# NATIONAL ADVISORY COMMITTEE
# FOR AERONAUTICS

## TECHNICAL NOTE 2764

ACCURACY OF APPROXIMATE METHODS FOR PREDICTING

PRESSURES ON POINTED NONLIFTING BODIES

OF REVOLUTION IN SUPERSONIC FLOW

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SUMMARY

The accuracy and range of applicability of the linearized theory, second-order theory, tangent-cone method, conical-shock-expansion theory, and Newtonian theory for predicting pressure distributions on pointed bodies of revolution at zero angle of attack are investigated. Pressure distributions and integrated pressure drag obtained by these methods are compared with standard values obtained by the method of characteristics and the theory of Taylor and Maccoll. Three shapes, cone, ogive, and a modified optimum body, are investigated over a wide range of fineness ratios and Mach numbers.

It is found that the linearized theory is accurate only at low values of the hypersonic similarity parameter (the ratio of free-stream Mach number to body fineness ratio) and that second-order theory appreciably extends the range of accurate application. The second-order theory gives good results on ogives when the ratio of the tangent of maximum surface angle to the tangent of the Mach angle is less than 0.9. Tangent-cone methods cannot be widely applied with good accuracy. In general, the conical-shock-expansion theory predicts pressure and drag within engineering accuracy when the hypersonic similarity parameter is greater than 1.2. Although Newtonian theory gives good accuracy, except for cones, at the highest values of the hypersonic similarity parameter investigated, it is less accurate than the conical-shock-expansion theory.

INTRODUCTION

Various methods have been proposed for predicting pressure distributions on bodies of revolution at zero angle of attack in supersonic flow. The method of characteristics, which can be carried to any degree of

accuracy, is too time consuming to be practical for many engineering needs. Other methods, although requiring less time, involve varying degrees of approximation which limit their accuracy and range of applicability. This investigation was undertaken to ascertain the range of applicability and accuracy of a few of the approximate methods. Pressure distributions determined from the method of characteristics and from the theory of Taylor and Maccoll are used as standards for determining the accuracy of these approximate methods. Wide ranges of Mach number and fineness ratio are investigated in order to determine the range of values of the hypersonic similarity parameter, the ratio of Mach number to body fineness ratio, for which each of the various approximate methods is useful.

## SYMBOLS

$C_p$    pressure coefficient $\left( \dfrac{p - p_0}{q_0} \right)$

$C_D$    aerodynamic coefficient of drag based on body frontal area

d       diameter of body

H       total pressure

K       similarity parameter, ratio of free-stream Mach number to body fineness ratio

$l$      length of body

M       Mach number, ratio of local velocity to local speed of sound

p       static pressure

q       dynamic pressure

r       body coordinate normal to axis of symmetry

x       body coordinate parallel to axis of symmetry

$\theta$    local slope of body

$\theta_s$   semivertex angle of body

$\gamma$    ratio of specific heats

Subscript

o   free-stream condition

## METHODS CONSIDERED

There are a number of theories for determining pressure and drag coefficients on pointed bodies of revolution in various ranges of supersonic speeds. The apparent number is increased because, in some cases, several different methods of applying the same basic idea have been proposed. A detailed study of all the theories would involve more time than seemed warranted. Therefore the following five methods have been chosen for investigation because of their anticipated usefulness and simplicity of application.

### Linearized Theory

Probably the best-known method of determining pressure distributions at supersonic speeds is the linearized or small perturbation theory presented by von Kármán and Moore (reference 1). This potential theory derived for slender bodies at low supersonic speeds can be applied when the free-stream Mach angle is greater than the maximum angle of flow deflection.

### Second-Order Theory

The second-order theory of Van Dyke refines the linearized (or first-order) theory by iteration. It has the same analytical limit; that is, the ratio of the tangent of the semivertex angle to the tangent of the Mach angle must be less than 1. (The practical limit is about 0.9.) The second-order theory has been set up for easy use with adequate tables and sample computing sheets in reference 2.

### Tangent-Cone Method

One method for estimating pressures on curved bodies is to make use of flow solutions for cones whose slopes correspond to these of the body surface at the points in question. Two procedures can be used in applying this method.

One of the procedures simply uses the pressure coefficients for cones of semivertex angle equal to the angle of the body at various stations. This method gives the correct pressure at the vertex but fails to predict any negative pressure coefficients. Estimating the pressure coefficients for the body in this way involves a different total-head ratio for each station and, hence, will be referred to as the tangent-cone method with local total-head ratio.

It was conjectured that better results might be obtained if the vertex total-head ratio, namely, that across the bow wave of the body, were used to calculate the surface pressures from the Mach number distribution. In this case the local Mach numbers are taken to be the surface Mach numbers for cones tangent to the body at various stations. Either of these methods can be applied rapidly by using tables such as those in reference 3.

## Conical-Shock-Expansion Theory

Recently Eggers and Savin (reference 4) have shown that the equations for variation of Mach number with stream angle downstream of the vertex on bodies of revolution reduce approximately to the Prandtl-Meyer equations for two-dimensional flow when the ratio of Mach number to body fineness ratio is greater than unity. Using this fact, they presented a conical-shock-expansion theory for determining the Mach number and thus the pressure distribution over bodies of revolution. To apply this method the surface Mach number on a cone of semivertex angle equal to that of the body is determined for the desired free-stream Mach number. This Mach number is determined from the approximate equations for conical flow presented by Eggers and Savin to provide a completely analytical solution, or it can be obtained from reference 3. The flow quantities downstream of the vertex are then obtained by applying the Prandtl-Meyer expansion equation. It will be noted that, according to this theory, the distribution of pressure as a function of $(\theta_s-\theta)/\theta_s$ is dependent only on the vertex angle and free-stream Mach number.

## Newtonian Theory

The Newtonian concept of flow assumes that the shock wave lies on the body surface, a condition which is reached in the limit as $M \rightarrow \infty$ and $\gamma \rightarrow 1$. The assumption is made that the component of momentum normal to the surface is lost and the tangential component is unchanged. This yields a pressure coefficient which depends only on the local slope. This simple analysis neglects the centrifugal forces due to body curvature. Equations which take into account the centrifugal forces were presented by Busemann

(reference 5) and were later rederived in reference 6. It has been suggested that either the Newtonian impact theory alone or with centrifugal forces considered might be applied at finite Mach numbers with reasonable accuracy when the shock wave lies close to the body. Newtonian theory does not predict the variation of pressure coefficient with Mach number but simply predicts the limiting value for very high Mach number.

<div align="center">PROCEDURE AND SCOPE</div>

The investigation included three body shapes, the cone, the tangent ogive,[1] and a modified nose of an optimum body (fig. 1). The forepart of a Haack optimum closed body defined by

$$r/r_{max} = \left[ 1 - \left( \frac{x-l}{l} \right)^2 \right]^{3/4}$$

was used as modified by the addition of a cone tangent at $x/l = 0.05$. The cone was used to replace the blunt nose in order to make it possible to apply the theories being investigated. For convenience, this modified body will be referred to as the optimum body in this report.

The theories were applied to various combinations of fineness ratio and Mach number. The following tables list the conditions investigated for each theory:

<div align="center">Linearized and Second-Order Theories</div>

| Cones | | | | | | Ogives | |
|---|---|---|---|---|---|---|---|
| $l/d$ | $\theta_s$ | $M_O$ | $l/d$ | $\theta_s$ | $M_O$ | $l/d$ | $M_O$ |
| 5.715 | 5° | 1.958 | 2.836 | 10° | 5.0 | 6 | 3.0 |
|  |  | 5.0 |  |  | 5.422 | 3 | 1.5 |
|  |  | 7.0 | 1.866 | 15° | 1.3 | 3 | 2.273 |
|  |  | 8.492 |  |  | 2.0 | 9 | 8.137 |
|  |  | 10.146 |  |  | 3.0 | 3 | 2.809 |
| 2.836 | 10° | 1.5 |  |  | 3.634 | 2 | 2.0 |
|  |  | 3.0 | 1.374 | 20° | 1.3 |  |  |
|  |  | 4.0 |  |  | 1.7 |  |  |
|  |  |  |  |  | 2.0 |  |  |
|  |  |  |  |  | 2.443 |  |  |

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[1]A tangent ogive is a pointed convex surface of revolution generated by rotation of a circular arc, the tangent at the maximum radius being parallel to the axis of symmetry.

Tangent-Cone Method (Total-head
ratio applied each way)

| Optimum bodies | | Ogives | | | |
|---|---|---|---|---|---|
| $l/d$ | $M_O$ | $l/d$ | $M_O$ | $l/d$ | $M_O$ |
| 3 | 3.0 | 3 | 1.5 | 4 | 6.0 |
| 4 | 6.0 | 2 | 2 | 1.5 | 3.0 |
| | | 3 | 3 | 3 | 6.0 |

Conical-Shock-Expansion Theory

| Optimum bodies | | Ogives | | | |
|---|---|---|---|---|---|
| $l/d$ | $M_O$ | $l/d$ | $M_O$ | $l/d$ | $M_O$ |
| 3 | 3.0 | 3 | 1.5 | 9 | 9.0 |
| 4 | 6.0 | 6 | 3 | 4 | 6.0 |
| | | 12 | 6 | 1.5 | 3.0 |
| | | 2 | 2 | 3 | 6.0 |
| | | 3 | 3 | 6 | 12.0 |
| | | 6 | 6 | | |

Newtonian Theory

| Cones | | | Cones | | | Optimum bodies | | Ogives | |
|---|---|---|---|---|---|---|---|---|---|
| $l/d$ | $\theta_s$ | $M_O$ | $l/d$ | $\theta_s$ | $M_O$ | $l/d$ | $M_O$ | $l/d$ | $M_O$ |
| 5.715 | 5° | 3.0 | 1.866 | 15° | 1.3 | 3 | 3.0 | 3 | 1.5 |
| | | 5.0 | | | 2.0 | 4 | 6.0 | 6 | 3.0 |
| | | 7.0 | | | 3.0 | | | 12 | 6.0 |
| | | 8.492 | | | 3.634 | | | 3 | 3.0 |
| | | 10.146 | 1.374 | 20° | 1.5 | | | 6 | 6.0 |
| 2.836 | 10° | 1.5 | | | 1.7 | | | 4 | 6.0 |
| | | 3.0 | | | 2.0 | | | 1.5 | 3.0 |
| | | 4.0 | | | 2.443 | | | 3 | 6.0 |
| | | 5.0 | | | | | | 6 | 12.0 |
| | | 5.422 | | | | | | | |

The accuracy of the methods was determined by comparing both the pressure distribution and the integrated pressure drag obtained by the chosen methods with those obtained from standard solutions. Standard values for cones were obtained from tables of solutions to the theory of Taylor and Maccoll (for example, reference 3). Solutions calculated by use of the method of characteristics which took into account the variation of entropy in the flow field were used as standards for curved bodies. Some of the characteristic solutions used were those presented in reference 7 or were obtained from the cross plots in that reference.

The validity of using pressure distributions from characteristic solutions as standards has been established by the close correlation of some available experimental pressure data with pressure distributions determined by the method of characteristics. The error in integrating the characteristic solutions to obtain pressure drag is estimated to be about 2 percent.

In applying the linearized and second-order theories, the approximate tangency condition and the exact isentropic equation for converting velocity to pressure were used, as was done in reference 2. In the calculations using conical-shock-expansion theory the vertex solution was obtained from reference 3 rather than from the approximate equations of reference 4. Both the simple Newtonian impact forces giving

$$C_p = 2 \sin^2\theta$$

and the expression including centrifugal forces were used in calculating the pressure distributions over the bodies investigated.

<div align="center">RESULTS AND DISCUSSION</div>

The results of this investigation are correlated on the basis of the hypersonic similarity parameter, the ratio of free-stream Mach number to body fineness ratio. The hypersonic similarity rule which was derived for slender bodies in hypersonic flow (reference 8) has been shown to hold over a wide range of Mach numbers and fineness ratios, but is not valid for low Mach numbers ($<2$) or small fineness ratios ($<2$) (reference 7). The rule states that pressure distributions in terms of $(p-p_0)/p_0$ are the same for related, pointed, axially symmetric bodies which have the same value of the hypersonic similarity parameter, $K$. This similarity indicates that flow solutions are not dependent on Mach number or fineness ratio separately but on their ratio. Thus, it would be expected that this ratio, or similarity parameter, would be a more important factor than either $M$ or $l/d$ in determining the range of applicability of a given theory.

The accuracy of linearized, second-order, and Newtonian theories as applied to cones is illustrated in figure 2 for cones of various semi-vertex angles over a range of values of $K$. Correlation for each method on the basis of this parameter is reasonably good, regardless of the inclusion of low Mach numbers (1.3) and fineness ratios (1.374). The linearized theory is accurate within 10 percent only for cones of $5^\circ$ and $10^\circ$ semivertex angle for $K < 0.7$. The second-order theory is accurate within 10 percent to $K = 1.2$ for cones $\lessapprox 20^\circ$ (semivertex angle) and to $K = 1.6$ for cones $\lessapprox 10^\circ$ (semivertex angle). Both these theories

become less accurate as the Mach angle approaches the semivertex angle and would also become less accurate near the Mach numbers of wave detachment. Newtonian theory applied to cones is always in error more than 17 percent for $K \leq 2$.

The results of applying the various methods for determining the pressure distributions over tangent ogives and optimum bodies are presented for values of $K$ of about 0.5, 1, and 2. In each $K$ range a representative Mach number and fineness-ratio combination, one for which the similarity law is applicable, is presented. Also, a test of the theories for each $K$ range is presented for a case with low Mach number or large vertex angle which gives conditions which are on the margin of or out of the range of applicability of the similarity rule.

A representative case for $K = 0.5$ ($l/d = 6$, $M = 3$) is shown in figure 3 for the tangent ogive. It is apparent that even at this low value of $K$ the linearized theory gives a pressure distribution appreciably different from the standard. The pressure coefficients predicted by this method are too low at the vertex and too high at the base. The compensating errors in pressure decrease the error in drag. Although the second-order theory predicts pressure coefficients which are slightly high at the vertex, the distribution is in fairly good agreement with the characteristic solution. The error in drag at this value of $K$ is small and the theory seems acceptable for most engineering purposes. As would be expected, neither the conical-shock-expansion nor the Newtonian theory (without centrifugal forces) gives good results at the low value of $K$. The conical-shock-expansion value is exact, of course, at the vertex but as $x/l$ increases the pressure coefficients fall increasingly lower than the standard values. The pressure predicted by Newtonian theory is 30 percent in error at the vertex and does not follow the general trends of the standard. The large errors in pressures and drag make these last two theories unsuitable for use at such low values of $K$. The pressure distributions determined by the equation including centrifugal forces are in greater error than those from Newtonian forces alone in all cases tested, and therefore are not shown except at a more favorable value of $K$ to be shown later.

Figure 4 shows the results for the same value of $K$ as shown in figure 3 ($K = 0.5$), but for low supersonic Mach number ($M = 1.5$). It is noted that the percentage error at the vertex is greater than in the preceding case for both the linearized and second-order theories. The errors in pressure distribution compensate to give no error in drag. The pressure distribution determined by tangent-cone method has been included in this figure. Since the total-head ratio for $K = 0.5$ is nearly unity, there is no distinguishable difference between the two procedures for applying the total-head ratio. The pressure coefficient at the vertex is, of course, exact but at the base, where the local slope

of the body is zero, the predicted pressure coefficient is zero. Thus the coefficients are all higher than the standard values. The drag error for this method at $K = 0.5$ is large.

To illustrate the trend in linearized and second-order theories on ogives as $K$ increases, figure 5 is presented for $K = 0.936$. The pressure coefficients predicted by both the theories are in greater error at this value than at the lower value of $K$. Even at this value of $K$, errors in pressure distribution compensate so that the integrated wave drag predicted by the second-order theory is within the accuracy of integration.

Figure 6 is presented as a representative combination of Mach number and fineness ratio for $K = 1$. This case is out of the range of practical applicability of the second-order and linearized theories (ratio of tangents of semivertex and Mach angle is greater than 0.9). Either of the tangent-cone methods gives a better approximation to the pressure distribution for this medium value of $K$ than for lower values. The procedure using a constant total-head ratio accurately predicts drag by underestimating the pressures on the forepart of the body and over-estimating the pressure near the base. The method using the local values of the total-head ratio is still consistently high and gives about 12-percent error in drag. The conical-shock-expansion method shows better agreement at this value than at lower values of $K$. The errors in drag for this method on tangent ogives ranged from 6 to 9 percent at $K = 1$ for practical combinations of Mach number and fineness ratio. An improvement is seen in Newtonian theory over the case for lower $K$, but the pressure distribution calculated by this theory is in greater error than those calculated by the other theories at $K = 1$.

Figure 7 gives the results of a test case of a body with a large vertex angle ($28°$) at a low Mach number (2). The trends of the predictions of the various approximate methods are consistent with those for other combinations with the same value of $K$, although the errors in some theories are more pronounced for this test case.

In order to give an indication of the effect of body shape, figure 8 is presented for the modified optimum body having the same values of Mach number (3) and fineness ratio (3) as the tangent ogive presented in figure 6. The tangent-cone method using local values of total-head ratio shows about the same accuracy as for the ogive. The error obtained by using the tangent-cone method with a vertex total-head ratio is not consistent with that for the corresponding ogive (drag is -12 percent in error compared to accurate value for ogive). The pressure distribution obtained from the conical-shock-expansion theory follows the same trends and the drag error is only slightly greater than that for the ogive. The integrated Newtonian pressure is in slightly less error than for the ogive.

Figure 9 gives representative results for K = 2 on a tangent
ogive. At this value of K, the tangent-cone method using local values
of total-head ratio gives pressures and drag which are within the
accuracy required for many engineering purposes. The other tangent-cone
method underestimates the pressures over most of the body and does not
give a reasonably accurate pressure distribution. It is evident that
the conical-shock-expansion method gives very accurate pressure distri-
butions at high values of K. The drag is within the accuracy of the
integrated drag obtained from characteristic solutions. Also at this
value of K, where the shock wave lies fairly close to the body, Newtonian
theory gives good results. The error in drag is 5 to 8 percent. In this
case, which should be most favorable to using infinite Mach number solu-
tions, the pressure distribution determined by the equation including
centrifugal forces is shown. According to reference 6, the curve would
be terminated at the axis where the absolute pressure would be zero for
M = ∞. Since application is made at a finite Mach number, the curve
might be terminated where the absolute pressure would be zero for the
given Mach number and for either $\gamma = 1$ or $\gamma = 1.4$. Zero pressure is not
reached on the surface of an ogive for $\gamma = 1$ for K = 2. As mentioned
previously, this equation was used for all cases, but the curves have not
been presented due to the large increase in error over that obtained by
using the Newtonian term alone. In the other cases, the pressure distri-
butions from the full equation were no more accurate than in this case.
The expression considering the centrifugal forces, rather than the
Newtonian term alone, should be more accurate in the limit (M = ∞, $\gamma = 1$).
Evidently there are some compensating errors when the centrifugal forces
are neglected and comparison is made at finite Mach number and $\gamma = 1.4$.

The effect of large vertex angle at this value of K (2) was investi-
gated by checking the methods on an ogive with a semivertex angle
of 36.87°. The pressure distributions on this ogive with a fineness
ratio of 1.5 are presented in figure 10. The percentage errors in the
tangent-cone methods are about the same as those obtained for the ogive
with a fineness ratio of 3. The error in the conical-shock-expansion
theory is somewhat larger than for the representative case ($l/d = 3$,
$M_O = 6$), but it is still less than 10 percent and therefore the theory
would be considered applicable. The error in Newtonian theory is about
the same as for other cases for K = 2.

Figure 11 summarizes the relative error in pressure drag determined
by the different methods. This figure, which gives error as a function
of K, may be used as a guide in determining which method will give
drag to the required accuracy for the body and velocity under considera-
tion. The curves have been faired for representative combinations of
Mach number and fineness ratio and are not expected to be valid for
extreme cases. For example, in some cases the drag for the ogive with
fineness ratio of 2 at a Mach number of 2 is not consistent with the
faired curves. It seems reasonable to assume that these curves can be

used as guides for other bodies of the same general shape. It should be remembered in using these curves that the error in drag is not necessarily indicative of the error in pressure coefficients at various body stations.

The plot also is indicative of the value of the ·similarity parameter as a correlating factor. It is seen that for the majority of cases the accuracy of the theory is dependent essentially on K and not on Mach number or fineness ratio.

A simplified means of obtaining drag may be used when values are desired for a given body over a range of Mach numbers. The second-order theory can be used to calculate drag for a few Mach numbers for which the semivertex angle is less than the free-stream Mach angle, and the conical-shock-expansion theory can be used for a few cases which have values of K greater than 1.2. A curve may be faired through values from both theories to give the drag of the body at any intermediate Mach number. This procedure would also be valuable if drag values for a given body were needed near K = 1 where neither theory is very accurate. As an example, this procedure has been followed for a tangent ogive with $l/d$ = 3, and the results are presented in figure 12. Drag was obtained by second-order theory for M = 1.5, 2.2734, and 2.809 and by conical-shock-expansion theory for M = 4, 5, and 6. The knowledge that conical-shock-expansion theory underestimates drag for K slightly greater than 1, and the second-order theory overestimates drag as K approached 1 (fig. 10) was used in fairing through the points. The drag read from the curve for M = 3 is within 5 percent of that calculated by characteristics as indicated in the figure.

The time consumed in applying the various theories considered is an important factor. The linearized theory takes only a few computing hours per solution. If tables and computing sheets as presented in reference 2 are used, a second-order solution can be done in about 10 hours. The conical-shock-expansion theory requires approximately an hour. Neither of the tangent-cone methods takes more than an hour or two. The Newtonian term alone is very quickly calculated while in the neighborhood of three hours is required if centrifugal forces are included. The time required for a characteristic solution would be expressed in weeks rather than hours.

## CONCLUDING REMARKS

The range of body shapes, fineness ratios, and Mach numbers for which any one of the foregoing theories gives results of acceptable accuracy (less than 10-percent error in integrated drag) is limited.

However, for most combinations of fineness ratio and Mach number, one of the approximate methods will give reasonable results. The important consideration is to choose the appropriate method for the case in question.

The second-order theory gives the most accurate pressure distribution for ogives when the ratio of the tangent of the semivertex angle to the tangent of the Mach angle is less than 0.9. (For ogives, this is equivalent to a value of K of about 0.9.) The drag determined by this method is acceptably accurate. For values of K greater than 1.2, the conical-shock-expansion theory gives the best results and is very accurate at high values of K.

The tangent-cone method using the local total-head ratios gives drag within 10 percent for K greater than 1.2 but is inferior to the conical-shock-expansion theory. The method using a constant total-head ratio gives very good results only for K = 1 on ogives and is not consistent for varying body shapes. The Newtonian theory gives acceptable coefficients at K = 2 for ogives and modified optimum body shapes, but it is not as accurate as the conical-shock-expansion theory.

Ames Aeronautical Laboratory
    National Advisory Committee for Aeronautics
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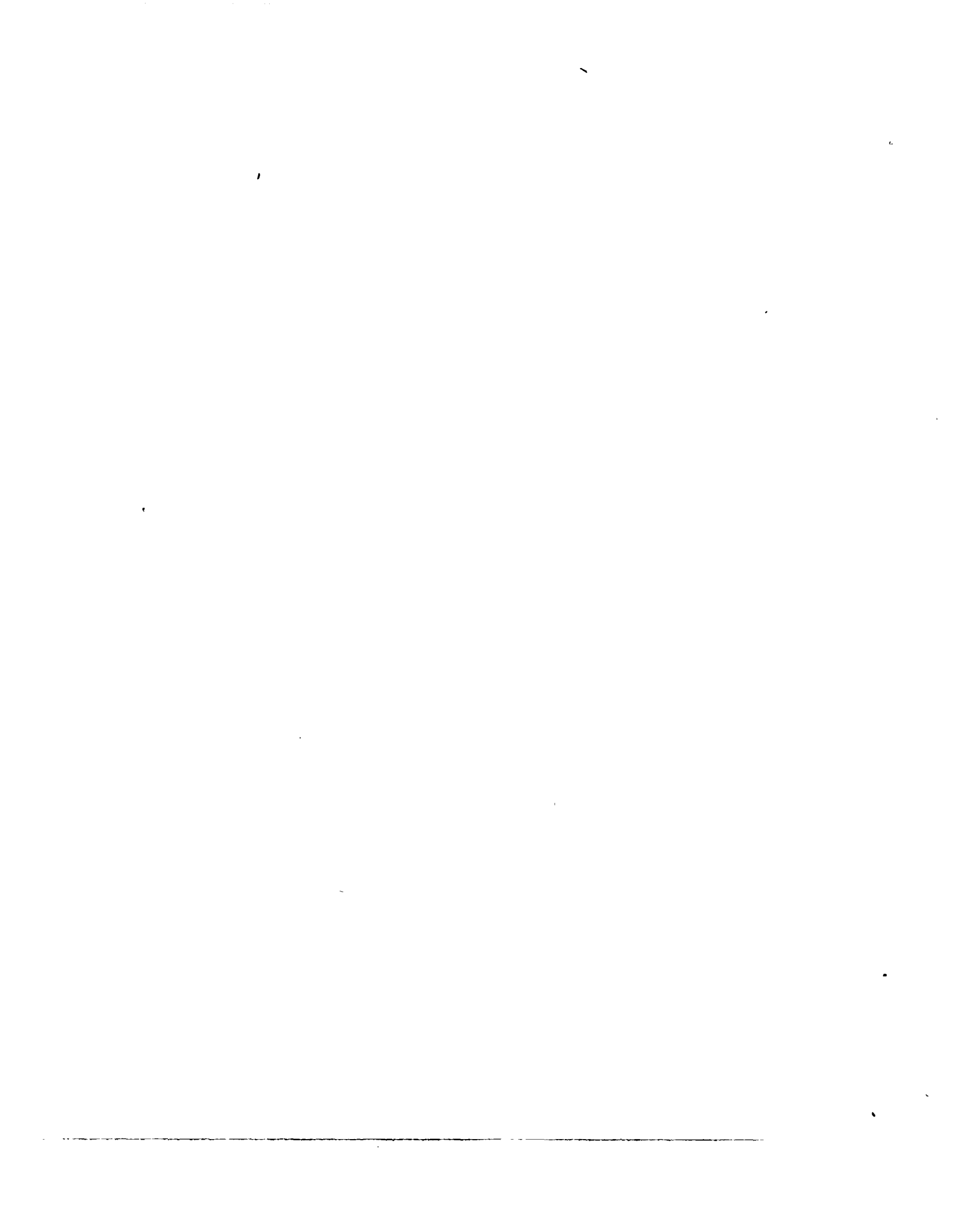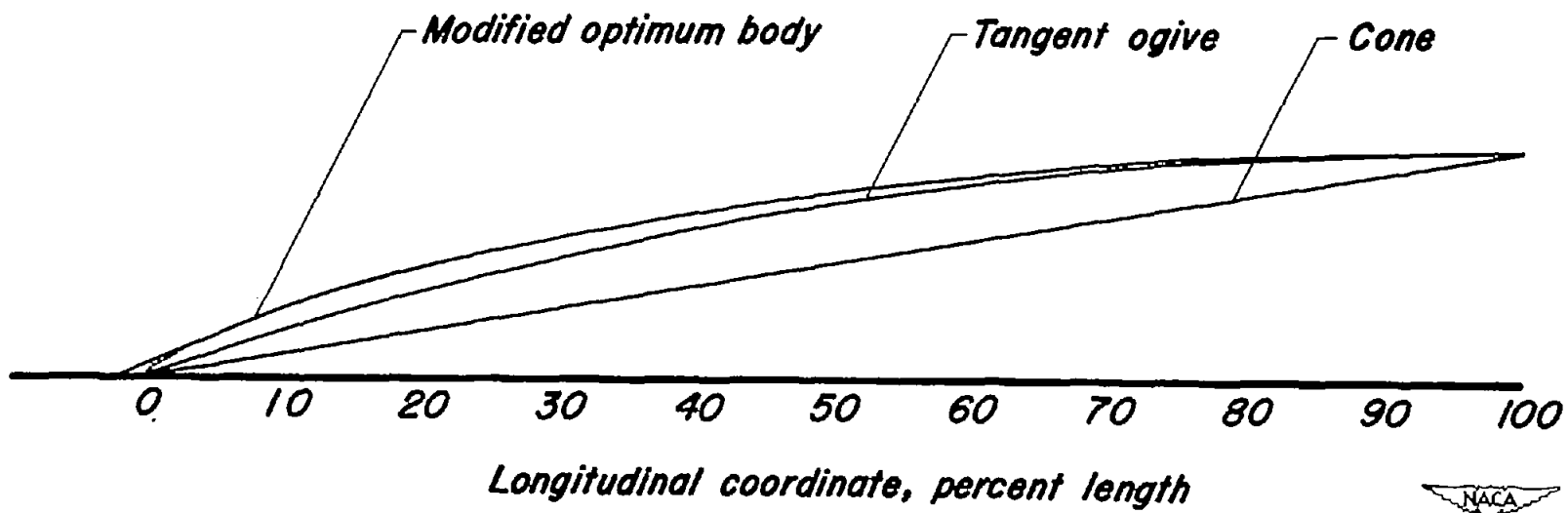
Figure I.—Body shapes used in investigation, comparison shown for l/d = 3.
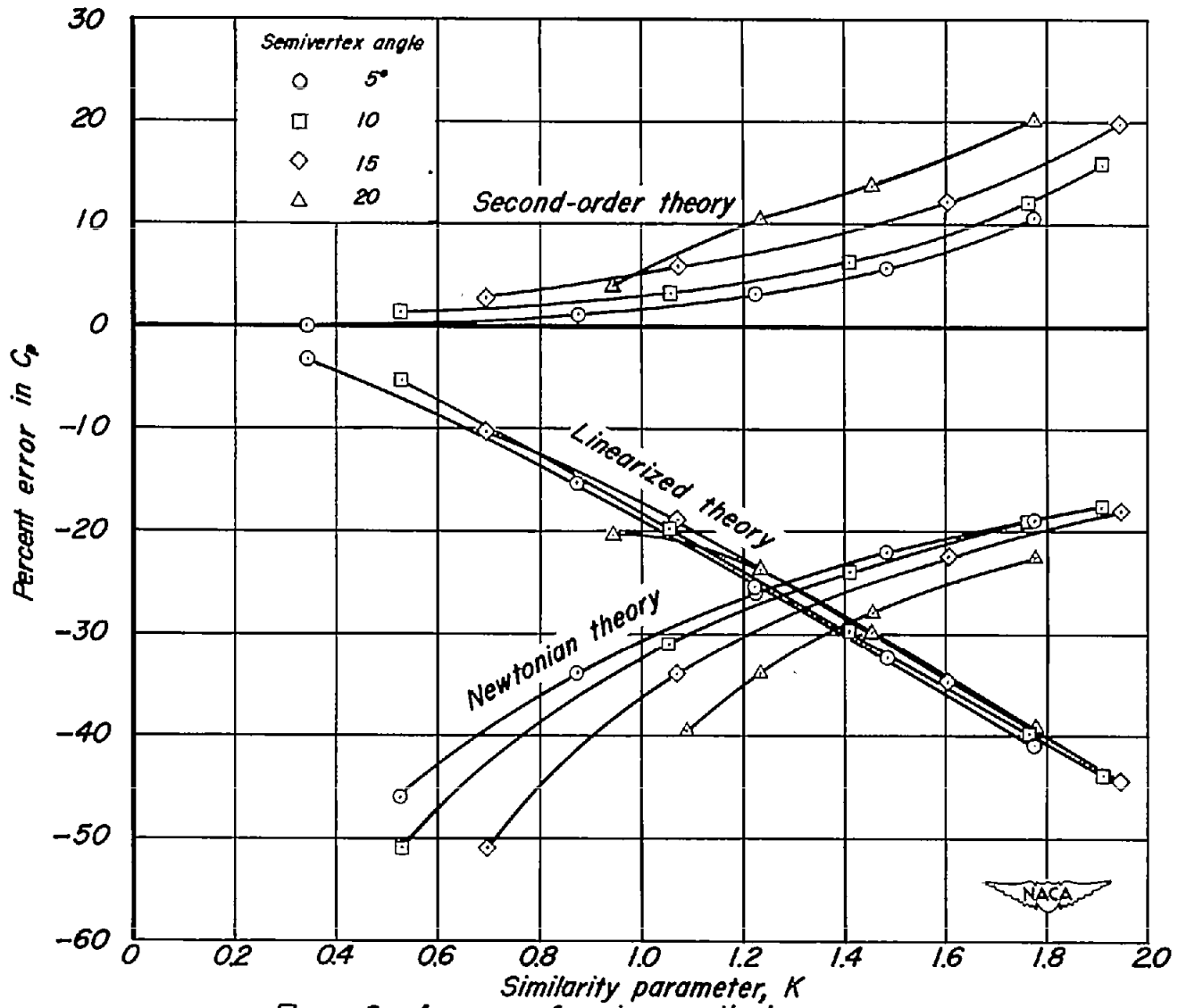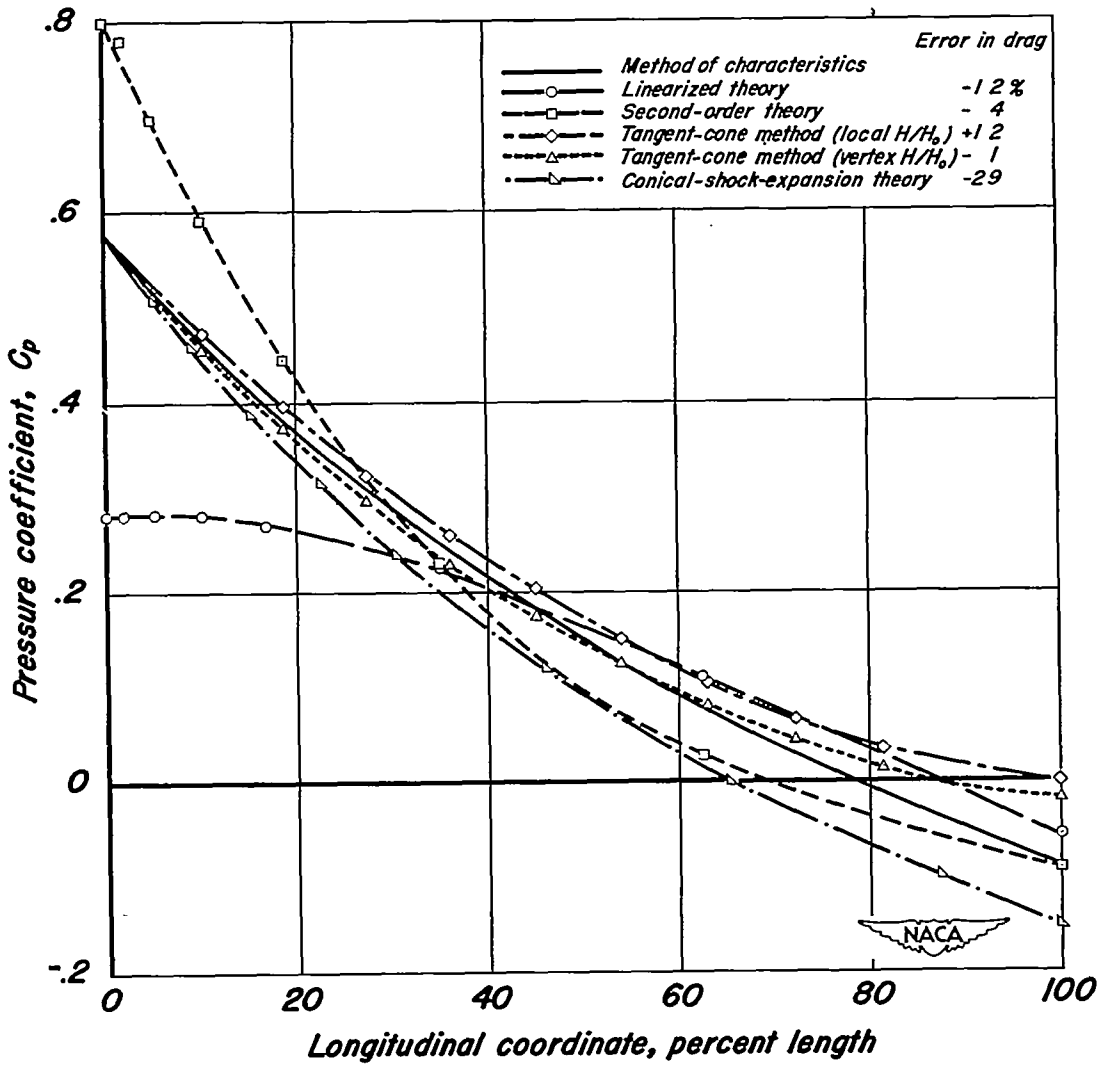
Figure 2.—Accuracy of various methods on cones.
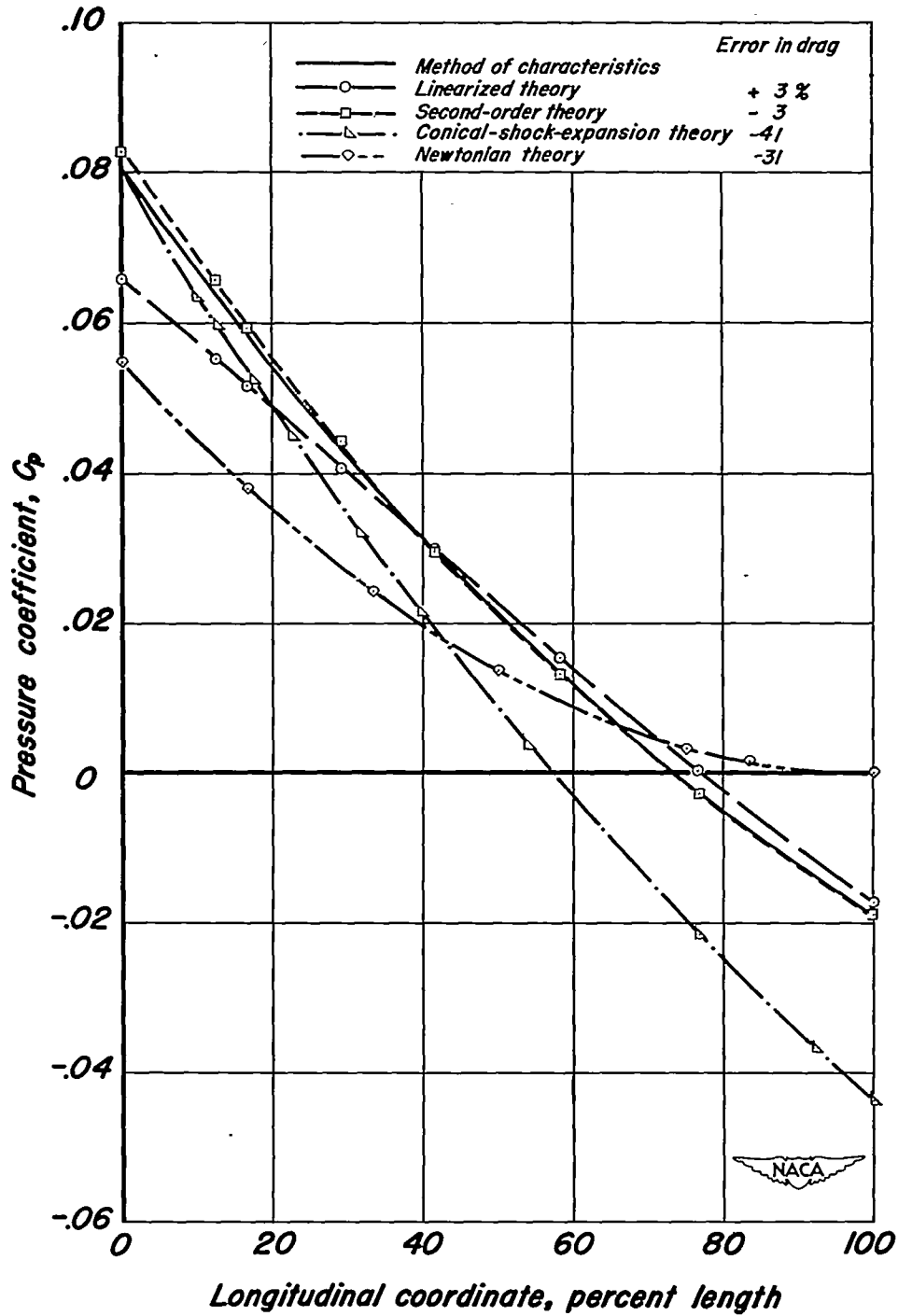
Figure 3.—Comparison of pressure distributions determined by various methods on a tangent ogive at K=0.5, l/d = 6, M₀= 3.
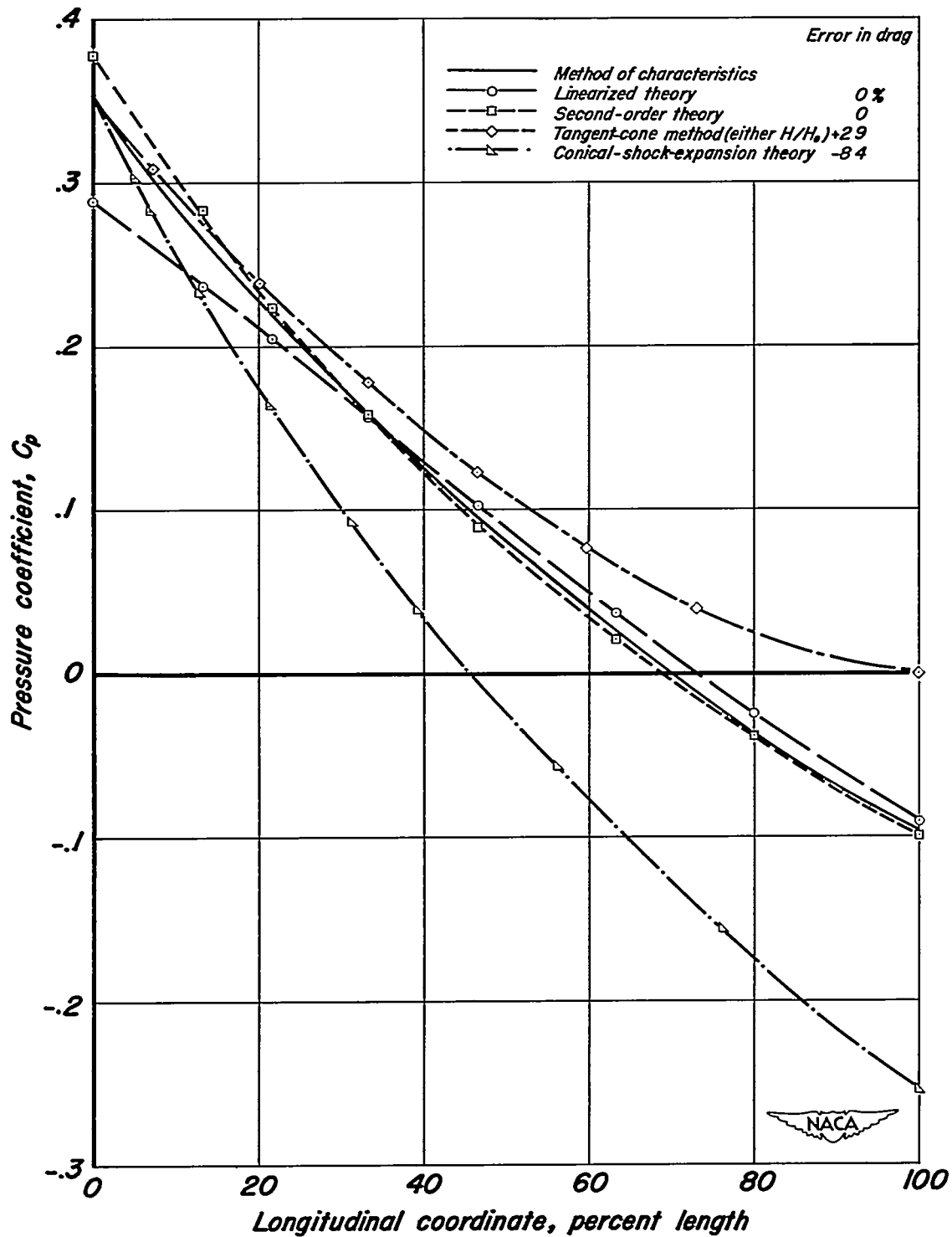
*Figure 4.—Comparison of pressure distributions determined by various methods on a tangent ogive at  K=0.5, l/d=3, M₀=1.5.*
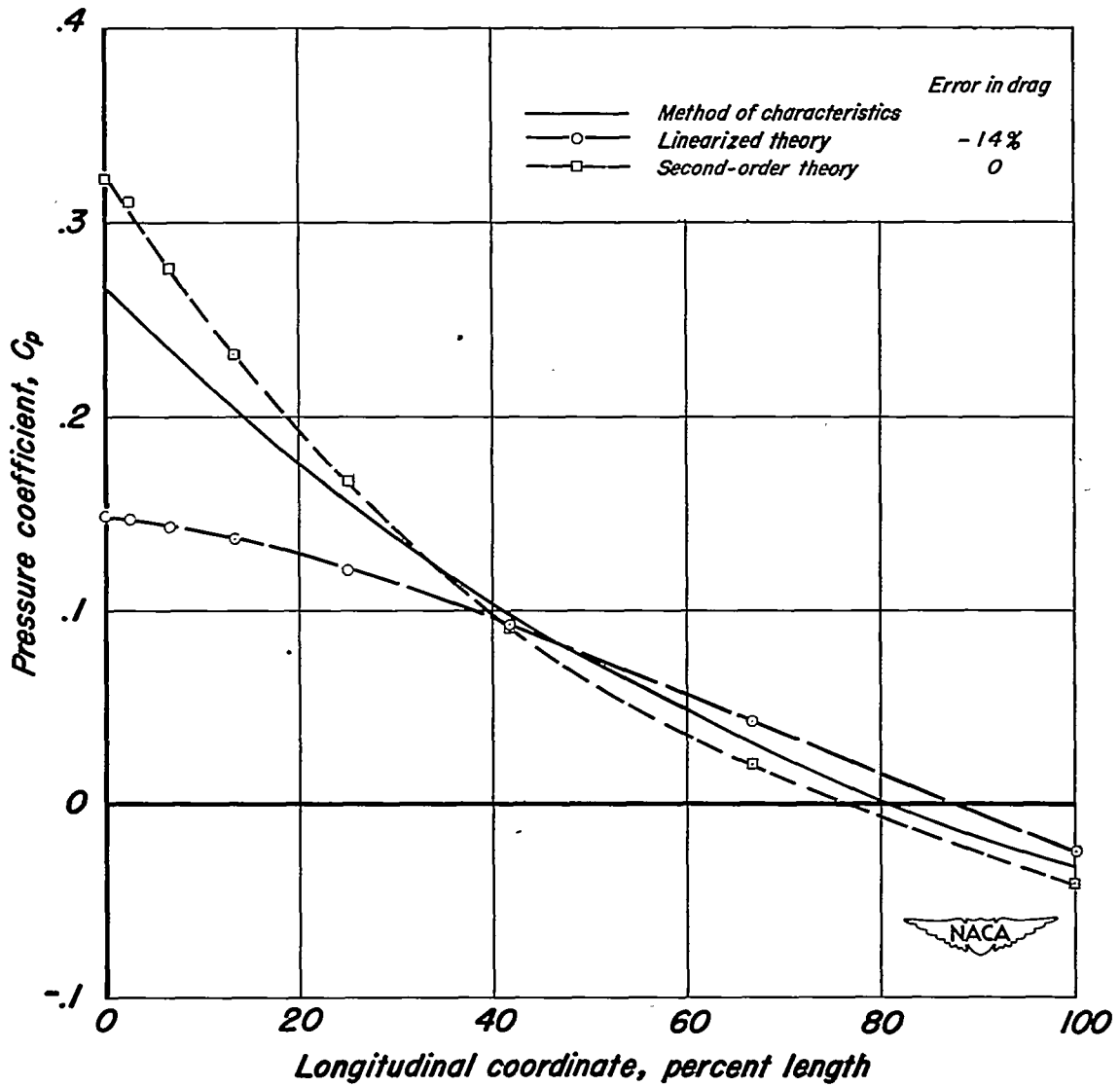
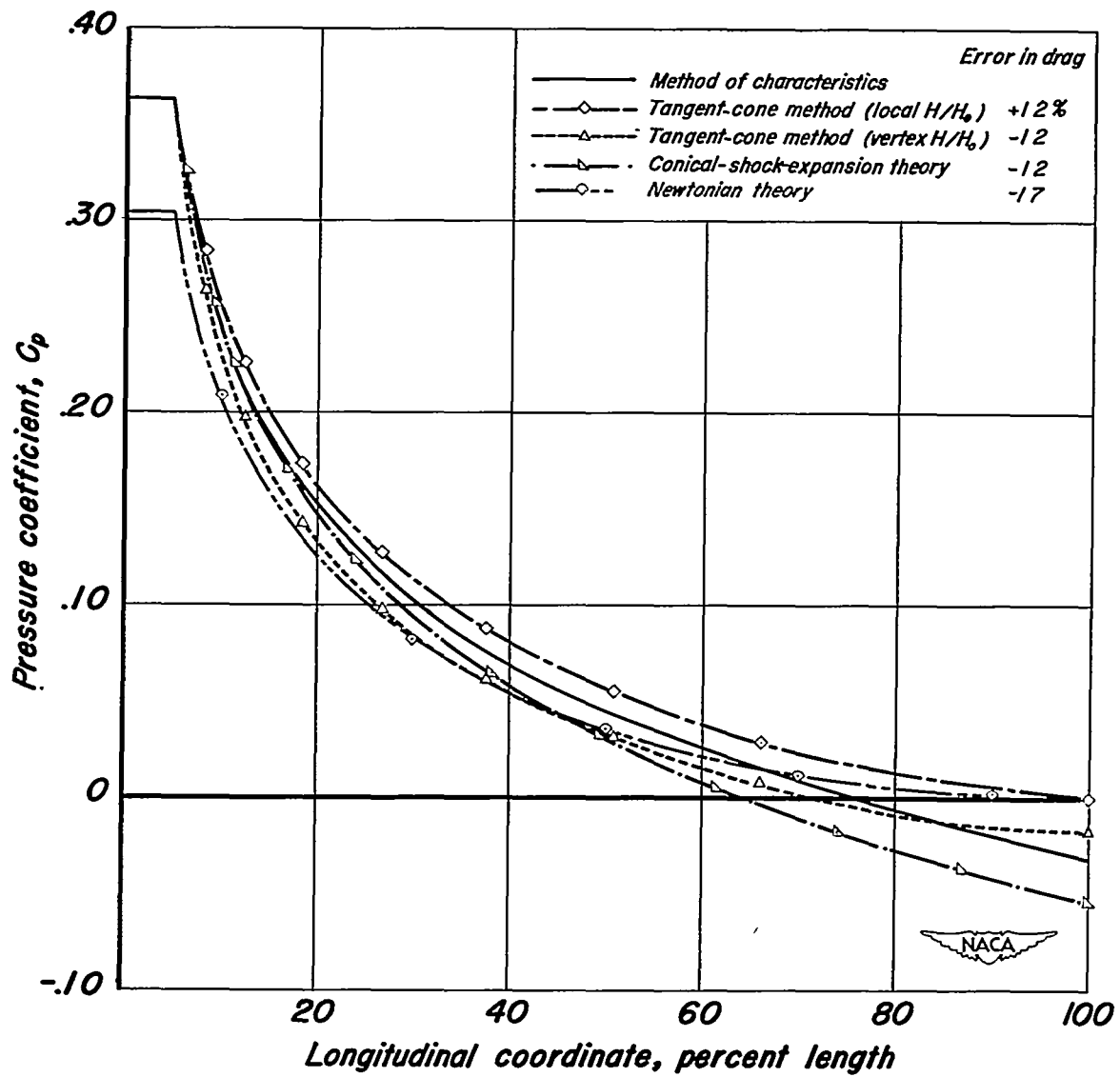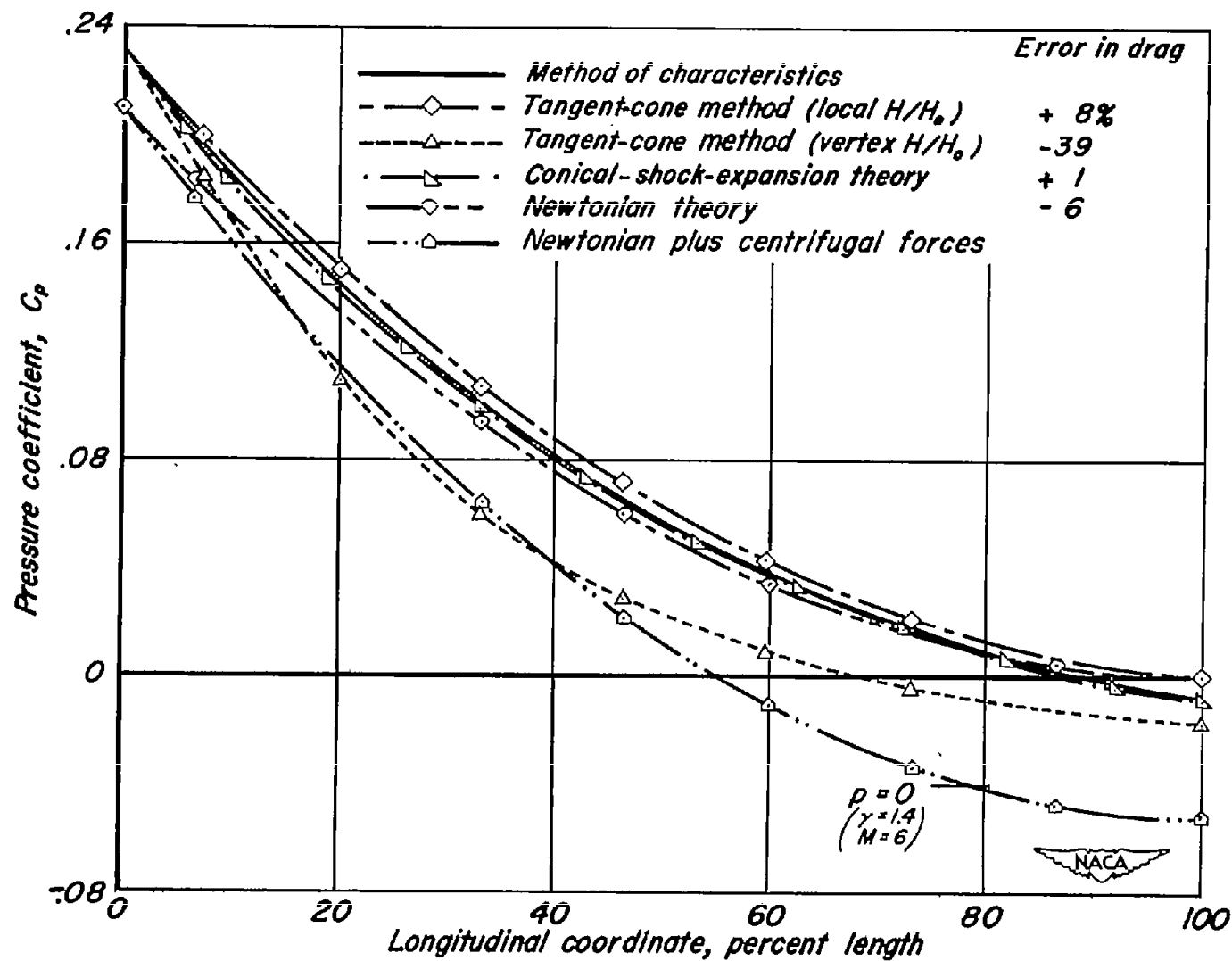Figure 5. — Comparison of pressure distributions determined by
various methods on a tangent ogive at $K = 0.936$, $l/d = 3$,
$M_o = 2.809$.
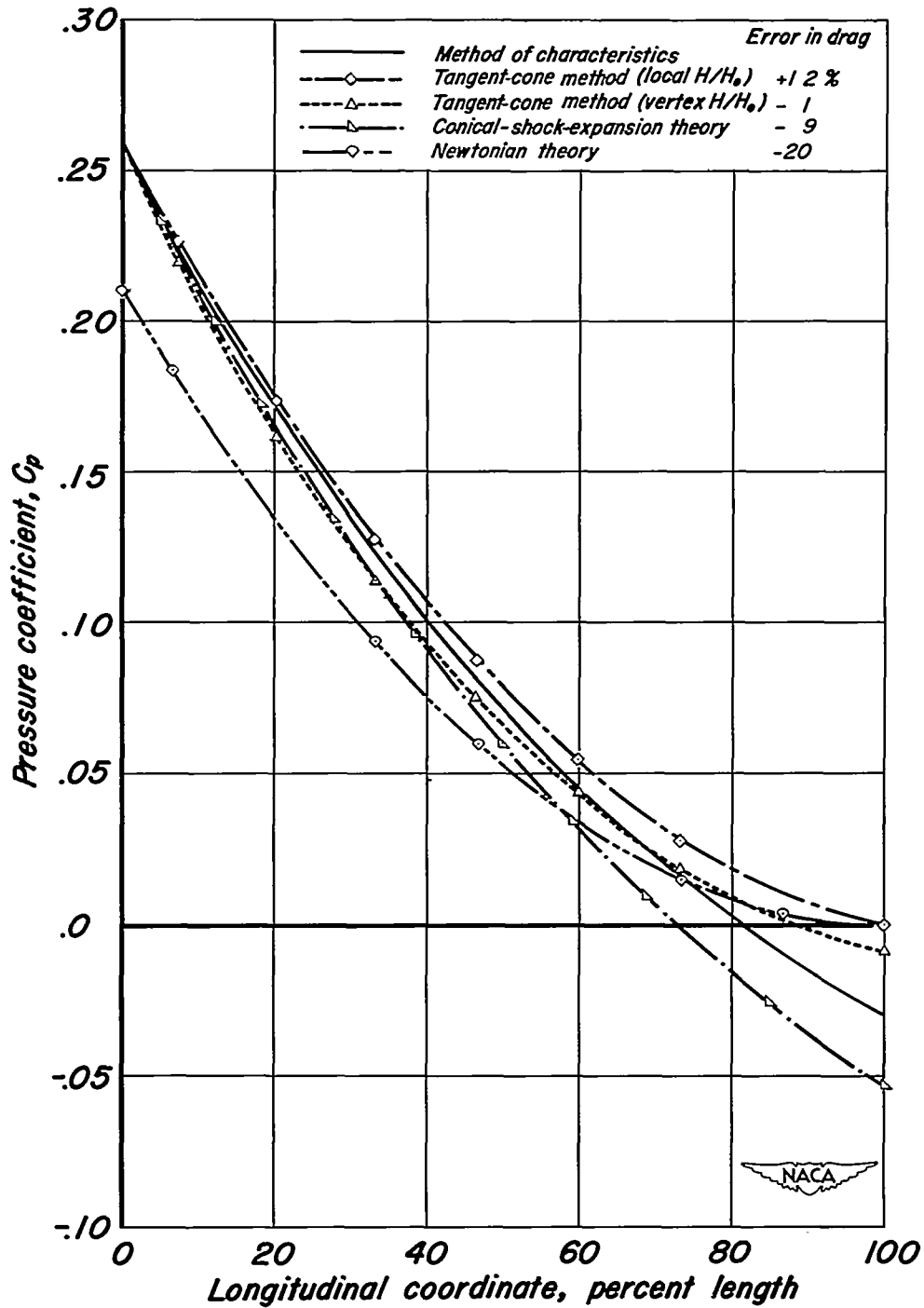
Figure 6. — Comparison of pressure distributions determined by various methods on a tangent ogive at K = 1, l/d = 3, $M_0$ = 3.

Figure 7.— Comparison of pressure distribution determined by various methods on a tangent ogive at $K = 1$, $l/d = 2$, $M_0 = 2$.

Figure 8. — Comparison of pressure distributions determined
by different methods on a modified optimum body at
$K = 1$, $1/d = 3$, $M_0 = 3$.

Figure 9.—Comparison of pressure distributions determined by different methods on a tangent ogive at $K = 2$, $l/d = 3$, $M_0 = 6$.
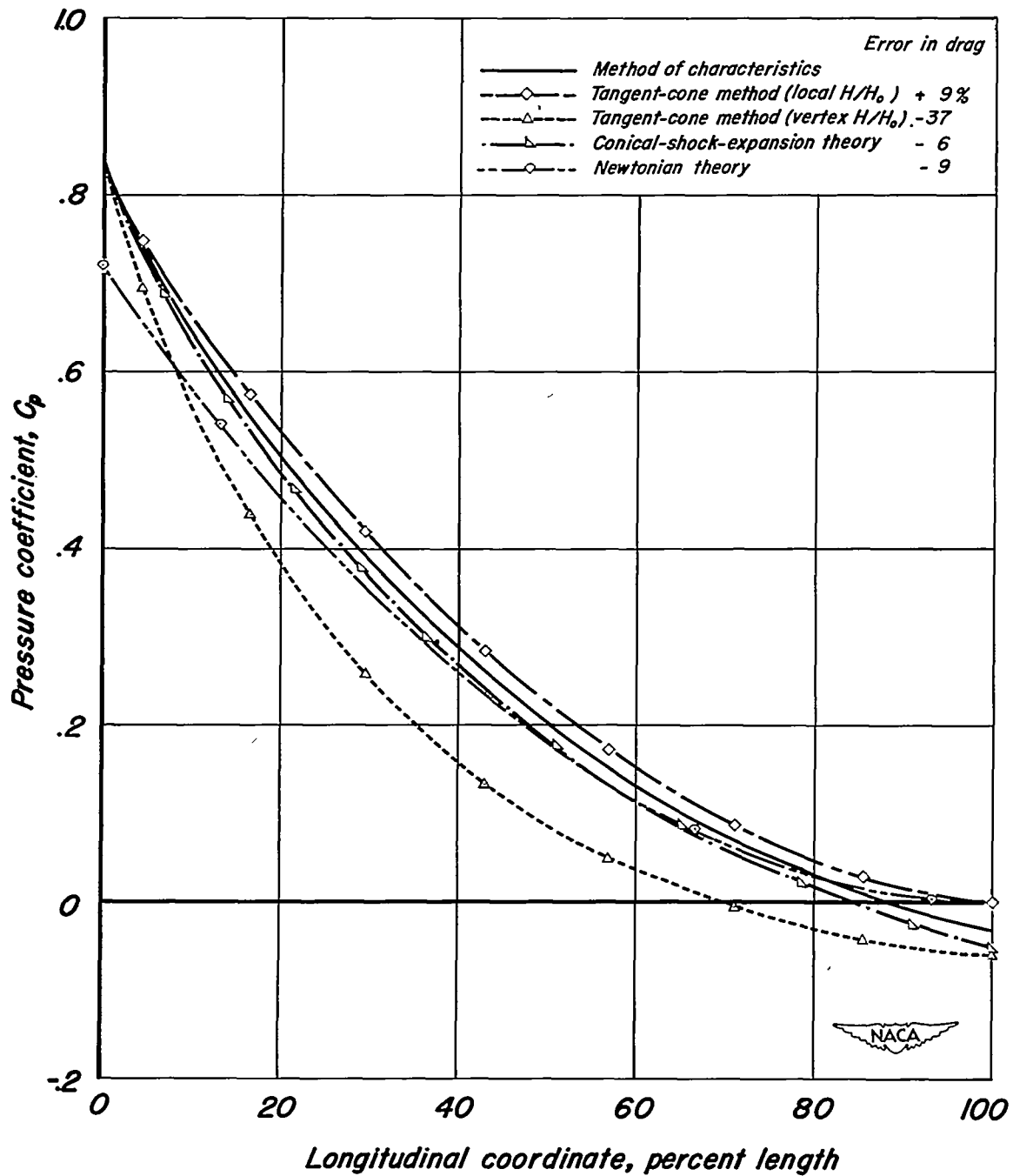
Figure 10.- Comparison of pressure distributions determined by various methods on a tangent ogive at $K=2$, $l/d=1.5$, $M_o=3$.
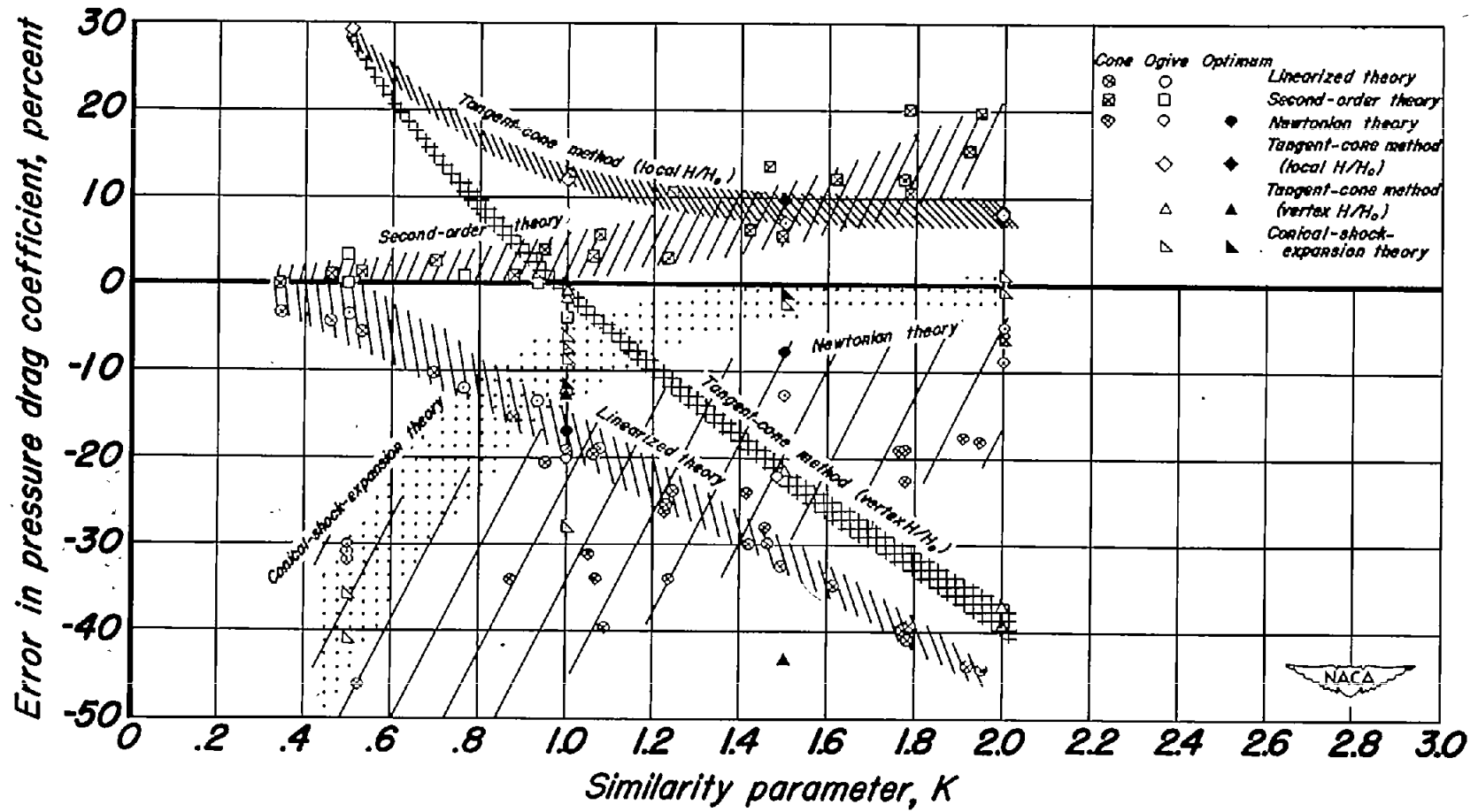
Figure 11.—Accuracy of various approximate methods in integrated pressure drag.
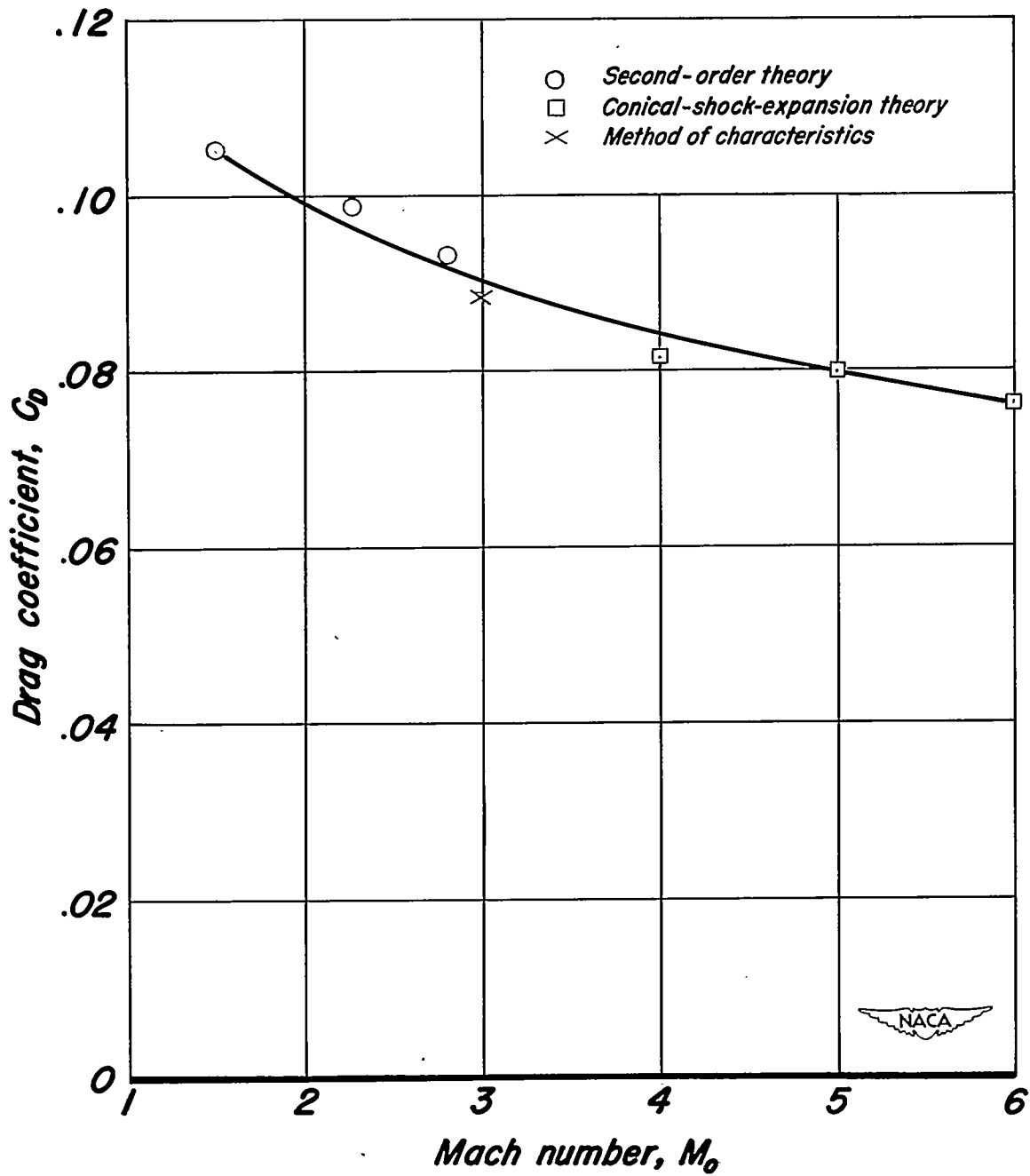
*Figure 12.—Example of interpolation for drag coefficient on tangent ogive, l/d = 3.*