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A STRUCTURAL-EFFICIENCY EVALUATION OF TITANIUM AT NORMAL AND ELEVATED TEMPERATURES

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SUMMARY

A structural-efficiency evaluation of titanium including comparisons with several other materials is given for compressive loading without buckling, for column buckling, and for the buckling of long plates in compression or shear. The methods of evaluation, based upon the use of stress-strain curves and structural indexes, are fully described. The comparisons indicate that the high-strength aluminum and magnesium alloys are generally more efficient on a unit-weight basis at normal temperatures than commercially pure titanium sheet having a compressive yield stress of about 106 ksi. For short-time loading conditions at temperatures beginning somewhat above 400° F, titanium sheet is more efficient than the aluminum alloys.

INTRODUCTION

As a result of extensive development programs in the last few years, titanium, a nonstrategic material, has become available. Titanium is a high-strength, corrosion-resistant, light-weight metal having favorable elevated-temperature properties and a density intermediate between that of steel and aluminum (references 1 and 2). These desirable characteristics have aroused extensive interest in the possibility of using titanium for aircraft structural applications. Some of the titanium alloys which are just becoming available look even more promising.

In order to indicate the potentialities of this new material, a structural-efficiency evaluation of titanium is given for compressive loading without buckling, for column buckling, and for the buckling of long plates in compression or shear. The evaluation includes comparisons with several materials at normal temperature (80° F) and with two high-strength aluminum alloys at elevated temperatures up to 600° F.

EVALUATION

A structural-efficiency evaluation of titanium is given for compressive loading without buckling, for column buckling, and for the buckling of long plates loaded in compression as well as in shear. The evaluation includes comparisons with various materials. The materials and sources of data are titanium sheet (reference 3), extruded ZK60A magnesium alloy (unpublished), extruded 75S-T6 aluminum alloy (reference 4), 24S-T3 aluminum alloy (reference 4), and Stainless W (reference 5). The comparisons are extended to elevated temperatures for titanium and the two aluminum alloys for which compressive stress-strain data are available. The evaluations at elevated temperatures are considered to be valid only for short-time loading and exposure conditions and do not apply to long-time loading conditions, where creep or exposure effects may be important.

In order to determine the structural efficiency on a unit-weight basis, the density as well as the stiffness and strength of the material has been taken into account. The efficiency is measured by the stress-density ratio, and the loading conditions and type of structure are covered by a structural parameter. Methods of analysis and principles involved at normal and elevated temperatures are described in the appendix, which includes derivations for the structural indexes for columns and for plates loaded in compression or in shear.

Compressive loading without buckling.- The efficiencies of the various materials for compressive loading without buckling are shown on a unit-weight basis in figure 1 for temperatures of 80° F, 400° F, and 600° F by plots of the stress-density ratio σ/d against the strain. Because of the marked anisotropy of titanium sheet in compression, individual curves are shown for loading this material in both the longitudinal (L) and the transverse (T) grain directions; the curves for the other materials are for loading in the longitudinal direction.

If the compressive strength of the structure is the criterion and buckling does not occur, the ratio of compressive yield stress to density may be taken as a measure of the efficiency of the material. On this basis, 75S-T6 is the most efficient of the materials at normal temperature, with titanium (T) and Stainless W next best (fig. 1). At 400° F, 24S-T3 is the most efficient and is followed closely by titanium (T) and 75S-T6. At 600° F, titanium is considerably more efficient than the aluminum alloys.

If, on the other hand, the stiffness of the structure is the criterion and buckling does not occur, the modulus-density ratios, given by the initial slopes of the curves of figure 1, can be taken as the measure of the efficiency. On this basis, all the materials have about the same efficiency at 80° F. At 600° F, however, titanium is more efficient with regard to stiffness than the aluminum alloys and is less affected by temperature.

If, however, a certain amount of distortion of the structure is the criterion and buckling does not occur, the curves of figure 1 can be taken directly as a measure of the efficiencies of the materials. On this basis, the material having the greatest value of the stress-density ratio for a given value of strain will be the most efficient. At 80° F, extruded 75S-T6 is the most efficient of these materials (fig. 1), with titanium (T) next best. At 400° F, 24S-T3 sheet is slightly more efficient than titanium (T) and appreciably more efficient than extruded 75S-T6. At 600° F, however, titanium is markedly more efficient on this basis than the aluminum alloys.

Column buckling.- The efficiencies of the various materials for column buckling are shown on a unit-weight basis in figure 2 for temperatures of 80° F, 400° F, and 600° F by plots of the ratio of buckling stress to density σ_{cr}/d against the structural index for columns $P_{cr}cf/L^2$. In this index, P_{cr} is the buckling load equal to the applied load, L is the distance over which the load is transmitted, c is the coefficient of end fixity, and f is a nondimensional shape factor for the column cross section. The material having the greatest value of the stress-density ratio for a given value of the structural index will have the least weight. The analysis is restricted to columns having cross sections of such proportions that local or torsional instability does not occur.

At 80° F, extruded 75S-T6 (fig. 2) is the most efficient material except for small values of the structural index where extruded ZK60A magnesium alloy is slightly more efficient. Titanium (T) is second best to extruded 75S-T6 for large values of the index. At 400° F, 24S-T3 sheet is the most efficient material with titanium (T) next in order for large values of the index. At 600° F, titanium is far more efficient than the aluminum alloys.

Plate buckling.- The efficiencies of the various materials for the buckling of long plates loaded in compression or shear are shown on a unit-weight basis in figure 3 for temperatures of 80° F, 400° F, and 600° F by plots of the ratios of the buckling stress to the density σ_{cr}/d or $1.73\tau_{cr}/d$ against the structural indexes for plates $P_{cr}k_c^{1/2}/b^2$ or $2.28Q_{cr}k_s^{1/2}/b^2$. In these indexes, P_{cr} and Q_{cr}, respectively, are the buckling loads in compression and in shear equal to the applied loads, b is the plate width, and k_c and k_s, respectively, are nondimensional plate-buckling coefficients for compression and shear loading which are dependent only upon the edge conditions for a long plate. The material having the highest value of the ratio of buckling stress to density for a given value of the structural index will be the most efficient and have the least weight.

At 80° F, extruded ZK60A magnesium alloy is the most efficient of the materials for values of the index up to about 3 ksi. For larger values of the index, extruded 75S-T6 is the most efficient with titanium (T) next in order. At 400° F, 24S-T3 sheet and extruded 75S-T6 are both more efficient than titanium. At 600° F, however, titanium shows up to much better advantage than the aluminum alloys.

CONCLUDING REMARKS

An evaluation of titanium including comparisons with several other materials has been given for compressive loading without buckling, for column buckling, and for the buckling of long plates in compression or shear. The procedure, based upon the use of structural indexes, provides a general method for determining the structural efficiency of a material for a particular structural application by means of a single curve. The comparisons indicate that the high-strength aluminum and magnesium alloys are generally more efficient on a unit-weight basis at normal temperatures than commercially pure titanium sheet having a compressive yield stress of about 106 ksi. For short-time loading conditions at temperatures somewhat above 400° F with an exposure time of about 1 hour, titanium is more efficient than the aluminum alloys. Longer or shorter exposure conditions would be expected to alter the results somewhat, particularly for the aluminum alloys.

If the compressive yield stress of titanium, or titanium alloys, should be increased to about 130 ksi, then for a given permissible strain, titanium would compare more favorably at normal temperatures with the high-strength lighter alloys for compressive loading without buckling. Such an increase in strength, however, would still not make titanium as efficient in general as the lighter alloys for column and plate buckling applications. For plate buckling, particularly, a substantial increase in the initial modulus-density ratio of titanium would be required in order to obtain efficiencies comparable to those for the lighter alloys. Because such an increase is improbable, the lighter high-strength alloys can be expected to be more efficient than titanium at normal temperatures except perhaps for cases in which the ratios of buckling stress to density for titanium are well above the ratios of yield stress to density for the lighter alloys.

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APPENDIX

METHODS OF ANALYSIS

Methods of analysis for determining the efficiency of a material for column buckling and for the buckling of long plates loaded in compression or shear are described. Derivations of the structural indexes for column and plate buckling are included. The same methods are used for the analysis at elevated temperatures; the basis for this extension is discussed in the last section of this appendix.

Column Buckling

The efficiency of a material for use in a column subject to buckling can be determined on a unit-weight basis by plots of the ratio of buckling stress to density against the structural index for columns. (See references 6 to 8.)

The derivation of the structural index for columns can be obtained by simultaneously satisfying equations (1) and (2):

$$\sigma_{cr} = \frac{c\pi^2 E_{tan}}{\left(\frac{L}{\rho}\right)^2} \tag{1}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} \tag{2}$$

Equation (1) is the generally accepted tangent-modulus column formula in which E_{tan} is the tangent modulus, c is the coefficient of end fixity (equal to 1 for pin-ended and 4 for fixed-ended columns), σ_{cr} is the buckling stress, L is the column length, and ρ is the radius of gyration. Equation (2) is the elementary formula relating the buckling stress σ_{cr}, the buckling load P_{cr} (equal to the applied load), and the cross-sectional area A.

The desired relationship between σ_{cr}, P_{cr}, and L can be obtained by multiplying and dividing the right-hand side of equation (1) by A, grouping the terms as follows

$$\sigma_{cr} = c\pi^2 E_{tan}\left(\frac{\rho^2}{A}\right)\left(\frac{A}{L^2}\right) \tag{3}$$

and recognizing that ρ^2/A is a nondimensional shape factor f for the column cross section. Eliminating the A in the second parenthetical term of equation (3) by means of equation (2) gives

$$\sigma_{cr} = c^{1/2}\pi E_{tan}{}^{1/2}f^{1/2}\left(\frac{P_{cr}}{L^2}\right)^{1/2} \tag{4}$$

Two structural indexes are evident from equation (4). The first is the usual index P_{cr}/L^2 defined by

$$\frac{P_{cr}}{L^2} = \frac{\sigma_{cr}{}^2}{c\pi^2E_{tan}f} \tag{5}$$

which is used chiefly for comparing the efficiencies of columns having a given shape factor and end-fixity coefficient. The second, which is a function only of stress, is $P_{cr}cf/L^2$:

$$\frac{P_{cr}cf}{L^2} = \frac{\sigma_{cr}{}^2}{\pi^2E_{tan}} \tag{6}$$

and is used herein for comparing the efficiencies of materials for column applications. The advantage of this latter index is that a single curve can indicate the efficiency of a material, whereas if the usual index P_{cr}/L^2 is used, the index varies with the shape factor and the end-fixity coefficient, and families of curves are required for each material. When such a curve of the ratio of buckling stress to density against the structural index is constructed, values of the structural index are calculated by means of equation (6) for assumed values of stresses, the corresponding tangent moduli being determined from the stress-strain curve.

Certain restrictions should be considered in the use of this structural index for columns. Any combination of values of P_{cr}, c, f, and L can satisfy an index value given by equation (6). In the actual column, however, certain load and length requirements must be met, and the cross-sectional shape will be determined by various structural and design requirements. The type of column instability which develops will depend upon this cross-sectional shape because a thin-walled column or one having an open-type section may develop local or torsional instability rather than bending instability. The column-index relations covered herein apply only to instability in bending.

Plate Buckling

The efficiency of a material used in long plates subject to buckling in compression or shear can also be determined on a unit-weight basis by plots of the ratio of buckling stress to density σ_{cr}/d or τ_{cr}/d against the structural index applicable for the type of loading (reference 6).

The derivation of the indexes for buckling in compression or shear parallels that for columns. The two equations that must be satisfied simultaneously are:

For compression,

$$\sigma_{cr} = \frac{k_c \pi^2 E_{rc}}{12\left(1 - \mu^2\right)}\left(\frac{t}{b}\right)^2 \tag{7}$$

$$\sigma_{cr} = \frac{P_{cr}}{bt} \tag{8}$$

For shear,

$$\tau_{cr} = \frac{k_s \pi^2 E_{rs}}{12\left(1 - \mu^2\right)}\left(\frac{t}{b}\right)^2 \tag{9}$$

$$\tau_{cr} = \frac{Q_{cr}}{bt} \tag{10}$$

In equations (7) to (10), σ_{cr} and τ_{cr} are, respectively, the buckling stresses in compression and shear, k_c and k_s are the plate buckling coefficients for compression and shear, E_{rc} and E_{rs} are the reduced moduli for compression and shear, P_{cr} and Q_{cr} are the critical buckling loads in compression and shear which are equal to the corresponding applied loads, t and b are the plate thickness and plate width, and μ is Poisson's ratio. In the case of long plates, the important design quantities are the stress, the load, and the plate width b over which the load is distributed; the length is not a factor as the strength is independent of the length for a long plate.

After t is eliminated from equations (7) and (8) for compression and from equations (9) and (10) for shear, the resulting expressions for σ_{cr} and τ_{cr} are

$$\sigma_{cr} = \left[\frac{k_c \pi^2}{12\left(1 - \mu^2\right)}\right]^{1/3} E_{rc}^{1/3}\left(\frac{P_{cr}}{b^2}\right)^{2/3} \tag{11}$$

$$\tau_{cr} = \left[\frac{k_s \pi^2}{12\left(1 - \mu^2\right)}\right]^{1/3} E_{rs}^{1/3}\left(\frac{Q_{cr}}{b^2}\right)^{2/3} \tag{12}$$

Inspection of equations (11) and (12) shows that, for each type of loading, the structural index may be defined in either of two ways.

For compression,

$$\frac{P_{cr}}{b^2} = \left[\frac{12(1 - \mu^2)}{k_c \pi^2}\right]^{1/2} \frac{\sigma_{cr}^{3/2}}{E_{rc}^{1/2}} \tag{13}$$

or

$$\frac{P_{cr} k_c^{1/2}}{b^2} = \left[\frac{12(1 - \mu^2)}{\pi^2}\right]^{1/2} \frac{\sigma_{cr}^{3/2}}{E_{rc}^{1/2}} \tag{14}$$

For shear,

$$\frac{Q_{cr}}{b^2} = \left[\frac{12(1 - \mu^2)}{k_s \pi^2}\right]^{1/2} \frac{\tau_{cr}^{3/2}}{E_{rs}^{1/2}} \tag{15}$$

or

$$\frac{Q_{cr} k_s^{1/2}}{b^2} = \left[\frac{12(1 - \mu^2)}{\pi^2}\right]^{1/2} \frac{\tau_{cr}^{3/2}}{E_{rs}^{1/2}} \tag{16}$$

The structural indexes P_{cr}/b^2 and Q_{cr}/b^2 (equations (13) and (15)) are useful chiefly for determining the efficiency of a plate structure having given loading and edge conditions. The structural indexes $P_{cr}k_c^{1/2}/b^2$ and $Q_{cr}k_s^{1/2}/b^2$ (equations (14) and (16)) include the plate buckling coefficients and are comparable to the structural index for columns $P_{cr}cf/L^2$ (equation (6)) in that they are also functions only of stress; these indexes are used herein for comparing the efficiencies of materials for plate applications.

For both compressive and shear loading of long plates, the secant modulus is the major factor in establishing the reduced modulus for plate buckling, according to Stowell's theory, regardless of type of edge condition for the plate (references 9 and 10). This theory, which gives approximately the same result as Bijlaard's (reference 11) and Ilyushin's (reference 12), has had experimental verification for buckling in compression of hinged flanges and simply supported plates (references 9 and 13), and for buckling in shear of clamped plates (reference 10). In view of the fact that the secant modulus is the major factor in the determination of the reduced modulus and that an approximation for the modulus is sufficiently accurate for comparative evaluations of materials, the secant modulus E_{sec} is substituted for E_{rc} and E_{rs} in equations (14) and (16). Then

$$\frac{P_{cr}k_c^{1/2}}{b^2} = \left[\frac{12(1 - \mu^2)}{\pi^2}\right]^{1/2} \frac{\sigma_{cr}^{3/2}}{E_{sec}^{1/2}} \tag{17}$$

and

$$\frac{Q_{cr}k_s^{1/2}}{b^2} = \left[\frac{12(1 - \mu^2)}{\pi^2}\right]^{1/2} \frac{\tau_{cr}^{3/2}}{E_{sec}^{1/2}} \tag{18}$$

A comparison of equations (17) and (18) shows that the structural indexes for compression and shear have the same form and that the right-hand sides are identical except that τ_{cr} replaces σ_{cr}. As $\sigma_{cr}/\sqrt{3}$ may be substituted for τ_{cr} (reference 10), the indexes have the same numerical value except for a factor of $\sqrt[4]{3^3}$. Thus

$$\frac{Q_{cr}k_s^{1/2}}{b^2} = \frac{1}{\sqrt[4]{3^3}} \frac{P_{cr}k_c^{1/2}}{b^2} \tag{19}$$

Consequently, the efficiency of a material for both compressive and shear loading can be measured by the same curve in a plot of the stress-density ratio against the structural index. The ordinate is σ_{cr}/d or $1.73\tau_{cr}/d$ and the abscissa $P_{cr}k_c^{1/2}/b^2$ or $2.28Q_{cr}k_s^{1/2}/b^2$. Such curves are constructed from values of the structural index calculated by means of equation (17) for assumed values of stresses, the corresponding secant moduli being determined from the stress-strain curve. In these indexes, values of k_c may vary from 4 for a simply supported plate to about 7 for a clamped edge condition; values of k_s vary from about 5.3 to 9 for corresponding conditions.

Analysis at Elevated Temperatures

The methods described for determining the efficiency of a material for application to structures that develop instability at normal temperatures have also been used for the analysis of structures at elevated temperatures. Research on the buckling of plates has shown that methods of calculating buckling stresses at normal temperatures can also be applied at elevated temperatures provided that the stress-strain curve used for the calculation is obtained under loading, temperature, and exposure conditions identical to those under which the plate is loaded (reference 14).

The stress-strain data used herein were obtained from short-time compressive stress-strain tests, the material being exposed to the test temperature approximately 1 hour before being loaded at a strain rate of about 0.002 per minute. The evaluations are considered valid only for such loading and exposure conditions and do not apply to long-time conditions, where creep or exposure effects may be important factors.

The increase in Poisson's ratio with temperature was considered to be negligible for titanium for the temperature range covered but was taken into account for the aluminum alloys by using values assumed in reference 14.

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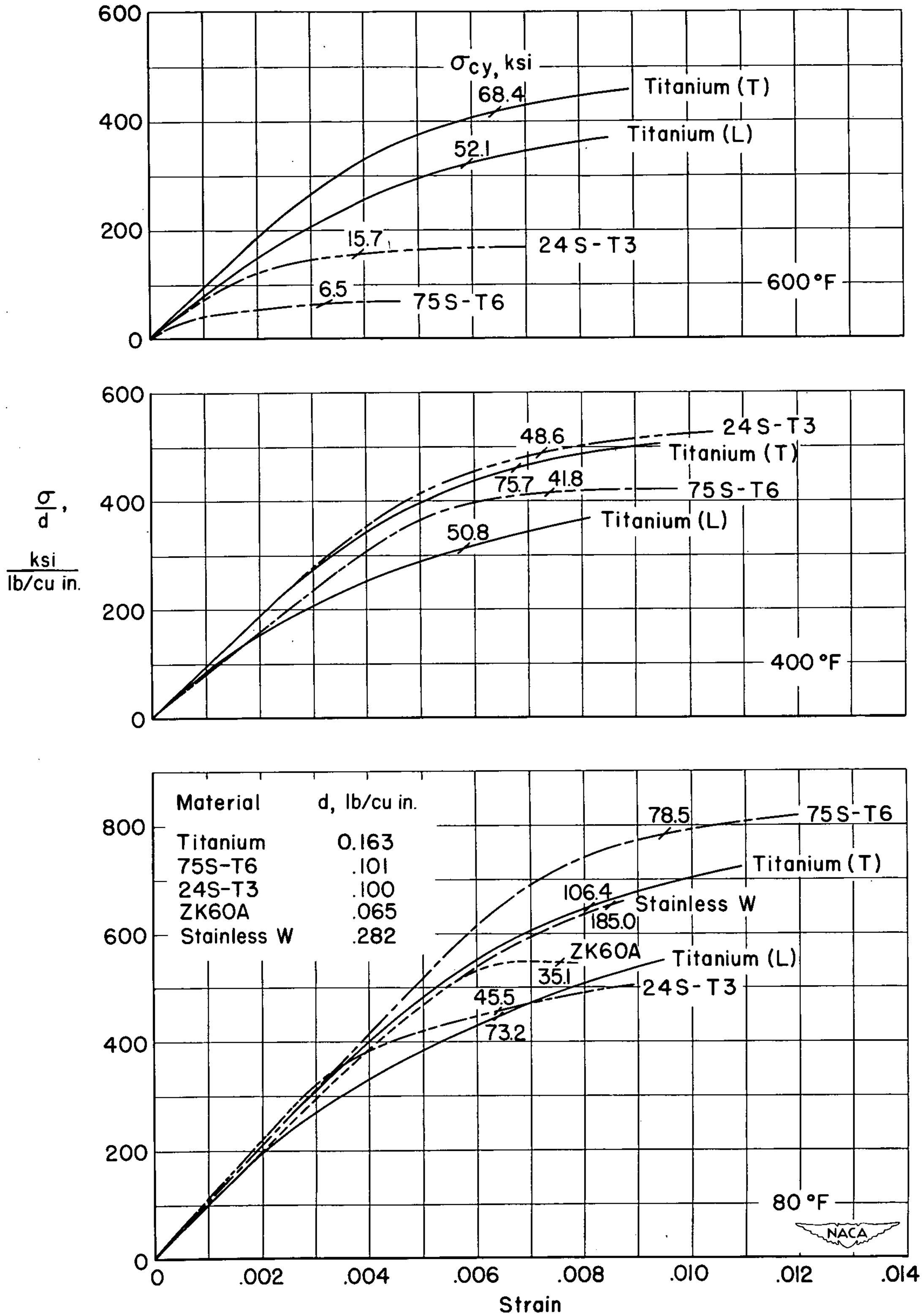

Figure 1.- Modified compressive stress-strain curves for titanium sheet, extruded 75S-T6 aluminum alloy, 24S-T3 aluminum-alloy sheet, extruded ZK60A magnesium alloy, and Stainless W sheet at 80°, 400°, and 600° F. (Compressive yield stress, 0.2 percent offset.)

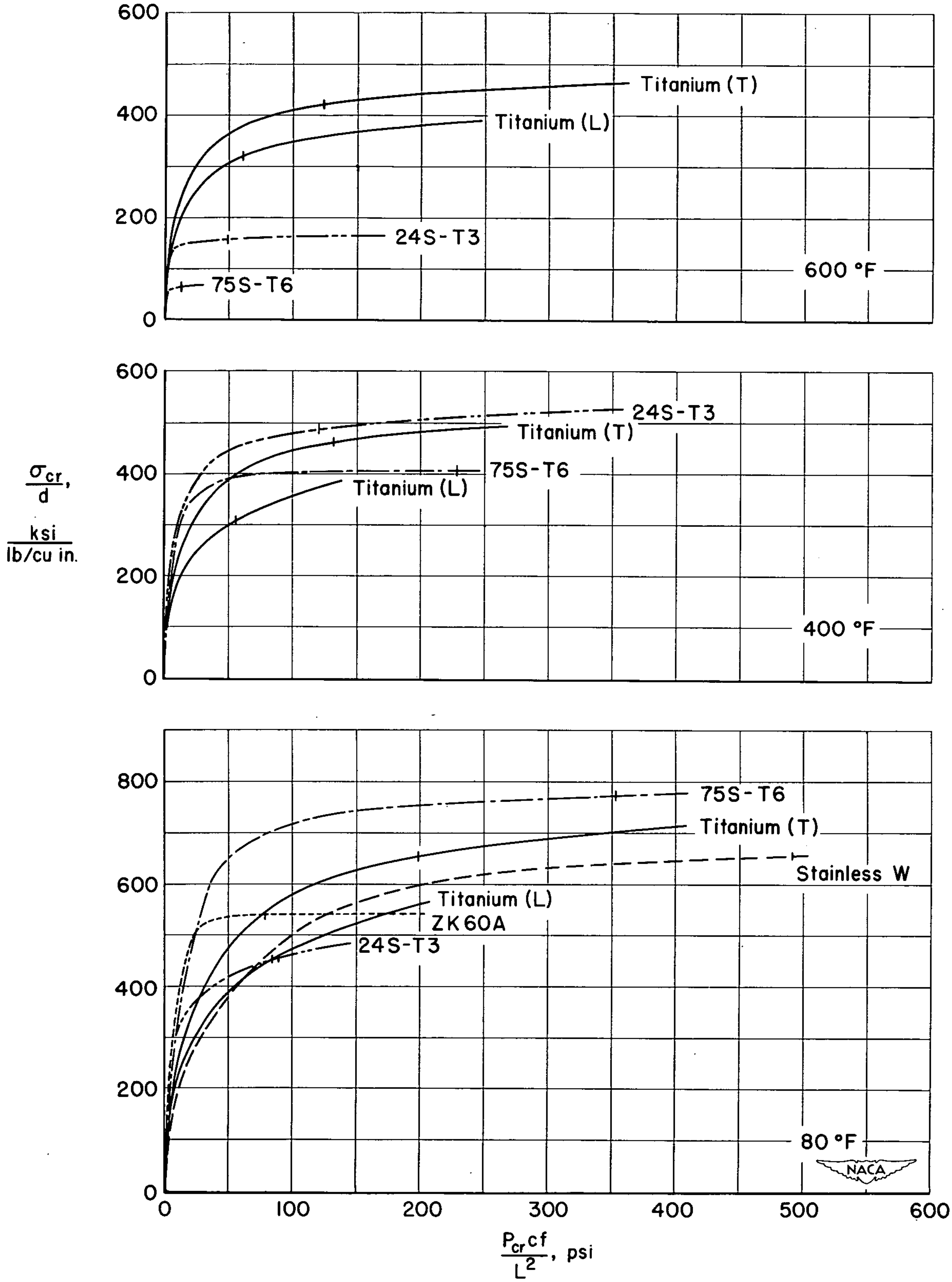

Figure 2.- Structural efficiencies of titanium and various other materials for column buckling at 80°, 400°, and 600° F. (Tab indicates the ratio of compressive yield stress to density.)

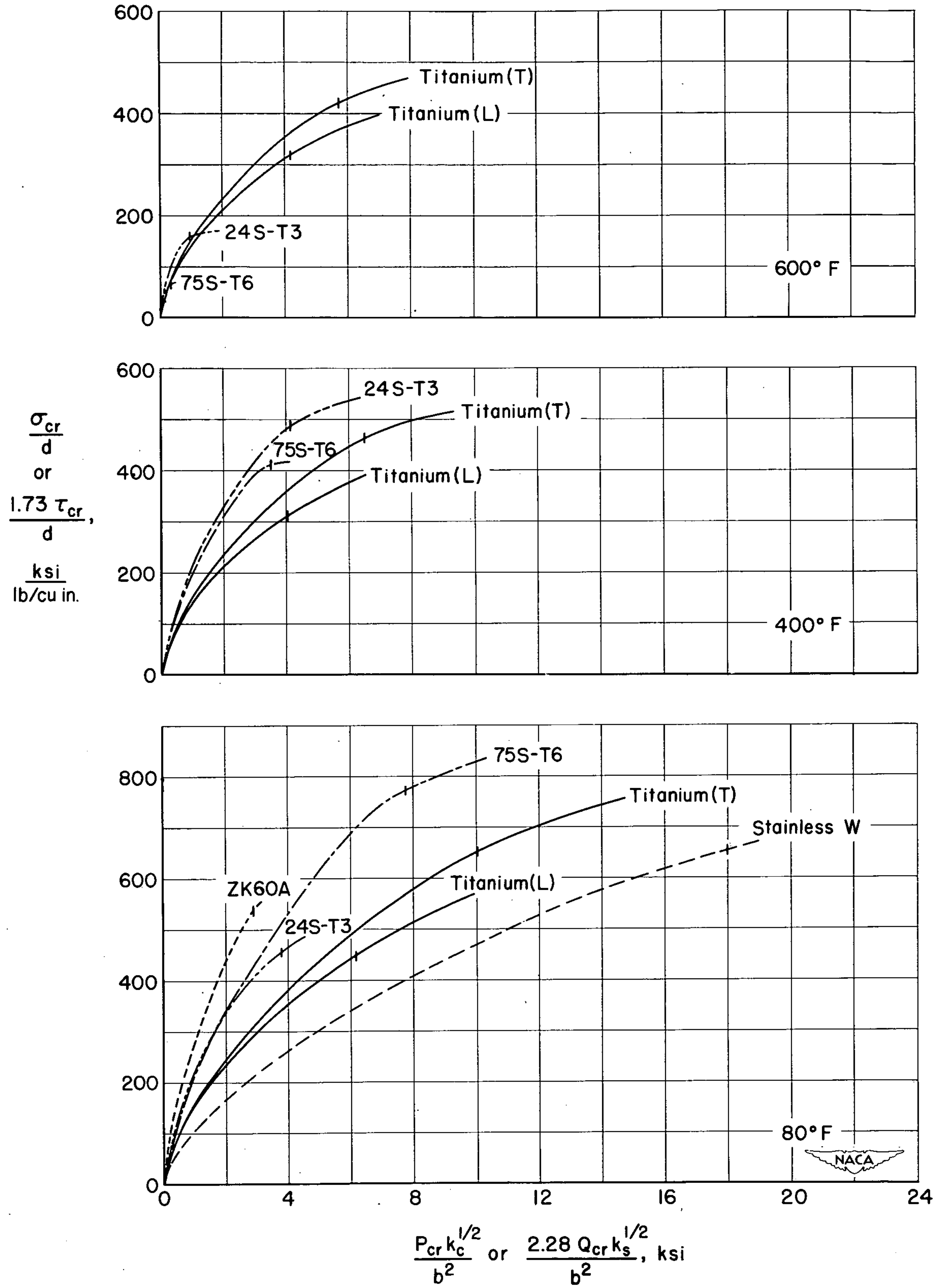

Figure 3.- Structural efficiencies of titanium and various other materials for the buckling of long plates loaded in compression or shear. (Tab indicates the ratio of compressive yield stress to density.)