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TECHNICAL NOTE 2193

EFFECT OF HEAT-CAPACITY LAG ON A VARIETY OF TURBINE-NOZZLE FLOW PROCESSES

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1326

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SUMMARY

As a necessary consideration in the application of the variable heat capacity of gases in the analysis of turbine-engine processes, the effect of heat-capacity lag in the adiabatic expansion of a gas (nitrogen plus water vapor) through a turbine nozzle was computed using a one-dimensional analysis of a representative flow process.

A number of parameter variations in the flow process were considered to determine the factors that amplify this effect and to show the magnitude of the deviation from equilibrium conditions. The flow parameters chosen to demonstrate this amplification were total temperature, outlet velocity, relaxation time, inlet pressure, nozzle length, and velocity distribution in the nozzle.

For nozzle-outlet Mach numbers of 0.9 and less and for nozzle-inlet temperatures of 3500° R and less, the ratio of outlet to inlet pressure across the nozzle necessary to obtain a specified outlet velocity was less than 1 percent smaller than that computed on the basis of the usual assumption that the gas is in temperature equilibrium at all times. The process, however, corresponded even more closely to one in which the vibrational energy of the gas was considered entirely unavailable or frozen. A value for the ratio of specific heats of 1.4 can thus be used to yield a closer approximation to the actual case than does the equilibrium value, which includes the effect of the varying heat capacity of the gas. Increases in nozzle-outlet Mach number led to correspondingly greater deviations in pressure ratio from the actual case if the calculations were based on an equilibrium expansion. At a turbine-nozzle outlet Mach number of approximately 4 and an inlet temperature of 3500° R, this deviation in pressure ratio was approximately 85 percent of the actual value. Calculations based on a condition of frozen vibrational energy yielded a pressure ratio differing by only 8.9 percent from the actual case.

A set of conditions approximating those for flow through the exhaust nozzle of a ram-jet engine resulted in a process that could be more closely approximated by equilibrium conditions in contrast with the conclusions drawn for turbine-nozzle processes.

INTRODUCTION

The range of combustion temperatures encountered in gas-turbine engines makes it necessary to consider the variation with temperature of the heat capacity of the working fluid when describing any flow process that takes place. In processes involving rapid expansion of the fluid (initially in temperature equilibrium) and hence rapid temperature variations, the mechanism contributing to the increase in heat capacity with temperature does not instantaneously follow the changes imposed by the process, but lags behind by an amount that increases as the time required for a given change in temperature decreases (reference 1). As a result, the instantaneous heat capacity of the exhaust gases during the flow process is less than the expected equilibrium value corresponding to the temperature of the gas. In some cases, the heat capacity of the exhaust gases approaches the low-temperature heat-capacity value (corresponding to a ratio of specific heats of 1.4 for diatomic molecules) even though the gas remains at a high temperature throughout the process. Consequently, in any investigation of the effect of variable heat capacity on a flow process, this additional effect, known as heat-capacity lag, must also be considered.

A gas can adjust to a change only in some finite time interval as evidenced by the dependency of the heat-capacity lag on the time required for a change in temperature. Thus, as a measure of the heat-capacity-lag effect to be expected in a process, the time required for the gas to approach equilibrium after an abrupt change in temperature, called the relaxation time of the gas, is used. The phenomenon was first studied in connection with its effect on the dispersion of sound waves. Landau and Teller give a theoretical treatment of the process in reference 2 where it is concluded that long relaxation times are the results of a very low probability of exchange in energy between vibrational systems and translational and rotational systems during molecular-collision processes.

The significance of this phenomenon in gas dynamics was investigated by Kantrowitz. In reference 1, he describes a gas-dynamic method for measuring relaxation times and demonstrates how the presence of certain impurities can appreciably affect the relaxation time of a gas.

The effect of variable heat capacity including heat-capacity lag was evaluated at the NACA Lewis laboratory by analyses of the flow conditions in typical gas-turbine nozzles using results of relaxation-time measurements presented in reference 3 by Kantrowitz and Huber. The results obtained are compared with calculations

made assuming equilibrium expansion of the gas (zero heat-capacity lag or zero relaxation time) and assuming a condition of frozen
1326 vibrational energy (infinite relaxation time or ratio of specific heats, 1.4). Equilibrium expansion and frozen vibrational energy represent the two possible extremes of the heat-capacity-lag effect. The results presented herein for comparison will indicate if in some cases the actual flow process can be closely approximated by one of these two cases.

SYMBOLS

The following symbols are used in the calculations and the figures:

C_p	heat capacity of gas at constant pressure
E	vibrational energy of gas per unit mass
E^*	equilibrium value for E at given temperature
E'	dimensionless vibrational energy, $E/R\theta$
exp( )	exponential function of quantity in parentheses
H_t	total enthalpy of gas per unit mass
H_t'	dimensionless total enthalpy, $H_t/R\theta$
K	dimensionless constant $L\alpha p_0/\theta\sqrt{R}$ used in equation (9c)
L	characteristic length, of order of nozzle length
M	Mach number
p	static pressure of gas (absolute)
R	gas constant per unit mass
s	entropy of gas per unit mass
T	static temperature of gas
T'	dimensionless temperature, T/θ
t	time, second

u velocity of gas

u' dimensionless velocity, $u/\sqrt{R\theta}$

w fraction of water vapor in gas mixture (by weight)

x distance along mean streamline in nozzle

x' dimensionless distance, x/L

α proportionality constant defined in equation (3)

γ ratio of specific heats of gas

θ characteristic vibrational temperature of gas

ρ density of gas

τ relaxation time, seconds

Subscripts:

0 nozzle inlet

1 nozzle outlet

s standard case

ANALYSIS

A brief discussion of the mechanism of the heat-capacity-lag phenomenon is presented as are the relations used to employ measured values of relaxation times to account for heat-capacity lag in flow processes. Equations are derived to determine the flow conditions in gas-turbine-nozzle expansion processes for the following three possible conditions:

(1) Variable heat capacity with heat-capacity lag

(2) Variable heat capacity with zero lag (equilibrium)

(3) Frozen vibrational energy ($\gamma = 1.4$)

<u>Heat-capacity-lag phenomenon</u>. - The kinetic theory of gases explains the variation of the heat capacity of a gas in terms of

the forms in which internal energy is stored by the gas molecules. In any ideal gas, the equilibrium internal energy per unit mass due to translational kinetic energy is (3/2)RT. That energy due to rotational kinetic energy in a diatomic gas, such as nitrogen, for example, at room temperature or above is RT. For this gas, the energy stored in the form of molecular vibrations, however, increases in proportion to the translational and rotational energy with rising temperature approaching a maximum RT at temperatures well above 6000° R.

The three forms of energy (translational, rotational, and vibrational) can be considered as separate energy-storing systems; each system is considered to be at a temperature determined by the internal energy stored in that system. When the gas is in equilibrium, the three systems will be in temperature equilibrium. If the internal energy of the gas molecules, however, is not divided among the three systems in the relative proportions required by a common temperature (equivalent to the three systems being at different temperatures) the systems are not in equilibrium. The mechanisms leading to thermal equilibrium are the molecular collisions that are effective in producing a net interchange of energy among the three systems. Those collisions that cause an interchange of translational and rotational energy in a gas are predominant so that the translational and rotational systems can be considered in equilibrium. Because of the high probability of exchange of similar energy quanta in molecular collisions, an equilibrium distribution of vibrational energy quanta is assumed to occur among the possible vibrational states of the molecules. Molecular vibrations, however, are excited only by certain types of collision. The result of the infrequency of these collisions is to decrease the rate at which energy can be transferred to and from the vibrational system. This effect is responsible for the observed heat-capacity lag and may be important in gas-flow processes because it places a limitation on the rate of availability of vibrational energy whenever the temperature of a gas is caused to change rapidly.

Relaxation-time relations used in analysis. - The principal component of the gas mixture considered in the analysis is nitrogen; water vapor is assumed present only in regard to its catalytic effect on the relaxation time of the nitrogen. None of the other exhaust-gas components are known to have an appreciable effect (reference 3). Even in the presence of carbon dioxide, which might be expected to exchange vibrational-energy quanta readily with nitrogen, little effect occurs other than that produced by water vapor because the water vapor is a necessary medium for the rapid transfer of vibrational energy from carbon dioxide as well.

Nitrogen was chosen as the single principal component because it is the main constituent of a mixture of exhaust gases, which makes the effect shown by nitrogen indicative of the magnitude of the effect in exhaust gases. In addition, only one characteristic temperature θ would be required to determine the equilibrium vibrational energy at a given temperature. The diatomic nitrogen molecules are approximated by rigid rotator - harmonic oscillators and only the heat capacity of the vibrational system of the molecules is considered as temperature dependent.

In the following analyses, the approach of the vibrational energy to equilibrium will be according to the relations presented in reference 1, which states that the relaxation time of a gas is inversely proportional to the collision frequency of a gas molecule and directly proportional to the average number of collisions per molecule necessary to approach equilibrium. The average number of collisions is nearly independent of temperature, according to the data of reference 3, when small quantities of water vapor are present as catalysts or agents to accelerate the adjustment of the vibrational energy to its equilibrium value.

When the vibrational energy E of the gas differs from its equilibrium value E^* where

$$E^* = RT\left(\frac{\theta/T}{e^{\theta/T}-1}\right) \tag{1}$$

it approaches that value according to the relation from reference 1

$$\frac{dE}{dt} = -\frac{1}{\tau}(E-E^*) \tag{2}$$

Here it is necessary to evaluate E^* at the apparent temperature T defined by the energy stored in the rotational and translational systems, which are assumed at all times to remain in equilibrium with each other. The temperature of the vibrational energy system thus approaches that of the translational and rotational systems which, in turn, establish what is defined as the gas temperature T.

The relaxation time τ, because it is directly proportional to the number of collisions per molecule required to approach equilibrium

1326

(herein considered a constant number because of the presence of water vapor) and inversely proportional to the collision rate between molecules of the gas and the catalyst, is shown in reference 1 and by the kinetic theory of gases to be

$$\frac{1}{\tau} = \alpha \frac{p}{\sqrt{T}} \tag{3}$$

The constant α can be determined from relaxation-time data for the mixture at a known temperature and pressure and is approximately proportional to the concentration of water vapor or the fraction of water vapor in the gas mixture w.

Flow process with variable heat capacity including heat-capacity lag. - The flow processes considered herein are representative of those occurring in gas-turbine nozzles and are investigated by a one-dimensional analysis for a gas of negligible viscosity. For each process considered, the gas is adiabatically accelerated to a specified velocity and the required pressure ratio is calculated.

The one-dimensional Euler equation for time-steady nonviscous flow is

$$u \frac{du}{dx} + \frac{1}{\rho} \frac{dp}{dx} = 0 \tag{4}$$

The equation of state for an ideal gas is

$$p = \rho RT \tag{5}$$

The enthalpy theorem applied to a diatomic gas composed of rigid rotator - harmonic oscillators at room temperature and above, where the rotational contribution to the heat capacity of the gas is considered as having reached its maximum value R, is

$$H_t = \frac{u^2}{2} + \frac{7}{2} RT + E \tag{6}$$

where E is the vibrational energy of the gas and T is the temperature of the rotational and translational systems, which are assumed to be in equilibrium. 1326

The equilibrium vibrational energy of nitrogen would constitute less than 30 percent of the total enthalpy H_t at an infinite temperature and only about 9.5 percent at a temperature of 3500° R. For the temperature range considered herein, changes in E due to relaxation effects will therefore be expected to have only a perturbing effect on the flow process. The magnitude of the effect, however, may change considerably when the gas is initially far from equilibrium. If the vibrational energy of the gas is initially greater than the equilibrium value, it is possible, in view of equation (2), for the rate of adjustment of vibrational energy to be as great as or greater than that occurring in an equilibrium-expansion process. The opposite case of lower than equilibrium initial vibrational energy would result in a transfer of energy into the vibrational system, thereby considerably magnifying the heat-capacity-lag effect. These possible nonequilibrium initial conditions of the gas result in a wide range of effects because of heat-capacity lag and are not considered herein.

Combining equations (4) and (5) yields

$$\frac{dp}{p} = -\frac{u}{RT}\frac{du}{dx}dx \tag{7a}$$

or

$$\frac{p}{p_0} = \exp\left(-\int_0^x \frac{1}{R}\frac{u}{T}\frac{du}{dx}dx\right) \tag{7b}$$

Substituting equations (1) and (3) into equation (2) and using the relation $\frac{d}{dt} = u\frac{d}{dx}$ applicable for time-steady one-dimensional flow gives

$$\frac{dE}{dx} = \frac{-\alpha\,(p/p_0)p_0}{u\sqrt{T}}\left(E - \frac{R\theta}{e^{\theta/T}-1}\right) \tag{8}$$

Equations (6), (7b), and (8) can be transformed into dimensionless form by the introduction of the variables:

$$T' = T/\theta \qquad u' = u/\sqrt{R\theta}$$

$$H_t' = H_t/R\theta \qquad E' = E/R\theta$$

$$x' = x/L \qquad K = \frac{L\alpha p_0}{\theta\sqrt{R}}$$

These substitutions yield the following set of equations:

$$T' = \frac{2}{7}\left[H_t' - E' - \frac{(u')^2}{2}\right] \tag{9a}$$

$$\frac{p}{p_0} = \exp\left(-\int_0^{x'} \frac{u'}{T'}\frac{du'}{dx'}\,dx'\right) \tag{9b}$$

$$\frac{dE'}{dx'} = \frac{-K\,(p/p_0)}{u'\sqrt{T'}}\left(E' - \frac{1}{e^{1/T'}-1}\right) \tag{9c}$$

A complete solution of the flow conditions can be obtained for the mixture by the simultaneous solution of equations (9) if the initial state and the constant K are known for the fluid and if the velocity distribution $u(x)$ is specified. In order to obtain the results presented herein, these equations were solved by numerical methods to give the required flow conditions of temperature, pressure ratio, and vibrational energy. The final values of these quantities then specify the inlet conditions for the next stage of the flow process. It should be noted that for certain velocity distributions the numerical solution is difficult to start if the initial velocity is assumed to be zero. This assumption also leads to a contradiction if the initial state of the fluid is specified as a nonequilibrium state because a value of $u' = 0$ in equation (9c) requires an equilibrium value of E' whenever the relaxation time is finite.

The constant K appearing in equation (9c) contains three important parameters of the flow process: the length of the nozzle, the absolute inlet pressure, and the relaxation time of the gas at some standard temperature and pressure. A variation in any of these quantities can thus be neutralized or simulated by a variation of one of the others.

As the value of K approaches zero, as it would for an infinitely long relaxation time ($\alpha = 0$), equation (9c) requires that dE'/dx' be zero or that the vibrational energy be "frozen" at its initial value. Similarly, if the relaxation time is negligible, K approaches the limiting value of infinity and equation (9c) requires that to have a finite value for dE'/dx' it is necessary for E' to be at all times equal to its equilibrium value.

<u>Flow process with variable heat capacity assuming zero heat-capacity lag.</u> - From equation (1) and the enthalpy theorem as given by equation (9a), the static temperature of a gas in equilibrium is related to its total enthalpy and flow velocity by

$$T'_1 = \frac{2}{7}\left[H_t' - \frac{\left(u'_1\right)^2}{2} - \frac{1}{e^{1/T'_1} - 1}\right] \tag{10}$$

For isentropic flow

$$ds = 0 = C_p \frac{dT}{T} - R \frac{dp}{p} \tag{11a}$$

or

$$0 = \frac{7}{2} R \frac{dT}{T} + \frac{1}{T} \frac{dE^*}{dT} dT - R \frac{dp}{p} \tag{11b}$$

When the value of E^* given in equation (1) is substituted in equation (11b) and equation (11b) is integrated, the following equation is obtained relating pressure ratio to the static temperatures:

$$\frac{7}{2}\log_e\frac{T_1}{T_0}+\left[\frac{\theta}{T}\left(\frac{1}{e^{\theta/T}-1}\right)-\log_e\left(1-e^{-\theta/T}\right)\right]_{T_0}^{T_1}=\log_e\frac{p_1}{p_0} \tag{12a}$$

In terms of dimensionless quantities, equation (12a) may be written as

$$\log_e\frac{p_1}{p_0}=\frac{7}{2}\log_e\frac{T'_1}{T'_0}+\left[\frac{1}{T'}\left(\frac{1}{e^{1/T'}-1}\right)-\log_e\left(1-e^{-1/T'}\right)\right]_{T'_0}^{T'_1} \tag{12b}$$

For specified values of total enthalpy and flow velocity, equations (10) and (12b) give the corresponding values of static temperature and pressure ratio.

<u>Flow process with frozen vibrational energy ($\gamma = 1.4$)</u>. - If the relaxation time is so long as to prevent any recovery of energy from the vibrational system, the vibrational contribution to the energy in equation (10) is a constant evaluated at the initial temperature T_0; therefore no change in vibrational energy occurs during the flow process and the flow variations obtained correspond to constant heat capacity with $\gamma = 1.4$. The static temperature is then related to the initial temperature, the total enthalpy, and the flow velocity by

$$T'_1=\frac{2}{7}\left[H_t'-E'_0-\frac{(u'_1)^2}{2}\right] \tag{13}$$

For zero change in vibrational energy during the flow process, equation (12b) reduces to

$$\log_e \frac{p_1}{p_0} = \frac{7}{2} \log_e \frac{T'_1}{T'_0} \tag{14}$$

1326

FLOW CONDITIONS APPLIED TO CALCULATIONS

The results of the foregoing analysis were used to compute the extent of adjustment of vibrational energy and to compute the pressure ratio required to obtain a specified flow velocity at the throat or outlet of the turbine nozzles considered and, in addition, to provide a comparison of the actual flow process with the flow process for equilibrium heat capacity and for frozen vibrational energy ($\gamma = 1.4$). In all cases, the initial velocity of the fluid in the nozzle was essentially zero and the necessary parameters, such as inlet pressure and temperature, relaxation time, and velocity distribution were specified. Calculations were made both for a standard set of gas-turbine conditions and for a range of the important parameters that would demonstrate the factors tending to amplify the heat-capacity-lag effect.

Calculations of flow in typical turbine nozzle. - A characteristic turbine nozzle in which the flow velocity was found to vary approximately linearly with distance along the mean streamline was chosen as a standard for the calculations. The distance from inlet to throat along this streamline was 1.4 inches, which is used as a characteristic length in the calculations. The gas mixture is thus accelerated from zero velocity to near sonic velocity in a distance of 1.4 inches. Other parameters chosen as representative of the turbine-nozzle flow processes in a turbojet engine operating at sea level were inlet pressure of 4 atmospheres and inlet temperature of 2000° R.

The exhaust gas was considered to contain only nitrogen with 2 percent of water vapor. The water vapor is approximately the amount that would be produced by the combustion of a fuel-air mixture containing 0.0175 pound of fuel per pound of air, the amount necessary to produce the desired inlet temperature. The relaxation time for this gas mixture has been experimentally measured by Kantrowitz and Huber (reference 3) to be 2.3×10^{-4} seconds at 1034° R and 1 atmosphere pressure. The catalytic action of the water vapor allows the use of equation (3) to give the variation

1326

of relaxation time with pressure and temperature because the quantity α is then independent of temperature. A list of the parameters of the flow processes considered in the calculations is presented in table I. In case 1, the standard case listed in table I, the gas is considered during the process of acceleration to a velocity corresponding to a Mach number somewhat greater than 0.9. Further acceleration of the gas to a Mach number of approximately 2 is presented as case 2 although the expansion beyond the throat of a turbine nozzle is essentially a corner-flow process, which cannot be approximated as accurately by a one-dimensional process as can flow within the nozzle.

A turbine operating with a nozzle-inlet temperature of 2500° R is considered in case 3 wherein all other parameters including K are identical to those in the standard case. The increased fuel-air ratio necessary to obtain the inlet temperature of 2500° R would result in a higher concentration of water vapor and hence a corresponding shorter relaxation time in the gas mixture considered. For the same turbine nozzle, p_0 must therefore be smaller for case 3 than for case 1. If the increased water-vapor concentration is assumed to increase the constant K by a factor of approximately 1.5, the required decrease in p_0 by this same factor would make case 3 correspond to operation of the turbojet engine at an altitude of approximately 12,000 feet.

A change in the parameter K from that employed in the standard case is presented as case 4 wherein $K = 0.25\ K_s$. This value of K represents, for example, the case for turbojet-engine operation at an altitude of approximately 33,000 feet (where $p_0 = 0.25$ of the value at sea level); all other parameters are fixed at the values used in the standard case.

Effect of various parameters on flow process. - In order to investigate the factors that might be expected to magnify the heat-capacity-lag effect, the flow conditions were calculated for various parameter values, which sometimes varied from those applicable to typical gas-turbine processes.

Cases 4 and 5 together with the standard case demonstrate the effect of several values of the constant K on the flow process. Case 4, as already described, considers $K = 0.25\ K_s$, whereas case 5 treats the opposite case, which differs from the standard case only in that $K = 4\ K_s$. The results of an increase in inlet pressure or in nozzle length by a factor of 4 (as may be the case in a large stationary gas-turbine installation) on the effect produced by heat-capacity lag on the flow process are thus demonstrated.

The effects of two nonstandard velocity distributions in a turbine nozzle are obtained in cases 6 and 7. The first distribution consists of a low initial acceleration followed by a rapid acceleration to the specified velocity in the neighborhood of the nozzle throat; the velocity distribution is chosen proportional to x^3. The second variation in flow is produced by prescribing a flow velocity proportional to $1 - (1-x)^3$. This distribution requires a rapid acceleration of the fluid at the inlet followed by a more gradual velocity increase near the throat.

The standard case and cases 3, 8, and 9 consider the effect on heat-capacity lag of inlet temperatures of 2000°, 2500°, 3000°, and 3500° R, respectively. The effect of continued acceleration of the gas mixture at the low temperature to a Mach number of approximately 2 is considered in case 2, whereas in case 10 the gas at an inlet temperature of 3500° R is accelerated more rapidly to a Mach number of approximately 0.9 in the distance of 1.4 inches. The continuation of this expansion process to Mach numbers of approximately 2 and 4 is made in cases 11 and 12, respectively. A linear velocity distribution during the entire expansion process is assumed for these cases.

The conditions chosen for case 13, similar to case 11 except for an increase in K/K_s to a value of 6, conform to those for ram-jet exhaust-nozzle flow. The nozzle is 18 inches in length and the ram-jet engine is operating at an altitude of 50,000 feet.

RESULTS OF COMPUTATIONS

For each case listed in table I, the variations in flow conditions with distance along the flow passage are computed for the three conditions of normal heat-capacity lag, zero heat-capacity lag, and frozen vibrational energy ($\gamma = 1.4$). Typical results obtained (case 11) are plotted in figure 1, which indicates the quantities used for evaluation of the magnitude of the heat-capacity-lag effect. Figure 1(a) shows a plot for the actual expansion process, whereas figure 1(b) is an enlarged plot of a section of figure 1(a) on which are shown the variations in pressure ratio and flow kinetic energy with distance in the nozzle. In addition, figure 1(b) shows curves for the cases of equilibrium expansion and for frozen vibrational energy. For a given final velocity at $x/L = 1.75$, the pressure ratios required are 0.136, 0.149, and 0.134 for actual, zero, and infinite relaxation times ($\gamma = 1.4$). For a given pressure drop corresponding to that required for the equilibrium expansion process, the respective flow kinetic energies (in dimensionless form) are 0.847, 0.878, and 0.842.

1326

The quantities chosen for determining the effect of heat-capacity lag are the percentage loss in available kinetic energy (defined as the percentage decrease in flow kinetic energy produced in an expansion process as compared with that produced by an equilibrium expansion process with the same pressure ratio) and the percentage deviation in pressure ratio in a process from that required for acceleration to the same velocity by an equilibrium expansion process that gives maximum utilization of available energy. The actual pressure ratio is also compared with that required for acceleration to the given velocity when frozen vibrational energy is assumed. These quantities are presented in table I for each of the 12 cases considered along with the pressure ratios and the percentage changes in vibrational energy, which is primarily responsible for the heat-capacity-lag effects.

The parameters, which have been studied in this investigation of the effect of heat-capacity lag, are nozzle-outlet Mach number M_1 or velocity u_1, the velocity distribution in the nozzle, the inlet temperature T_0 and pressure p_0, the nozzle length L, and the percentage water vapor in the gas mixture w. The last three parameters have been combined in a single factor K, which is listed in table I relative to a standard value K_s, so that

$$\frac{K}{K_s} = \frac{p_0}{p_{0,s}} \frac{L}{L_s} \frac{w}{w_s} \qquad (15)$$

The parameters chosen for the standard case in which a linear velocity distribution is assumed are: nozzle-outlet Mach number, approximately 0.9; nozzle-inlet temperature, 2000° R; nozzle-inlet pressure, 4 atmospheres; nozzle length, 1.4 inches; and fraction of water vapor by weight in gas mixture, 2 percent.

Any case containing parameters within the range of those considered can be compared with the calculated results by determining K/K_s for the case and finding the set of parameters in table I to which the case most closely coincides. The trends produced by various parameter changes from the standard case then indicate the expected effect of heat-capacity lag in the case considered. The results for the first or standard case demonstrate that: (1) The effect of heat-capacity lag on the usually assumed flow conditions is small in magnitude because only a small part of the internal energy of the gas is stored as vibrational energy and the process calls for release of only a fraction of this energy; (2) of greater importance, is the result that the actual process corresponds more closely to a process with frozen vibrational energy ($\gamma = 1.4$) than it does to the usually assumed equilibrium process.

Effect of variation in factor K. - Because the factor K directly reflects the rate at which vibrational energy can be trans- 1326
ferred, a reduction in K to 0.25 of the standard value makes the pressure ratio and loss in available energy, as shown in table I, closely approach the condition of frozen vibrational energy. The parameter K, if increased by a factor of 4, however, gives a corresponding increase in the availability of vibrational energy with the result that case 5 approaches an equilibrium expansion. The pressure ratio for the equilibrium process differs from the actual value by only 0.19 percent. A plot of the deviations in pressure ratio for the assumptions of equilibrium expansion and frozen vibrational energy as functions of K/K_s is shown in figure 2. The assumption of frozen vibrational energy is equivalent to $K/K_s = 0$. The assumption of frozen vibrational energy is equivalent to $K/K_s = 0$ because frozen vibrational energy corresponds to the condition of infinite relaxation time or $\alpha = 0$. The dependence of K on α then requires that $K = 0$ and $K/K_s = 0$.

Effect of variation in velocity distribution. - Both of the nonlinear velocity distributions considered in cases 6 and 7 required pressure ratios intermediate between the standard case and the condition of frozen equilibrium. The linear velocity distribution is thus shown to result in a smaller loss in available energy than either of the extreme velocity distributions treated. The time required for most of the acceleration in cases 6 and 7 was shorter than that in the standard case so that less adjustment of vibrational energy occurred during this part of the acceleration. The details of the calculation results are particularly interesting in that the larger change in vibrational energy indicated in case 7 than in case 6 occurred principally after the rapid acceleration. This energy was thus not applied to the kinetic energy of the gas during the acceleration process, but contributed only to the increase in entropy of the gas as it approached temperature equilibrium.

Effect of variation in initial temperature. - As the temperature is increased from that employed in the standard case (2000° R) to the value assumed in cases 3, 8, and 9 (2500°, 3000°, and 3500°, respectively), progressively smaller fractional changes in vibrational energy occur in the expansion process. This result is due principally to the fact that a smaller fraction of the vibrational energy is normally transferred in the acceleration of the gas to the specified velocity even under equilibrium conditions. The accompanying smaller deviations in pressure ratio and, at higher temperatures, losses in available energy are also shown in table I.

The pressure ratios for frozen vibrational energy differ by less than 0.14 percent from the actual values, whereas the pressure ratio for equilibrium expansion differs by approximately 0.4 percent from the actual value for a nozzle-inlet temperature of 2000° R.

Effect of variation in final velocity. - The effect of continued acceleration of the gas mixture is partly shown by case 2, which extends the standard acceleration process to a Mach number of approximately 2. Here, a larger portion of the vibrational energy (87 percent) would normally be released in an equilibrium expansion; however, because of heat-capacity lag, only 18 percent of this energy is made available. The resultant effect on the required pressure ratio is seen in table I to be a significant decrease from that determined by the usual assumption of an equilibrium expansion. Again, the actual case is much more closely approximated by a process with frozen vibrational energy than by equilibrium conditions. The deviation in pressure ratio from the actual value is less than 1.7 percent for frozen vibrational energy, whereas the deviation is approximately 4.5 percent for equilibrium expansion.

A change in case 9 to produce an acceleration to a Mach number approximately that attained in the standard case in the same length of nozzle and to cause a larger proportional temperature drop is presented as case 10. Here, both the deviation in pressure ratio and the loss in available energy are greater than in the standard case because of the increased vibrational energy storage at the higher temperature. The deviation in pressure ratio for equilibrium expansion is approximately 0.6 percent compared with a deviation of less than 0.14 percent for frozen vibrational energy at a temperature of 3500° R. A continued acceleration of the gas to a Mach number of approximately 2, considered in case 11, shows a significant further increase in the deviation in pressure ratio from the equilibrium value as well as a greater loss in available energy. The deviations in pressure ratio from the values for equilibrium expansion and for frozen vibrational energy are approximately 9.9 and 1.7 percent, respectively. In cases such as 12 at a temperature of 3500° R and acceleration to a Mach number of 4, the consideration of heat-capacity lag is evidently of primary importance as evidenced by the large change in pressure ratio and loss in available energy. The pressure ratio for frozen vibrational energy deviates from the actual value by approximately 8.9 percent; whereas the pressure ratio for equilibrium expansion deviates by approximately 85 percent from the actual value. Although the deviation in pressure ratio from the equilibrium condition becomes very large, the loss in available energy is limited to less than the amount of energy stored in the vibrational system of the gas molecules.

<u>Effect of variation in factor K for high temperature and high final velocity.</u> - Increasing the value of K/K_s to 6, as was done in proceeding from case 11 to case 13, produced a process differing markedly from that described in the previous paragraph. Although the conditions were such as to make the process considerably dependent on vibrational energy, the availability of this energy, as reflected by the larger value of K/K_s, made the process differ only slightly from equilibrium conditions. The pressure ratio assuming equilibrium expansion deviates by only 0.54 percent from the actual case, whereas the assumption of frozen vibrational energy would result in a pressure ratio differing by 10 percent from the actual process. This case, representative of ram-jet exhaust-nozzle flow processes, thus cannot be better approximated by using the value $\gamma = 1.4$.

The results described in this section and listed in table I demonstrate that the greatest effect of heat-capacity lag is observed in cases where a relatively large amount of energy is stored in the vibrational system of the gas, as occurs at high temperatures, at the time when the gas undergoes a large change in temperature. If this change takes place in a time short compared with the relaxation time of the gas, as would occur in expansion processes at high sonic or supersonic velocities in gas turbines, very little of the vibrational energy is utilized.

SUMMARY OF RESULTS

The effect of heat-capacity lag on the flow conditions for expansion of a gas mixture of nitrogen and water vapor through a turbine nozzle has been calculated for a set of representative flow conditions in a typical turbojet engine and for a number of variations of this standard flow process. This gas mixture was chosen because the results were felt to be indicative of the importance of heat-capacity lag in gas-turbine engines; chemical dissociation was not considered in this investigation. The pressure ratios (outlet to inlet) across the nozzle to provide a given nozzle jet velocity were compared for three cases, namely:

(a) Actual case, in which heat-capacity lag is considered in the expansion process

(b) Equilibrium case, in which heat-capacity lag is assumed to be zero

(c) Frozen-vibrational-energy case, in which the heat-capacity lag is assumed infinite. (For this case, the ratio of specific heats is constant and for diatomic gases is equal to 1.40.)

For nozzle-outlet Mach numbers of 0.9 and less and for nozzle-inlet temperatures of 3500° R and less, the three cases to produce the same jet velocity required pressure ratios differing by less than 1 percent. All cases in which 2 percent of water vapor or less was present in the gas mixture showed that a condition of frozen vibrational energy gave a closer approximation to the actual case than did the equilibrium expansion. For 2 percent water vapor and a nozzle-outlet Mach number of approximately 0.9, the pressure ratio for frozen vibrational energy differed by less than 0.14 percent from the actual value; whereas that pressure ratio required for equilibrium expansion differed by approximately 0.4 and 0.6 percent for temperatures of 2000° and 3500° R, respectively.

At nozzle-outlet Mach numbers of approximately 2, the pressure ratios for frozen vibrational energy deviated from the actual values by less than 1.7 percent for nozzle-inlet temperatures of 2000° and 3500° R, whereas those for an equilibrium expansion deviated by approximately 4.5 and 9.9 percent, respectively. An approximation to the continued acceleration of the gas with an inlet temperature of 3500° R to a Mach number of approximately 4 when equilibrium expansion was assumed deviated in pressure ratio from the actual case by approximately 85 percent; whereas an assumption of frozen vibrational energy led to a pressure ratio deviating from the actual case by 8.9 percent.

Conditions closely approximated by equilibrium expansion were obtained for $K/K_s = 4$ for a nozzle-inlet temperature of 2000° R and a nozzle-outlet Mach number of approximately 0.9. A similar result was obtained for a nozzle with inlet temperature of 3500° R and outlet Mach number of approximately 2 when $K/K_s = 6$. The respective deviations in pressure ratio from actual values by assuming equilibrium expansions were only 0.19 percent and 0.54 percent.

CONCLUSION

As based on the calculations described herein, the effect of heat-capacity lag on nozzle expansion processes is dependent principally on a parameter K, which is proportional to the nozzle-inlet pressure, the nozzle length, and the amount of water vapor

1326

present in the exhaust gases. For a range of values of K representative of turbine-nozzle flow processes, an assumption of frozen vibrational energy or constant heat capacity gives smaller deviations from the actual flow conditions than does the equilibrium variable heat capacity of the gas. Larger values of K dictated by ram-jet operating conditions, however, lead to flow processes, which can be closely approximated by equilibrium conditions.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, March 2, 1950.

REFERENCES

1. Kantrowitz, Arthur: Effects of Heat-Capacity Lag in Gas Dynamics. NACA ARR 4A22, 1944.

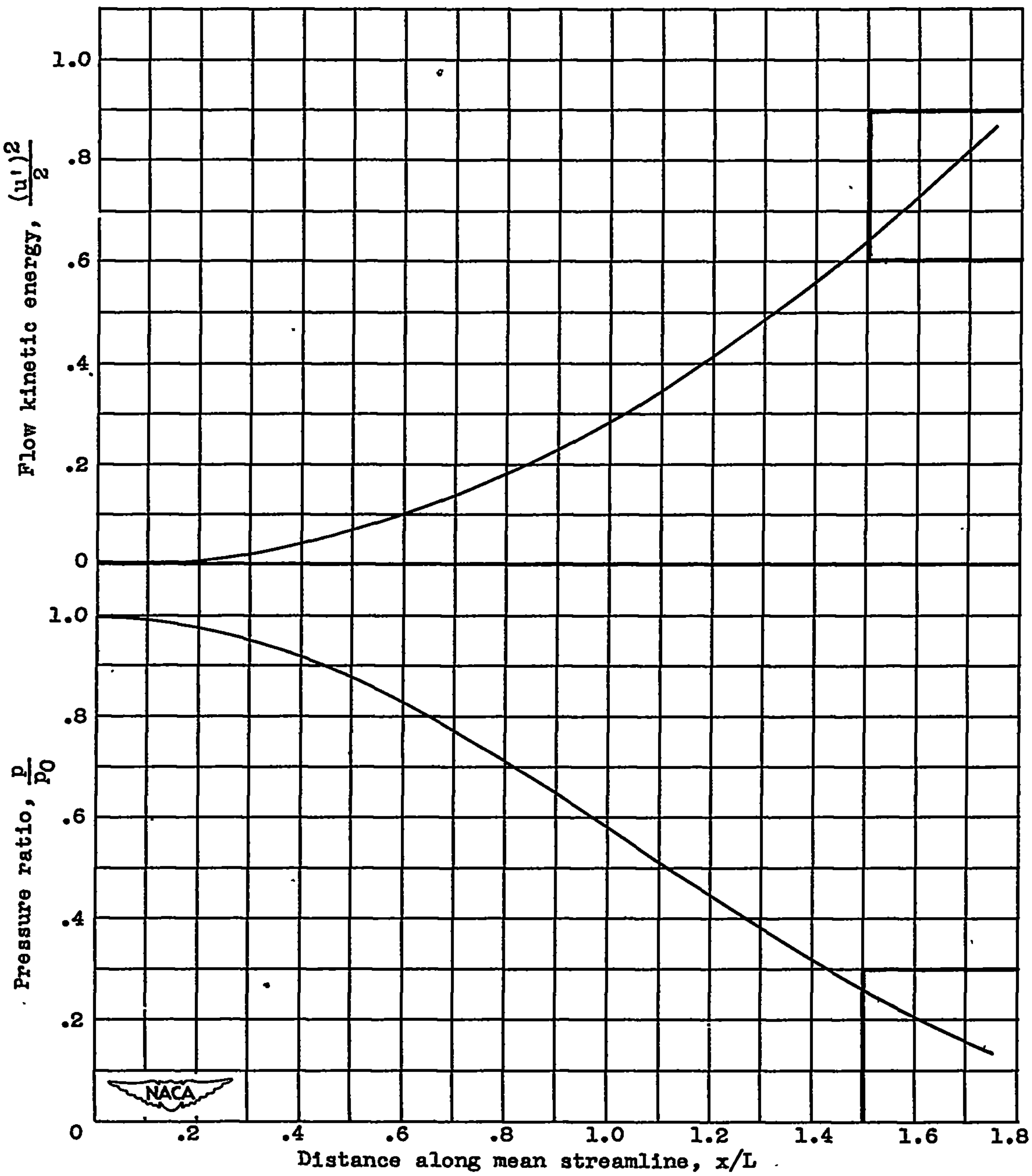
2. Landau, L., und Teller, E.: Zur Theorie der Schalldispersion. Phys. Zeitschr. d. Sowjetunion, Bd. 10, Heft 1, 1936, S. 34-43.

3. Kantrowitz, Arthur, and Huber, Paul W.: Heat-Capacity Lag in Turbine-Working Fluids. NACA RB L4E29, 1944.

TABLE I - RESULTS OF CALCULATIONS FOR COMPARISON OF FLOW PROCESSES

Case	Velocity distribution	Ratio of K factor to standard value	Inlet temperature (°R)	Final velocity (dimensionless)	Final Mach number (approximate)	Pressure ratio required			Percent deviation of p_1/p_0 from actual value for a given kinetic energy		Loss in available energy for given p_1/p_0, percent		Ratio of initial vibrational energy to total enthalpy	Percent change in vibrational energy	
						Actual	Frozen vibrational energy	Equilibrium	Equilibrium	Frozen vibrational energy	Actual	Frozen vibrational energy		Actual	Equilibrium
1[a]	Linear	1	2000	0.572	0.9	0.5838	0.5830	0.5859	0.360	0.137	0.627	0.867	0.0414	9.68	37.0
2[a]	Linear	1	2000	1.001	2	.1358	.1335	.1417	4.50	1.55	1.69	2.27	.0414	18.0	87.5
3[a]	Linear	1	2500	.572	.8	.6546	.6539	.6566	.290	.107	.644	.877	.0622	6.28	24.9
4[a]	Linear	.25	2000	.572	.9	.5833	.5830	.5859	.446	.051	.790	.867	.0414	3.07	37.0
5	Linear	4	2000	.572	.9	.5848	.5830	.5859	.188	.308	.351	.867	.0414	21.2	37.0
6	X^3	1	2000	.572	.9	.5835	.5830	.5859	.411	.086	.730	.867	.0414	5.36	37.0
7	$1-(1-x)^3$	1	2000	.572	.9	.5836	.5830	.5859	.394	.103	.687	.867	.0414	13.3	37.0
8	Linear	1	3000	.572	.7	.7050	.7045	.7066	.227	.071	.615	.826	.0803	4.42	18.1
9	Linear	1	3500	.572	.7	.7427	.7423	.7440	.175	.054	.574	.761	.0954	3.29	13.9
10	Linear	1	3500	.757	.9	.5839	.5831	.5876	.634	.137	1.10	1.35	.0954	4.23	24.1
11	Linear	1	3500	1.324	2	.1358	.1335	.1492	9.87	1.69	3.73	4.36	.0954	8.22	67.8
12	Linear	1	3500	1.741	4	.0079	.0072	.0146	84.8	8.86	6.87	7.72	.0954	9.01	96.9
13[b]	Linear	6	3500	1.324	2	.1484	.1335	.1492	.539	10.04	.222	4.36	.0954	56.2	67.8

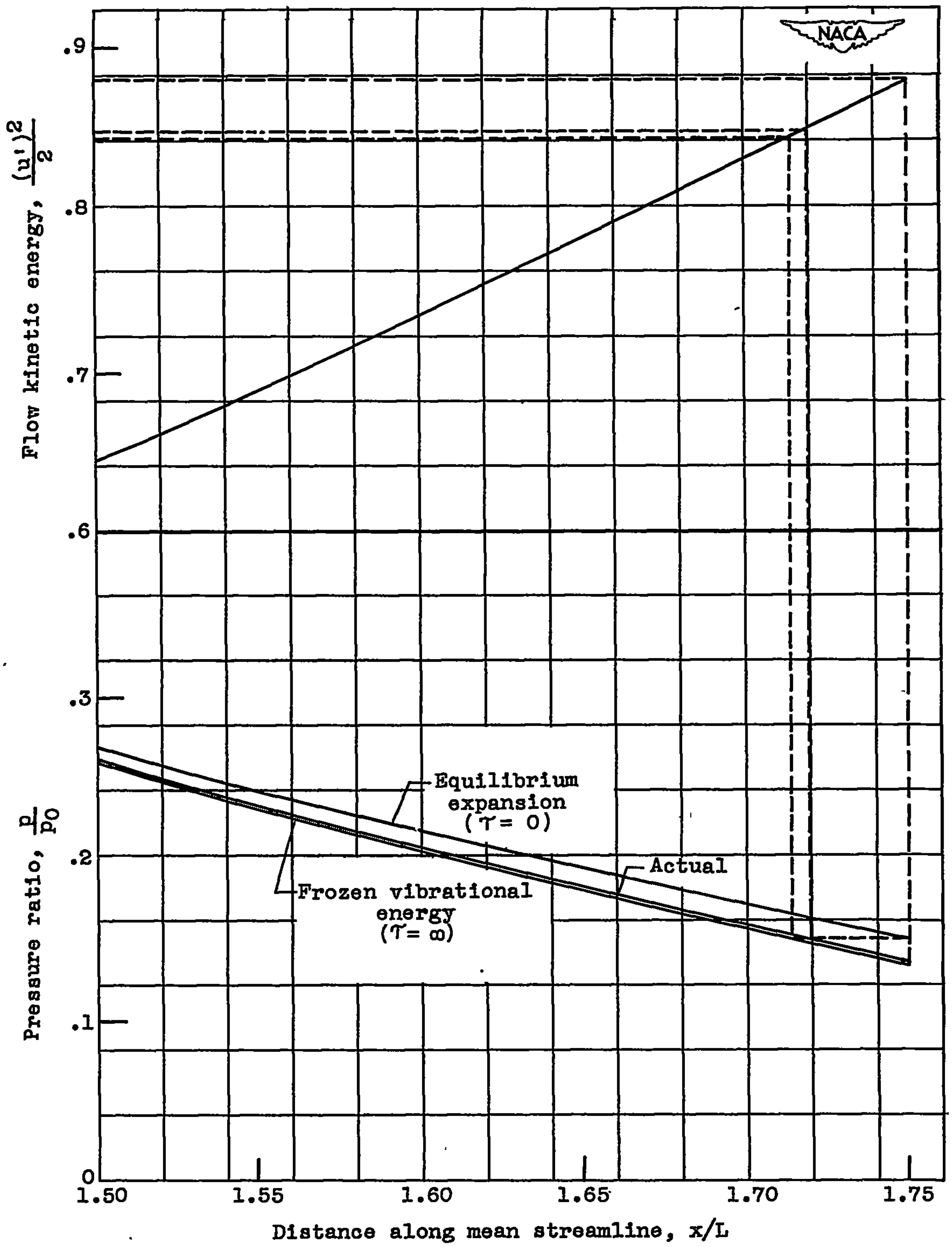
[a]Representative of various turbine-nozzle operating conditions.
[b]Representative ram-jet exhaust-nozzle condition.



(a) Actual expansion process.

Figure 1. - Flow kinetic energy and pressure ratio of gas as functions of position in nozzle (case (11)).

1526



(b) Enlarged portions of figure 1(a). Actual, zero, and infinite relaxation time.

Figure 1. - Concluded. Flow kinetic energy and pressure ratio of gas as functions of position in nozzle (case (11)).

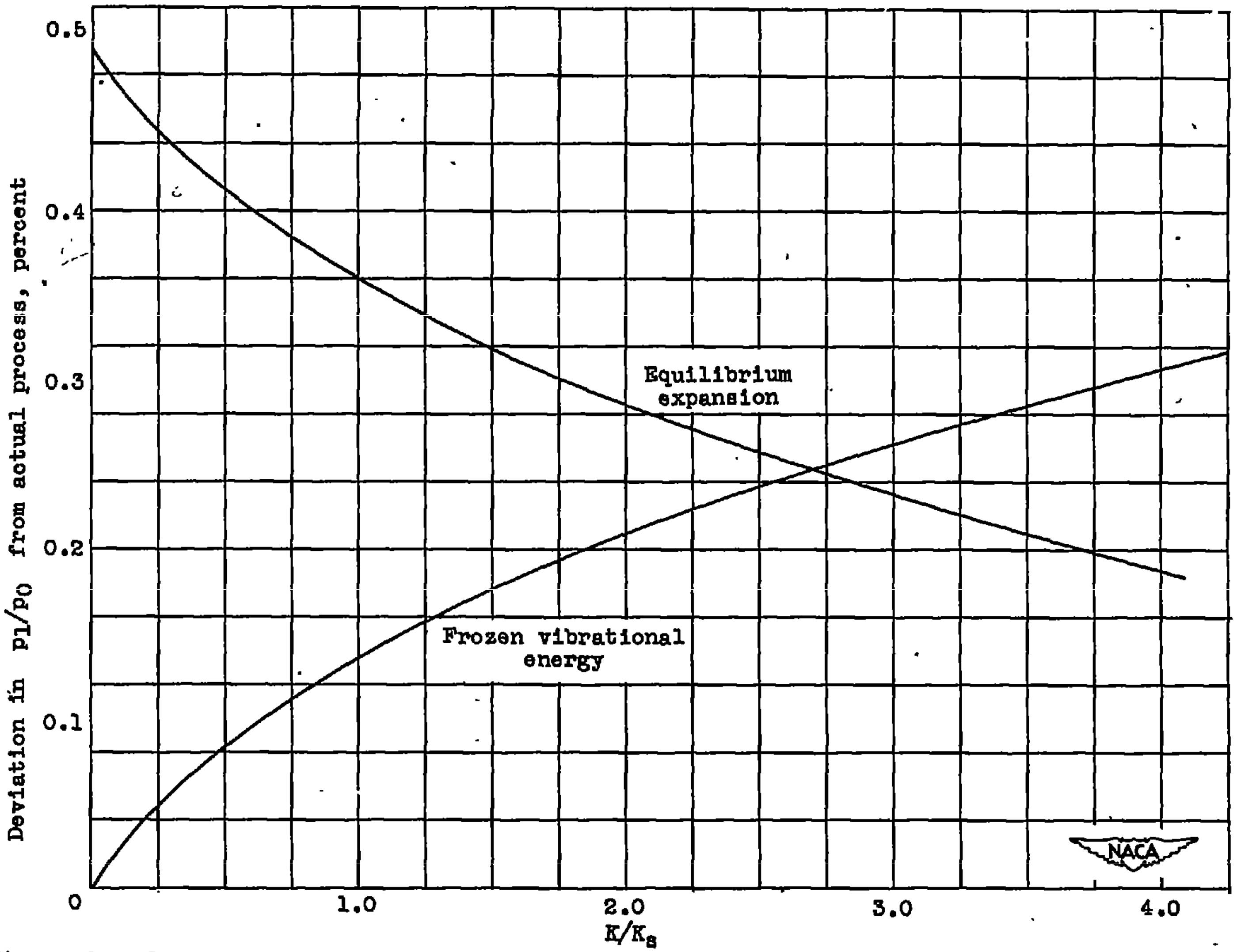


Figure 2. - Deviation in pressure ratio from actual value as function of K/K_s for assumptions of equilibrium expansion and frozen vibrational energy. Nozzle-inlet temperature, 2000° R; nozzle-outlet Mach number, 0.9; nozzle-inlet pressure, 4 atmospheres.