\Delta N}{N} \qquad \text{(B19)}$$

The linear form of equation (B15) is

$$\frac{\Delta P_5}{P_5} = 2\left[1 - \left(\frac{p_0}{P_4}\right)\left(\frac{P_4}{P_5}\right)\right]\frac{\Delta w_a}{w_a} + \left[1 - \left(\frac{p_0}{P_4}\right)\left(\frac{P_4}{P_5}\right)\right]\frac{\Delta H_5}{H_5}$$

From equation (B5) and the definition of ϕ

$$\frac{P_5}{P_4} = \left[1 - \frac{1}{\eta_t}\left(1 - \frac{H_5}{H_4}\right)\right]^{\frac{1}{0.248}} = \left[1 - \left(\frac{\phi-1}{\eta_t\phi}\right)\right]^{\frac{1}{0.248}}$$

therefore

$$\frac{\Delta P_5}{P_5} = 2\left[1 - \frac{1}{\frac{P_4}{p_0}\left(1 - \frac{\phi-1}{\eta_t\phi}\right)^{\frac{1}{0.248}}}\right]\frac{\Delta w_a}{w_a} +$$

$$\left[1 - \frac{1}{\frac{P_4}{p_0}\left(1 - \frac{\phi-1}{\eta_t\phi}\right)^{\frac{1}{0.248}}}\right]\frac{\Delta H_5}{H_5} \tag{B20}$$

Five linear engine equations are represented in equations (B16) to (B20) and can be written in the form

$$\frac{N}{h\eta_b w_f}\Delta Q = \frac{\Delta w_f}{w_f} + \frac{\Delta\eta_b}{\eta_b} - \frac{\Delta w_a}{w_a} - C_1\frac{\Delta H_5}{H_5} \tag{B21}$$

$$\frac{\Delta H_5}{H_5} = \left(1 - \frac{1}{2}C_2\right)\frac{\Delta H_4}{H_4} - C_3\frac{\Delta\eta_t}{\eta_t} + C_2\frac{\Delta P_5}{P_5} - C_2\frac{\Delta w_a}{w_a} \tag{B22}$$

$$\frac{\Delta w_a}{w_a} = \frac{\Delta w_f}{w_f} + \frac{\Delta\eta_b}{\eta_b} + C_4\frac{\Delta N}{N} - C_5\frac{\Delta H_4}{H_4} \tag{B23}$$

$$\frac{\Delta H_4}{H_4} = -2\,\frac{\Delta w_a}{w_a} - C_6\,\frac{\Delta \eta_c}{\eta_c} + 2\,C_6\,\frac{\Delta N}{N} \qquad \text{(B24)}$$

$$\frac{\Delta P_5}{P_5} = 2\,C_7\,\frac{\Delta w_a}{w_a} + C_7\,\frac{\Delta H_5}{H_5} \qquad \text{(B25)}$$

where

$$C_1 = \frac{1}{1-\epsilon}$$

$$C_2 = 0.248\left[(\eta_t-1)\,\phi + 1\right]$$

$$C_3 = \phi - 1$$

$$C_4 = 2\left(\frac{\phi-1}{1-\epsilon}\right)$$

$$C_5 = \frac{\phi}{1-\epsilon}$$

$$C_6 = \frac{\dfrac{2}{0.283}}{1 + \dfrac{\dfrac{\epsilon}{\eta_c}}{\phi-1}}$$

and

$$C_7 = 1 - \frac{1}{\dfrac{P_4}{p_0}\left(1 - \dfrac{\phi-1}{\eta_t\phi}\right)^{\frac{1}{0.248}}}$$

The following relations are also known from equation (B10):

$$\frac{\Delta\eta_c}{\eta_c} = \left(\frac{\frac{\partial\eta_c}{\eta_c}}{\frac{\partial N}{N}}\right)_{w_f} \frac{\Delta N}{N} + \left(\frac{\frac{\partial\eta_c}{\eta_c}}{\frac{\partial w_f}{w_f}}\right)_N \frac{\Delta w_f}{w_f} \tag{B26}$$

$$\frac{\Delta\eta_t}{\eta_t} = \left(\frac{\frac{\partial\eta_t}{\eta_t}}{\frac{\partial N}{N}}\right)_{w_f} \frac{\Delta N}{N} + \left(\frac{\frac{\partial\eta_t}{\eta_t}}{\frac{\partial w_f}{w_f}}\right)_N \frac{\Delta w_f}{w_f} \tag{B27}$$

$$\frac{\Delta\eta_b}{\eta_b} = \left(\frac{\frac{\partial\eta_b}{\eta_b}}{\frac{\partial N}{N}}\right)_{w_f} \frac{\Delta N}{N} + \left(\frac{\frac{\partial\eta_b}{\eta_b}}{\frac{\partial w_f}{w_f}}\right)_N \frac{\Delta w_f}{w_f} \tag{B28}$$

Combining equations (B21) to (B28) and eliminating all variables except ΔQ, ΔN, and Δw_f results in the following linear equation:

$$\begin{aligned}
\Delta Q = \frac{h\eta_b w_f}{N^2} \Bigg\{ & 2\,C_6 \left[\frac{C_5\,(1-2\,C_1)}{1-2\,C_5} - C_1\left(\frac{1-\frac{1}{2}\,C_2}{1-C_2C_7}\right)\right]\left[1-\frac{1}{2}\left(\frac{\frac{\partial\eta_c}{\eta_c}}{\frac{\partial N}{N}}\right)_{w_f}\right] - \\
& C_4\left(\frac{1-2\,C_1}{1-2\,C_5}\right) + 2\left(\frac{C_1-C_5}{1-2\,C_5}\right)\left[\left(\frac{\frac{\partial\eta_b}{\eta_b}}{\frac{\partial N}{N}}\right)_{w_f}\right] + \frac{1}{2}\left(\frac{C_4}{1-C_2C_7}\right)\left(\frac{\frac{\partial\eta_t}{\eta_t}}{\frac{\partial N}{N}}\right)_{w_f}\Bigg\}\Delta N + \\
& \frac{h\eta_b}{N}\Bigg\{2\,C_6\left[\frac{C_5(1-C_1)}{1-2\,C_5} - C_1\left(\frac{1-\frac{1}{2}\,C_2}{1-C_2C_7}\right)\right]\left[-\frac{1}{2}\left(\frac{\frac{\partial\eta_c}{\eta_c}}{\frac{\partial w_f}{w_f}}\right)_N\right] + \\
& 2\left(\frac{C_1-C_5}{1-2\,C_5}\right)\left[1-\left(\frac{\frac{\partial\eta_b}{\eta_b}}{\frac{\partial w_f}{w_f}}\right)_N\right] + \frac{1}{2}\left(\frac{C_4}{1-C_2C_7}\right)\left(\frac{\frac{\partial\eta_t}{\eta_t}}{\frac{\partial w_f}{w_f}}\right)_N\Bigg\}\Delta w_f\,.
\end{aligned} \tag{B29}$$

If $\Delta\eta_c$ and $\Delta\eta_t$ are assumed equal to zero, equations (B21) to (B25) can be combined by eliminating all variables except Q, ΔN, and ΔH_4. If the specific heat is assumed constant, however, $\Delta T_4/T_4$ is equal to $\Delta H_4/H_4$. Making this substitution in the combined equation results in

$$\Delta Q = \frac{h\eta_b w_f}{N^2}\left[C_1C_2C_6\left(\frac{1-2\,C_7}{1-C_2C_7}\right) - C_4\right]\Delta N + \frac{1}{2}\,\frac{h\eta_b w_f}{NT_4}\,C_4\Delta T_4 \tag{B30}$$

Only the coefficients of ΔN in equations (B29) and (B30) were used in this analysis. The coefficients are expressed in terms of the variables ϕ, ϵ, P_4/p_0, η_c, η_t, and $(h\eta_b w_f)/N^2$ and equation (B29) contains the additional terms $\left(\frac{\partial\eta_c}{\eta_c}\Big/\frac{\partial N}{N}\right)_{w_f}$, $\left(\frac{\partial\eta_t}{\eta_t}\Big/\frac{\partial N}{N}\right)_{w_f}$, and $\left(\frac{\partial\eta_b}{\eta_b}\Big/\frac{\partial N}{N}\right)_{w_f}$. The first of these variables can be expressed entirely in terms of primary engine variables and corrected engine speed by use of the thermodynamic equations assumed.

Combining equations (B3), (B4), (B6), and (B8) yields

$$\frac{N^2}{h\eta_b w_f} =$$

$$\frac{1.02\sqrt{126}}{2116 \times 0.274 \times 9.48 \times 10^{-6}}\sqrt{\frac{H_4}{H_2}}\left(\frac{1.02H_4-H_2-\frac{h\eta_b w_f}{w_a}}{\frac{h\eta_b w_f}{w_a}}\right)\frac{p_0}{P_4}\sqrt{\frac{H_2}{H_0}}\;\frac{\sqrt{\theta}}{\delta A_t D^2}$$

where 126 is the static sea-level enthalpy and 2116 is the static sea-level pressure. From the definition of ϕ and ϵ, the preceding equation can be written in the form

$$\frac{N^2}{h\eta_b w_f} = 2.08 \times 10^3\,\frac{\sqrt{\theta}}{\delta A_t D^2}\sqrt{\frac{\phi}{\epsilon}}\left(\frac{\phi-1}{1-\epsilon}\right)\left(\frac{p_0}{P_4}\right)\sqrt{\frac{H_2}{H_0}} \qquad \text{(B31)}$$

In equation (B31), the ratio H_2/H_0 can be evaluated from the temperature-rise equation for a given ram pressure ratio by the equation

$$\frac{H_2-H_0}{H_0} = \frac{1}{0.90}\left[\left(\frac{P_2}{p_0}\right)^{0.283} - 1\right] \qquad \text{(B32)}$$

where 0.90 is the assumed ram efficiency.

An expression for compressor-tip Mach number in terms of primary engine variables is required.

Combining equations (B2), (B3), and (B9) gives

$$\frac{DN}{2} = \sqrt{\frac{gJH_2}{0.95\eta_c}\left[\left(\frac{1}{0.95}\,\frac{P_4p_0}{p_0P_2}\right)^{0.283} - 1\right]}$$

The preceding equation is an expression for the compressor-tip velocity. The following expression is obtained for the compressor-tip Mach number by dividing this expression by the velocity of sound at the compressor inlet:

$$M = 1.62\sqrt{\frac{\left(\frac{1}{0.95}\,\frac{P_4}{p_0}\,\frac{p_0}{P_2}\right)^{0.283} - 1}{\eta_c}} \qquad \text{(B33)}$$

The expression for ϕ and ϵ is obtained from the solution of two equations that are also derived from the assumed thermodynamic equations.

The following equation is obtained when equations (B2), (B4), (B8), and (B9) are combined:

$$\frac{1.02\,H_4 - \eta_b h\,\frac{W_f}{W_a} - H_2}{H_2} = \frac{1}{\eta_c}\left[\left(\frac{1}{0.95}\,\frac{P_4}{p_0}\,\frac{p_0}{P_2}\right)^{0.283} - 1\right]$$

or

$$\frac{\phi-1}{\epsilon} = \frac{1}{\eta_c}\left[\left(\frac{1}{0.95}\,\frac{P_4}{p_0}\,\frac{p_0}{P_2}\right)^{0.283} - 1\right] \qquad \text{(B34)}$$

From equation (B5)

$$1 - \frac{H_5}{H_4} = \frac{\phi-1}{\phi} = \eta_t\left[1 - \left(\frac{P_5}{P_4}\right)^{0.248}\right]$$

Eliminating w_g from equations (B6) and (B7) yields

$$\frac{P_5}{P_4} = \frac{\left(\frac{0.274}{0.563}\right)^2 \left(\frac{A_t}{A_e}\right)^2 \frac{H_5}{H_4}}{\frac{p_0}{P_4}} + \frac{p_0}{P_4}$$

therefore

$$\frac{\phi-1}{\phi} = \eta_t \left\{ 1 - \left[\frac{\left(\frac{0.274}{0.563}\right)^2 \left(\frac{A_t}{A_e}\right)^2 \frac{1}{\phi}}{\frac{p_0}{P_4}} + \frac{p_0}{P_4} \right]^{0.248} \right\} \tag{B35}$$

From equations (B31) to (B35), it can be seen that equations (B29) and (B30) can be written in the form

$$\Delta Q = \frac{A_t D^2 \delta}{\sqrt{\theta}} \left\{ f_1 \left[\frac{N}{\sqrt{\theta_2}} D, \frac{P_2}{p_0}, \frac{A_t}{A_e}, \eta_c, \eta_t, \left(\frac{\frac{\partial \eta_c}{\eta_c}}{\frac{\partial N}{N}} \right)_{w_f}, \left(\frac{\frac{\partial \eta_t}{\eta_t}}{\frac{\partial N}{N}} \right)_{w_f}, \left(\frac{\frac{\partial \eta_b}{\eta_b}}{\frac{\partial N}{N}} \right)_{w_f} \right] \right\} \Delta N + f_2(v_1) \Delta w_f \tag{B36}$$

and

$$\Delta Q = \frac{A_t D^2 \delta}{\sqrt{\theta}} \left[f_3 \left(\frac{N}{\sqrt{\theta_2}} D, \frac{P_2}{p_0}, \frac{A_t}{A_e}, \eta_c, \eta_t \right) \right] \Delta N + f_4(v_2) \Delta T_4 \tag{B37}$$

In these equations, v_1 and v_2 represent a grouping of variables and are designated as such inasmuch as they are insignificant in the analysis.

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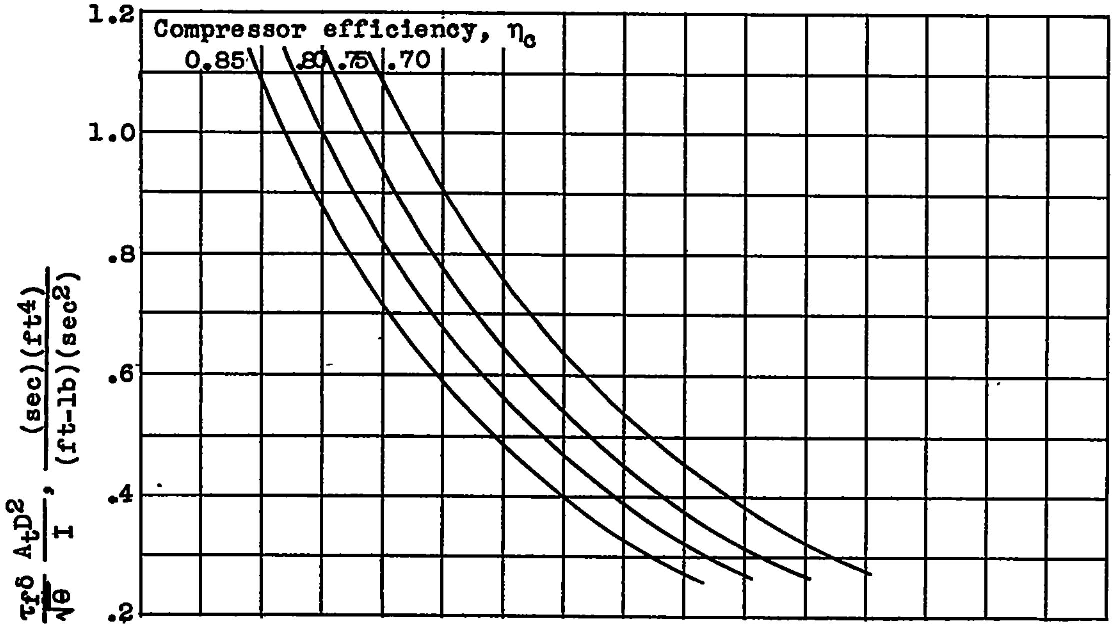
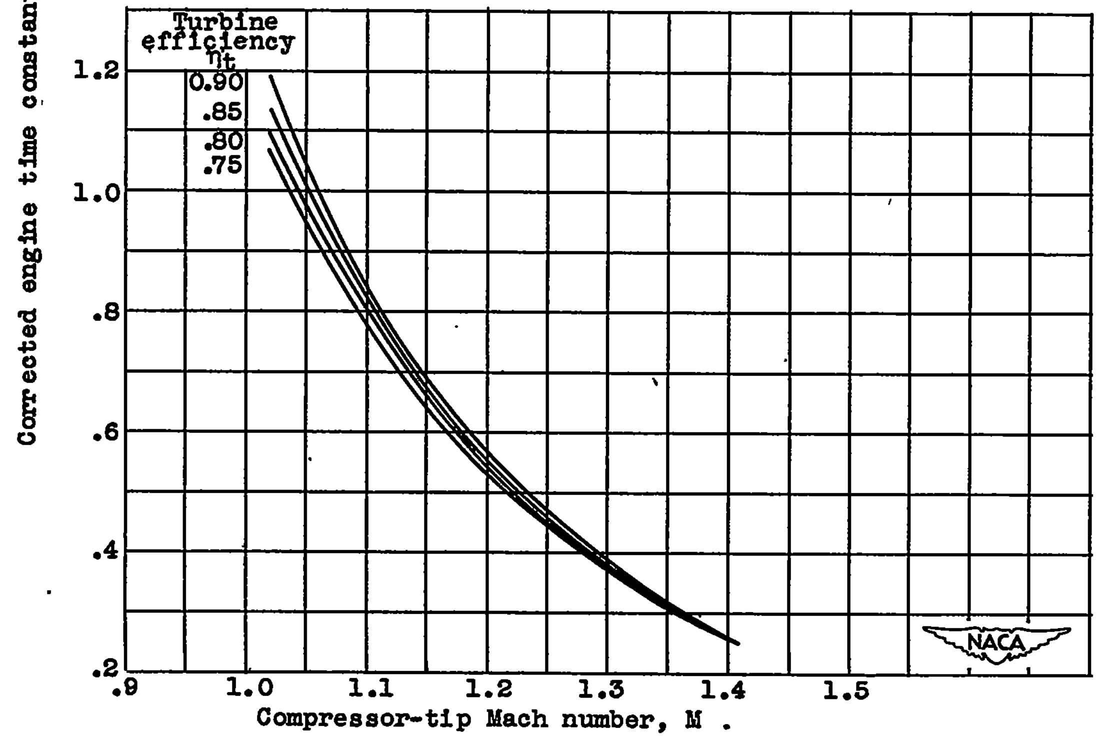
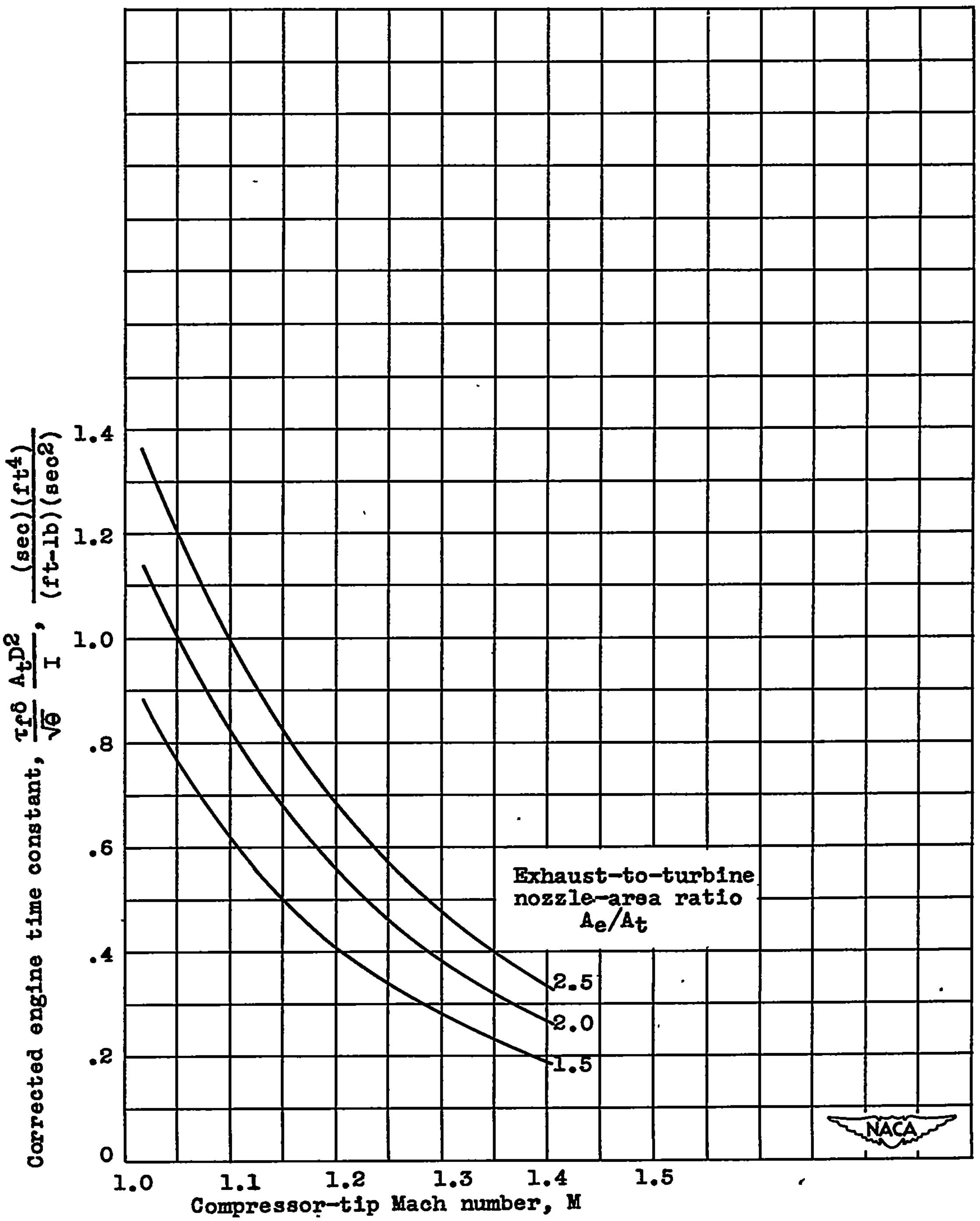


(a) Variation of compressor efficiency.



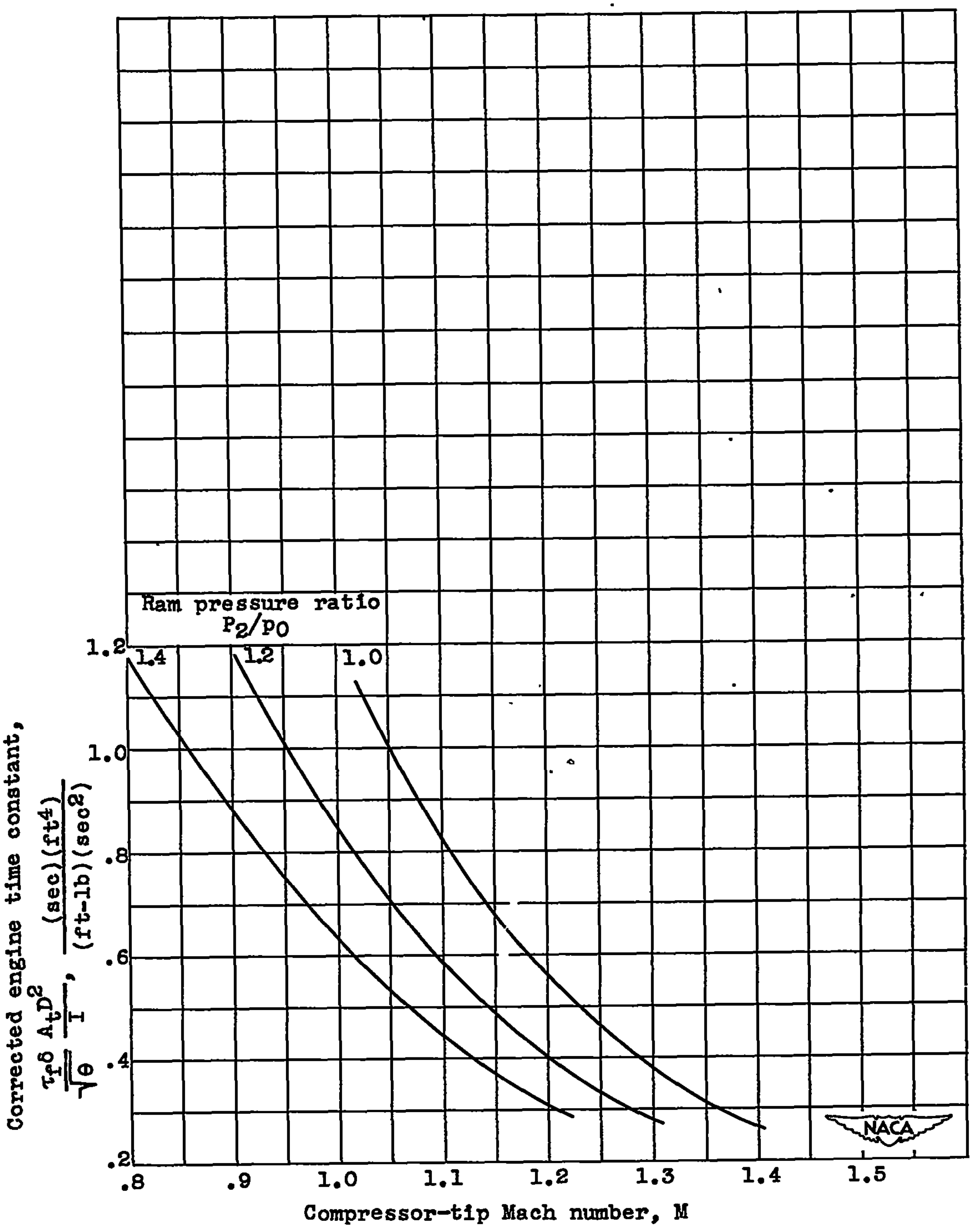
(b) Variation of turbine efficiency.

Figure 1. - Effect of independent variation of primary engine variables on relation between corrected engine time constant and compressor-tip Mach number.



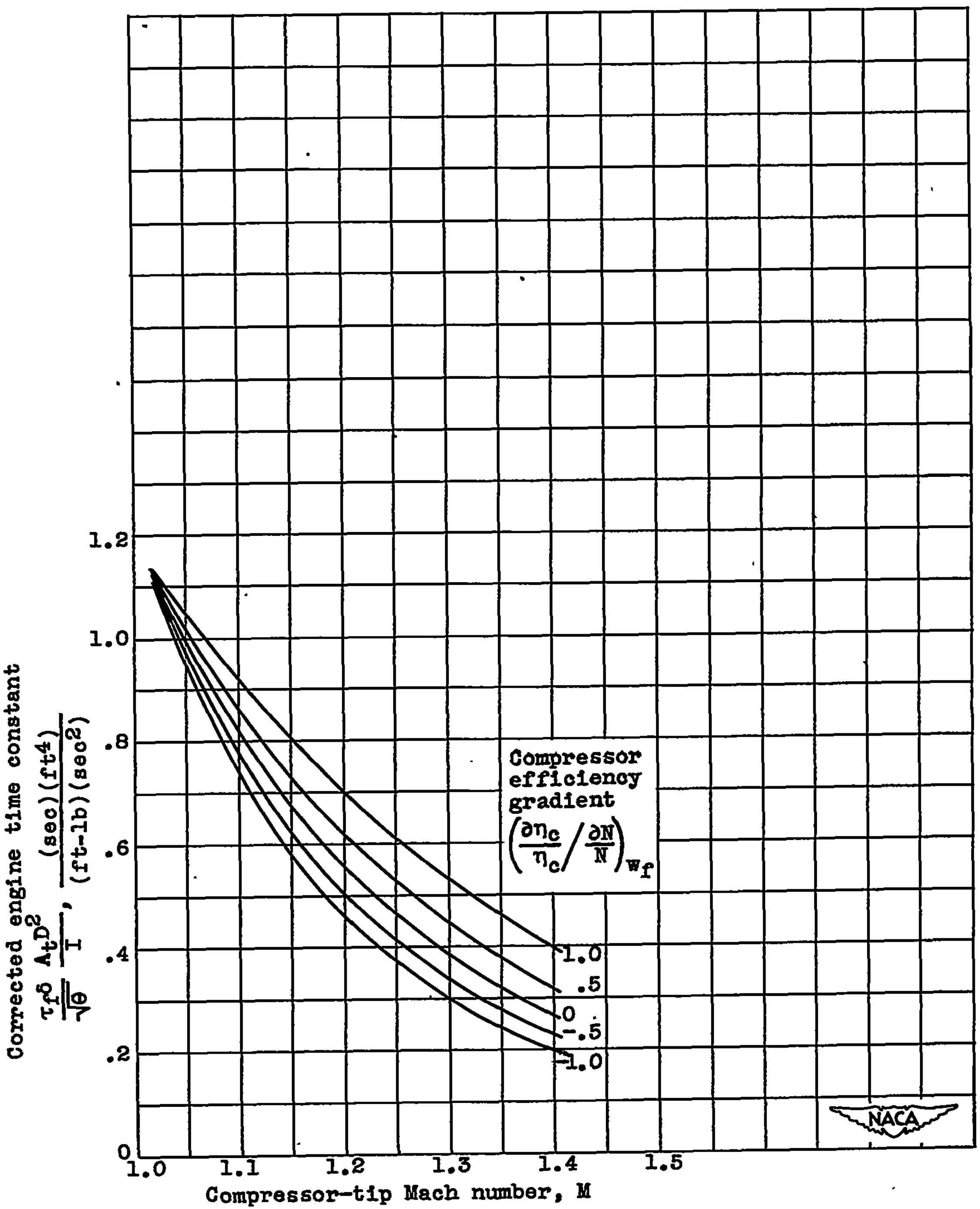
(c) Variation of exhaust-to-turbine nozzle-area ratio.

Figure 1. - Continued. Effect of variation of primary engine variables on relation between corrected engine time constant and compressor-tip Mach number.



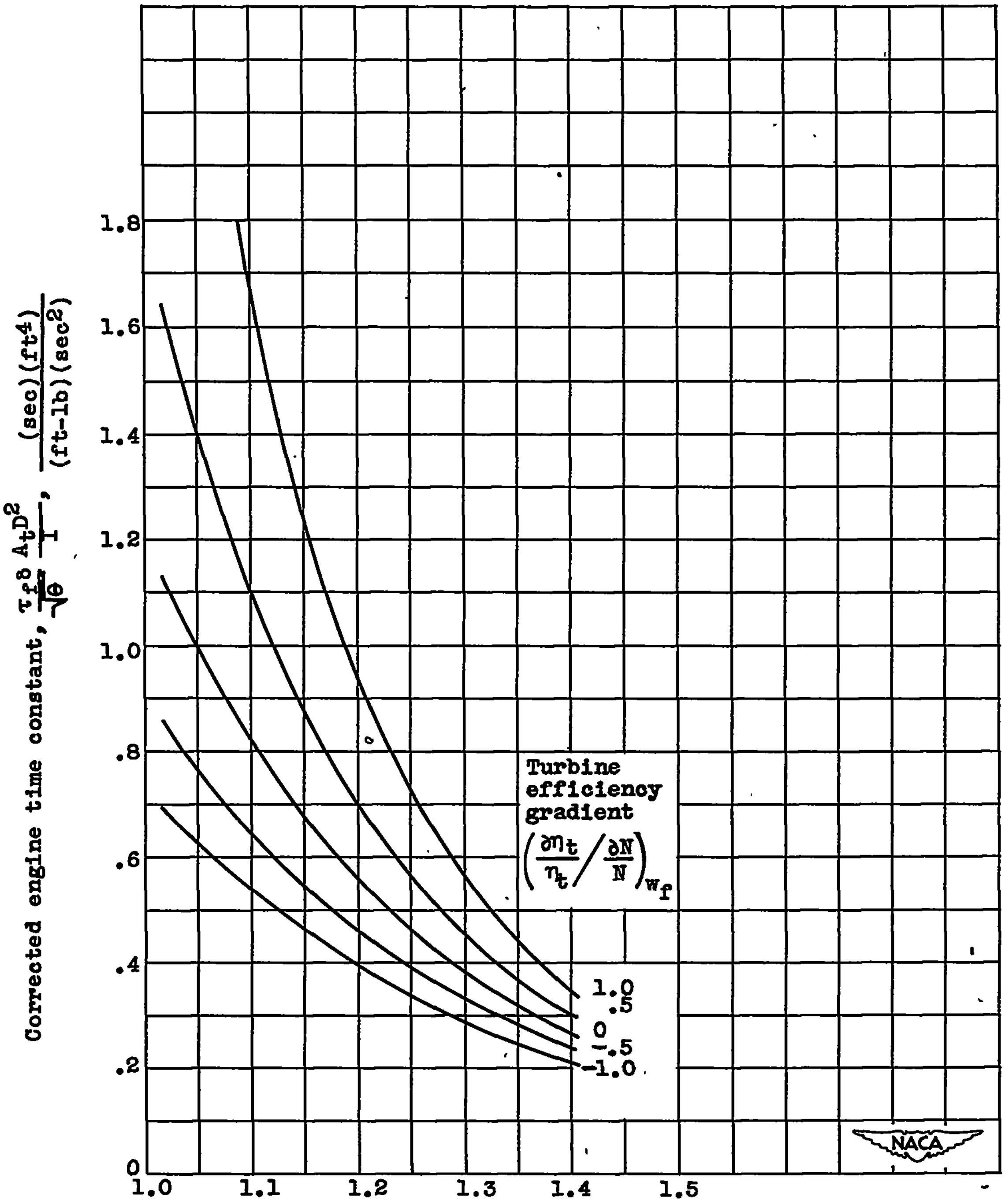
(d) Variation of ram pressure ratio.

Figure 1. - Continued. Effect of independent variation of primary engine variables on relation between corrected engine time constant and compressor-tip Mach number.



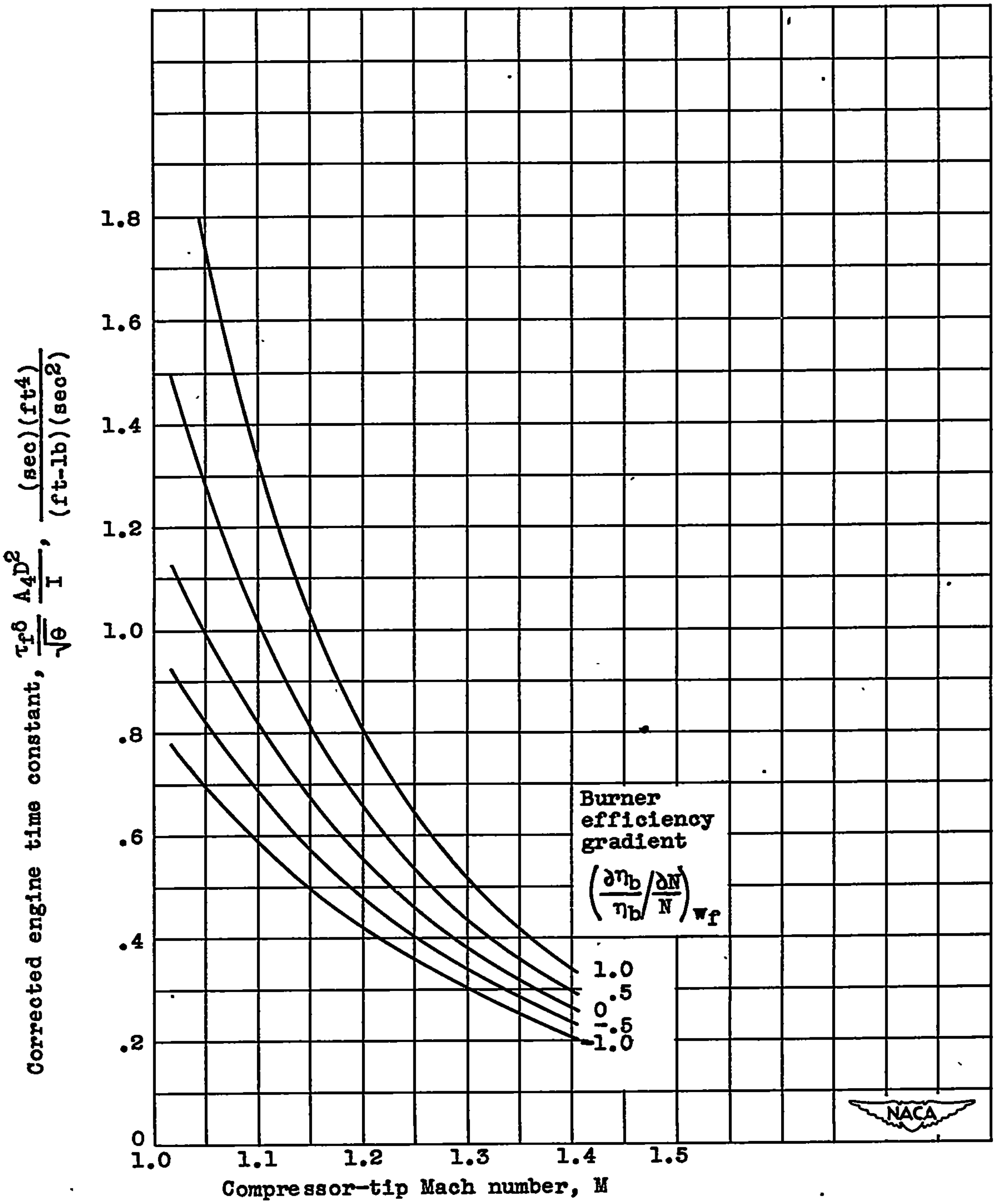
(e) Variation of compressor efficiency gradient.

Figure 1. - Continued. Effect of independent variation of primary engine variables on relation between corrected engine time constant and compressor-tip Mach number.



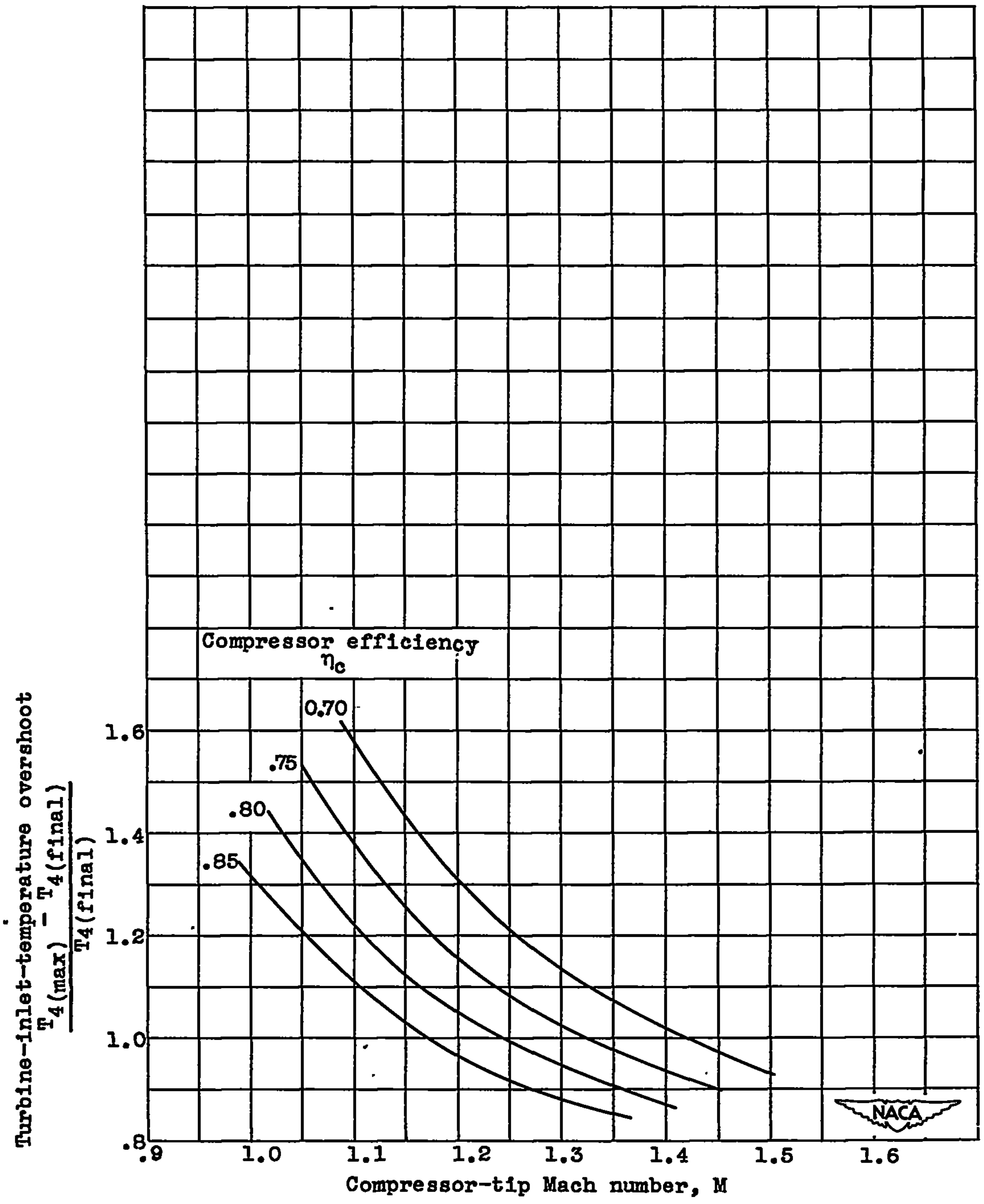
(f) Variation of turbine efficiency gradient.

Figure 1. - Continued. Effect of independent variation of primary engine variables on relation between corrected engine time constant and compressor-tip Mach number.



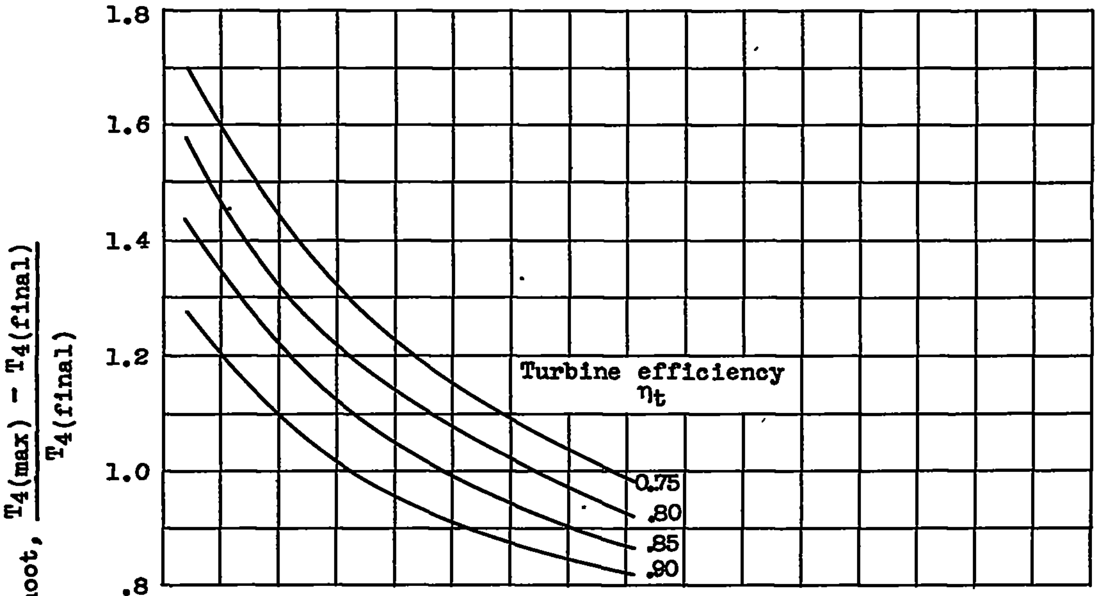
(g) Variation of burner efficiency gradient.

Figure 1. - Concluded. Effect of independent variation of primary engine variables on relation between corrected engine time constant and compressor-tip Mach number.

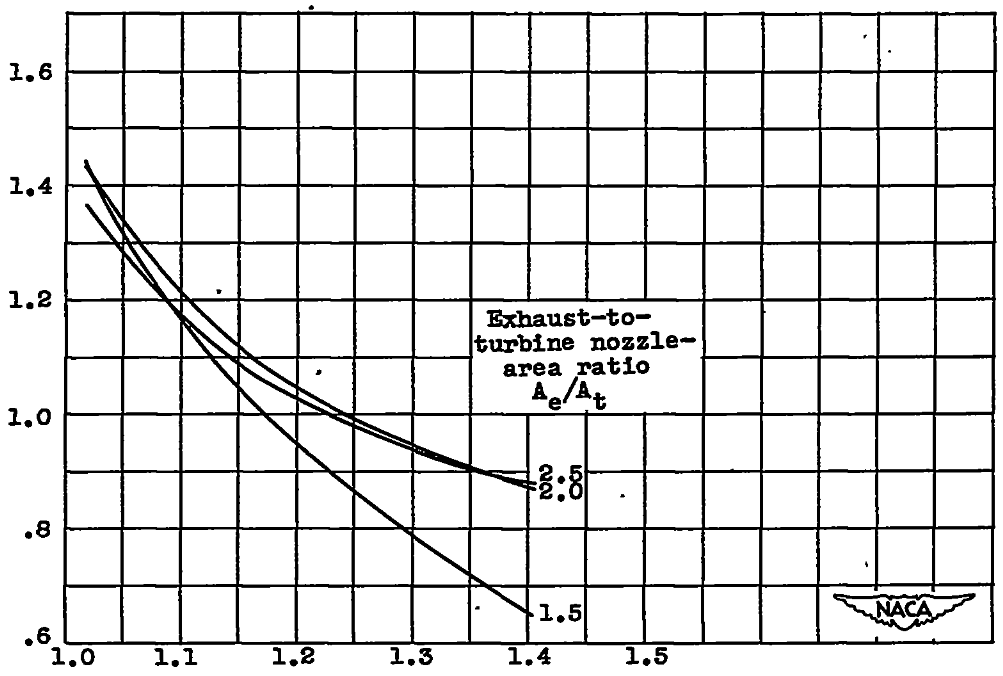


(a) Variation of compressor efficiency.

Figure 2. - Effect of independent variation of primary engine variables on relation between turbine-inlet-temperature overshoot and compressor-tip Mach number.

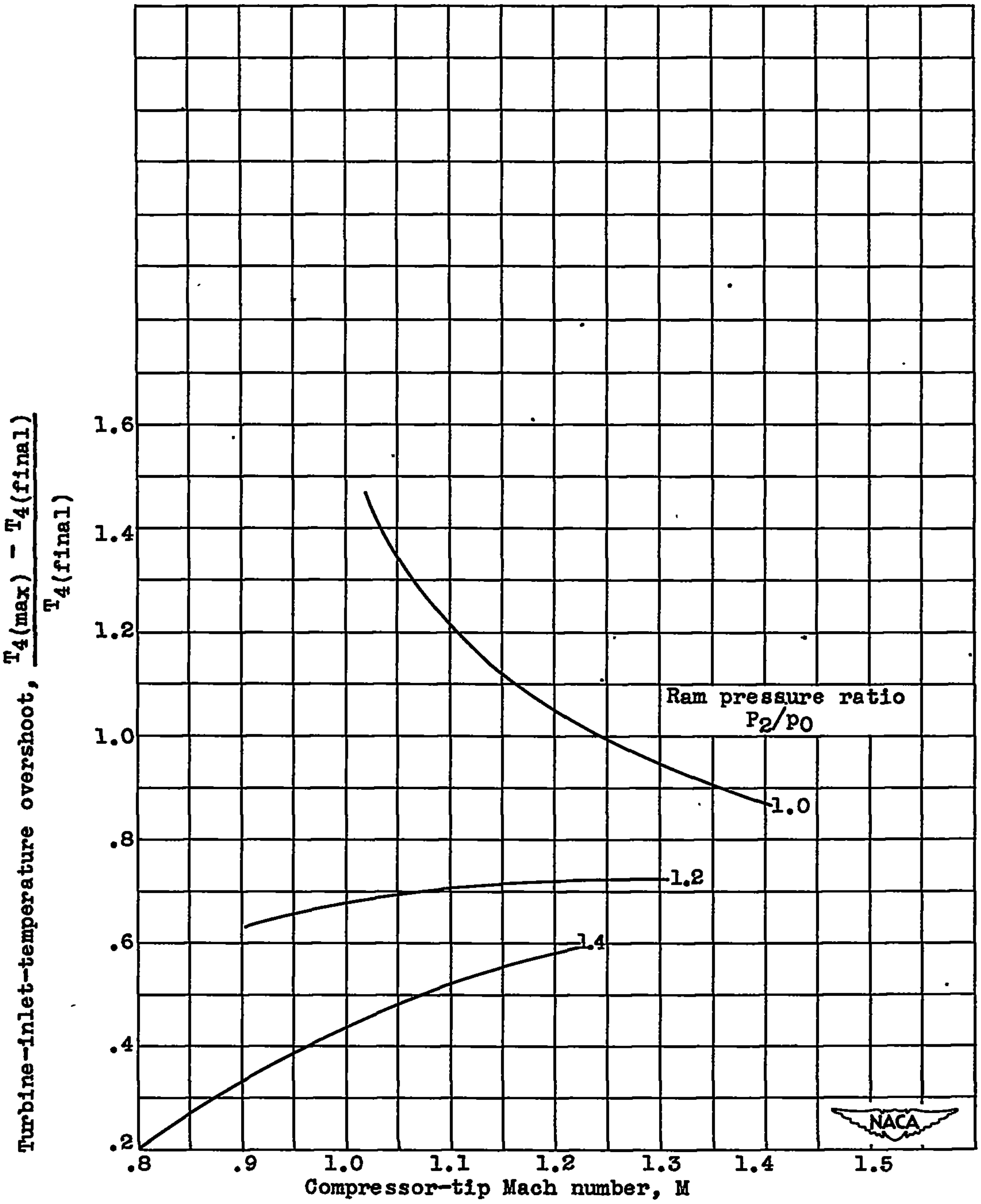


(b) Variation of turbine efficiency.



(c) Variation of exhaust-to-turbine nozzle-area ratio.

Figure 2. - Continued. Effect of independent variation of primary engine variables on relation between turbine-inlet-temperature overshoot and compressor-tip Mach number.



(d) Variation of ram pressure ratio.

Figure 2. - Concluded. Effect of independent variation of primary engine variables on relation between turbine-inlet-temperature overshoot and compressor-tip Mach number.