NACA TN 2938

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2938

ANALYSIS OF HEAT ADDITION IN A CONVERGENT-DIVERGENT NOZZLE

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Washington
April 1953

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SUMMARY

The effect of heat addition to a divergent stream with initially sonic flow is analyzed and the variation of exit Mach number, static pressure, and total pressure is presented. Application of these results to the diverging portion of a convergent-divergent nozzle indicated that nozzle heat addition delays nozzle overexpansion and affects the jet thrust appreciably. Moreover, misleading nozzle coefficients will be computed if heat addition in the nozzle is ignored.

INTRODUCTION

It has been shown (ref. 1) that an increase in jet thrust, over that achieved with a sonic outlet, can be realized by complete expansion of the exhaust gases in a convergent-divergent nozzle. In the calculation of this process and in the evaluation of experimental nozzle data the flow process in the nozzle is usually assumed to be adiabatic. However, for nozzles located directly downstream of combustion chambers (such as ram-jet, rocket, or turbojet afterburner nozzles), heat may be released to the working fluid during the expansion process in the divergent portion of the nozzle as a result of inefficient or delayed combustion. The purpose of this report is to treat analytically heat addition to a divergent stream with initially sonic flow and to employ the results thus obtained in evaluating the effect of delayed combustion on convergent-divergent nozzle performance. In this analysis, which was conducted at the NACA Lewis laboratory, use is made of the general equations developed in reference 2 for heat addition to a subsonic divergent stream.

SYMBOLS

The following symbols are used in this report:

A area, sq ft

F jet thrust, $mV + A(p - p_0)$, lb

K constant, defined in equation (3)

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M Mach number

m mass flow, ρAV, slug/sec

P total pressure, lb/sq ft

p static pressure, lb/sq ft

T total temperature, $^{\circ}F$ abs

V velocity, ft/sec

x constant, defined in equation (5)

γ ratio of specific heats

ρ static density, slug/cu ft

τ total-temperature ratio

Subscripts:

c combustion chamber

n diverging nozzle

0 free stream

1 intermediate combustion-chamber station

2 combustion-chamber outlet

3 nozzle exit

$*$ nozzle throat

Superscripts:

$'$ conditions for incremental heat addition prior to nozzle

ANALYSIS

A schematic diagram of the case to be considered, a combustion chamber followed by a convergent-divergent exit nozzle, is shown in figure 1. Temperature ratios of practical interest are considered and for analysis purposes all nozzle heat addition is assumed to occur in the diverging portion of the nozzle. That heat addition which occurs in the convergent portion of the nozzle is treated as part of the combustion-chamber temperature rise, thereby influencing slightly the total pressure available at the nozzle throat.

Assumptions

The following assumptions have been made in the analysis:

1. The flow can be considered as inviscid and one-dimensional.

2. The working fluid is a perfect gas with a ratio of specific heats constant at a value of 1.30. (A negligible effect on the computed results was observed by using constant γ values of 1.32 and 1.28. If, however, the value of γ changes during the process, the results would be somewhat affected. For the range of conditions considered, the value of γ increases during the expansion process. The inaccuracy introduced by the simplifying assumption of a constant γ was not considered of sufficient magnitude to justify additional refinement of the computations.)

3. The expansion process in the convergent portion of the nozzle is isentropic.

4. The nozzle throat is choked $(M_* = 1.0)$ at all times.

Heat Addition in the Divergent Portion of a Choked Convergent-Divergent Nozzle

An analysis of subsonic heat addition in a diverging channel has been made (ref. 2), and, in general, the equations developed therein also apply to the supersonic case. In the development of these equations, the differential forms of the equations for conservation of momentum and energy are used in conjunction with the equation of state for a perfect gas. The following expressions relating the changes in static pressure, area, total temperature, and Mach number can be obtained (see ref. 2):

$$\frac{dp}{p} = \frac{\gamma M^2}{1-\gamma M^2}\left(\frac{dA}{A} - \frac{dT}{T} + \frac{\frac{\gamma-1}{2}\, dM^2}{1 + \frac{\gamma-1}{2} M^2}\right) \tag{1}$$

$$\frac{dA}{A} = \frac{(1 + \gamma M^2)dT}{2T} - \frac{(1 - M^2)dM^2}{2M^2\left(1 + \frac{\gamma-1}{2} M^2\right)} \tag{2}$$

Simple integration of these equations is possible if it is assumed (as in ref. 2) that

$$\frac{dA}{A} = K \frac{dT}{T} \tag{3}$$

The integrated form of equation (3) is:

$$\frac{A_3}{A_*} = \left(\frac{T_3}{T_*}\right)^K = \left(\tau_n\right)^K \tag{3a}$$

The rate of heat addition may be changed by altering the constant K, the value of which is determined from equation (3a) by the temperature rise considered and the over-all change in flow area through which the temperature rise occurs. Equation (3) may not necessarily describe the actual way in which heat is added but was considered adequate for obtaining indications of trends in nozzle performance.

When equations (1), (2), and (3) are combined with the equation for conservation of mass, the following expressions relating the static pressure ratio, temperature ratio, and Mach number change across the diverging portion of the nozzle can be derived (see ref. 2):

$$\frac{p_3}{p_*} = \left(\frac{1 + \frac{\gamma-1}{2} M_*^2}{1 + \frac{\gamma-1}{2} M_3^2}\right)^x \left(\frac{1 - 2K + \gamma M_*^2}{1 - 2K + \gamma M_3^2}\right)^{1-x} \tag{4}$$

where

$$x = \frac{2K\gamma}{\gamma + 1 + 2K(\gamma-1)} \tag{5}$$

and

$$\tau_n = \left(\frac{p_3}{p_*}\right)^{\frac{2}{1-2K}} \left(\frac{M_3}{M_*}\right)^{\frac{2}{1-2K}} \left(\frac{1 + \frac{\gamma-1}{2} M_3^2}{1 + \frac{\gamma-1}{2} M_*^2}\right)^{\frac{1}{1-2K}} \tag{6}$$

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The ratio of total pressure at the two stations, * and 3, is also of interest and is found by expressing the total pressures in terms of the static pressures and Mach numbers:

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$$\frac{P_3}{P_*} = \left(\frac{p_3}{p_*}\right)\left[\frac{\left(1 + \frac{\gamma-1}{2} M_3^{\,2}\right)^{\frac{\gamma}{\gamma-1}}}{\left(1 + \frac{\gamma-1}{2} M_*^{\,2}\right)^{\frac{\gamma}{\gamma-1}}}\right] \tag{7}$$

For the special case wherein the rate of area change with heat addition is such as to maintain a constant Mach number throughout the heated region, the following expressions are obtained from equations (1) and (2):

$$\frac{A_{final}}{A_{initial}} = \left(\frac{T_{final}}{T_{initial}}\right)^{\frac{\gamma M^2+1}{2}} \tag{8}$$

$$\frac{p_{final}}{p_{initial}} = \frac{P_{final}}{P_{initial}} = \left(\frac{T_{final}}{T_{initial}}\right)^{-\frac{\gamma M^2}{2}} \tag{9}$$

Since K for this case becomes $(\gamma M^2+1)/2$, the value of the static pressure ratio p_3/p_* of equation (4) is indeterminate. The value of this ratio is obtainable, however, from equation (6) which for a constant Mach number reduces to equation (9).

If the initial Mach number is unity, and this Mach number is maintained throughout the divergent portion of the nozzle, equations (8) and (9) reduce to the following:

$$\frac{A_3}{A_*} = (\tau_n)^{\frac{\gamma+1}{2}} \tag{8a}$$

$$\frac{p_3}{p_*} = \frac{P_3}{P_*} = (\tau_n)^{-\frac{\gamma}{2}} \tag{9a}$$

For this limiting case where $M_3 = M_* = 1.0$, the value of K is 1.15 ($\gamma = 1.30$). For values of $K > 1.15$, the exit Mach number, as well as the Mach number at any point downstream of the throat, is supersonic and greater than M_*.

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Thrust Equations

Jet thrust is defined as

$$F = mV + A(p - p_0) \tag{10}$$

If equation (10) is expressed in terms of Mach number and total pressure, a ratio can be derived relating the jet thrust at the outlet of a convergent-divergent nozzle to the jet thrust at the nozzle throat. This latter value is the thrust for a sonic discharge. Thus,

$$\frac{F_3}{F_*} = \left(\frac{A_3}{A_*}\right)\left[\frac{\dfrac{P_3}{p_0}\dfrac{\left(1 + \gamma M_3^{\,2}\right)}{\left(1 + \dfrac{\gamma-1}{2}M_3^{\,2}\right)^{\frac{\gamma}{\gamma-1}}} - 1}{\dfrac{P_*}{p_0}\dfrac{\left(1 + \gamma M_*^{\,2}\right)}{\left(1 + \dfrac{\gamma-1}{2}M_*^{\,2}\right)^{\frac{\gamma}{\gamma-1}}} - 1}\right] \tag{11}$$

For $M_* = 1.0$ and complete isentropic expansion $(P_* = P_3,\ T_* = T_3,$ and $p_3 = p_0)$, equation (11) reduces to:

$$\frac{F_3}{F_*} = \left(\frac{A_3}{A_*}\right)\left[\frac{\gamma M_3^{\,2}}{\dfrac{P_*}{p_0}\dfrac{(1 + \gamma)}{\left(1 + \dfrac{\gamma-1}{2}\right)^{\frac{\gamma}{\gamma-1}}} - 1}\right] \tag{12}$$

For this process, the exit Mach number M_3 and the area ratio A_3/A_* are determined from the nozzle pressure ratio P_*/p_0 by making use of isentropic flow relations.

If, however, heat is released to the working fluid as it passes through the diverging section of the nozzle, the expansion process is no longer isentropic and it is necessary to solve equation (11) for the change in jet thrust. For given values of τ_n and A_3/A_*, a simultaneous solution of equations (3), (4), and (6) is required. Appropriate substitution in equations (7) and (11) then yields the desired thrust ratio.

Thus far, the incremental temperature rise has been considered as occurring supersonically in a diverging nozzle. It is also of interest to consider this incremental temperature rise as occurring subsonically ahead of the exhaust nozzle between stations 1 and 2. From the conservation of mass, equation (13) can be derived relating the conditions at the nozzle throat with and without the incremental combustion-chamber heat addition τ_c.

$$\frac{A_*'}{A_*} = \frac{p_*}{p_*'}\sqrt{\tau_c} \tag{13}$$

If equation (13) is used in conjunction with an expression similar to equation (11), a jet thrust ratio for a sonic outlet nozzle with and without the incremental τ_c occurring prior to the nozzle can be obtained:

$$\frac{F_*'}{F_*} = \sqrt{\tau_c}\left[\frac{(1+\gamma) - \frac{p_0}{P_*'}\left(1+\frac{\gamma-1}{2}\right)^{\frac{\gamma}{\gamma-1}}}{(1+\gamma) - \frac{p_0}{P_*}\left(1+\frac{\gamma-1}{2}\right)^{\frac{\gamma}{\gamma-1}}}\right] \tag{14}$$

It should be noted that the absolute value of the over-all τ at station 1 before the addition of the incremental τ_c does not influence the results obtained.

The solution of equation (14) for various values of τ_c requires that the reduction in total pressure at the nozzle throat $(P_*' < P_*)$ due to the incremental subsonic heat addition be evaluated. From the conservation of mass and momentum, the following equations were derived in a manner similar to that presented in reference 3:

$$\frac{P_*'}{P_*} = \frac{P_2}{P_1} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)\left[\frac{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}}}\right] \tag{15}$$

$$\frac{M_2^2\left(1 + \frac{\gamma-1}{2} M_2^2\right)}{(1 + \gamma M_2^2)^2} = \tau_c \frac{M_1^2\left(1 + \frac{\gamma-1}{2} M_1^2\right)}{(1 + \gamma M_1^2)^2} \tag{16}$$

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In the examples computed, M_1 was assumed equal to 0.5.

Equation (14) indicates the gains in thrust at a sonic throat due to the incremental heat addition made prior to the nozzle. If this incremental temperature rise is accompanied by isentropic expansion to ambient static pressure in a diverging nozzle (eq. 12), a further increase in thrust occurs. When this result is compared with the result from equation (11) for the same incremental τ, the effect of incomplete combustion with the resulting heat addition in the nozzle can be determined.

DISCUSSION OF RESULTS

Heat Addition to a Diverging Stream with Initially Sonic Flow

Presented in figures 2, 3, and 4 as a function of expansion ratio and total-temperature ratio are variation of exit Mach number, static pressure ratio, and total-pressure ratio for heat addition to a diverging stream with initially sonic flow. The trends are generally the same as for supersonic heat addition in a constant-area duct (ref. 4). For example, as shown in figure 2, the exit Mach number approaches unity as heat is added to the supersonic stream. However, increasing amounts of heat addition are required as the expansion ratio is increased in order to achieve an exit Mach number M_3 equal to 1.0.

The effect of this mode of heat addition on exit static pressure is shown by figure 3, which indicates that the addition of heat to a diverging supersonic stream causes an increase in the exit static pressure. Therefore, a convergent-divergent nozzle, when operating at the design pressure ratio for complete isentropic expansion, becomes underexpanded as heat is released in the divergent portion of the nozzle. Therefore, to prevent overexpansion when the nozzle is operating at lower than design pressure ratios, heat may intentionally be added in the diverging portion of a fixed-geometry nozzle.

As heat is added in the manner being considered, a loss in total pressure occurs (fig. 4) which is considerably greater than the loss accompanying subsonic heat addition. This loss in total pressure increases as the area ratio is increased because of the succeedingly higher Mach number at which the heat addition occurs.

The effect on jet thrust of expanding the flow from the sonic station and simultaneously adding heat during the expansion process is shown in figure 5. Presented (as determined by equation (11)) is the ratio of the jet thrust at the nozzle exit to the jet thrust at the throat F_3/F_*, for various temperature ratios τ_n and exit-to-throat area ratios A_3/A_*. Also included on the figure are lines indicating the pressure ratios P_*/p_0 required for complete expansion at the operating conditions being considered.

The gain in jet thrust due to expanding the flow isentropically to ambient static pressure as compared with sonic discharge is shown by the zero heat addition curve, $\tau_n = 1.0$. This is the conventional thrust gain usually computed for convergent-divergent nozzle performance and varies from approximately 4 percent at a pressure ratio of 5 to approximately 17 percent at a pressure ratio of 25. If, however, heat is released in the divergent portion of the nozzle (increasing values of τ_n), additional thrust gains result. At a τ_n of 1.20, for example, a thrust gain from 3 to 4 percent results over the area ratio range considered. It should be emphasized, however, that the thrust gains indicated on this figure due to increasing value of τ_n occur only because additional energy is being added to the working fluid and that the heat released prior to the nozzle remains unchanged.

The following example illustrates the use of figure 5: A nozzle with a pressure ratio $P_*/p_0 = 7.0$ and an expansion ratio $A_3/A_* = 1.70$ (point A) is considered. At these conditions the jet thrust has been increased 6 percent over that of a sonic outlet by complete isentropic expansion. If, however, additional energy is added to the working fluid in the form of heat released in the diverging section of the nozzle such that $\tau_n = 1.20$, the jet thrust can be increased 3.5 percent (point B) and the nozzle becomes underexpanded (fig. 3). In order to achieve complete expansion, the area ratio may be increased to a value of 2.13 (point C). At this point the jet thrust has been increased 13 percent over that of a sonic outlet (a gain of 7 percent over the isentropic expansion case) and the nozzle is at its optimum operating condition for the τ_n and the pressure ratio considered. The family of pressure ratio curves shown in figure 5 not only indicates the area ratio variation required to maintain complete expansion with nozzle heat addition at any given pressure ratio but also shows the maximum thrust possible as compared with a sonic outlet at each pressure ratio if additional energy is added to the system by releasing heat in the diverging portion of the nozzle.

The example shown in figure 5 also illustrates the use of nozzle heat addition to prevent overexpansion. A fixed geometry nozzle is considered to be designed for complete isentropic expansion at a pressure ratio of approximately 10 and an area ratio of 2.12 (point D).

If the pressure ratio is reduced to 7.0 (as by a decrease in altitude) and since the nozzle area ratio is fixed at 2.12, the nozzle would be overexpanded if the flow were expanding isentropically. However, if heat were intentionally added in the nozzle so that $\tau_n = 1.20$, the static pressure at the exit would be increased to ambient static pressure so that complete expansion would then exist (point C).

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At any given pressure ratio, the jet thrust ratio increases essentially linearly with τ_n. Also, for a given value of τ_n the rate of the increase in thrust ratio decreases as the nozzle pressure ratio is raised. In addition, a larger area ratio change is required to maintain complete expansion at the higher pressure ratios than at the lower pressure ratios.

Thrust Comparison Between Nozzle and Combustor Heat Addition

A comparison on a jet thrust basis between heat addition in the diverging section of a nozzle and in the combustor prior to the nozzle is presented in figure 6. The results are presented in terms of a ratio between the jet thrust with incremental heat addition and the jet thrust with a sonic outlet and no incremental heat addition. Although only results for a nozzle pressure ratio P_1/p_0 ($P_1 = P_*$ for $\tau_c = 1.0$) of 10 are shown, similar trends were found for all the pressure ratios considered.

Case I in figure 6 indicates the gains in thrust possible if the additional energy were added in the form of heat released prior to the nozzle and the flow expanded isentropically to ambient static pressure through the nozzle. If the additional energy were added in the diverging portion of a convergent-divergent nozzle designed for complete isentropic expansion (T = constant), the variation in thrust would be as indicated by case II. Since subsonic heat addition is more efficient than supersonic heat addition, the gains in thrust for the same energy input are greater for case I than for case II. However, this difference can be appreciably reduced if the area ratios are increased to maintain complete expansion as heat is added in the nozzle (case III).

Results such as presented in figure 6 have been used to determine the effect of incomplete combustion on jet thrust. In the usual theoretical jet thrust calculations it is assumed that all the energy has been added prior to the nozzle and that the nozzle expands the flow isentropically to ambient static pressure. Differences between measured and theoretical values of thrust are then usually ascribed to inefficiencies in the expansion process and are presented in the form of nozzle coefficients. However, if all of the heat released by combustion is not added to the flow in the combustion chamber, but instead a portion of it is released in the divergent portion of a convergent-divergent exit nozzle (as by delayed combustion), the usual nozzle coefficients do not adequately describe the efficiency of the nozzle.

An example of the difficulty encountered in assigning coefficients to nozzles used with ram jets, afterburners, and rockets is indicated in figure 7. The jet thrust was computed for an ideal frictionless convergent-divergent exit nozzle with an area ratio corresponding to complete expansion with zero nozzle heat addition at each pressure ratio considered. It was assumed that in one case F_3, an incremental amount of the total-temperature rise across the engine, occurred in the diverging portion of the nozzle. In the other case computed, F_3', this increment of heat addition was assumed to occur ahead of the nozzle throat. The ratio of these two jet thrusts indicates the effect of delayed combustion and is presented as a function of the incremental total-temperature ratio and nozzle pressure ratio (fig. 7). It is apparent that if part of the temperature rise occurs in the nozzle instead of in the combustion chamber, the jet thrust may be reduced considerably. This thrust loss may be erroneously assigned as exit nozzle inefficiency and the nozzle given misleading coefficients.

SUMMARY OF RESULTS

An analysis of heat addition to a diverging stream with initially sonic flow yields the following:

1. Heat addition in the diverging portion of a convergent-divergent nozzle appreciably influences the jet thrust and must be considered when evaluating nozzle performance.

2. As heat is added to a diverging stream with initially sonic flow, the exit Mach number (which is supersonic) approaches unity, the exit static pressure increases, the exit total pressure decreases, and the total momentum at the exit increases.

3. Although heat addition in the diverging portion of a convergent-divergent nozzle produces an increase in jet thrust, the increase is greater if the same heat is added prior to a completely expanding nozzle.

4. Overexpansion of a convergent-divergent nozzle may be reduced by the addition of heat in the divergent portion of such a nozzle.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, February 24, 1953

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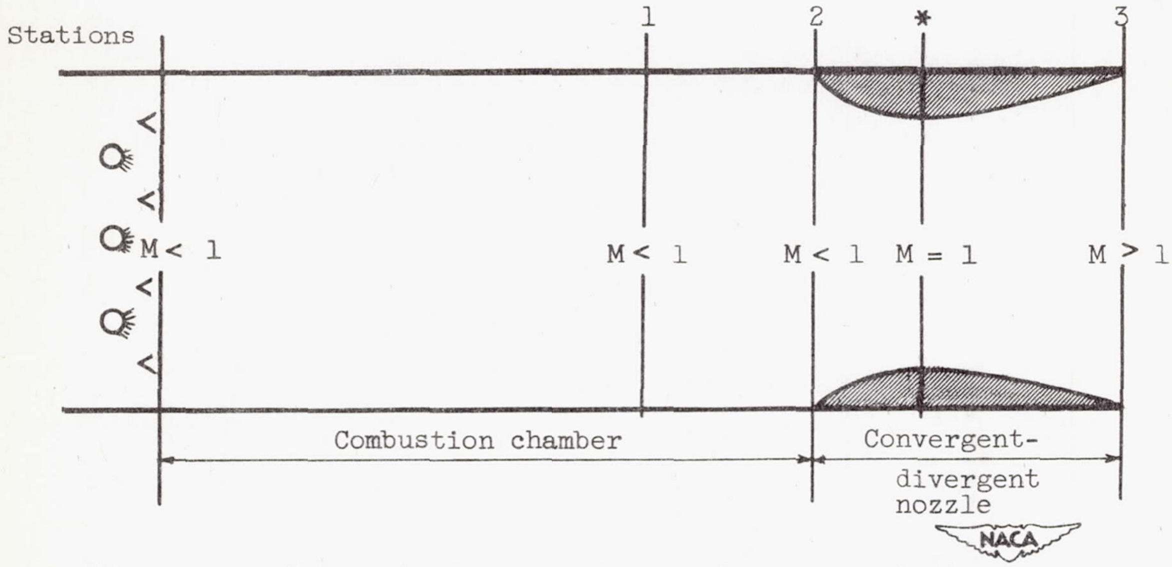

Figure 1. - Schematic diagram of typical convergent-divergent nozzle application.

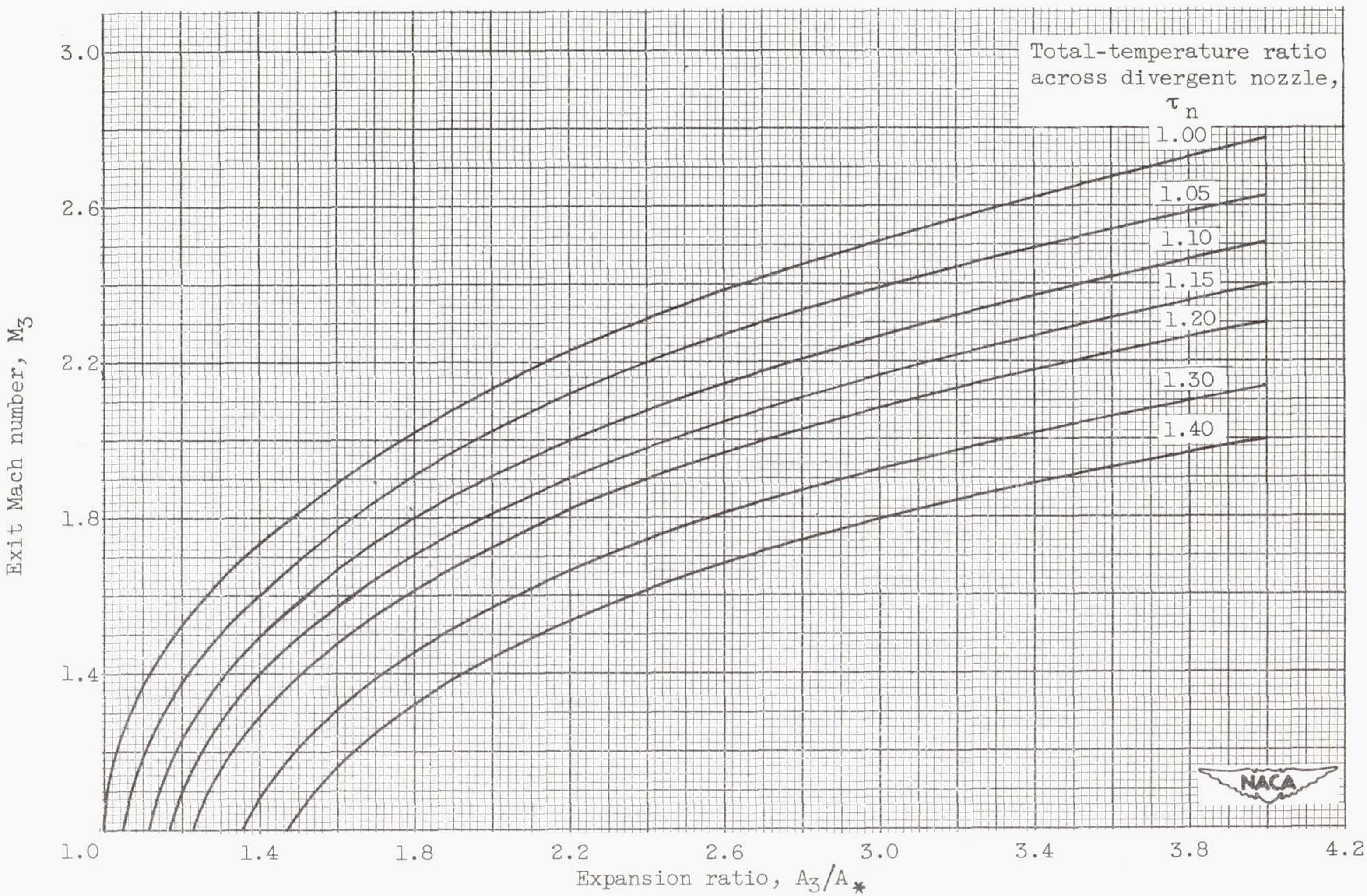

Figure 2. - Variation of exit Mach number with expansion ratio for various amounts of supersonic heat addition.

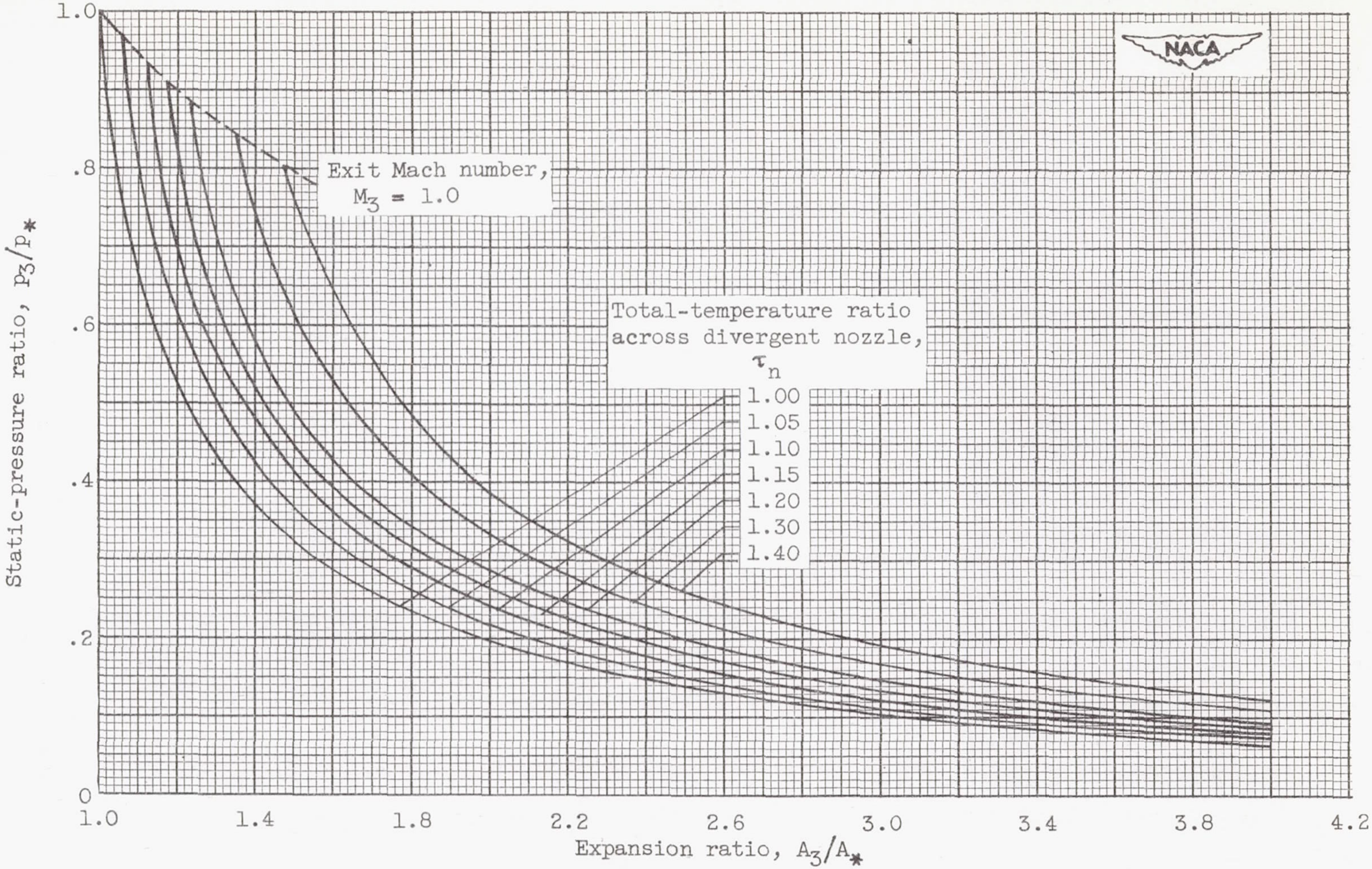

Figure 3. - Variation of static-pressure ratio with expansion ratio for various amounts of supersonic heat addition.

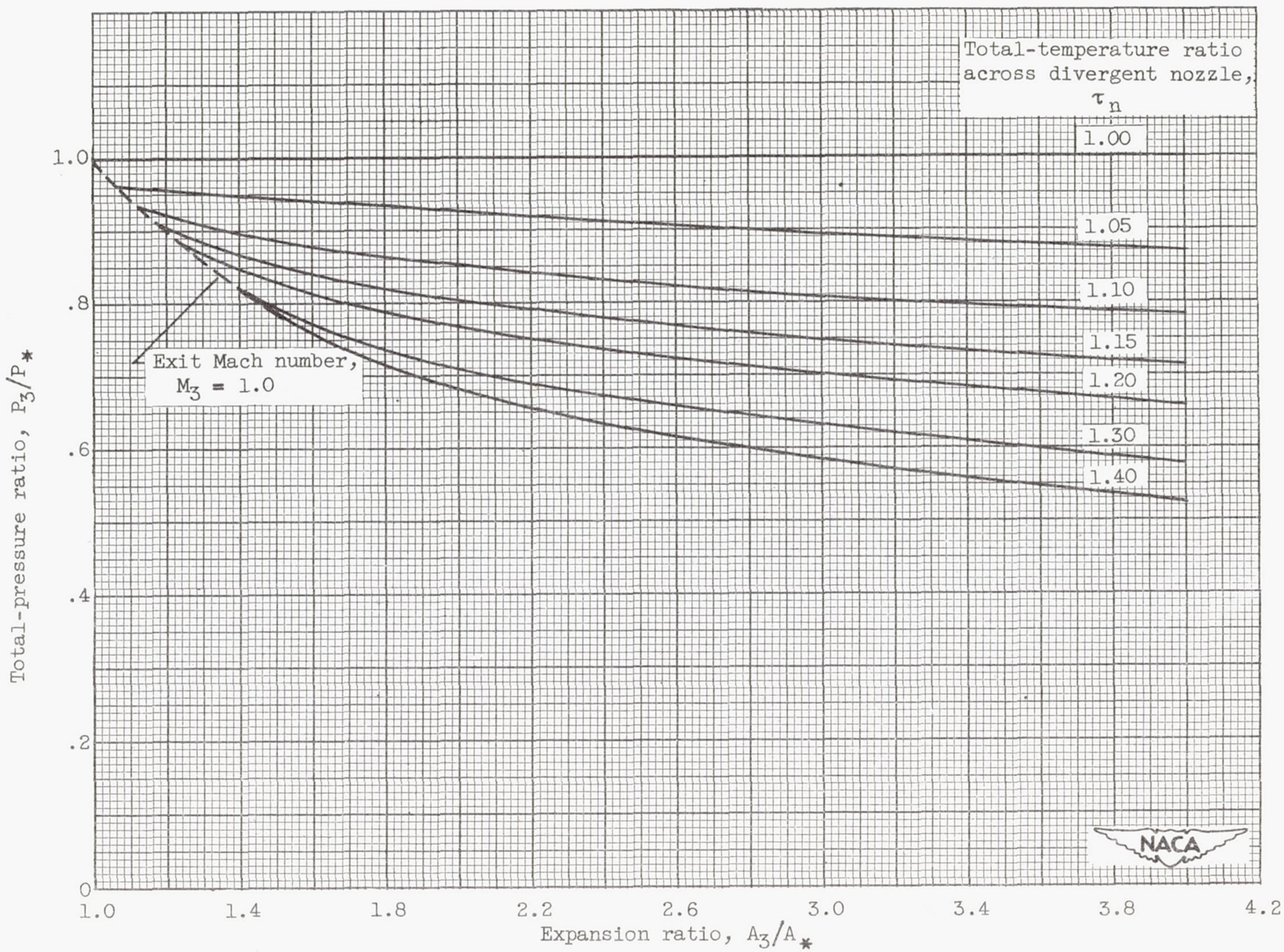

Figure 4. - Variation of total-pressure ratio with expansion ratio for various amounts of supersonic heat addition.

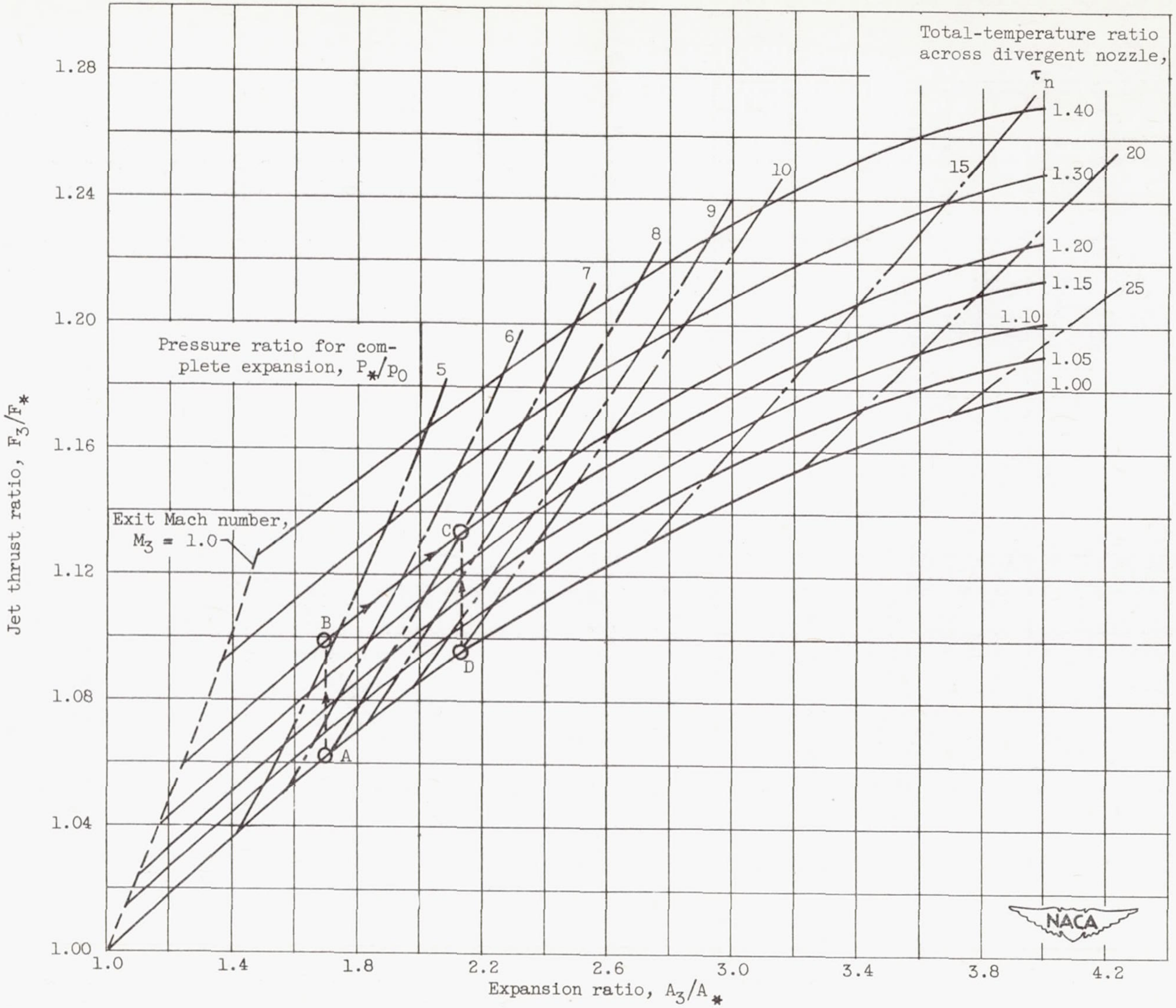

Figure 5. - Variation of jet thrust with expansion ratio for various amounts of supersonic heat addition.

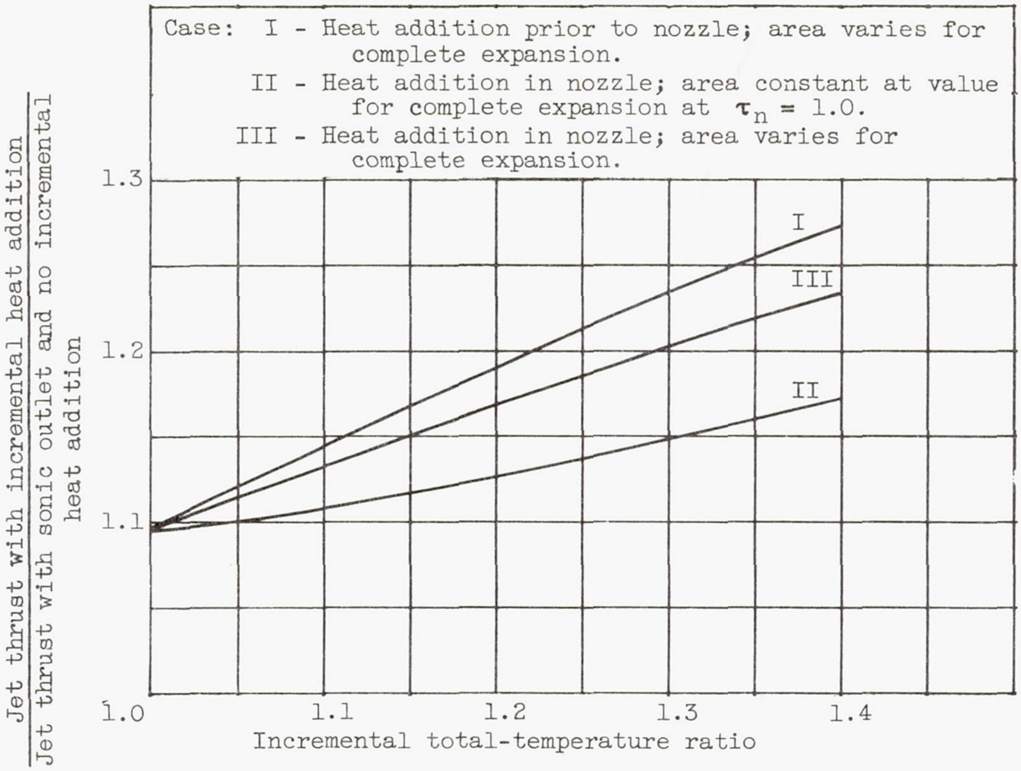

Figure 6. - Thrust comparison of several methods of heat addition and flow expansion. $P_1/p_0 = 10.0$.

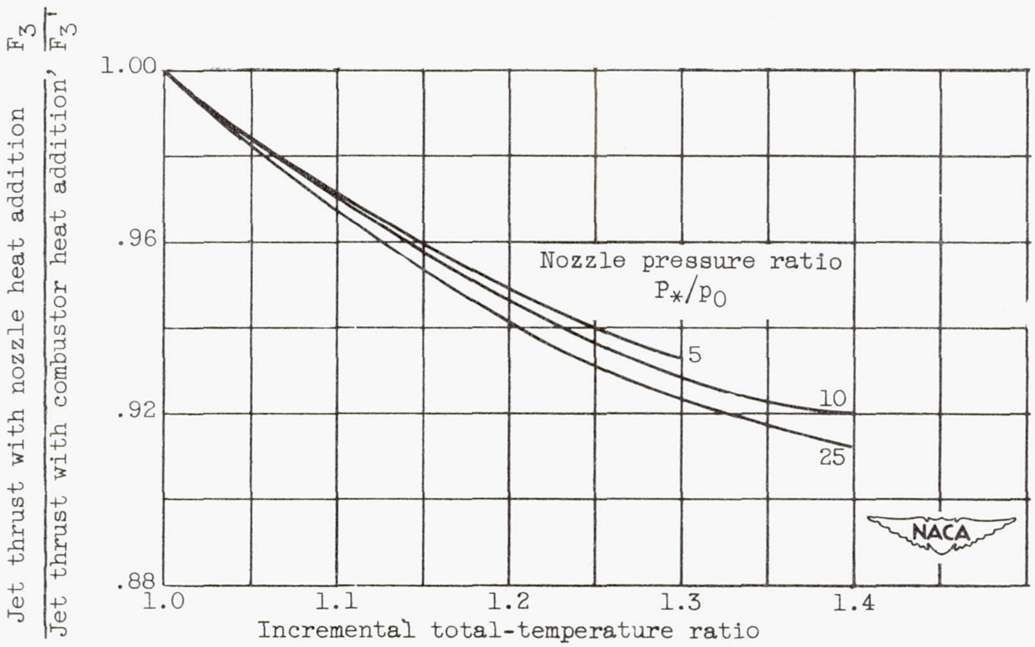

Figure 7. - Effect of supersonic heat addition on jet thrust of ideal nozzle.

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