NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3103

COOLING REQUIREMENTS FOR STABILITY OF LAMINAR BOUNDARY LAYER WITH SMALL PRESSURE GRADIENT AT SUPERSONIC SPEEDS

By George M. Low

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

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SUMMARY

The amount of cooling required to stabilize the two-dimensional supersonic laminar boundary layer for all Reynolds numbers is calculated for flows with pressure gradients of a magnitude usually encountered over slender aerodynamic shapes. Only two-dimensional disturbances are treated in the stability calculations.

It is determined that small pressure gradients have an appreciable effect on stability. The cooling due to radiation alone may suffice, at moderate supersonic Mach numbers, to completely stabilize the boundary layer over wings with favorable pressure gradients. For flows with adverse pressure gradients, the cooling required for complete stability is considerably greater than that for flat-plate flows.

INTRODUCTION

Because an appreciable portion of the total drag of an airplane or missile can be attributed to friction, a sizable reduction in friction drag is desirable. Such a reduction can be realized by delaying the transition from laminar to turbulent flow.

Two possible explanations for the transition from laminar to turbulent flow have been advanced in the literature. Taylor (ref. 1) proposed that the local pressure gradients accompanying a disturbance in the flow cause intermittent separation of the laminar boundary layer. Eddies formed in the separated region soon diffuse and lead to turbulent flow. Transition by this mechanism might be expected in high turbulence level wind tunnel tests or for flows where the surface roughness is appreciable.

Tollmien and Schlichting (refs. 2 and 3), on the other hand, suggest that infinitesimal wave-like disturbances in the laminar layer are directly responsible for transition. If conditions are such that these infinitesimal disturbances are amplified, they will undergo increasing amounts of distortion and eventually lead to transition from laminar to turbulent motion. Schubauer and Skramstad (ref. 4) have experimentally verified the Tollmien-Schlichting hypothesis for low speed flows. It is believed that transition is caused by laminar instability of this kind whenever the free-stream turbulence level is very low, and no extraneous disturbances such as excessive roughness exist. In free flight the turbulence level of the air is generally low, so that transition as a consequence of laminar instability would be expected whenever the aircraft surfaces are smooth.

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Lees and Lin (refs. 5 and 6) have developed a theory for the stability of the compressible laminar boundary layer based on the Tollmien-Schlichting concept. Their theory predicts that withdrawal of heat from the boundary layer has a stabilizing effect, and suggests that sufficient cooling will stabilize the boundary layer regardless of Reynolds number. Accurate and detailed calculations by Van Driest (ref. 7) based on the theory of Lees and Lin show that the laminar boundary layer on a flat plate can be completely stabilized at Mach numbers between 1 and 9. These results have been qualitatively substantiated by Sternberg (ref. 8), who observed laminar boundary layers at Reynolds numbers as high as 50×10^6.

The theory of Lees and Lin is limited to the flow over a flat plate (zero pressure gradient). It was subsequently shown by Laurmann (ref. 9) and Cheng (ref. 10) that the criteria derived by Lees and Lin apply also to flows over curved surfaces provided the local velocity and temperature profiles are considered.

In the present report calculations based on the theory of Lees and Lin are made for flows with small constant pressure gradients. In particular, the cooling required to completely stabilize the laminar boundary layer for flows with pressure gradients of a magnitude usually encountered over thin aerodynamic shapes at supersonic speeds is calculated. The pressure gradients are considerably smaller than the pressure gradients required for laminar separation. Velocity and temperature profiles for this calculation are obtained from reference 11. The calculations were made at the NACA Lewis laboratory during the summer of 1953.

ASSUMPTIONS AND LIMITATIONS

The present work employs the results of reference 11 and therefore contains the same assumptions, discussed in detail therein, concerning the boundary-layer flow. These include constant specific heat, constant Prandtl number (Pr = 0.72), constant wall temperature, and viscosity proportional to temperature. Van Driest's calculations for the stability of flat-plate boundary layers show that these assumptions do not lead to large errors at Mach numbers less than 3.

The velocity at the outer edge of the boundary layer u_e is postulated to differ only slightly from a reference velocity u_r:

$$\frac{u_e}{u_r} = 1 + \varepsilon x \qquad (1)$$

(All symbols used in this report are defined in appendix A.) The quantity εx is small compared with unity so that the square of this quantity may be neglected. Equation (1) represents velocities which are increasing or decreasing linearly along the body and requires that the pressure gradient dp/dx be constant and equal to $-\gamma \varepsilon M_r^2$. Equation (1) leads to a pressure gradient parameter $\frac{x}{u_r}\frac{du_e}{dx} = \varepsilon x$, which was used in the stability calculations. This parameter is a function not only of the pressure gradient, but also of the distance over which the pressure gradient acts.

It should be noted that the pressure gradients used in this report are the simplest case of the more general pressure distributions treated in reference 11. Although the stability analysis could be carried out for the more general pressure gradients, it was believed that the relative effects of pressure gradients on laminar boundary layer stability could adequately be demonstrated for the special case of a constant pressure gradient.

Because the calculations of Van Driest indicate that the assumption of a linear viscosity-temperature relation leads to appreciable errors in the stability calculations for Mach numbers greater than 3, and because the method of reference 11 becomes questionable at $M_r > 3$ for the values of $\frac{x}{u_r}\frac{du_e}{dx}$ considered herein, all calculations were limited to values of $M_r < 3$.

The assumptions utilized in the solution of the stability differential equations are discussed in references 5 and 6. Although these assumptions originated from a study of the flow over flat plates, it was shown in references 9 and 10 that the equations derived by Lees and Lin apply equally well to flows with pressure gradients provided that local velocity and temperature profiles are considered in the analysis.

STABILITY ANALYSIS

Lees (ref. 6) suggested the possibility that, if sufficient heat is withdrawn from a supersonic laminar boundary layer, the flow will be stable at all Reynolds numbers. This means that no self-excited disturbances can exist that satisfy the stability differential equations, their boundary conditions, and the physical requirement that the dimensionless phase velocity of the disturbance c be greater than $1 - \frac{1}{M_e}$.

The stability differential equations are obtained by linearizing the equations of motion and energy for a flow with time-dependent fluctuating components superposed on the steady mean flow quantities. If the effect of viscosity is neglected in the solution of these equations, the stability of the fluid is found to be governed by the distribution of the product of density and vorticity through the boundary layer. This "inviscid" solution applies only at infinite Reynolds numbers. It was shown in reference 6, however, that the effect of viscosity is destabilizing; a flow which is stable at $Re = \infty$ may not be stable at all other Reynolds numbers. In the so-called viscous solution of the stability differential equations, the effect of viscosity is considered to the first order; these solutions therefore apply at large, but finite, Reynolds numbers. 3117

Because the purpose of the present report is to find a wall temperature ratio below which the flow will be stable at all flight Reynolds numbers, the viscous solution of the stability equations is required. The problem reduces to finding a wall temperature ratio t_w which, for a given free-stream Mach number and pressure gradient, simultaneously satisfies the following equations:

$$V = -\frac{\pi c u_e^* u^{*\prime}(0)}{(t_w^*)^2}\left\{\frac{t^{*2}}{(u^{*\prime})^2}\left[\frac{u^{*\prime\prime}}{u^{*\prime}} - \frac{2t^{*\prime}}{t^*}\right]\right\}_{\frac{u^*}{u_e^*} = c} \tag{2}$$

$$V = \frac{0.580(1 + \lambda)}{1 - 0.960\lambda + 0.570\lambda^2} \tag{3}$$

where

$$\lambda = \frac{u^{*\prime}(0)\ It^*(\eta_c)}{u_e^* t_w^* c} - 1 \tag{4}$$

$$c = 1 - \frac{1}{M_e} \tag{5}$$

Equation (2) is equivalent to equation (24) of reference 6, while equation (3) is obtained by combining equations (19) and (20) of reference 6, as suggested by Bloom (ref. 12). Equations (2), (3), and (4) are valid in the transformed coordinate system of reference 11. A table of relations between the terms appearing in the present report and those of reference 6 is presented in appendix B.

Once the free-stream Mach number and pressure gradient are specified and a wall temperature ratio is assumed, all terms in equations (2) and (3) can be found from reference 11. (The relations between these terms and the functions tabulated in ref. 11 are presented in appendix C.) The correct wall temperature ratio satisfying both equations was found by trial and error.[1]

EFFECT OF PRESSURE GRADIENT ON COOLING REQUIREMENTS FOR STABILITY

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Equations (2) and (3) were solved for Mach numbers between 1 and 3, and for values of the pressure gradient parameter $\frac{x}{u_r}\frac{du_e}{dx}$ of 0, ± 0.05, and ± 0.10. The limiting wall temperature ratio for complete stability t_w/t_e is presented in figure 1 as a function of local stream Mach number M_e. At a given pressure gradient the laminar boundary layer is stable for all Reynolds numbers if the wall temperature ratio is less than the value given on the curve for that particular gradient. If t_w/t_e is greater than that value, self-excited disturbances will exist if the Reynolds number is sufficiently high. Also shown in figure 1 are the wall temperature ratio for zero heat transfer and the wall temperature ratio 5 feet aft of the leading edge of a flat plate for black-body radiation at an altitude of 50,000 feet. (Both the zero heat transfer curve and the radiation curve are altered only slightly by the pressure gradient.) The wall temperature ratio for radiation is obtained from a balance of the heat lost by the surface through radiation and the heat gained by the surface through convection and conduction. It was assumed, as is conventional, that the surface radiates to a mean receptor temperature equal to the ambient temperature. As a comparison, this heat balance was also made for a point $2\frac{1}{2}$ feet aft of the leading edge, and it was found that the resulting equilibrium wall temperature ratio at $M_e = 3$ was 4 percent higher than at the 5-foot station. This difference in temperature ratios decreases as the Mach number decreases.

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[1]Dr. C. C. Lin and Dr. D. W. Dunn have informed the author that they have obtained improved viscous solutions of the stability differential equations, but that calculations based on these solutions agree to within 2 percent with the present results for the case of zero pressure gradient. For the case of $\frac{x}{u_r}\frac{du_e}{dx} = 0.10$ and $M_e = 3.06$, the new solutions yielded a wall temperature ratio 0.5 percent higher than reported herein. It is therefore believed that all the present results are in error by no more than about 2 percent.

The pressure gradients represented in this figure are of a magnitude that might be encountered over thin wings at supersonic speeds. For example, the value of $\frac{x}{u_r}\frac{du_e}{dx}$ at the midchord station of a 5 percent thick circular arc airfoil at a Mach number of 2 is 0.06. The adverse pressure gradients $\left(\frac{x}{u_r}\frac{du_e}{dx}\ \text{negative}\right)$ might be found on compression ramps of supersonic engine inlets. It should be noted that the parameter $\frac{x}{u_r}\frac{du_e}{dx}$ will not be a constant along a surface when the pressure gradient is constant. Thus a given stability limit, as plotted in figure 1, applies only at one point on a wing, and the boundary layer may be more stable (or less stable) at other chordwise stations.

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It is evident that the effect of a small pressure gradient on the cooling requirements for complete stability is appreciable. At a local stream Mach number of 2, and for a value of $\frac{x}{u_r}\frac{du_e}{dx}$ equal to 0.10, the wall temperature ratio for stability is not far below the zero-heat-transfer temperature ratio. For this pressure gradient the cooling due to radiation alone is seen to stabilize the boundary layer for $M_e > 1.7$. For flows with adverse pressure gradients, the wall temperature ratio for complete stability is considerably lower than for flat-plate flows.

The effects of pressure gradient may be demonstrated in terms of the relative rate of heat transfer required, in the absence of radiation, to stabilize flows with pressure gradients as compared with flat-plate flows. The ratio of heat-transfer rates q/q_{FP}, as obtained from the temperature ratios of figure 1 and from equation (63) of reference 11, is plotted as a function of local stream Mach number in figure 2, and applies at an ambient air temperature of -67° F. At a Mach number of 2.5, only 5 percent of the cooling required to stabilize the flow over a flat plate will suffice to stabilize the flow over a wing when $\frac{x}{u_r}\frac{du_e}{dx} = 0.10$. For the most adverse pressure gradient considered, on the other hand, the ratio q/q_{FP} reaches a peak value of about 2.2.

There appears to be an optimum Mach number, in the neighborhood of 2, where pressure gradients have a large effect on the cooling requirements for complete stability. At higher Mach numbers the frictional heating terms in the boundary layer energy equation become of greater importance, and the relative effect of pressure gradient becomes less important.

A possible shortcoming of the present calculations, in common with the shortcoming of previous papers, is that only two-dimensional disturbances are treated. It was shown by Squire (ref. 13) that, for incompressible flow, two-dimensional disturbances are always more destabilizing than three-dimensional disturbances. Since the present calculations have been completed, however, it has been reported by Dunn and Lin (ref. 14) that Squire's theorem cannot be extended to supersonic flows, and that under certain conditions three-dimensional disturbances will be more destabilizing than two-dimensional disturbances. The trends of the two-dimensional theory have, however, been verified experimentally (ref. 8). It is believed, therefore, that the present results at least qualitatively describe the effects of pressure gradients on boundary layer stability.

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CONCLUDING REMARKS

The amount of cooling required to completely stabilize a two-dimensional laminar boundary layer with small pressure gradients in supersonic flow has been calculated. It was found that the wall temperature ratio for complete stability approaches the zero-heat-transfer temperature ratio for reasonably small favorable pressure gradients. At a local stream Mach number of 2.5, for example, only 5 percent of the cooling required to stabilize the flow over a flat plate will suffice to stabilize the flow over a wing when the pressure gradient parameter $\frac{x}{u_r}\frac{du_e}{dx}$ equals 0.10. Under these conditions the cooling due to radiation may be adequate to stabilize the boundary layer for all Reynolds numbers.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, November 3, 1953

APPENDIX A

SYMBOLS

The following symbols are used in this report:

c	phase velocity of disturbance divided by u_e
M	Mach number
q	local rate of heat transfer
t	static temperature
t*	t/t_r
u	velocity in x-direction
u*	u/u_r
V	function appearing in eqs. (2) and (3)
x	distance along surface measured from leading edge
γ	ratio of specific heats
ε	small quantity - measure of shape and magnitude of velocity distribution at outer edge of boundary layer
η	characteristic variable (see appendix B)
λ	wave length of disturbance

Subscripts:

c	value of function when $\frac{u}{u_e} = c$
e	conditions at outer edge of boundary layer
FP	equivalent flat-plate value
r	reference condition (see ref. 11)
w	conditions at wall or surface

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Superscripts:

' differentiation with respect to η

* dimensionless quantity

Special notation:

The symbol I preceding a quantity indicates integration from zero to η; for example,

$$It^*(\eta_c) = \int_0^{\eta_c} t^*(\xi)d\xi$$

APPENDIX B

RELATION BETWEEN NOTATION OF REFERENCE 6 AND NOTATION EMPLOYED IN PRESENT REPORT

Because the present stability calculations are based on the velocity and temperature distributions of reference 11, it was necessary to obtain the stability equations in terms of the variables used in that reference. A table of equivalent relations which are required in order to obtain equations (2), (3), and (4) from the equations of reference 6 is therefore presented: 3117

Lees' notation (ref. 6)	Present notation
c	c
T	t^*/t_e^*
v	V
w	u^*/u_e^*
y	$It^*(\eta)$
λ	λ
$' = \frac{d}{dy}$	$' = \frac{d}{d\eta}, \quad \frac{d}{dy} = \frac{1}{t^*}\frac{d}{d\eta}$
$'' = \frac{d^2}{dy^2}$	$'' = \frac{d^2}{d\eta^2}, \frac{d^2}{dy^2} = \frac{1}{(t^*)^2}\frac{d^2}{d\eta^2} - \frac{t^{*\prime}}{(t^*)^3}\frac{d}{d\eta}$
Subscript 1	Subscript w
Subscript 0	Subscript e

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APPENDIX C

DEFINITION OF TERMS USED IN STABILITY CALCULATION

The terms required in the stability calculation for flows with pressure gradients are related to the functions tabulated in reference 11 as follows:

$$u^* = \frac{1}{2} f'(\eta) + \epsilon x\ g'(\eta)$$

$$u^{*\prime} = \frac{1}{2} f''(\eta) + \epsilon x\ g''(\eta)$$

$$u^{*\prime\prime} = \frac{1}{2} f'''(\eta) + \epsilon x\ g'''(\eta)$$

$$t^* = 1 + K\ s(\eta) + \frac{\gamma-1}{2} M_r^2 \left[r(\eta) - 2\epsilon x\ H(\eta)\right]$$

$$t^{*\prime} = K\ s'(\eta) + \frac{\gamma-1}{2} M_r^2 \left[r'(\eta) - 2\epsilon x\ H'(\eta)\right]$$

$$It^*(\eta) = \eta + K\ Is(\eta) + \frac{\gamma-1}{2} M_r^2 \left[Ir(\eta) - 2\epsilon x\ IH(\eta)\right]$$

where

$$g(\eta) = g_{11}(\eta) + M_r^2\ g_{12}(\eta) + Kg_{13}(\eta)$$

$$H(\eta) = H_{11}(\eta) + M_r^2\ H_{12}(\eta) + KH_{13}(\eta) + \frac{K}{M_r^2} H_{14}(\eta) + \frac{K^2}{M_r^2} H_{15}(\eta)$$

and

$$K = \frac{1}{s(0)} \left[\frac{t_w}{t_r} - 1 - \frac{\gamma-1}{2} M_r^2\ r(0)\right]$$

All functions of η with the exception of $f'''(\eta)$ and $g'''(\eta)$ are tabulated in reference 11. The function $g'''(\eta)$ is presented in table I of the present report, while f''' is obtained from the Blasius equation

$$f''' = -ff''$$

The relation between the local stream Mach number M_e and the reference Mach number M_r is obtained from equation (1) and the corresponding equation for the local stream temperature

$$t_e^* = 1 - (\gamma-1)\, M_r^2\, \varepsilon x$$

Thus

$$M_e = M_r \left[1 + \varepsilon x \left(1 + \frac{\gamma-1}{2} M_r^2\right)\right]$$

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TABLE I. - THE FUNCTION OF $g_1'''(\eta)$

η	g_{11}'''	g_{12}'''	g_{13}'''
0	-4.0000	-0.6781	-8.2990
.1	-4.0000	-.6694	-7.8084
.2	-3.9996	-.6433	-7.3179
.3	-3.9972	-.6001	-6.8275
.4	-3.9887	-.5404	-6.3352
.5	-3.9670	-.4656	-5.8374
.6	-3.9219	-.3779	-5.3284
.7	-3.8409	-.2807	-4.8016
.8	-3.7105	-.1781	-4.2507
.9	-3.5178	-.0755	-3.6715
1.0	-3.2538	.0214	-3.0637
1.1	-2.9147	.1068	-2.4326
1.2	-2.5047	.1757	-1.7898
1.3	-2.0358	.2241	-1.1526
1.4	-1.5283	.2501	-.5428
1.5	-1.0080	.2539	.0163
1.6	-.5037	.2378	.5023
1.7	-.0432	.2060	.8973
1.8	.3501	.1636	1.1902
1.9	.6597	.1163	1.3781
2.0	.8780	.0693	1.4661
2.1	1.0060	.0270	1.4663
2.2	1.0522	-.0078	1.3957
2.3	1.0306	-.0336	1.2738
2.4	.9583	-.0503	1.1202
2.5	.8527	-.0587	.9525
2.6	.7299	-.0603	.7850
2.7	.6033	-.0571	.6284
2.8	.4828	-.0508	.4894
2.9	.3748	-.0430	.3712
3.0	.2828	-.0348	.2746
3.1	.2074	-.0271	.1981
3.2	.1485	-.0204	.1400
3.3	.1028	-.0147	.0960
3.4	.0697	-.0103	.0647
3.5	.0463	-.0071	.0427
3.6	.0298	-.0047	.0276
3.7	.0189	-.0030	.0175
3.8	.0116	-.0020	.0108
3.9	.0071	-.0012	.0067
4.0	.0041	-.0006	.0040
4.1	.0024	-.0004	.0024
4.2	.0014	-.0002	.0014
4.3	.0007	-.0001	.0008
4.4	.0004	-.0001	.0004
4.5	.0002	0	.0002

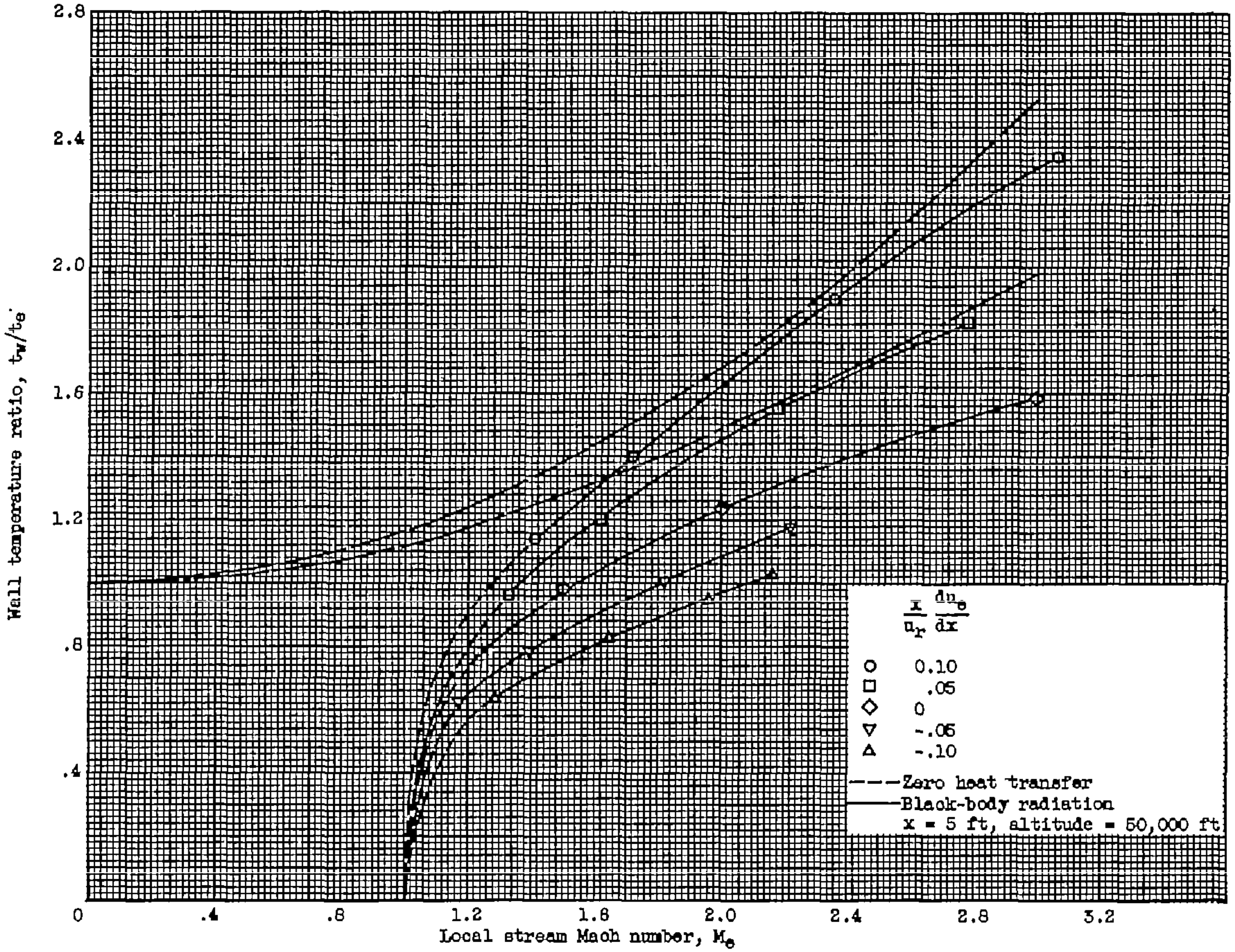

Figure 1. - Limiting wall temperature required for complete stabilization of laminar boundary layer.

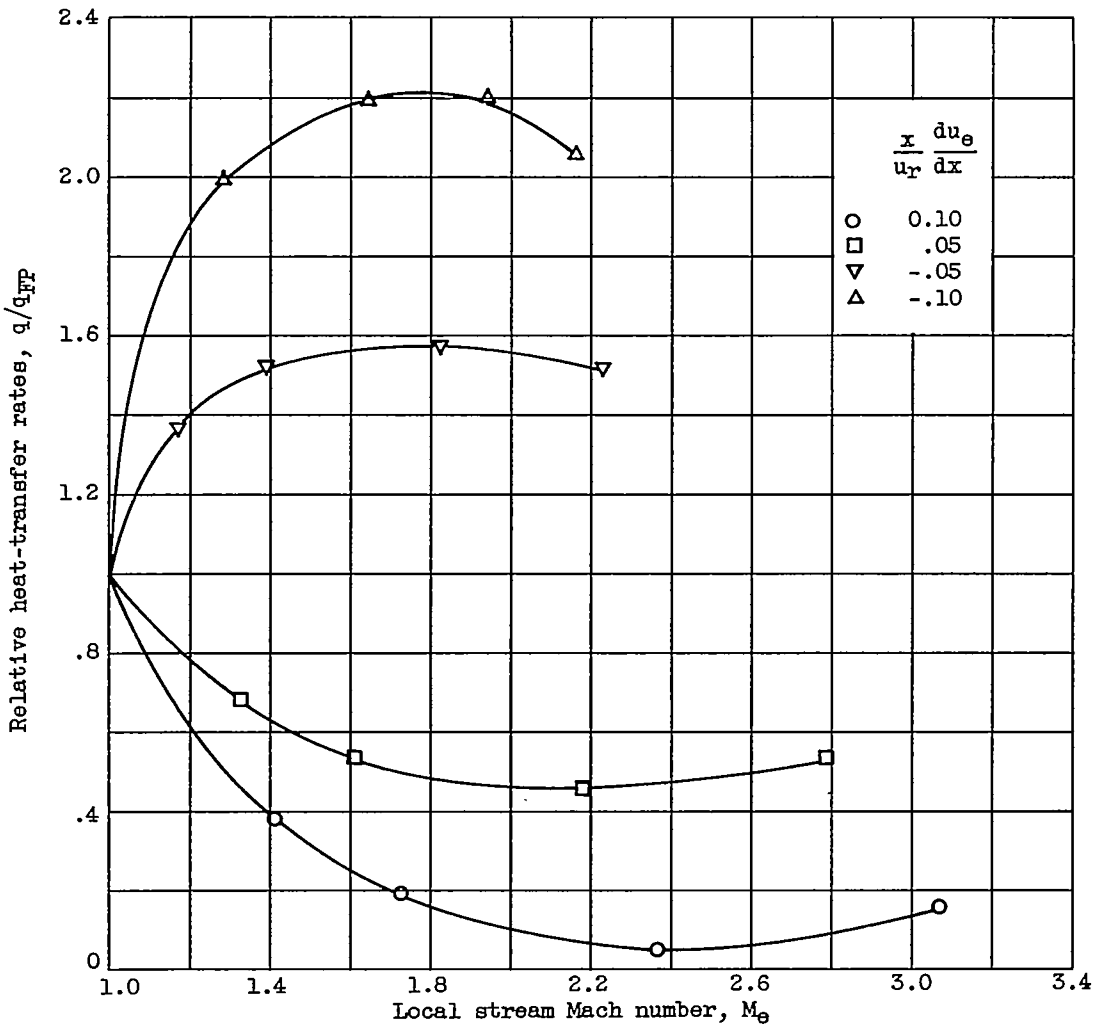

Figure 2. - Cooling requirements for flows with pressure gradients referred to cooling requirements for flat-plate flows.

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