NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2856

ESTIMATED POWER REDUCTION BY WATER INJECTION IN A NONRETURN SUPERSONIC WIND TUNNEL

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Washington
January 1953

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SUMMARY

A simplified analysis has been made to estimate the extent to which the pressure ratio and power of a nonreturn supersonic wind tunnel operating in the low supersonic Mach number range can be reduced by the evaporation of water injected into the diffuser. It appears to be theoretically possible to reduce the power by as much as 20 percent for a typical example of a tunnel operating at a Mach number of 1.4 and at the following stagnation conditions: pressure, 15 pounds per square inch; temperature, 200° F; and dew point, 0° F or less. For a tunnel having a test section of 50 square feet, the amount of water injected would be about 300 gallons per minute and the power saved, about 7,000 horsepower. The power required to provide the necessary water and the possible increases in diffuser losses associated with water injection must, of course, be weighed against the theoretical power saving.

INTRODUCTION

The recent increased interest in experimental research at transonic and supersonic speeds has resulted in the design and operation of large-scale wind tunnels having operating powers in excess of 50,000 horsepower. In some proposed designs considerably greater powers have been contemplated. The use of methods for improving the operating efficiencies, even though the improvement may amount to but a few percent, would therefore result in significant power savings. The purpose of this paper is to determine the extent to which cooling by the evaporation of water in the diffuser of a wind tunnel may theoretically improve the operating efficiencies of the tunnel at transonic and supersonic speeds. Such a method presupposes a nonreturn tunnel, a type which is currently in favor for propulsion investigations.

Although the method in principle is not unlike the thermodynamic drive discussed, for example, in reference 1, it differs considerably in application. The thermodynamically operated wind tunnel employs both heaters and coolers (water evaporation), which serve as a primary drive and eliminate the usual compressor and electric motors, and thereby constitutes a tunnel radically different from the type now in operation. The present scheme is merely a device supplementary to the conventional drive and utilizes the dry tunnel air which is otherwise wasted. No basic tunnel-configuration changes are required other than those needed to introduce the water.

SYMBOLS

ρ	mass density
p	pressure
v	velocity
T	temperature
c_p	specific heat at constant pressure
γ	ratio of specific heat at constant pressure to specific heat at constant volume
a	speed of sound
M	Mach number, v/a
h	total energy per unit mass
i	momentum per unit area
m	mass flow per unit area
Q	heat added per unit mass
q	energy ratio, Q/c_pT_0
R	gas constant

Subscripts:

0	stagnation conditions ahead of shock or heat source or both
1	reference station ahead of shock or heat source or both
2	reference station behind shock or heat source or both
3	stagnation conditions behind shock or heat source or both

ANALYSIS

In order to determine the possible gains to be realized by heat removal in the diffuser of a wind tunnel, a simplified analysis is made in which one-dimensional flow through a tube of constant cross section is assumed. The two reference stations considered are station 1 located upstream of the heat source (or shock or both) and station 2 located downstream of the heat source. The flow parameters at the two stations are related by the following equations:

Energy equation,

$$h_0 = \frac{v_1^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_1}{\rho_1} = \frac{v_2^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_2}{\rho_2} - Q = h_3 - Q \tag{1}$$

Momentum equation,

$$i = p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2 \tag{2}$$

Continuity equation,

$$m = \rho_1 v_1 = \rho_2 v_2 \tag{3}$$

In order to discuss the basic concepts of flows with heat addition (qualitatively at first), the pressure and density are eliminated from equation (1) by means of equations (2) and (3) to result in the following equation (ref. 2):

$$h = \frac{\gamma}{\gamma - 1}\frac{i}{m}v - \frac{1}{2}\frac{\gamma + 1}{\gamma - 1}v^2 \tag{4}$$

where the subscripts for h and v may be either 0 and 1 or 3 and 2, respectively. Equation (4), which relates the stagnation energy per unit mass to the velocity for given values of the impulse per unit mass flow i/m, is plotted illustratively in figure 1 (ref. 2). Although only a constant value of the ratio i/m is required to make the curve unique, actually, from the assumed flow, both i and m are individually constant. Several of the salient features of this curve are labeled in figure 1. The velocity occurring at maximum stagnation energy, point Ⓢ, is (from ref. 2) sonic velocity so that points on the upper branch of the curve represent supersonic flows and lower-branch points represent subsonic flows. Point (E) (from eqs. (1) and (4)) represents the maximum velocity attainable and corresponds to a condition of zero pressure, density, and temperature.

Relationship Between Velocity and Total Energy as Illustrated in Figure 1

No heat addition (ref. 2).- In order for the flow to become subsonic when a supersonic stream exists at point (1) (fig. 1), a shock must occur (the total energy of the flow remains constant) and, as a result, an abrupt change in velocity occurs between point (1) and point (2a). As pointed out in reference 2, however, the jump at constant total energy can be replaced by the continuous process from (1) to (A) to (S) to (2a) where, by definition, proceeding in the direction of increasing total energy means the introduction of heat and proceeding in the opposite direction means heat removal. For the case of the shock, the amount of heat added from (1) to (A) to (S) is exactly counterbalanced by the heat removed during the change from (S) to (2a), so that the flow remains adiabatic. The increase in entropy occurring from (1) to (A) to (S) outweighs, of course, the decrease from (S) to (2a), with the net amount determining the stagnation-pressure decrease across the normal shock.

Heat addition.- If the supersonic stream at point (1) in figure 1 becomes subsonic with heat being added in the presence of the shock, the final subsonic flow condition (2b) must (by definition of the abscissa scale) be to the right of the adiabatic case (2a). Furthermore, the process from (1) to (2b) must be less efficient than that from (1) to (2a) since the decrease in entropy associated with moving from (2b) to (2a) is lost. This result is independent of whether the heat is added supersonically so that the path is (1) to (A) to (2b) or whether it is added subsonically behind the shock so that the path is (1) to (2a) to (2b); hence, heat addition corresponds to added losses and results in decreased pressure recoveries. Such a process as just outlined occurs during condensation in wind tunnels.

Heat removal.- If the supersonic stream at point (1) (fig. 1) is cooled in the presence of the shock, the final condition (2c) must be to the left of the adiabatic case (2a); hence, a gain in efficiency occurs and is associated with the added decrease in entropy between (2a) and (2c). Again, this result is independent of whether the heat is removed supersonically so that the path is (1) to (B) to (2c) or subsonically so that the path is (1) to (2a) to (2c). Improved "diffusion" efficiencies are therefore associated with cooling, and although a fixed amount of heat removed is equally effective subsonically or supersonically, heat removal subsonically is, by far, the simpler process, as is shown subsequently.

Basic Relations

The basic relations between the flow parameters upstream and downstream of the heat source can be evaluated from equations (1) to (3) as a function of the initial Mach number and the ratio of the heat added to the initial stagnation energy. These relations are (with the assumption of a perfect gas, $p = \rho RT$)

$$1 + \frac{Q}{c_p T_0} = \left[\left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}\right)\frac{M_2}{M_1}\right]^2 \frac{1 + \frac{\gamma - 1}{2} M_2^2}{1 + \frac{\gamma - 1}{2} M_1^2} \tag{5}$$

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \tag{6}$$

and

$$\frac{T_2}{T_1} = \left[\left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}\right)\frac{M_2}{M_1}\right]^2 \tag{7}$$

Although the final Mach number M_2 can be solved for explicitly as a function of M_1 and $Q/c_p T_0$ from equation (5), the following form of equation (5) is usually much more convenient:

$$f(M_2) = \frac{f(M_1)}{1 + \frac{Q}{c_p T_0}} \tag{8}$$

In order to facilitate the use of equation (8), values of $f(M)$ where

$$f(M) = \frac{\left(1 + \gamma M^2\right)^2}{M^2\left(1 + \frac{\gamma - 1}{2} M^2\right)}$$

are presented in table I. By means of this table, M_2 can be determined directly when M_1 and Q/c_pT_0 are the independent variables. These general relationships for flows with heat addition have been determined previously. (See, for example, refs. 1, 3, and 4.)

Equation (5) is also plotted in figure 2 for representative values of q for both the subsonic and supersonic Mach number ranges. It is to be noted that the basic energy-ratio parameter Q/c_pT_0 in equation (5) is (from eq. (1)) equivalent to $\frac{T_3 - T_0}{T_0}$ so that the quantity $1 + \frac{Q}{c_pT_0}$ is simply the stagnation temperature ratio T_3/T_0.

Figure 3 presents the stagnation pressure ratio as a function of Q/c_pT_0 for several supersonic Mach numbers and indicates the pressure-ratio gains to be realized by cooling. The curves presented in this figure have been obtained under the assumption of isentropic flow on each individual side of the heat source and shock and by the use of equations (5) to (7) to join the two stations. It is to be noted that, in figure 3, the condition of adiabatic flow $\frac{Q}{c_pT_0} = 0$ corresponds to the normal shock, and the dashed line in the figure represents the maximum amount of heat that can be added at a given Mach number. (See, for example, ref. 4.)

APPLICATION

The application of the analysis presented in the previous section is now considered in order to study the possibilities of reducing the pressure ratio and, hence, the power of supersonic wind tunnels. One of the simplest and most effective methods of cooling air is by the evaporation of water. Because of the high value of its latent heat of vaporization, the use of water is found to be extremely effective. The evaporation process is particularly suited for this application because, in order to conduct tests at supersonic speeds, dry air is necessarily used to eliminate adverse condensation conditions in the supersonic nozzle and test section; hence, the proper location for injecting the water without affecting the tests is downstream of the test section. The subsonic flow region in the diffuser or a wind tunnel, as illustrated schematically in figure 4, is a much better location for evaporation than the supersonic flow region rearward of the test section because subsonically the static temperature of the air is much higher and, therefore, much larger quantities of water can be evaporated. In reality, the evaporation of water in the supersonic region would be almost impossible since most tunnels operate at or below the saturation temperature in the test section. In any case, the amount of water that can be evaporated is assumed to be limited by saturation conditions locally.

The possible advantages of evaporating water to improve tunnel efficiencies by reducing the operating pressure ratios can be inferred from figure 3 by interpreting the energy ratio Q/c_pT_O in terms of physical quantities such as the initial stagnation conditions of flow and the amount of water (assumed to be at 65° F) that can be evaporated. Such an interpretation is shown in figure 5 where the stagnation pressure ratio is presented as a function of free-stream Mach number for various percentages of water saturation for the following upstream stagnation conditions: pressure, 15 pounds per square inch; temperature, 100° F and 200° F; and dew point, -30° F and 0° F. The effects of increasing the dew point from -30° F to 0° F are seen to be exceedingly small (the amount of water associated with this increase is negligible) so that further discussion is limited to the condition of 0° F dew point. The higher stagnation pressure ratios associated with both the 100-percent-saturated and 50-percent-saturated conditions are clearly significant, as is the increase in pressure ratio resulting from the change in stagnation temperature from 100° F to 200° F. For some conditions at and below a Mach number of 1.4, stagnation pressure ratios in excess of 1.0 are evidenced. In order to indicate the amounts of water required to realize these gains, the weight ratios of water to air are presented in figure 6. The amount of water actually required becomes more physically evident in the following specific example:

Assume a nonreturn supersonic tunnel having a test section of 50 square feet and stagnation temperatures of 100° F and 200° F. In order to facilitate calculations, a constant stagnation pressure of 15 pounds per square inch is assumed since variations in test-section stagnation pressure with Mach number are of secondary importance in determining the percent power reduction. (The amount of water required will, however, be approximately proportional to the stagnation pressure.) The operating horsepowers would be as shown in figure 7. In order to obtain a realistic estimate of the power, based on previous experience from other tunnels, the compression ratio required is assumed to be given by the product of the adiabatic stagnation pressure ratio across a normal shock at the test Mach number and the compression index given in figure 7. Furthermore, a compressor efficiency of 85 percent is assumed. The resultant horsepower presented in figure 7 is primarily illustrative and is shown solely to establish the magnitudes of the operating powers. The gains to be realized in power savings (fig. 8) are relatively large for the amounts of water required (fig. 9). In the computation of the power savings, the compression index of figure 7 has again been assumed. In addition, the inlet temperature at the compressor is assumed constant, regardless of whether water is added. Such will be the case for a compressor located upstream of the test section. If the compressor is downstream of the source of water injection, added power savings will result because of the drop in stagnation temperature caused by cooling. For the latter case, however, the initial stagnation temperatures will be much lower (usually atmospheric temperature) so that the method of water injection appears less promising for this type of installation.

Figures 7, 8, and 9 show that, at a Mach number of about 1.4, possibly 20 percent of the power (that is, about 7,000 horsepower) might be saved at a stagnation temperature of 200° F for water additions of 300 gallons per minute in magnitude. This percentage savings varies inversely with Mach number. It is of interest to note that the operating power actually will vanish at some subsonic Mach number. This fact, however, appears to be of only academic interest since, besides the starting problem, the cost of drying and heating equipment, not usual for subsonic tunnels, would invariably outweigh the cost of conventional drives. Even in applications at supersonic speeds, the power required to supply the necessary water and the possible increases in diffuser losses associated with physically introducing water into the diffuser must, of course, be weighed against the theoretical power saving.

CONCLUDING REMARKS

A simplified analysis has been made to estimate the extent to which the pressure ratio and power of a nonreturn supersonic wind tunnel operating in the low supersonic Mach number range can be reduced by the evaporation of water injected into the diffuser. It appears to be theoretically possible to reduce the power by as much as 20 percent for a typical example of a tunnel operating at a Mach number of 1.4 and at the following stagnation conditions: pressure, 15 pounds per square inch; temperature, 200° F; and dew point, 0° F or less. For a tunnel having a test section of 50 square feet, the amount of water injected would be about 300 gallons per minute and the power saved about 7,000 horsepower. The power required to provide the necessary water and the possible increases in diffuser losses associated with water injection must, of course, be weighed against the theoretical power saving.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., October 10, 1952.

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1. Shapiro, Ascher H., and Hawthorne, W. R.: The Mechanics and Thermodynamics of Steady One-Dimensional Gas Flow. Jour. Appl. Mech., vol. 14, no. 4, Dec. 1947, pp. A-317 - A-336.

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3. Bailey, Neil P.: The Thermodynamics of Air at High Velocities. Jour. Aero. Sci., vol. 11, no. 3, July 1944, pp. 227-238.

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TABLE I.- VALUES OF MACH NUMBER PARAMETER $f(M) = \frac{(1 + \gamma M^2)^2}{M^2\left(1 + \frac{\gamma - 1}{2} M^2\right)}$

FOR VALUES OF MACH NUMBER M FROM 0 TO 2.00

M	f(M)	M	f(M)	M	f(M)	M	f(M)
0	∞						
.02	2503	0.52	6.668	1.02	4.802	1.52	5.309
.04	627.6	.54	6.426	1.04	4.805	1.54	5.338
.06	280.4	.56	6.214	1.06	4.811	1.56	5.368
.08	158.9	.58	6.027	1.08	4.819	1.58	5.398
.10	102.6	.60	5.861	1.10	4.830	1.60	5.429
.12	72.06	.62	5.715	1.12	4.841	1.62	5.459
.14	53.65	.64	5.587	1.14	4.855	1.64	5.490
.16	41.70	.66	5.473	1.16	4.870	1.66	5.521
.18	33.51	.68	5.372	1.18	4.887	1.68	5.552
.20	27.66	.70	5.283	1.20	4.904	1.70	5.583
.22	23.33	.72	5.206	1.22	4.924	1.72	5.615
.24	20.04	.74	5.137	1.24	4.944	1.74	5.646
.26	17.49	.76	5.077	1.26	4.965	1.76	5.677
.28	15.47	.78	5.025	1.28	4.987	1.78	5.708
.30	13.84	.80	4.979	1.30	5.011	1.80	5.740
.32	12.51	.82	4.941	1.32	5.035	1.82	5.771
.34	11.41	.84	4.908	1.34	5.060	1.84	5.803
.36	10.50	.86	4.880	1.36	5.085	1.86	5.833
.38	9.727	.88	4.857	1.38	5.111	1.88	5.865
.40	9.073	.90	4.838	1.40	5.138	1.90	5.896
.42	8.514	.92	4.824	1.42	5.165	1.92	5.927
.44	8.034	.94	4.813	1.44	5.193	1.94	5.957
.46	7.618	.96	4.806	1.46	5.221	1.96	5.989
.48	7.257	.98	4.802	1.48	5.250	1.98	6.019
.50	6.943	1.00	4.800	1.50	5.279	2.00	6.051

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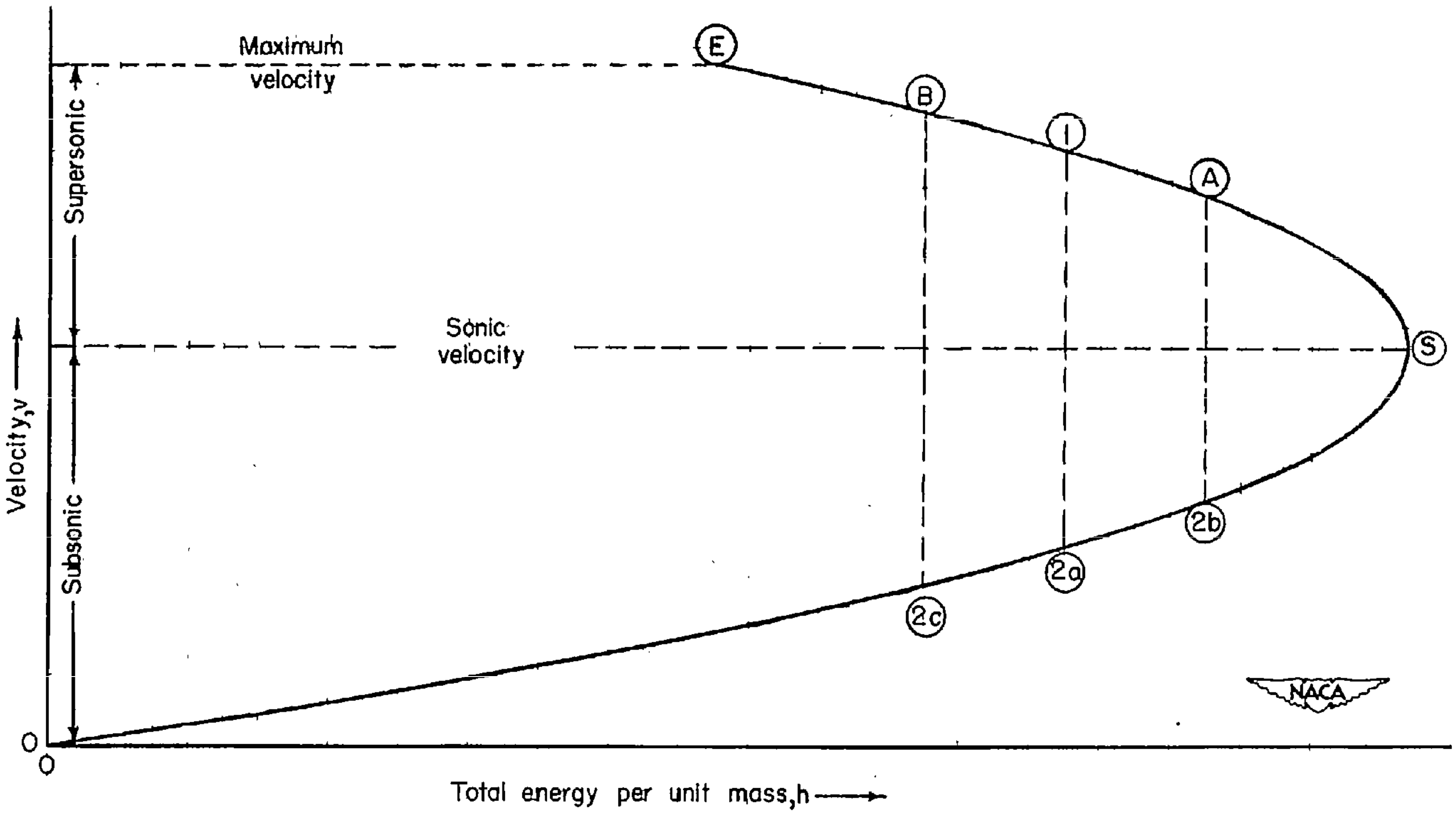


Figure 1.- Illustrative relationship between velocity and total energy for a given impulse per unit mass flow (ref. 2).

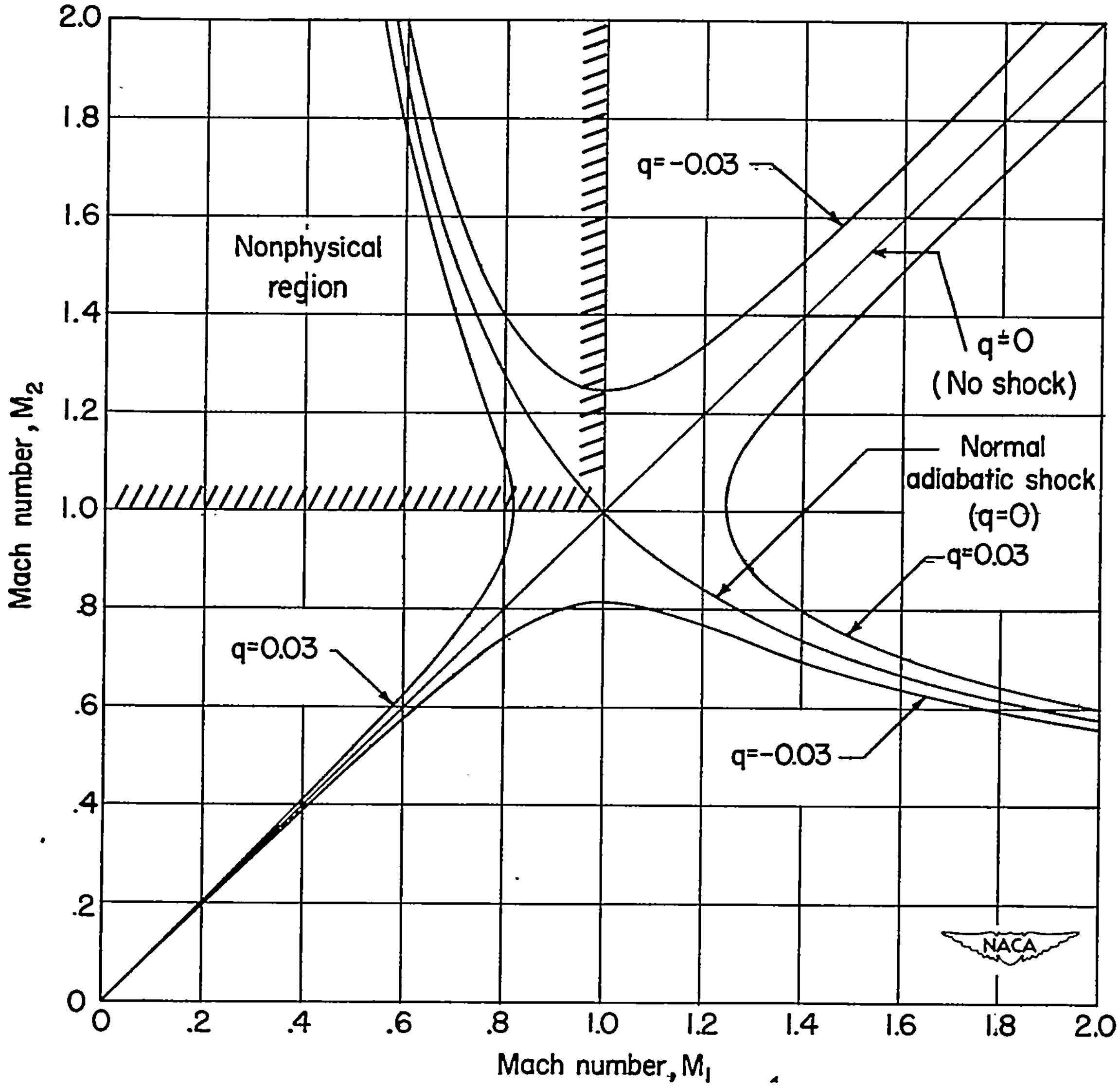


Figure 2.- The effects of heat on the stream Mach number for one-dimensional flow.

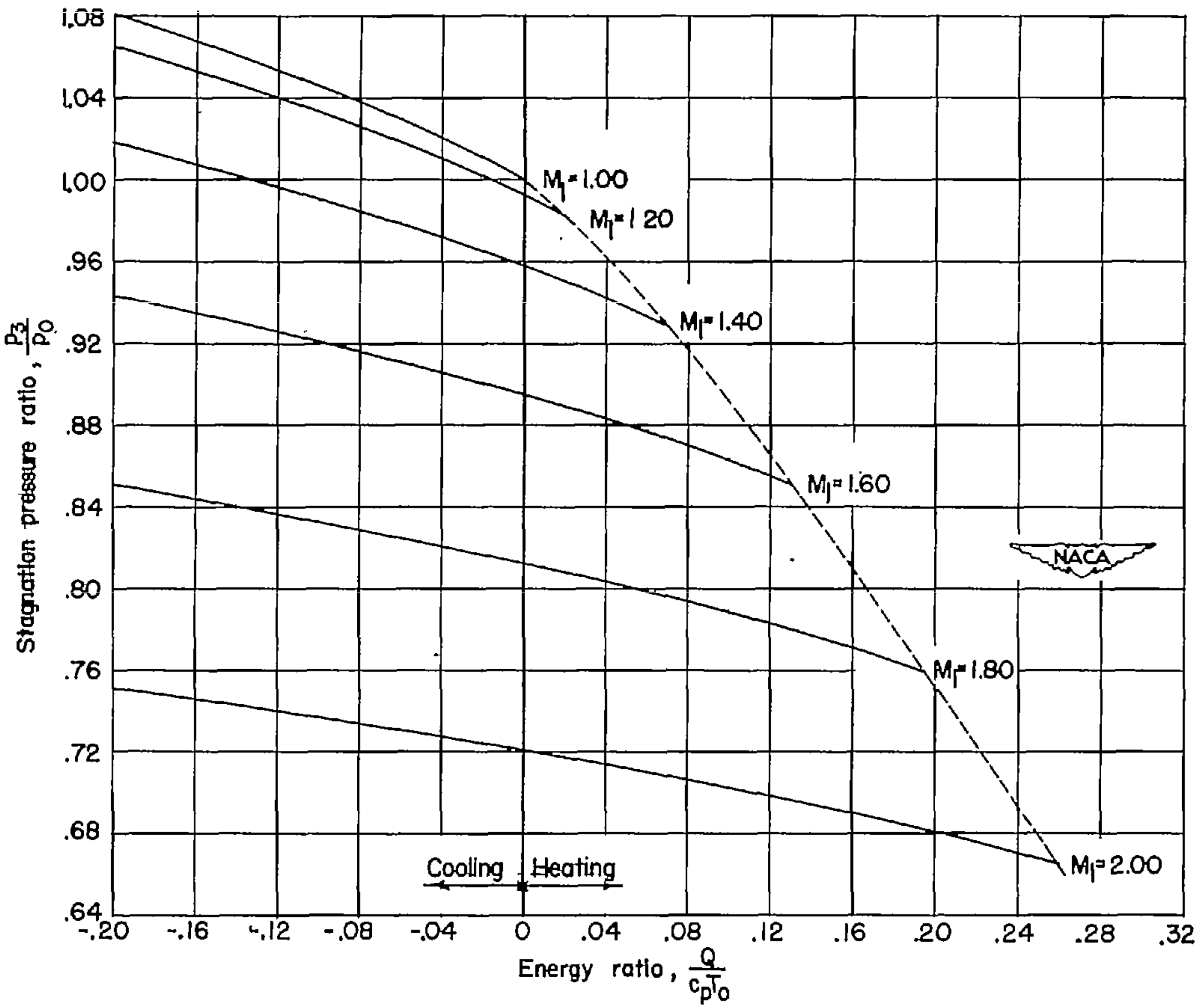


Figure 3.- Stagnation pressure ratio as a function of energy ratio.

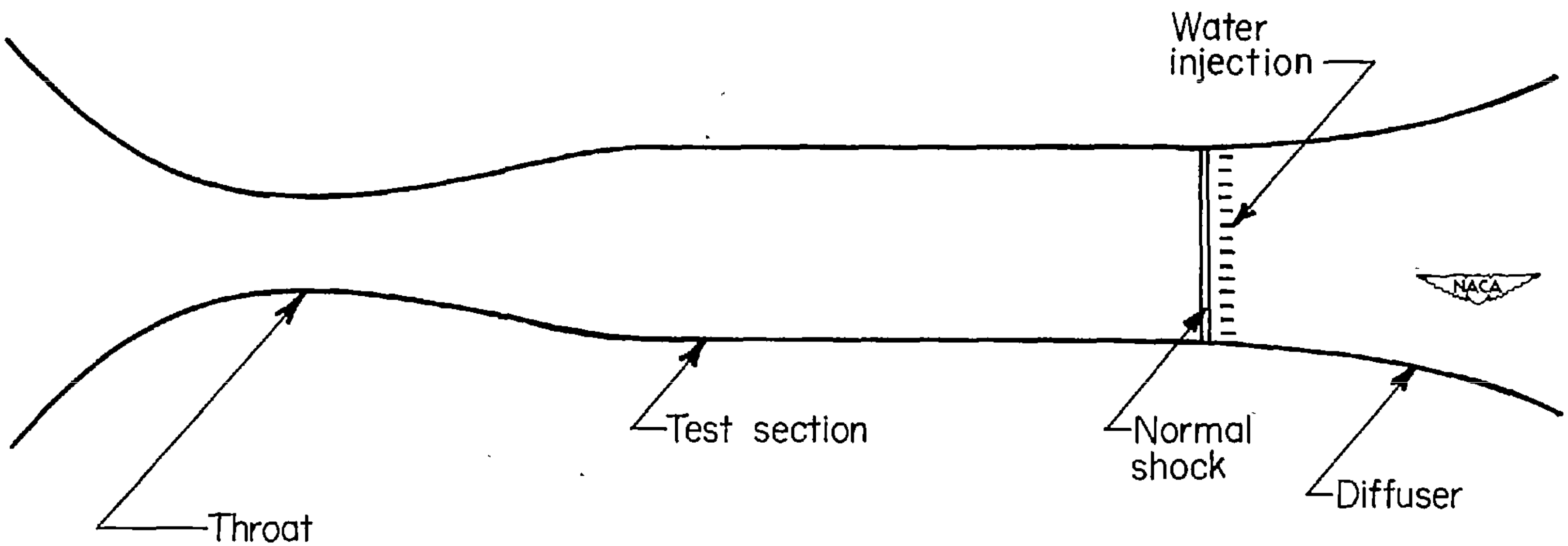
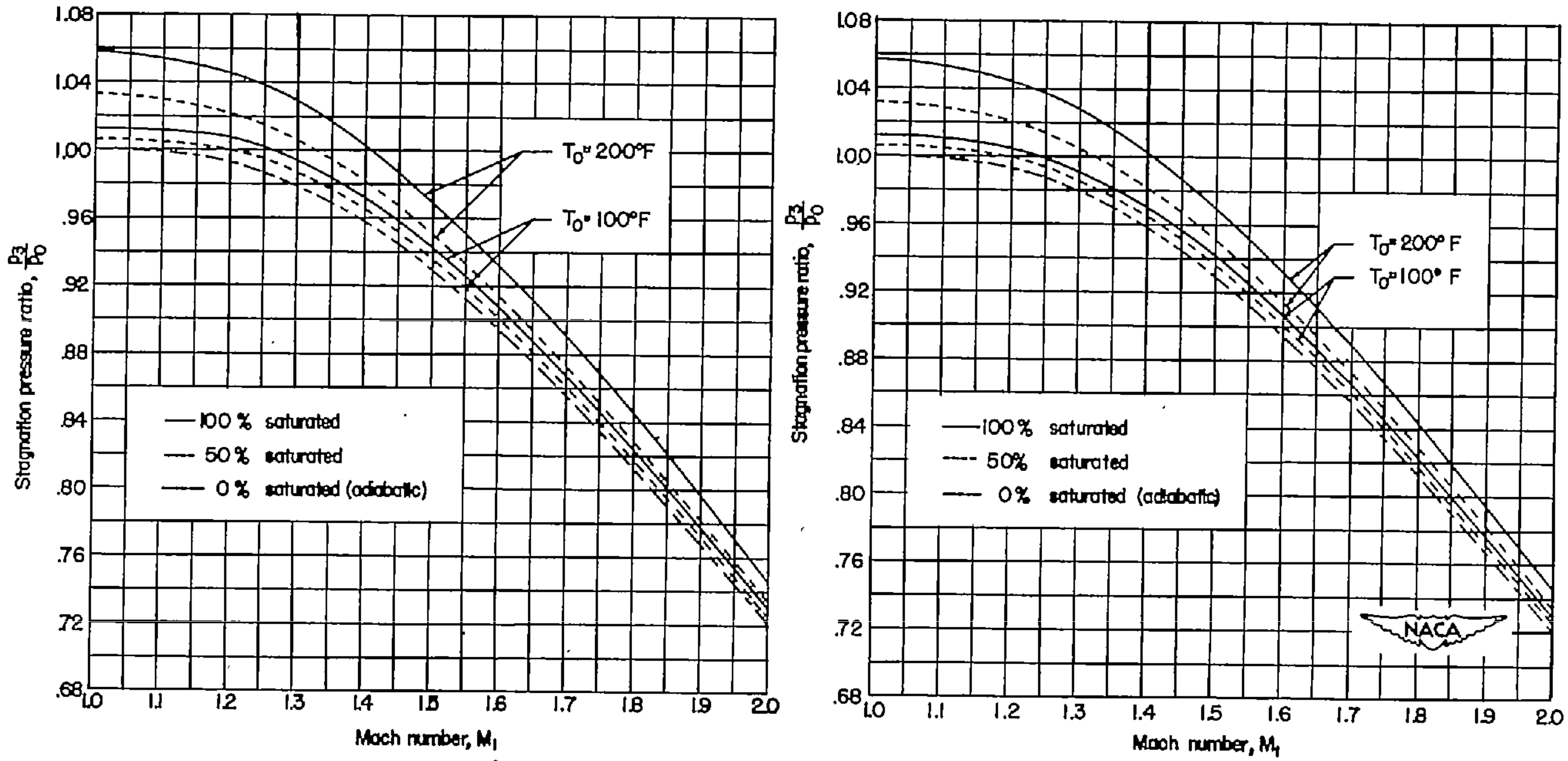


Figure 4.- Schematic arrangement of nonreturn supersonic tunnel with water injection.



(a) Dew point, -30° F.

(b) Dew point, 0° F.

Figure 5.- Stagnation pressure ratio as a function of Mach number for various amounts of water injection at stagnation temperatures of 100° F and 200° F and dew points of -30° F and 0° F. p_0 = 15 pounds per square inch.

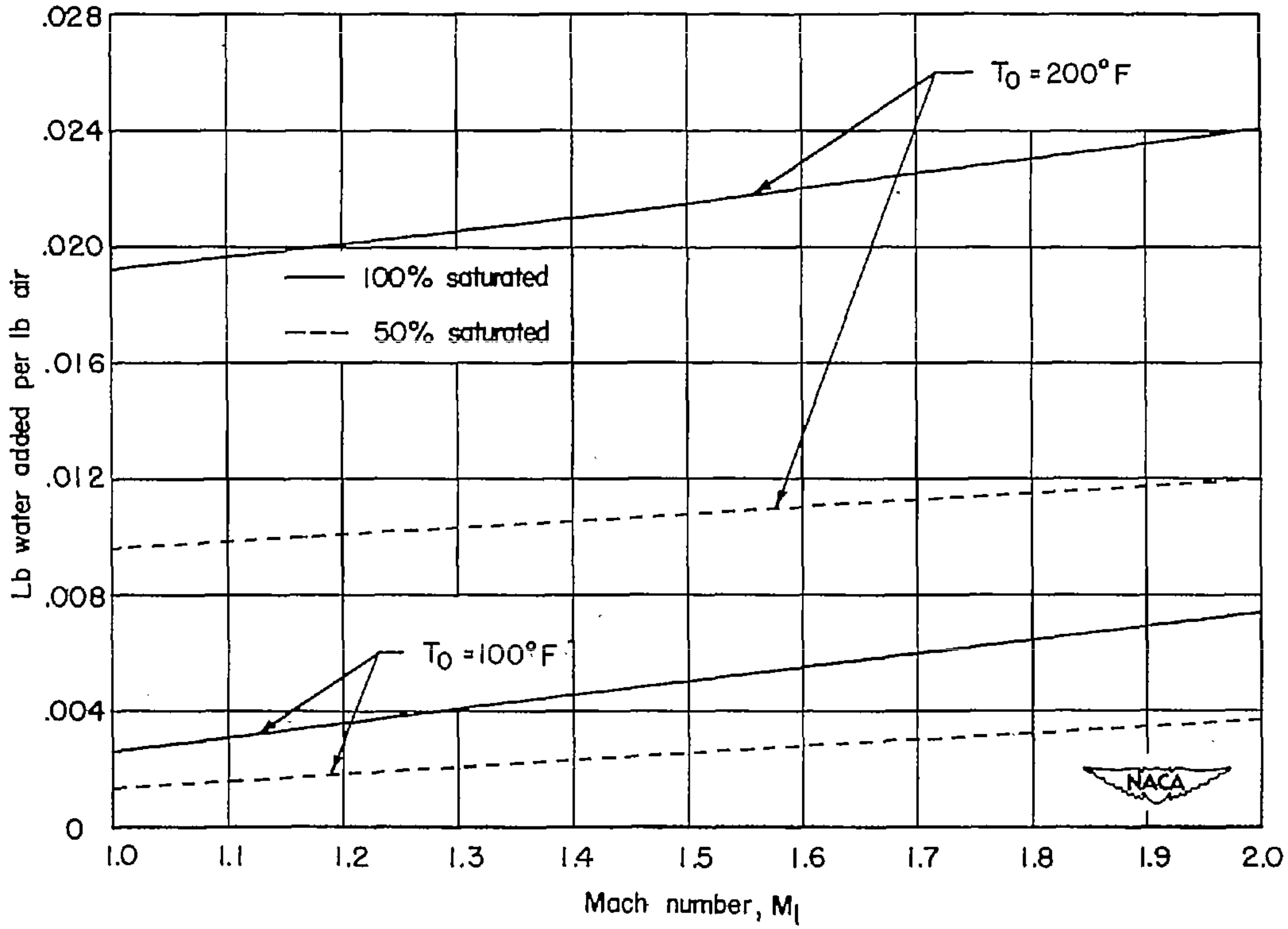


Figure 6.- Amount of water added as a function of Mach number for complete saturation and 50-percent saturation. Stagnation temperatures, 100° F and 200° F; dew point, 0° F; p_0 = 15 pounds per square inch.

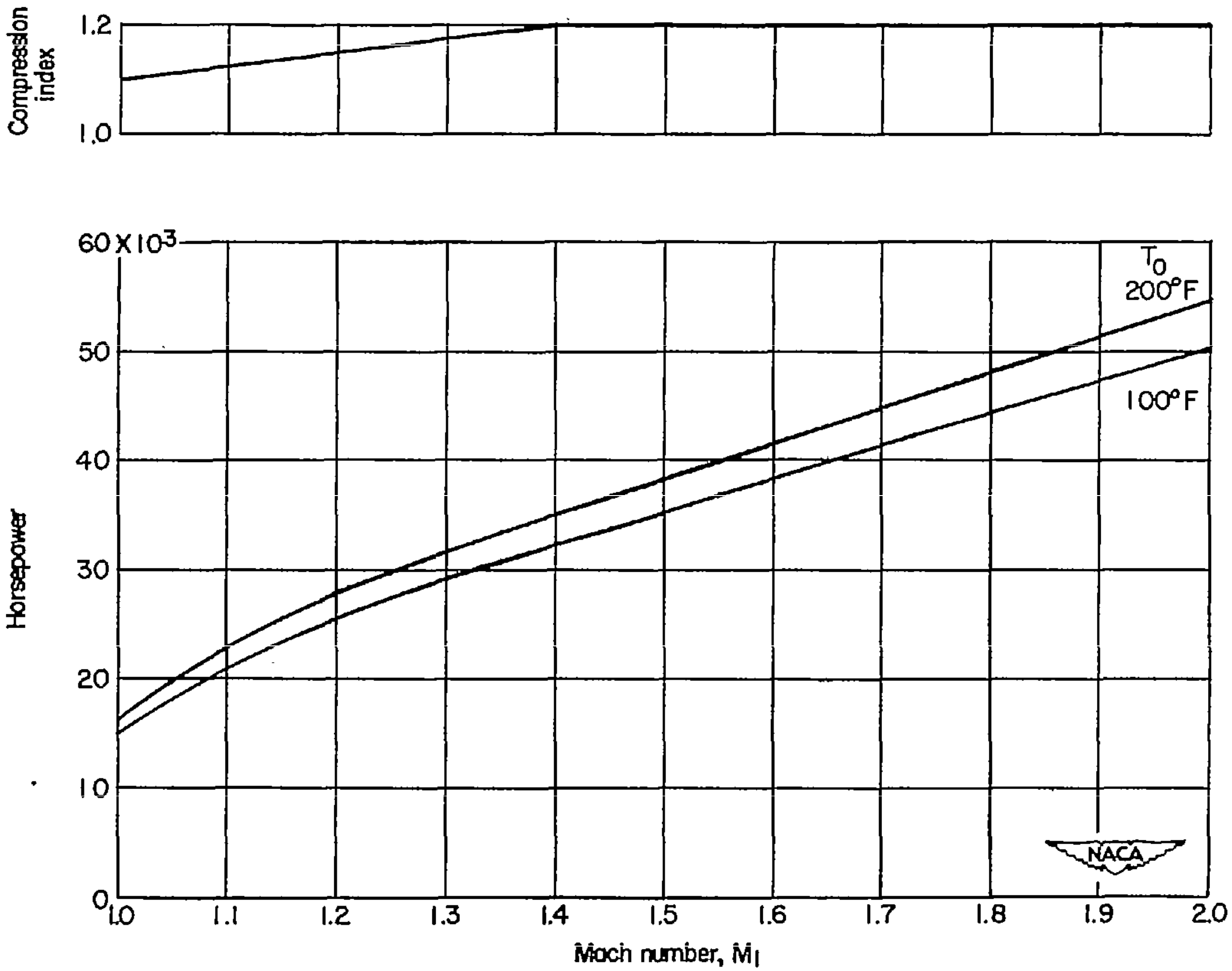


Figure 7.- Horsepower and compression index for assumed wind-tunnel configuration. p_0 = 15 pounds per square inch; test-section area, 50 square feet.

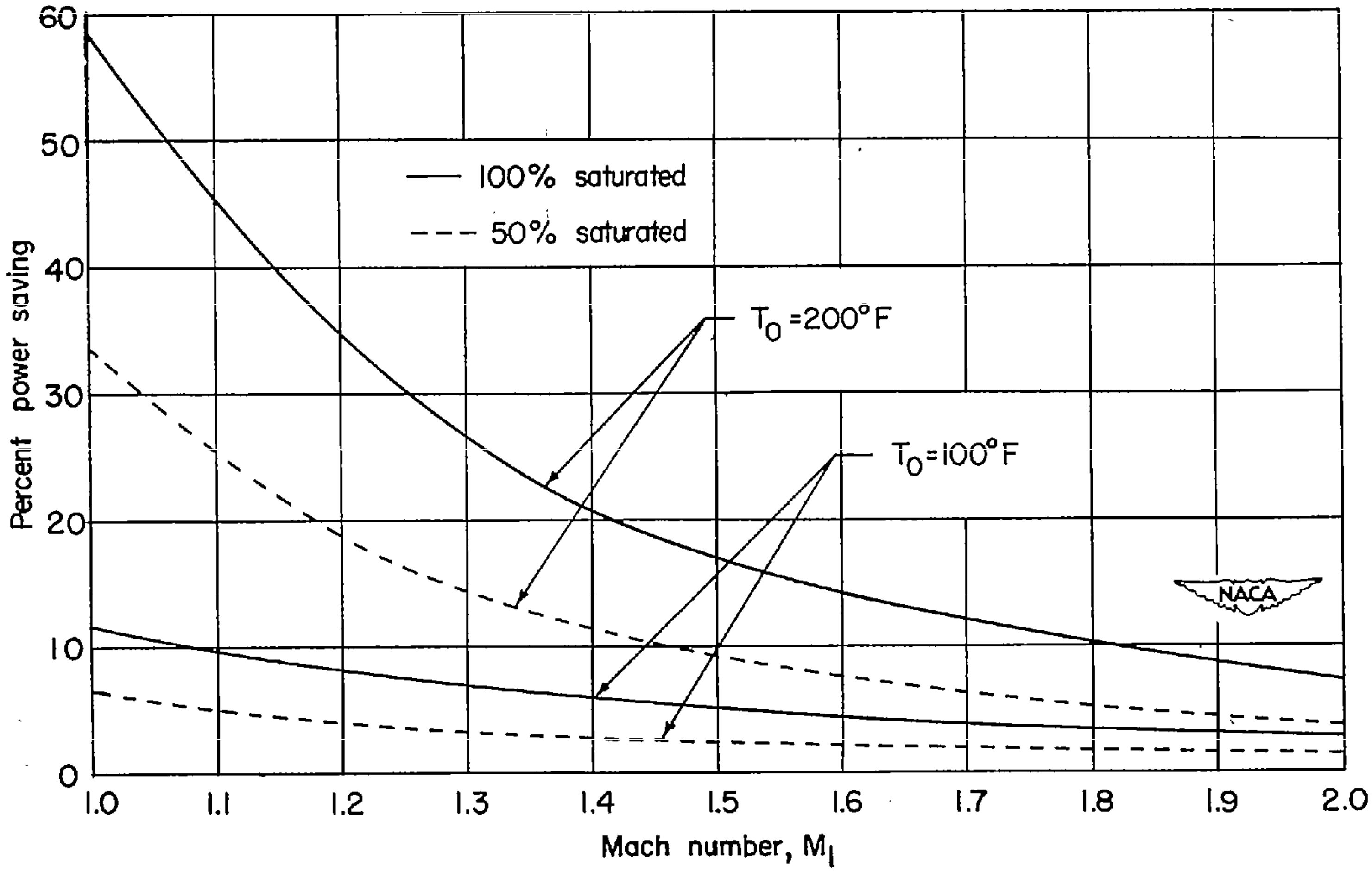


Figure 8.- Percent power savings as a function of Mach number for complete saturation and 50-percent saturation. Stagnation temperatures, 100° F and 200° F; dew point, 0° F; p_0 = 15 pounds per square inch.

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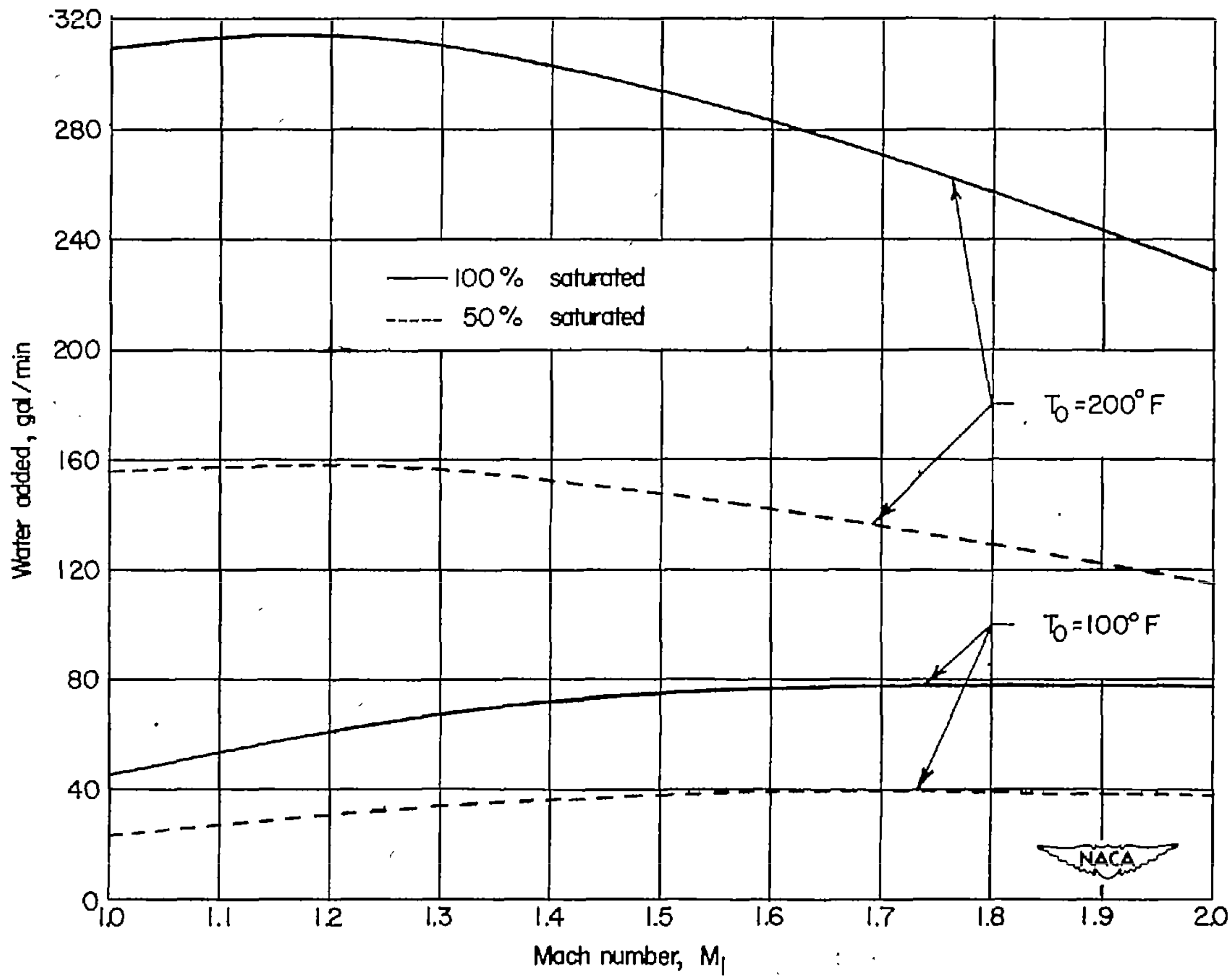

Figure 9.- Quantity of water added for assumed wind-tunnel configuration.