NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2679

THE STABILITY UNDER LONGITUDINAL COMPRESSION OF FLAT SYMMETRIC CORRUGATED-CORE SANDWICH PLATES WITH SIMPLY SUPPORTED LOADED EDGES AND SIMPLY SUPPORTED OR CLAMPED UNLOADED EDGES

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Washington
April 1952

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SUMMARY

A theory for the elastic behavior of orthotropic sandwich plates is used to determine the compressive-buckling-load parameters of flat symmetric corrugated-core sandwich plates with simply supported loaded edges and simply supported or clamped unloaded edges. Charts are presented for corrugated-core sandwich plates for which the transverse shear stiffness in planes parallel to the axis of the corrugations may be assumed infinite. The limits of validity of this assumption are investigated for simply supported plates.

INTRODUCTION

Considerable work has been done on the problem of the stability of sandwich plates with isotropic faces and isotropic non-stress-carrying core materials such as end-grain balsa or cellular cellulose acetate. The corrugated-core sandwich plate, which consists of a corrugated metal sheet fastened between two flat sheets, is of a different type in that the core has orthotropic flexural and transverse shear properties. The transverse shear stiffness in planes parallel to the axis of the corrugations is usually many times the stiffness in planes perpendicular to the axis of the corrugations and may be considered infinite for many practical constructions. The flexural properties are such that the corrugated core can to some extent resist bending moments applied in planes parallel to the axis of the corrugations, whereas its resistance to bending moments applied in planes perpendicular to the axis of the corrugations is negligible.

The theory of reference 1, together with the physical constants derived in reference 2, makes possible the determination of the elastic over-all buckling loads of flat corrugated-core sandwich plates with symmetric corrugated cores. By over-all buckling is meant buckling of the sandwich plate as a whole, without regard to local buckling of the faces between corrugation crests or of the corrugation walls. In the present paper the theory is applied to the problem of the stability under longitudinal compression of flat symmetric corrugated-core sandwich plates with simply supported loaded edges and simply supported or clamped unloaded edges.

Stability criterions are derived for these two problems in an appendix. Numerical results are presented in the form of charts which show the variation of the compressive-buckling-load parameter with plate aspect ratio for different values of parameters involving the flexural and transverse shear stiffness of the corrugated core. For these charts the transverse shear stiffness in planes parallel to the axis of the corrugations was taken as infinite. The limits of validity of the charts for corrugated-core sandwich plates having finite transverse shear stiffness in both directions are investigated for simply supported plates. Some of the numerical results of the present paper have been published separately in references 3 and 4.

SYMBOLS

A, B, C	coefficients in expressions for w, Q_x, and Q_y, respectively, for a plate with simply supported unloaded edges
A_i, B_i, C_i	coefficients in expressions for w, Q_x, and Q_y, respectively, for a plate with clamped unloaded edges $(i = 1, 2, 3)$
a	plate length
b	plate width
D_{Q_x}, D_{Q_y}	transverse shear stiffnesses per unit width of a beam cut from plate in x- and y-directions, respectively (formulas and charts for calculation of D_{Q_x} and D_{Q_y} are given in reference 2)
E_C	Young's modulus of elasticity of core material
E_S	Young's modulus of elasticity of face material
h	distance between middle surfaces of face sheets

$\bar{I}_C$ — moment of inertia per unit width of corrugation cross section in planes perpendicular to x-axis, taken about sandwich-plate middle surface

$\bar{I}_S$ — moment of inertia per unit width of faces, considered as membranes, taken about sandwich-plate middle surface $\left(\frac{1}{2} t_S h^2\right)$

m — number of buckle half-waves in x-direction

N — longitudinal compressive buckling load per unit width of sandwich plate

n — number of buckle half-waves in y-direction of a plate with simply supported unloaded edges

n_i — arbitrary quantities in expressions for w, Q_x, and Q_y for a plate with clamped unloaded edges $(i = 1, 2, 3)$

Q_x, Q_y — transverse shear forces in planes perpendicular to the x- and y-axes, respectively

t_S — face thickness

w — deflection of sandwich-plate middle surface

x, y — coordinate axes (see fig. 1)

μ_S — Poisson's ratio of face material

∇^2 — differential operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$

Parameters used in the presentation of results:

a/b — plate aspect ratio

$\frac{b^2 N}{\pi^2 E_S \bar{I}_S}$ — compressive-buckling-load parameter

$\frac{E_C \bar{I}_C}{E_S \bar{I}_S}$ — flexural-stiffness ratio

$\frac{\pi^2 E_S \bar{I}_S}{b^2 D_{Q_x}}$, $\frac{\pi^2 E_S \bar{I}_S}{b^2 D_{Q_y}}$ transverse-shear-flexibility parameters

$\frac{D_{Q_x}}{D_{Q_y}}$ transverse-shear-stiffness ratio

For convenience, these parameters are abbreviated in the appendix as follows

β plate aspect ratio (a/b)

k compressive-buckling-load parameter $\left(\frac{b^2 N}{\pi^2 E_S \bar{I}_S}\right)$

η flexural-stiffness ratio $\left(\frac{E_C \bar{I}_C}{E_S \bar{I}_S}\right)$

r_x, r_y transverse-shear-flexibility parameters $\left(\frac{\pi^2 E_S \bar{I}_S}{b^2 D_{Q_x}}\right.$ and $\left.\frac{\pi^2 E_S \bar{I}_S}{b^2 D_{Q_y}}\text{, respectively}\right)$

CORRUGATED-CORE SANDWICH-PLATE THEORY

The sandwich-plate theory of reference 1 deals with elastic plates of continuous construction which have orthotropic flexural and transverse-shear properties. Straight lines in the plate that are originally perpendicular to the undeformed plate middle surface are assumed to remain straight but not necessarily perpendicular to the plate middle surface after bending occurs. The plate is also assumed to have no local deformations. This last assumption permits the analysis of only the over-all stability of sandwich plates.

Corrugated-core sandwich plates can be analyzed by this theory provided that limitations are imposed on the relative dimensions of the sandwich faces and of the core. The pitch of the core corrugations should be small compared with the plate width perpendicular to the axis of the corrugations so that the plate can be treated adequately as a continuous orthotropic medium. The thickness of the faces should be small compared with the over-all plate thickness in order that bending of each face about its own middle surface may be neglected. The core should be sufficiently stiff so that changes in the plate thickness are negligible.

The investigations of reference 2 indicate that limitations should also be placed on the type of corrugated-core sandwich plate that may be analyzed by the sandwich-plate theory of reference 1. The Poisson's ratios of the materials of the two faces must be equal, the neutral plane of bending of the faces alone must coincide with the plane passing through the centroidal axis of the corrugation cross section, and load resultants must be applied in specified planes between the plate faces. Symmetric corrugated-core sandwich plates - that is, plates with faces of equal thickness and of the same material and with a corrugated core having symmetrical corrugations - satisfy these conditions, provided that load resultants are applied in the plane of the plate middle surface.

RESULTS AND DISCUSSION

In the present paper the compressive buckling of flat symmetric corrugated-core sandwich plates with simply supported loaded edges and simply supported or clamped unloaded edges (see fig. 1) is investigated. For symmetric corrugated-core sandwich plates the elastic constants derived in reference 2 reduce to the relatively simple forms given in the appendix. The solution can be expressed in terms of six nondimensional quantities

$\dfrac{b^2N}{\pi^2 E_S \bar{I}_S}$	compressive-buckling-load parameter
a/b	plate aspect ratio
$\dfrac{E_C \bar{I}_C}{E_S \bar{I}_S}$	flexural-stiffness ratio
$\dfrac{\pi^2 E_S \bar{I}_S}{b^2 D_{Q_x}}$, $\dfrac{\pi^2 E_S \bar{I}_S}{b^2 D_{Q_y}}$	transverse-shear-flexibility parameters
μ_S	Poisson's ratio of face material

Stability criterions which relate these six parameters for the two problems considered are derived in the appendix (equation (6) for plates with simply supported unloaded edges and equations (11) and (15) for plates with clamped unloaded edges).

For many practical structures the transverse shear stiffness in planes parallel to the axis of the corrugations is very much greater than the transverse shear stiffness in planes perpendicular to the axis of the corrugations and may be assumed infinite. In the present paper the corrugations are taken parallel to the x-axis, in which case D_{Q_x} is the transverse shear stiffness that may be assumed infinite. The transverse-shear-flexibility parameter $\frac{\pi^2 E_S \overline{I}_S}{b^2 D_{Q_x}}$ is then equal to zero and the stability criterions reduce to equation (7) for plates with simply supported unloaded edges and to equations (16) and (17) for plates with clamped unloaded edges.

Charts have been prepared for the case of infinite D_{Q_x} and show the variation of the compressive-buckling-load parameter $\frac{b^2 N}{\pi^2 E_S \overline{I}_S}$ with plate aspect ratio a/b for the transverse-shear-flexibility parameter $\frac{\pi^2 E_S \overline{I}_S}{b^2 D_{Q_y}}$ equal to 0, 0.1, 0.25, 0.5, 1.0, and ∞ and the flexural-stiffness ratio $\frac{E_C \overline{I}_C}{E_S \overline{I}_S}$ equal to 0, 0.5, and 1.0. Poisson's ratio for the face material μ_S has been taken as 1/3. In figure 2, the charts for finite plates with simply supported unloaded edges are presented. The compressive-buckling-load parameter for infinitely long plates with simply supported edges is plotted against the transverse-shear-flexibility parameter $\frac{\pi^2 E_S \overline{I}_S}{b^2 D_{Q_y}}$ in figure 3. These curves were obtained by replotting the minimum values of compressive-buckling-load parameter of the curves of figure 2. Similar charts are presented for plates with clamped unloaded edges in figures 4 and 5.

The trends of the curves of figures 2 to 5 are similar in some respects to those of the curves of references 5 and 6 for isotropic sandwich plates. As the transverse-shear-flexibility parameter $\frac{\pi^2 E_S \overline{I}_S}{b^2 D_{Q_y}}$ increases, the compressive-buckling-load parameters are materially reduced. The reductions are greater for plates with clamped unloaded edges, because the effect of edge clamping is lessened with increasing transverse shear flexibility and the values of the compressive-buckling-load parameter for plates with clamped unloaded edges approach those of plates with simply supported unloaded edges. Because the corrugated-core sandwich plates are assumed to have unequal transverse shear stiffnesses, the transverse shear stiffness D_{Q_x} being greater than the transverse shear stiffness D_{Q_y}, the reductions in compressive-buckling-load parameters are not so great as those for isotropic-core sandwich plates. For example, for the case of a corrugated-core sandwich plate with infinite transverse shear stiffness D_{Q_x} and zero transverse shear stiffness D_{Q_y}, the compressive-buckling-load parameter is finite, rather than being equal to zero, and varies with plate aspect ratio.

The longitudinal buckle half wave lengths of isotropic-core sandwich plates decrease with decreasing transverse shear stiffness; the buckle wave lengths of corrugated-core sandwich plates with infinite transverse shear stiffness D_{Q_x}, however, tend to increase with decreasing transverse shear stiffness D_{Q_y}. Calculations indicate that a corrugated-core sandwich plate with zero transverse shear stiffness D_{Q_y} buckles in only one longitudinal half wave, regardless of the plate aspect ratio. The buckle half wave lengths of infinitely long corrugated-core sandwich plates with simply supported edges remain relatively constant over a large range of values of the transverse-shear-flexibility parameter and flexural-stiffness ratio. The half wave length is approximately equal to the plate width for values of the transverse-shear-flexibility parameter $\frac{\pi^2 E_S \bar{I}_S}{b^2 D_{Q_y}}$ varying from 0 to 1.0 and for values of the flexural-stiffness ratio varying from 0 to 1.0. In this range the half wave length is 1.0 to 1.3 times the plate width. For plates with clamped unloaded edges, the half wave length of buckle varies somewhat more (from 0.66 to 1.2 times the plate width).

EFFECT OF FINITE TRANSVERSE SHEAR STIFFNESS D_{Q_x}

Although it is customary to assume in the analysis of corrugated-core sandwich plates that the transverse shear stiffness D_{Q_x} is infinite, little or no information as to the limits of validity of this assumption is available. Calculations have been made for the present paper to determine the minimum value of D_{Q_x}/D_{Q_y} for which the assumption of infinite D_{Q_x} is adequate.

Values of $\frac{b^2 N}{\pi^2 E_S \bar{I}_S}$ for infinitely long simply supported plates with various values of D_{Q_x}/D_{Q_y} are shown in figures 6(a) to 6(c). These values were obtained by plotting values of $\frac{b^2 N}{\pi^2 E_S \bar{I}_S}$ given by equation (6) for several values of the buckle aspect ratio a/mb and by picking off the minimum of the curve so defined. This procedure was repeated for various sets of values of $\frac{\pi^2 E_S \bar{I}_S}{b^2 D_{Q_y}}$, $\frac{E_C \bar{I}_C}{E_S \bar{I}_S}$, and $\frac{\pi^2 E_S \bar{I}_S / b^2 D_{Q_y}}{\pi^2 E_S \bar{I}_S / b^2 D_{Q_x}}$ or $\frac{D_{Q_x}}{D_{Q_y}}$. Since the transverse-shear-flexibility parameter $\frac{\pi^2 E_S \bar{I}_S}{b^2 D_{Q_y}}$ for plates of practical dimensions is less than about 0.5, it may be concluded that values of $\frac{b^2 N}{\pi^2 E_S \bar{I}_S}$ are given with little error by the curve for D_{Q_x} equal to infinity $\left(\frac{D_{Q_x}}{D_{Q_y}} = \infty\right)$ if the transverse-shear-stiffness ratio D_{Q_x}/D_{Q_y} is greater than about 10. Calculations indicate that this conclusion applies also to plates of finite length, provided that the plate aspect ratio a/b is not less than about 0.6.

Because of the complexity of the stability criterion for compressive buckling of corrugated-core sandwich plates with simply supported loaded edges and clamped unloaded edges (equations (11) and (15)), no attempt was made to obtain information as to the limits of validity of the assumption of infinite transverse shear stiffness D_{Q_x}. It seems reasonable, however, to expect a transverse-shear-stiffness ratio D_{Q_x}/D_{Q_y} of about 10 to be a lower limit for the assumption to be adequate.

CONCLUDING REMARKS

The over-all stability of flat symmetric corrugated-core sandwich plates with simply supported loaded edges and simply supported or clamped unloaded edges has been investigated by the use of the orthotropic-sandwich-plate theory of NACA Rep. 899 in conjunction with the physical constants for symmetric corrugated-core sandwich plates derived in NACA TN 2289. Charts showing the variation of the compressive-buckling-load coefficient with plate aspect ratio, transverse-shear-flexibility parameter, and flexural-stiffness ratio have been prepared for plates for which the transverse shear stiffness in planes parallel to the axis of the corrugations can be assumed to be infinite.

By use of the more general equations derived for corrugated-core sandwich plates with finite transverse shear stiffness in both directions, it is concluded that these charts may be considered adequate for plates of practical dimensions for which the transverse shear stiffness in planes parallel to the axis of the corrugations is 10 or more times the transverse shear stiffness in planes perpendicular to the axis of the corrugations, provided that the plate aspect ratio is greater than about 0.6.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., January 22, 1952

APPENDIX

DERIVATION OF STABILITY CRITERIONS

Differential Equations

Equations that may be used for the determination of the compressive buckling loads of flat symmetric corrugated-core sandwich plates are given in reference 1. The seven physical constants of corrugated-core sandwich plates required for the use of these equations are derived in reference 2 and are as follows for plates having thin faces of equal thickness and symmetrical cores

$$\left.\begin{aligned}
D_x &= E_S\bar{I}_S\left(1 + \frac{E_C\bar{I}_C}{E_S\bar{I}_S}\right)\\
D_y &= E_S\bar{I}_S\left[\frac{1 + \frac{E_C\bar{I}_C}{E_S\bar{I}_S}}{1 + \left(1 - \mu_S{}^2\right)\frac{E_C\bar{I}_C}{E_S\bar{I}_S}}\right]\\
D_{xy} &= E_S\bar{I}_S\left(\frac{1}{1 + \mu_S}\right)\\
\mu_x &= \mu_S\\
\mu_y &= \mu_S\left[\frac{1}{1 + \left(1 - \mu_S{}^2\right)\frac{E_C\bar{I}_C}{E_S\bar{I}_S}}\right]\\
&D_{Q_x}\\
&D_{Q_y}
\end{aligned}\right\} \qquad (1)$$

The formulas for the calculation of D_{Q_x} and D_{Q_y}, derived in reference 2, are rather cumbersome and are not presented here.

The differential equations of reference 1 are given for the present problem as

$$
\left.
\begin{aligned}
&N \frac{\partial^2 w}{\partial x^2} - \frac{\partial Q_x}{\partial x} - \frac{\partial Q_y}{\partial y} = 0 \\[2ex]
&\left(\frac{E_S\bar{I}_S}{1 - \mu_S{}^2} \frac{\partial}{\partial x} \nabla^2 + E_C\bar{I}_C \frac{\partial^3}{\partial x^3}\right)w + \left[1 - \frac{E_S\bar{I}_S}{2(1 + \mu_S)} \frac{1}{D_{Q_x}} \frac{\partial^2}{\partial y^2} - \right. \\
&\qquad \left. \left(\frac{E_S\bar{I}_S}{1 - \mu_S{}^2} + E_C\bar{I}_C\right)\frac{1}{D_{Q_x}} \frac{\partial^2}{\partial x^2}\right]Q_x - \frac{E_S\bar{I}_S}{2(1 - \mu_S)} \frac{1}{D_{Q_y}} \frac{\partial^2 Q_y}{\partial x\, \partial y} = 0 \\[2ex]
&\frac{E_S\bar{I}_S}{1 - \mu_S{}^2} \frac{\partial}{\partial y} \nabla^2 w - \frac{E_S\bar{I}_S}{2(1 - \mu_S)} \frac{1}{D_{Q_x}} \frac{\partial^2 Q_x}{\partial x\, \partial y} + \left[1 - \frac{E_S\bar{I}_S}{2(1 + \mu_S)} \frac{1}{D_{Q_y}} \frac{\partial^2}{\partial x^2} - \right. \\
&\qquad \left. \frac{E_S\bar{I}_S}{1 - \mu_S{}^2} \frac{1}{D_{Q_y}} \frac{\partial^2}{\partial y^2}\right]Q_y = 0
\end{aligned}
\right\} \qquad (2)
$$

Simply Supported Unloaded Edges

The conditions that are satisfied at the edges of the simply supported plate (fig. 1(a)) are those of zero deflection of the middle surface, zero moment normal to the edges, and no relative movement parallel to the edges of points in the boundary. These conditions may be expressed in terms of w, Q_x, and Q_y, at $x = 0$ and $x = a$, as

$$
w = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x} - \frac{Q_x}{D_{Q_x}}\right) + \frac{\mu_S}{1 + \left(1 - \mu_S{}^2\right)\dfrac{E_C\bar{I}_C}{E_S\bar{I}_S}} \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y} - \frac{Q_y}{D_{Q_y}}\right) = \frac{Q_y}{D_{Q_y}} = 0 \qquad (3a)
$$

and, at $y = 0$ and $y = b$, as

$$w = \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y} - \frac{Q_y}{D_{Q_y}}\right) + \mu_S \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x} - \frac{Q_x}{D_{Q_x}}\right) = \frac{Q_x}{D_{Q_x}} = 0 \tag{3b}$$

Functions that satisfy these boundary conditions are

$$\left.\begin{aligned} w &= A \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \\ Q_x &= B \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \\ Q_y &= C \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \end{aligned}\right\} \tag{4}$$

The differential equations will be satisfied also if the following set of simultaneous homogeneous equations, obtained by substituting equations (4) into equations (2) and rearranging terms, is satisfied:

$$\left.\begin{aligned} &\frac{m^2}{\beta^2} kA - \frac{m}{\beta} \frac{1}{E_S \bar{I}_S}\left(\frac{b}{\pi}\right)^3 B - n \frac{1}{E_S \bar{I}_S}\left(\frac{b}{\pi}\right)^3 C = 0 \\ &\frac{m}{\beta}\left\{n^2 + \frac{m^2}{\beta^2}\left[1 + \left(1 - \mu_S^2\right)\eta\right]\right\} A - \left(1 - \mu_S^2 + \left\{\frac{1 - \mu_S}{2} n^2 + \frac{m^2}{\beta^2}\left[1 + \right.\right.\right. \\ &\qquad \left.\left.\left.\left(1 - \mu_S^2\right)\eta\right]\right\} r_x\right)\frac{1}{E_S \bar{I}_S}\left(\frac{b}{\pi}\right)^3 B - \frac{1 + \mu_S}{2} n \frac{m}{\beta} r_y \frac{1}{E_S \bar{I}_S}\left(\frac{b}{\pi}\right)^3 C = 0 \\ &n\left(\frac{m^2}{\beta^2} + n^2\right)A - \frac{1 + \mu_S}{2} n \frac{m}{\beta} r_x \frac{1}{E_S \bar{I}_S}\left(\frac{b}{\pi}\right)^3 B - \\ &\qquad \left[1 - \mu_S^2 + \left(\frac{1 - \mu_S}{2} \frac{m^2}{\beta^2} + n^2\right) r_y\right] \frac{1}{E_S \bar{I}_S}\left(\frac{b}{\pi}\right)^3 C = 0 \end{aligned}\right\} \tag{5}$$

The condition that A, B, and C have values other than zero - that is, that the determinant of their coefficients in equations (5) vanish - yields the stability criterion for simply supported corrugated-core sandwich plates under longitudinal compression

$$k = \frac{\left\{\left(\frac{m}{\beta} + n^2 \frac{\beta}{m}\right)^2 + \left(1 - \mu_S{}^2\right)\eta \frac{m^2}{\beta^2} + \left[\frac{1}{2\left(1 + \mu_S\right)}\left(\frac{m}{\beta} + n^2 \frac{\beta}{m}\right)^2 + \eta\left(n^2 + \frac{1 - \mu_S}{2} \frac{m^2}{\beta^2}\right)\right]\left(n^2 r_x + \frac{m^2}{\beta^2} r_y\right)\right\}}{\left\{1 - \mu_S{}^2 + \left[\frac{1 - \mu_S}{2} n^2 + \frac{m^2}{\beta^2} + \left(1 - \mu_S{}^2\right)\eta \frac{m^2}{\beta^2}\right]r_x + \left(n^2 + \frac{1 - \mu_S}{2} \frac{m^2}{\beta^2}\right)r_y + \left[\frac{1}{2\left(1 + \mu_S\right)}\left(\frac{m^2}{\beta^2} + n^2\right)^2 + \eta \frac{m^2}{\beta^2}\left(n^2 + \frac{1 - \mu_S}{2} \frac{m^2}{\beta^2}\right)\right]r_x r_y\right\}} \tag{6}$$

In the analysis of corrugated-core sandwich plates in which the corrugations are oriented in the direction of the x-axis, it is often assumed that D_{Q_x} is infinite. Then r_x is zero and the stability criterion is simplified to

$$k = \frac{\left(\frac{m}{\beta} + n^2 \frac{\beta}{m}\right)^2}{1 - \mu_S{}^2 + \frac{n^2}{\frac{1}{r_y} + \frac{1}{2\left(1 + \mu_S\right)} \frac{m^2}{\beta^2}}} + \eta \frac{m^2}{\beta^2} \tag{7}$$

When equation (6) or equation (7) is used, m and n are assigned different integral values until the lowest value of the compressive-buckling-load parameter k is obtained. Computations indicate that n should always be given the value 1 in these calculations so that the corrugated-core sandwich plate buckles with one sinusoidal half-wave in the y-direction.

Clamped Unloaded Edges

In the problem for plates with clamped unloaded edges (fig. 1(b)), the conditions that are satisfied along the simply supported edges are those of zero deflection of the middle surface, zero moment normal to the edges, and zero relative movement parallel to the edges of points in the boundary. These conditions are expressed by equation (3a). Along the clamped edges, the boundary conditions are those of zero deflection of the middle surface, zero relative movement normal to the edges of points in the boundary, and zero relative movement parallel to the edges of points in the boundary. At $y = \pm\frac{b}{2}$,

$$w = \frac{\partial w}{\partial y} - \frac{Q_y}{D_{Q_y}} = \frac{Q_x}{D_{Q_x}} = 0 \tag{8}$$

Solutions of the differential equations (2) for the middle-surface deflection w and the shear forces Q_x and Q_y exist in the form

$$\left.\begin{aligned} w &= \sin\frac{m\pi x}{a}\sum_i A_i \cosh\frac{\pi n_i y}{b} \\ Q_x &= \cos\frac{m\pi x}{a}\sum_i B_i \cosh\frac{\pi n_i y}{b} \\ Q_y &= \sin\frac{m\pi x}{a}\sum_i C_i \sinh\frac{\pi n_i y}{b} \end{aligned}\right\} \tag{9}$$

where values of n_i, A_i, B_i, and C_i are to be determined from the differential equations (2). Equations (2) are satisfied by equations (9) if, for each value of i, the following set of simultaneous equations is satisfied:

$$\frac{m^2}{\beta^2} kA_i - \frac{m}{\beta}\frac{1}{E_S\overline{I}_S}\left(\frac{b}{\pi}\right)^3 B_i + n_i \frac{1}{E_S\overline{I}_S}\left(\frac{b}{\pi}\right)^3 C_i = 0 \tag{10a}$$

$$\frac{m}{\beta}\left\{n_i^2 - \frac{m^2}{\beta^2}\left[1 + \left(1 - \mu_S^2\right)\eta\right]\right\} A_i + \left(1 - \mu_S^2 - \left\{\frac{1 - \mu_S}{2} n_i^2 - \right.\right.$$

$$\left.\left.\frac{m^2}{\beta^2}\left[1 + \left(1 - \mu_S^2\right)\eta\right] r_x\right\}\right)\frac{1}{E_S\bar{I}_S}\left(\frac{b}{\pi}\right)^3 B_i - \frac{1 + \mu_S}{2} n_i \frac{m}{\beta} r_y \frac{1}{E_S\bar{I}_S}\left(\frac{b}{\pi}\right)^3 C_i = 0 \quad \text{(10b)}$$

$$n_i\left(\frac{m^2}{\beta^2} - n_i^2\right)A_i - \frac{1 + \mu_S}{2} n_i \frac{m}{\beta} r_x \frac{1}{E_S\bar{I}_S}\left(\frac{b}{\pi}\right)^3 B_i -$$

$$\left[1 - \mu_S^2 + \left(\frac{1 - \mu_S}{2}\frac{m^2}{\beta^2} - n_i^2\right) r_y\right]\frac{1}{E_S\bar{I}_S}\left(\frac{b}{\pi}\right)^3 C_i = 0 \quad \text{(10c)}$$

The condition that A_i, B_i, and C_i have values other than zero - that is, that the determinant of their coefficients in equations (10) vanish - yields an equation for the determination of n_i

$$\left[\frac{1}{2\left(1 + \mu_S\right)} r_x\right] n_i^6 + \left[1 + \frac{1}{2\left(1 + \mu_S\right)}\frac{m^2}{\beta^2} r_y + \left(\frac{1}{1 + \mu_S} + \eta\right)\frac{m^2}{\beta^2} r_x - \right.$$

$$\left.\frac{1}{2\left(1 + \mu_S\right)}\frac{m^2}{\beta^2} k r_x r_y\right] n_i^4 + \frac{m^2}{\beta^2}\left\{2 + \left[\left(\frac{1}{1 + \mu_S} + \eta\right)\frac{m^2}{\beta^2} - k\right] r_y + \right.$$

$$\left.\frac{1 - \mu_S}{2}\left[\left(\frac{1}{1 - \mu_S^2} + \eta\right)\frac{m^2}{\beta^2} - k\right] r_x - \left(\frac{1}{1 + \mu_S} + \eta\right)\frac{m^2}{\beta^2} k r_x r_y\right\} n_i^2 +$$

$$\frac{m^2}{\beta^2}\left[1 + \frac{1}{2\left(1 + \mu_S\right)}\frac{m^2}{\beta^2} r_y\right]\left\{\left(1 - \mu_S^2\right)k - \right.$$

$$\left.\frac{m^2}{\beta^2}\left[1 + \left(1 - \mu_S^2\right)\eta\right]\left(1 - k r_x\right)\right\} = 0 \quad \text{(11)}$$

Thus n_i has the values $\pm n_1$, $\pm n_2$, and $\pm n_3$, the six roots of equation (11). Only the positive roots need be considered. Each of the coefficients B_i and C_i may be given in terms of the corresponding coefficient A_i. From equations (10a) and (10c), the following relations are obtained:

$$\left.\begin{aligned}
\frac{1}{E_S\overline{I}_S}\left(\frac{b}{\pi}\right)^3 B_i &= \lambda_i A_i \\
\frac{1}{E_S\overline{I}_S}\left(\frac{b}{\pi}\right)^3 C_i &= -\phi_i A_i
\end{aligned}\right\} \qquad (12)$$

where

$$\lambda_i = \frac{m}{\beta} k - \frac{\beta}{m} n_i^2 \frac{n_i^2 - \frac{m^2}{\beta^2} + \frac{1 + \mu_S}{2} \frac{m^2}{\beta^2} kr_x}{1 - \mu_S^2 + \left(\frac{1 - \mu_S}{2} \frac{m^2}{\beta^2} - n_i^2\right) r_y + \frac{1 + \mu_S}{2} n_i^2 r_x}$$

$$\phi_i = \frac{n_i^2 - \frac{m^2}{\beta^2} + \frac{1 + \mu_S}{2} \frac{m^2}{\beta^2} kr_x}{1 - \mu_S^2 + \left(\frac{1 - \mu_S}{2} \frac{m^2}{\beta^2} - n_i^2\right) r_y + \frac{1 + \mu_S}{2} n_i^2 r_x} n_i$$

Equation (9) may now be written as

$$\left.\begin{aligned}
w &= \sin \frac{m\pi x}{a}\left(A_1 \cosh \frac{\pi n_1 y}{b} + A_2 \cosh \frac{\pi n_2 y}{b} + A_3 \cosh \frac{\pi n_3 y}{b}\right) \\
Q_x &= \frac{1}{E_S\overline{I}_S}\left(\frac{b}{\pi}\right)^3 \cos \frac{m\pi x}{a}\left(\lambda_1 A_1 \cosh \frac{\pi n_1 y}{b} + \lambda_2 A_2 \cosh \frac{\pi n_2 y}{b} + \right. \\
&\qquad \left. \lambda_3 A_3 \cosh \frac{\pi n_3 y}{b}\right) \\
Q_y &= -\frac{1}{E_S\overline{I}_S}\left(\frac{b}{\pi}\right)^3 \sin \frac{m\pi x}{a}\left(\phi_1 A_1 \sinh \frac{\pi n_1 y}{b} + \phi_2 A_2 \sinh \frac{\pi n_2 y}{b} + \right. \\
&\qquad \left. \phi_3 A_3 \sinh \frac{\pi n_3 y}{b}\right)
\end{aligned}\right\} \qquad (13)$$

Equations (13) already satisfy the boundary conditions along the simply supported edges (equation (3a)). The boundary conditions along the clamped edges (equation (8)) must now be satisfied. The substitution of equations (13) into equation (8) yields the following set of equations:

$$\left.\begin{aligned}
&A_1 \cosh \frac{\pi n_1}{2} + A_2 \cosh \frac{\pi n_2}{2} + A_3 \cosh \frac{\pi n_3}{2} = 0 \\
&\lambda_1 A_1 \cosh \frac{\pi n_1}{2} + \lambda_2 A_2 \cosh \frac{\pi n_2}{2} + \lambda_3 A_3 \cosh \frac{\pi n_3}{2} = 0 \\
&\left(n_1 + \phi_1 r_y\right) A_1 \sinh \frac{\pi n_1}{2} + \left(n_2 + \phi_2 r_y\right) A_2 \sinh \frac{\pi n_2}{2} + \\
&\quad \left(n_3 + \phi_3 r_y\right) A_3 \sinh \frac{\pi n_3}{2} = 0
\end{aligned}\right\} \tag{14}$$

In order to assure the existence of values of A_1, A_2, and A_3 other than zero, the determinant of the coefficients of A_1, A_2, and A_3 is set equal to zero. When the determinant is expanded, the following equation is obtained:

$$\left(\lambda_3 - \lambda_2\right)\left(n_1 + \phi_1 r_y\right)\tanh \frac{\pi n_1}{2} + \left(\lambda_1 - \lambda_3\right)\left(n_2 + \phi_2 r_y\right)\tanh \frac{\pi n_2}{2} +$$

$$\left(\lambda_2 - \lambda_1\right)\left(n_3 + r_y \phi_3\right)\tanh \frac{\pi n_3}{2} = 0 \tag{15}$$

Equation (15) in conjunction with equation (11) is the criterion for the compressive buckling of corrugated-core sandwich plates with simply supported loaded edges and clamped unloaded edges.

When values of the compressive-buckling-load-parameter k are computed with equations (11) and (15), a trial and error process is used. For given values of β, r_x, r_y, and η, a value of k is assumed and m is assigned some integral value. Equation (11) is solved to give the values of n_1, n_2, and n_3. These values are then substituted into equation (15). If the left-hand side of equation (15) does not vanish, other values of k must be chosen and the process repeated. This procedure yields a series of values of k and corresponding values of the left-hand side of equation (15). The correct value of k may now be obtained by plotting these values as ordinate and abscissa and picking off the value of k at which the left-hand side of equation (15) is equal to zero. The entire process is then repeated for other integral values of m until the lowest value of k is obtained.

For the case of infinite D_{Q_x}, the following stability criterion is obtained:

$$\frac{n_1}{1 - \mu_S^2 + \left(\frac{1 - \mu_S}{2}\frac{m^2}{\beta^2} - n_1^2\right)r_y}\tanh\frac{\pi n_1}{2} - \frac{n_2}{1 - \mu_S^2 + \left(\frac{1 - \mu_S}{2}\frac{m^2}{\beta^2} - n_2^2\right)r_y}\tanh\frac{\pi n_2}{2} = 0 \tag{16}$$

where n_1 and n_2 are the positive values of the roots $\pm n_1$ and $\pm n_2$ of the equation

$$\left[1 + \frac{1}{2(1 + \mu_S)}\frac{m^2}{\beta^2}r_y\right]n_1^4 - \frac{m^2}{\beta^2}\left\{2 + \left[\left(\frac{1}{1 + \mu_S} + \eta\right)\frac{m^2}{\beta^2} - k\right]r_y\right\}n_1^2 - \frac{m^2}{\beta^2}\left[1 + \frac{1}{2(1 + \mu_S)}\frac{m^2}{\beta^2}r_y\right]\left\{\left(1 - \mu_S^2\right)k - \frac{m^2}{\beta^2}\left[1 + \left(1 - \mu_S^2\right)\eta\right]\right\} = 0 \tag{17}$$

The same procedure is used in solving equations (16) and (17) for values of k as is used for equations (11) and (15).

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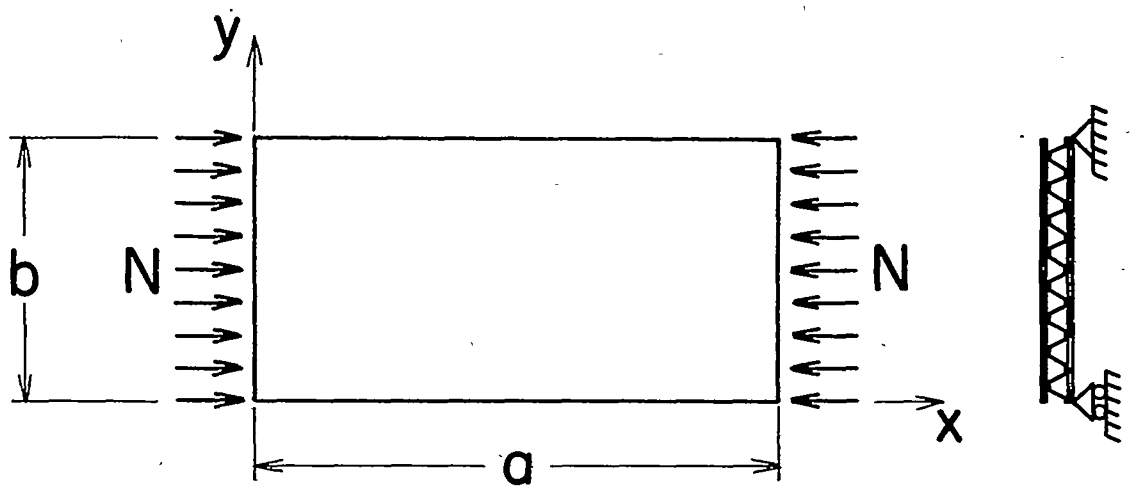
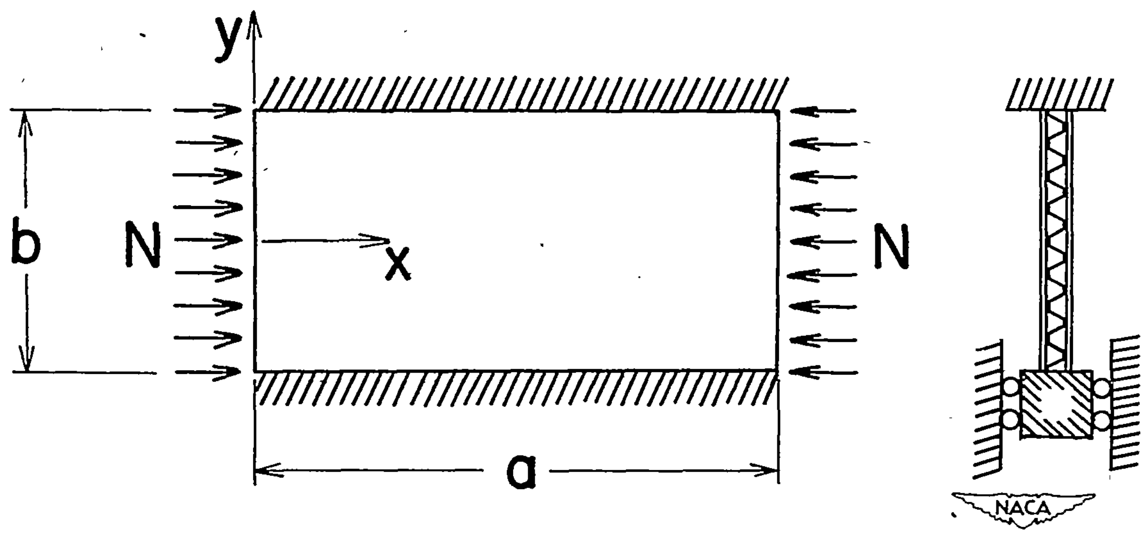
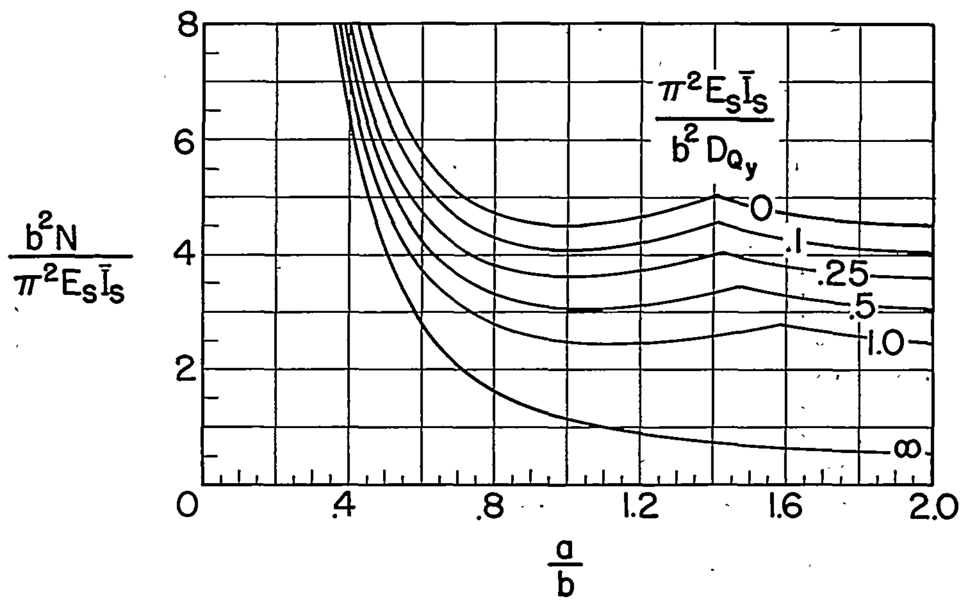


(a) Simply supported unloaded edges.

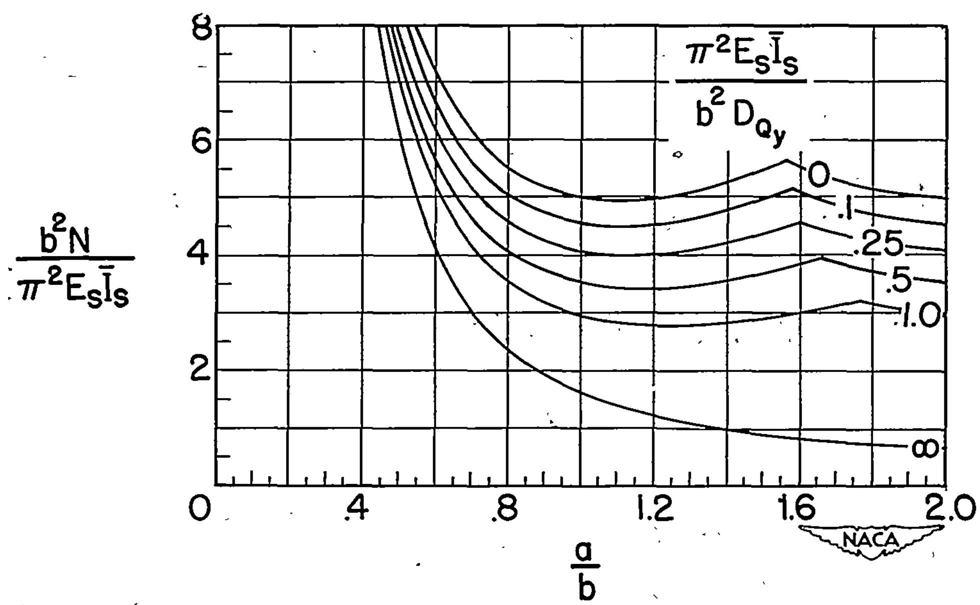


(b) Clamped unloaded edges.

Figure 1.- Flat symmetric corrugated-core sandwich-plate buckling problems solved in the present paper.

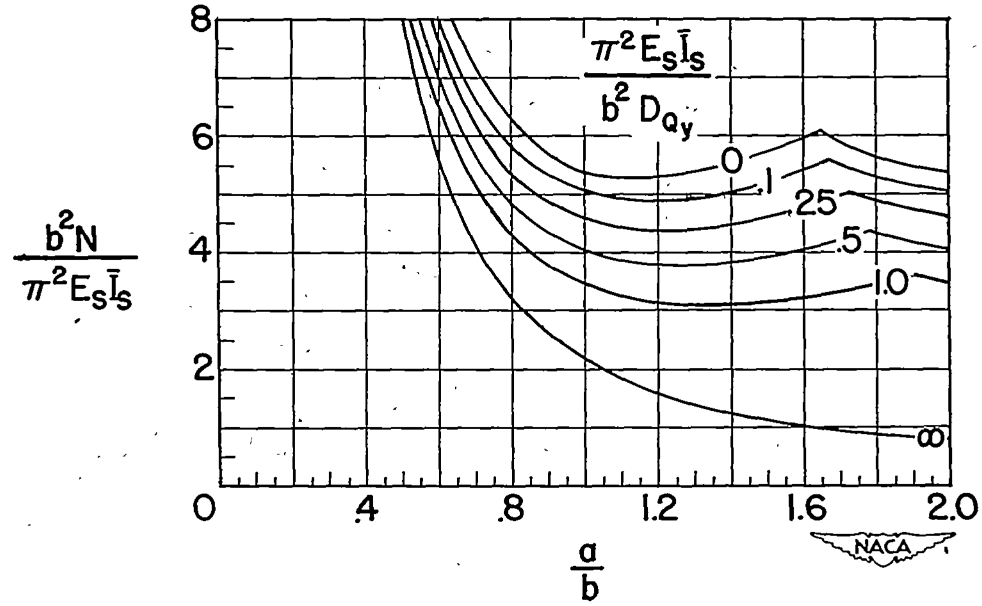


(a) $\frac{E_C \bar{I}_C}{E_S \bar{I}_S} = 0.$



(b) $\frac{E_C \bar{I}_C}{E_S \bar{I}_S} = 0.5.$

Figure 2.- Compressive-buckling-load parameters for corrugated-core sandwich plates with simply supported unloaded edges.



(c) $\frac{E_C \bar{I}_C}{E_S \bar{I}_S} = 1.0.$

Figure 2.- Concluded.

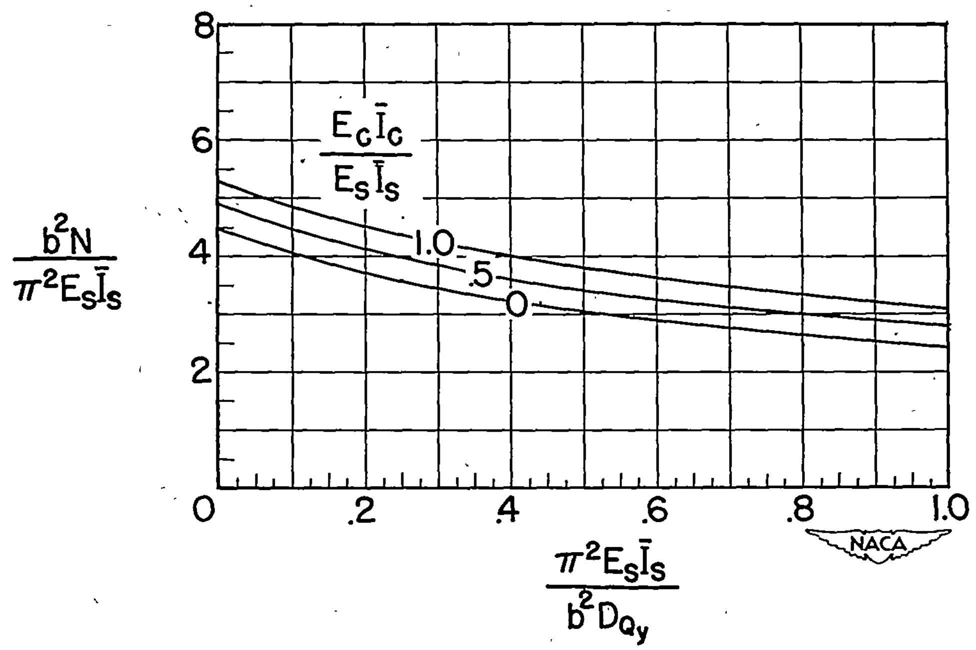
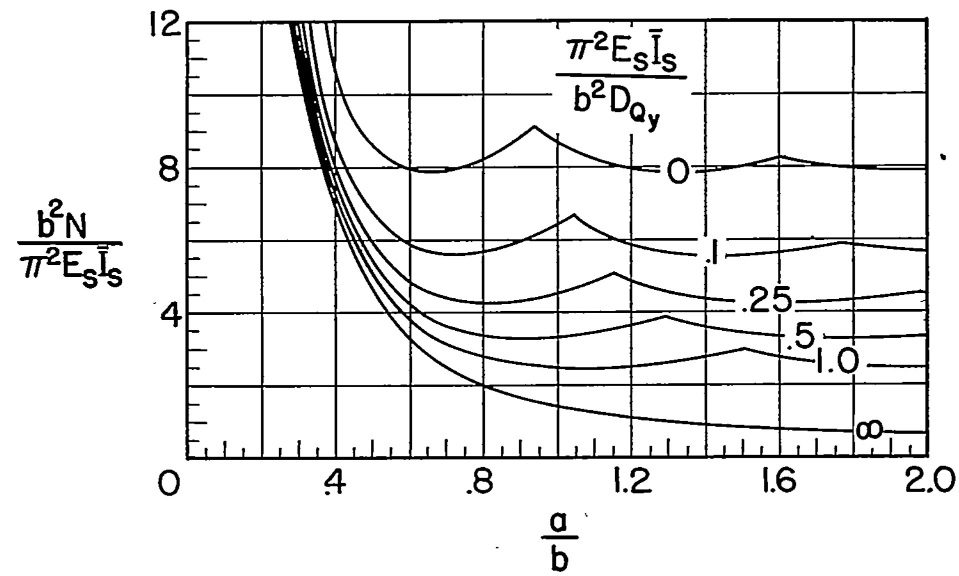
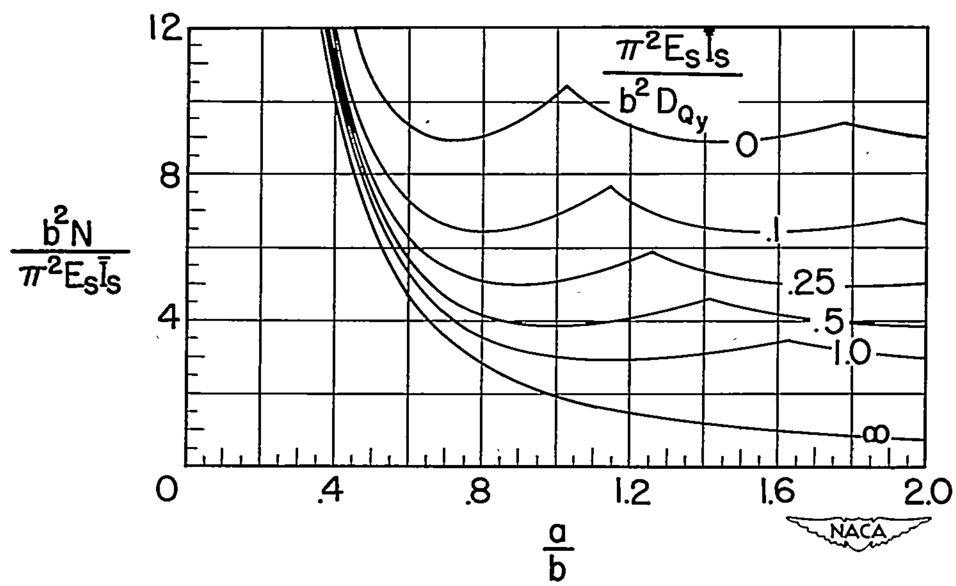


Figure 3.- Compressive-buckling-load parameters for infinitely long simply supported corrugated-core sandwich plates.

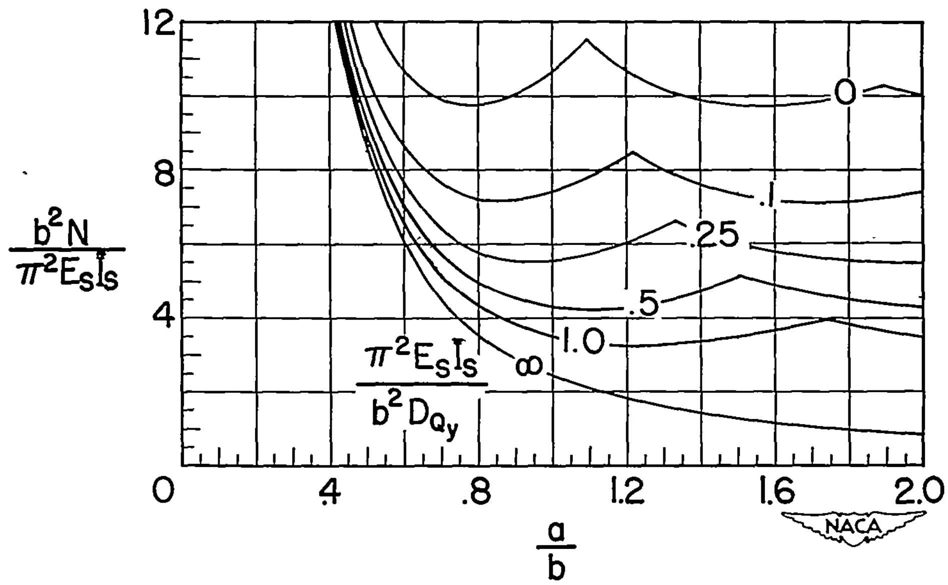


(a) $\frac{E_C \bar{I}_C}{E_S \bar{I}_S} = 0.$



(b) $\frac{E_C \bar{I}_C}{E_S \bar{I}_S} = 0.5.$

Figure 4.- Compressive-buckling-load parameters for corrugated-core sandwich plates with clamped unloaded edges.



(c) $\frac{E_C \bar{I}_C}{E_S \bar{I}_S} = 1.0.$

Figure 4.- Concluded.

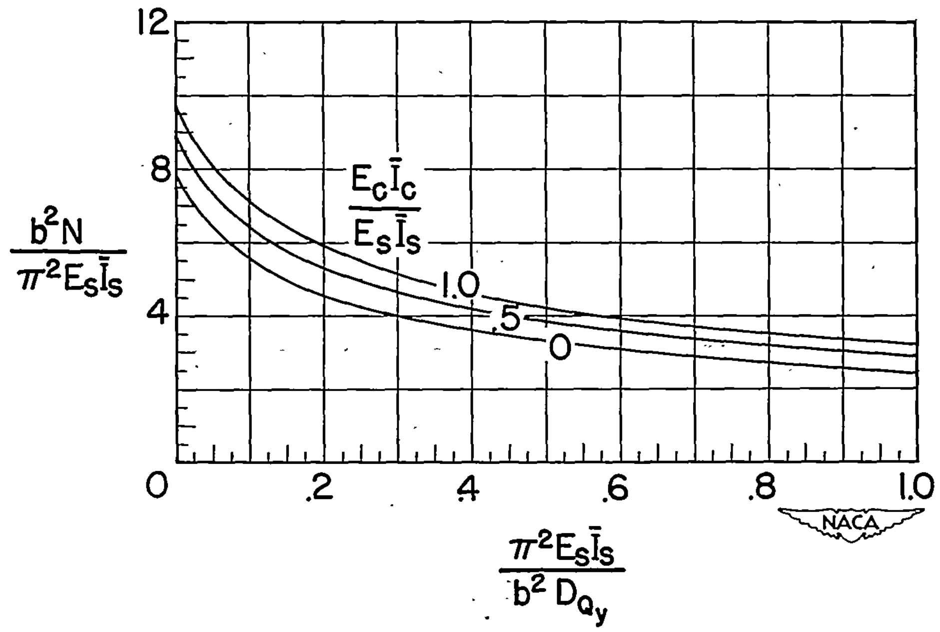
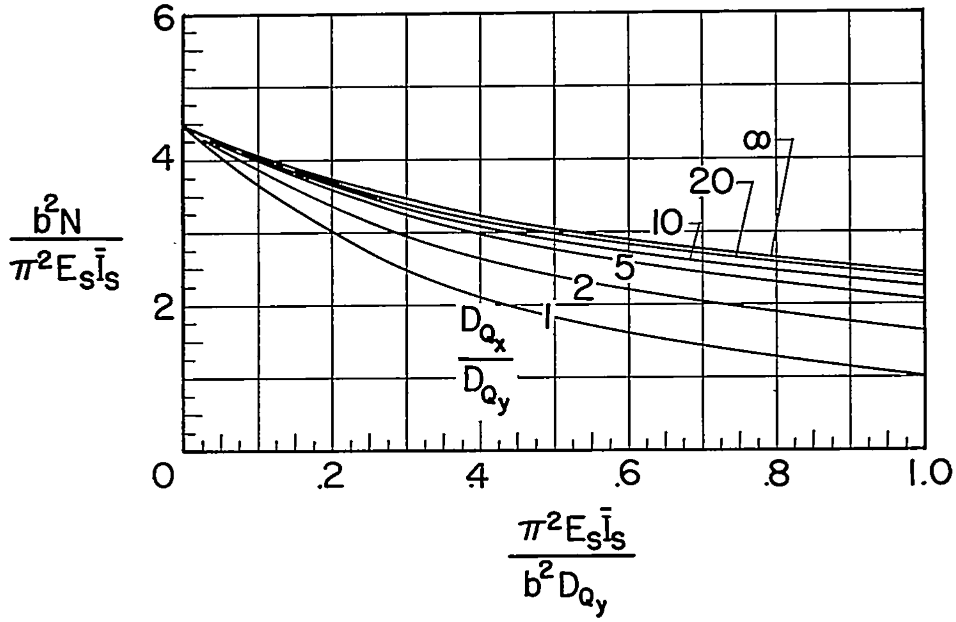
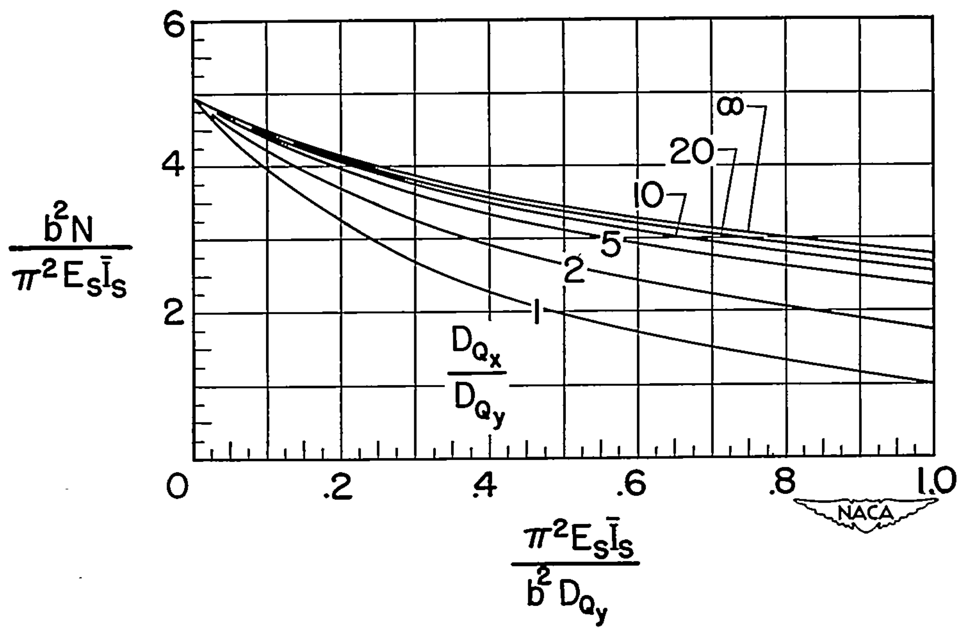


Figure 5.- Compressive-buckling-load parameters for infinitely long clamped corrugated-core sandwich plates.

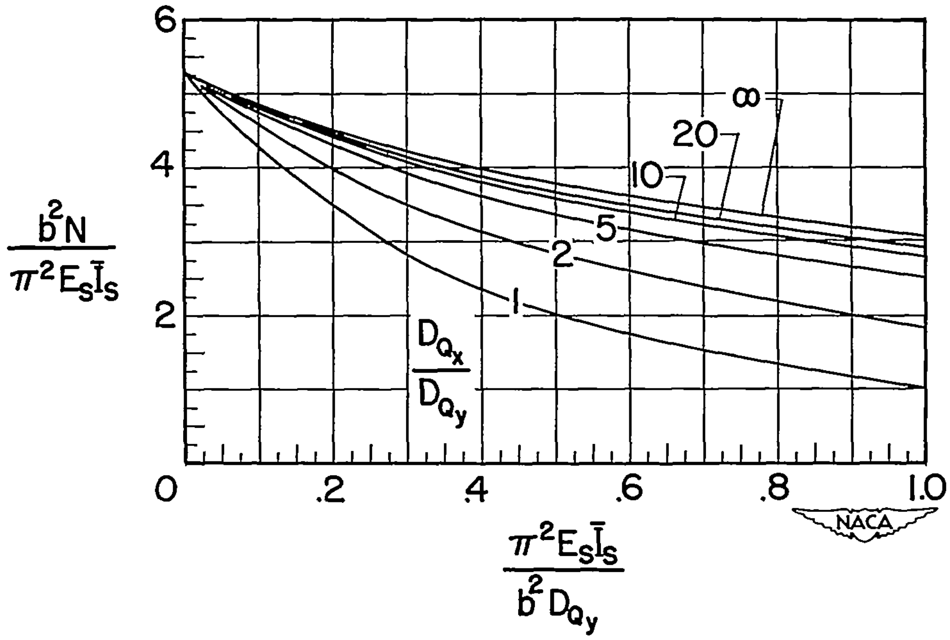


(a) $\frac{E_C \bar{I}_C}{E_S \bar{I}_S} = 0.$



(b) $\frac{E_C \bar{I}_C}{E_S \bar{I}_S} = 0.5.$

Figure 6.- Effect of finite transverse shear stiffness D_{Q_X} on the compressive-buckling-load parameters of infinitely long simply supported corrugated-core sandwich plates.



(c) $\frac{E_C \bar{I}_C}{E_S \bar{I}_S} = 1.0.$

Figure 6.- Concluded.

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