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EFFECT OF SLIP ON FLOW NEAR A STAGNATION POINT AND IN A BOUNDARY LAYER

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SUMMARY

Theoretical analyses are presented of the effect of slip on the flow of a rarefied gas near a stagnation point and in a boundary layer on a flat plate. The results indicate that the stagnation pressure is increased because of the effect of slip but that there is a negligible effect on the flat-plate skin-friction coefficient in the range of application of the analysis.

INTRODUCTION

In the rarefied-gas-dynamics regime it is known that the ordinary condition of no slip at a wall is altered and should be replaced by a relation between the wall shear and a slip velocity, usually taken to be given in the form indicated in equation (4) or equation (13) (reference 1). It is also believed that the Navier-Stokes equations of gas dynamics are inadequate for accurate description of the flow field. At least three different systems of supposedly more adequate equations have been proposed in references 2 to 6 but their validity remains open to question. It is the purpose of this note to determine theoretically the rarefaction effect of slip in the boundary condition, using the Navier-Stokes differential equations. As will be indicated below, it is probable that the rarefaction effect associated with the boundary condition produces terms of the same order as the still undetermined rarefaction effect associated with the differential equations. Hence, the present analysis probably furnishes only a partial improvement over the normal density treatment. The relatively simple boundary-layer case is considered first and then the more complicated stagnation-point flow.

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SYMBOLS

b radius of curvature of nose of impact tube or radius of sphere

$C_\mu = P_\mu \Big/ \frac{1}{2} V_1^{\,2}$

$C_{\mu_o} = P_{\mu_o} \Big/ \frac{1}{2} V_1^{\,2}$

f stagnation-point function, defined in equations (16) and (17)

f_o stagnation-point function for no-slip case

f_1 slip correction to stagnation-point function, equation (24)

K constant in equation (18)

L molecular mean free path

M Mach number

p gas pressure

p_{st} mean hydrostatic stagnation pressure

p_{zz} normal pressure in z-direction

$\left(p_{zz}\right)_{st}$ normal stagnation pressure

P_μ viscous correction to impact pressure, given by equation (53)

P_{μ_o} viscous correction to impact pressure for case of no slip

r distance from z-axis

Re Reynolds number

Re_x Reynolds number based on x $\left(u_o x/\nu\right)$

u tangential velocity component, that is, velocity component in x-direction; also radial velocity component, that is, velocity component in r-direction

u_o	free-stream velocity past flat plate
v	normal velocity component, that is, velocity component in y-direction; also axial velocity component, that is, velocity component in z-direction
V_1	undisturbed free-stream velocity of gas
x	distance tangential to flat plate
y	distance normal to flat plate
z	distance normal to wall
α	coefficient of sliding friction
β	velocity expansion parameter, given by equations (13)
γ	ratio of specific heats of gas
δ^*	boundary-layer displacement thickness
δ_o^*	no-slip boundary-layer displacement thickness
η	dimensionless independent variable, defined by equation (15)
λ	dimensionless parameter, defined by equation (20)
μ	mechanical-viscosity coefficient
ν	kinematic-viscosity coefficient (μ/ρ)
ξ	coefficient of slip, defined by equations (5) and (12)
ρ	gas density
σ	Maxwell's reflection coefficient
ψ	stream function, equations (2)
ψ_o	Blasius stream function
ψ_1	slip correction to Blasius stream function

Subscripts:

B condition at edge of boundary layer

1 free-stream conditions

o no-slip conditions

BOUNDARY-LAYER FLOW

The stream function for incompressible boundary layer on a flat plate is governed by the Blasius differential equation (reference 7)

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\,\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = \nu\frac{\partial^3\psi}{\partial y^3} \tag{1}$$

where

$$\left.\begin{aligned} u &= \frac{\partial\psi}{\partial y} \\ v &= -\frac{\partial\psi}{\partial x} \end{aligned}\right\} \tag{2}$$

The corresponding boundary conditions are

$$\left.\begin{aligned} v &= 0, \quad y = 0 \\ u &= u_o, \quad y = \infty \end{aligned}\right\} \tag{3}$$

and for the rarefied-gas-dynamics regime (reference 1)

$$\xi\frac{\partial u}{\partial y} = u, \quad y = 0 \tag{4}$$

$$\xi = 0.998\,\frac{2 - \sigma}{\sigma}\,L \tag{5}$$

where σ is the Maxwell reflection coefficient and L, the molecular mean free path. For the case of diffuse reflection $\xi \approx L$. In terms of the stream function ψ

$$\left.\begin{aligned} &\psi = 0, \quad y = 0 \\ &L \frac{\partial^2\psi}{\partial y^2} = \frac{\partial\psi}{\partial y}, \quad y = 0 \qquad \frac{\partial\psi}{\partial y} = u_o, \quad y = \infty \end{aligned}\right\} \tag{6}$$

where equation (6) reduces to the ordinary density case if $L = 0$. Write

$$\psi = \psi_o + L\psi_1$$

Substitute into equations (1) and (6) and equate coefficients of L to 0 and obtain

$$\left.\begin{aligned} &\frac{\partial\psi_o}{\partial y}\frac{\partial^2\psi_o}{\partial x\,\partial y} - \frac{\partial\psi_o}{\partial x}\frac{\partial^2\psi_o}{\partial y^2} = \nu\frac{\partial^3\psi_o}{\partial y^3} \\ &\psi_o = 0 \text{ and } \frac{\partial\psi_o}{\partial y} = 0 \text{ for } y = 0, \text{ and } \frac{\partial\psi_o}{\partial y} = u_o \text{ for } y = \infty \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} &\frac{\partial^2\psi_o}{\partial x\,\partial y}\frac{\partial\psi_1}{\partial y} + \frac{\partial\psi_o}{\partial y}\frac{\partial^2\psi_1}{\partial x\,\partial y} - \frac{\partial\psi_o}{\partial x}\frac{\partial^2\psi_1}{\partial y^2} - \frac{\partial^2\psi_o}{\partial y^2}\frac{\partial\psi_1}{\partial x} = \nu\frac{\partial^3\psi_1}{\partial y^3} \\ &\psi_1 = 0 \text{ and } \frac{\partial\psi_1}{\partial y} = \frac{\partial^2\psi_o}{\partial y^2} \text{ for } y = 0, \text{ and } \frac{\partial\psi_1}{\partial y} = 0 \text{ for } y = \infty \end{aligned}\right\} \tag{8}$$

Clearly the system of equation (7) is exactly the ordinary Blasius case and the solution is well-known. The solution of equation (8) may be directly verified to be given by

$$\psi_1 = \frac{\partial\psi_o}{\partial y} \tag{9}$$

so that ψ is immediately determined, to order L, from the known Blasius solution. The displacement thickness is given by

$$\delta^* = \int_0^\infty \left(1 - \frac{1}{u_o}\frac{\partial\psi}{\partial y}\right) dy$$

$$= \int_0^\infty \left(1 - \frac{1}{u_o}\frac{\partial\psi_o}{\partial y}\right) dy - \frac{L}{u_o}\int_0^\infty \frac{\partial^2\psi_o}{\partial y^2}\, dy$$

This may be written in the form

$$\frac{\delta^*}{x} = \frac{1.732}{\sqrt{Re_x}}\left(1 - 0.867\,\frac{M}{\sqrt{Re_x}}\right) \tag{11}$$

There is no change in the drag coefficient to the order of terms retained. This somewhat surprising result arises from the special nature of the assumptions made and it should not be inferred that there is no decrease in skin friction due to slip effects in a physical rarefied-gas flow. The mathematical expression indicated in equation (9) is a solution to the incompressible boundary-layer equation with a slip-flow boundary condition. Terms of the order $1/\sqrt{Re_x}$ as compared with terms retained have been systematically neglected in reducing the complete Navier-Stokes equations to the boundary-layer equation. The assumption of incompressibility also limits the applicable range of Mach numbers. Thus the assumptions implicit in the use of equation (1) restrict the Mach and Reynolds numbers to roughly

$$M < 0.5$$

$$Re_x > 10^4$$

The effect of slip on the boundary-layer displacement thickness, for example, the term $0.867M/\sqrt{Re_x}$ in equation (11), is thus less than 1/2 percent in the range of applicability. A more important fact, however, is that this term is only one of several different terms of the form

$$\text{Constant}/\sqrt{Re_x} \quad \text{or} \quad \text{Constant } M/\sqrt{Re_x}$$

which would presumably be obtained in a complete analysis, that is, a solution of the complete Navier-Stokes equations for compressible flow. The present result with respect to skin friction thus seems to indicate only that the slip boundary condition, by itself, is initially of less importance than low Reynolds number modifications of boundary-layer theory in determining skin friction in a slightly rarefied gas. In this connection it should be noted that the somewhat cruder analyses of references 8 and 9 as well as the experimental data obtained at Berkeley and presented in references 10 and 11 (see fig. 1) all indicate a relative decrease in skin friction due to slip effects at sufficiently low Reynolds numbers but only after the skin friction has first increased above the value it has at high Reynolds numbers. This is presumably associated with the breakdown of boundary-layer theory, that is, to the increasing importance of additional terms of the form $\text{Constant}/\sqrt{Re_x}$.

A further assumption implicit in the use of equation (1) is that rarefied-gas-dynamic terms, for example, Burnett terms, in the basic differential equations such as those discussed in references 2 to 6 may be neglected. Inspection of these equations suggests that additional terms of the form $\text{Constant}/\sqrt{Re}$ would also arise in this connection. There are also additional terms in the boundary condition (reference 4) but these do not affect the present results to the order of terms retained. In the stagnation-point-flow analysis presented in the next section, such effects have been neglected; the complete Navier-Stokes equations have been solved to the order of terms retained, however.

STAGNATION-POINT FLOW

It is the purpose of this section to provide a theoretical basis for the interpretation of impact-tube measurements taken in a high-velocity gas stream of sufficiently low density so that the effect of velocity slip on the reading of the instrument can no longer be ignored.

Considered first is the problem of finding the flow in the vicinity of a forward stagnation point for a viscous fluid having the plane $z = 0$ as a fixed wall. At the surface of the wall are prescribed the boundary conditions that the normal velocity is zero and that the tangential velocity - that is, the velocity of slip - is proportional to the velocity gradient in the normal direction. It is known from the kinetic theory of gases that the factor of proportionality ξ is essentially (reference 1):

$$\xi = 0.998\left(\frac{2 - \sigma}{\sigma}\right) L \tag{12}$$

where σ is the Maxwell reflection coefficient and L, the molecular mean free path. At large distances from the wall the motion is essentially the same as that given by the potential flow. It follows that the boundary conditions for the problem are:

$$\left.\begin{aligned}
&\text{For } z = 0: \\
&\qquad u = \xi \frac{\partial u}{\partial z} \quad \text{and} \quad v = 0 \\
&\text{For } z = \infty: \\
&\qquad u = \beta r \quad \text{and} \quad v = -2\beta(z - \delta^*)
\end{aligned}\right\} \tag{13}$$

On assuming that the flow is steady, incompressible, and free from external forces, the forward stagnation-point-flow problem can be reduced to that of solving the third-order nonlinear ordinary differential equation (reference 12),

$$f''' + 2ff'' - (f')^2 + 1 = 0 \tag{14}$$

satisfying certain conditions on the boundary with

$$\eta = (\beta/\nu)^{1/2}z \tag{15}$$

$$u = \beta r f'(\eta) \tag{16}$$

$$v = -2(\beta\nu)^{1/2}f(\eta) \tag{17}$$

and

$$p = K - \frac{1}{2}\rho(\beta^2r^2 + v^2) + \mu\frac{\partial v}{\partial z} \tag{18}$$

The primes denote differentiation with respect to η. For the present problem, upon using equations (15) to (17), the boundary conditions (equations (13)) become

$$\left.\begin{aligned}
&f(0) = 0 \\
&f'(0) = \lambda f''(0) \\
\text{and}\quad &f'(\infty) = 1
\end{aligned}\right\} \tag{19}$$

where

$$\lambda = \xi(\beta/\nu)^{1/2} = 0.998\left(\frac{2 - \sigma}{\sigma}\right)L(\beta/\nu)^{1/2} \tag{20}$$

using equation (12).

Now if f_o denotes the function f for the nonslip case, one has (reference 12)

$$f_o''' + 2f_of_o'' - \left(f_o'\right)^2 + 1 = 0 \tag{21}$$

and

$$\left.\begin{aligned} f_o(0) &= 0 \\ f_o'(0) &= 0 \\ f_o'(\infty) &= 1 \end{aligned}\right\} \tag{22}$$

From Homann's solution to f_o (reference 13)

$$\left.\begin{aligned} f_o(\infty) &= \eta - 0.5576 \\ f_o''(0) &= 1.317 \\ f_o''(\infty) &= 0 \end{aligned}\right\} \tag{23}$$

For λ small, one may write

$$f = f_o + \lambda f_1 \tag{24}$$

accurate to the first power of λ. Then from equations (14), (19), (21), (22), and (24)

$$f_1''' + 2f_of_1'' - 2f_o'f_1' + 2f_o''f_1 = 0 \tag{25}$$

and

$$\left.\begin{aligned} f_1(0) &= 0 \\ f_1'(0) &= f_o''(0) \\ f_1'(\infty) &= 0 \end{aligned}\right\} \qquad (26)$$

also accurate to the first power of λ. Differentiating equation (21) with respect to η once, one obtains

$$f_o^{iv} + 2f_of_o''' = 0 \qquad (27)$$

From equations (22) and (23) it is seen that f_o satisfies the conditions

$$\left.\begin{aligned} f_o'(0) &= 0 \\ f_o''(\infty) &= 0 \end{aligned}\right\} \qquad (28)$$

If f_1 is replaced by f_o', equations (25) and (26) are reduced identically to equations (27) and (28), respectively. Therefore

$$f_1 = f_o' \qquad (29)$$

is the solution to equations (25) and (26). Hence from equation (24)

$$f = f_o + \lambda f_o' \qquad (30)$$

Thus, besides the parameter λ, the function f of the slip case is expressed in terms of f_o and its derivative f_o' of the nonslip case. Making use of equation (30) and Homann's solution to f_o (reference 13), one obtains in figure 2 the curves of f, f', and f'' for $\lambda = 0.1$. Referring to equation (50), $\lambda \approx 2M/\sqrt{Re}$ (see fig. 3). The corresponding curves for the nonslip case and the potential case are also shown in figure 2 for comparison.

At the stagnation point $r = z = v = 0$ and $p = p_{st}$. Substituting these into equation (18)

$$p_{st} = K + \mu\left(\frac{\partial v}{\partial z}\right)_{z=0} \tag{31}$$

where p_{st} is the mean hydrostatic pressure, or the mean of the normal pressures across any three orthogonal planes, at the stagnation point. For impact-tube measurements the normal stress across the wall p_{zz} at the stagnation point is of special interest. For incompressible flow, in general (reference 14),

$$p_{zz} = -p + 2\mu \frac{\partial v}{\partial z} \tag{32}$$

At the stagnation point

$$p_{zz} = \left(p_{zz}\right)_{st}$$

From equations (32) and (31)

$$\left(p_{zz}\right)_{st} = -K + \mu\left(\frac{\partial v}{\partial v}\right)_{z=0} \tag{33}$$

From equations (15) and (17)

$$\left(\frac{\partial v}{\partial z}\right)_{z=0} = -2\beta f'(0) \tag{34}$$

From equation (30), upon using equations (23) and (22)

$$f'(0) = 1.317\lambda \tag{35}$$

Substituting equation (34) into equation (33), and using equation (35),

$$\left(p_{zz}\right)_{st} = -(K + 1.317 \times 2\mu\beta\lambda) \tag{36}$$

where the negative sign indicates compressive stress.

The above equation is a solution to the Navier-Stokes equations with slip. So far no boundary-layer assumption has been made.

IMPACT TUBE

Solution (36) will be applied to the case of an impact tube with a half-spherical nose, tested at subsonic speed at low pressure conditions, with its longitudinal axis placed parallel to the direction of the flow. Let the flow along the central streamline leading to the stagnation point decelerate isentropically to the outside seam of the boundary layer, which is presumed to form in the neighborhood of the stagnation point. It is assumed that in the boundary layer in the neighborhood of the stagnation point the flow can be treated as viscous and incompressible and that outside the boundary layer the effects of viscosity can be neglected.

At the point of the boundary layer where the stagnation streamline enters, that is, $z = z_B$, $r = 0$ and $\left(\frac{\partial v}{\partial z}\right)_{z=z_B} = -2\beta$ (reference 15) so that from equation (18)

$$K = \left(p_B + \frac{1}{2}\rho_B V_B^2\right) + 2\mu\beta \tag{37}$$

where p_B and V_B are the pressure and velocity at the point z_B. The term in parentheses represents the impact pressure p_R recorded by an impact tube placed in a gas stream if the effect of compressibility in the boundary layer is zero. Since this was previously assumed for a subsonic flow, the impact pressure is given by the conventional isentropic-flow equation (reference 16)

$$\begin{aligned} p_R &= p_B + \frac{1}{2}\rho_B V_B^2 \\ &= p_1\left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} \end{aligned} \tag{38}$$

Substituting these into equation (36)

$$\left(p_{zz}\right)_{st} = -\left[p_1\left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} + 2\mu\beta(1 + 1.317\lambda)\right] \tag{39}$$

The constant β must be determined from the velocity distribution at the seam of the boundary layer which surrounds the body at the stagnation point. Since the flow outside the boundary layer is assumed inviscous and compressible, β can be deduced from the corresponding ideal subsonic compressible pattern. For a half-spherical impact tube of radius b, β has the form (reference 17)

$$\beta = \frac{V_1}{2b}\left(\frac{29}{8} - \frac{31}{34} M_1{}^2\right) \tag{40}$$

where V_1 is the undisturbed free-stream velocity.

Because of the presence of a boundary layer of average displacement thickness δ^*, the effective radius of curvature of the nose of the impact tube becomes $b + \delta^*$. For viscous incompressible flow passing a sphere, the average displacement thickness $\delta_o{}^*$ of the boundary layer of the nonslip case can be expressed as (reference 13)

$$\frac{\delta_o{}^*}{b} = \frac{\eta_o{}^*}{\sqrt{\frac{3}{2}\,\mathrm{Re}}} \tag{41}$$

where

$$\eta_o{}^* = \eta - f_o(\infty) \tag{42}$$

$$\mathrm{Re} = \frac{\rho V_1 b}{\mu} \tag{43}$$

Similarly

$$\frac{\delta^*}{b} = \frac{\eta^*}{\sqrt{\frac{3}{2}\,\mathrm{Re}}}$$

with

$$\eta^* = \eta - f(\infty)$$

From equation (30), upon using equations (23) and (22),

$$f(\infty) = \eta - 0.5576 + \lambda \tag{44}$$

so that

$$\eta^* = 0.5576 - \lambda \tag{45}$$

and

$$\frac{\delta^*}{b} = \frac{(0.455 - 0.816\lambda)}{\sqrt{Re}} \tag{46}$$

Therefore equation (40) becomes

$$\beta = \frac{V_1}{2b} \frac{\left(\frac{29}{8} - \frac{31}{34} M_1{}^2\right)}{\left(1 + \frac{0.455 - 0.816\lambda}{\sqrt{Re}}\right)} \tag{47}$$

Substituting this into equation (39)

$$\left(p_{zz}\right)_{st} = -\left[p_1\left(1 + \frac{\gamma - 1}{2} M_1{}^2\right)^{\frac{\gamma}{\gamma-1}} + \frac{\mu V_1}{b} \frac{\left(\frac{29}{8} - \frac{31}{34} M_1{}^2\right)\left(1 + 1.317\lambda\right)}{\left(1 + \frac{0.455 - 0.816\lambda}{\sqrt{Re}}\right)}\right] \tag{48}$$

Consider next the expression for λ. Substituting equation (48) into equation (20) and for the present, since λ is small, neglecting the slip correction to the effective radius of the impact tube,

$$\lambda = 1.343\left(\frac{2 - \sigma}{\sigma}\right)\left(\frac{1 - 0.252M_1{}^2}{1 + 0.455/\sqrt{Re}}\right)^{1/2} \frac{L}{b} \sqrt{Re} \tag{49}$$

Upon using the relation (reference 18)

$$L/b = 1.255\sqrt{\gamma}\, M/Re \tag{50}$$

with $\gamma = 1.405$ and $\sigma = 1$, equation (49) becomes

$$\lambda = 1.997\left(\frac{1 - 0.252M_1^2}{1 + 0.455/\sqrt{Re}}\right)^{1/2} \frac{M_1}{\sqrt{Re}} \tag{51}$$

The variation of λ with the rarefaction parameter M_1/Re for various values of the Mach number M_1 is shown in figure 3. Substituting equation (51) into equation (48)

$$(p_{zz})_{st} = -p_1\left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} -$$

$$\mu \frac{V_1}{b} \frac{\left(\frac{29}{8} - \frac{31}{34} M_1^2\right)\left[1 + 2.63\left(\frac{1 - 0.252M_1^2}{1 + 0.455/\sqrt{Re}}\right)^{1/2}\frac{M_1}{\sqrt{Re}}\right]}{1 + \frac{0.455}{\sqrt{Re}} - 1.63\left(\frac{1 - 0.252M_1^2}{1 + 0.455/\sqrt{Re}}\right)^{1/2} \frac{M_1}{Re}} \tag{52}$$

If P_μ denotes the viscous correction to the impact-tube pressure, from equation (48)

$$P_\mu = \frac{\mu V_1}{b} \frac{\left(\frac{29}{8} - \frac{31}{34} M_1^2\right)(1 + 1.317\lambda)}{1 + \frac{0.455 - 0.816\lambda}{\sqrt{Re}}} \tag{53}$$

For the nonslip case $\lambda = 0$, $P_\mu = P_{\mu_o}$. From equation (53)

$$P_{\mu_o} = \frac{\mu V_1}{b} \frac{\left(\frac{29}{8} - \frac{31}{34} M_1^2\right)}{1 + \frac{0.455}{\sqrt{Re}}} \tag{54}$$

so that, upon using equation (51)

$$\frac{P_\mu}{P_{\mu_o}} = \frac{1 + 2.63 \dfrac{\left(1 - 0.252M_1^{\,2}\right)^{1/2}}{\left(1 + 0.455/\sqrt{Re}\right)^{1/2}} \dfrac{M_1}{\sqrt{Re}}}{1 - 1.63 \dfrac{\left(1 - 0.252M_1^{\,2}\right)^{1/2}}{\left(1 + 0.455/\sqrt{Re}\right)^{3/2}} \dfrac{M_1}{Re}} \tag{55}$$

For M_1/Re small and M_1 small the above expression is larger than unity. It follows that the effect of slip increases the viscous correction to the impact-tube measurements. The curves of P_μ/P_{μ_o} against M_1/Re, $1/Re$, and Re for various values of M_1 are given in figures 4, 5, and 6, respectively.

Define

$$\left.\begin{aligned} C_\mu &= \frac{P_\mu}{\frac{1}{2}\rho V_1^{\,2}} \\ C_{\mu_o} &= \frac{P_{\mu_o}}{\frac{1}{2}\rho V_1^{\,2}} \end{aligned}\right\} \tag{56}$$

Upon using equations (56), (53), (52), and (51), the variations of C_μ and C_{μ_o} with Re and $1/Re$ for $M_1 = 0.4$ and 0.8 are shown in figures 7 and 8, respectively.

SLIP FLOW PAST A SPHERE

It is interesting to note that at the stagnation point the normal pressure across the wall increases with the coefficient of slip, although at the same point the mean hydrostatic pressure, that is, the mean of the normal pressures across any three orthogonal planes, decreases. This is due to the fact that for the slip case the normal velocity gradient (dv/dz) at the stagnation point is no longer zero. It follows that the effect of slip increases the viscous correction (reference 15) to the impact-tube measurements. A similar result is obtained for the case of the slow translation of a sphere through a viscous fluid with slip from the well-known solution given by Basset (reference 19 or reference 14, pp. 603-604). In his case the inertia terms are neglected.

For the case of the slow translation of a sphere through a viscous fluid with slip the impact pressure p_i at the forward stagnation point has been obtained by Basset (reference 19), who neglected the inertia terms which are small in comparison with the viscous and pressure terms. If b is the radius of the sphere, V_1, the velocity, α, the coefficient of sliding friction, and p_1, the hydrostatic pressure, then

$$p_{st} = p_1 + \frac{3}{2}\frac{\mu V_1}{b}\frac{1 + \frac{6\mu}{\alpha b}}{1 + \frac{3\mu}{\alpha b}} \tag{57}$$

On using the relation (reference 18)

$$\frac{\mu}{\alpha b} = 1.253\sqrt{\gamma}\left(\frac{2 - \sigma}{\sigma}\right)\frac{M_1}{Re} \tag{58}$$

equation (57) becomes, for $\gamma = 1.405$ and $\sigma = 1$,

$$p_{st} = p_1 + \frac{3}{2}\frac{\mu V_1}{b}\frac{1 + 8.88M_1/Re}{1 + 4.44M_1/Re} \tag{59}$$

The viscous correction to the impact pressure is

$$P_\mu = \frac{3}{2}\frac{\mu V_1}{b}\frac{1 + 8.88M_1/Re}{1 + 4.44M_1/Re} \tag{60}$$

For the nonslip case $\alpha = \infty$, from equation (57)

$$P_{\mu_o} = \frac{3}{2}\frac{\mu V_1}{b} \tag{61}$$

Therefore

$$\frac{P_\mu}{P_{\mu_o}} = \frac{1 + 8.88M_1/Re}{1 + 4.44M_1/Re} \tag{62}$$

As a comparison the curve of P_μ/P_{μ_o} for the Basset sphere against M_1/Re is shown in figure 4.

CONCLUSIONS

A method has been developed for determining some of the rarefaction effects in slip-regime flow for the flat plate, a wall, and a hemispherical impact tube, using a reduced form of the Navier-Stokes equations and a slip boundary condition. With this method it was found that:

1. The effect of slip on the viscous flow about an impact tube is to increase the viscous correction to the ideal impact pressure.

2. The effect of slip on the flat plate is to reduce the boundary-layer displacement thickness. There is no effect on the drag coefficient to the order of terms retained in the analysis.

University of California
Berkeley, Calif., October 20, 1950

REFERENCES

1. Kennard, Earle H.: Kinetic Theory of Gases. First ed., McGraw-Hill Book Co., Inc., 1938, p. 296.

2. Burnett, D.: The Distribution of Molecular Velocities and the Mean Motion in a Non-Uniform Gas. Proc. London Math. Soc., vol. 40, 1935, pp. 382-435.

3. Chapman, Sydney, and Cowling, T. G.: The Mathematical Theory of Non-Uniform Gases. First ed., The Univ. Press (Cambridge), 1939.

4. Schamberg, Richard: The Fundamental Differential Equations and the Boundary Conditions for High Speed Slip-Flow and Their Application to Several Specific Problems. Ph. D. Thesis, C.I.T., May 1947.

5. Truesdell, C.: On the Differential Equations of Slip Flow. Proc. Nat. Acad. Sci., vol. 34, no. 7, July 1948, pp. 342-347.

6. Grad, H.: On the Kinetic Theory of Rarefied Gases. Communications on Pure and Appl. Math., vol. II, no. 4, 1949, pp. 331-407.

7. Blasius, H.: Grenzschichten in Flüssigkeiten mit kleiner Reibung. Zeitschr. Math. und Phys., Bd. 56, Heft 1, 1908, pp. 1-37. (Available in translation at NACA TM 1256, 1950.)

8. Donaldson, Coleman duP.: An Approximate Method for Estimating the Incompressible Laminar Boundary-Layer Characteristics on a Flat Plate in Slipping Flow. NACA RM L9C02, 1949.

9. Schaaf, S. A.: A Note on the Flat Plate Drag Coefficient. Rep. No. HE-150-66, Contract No. N7-ONR-295-Task 3, Office of Naval Research and Eng. Res. Projects, Univ. of Calif., Feb. 20, 1950.

10. Sherman, F. S.: Skin Friction in Supersonic Low Density Gas Flow. Rep. No. HE-150-70, Contract No. N7-ONR-295-Task 3, Office of Naval Research and Eng. Res. Projects, Univ. of Calif., July 26, 1950.

11. Sherman, F. S.: Skin Friction in Supersonic Low Density Gas Flow - Part 2. Rep. No. HE-150-81, Contract No. N7-ONR-295-Task 3, Office of Naval Research and Eng. Res. Projects, Univ. of Calif., March 15, 1951.

12. Fluid Motion Panel of the Aeronautical Research Committee and Others (S. Goldstein, ed.): Modern Developments in Fluid Dynamics. Vol. I. The Clarendon Press (Oxford), 1938.

13. Homann, Fritz: Der Einfluss grosser Zähigkeit bei der Strömung um den Zylinder und um die Kugel. Z.f.a.M.M., Bd. 16, Heft 3, June 1936, pp. 153-164.

14. Lamb, Horace: Hydrodynamics. Sixth ed., Dover Publications, 1945, p. 574.

15. Chambré, P. L., and Smith, H. R.: The Impact Tube in a Viscous Compressible Gas. Rep. No. HE-150-63, Contract No. N7-ONR-295-Task 3, Office of Naval Research and Eng. Res. Projects, Univ. of Calif., Aug. 29, 1949. (Also Heat Transfer and Fluid Mechanics Institute, A.S.M.E., 1949, pp. 271-278.)

16. Liepmann, Hans Wolfgang, and Puckett, Allen E.: Introduction to Aerodynamics of a Compressible Fluid. John Wiley & Sons, Inc., 1947, p. 26.

17. Chambré, P. L.: The Theory of the Impact Tube in a Viscous Compressible Gas. Rep. No. HE-150-50, Contract No. N7-ONR-295-Task 3, Office of Naval Research and Eng. Res. Projects, Univ. of Calif., Nov. 1, 1948.

18. Tsien, Hsue-Shen: Superaerodynamics, Mechanics of Rarefied Gases. Jour. Aero. Sci., vol. 13, no. 12, Dec. 1946, pp. 653-664.

19. Basset, A. B.: Hydrodynamics. Vol. 2. Deighton, Bell and Co. (Cambridge), 1888, pp. 270-271.

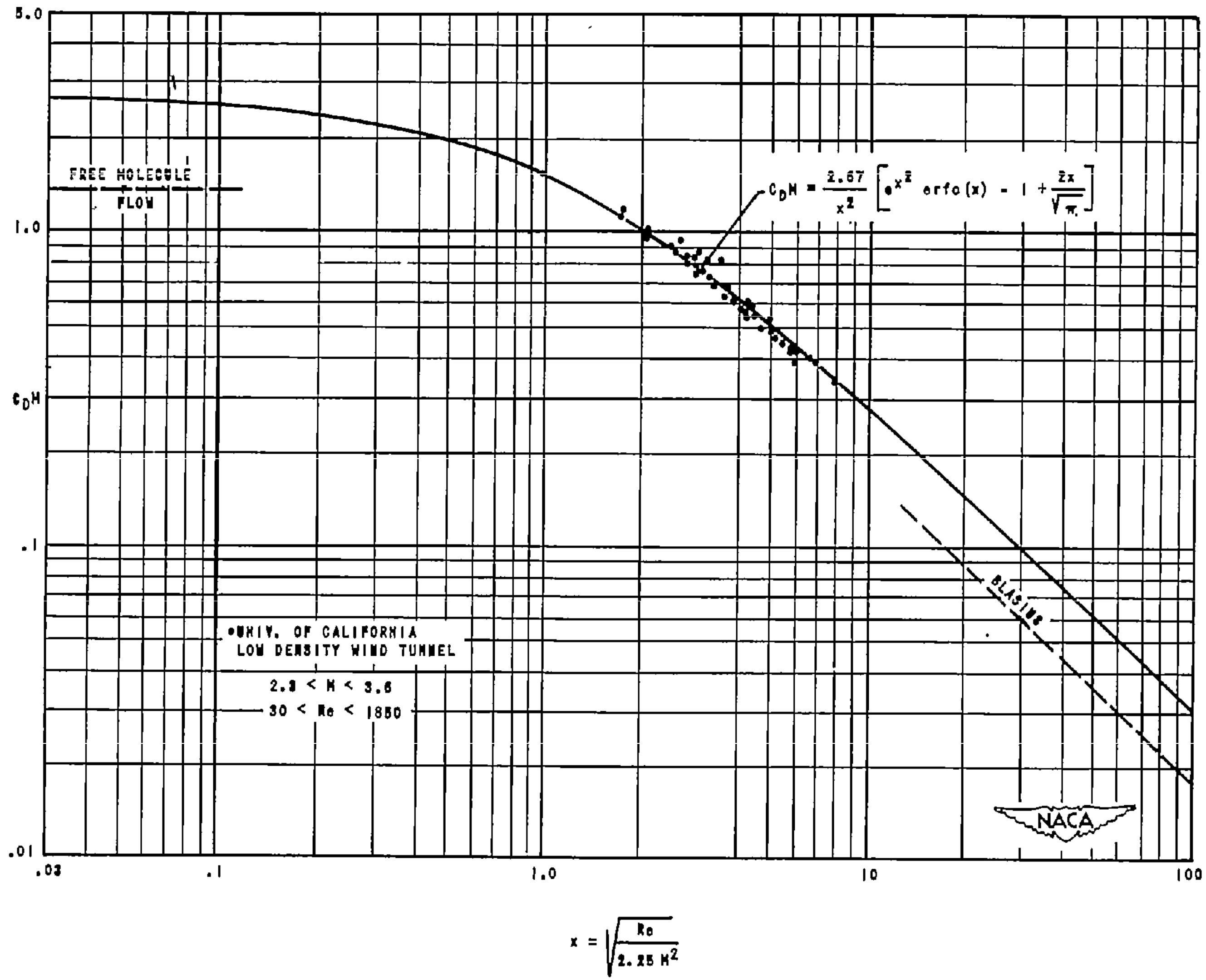

Figure 1.- Flat-plate drag coefficient.

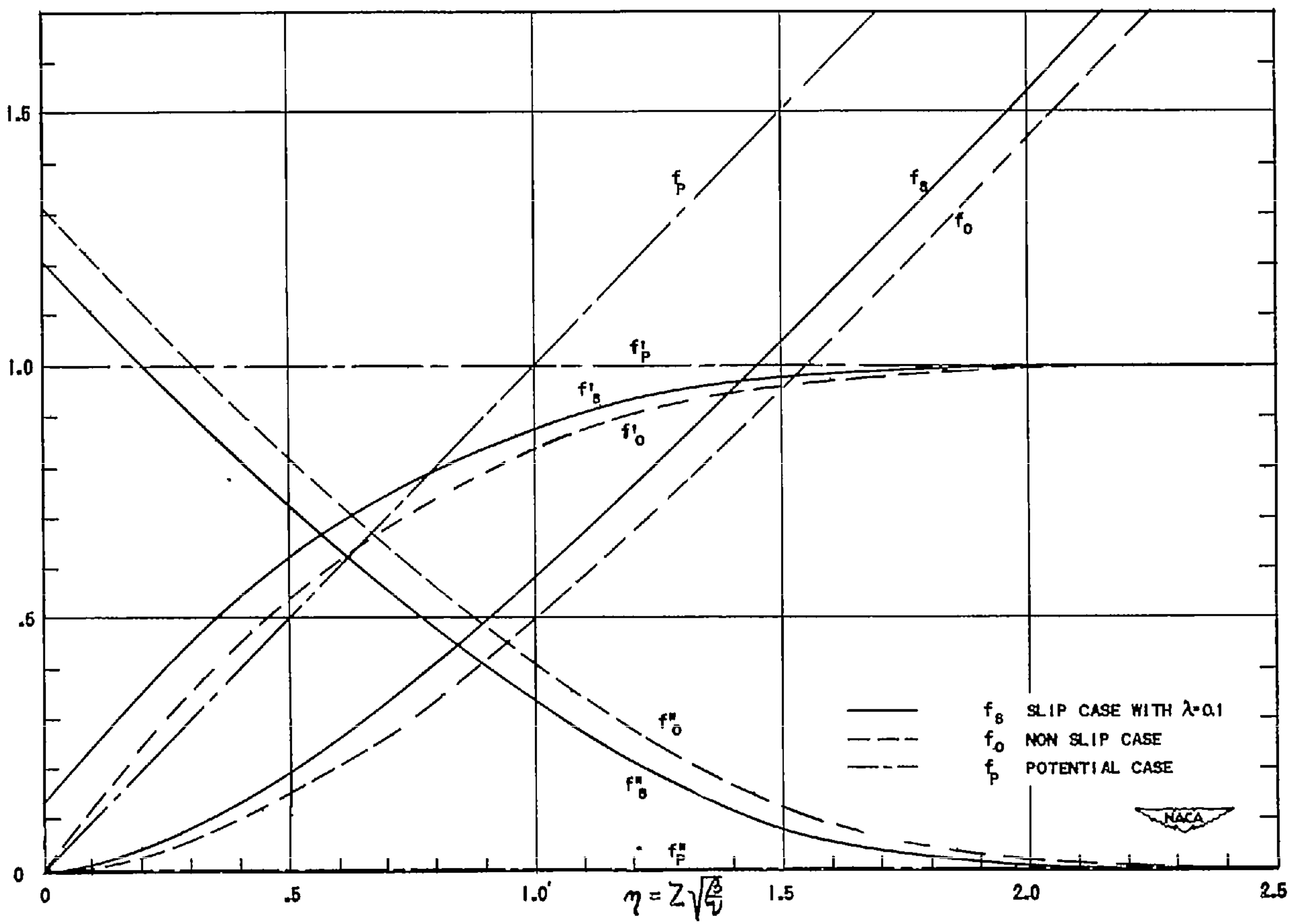

Figure 2.- Functions of forward stagnation-point flow. $u = \beta r f'(\eta)$.

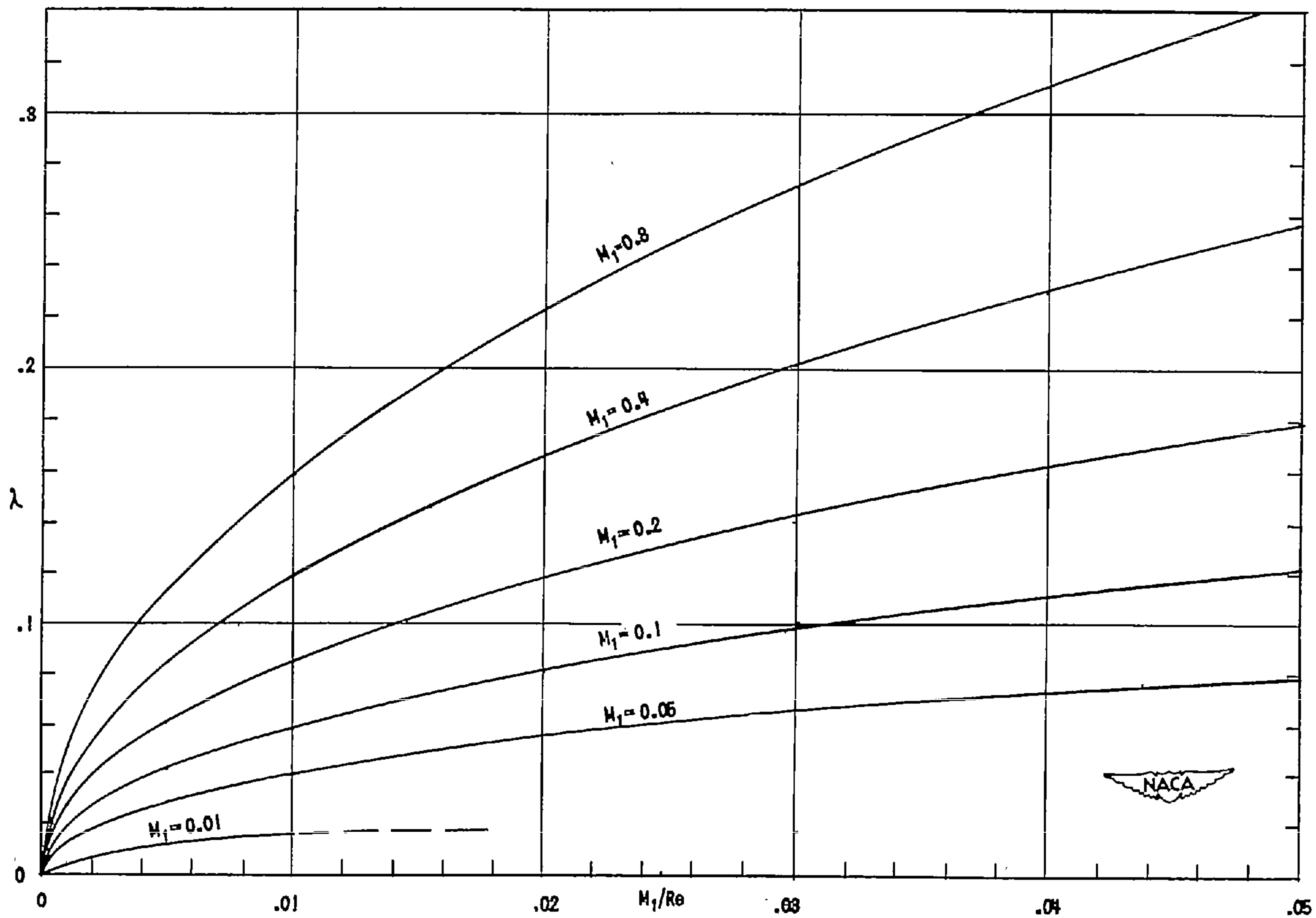

Figure 3.- Parameter λ against M_1/Re for various Mach numbers.

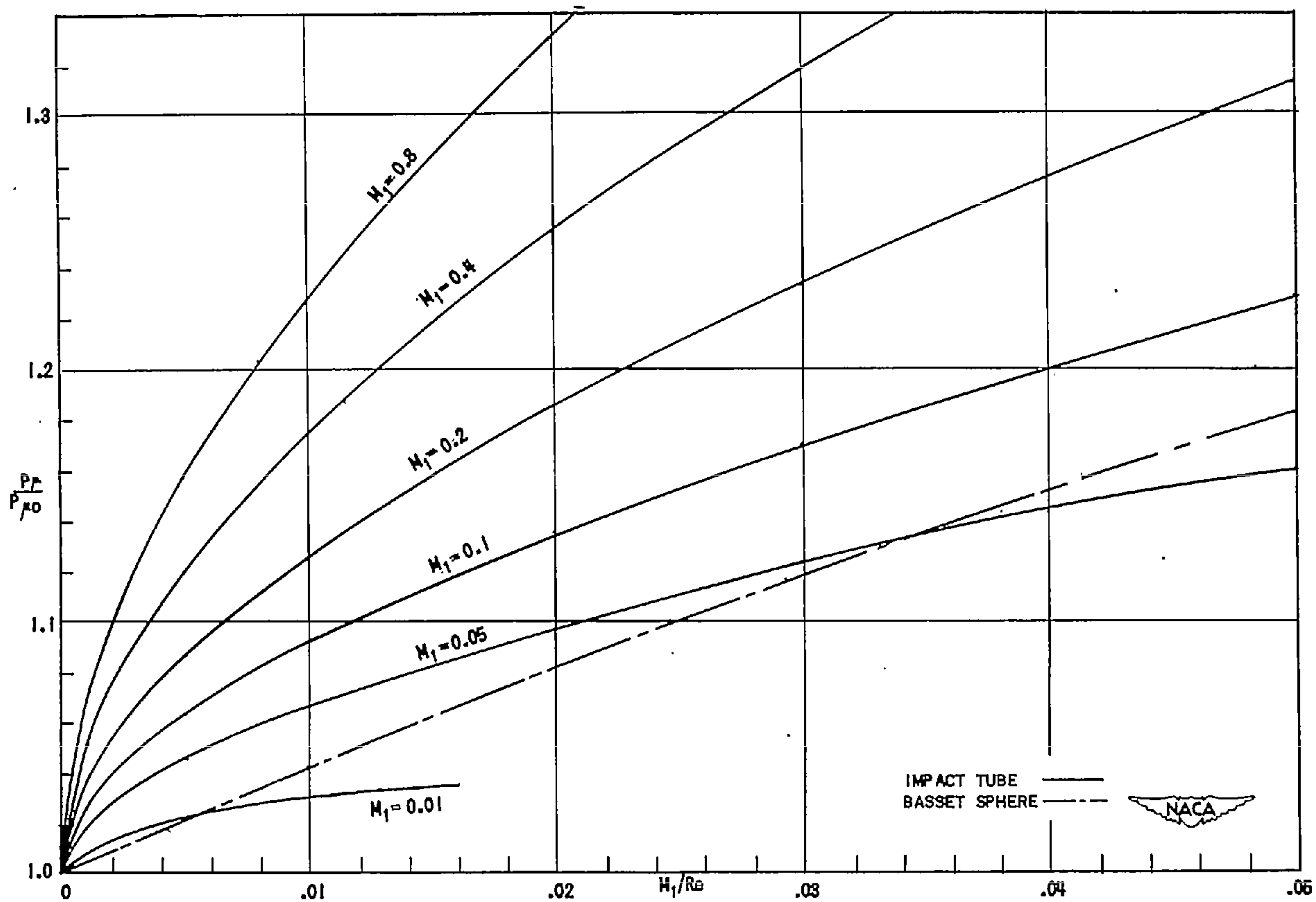

Figure 4.- Effect of slip on viscous correction P_μ/P_{μ_o} against M_1/Re at various Mach numbers.

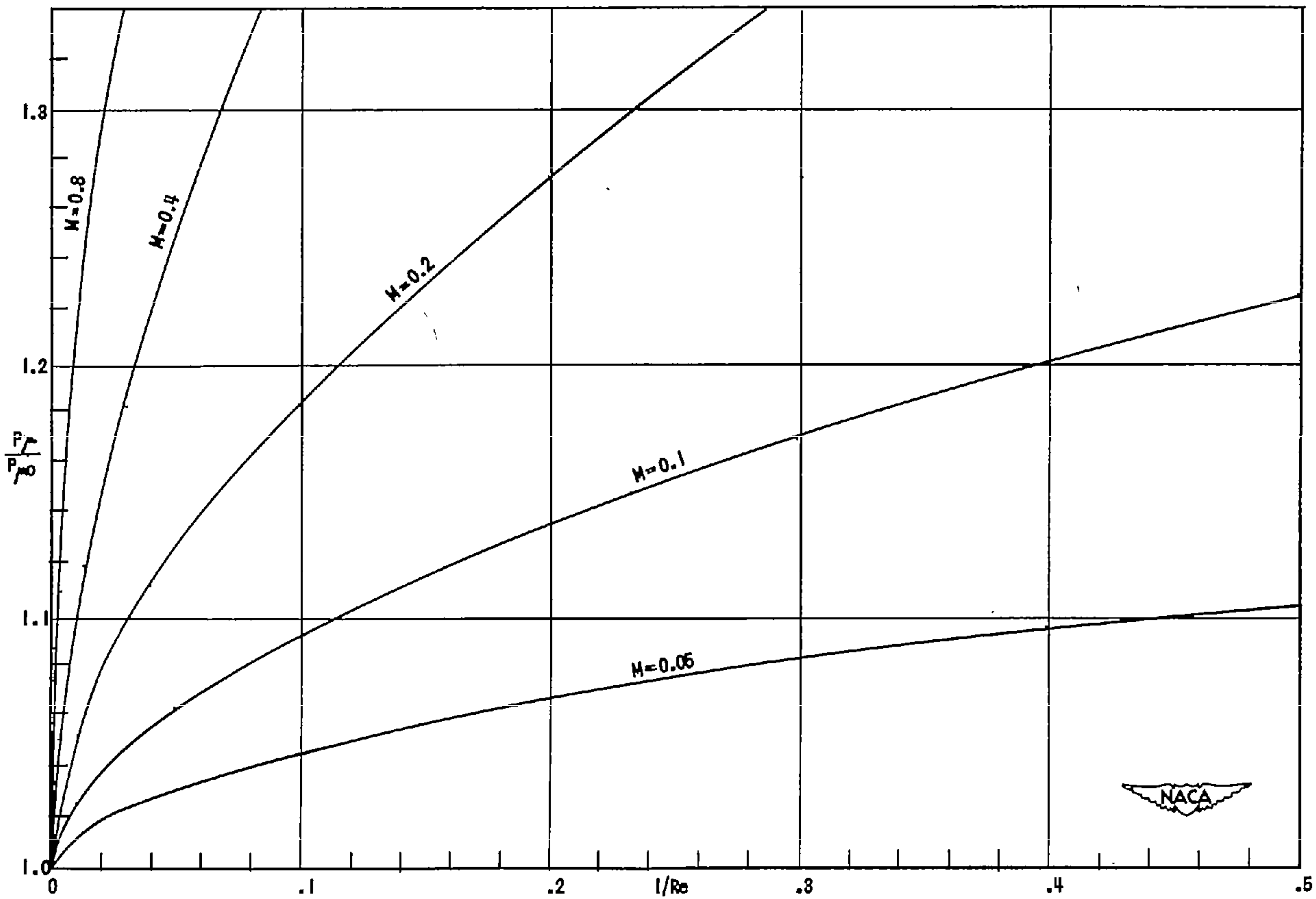

Figure 5.- Effect of slip on viscous correction P_μ/P_{μ_0} against 1/Re at various Mach numbers.

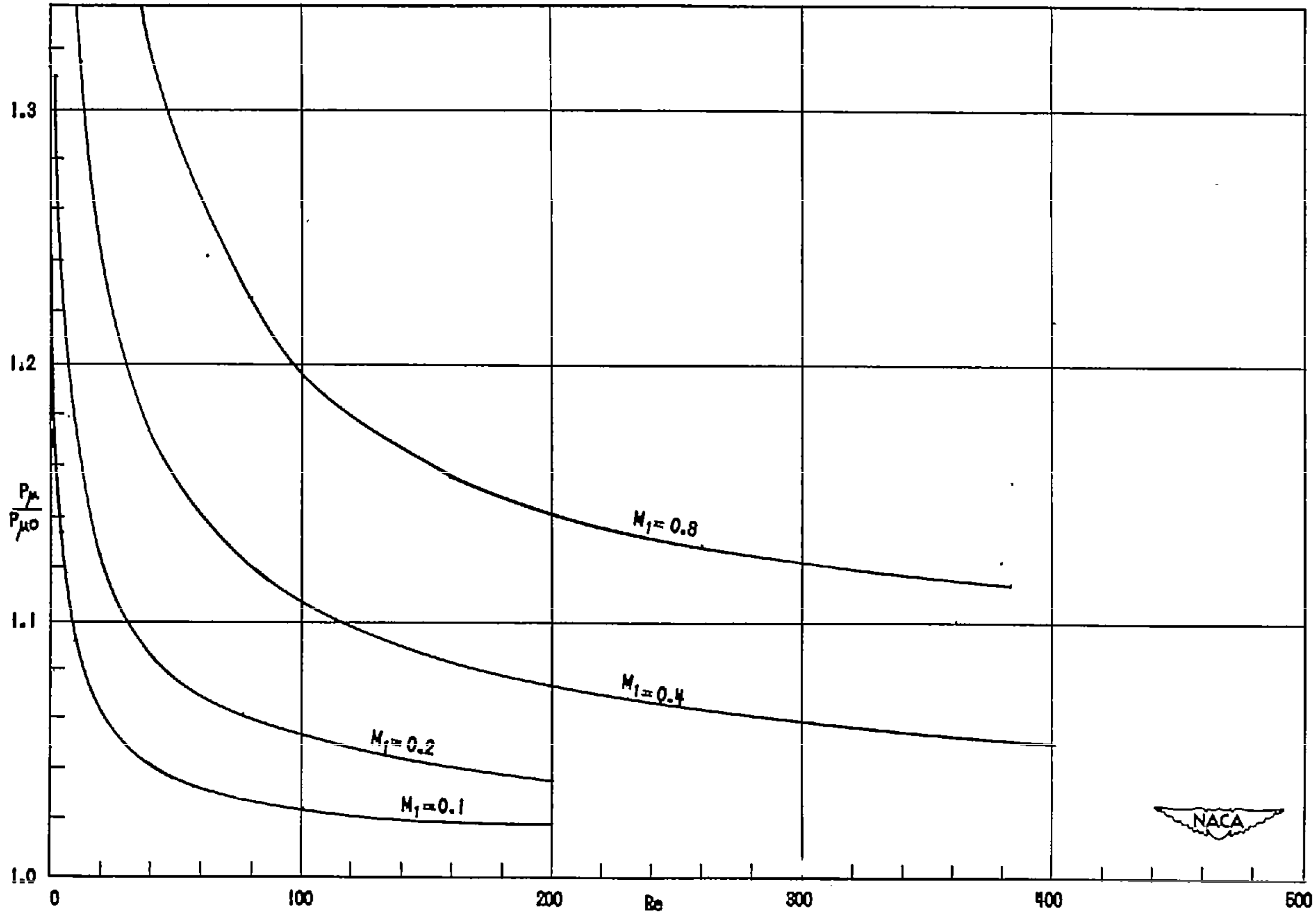

Figure 6.- Effect of slip on viscous correction P_μ/P_{μ_o} against Re at various Mach numbers.

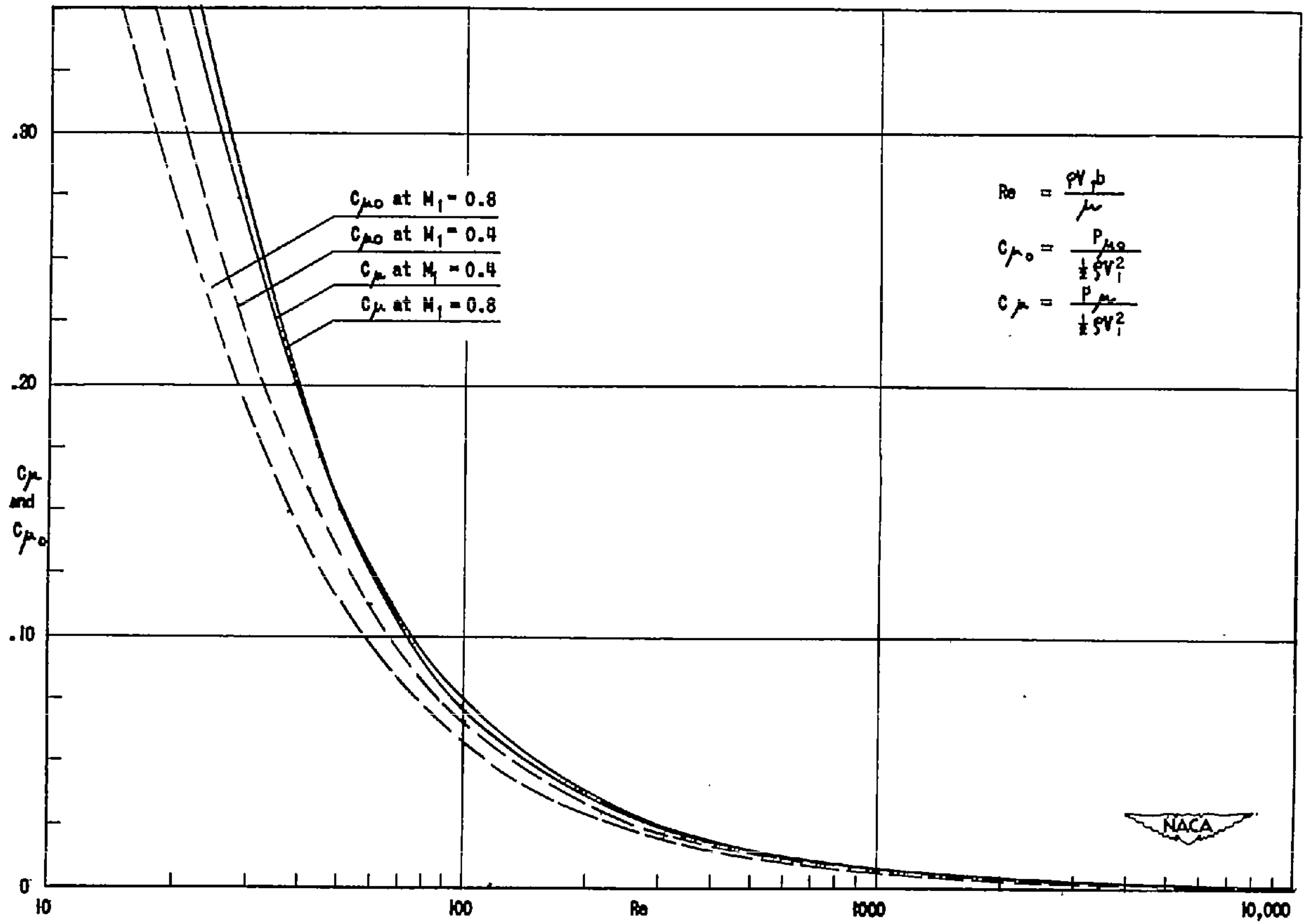

Figure 7.- Viscous correction with and without slip, C_μ and C_{μ_o}, against Re.

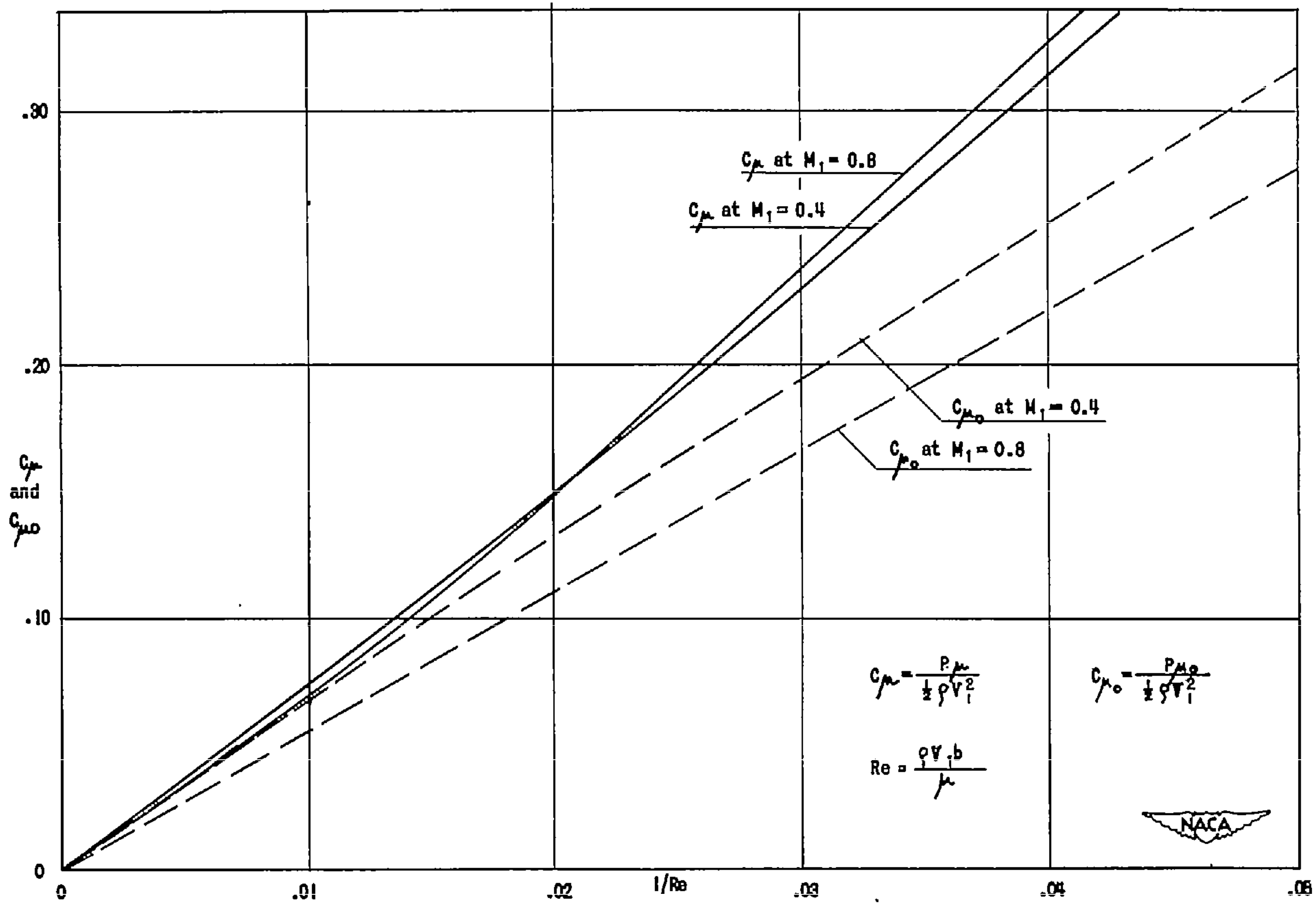

Figure 8.- Viscous correction with and without slip, C_μ and C_{μ_o}, against 1/Re.

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