NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2882

THEORETICAL INVESTIGATION OF THE LONGITUDINAL RESPONSE CHARACTERISTICS OF A SWEPT-WING FIGHTER AIRPLANE HAVING A PITCH-ATTITUDE CONTROL SYSTEM

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SUMMARY

A theoretical analysis is made of a pitch-attitude control system as applied to a swept-wing fighter airplane. The system is investigated both with and without pitch-rate feedback. The effects that changes in altitude and Mach number have on the response characteristics of the airplane-autopilot combination are investigated, as are the effects of changes in the rate and error gain settings of the system.

The results are discussed on the basis of the characteristics of the frequency and transient responses in pitch attitude, such as the frequency at which the peak in the amplitude response occurs, the time for an error to reduce to and stay within 5 percent of the original command increment, and the cycles to damp to one-half amplitude. Also discussed are the variations of elevator deflection, normal acceleration, and flight path encountered at various gain settings and flight conditions when an approximate step command in pitch attitude is impressed on the system.

The primary conclusions reached in this theoretical investigation are that (1) the airplane-autopilot combination incorporating pitch-attitude and pitch-rate feedback can be made to perform well as far as attitude response is concerned, for the flight conditions investigated, provided some means is available for changing the gain settings, (2) with the gain settings which give the best response characteristics the elevator deflections encountered for the high-altitude conditions would be very large, and the magnitude of these gain settings may be limited by factors not considered in this paper, and (3) the low levels of normal acceleration sustained after an approximate step command in pitch attitude would indicate that the pitch-attitude autopilot is not particularly suited to tight control of the flight path.

INTRODUCTION

The addition of an autopilot to an airplane introduces many possible variations in the manner in which the airplane motions and loads can be controlled. The National Advisory Committee for Aeronautics has undertaken a theoretical investigation, the general purposes of which are (1) to study the response characteristics of an airplane having various autopilots in order to determine the effects of the basic types of longitudinal stabilization, such as pitch-attitude, normal-acceleration, and angle-of-attack, (2) to determine the effects of change of altitude and Mach number on the response characteristics of the various systems, and (3) to determine the effects of changing the gain settings on the response characteristics of the various systems. In the present paper pitch-attitude stabilization and control is investigated, and the effects on the performance of the airplane-autopilot combination of changing the flight conditions of the airplane and the gain settings in the system are discussed.

The types of controls analyzed herein incorporate pitch-attitude feedback alone and pitch-attitude plus pitch-rate feedback. The gain settings associated with these two types of controls were initially determined by a well-known technique which involves adjusting the peak magnification of the closed loop to a specified value. The gain settings then were altered to find the effects on the performance of the system.

The results presented are discussed on the basis of the characteristics of the frequency and transient responses in pitch attitude, such as the frequency at which the peak amplitude response occurs (hereinafter called the peak frequency), the time for an error to reduce to and stay within 5 percent of the original command increment (hereinafter called the response time), and the cycles to damp to one-half amplitude. Also discussed are the variations of elevator deflection, normal acceleration, and flight path encountered at various gain settings and flight conditions when an approximate step command in pitch attitude is impressed on the system.

SYMBOLS

α angle of attack, radians

γ angle of flight path with horizontal, radians

δ elevator deflection, positive when trailing edge is up, deg

ϵ error signal, $\theta_i - \theta_o$

ϵ_1	autopilot input
θ	angle of pitch, radians
Λ	angle of sweepback of 0.25-chord line, deg
Φ	phase angle, deg
μ	relative-density factor, $m/\rho S\bar{c}$
ρ	atmospheric density, slugs/cu ft
ω	circular frequency, radians/sec
ω'	nondimensional circular frequency, $\frac{\bar{c}}{V}\omega$
ω_p	frequency at which the peak in amplitude response occurs, radians/sec
A	aspect ratio
$C_{1/2}$	cycles for oscillations to reach one-half amplitude
D	nondimensional differential operator, $\frac{\bar{c}}{V}\frac{d}{dt}$
I_Y	moment of inertia about Y-axis, slug-ft^2
K_δ	autopilot gain setting for position signal fed back from control surface
K_ϵ	pitch-attitude error-signal gain setting
K_x	autopilot gain setting for position signal fed back from pilot piston
K_Y	nondimensional radius of gyration about lateral stability axis
K_R	rate-signal gain setting, $\frac{\text{radians}}{\text{radian/sec}}$
L	lift, lb
M	Mach number; pitching moment in figure 1

S	wing area, sq ft
$T_{5\%}$	time for an error to reduce to and stay within 5 percent of the command increment, sec
V	airspeed, ft/sec
W	airplane weight, lb
X	longitudinal axis of reference fixed in airplane
Y_1	autopilot transfer function
Y_2	airplane transfer function relating angle of pitch to elevator deflection
Z	normal axis of reference fixed in airplane
b	wing span, ft
$\bar{c}$	mean aerodynamic chord, ft
g	acceleration due to gravity, ft/sec^2
h_p	pressure altitude, ft
$j = \sqrt{-1}$	
l	tail length, measured from $0.25\bar{c}$ of the wing to $0.25\bar{c}$ of the tail, ft
m	mass of airplane, slugs
n	normal acceleration, positive up, g units
q	$D\theta$; dynamic pressure, $\frac{1}{2}\rho V^2$, lb/sq ft
t	time, sec
C_L	lift coefficient, L/qS
C_{L_α}	rate of change of lift coefficient with angle of attack, per radian
$C_{L_{\alpha t}}$	rate of change of lift coefficient with angle of attack of tail, per radian

C_{L_q} rate of change of lift coefficient with pitching velocity

$C_{L_{D\alpha}}$ rate of change of lift coefficient with rate of change of angle of attack

C_{L_δ} rate of change of lift coefficient with elevator deflection, per radian

$C_{m_{C_L}}$ rate of change of pitching-moment coefficient with lift coefficient

C_{m_α} rate of change of pitching-moment coefficient with angle of attack, per radian

$C_{m_{\alpha_t}}$ rate of change of pitching-moment coefficient with angle of attack of the tail, per radian

C_{m_q} rate of change of pitching-moment coefficient with pitching velocity

$C_{m_{D\alpha}}$ rate of change of pitching-moment coefficient with rate of change of angle of attack

C_{m_δ} rate of change of pitching-moment coefficient with elevator deflection, per radian

$d\epsilon/d\alpha$ rate of change of downwash with angle of attack

$\left|\theta_o/\theta_i\right|$ absolute value of amplitude ratio

Subscripts:

i input

o output

ss steady state

ANALYSIS

The analysis throughout this paper was made by conventional techniques in which the concept of the transfer function was utilized.

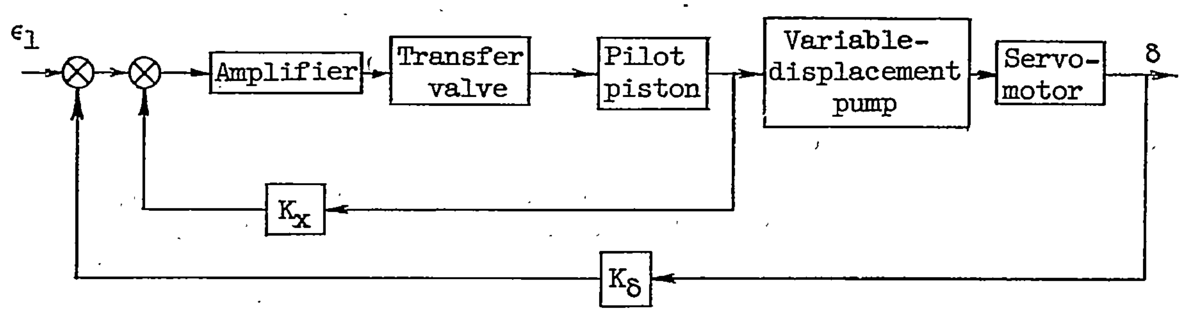
Airplane Transfer Function

The transfer function of the airplane relating angle of pitch to elevator deflection was obtained from an "equation of motion" type of analysis by use of stability derivatives estimated from theory, wind-tunnel data, and flight-test data. The various transfer functions used in this analysis are presented in the appendix. The system of axes and sign conventions used herein is presented in figure 1. In the analysis the degree of freedom involving changes in longitudinal velocity was neglected, as this paper is concerned primarily with short-period command characteristics.

Since the coefficients of the transfer function vary with airspeed, altitude, Mach number, and other conditions, it is necessary to study the control characteristics for flight conditions that represent the normal speed and altitude range of the airplane being considered. The four conditions selected were: condition I, $M = 0.5$ and $h_p = 35,000$ feet; condition II, $M = 0.7$ and $h_p = 0$; condition III, $M = 0.7$ and $h_p = 35,000$ feet; and condition IV, $M = 0.9$ and $h_p = 35,000$ feet. These flight conditions, the basic airplane dimensions, and the corresponding airplane stability parameters are presented in table I. Figure 2 shows the airplane frequency-response curves for the flight conditions investigated. These curves are in fairly good agreement with similar curves subsequently obtained from actual flight tests.

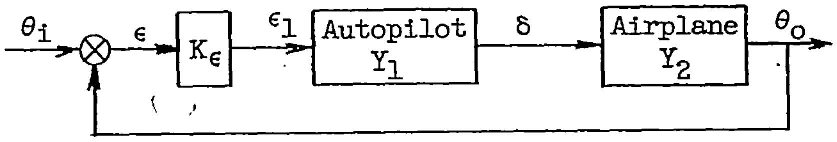
Autopilot Transfer Function

The frequency response of the autopilot used throughout this analysis was obtained experimentally from a bench setup of a special autopilot system whose characteristics made it suitable for use in a high-speed fighter airplane. The autopilot considered had essentially constant amplitude-response characteristics up to a frequency of about 6 cycles per second. The frequency response of this system is shown in figure 3 and a block diagram of the system is as follows:



Airplane-Autopilot Combination Without Rate Feedback

A block diagram of the airplane-autopilot combination without rate feedback is as follows:



The open-loop transfer function, from which the pitch-attitude response characteristics may be obtained, appears symbolically as

$$\frac{\theta_o}{\epsilon} = K_\epsilon Y_1 Y_2$$

Dynamically, the feedback element which measures pitch attitude was assumed to have an amplitude ratio of 1 and zero phase lag.

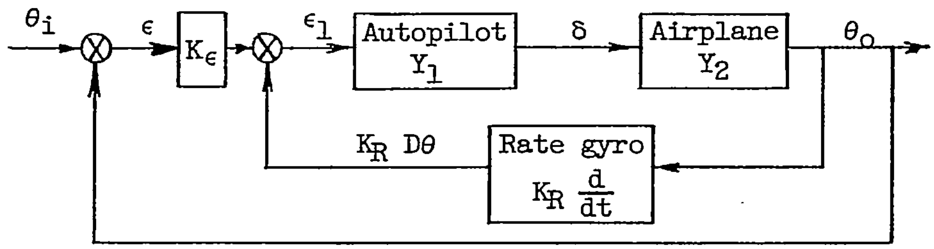
In servomechanism synthesis, a peak magnification of the closed-loop frequency response ranging from 1.2 to 1.6 usually gives the best response characteristics (ref. 1, p. 107) for a given system, although this does not necessarily mean that the system is satisfactory. In this analysis the error gain setting K_ϵ producing a peak magnification of approximately 1.2 was determined graphically in the manner described in reference 1, pages 185 to 188. With this gain setting and the known transfer function of both the airplane and the autopilot, the closed-loop frequency response of the system was calculated and plotted. The closed-loop frequency response is given by the expression

$$\frac{\theta_o}{\theta_i} = \frac{K_\epsilon Y_1 Y_2}{1 + K_\epsilon Y_1 Y_2}$$

The transient-response characteristics of the system were determined from the frequency response with the aid of a Fourier synthesizer of the type described in reference 2. The input quantity, pitch-attitude command θ_i, was in the form of an approximate square wave utilizing the first 24 harmonics of a Fourier series, as shown in figure 4. Because of the poor response characteristics for the system without rate feedback, variations of K_ϵ from the values giving a peak magnification of 1.2 were not investigated.

Airplane-Autopilot Combination With Rate Feedback

It was assumed that a rate gyro would supply perfectly the additional derivative signal. When the rate gyro is placed in the feedback loop of the airplane-autopilot combination the block diagram of the system is altered as follows:



The open-loop transfer function now appears symbolically as

$$\frac{\theta_o}{\epsilon} = \frac{K_\epsilon Y_1 Y_2}{1 + K_R Y_1 Y_2 D}$$

The procedure used in establishing the gearing constants for this system was as follows: At each flight condition investigated the gain setting K_R was determined so that the peak magnification of the inner loop was adjusted to a value of 1.2, and then the error gain setting K_ϵ was determined in a similar manner so that the peak magnification of the outer loop was adjusted to approximately 1.2. The effects of changing K_R and K_ϵ were also investigated. The rate-signal gain setting K_R was increased by 25 and 50 percent and decreased by 50 percent of its original value, and K_ϵ was changed simultaneously to give a peak magnification of the outer loop of 1.2, on the assumption that this value closely approximates the optimum value for the outer loop. In this manner the trend of the frequency-response curves could be seen. In order to verify that a peak magnification of 1.2 more closely approximates the optimum value for the outer loop, calculations were also made with values of K_ϵ that would give a peak magnification of 1.6. In addition to varying the gain settings for each flight condition, the effect of holding the system gains constant while varying the flight conditions was investigated. Initially the gain settings which gave the best results at the altitude cruising condition, condition III, were used, and finally the gain settings which gave the best results at the sea-level condition, condition II, were used.

The Fourier synthesizer was also employed to establish the transient-response characteristics for the cases of the autopilot with rate feedback.

The response characteristics that were used to evaluate the various conditions were (1) the peak frequency ω_p, (2) the response time $T_{5\%}$, and (3) the number of cycles to damp to one-half amplitude $C_{1/2}$. In addition to the transient responses in pitch attitude, calculations were made of the variations of elevator deflection needed to produce these responses, the variation of the normal acceleration imposed upon the airframe, and the response in flight path when the system was subjected to the pitch-attitude command. Expressions for the transfer functions δ/θ_i, n/θ_i, and γ/θ_i are presented in the appendix. These relations were employed to determine the associated transient-response characteristics through use of the Fourier synthesizer, as described in reference 2. This procedure was applied at all four flight conditions when the best gains obtained at each condition were used, when the gains were held constant throughout all conditions at the best value obtained for the high-altitude cruising condition, and when the gains were held constant throughout all conditions at the best value for the sea-level condition.

RESULTS AND DISCUSSION

Airplane-Autopilot Combination Without Rate Feedback

The best frequency and transient responses obtained for the system without rate feedback at all four flight conditions are shown in figure 5. For comparison, the approximate square-wave input produced by the synthesizer also is plotted. For simplicity, this input curve in figure 5 and subsequent figures was smoothed so that the ripples existing in the actual input would not appear. The fairing of this curve was justified because the frequency of the ripple was high enough to have no significant effect on the output. The small ripples were caused by the limited number of harmonics utilized by the Fourier synthesizer. In addition, the parameters ω_p, $T_{5\%}$, and $C_{1/2}$ associated with each transient and frequency response are tabulated in figure 5.

It is obvious from figure 5 that in all four conditions the dynamic characteristics of the airplane-autopilot combination would be undesirable because of the long time required for the airplane to respond to an applied command and the very low damping of the system. For a particular condition, better damping could be obtained at the expense of a more sluggish response by decreasing K_ϵ in cases where a still slower response can be tolerated.

Improvement in the transient response could be effected by increasing the magnification of the low-frequency end of the frequency-response curves without increasing the peak magnification of the curves. As is well-known, the addition of derivative feedback would accomplish this purpose.

Airplane-Autopilot Combination With Rate Feedback

The results obtained with rate feedback incorporated for the four flight conditions are presented in figures 6 to 9. By comparing these figures with figure 5, it can be seen that the incorporation of rate feedback improved the system considerably, and the results presented herein indicate that the present airplane-autopilot combination incorporating rate feedback can be made to have a very rapid attitude response, provided some means of changing the gain settings is available. In fact, a comparison of figures 6 to 9 with figure 2 would indicate that in some cases the natural frequency of the airplane-autopilot combination can be made 10 times that of the basic airplane without loss of adequate damping.

Effects of Varying Gain Settings

In figures 5 to 9, the original gain settings were determined by adjusting the gain settings of both the inner and the outer loop for a peak magnification of approximately 1.2. This case corresponds to the curves in the second row from the top of each figure. For the purpose of determining the effects of variations in the gain settings at each condition, when the peak magnification of the outer loop was maintained at approximately 1.2, the original values of rate gain were (1) decreased 50 percent, (2) increased 25 percent, and (3) increased 50 percent. The results for these settings are also presented in each figure. As these gains were altered, the trends of the frequency-response curves could be observed. For perfect following, the amplitude response would have a value of unity from zero to infinite frequency and the phase-angle curve would be zero for all values of ω. This result obviously is impossible to achieve, but any modification which serves to increase the peak frequency ω_p or raise the low-frequency dip in the frequency-response curve without encountering high peak magnifications makes possible a closer approach to this ideal curve. Similarly, as perfect following would involve no phase lag, any modification that generally reduces the phase-angle variation with frequency would improve the response.

To a limited extent, increasing the rate gain made possible an increase in the error gain without an increase in the peak magnification; thereby the peak frequency was increased and the low-frequency dip was raised. These adjustments, therefore, afford further increases in the performance of the system. As the rate gain settings were increased beyond a certain value, the inner loop containing the rate feedback began to approach instability, and, as indicated in figures 6 to 9, further increases in rate gain did not afford improved performance.

The response time was not significantly altered by increasing the rate gain 50 percent. The response time for condition IV (high altitude and a Mach number of 0.9) was increased appreciably when the rate gain was reduced 50 percent, but for the other flight conditions reduction in rate gain did not produce any large changes in the response time. The effects on peak frequency, which is an indication of the rapidity of the initial response, are more predominant. The peak frequency increases as the rate gain increases. The effect on the stability of the system in terms of $C_{1/2}$ was also more pronounced than the effect on $T_{5\%}$, but in general the response characteristics for the various flight conditions were not very sensitive to gain changes of this magnitude.

Figure 10 shows the results obtained when the peak magnification of the outer loop is increased to 1.6 for condition IV. A comparison with the corresponding curves of figure 9 shows the effects of changes in error gain alone. The changes in error gain associated with the change in peak magnification from 1.2 to 1.6 are in all cases small; hence the stability of the system is rather sensitive to changes in error gain alone. The response characteristics, as indicated by ω_p and $T_{5\%}$, are not significantly changed by the change in peak magnification, but considerations of stability (in terms of $C_{1/2}$) would tend to make a peak magnification of 1.2 preferable.

Figures 6 to 9 reveal, further, that the magnitudes of the gain settings for the altitude conditions (conditions I, III, and IV) were extremely high as compared with those being currently used in airplane autopilots. At these altitude conditions, the error gain K_ϵ reached magnitudes as high as 40, which means that for 1° of steady attitude error the elevator would deflect 40°. In practice, gains might very well be limited to lower values by other considerations, such as servo power, loads, saturation, and the possibility of exciting high-frequency chatter. The limited considerations involved in the analysis, however, allowed a very rapid response with a good degree of stability. As mentioned previously in connection with the autopilot without rate feedback, the gains could be relaxed at the expense of the response if it were established that a poorer response could be tolerated.

Effects of Mach Number and Altitude

The variation of the gain settings with flight condition is necessitated by the alterations in the airplane frequency response, as shown in figure 2. The effects of Mach number change at a constant altitude can be seen by comparing figures 6, 8, and 9. As the Mach number is increased from 0.5 to 0.9, the gain settings decrease in magnitude by a factor of about 2.5. This decrease in gain is enabled by the fact that at subsonic speeds the frequency response of airplanes usually improves with increased airspeed. The expected trend is for the frequency-response curves of the airplane to be stretched along the frequency axis by an amount proportional to the increase in airspeed. This stretching reflects an increase in the natural frequency of the airplane. A point worth noting, however, is that with the best gain adjustment obtained at each high-altitude condition the response time and degree of stability of the airplane-autopilot combination correspond closely.

A comparison of figures 7 and 8 shows the effects of altitude change at constant Mach number. In going from sea level to a 35,000-foot altitude the best gain settings obtained increase by a factor of about 4. The reasons for this necessary increase in gain setting can be seen by comparing the airplane frequency-response curves for the two altitudes (see fig. 2). At sea level the peak of the amplitude-response curve occurs at about twice the frequency of the peak for a 35,000-foot altitude at the same Mach number. The rapid phase shift in the airplane frequency response also is delayed to higher frequencies for the sea-level condition. These differences reflect an increase in the natural frequency of the airplane with decrease in altitude, with the result that the high-frequency response generally improves. Also, at low frequency ($\omega < 1$ radian per second) the amplitude response is greater for the sea-level condition. The differences in the airplane frequency-response curves can be attributed primarily to the fact that the density decreases by a factor of 3 in going from sea level to 35,000 feet. Another effect of altitude which usually occurs can be seen from figure 2. This effect is an increase in the ratio of the peak amplitude response to the minimum amplitude response at low frequency as the operating altitude is increased at constant Mach number. An increase in this ratio indicates a decrease in airplane damping. This reduction in damping can be offset by increasing the rate feedback of the output. It is of interest to note that although the gains at low altitude are decidedly lower, the response time at the best gain settings obtained tends to be larger at sea level than at 35,000 feet. This condition results from the tendency of the inner loop to approach instability at low rate gains for the sea-level condition. This limitation on the usable rate gain in turn limits the maximum error gain. The comparative response times might be changed if other possible limitations on the high gain at altitude were considered.

The analysis thus far has shown that the gain settings of the airplane-autopilot combination with rate feedback can be adjusted to give high performance in each flight condition. Manual adjustment of the gains would create extra work for the pilot, and automatic adjustment would introduce added complications in the autopilot. Therefore, the effects of holding the gain settings constant while varying the flight conditions were investigated, and the results are shown in figures 11 and 12. In figure 11, the gain settings that were chosen were the best gain settings obtained for the altitude cruising condition (see fig. 8). The system became unstable at sea level with these gain settings (condition II of fig. 11). The gain settings of condition II of figure 11 are much larger than the best gain settings obtained in figure 7, and this large increase in gain causes instability of the inner loop of the system. Condition IV of figure 11 was almost unstable too, because for this case the gains were increased greatly over the best gains obtained in figure 9. The peak magnification for condition IV of figure 11 increased to 5.0 as a result of the gain increases, furnishing another indication that the system would be poorly damped.

In figure 12, the gain settings that were held constant as the flight conditions were varied were the best gain settings obtained for the sea-level condition (see fig. 7). Since these gain settings were in all cases lower than the best values at the other flight conditions, there was a general trend toward improved stability, but the responses were more sluggish. The natural frequency of the airplane-autopilot combination for these cases was still roughly twice the natural frequency of the airplane alone for the altitude flight conditions. This fact indicates a possibility that a constant gain setting could be used if it were established that a considerably poorer response than the best obtainable could be tolerated.

Variations of Elevator Deflection, Normal Acceleration, and Flight Path

The response characteristics of the basic controlled quantity, in this case pitch attitude, do not alone determine the adequacy of a given system. The control motions involved in obtaining the response have effects on the maximum rate, force, and power output of the servomotor in addition to its frequency-response characteristics, and these motions also determine the magnitude of the loads impressed on the airplane structure as a result of control applications. Of equal importance is the variation of total load on the airplane as indicated by the variation of normal acceleration during the response. Another aspect involved in determining the suitability of an autopilot is that the intent of the system may be to control some quantity other than the basic quantity. For example, a pitch-attitude system may be used primarily to control the flight path of an airplane rather than its attitude.

Because of these aspects, the variations of elevator deflection, normal acceleration, and flight path in response to the approximate step command in pitch attitude were investigated for each flight condition. Results for the best gains obtained at each flight condition are presented in figure 13; results for constant gain with the best gain values obtained for condition III (altitude cruising) are presented in figure 14, and results for constant gain with the best gain values obtained for condition II (sea level) are presented in figure 15.

The ratios of elevator deflection to input attitude command are plotted in figure 13 with the best gain values obtained, as shown in figures 6 to 9. For an input of the abruptness shown, these ratios approach closely the gain values, which, as previously mentioned, are very high for the altitude conditions. In all instances the elevator oscillates at fairly high frequency and the deflection amplitude rapidly approaches zero as the attitude error is reduced. As a result, normal accelerations of significant magnitude are imposed on the airplane only for a short period of time, as shown in figures 13 to 15. Since the flight-path angle is proportional to the time integral of the normal acceleration, the flight-path response is poor as compared with the response in pitch attitude. The flight-path response is most rapid for the sea-level condition, which is the condition that has the lowest gain settings and gives by far the lowest ratio of elevator deflection to input pitch-attitude command in figure 13. This result and the fact that the pitch-attitude responses are very similar for all four flight conditions, whereas the flight-path responses differ considerably, are indications that the effects of gain and of flight condition on the flight-path response characteristics are considerably different from the effects of these factors on the pitch-attitude response characteristics. The occurrence of initial acceleration and flight-path change in the wrong direction results from the fact that the maneuvering tail load opposes and leads the load due to the angle-of-attack change.

The results for constant gain with altitude cruising settings are presented in figure 14. Comparison of the results for a Mach number of 0.5 (condition I) with those presented in figure 6 shows that the decreased gains associated with the constant-gain case reduced the control deflections encountered, with a resulting deterioration in pitch-attitude response. The maximum amplitude of the normal acceleration also decreases but the average level of the curve is not effectively altered, with the result that the over-all flight-path response is little changed.

The appreciable increase in gain for condition II (the sea-level condition) for the case presented in figure 14, as compared with the case presented in figure 13, results in an unstable system, as mentioned previously. The gain increase for condition IV results in violent oscillations of the elevator. The oscillations in pitch attitude and normal acceleration are attenuated somewhat, and the oscillations in the flight-path response are attenuated appreciably, a fact which indicates poor high-frequency response in flight path for the airplane.

Further insight into the effect of autopilot gain settings on the flight-path response may be had by comparing the results for the sea-level constant-gain case in figure 15 with the cases presented in figures 13 and 14. The gains for flight conditions I, III, and IV are appreciably lower for the case presented in figure 15 than for those presented in figures 13 and 14. Although the pitch-attitude response is deteriorated by these lower gains, the flight-path response is actually improved slightly by decreasing the gain. This effect is the result of a slower reduction in the attitude error, which causes the elevator deflections to be held longer and the normal accelerations to be sustained longer. These combined effects, as reflected in the frequency response, tend to improve the low-frequency amplitude response in flight path even though the high-frequency amplitude response drops off somewhat with decrease in gain.

In order to investigate the possibilities for improving the flight-path response when a pitch-attitude autopilot is used, a comparison has been made of the flight-path response obtained with the present airplane-autopilot combination and that obtained when a perfect pitch-attitude response is assumed. For the latter case the transfer function relating the flight-path response to the pitch-attitude command is derived in the appendix. By means of this transfer function, the frequency response was obtained. This frequency response was used in conjunction with the Fourier synthesizer to determine the response of the flight path to the approximate step command in pitch attitude used herein, for the case of perfect pitch-attitude response. The results are presented in figure 16 as a comparison of the flight-path response obtained for the airplane-autopilot combination and the flight-path response obtained for the case of perfect pitch-attitude response. Figure 16 shows that even if the pitch-attitude response were improved the flight-path response would not be significantly improved. Therefore, in view of the low levels of normal acceleration sustained, a pitch-attitude autopilot is not particularly suited to tight control of the flight path.

SUMMARY OF RESULTS

A theoretical analysis has been made of the longitudinal response characteristics of a swept-wing fighter airplane having a pitch-attitude control system. The system was investigated with pitch-attitude feedback alone and pitch-attitude plus pitch-rate feedback. The effects of varying the autopilot gains and the airplane flight conditions were investigated. The flight conditions considered were: condition I, $M = 0.5$ and $h_p = 35{,}000$ feet; condition II, $M = 0.7$ and $h_p = 0$; condition III, $M = 0.7$ and $h_p = 35{,}000$ feet; and condition IV, $M = 0.9$ and $h_p = 35{,}000$ feet. (M denotes Mach number and h_p denotes pressure altitude.) From this analysis the following conclusions were reached:

1. When the autopilot incorporated pitch-attitude feedback alone, the gain in the autopilot error signal was limited to low values for all four flight conditions by considerations of system stability. As a result, the response to pitch-attitude commands was sluggish. When rate feedback was incorporated, the airplane-autopilot combination could be made to have a very rapid pitch-attitude response, provided some means of changing the gain settings with flight conditions were made available. For some cases the natural frequency of the airplane-autopilot combination could be 10 times that for the basic airplane without loss of adequate damping.

Conclusions 2 to 7 are concerned with the system incorporating both pitch-attitude and pitch-rate feedback.

2. The peak elevator deflections which occurred in response to a fairly abrupt steplike command in pitch attitude were very large for the high-altitude conditions when the best gain settings obtained were used. In some cases the error gain settings would produce 40° of elevator deflection for 1° of steady pitch-attitude error. In practice, use of such high gains may be limited by factors other than those considered in this paper. For the abrupt steplike command, the ratio of peak elevator deflections to input command closely approached the error gain values.

3. The effect on the response time of varying the rate gain ±50 percent from the original value was reasonably small for each flight condition investigated when a procedure for simultaneous adjustment of error gain was employed. In all cases the stability of the system was very sensitive to changes in error gain alone.

4. An increase in Mach number from 0.5 to 0.9 at an altitude of 35,000 feet decreased the magnitude of the best gain settings obtained by a factor of about 2.5. This reduction was brought about by a general improvement in the airplane frequency response with increase in airspeed in the subsonic speed range.

5. When the best gain adjustments obtained were used, the response time and degree of stability of the airplane-autopilot combination corresponded closely for all the high-altitude flight conditions investigated, but the response time tended to be larger at sea level than at high altitude. The autopilot gains were much higher at high altitude, however, because both the high-frequency and the low-frequency amplitude response of an airplane decrease in proportion to the decrease in air density.

6. When operation was attempted at sea level with the best gain settings obtained for cruising speed at high altitude, the system became unstable because of the large increase in gain settings over the best sea-level values obtained. When attempts were made to operate at high altitude with the best gain settings obtained for the sea-level condition, in all cases the gain setting was lower than the best values

at the high-altitude conditions. The result was a general trend toward improved stability but a more sluggish response.

7. In view of the low levels of normal acceleration sustained by the pitch-attitude autopilot, this autopilot is not particularly suited to tight control of the flight path since the change in flight-path angle is proportional to the time integral of the normal acceleration. The flight-path response was not significantly affected by changes in gain settings. In some cases the response time was actually improved slightly by decreasing the gain.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., November 13, 1952.

APPENDIX

TRANSFER FUNCTIONS APPLICABLE TO PRESENT ANALYSIS

The longitudinal equations of motion in nondimensional form as used in the analysis of this paper are

$$\left(2\mu K_Y{}^2 D - C_{m_q}\right)q + \left(-C_{m_\alpha} - C_{m_{D\alpha}}D\right)\alpha = C_{m_\delta}\delta \tag{1}$$

$$\left(2\mu - C_{L_q}\right)q + \left(-2\mu D - C_{L_\alpha} - C_{L_{D\alpha}}D\right)\alpha = C_{L_\delta}\delta \tag{2}$$

The transfer function of the airplane that relates pitching velocity to elevator-deflection input in terms of airplane stability derivatives is derived from equations (1) and (2) in the form:

$$\frac{q}{\delta} = \frac{AD + B}{CD^2 + ED + F}$$

where

$$A = C_{m_\delta}C_{L_{D\alpha}} - C_{m_{D\alpha}}C_{L_\delta} + 2\mu C_{m_\delta}$$

$$B = C_{L_\alpha}C_{m_\delta} - C_{m_\alpha}C_{L_\delta}$$

$$C = 2\mu K_Y{}^2 C_{L_{D\alpha}} + 4\mu^2 K_Y{}^2$$

$$E = -2\mu C_{m_{D\alpha}} + C_{L_q}C_{m_{D\alpha}} - 2\mu C_{m_q} + 2\mu K_Y{}^2 C_{L_\alpha} - C_{m_q}C_{L_{D\alpha}}$$

$$F = -2\mu C_{m_\alpha} + C_{m_\alpha}C_{L_q} - C_{L_\alpha}C_{m_q}$$

Since

$$q = D\theta$$

then

$$\frac{\theta}{\delta} = \frac{AD + B}{D\left(CD^2 + ED + F\right)}$$

The frequency-response form of the transfer function may be obtained by substituting $j\omega'$ for the operator D. Then the airplane amplitude ratio becomes

$$\left|\frac{\theta_o}{\delta}\right| = \frac{\sqrt{G^2 + H^2}}{I}$$

and the phase angle becomes

$$\Phi = \tan^{-1}\frac{+H}{+G}$$

where

$$G = AC(\omega')^3 + (BE - AF)\omega'$$

$$H = (AE - BC)(\omega')^2 + BF$$

$$I = -\omega'\left[C^2(\omega')^4 - 2FC(\omega')^2 + E^2(\omega')^2 + F^2\right]$$

The frequency response of the autopilot was determined experimentally by applying known sinusoidal inputs of various frequencies and measuring the amplitude and phase angle of the sinusoidal output.

A block diagram of the automatic control system under consideration is presented in the section entitled "Analysis." The symbols Y_1 and

Y_2 are the autopilot and airplane transfer functions, respectively. The open-loop transfer function of the system is

$$\frac{\theta_o}{\epsilon} = \frac{K_\epsilon Y_1 Y_2}{1 + K_R Y_1 Y_2 D}$$

The closed-loop transfer function of the system is

$$\frac{\theta_o}{\theta_i} = \frac{\dfrac{\theta_o}{\epsilon}}{1 + \dfrac{\theta_o}{\epsilon}}$$

The known transfer functions of the airplane and the autopilot, together with the gain settings determined by means of the technique described in reference 1, pages 185 to 188, were used to find the frequency response of the closed-loop transfer function of the system. The frequency-response data were fed into the Fourier synthesizer for the determination of the transient-response characteristics of the system in pitch attitude.

The transfer function relating elevator deflection to the pitch-attitude input command of the closed-loop system was obtained from the airplane transfer function relating pitch attitude to elevator deflection (previously described) and from the closed-loop transfer function of the system relating pitch-attitude output to the pitch-attitude input command. Thus,

$$\frac{\delta}{\theta_i} = \frac{\theta_o/\theta_i}{\theta_o/\delta}$$

In a similar manner the transfer function relating normal acceleration to elevator deflection was used to establish the transfer function relating normal acceleration to the pitch-attitude input command for the closed-loop system. The n/δ transfer function in terms of airplane stability derivatives, as derived from equations (1) and (2) and the expression

$$n = \frac{V^2}{g\bar{c}} (q - D\alpha)$$

is of the form:

$$\frac{n}{\delta} = \frac{JD^2 + KD + L}{CD^2 + ED + F}$$

where

$$J = -2\mu K_Y{}^2 C_{L_\delta}$$

$$K = -C_{m_\delta} C_{L_q} + C_{L_\delta} C_{m_q} - C_{m_\delta} C_{L_{D\alpha}} + C_{m_{D\alpha}} C_{L_\delta}$$

$$L = C_{m_\alpha} C_{L_\delta} - C_{L_\alpha} C_{m_\delta}$$

The n/θ_i transfer function for the closed loop is

$$\frac{n}{\theta_i} = \frac{\frac{n}{\delta}\frac{\theta_o}{\theta_i}}{\frac{\theta_o}{\delta}}$$

The change in flight-path angle is proportional to the time integral of the normal acceleration:

$$n = \frac{V^2}{g\bar{c}} D\gamma$$

Substituting this expression for n in the n/θ_i transfer function gives the transfer function relating the flight-path response to an input pitch-attitude command for the system:

$$\frac{\gamma}{\theta_i} = \frac{g\bar{c}}{V^2} \frac{1}{D} \frac{n}{\theta_i}$$

The frequency-response data determined for δ/θ_i, n/θ_i, and γ/θ_i were also fed into the Fourier synthesizer to establish the transient-response characteristics of these output quantities in terms of the input pitch-attitude command.

The transfer function relating the flight-path angle of an airplane to its angle of pitch is derived from equations (1) and (2) and the following relations

$$D\gamma + D\alpha = D\theta \tag{3}$$

$$C_{m_\delta} = -C_{L_\delta}\frac{l}{\bar{c}} \tag{4}$$

$$D\theta = q \tag{5}$$

The transfer function γ/θ is:

$$\frac{\gamma}{\theta} = \frac{-2\mu K_Y{}^2 D^2 + \left(C_{m_q} + C_{m_{D\alpha}} + C_{L_q}\frac{l}{\bar{c}} + C_{L_{D\alpha}}\frac{l}{\bar{c}}\right)D + C_{m_\alpha} + C_{L_\alpha}\frac{l}{\bar{c}}}{\left(2\mu\frac{l}{\bar{c}} + C_{m_{D\alpha}} - C_{L_{D\alpha}}\frac{l}{\bar{c}}\right)D + C_{L_\alpha}\frac{l}{\bar{c}} + C_{m_\alpha}} \tag{6}$$

Equation (6) is of the form

$$\frac{\gamma}{\theta} = \frac{MD^2 + ND + O}{PD + Q} \tag{7}$$

where

$$M = -2\mu K_Y{}^2$$

$$N = C_{m_q} + C_{m_{D\alpha}} + C_{L_q}\frac{l}{\bar{c}} + C_{L_{D\alpha}}\frac{l}{\bar{c}}$$

$$O = C_{m_{\alpha}} + C_{L_{\alpha}} \frac{l}{\bar{c}}$$

$$P = 2\mu \frac{l}{\bar{c}} + C_{m_{D\alpha}} - C_{L_{D\alpha}} \frac{l}{\bar{c}}$$

$$Q = C_{L_{\alpha}} \frac{l}{\bar{c}} + C_{m_{\alpha}}$$

This transfer function γ/θ may be interpreted as that which relates the flight-path angle to a pitch-attitude command when the attitude response is perfect. The frequency-response form of the transfer function may be obtained by substituting $D = j\omega'$ into equation (7) and rationalizing the equation. Then the amplitude ratio and phase angle become

$$\frac{\gamma}{\theta} = \frac{\sqrt{R^2 + S^2}}{T}$$

$$\Phi = \tan^{-1} \frac{+S}{+R}$$

where

$$R = (NP - MQ)(\omega')^2 + OQ$$

$$S = \omega'\left[MP(\omega')^2 - OP + NQ\right]$$

$$T = P^2(\omega')^2 + Q^2$$

The frequency-response data thus obtained for γ/θ were fed into the Fourier synthesizer to establish the transient-response characteristics of the system when perfect attitude response is assumed.

REFERENCES

1. Brown, Gordon S., and Campbell, Donald P.: Principles of Servomechanisms. John Wiley & Sons, Inc., 1948.

2. Seamans, R. C., Jr., Blasingame, B. P., and Clementson, G. C.: The Pulse Method for the Determination of Aircraft Dynamic Performance. Jour. Aero. Sci., vol. 17, no. 1, Jan. 1950, pp. 22-38.

TABLE I.- AIRPLANE CHARACTERISTICS, FLIGHT CONDITIONS,

AND STABILITY PARAMETERS

Symbol	Condition			
	I	II	III	IV
M	0.5	0.7	0.7	0.9
h_p, ft	35,000	0	35,000	35,000
W, lb	15,291	15,291	15,291	15,291
S, sq ft	288	288	288	288
b, ft	37.1	37.1	37.1	37.1
V, ft/sec	485	779	682	877
$\bar{c}$, ft	8.085	8.085	8.085	8.085
Λ, deg	35.23	35.23	35.23	35.23
$l/\bar{c}$	2.4	2.4	2.4	2.4
C_L	0.611	0.0735	0.312	0.189
A	4.79	4.79	4.79	4.79
μ	277.24	85.807	277.24	277.24
C_{m_δ}	0.427	0.398	0.398	0.344
$C_{m_{C_L}}$	-0.100	-0.152	-0.126	-0.175
C_{L_δ}	-0.174	-0.162	-0.162	-0.140
C_{L_α}	4.355	4.928	4.584	6.016
C_{m_α}	-0.435	-0.751	-0.579	-1.054
$C_{m_{\alpha t}}$	-0.791	-0.825	-0.825	-0.848
$C_{L_{\alpha t}}$	0.323	0.337	0.337	0.346
C_{L_q}	0.791	0.825	0.825	0.848
C_{m_q}	-1.897	-2.017	-2.017	-2.074
$C_{L_{D\alpha}}$	0.1583	0.272	0.404	0.392
$C_{m_{D\alpha}}$	-0.3876	-0.664	-0.991	-0.962
$d\epsilon/d\alpha$	0.35	0.395	0.368	0.483
K_Y	0.308	0.308	0.308	0.308
I_Y	29,448	29,448	29,448	29,448

NACA

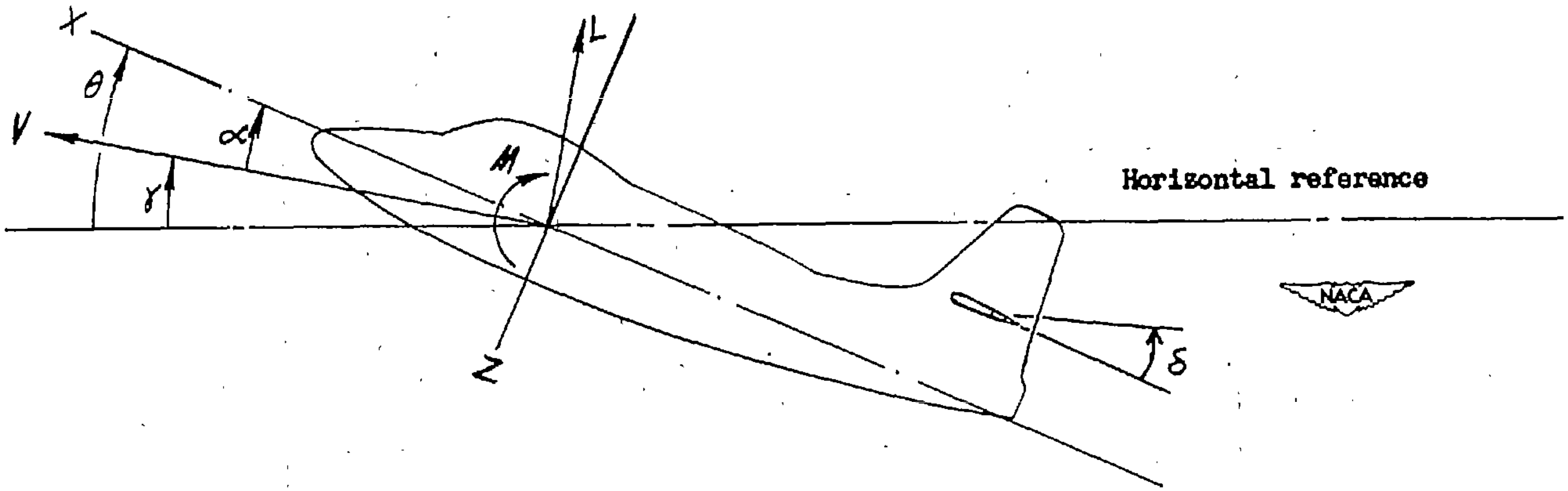


Figure 1.- System of axes and angular relationship in flight. Arrows indicate positive directions of forces and moments.

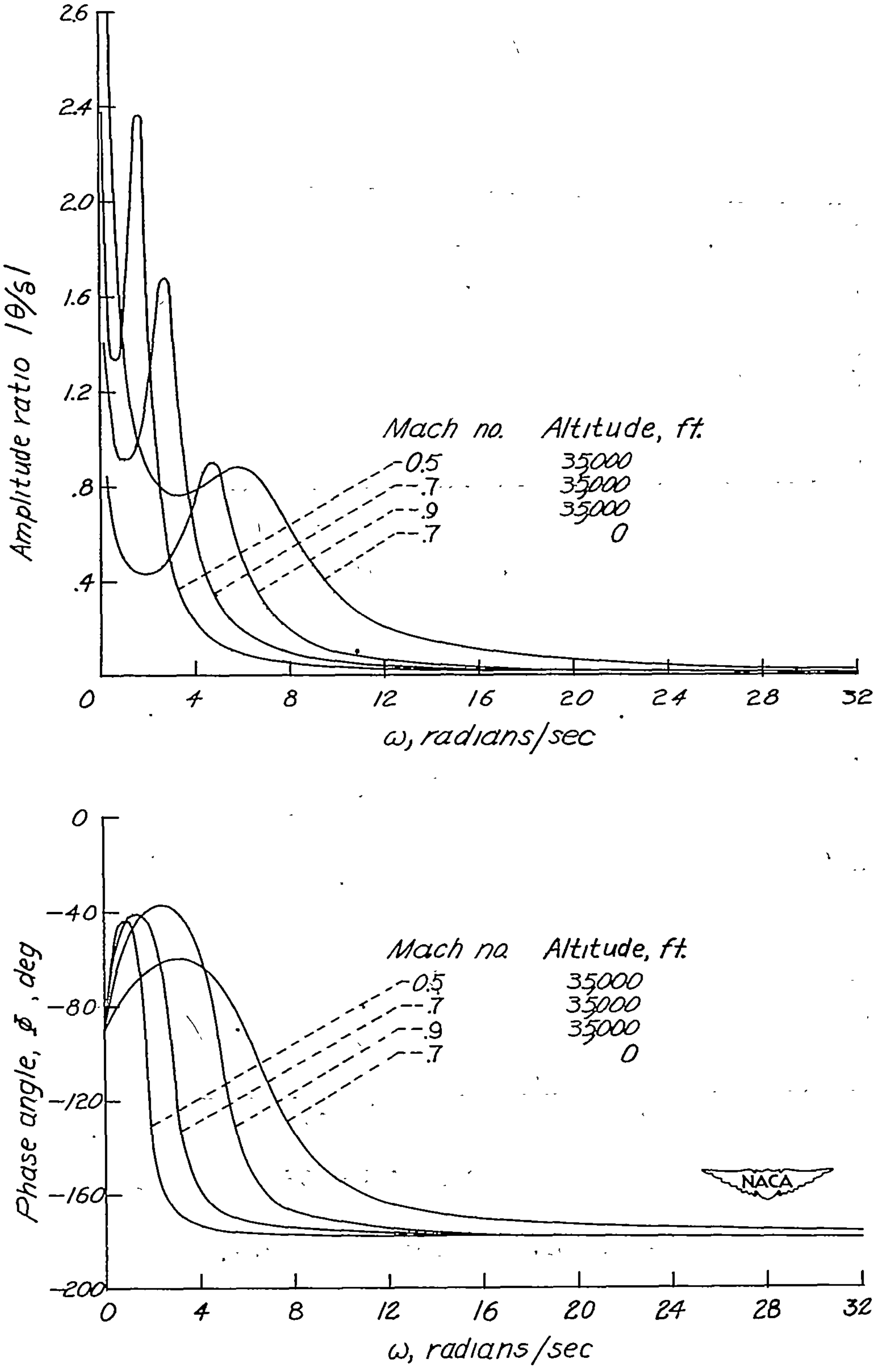


Figure 2.- Airplane frequency response.

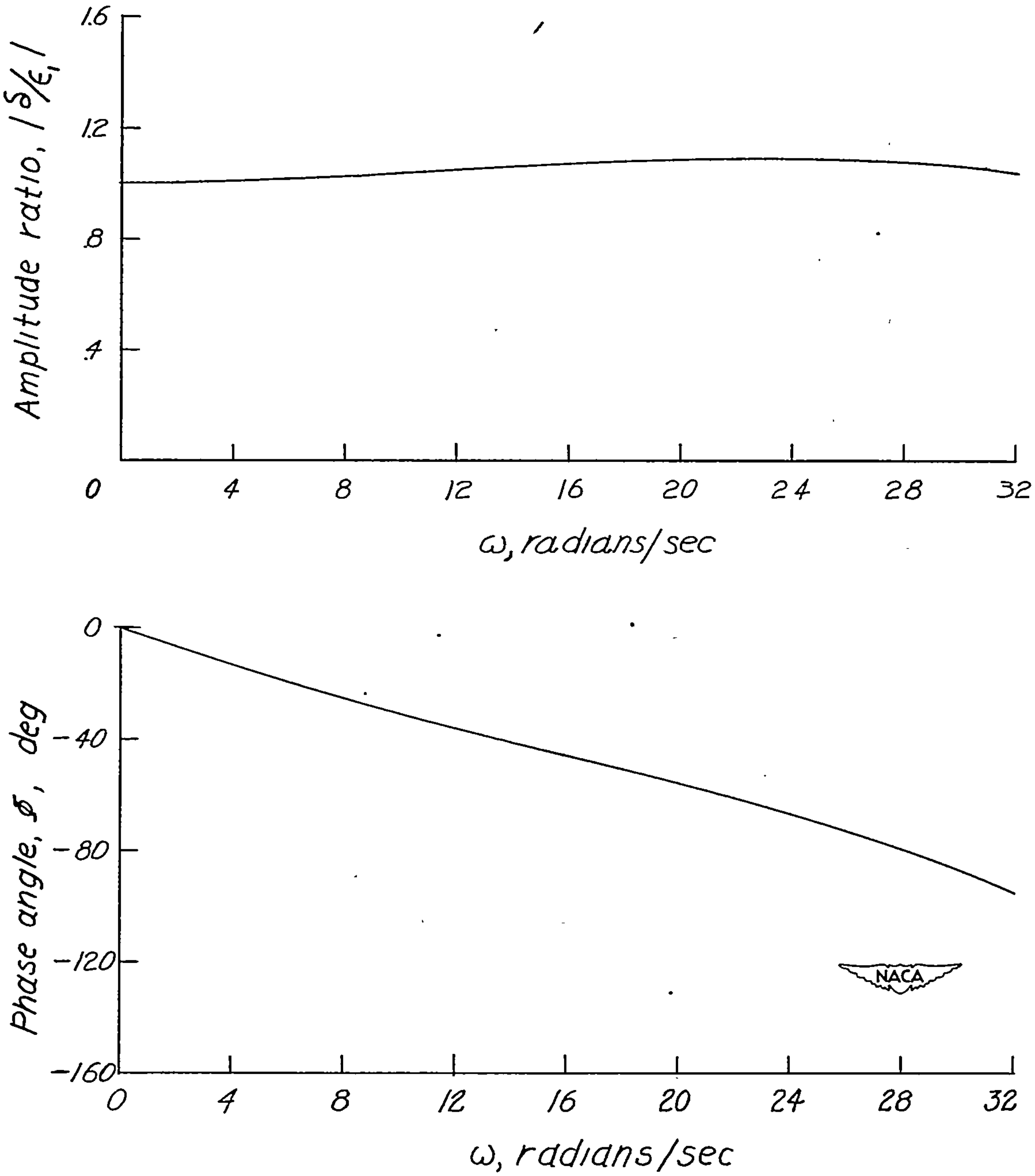


Figure 3.- Autopilot frequency response.

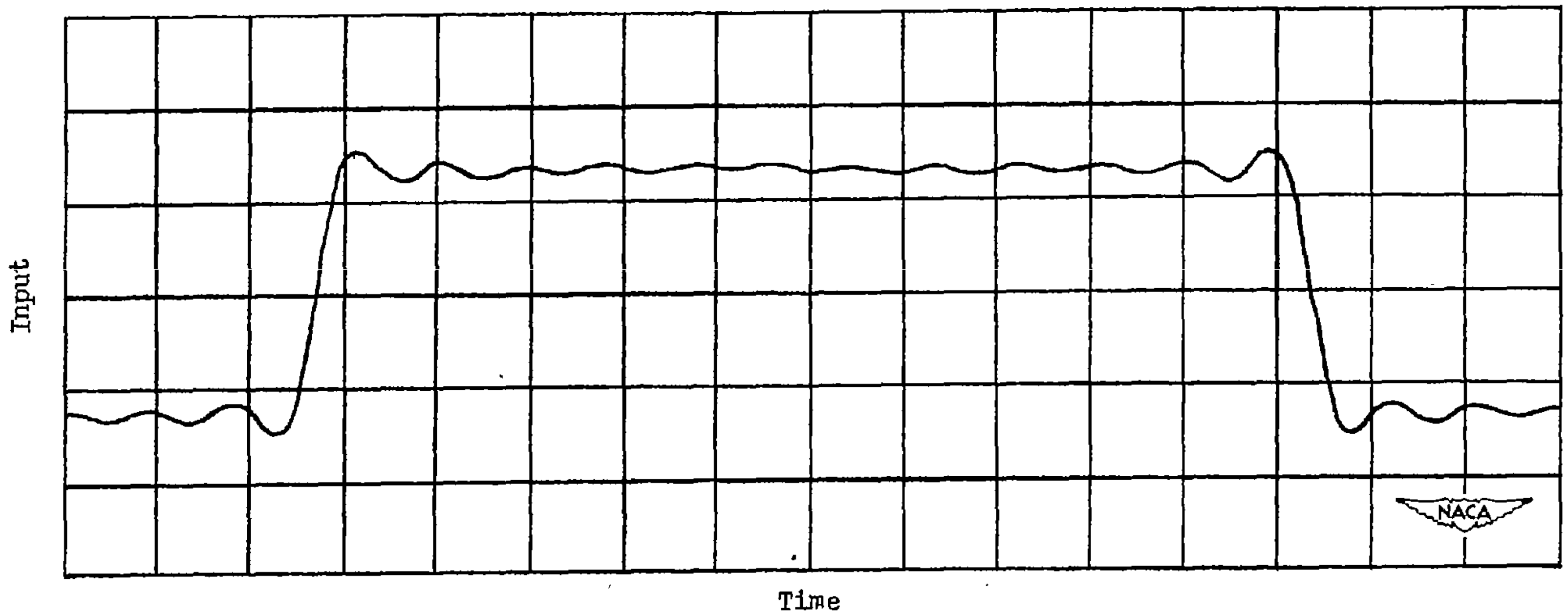


Figure 4.- Approximate square-wave input produced by the Fourier synthesizer.

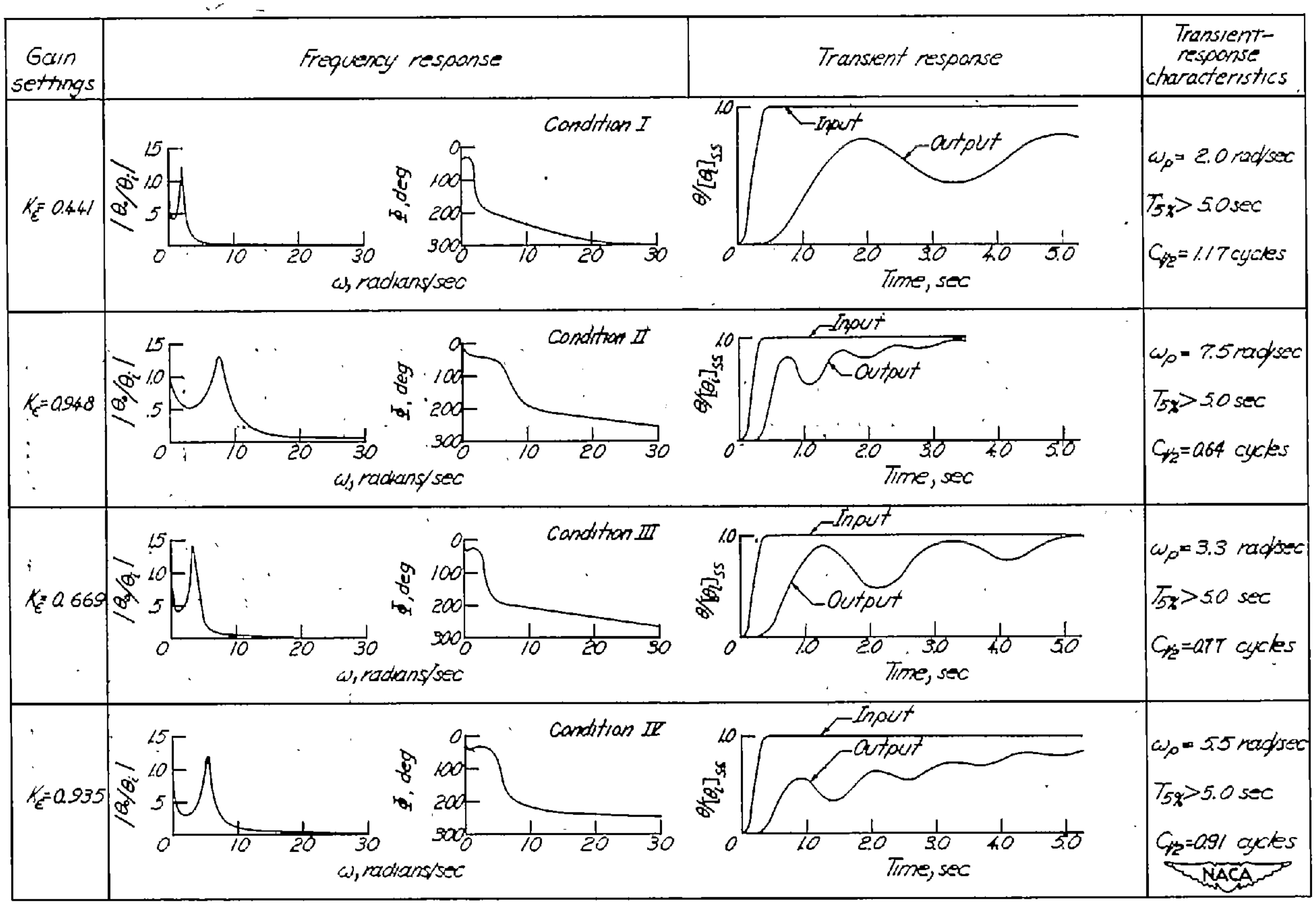


Figure 5.- Frequency response and transient response of airplane-autopilot combination without rate gyro for conditions I, II, III, and IV.

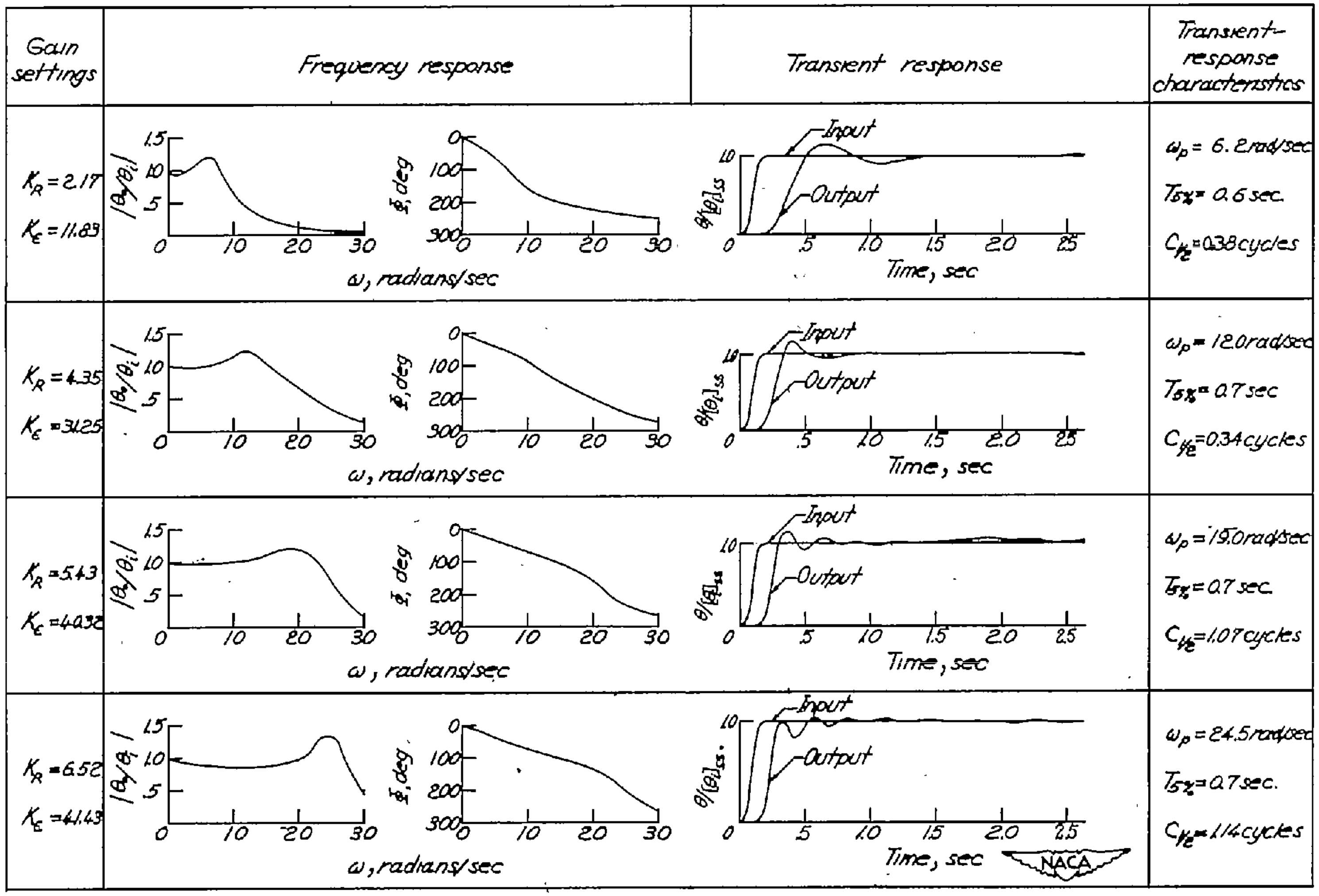


Figure 6.- Condition I: Effects of gain settings upon frequency response and transient response of airplane-autopilot combination with rate gyro. Mach number, 0.5; altitude, 35,000 feet; $|\theta_o/\theta_i|_{max} \approx 1.2$.

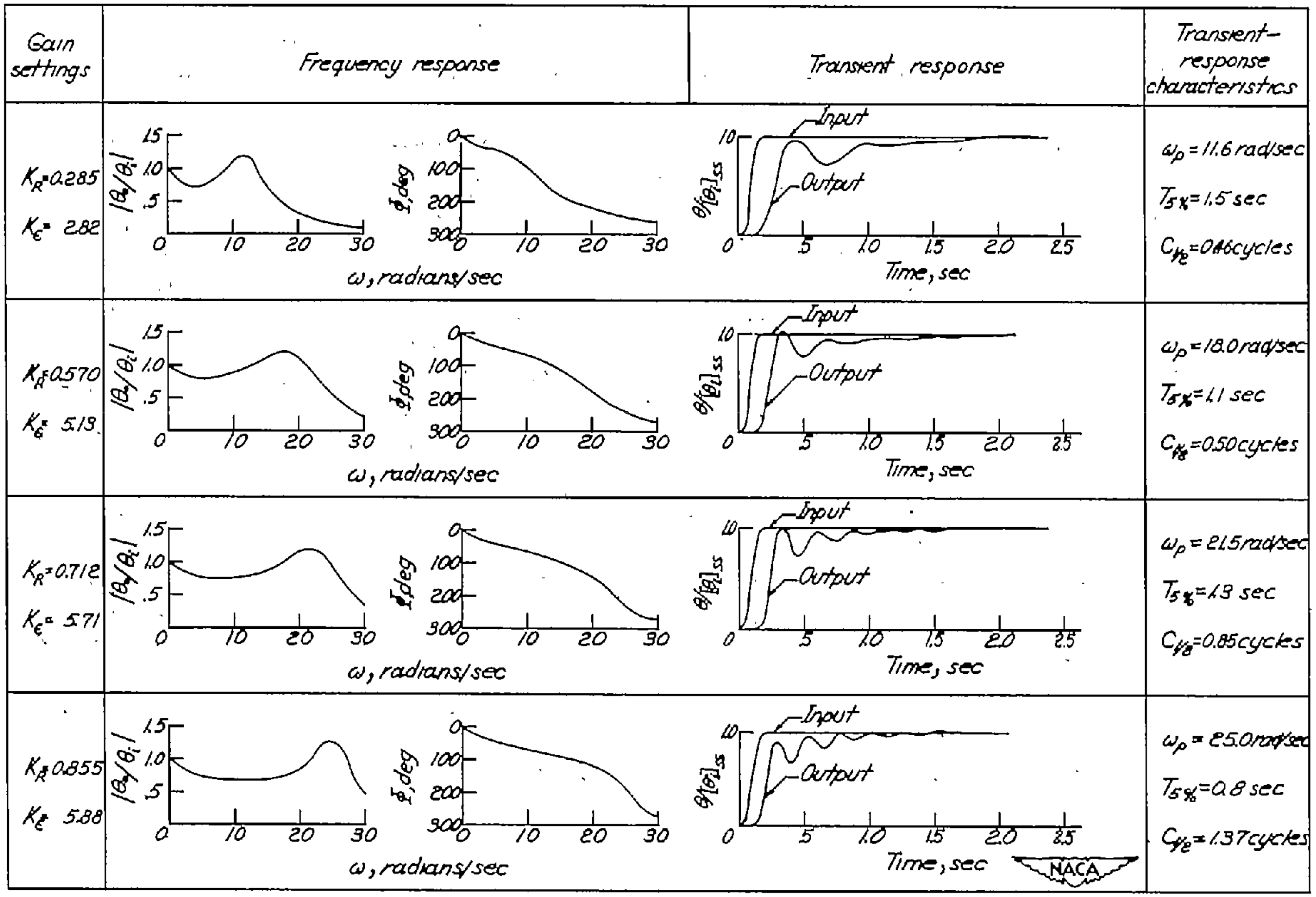


Figure 7.- Condition II: Effects of gain settings upon frequency response and transient response of airplane-autopilot combination with rate gyro. Mach number, 0.7; altitude, 0; $|\theta_o/\theta_i|_{max} \approx 1.2$.

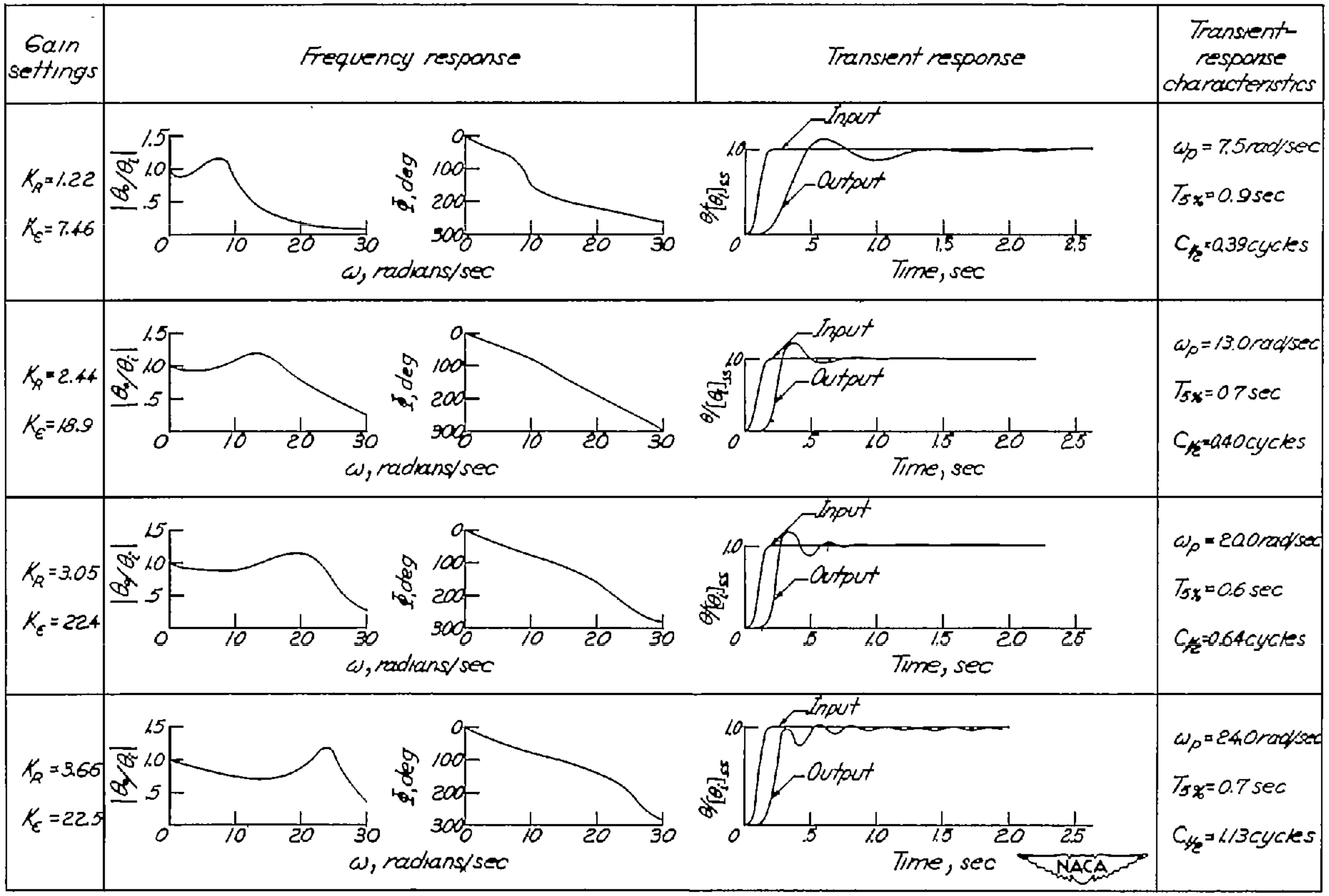


Figure 8.- Condition III: Effects of gain settings upon frequency response and transient response of airplane-autopilot combination with rate gyro. Mach number, 0.7; altitude, 35,000 feet; $|\theta_o/\theta_i|_{max} \approx 1.2$.

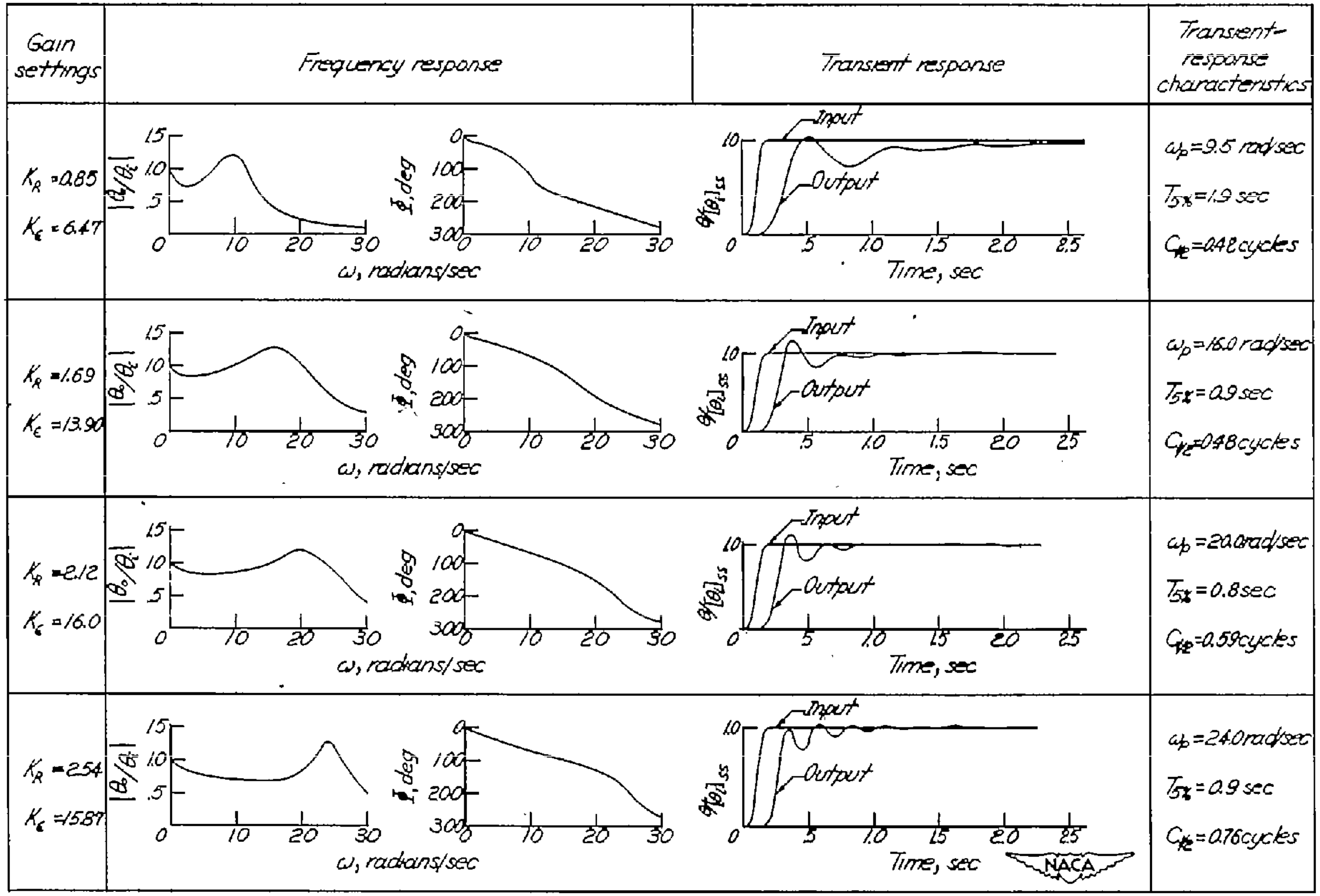


Figure 9.- Condition IV: Effects of gain settings upon frequency response and transient response of airplane-autopilot combination with rate gyro. Mach number, 0.9; altitude, 35,000 feet; $|\theta_o/\theta_i|_{max} \approx 1.2$.

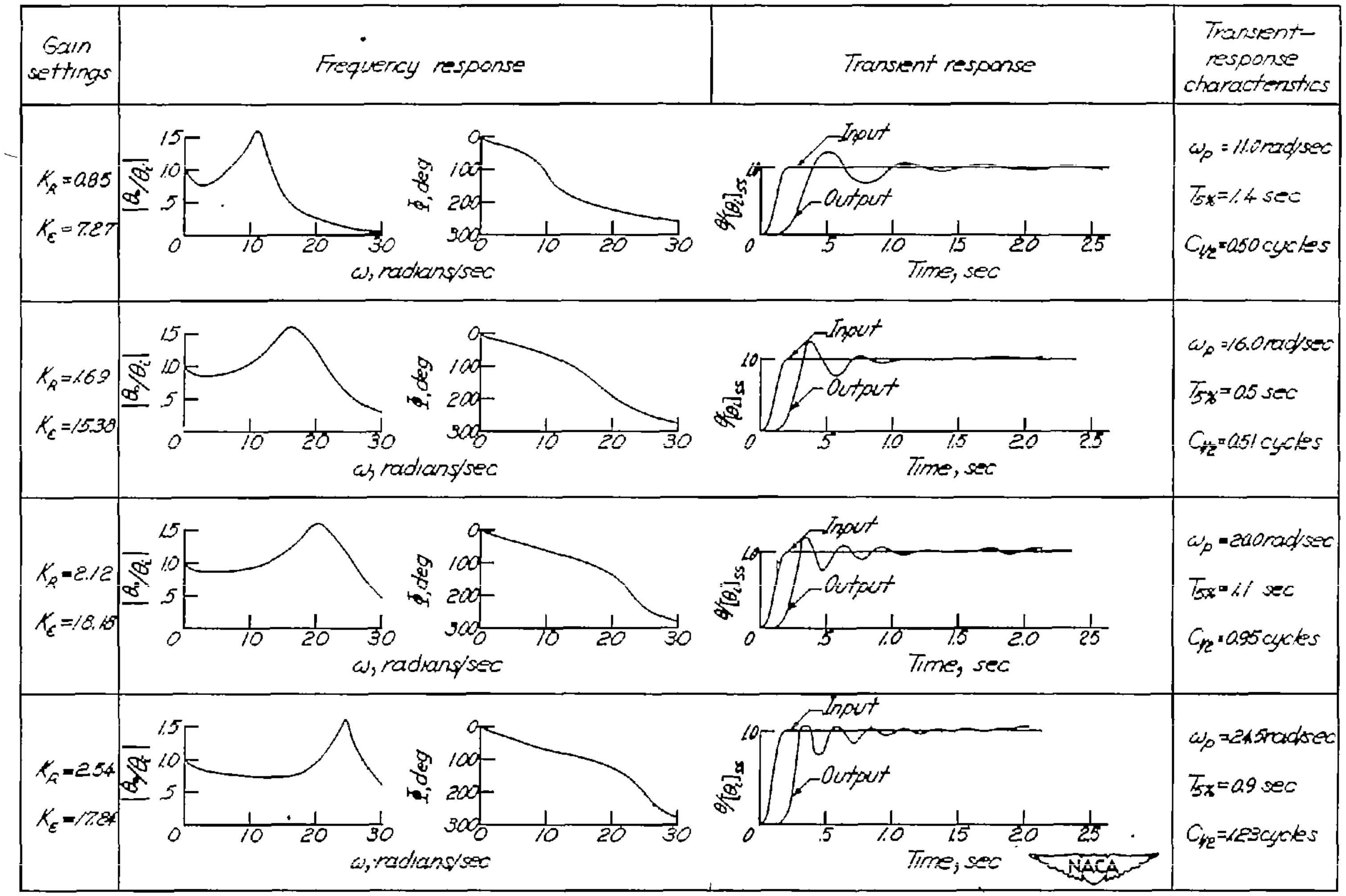


Figure 10.- Condition IV: Effects of gain settings upon frequency response and transient response of airplane-autopilot combination with rate gyro. Mach number, 0.9; altitude, 35,000 feet; $|\theta_o/\theta_i|_{max} \approx 1.6$.

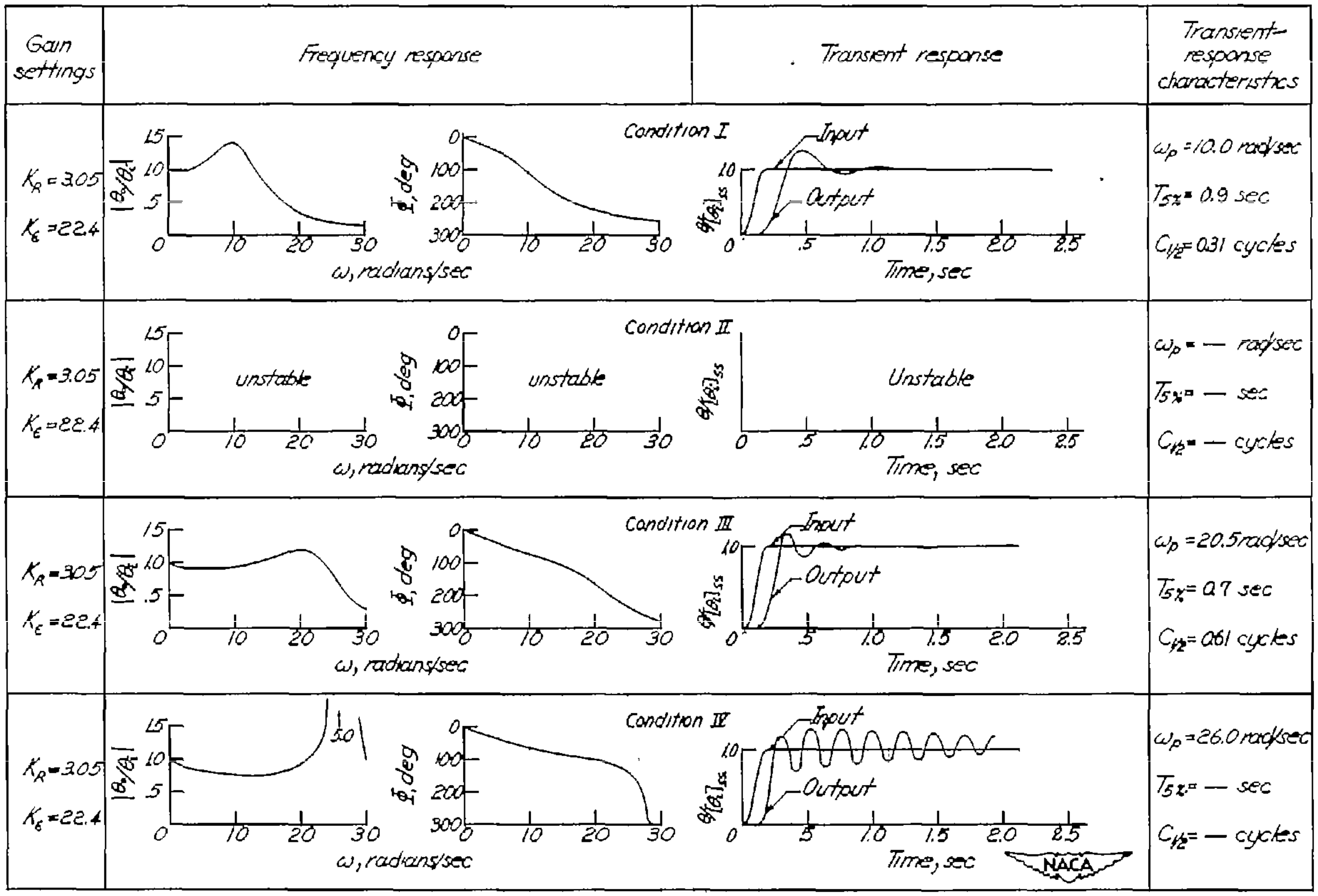


Figure 11.- Frequency response and transient response of airplane-autopilot combination with rate gyro when the best values obtained for K_R and K_ϵ at condition III are used. Mach number, 0.7; altitude, 35,000 feet.

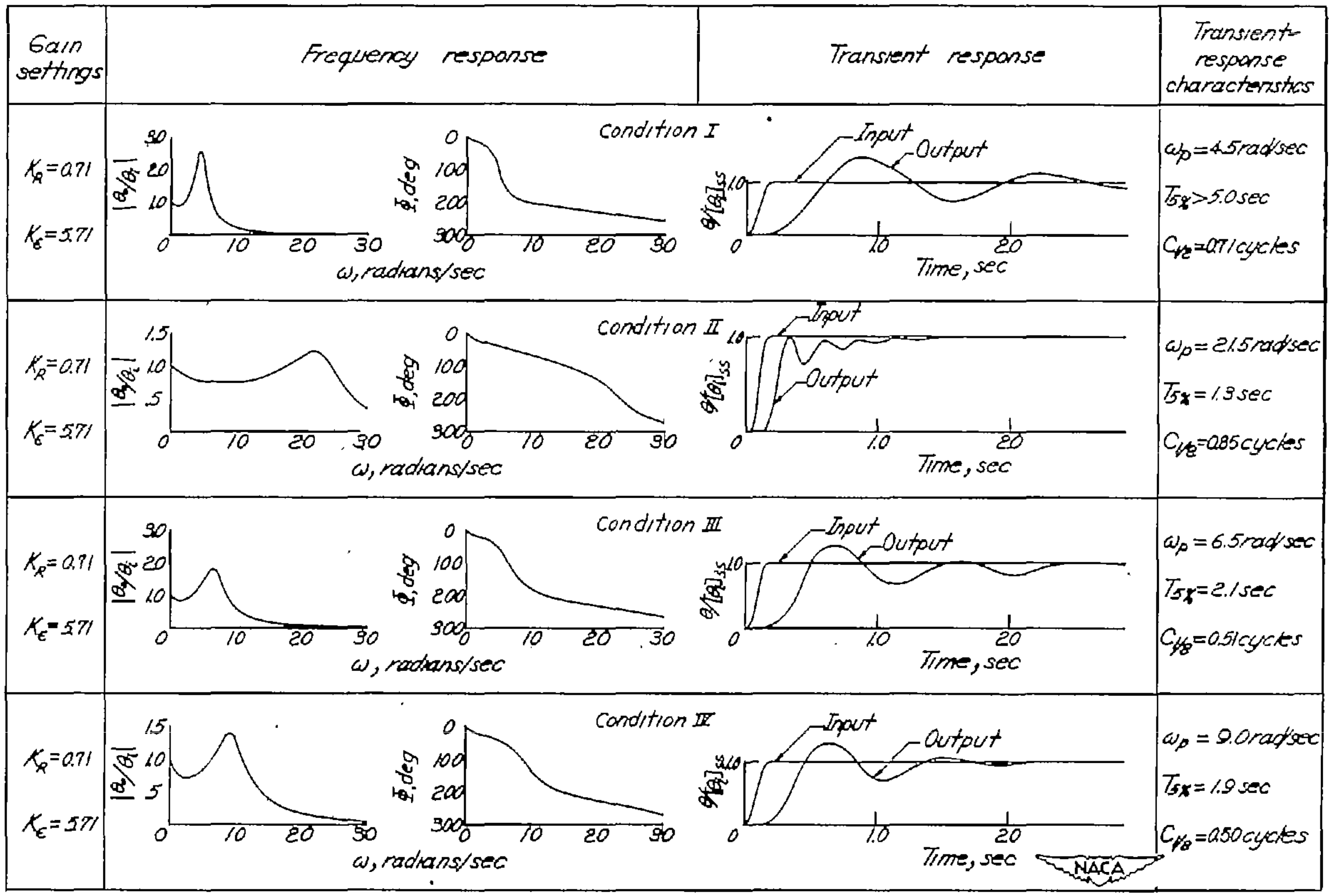


Figure 12.- Frequency response and transient response of airplane-autopilot combination with rate gyro when the best values obtained for K_R and K_ϵ at condition II are used. Mach number, 0.7; altitude, 0.

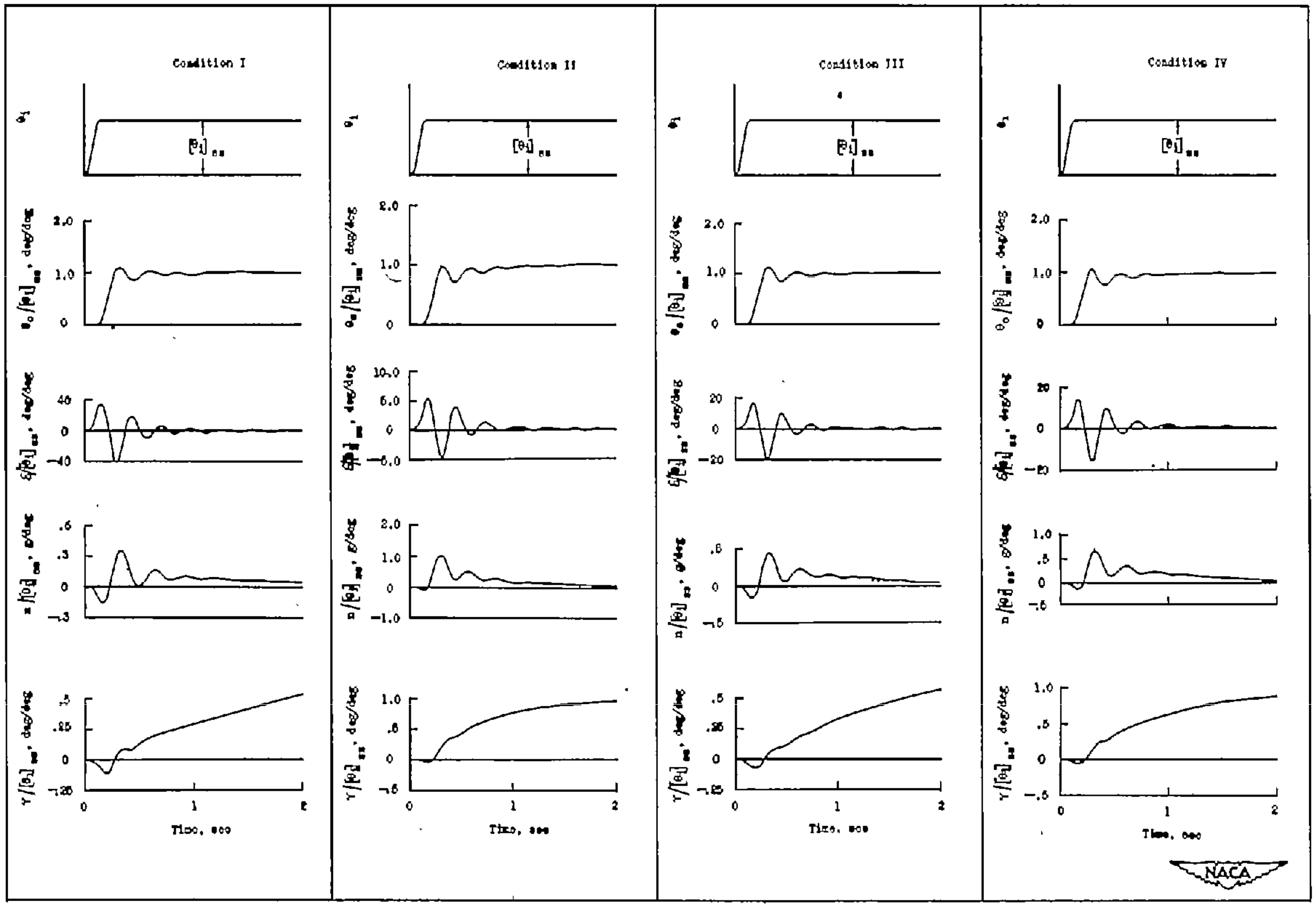


Figure 13.- Response in pitch attitude, elevator deflection, normal acceleration, and flight path to pitch-attitude command when best gains obtained at each condition are used.

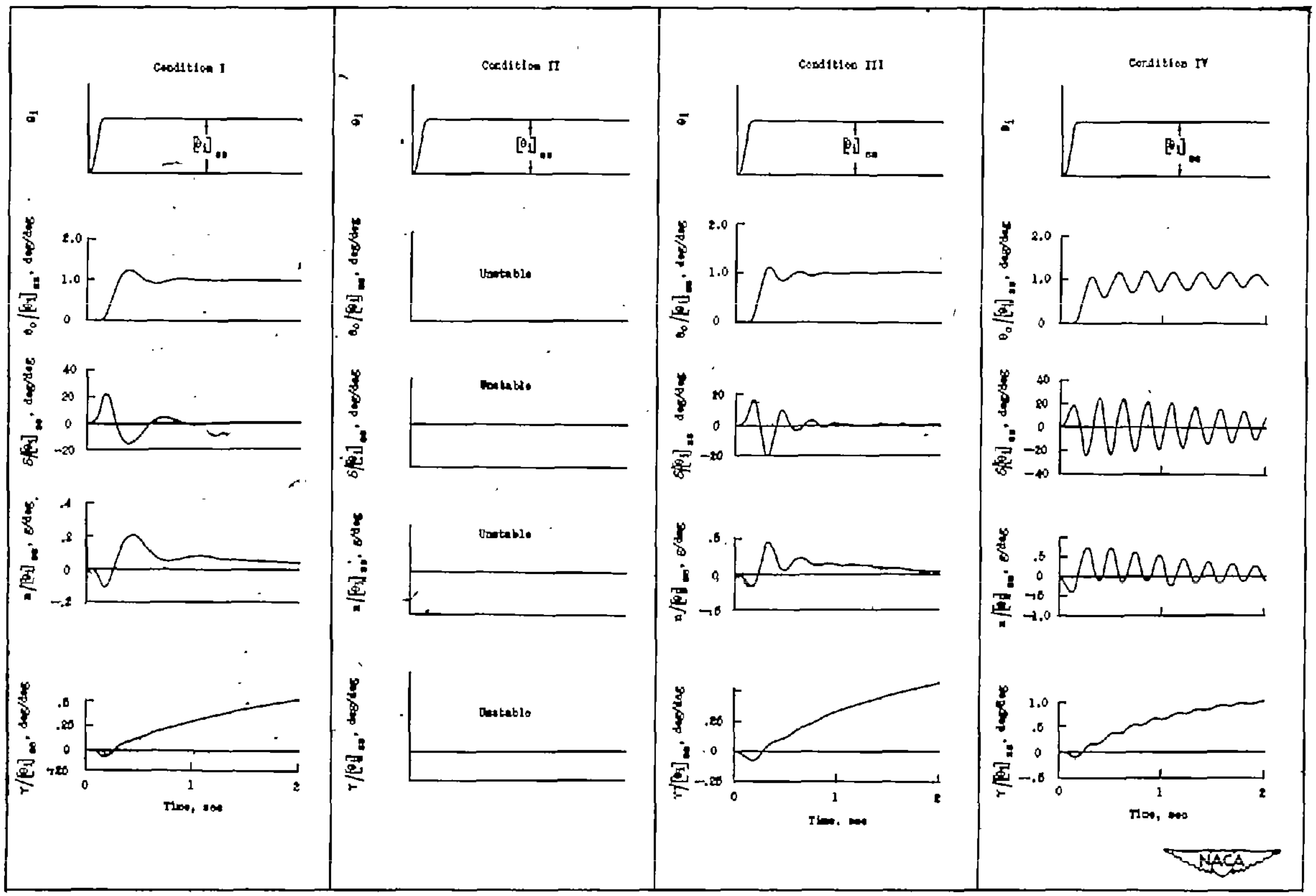


Figure 14.- Response in pitch attitude, elevator deflection, normal acceleration, and flight path to pitch-attitude command when best gains obtained at condition III are used. Mach number, 0.7; altitude, 35,000-feet.

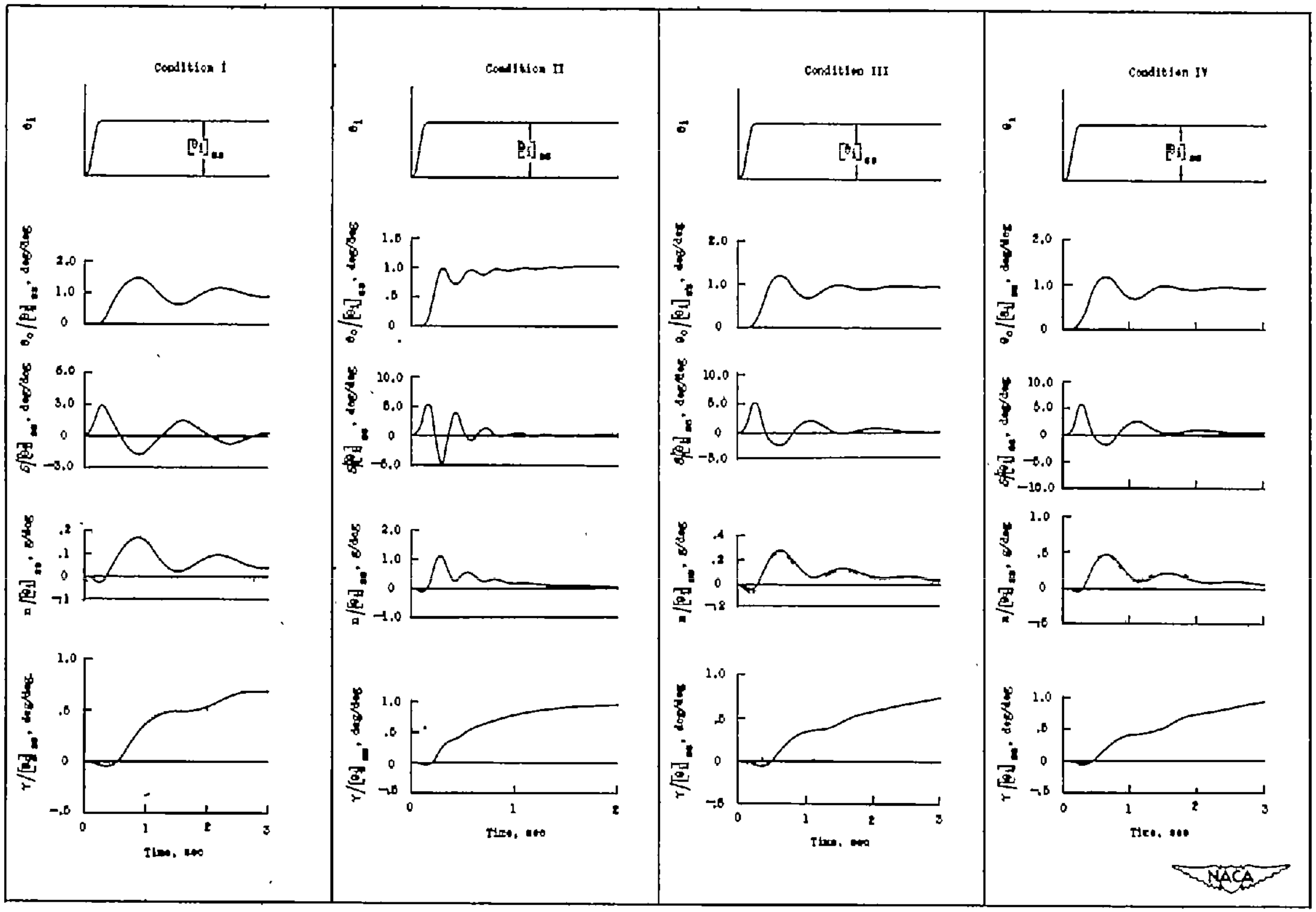


Figure 15.- Response in pitch attitude, elevator deflection, normal acceleration, and flight path to pitch-attitude command when best gains obtained at condition II are used. Mach number, 0.7; altitude, 0.

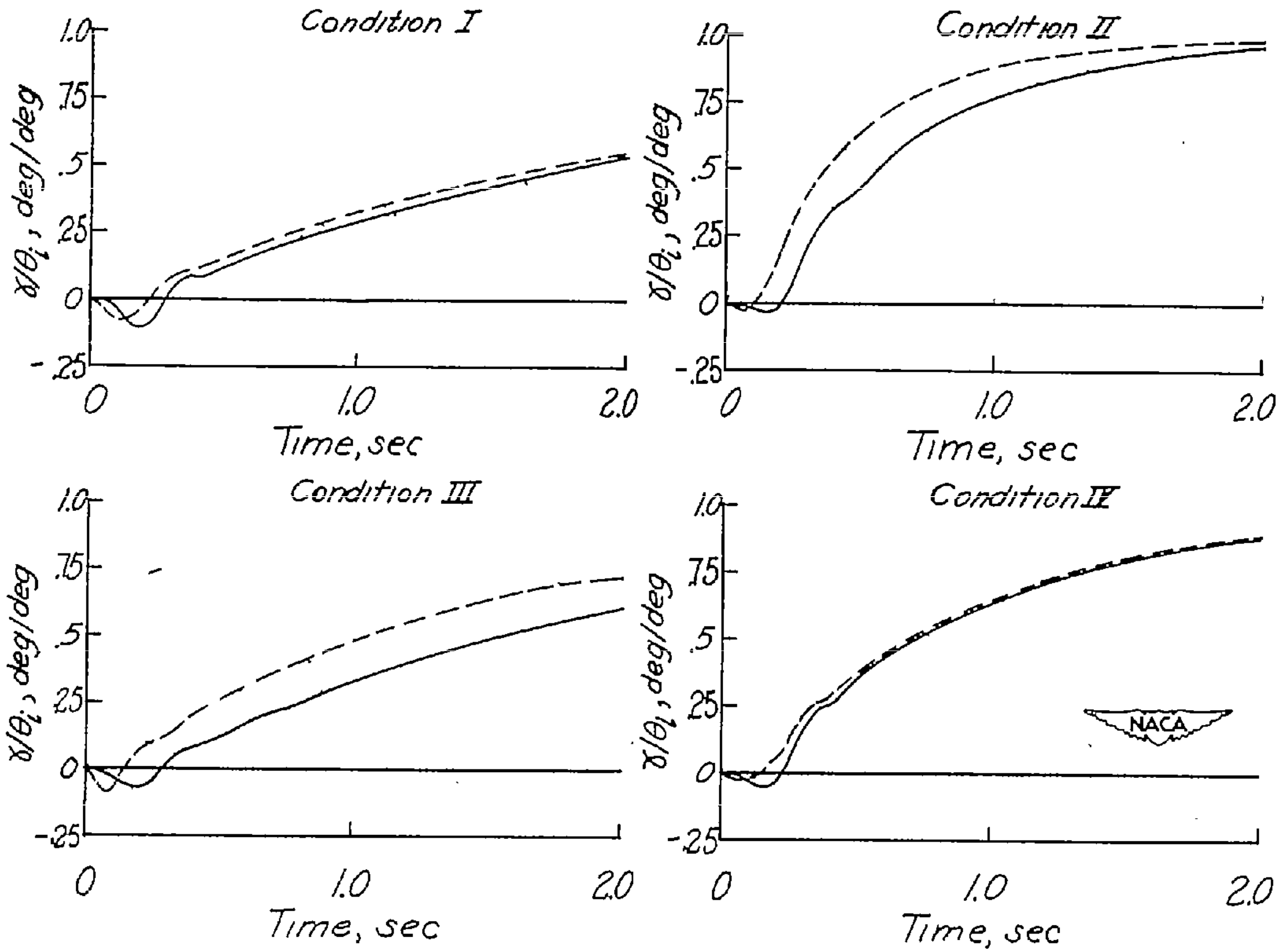


Figure 16.- Comparison between flight-path response as presented in figure 13 for the airplane-autopilot combination with rate gyro and the flight-path response which would be obtained with perfect attitude response. Solid line indicates airplane-autopilot response. Dashed line indicates perfect attitude response.