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EXACT SOLUTIONS OF LAMINAR-BOUNDARY-LAYER EQUATIONS WITH CONSTANT PROPERTY VALUES FOR POROUS WALL WITH VARIABLE TEMPERATURE

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SUMMARY

Exact solutions of the laminar-boundary-layer equations for wedge-type flow with constant property values are presented for transpiration-cooled surfaces with variable wall temperatures. The difference between wall and stream temperature is assumed proportional to a power of the distance from the leading edge. Solutions are given for a Prandtl number of 0.7 and ranges of pressure-gradient, cooling-air-flow, and wall-temperature-gradient parameters. Boundary-layer profiles, dimensionless boundary-layer thicknesses, and heat-transfer coefficients are given in both tabular and graphical form. Corresponding results for constant wall temperature and for impermeable surfaces are included for comparison purposes.

The results indicate that increasing the wall-temperature gradient yields steeper temperature profiles in the boundary layer for a given coolant flow. The steeper temperature profiles produce increased local heat-transfer coefficients. These effects of the wall-temperature gradient were reduced as the coolant flow was increased. Wall-temperature variations resulting in zero boundary-layer temperature gradients at the wall were found to be increased by increased pressure gradient and decreased by increased coolant flow.

INTRODUCTION

A knowledge of the behavior of the boundary layer adhering to cooled or heated bodies immersed in a moving fluid is essential for accurate prediction of heat transfer or skin friction. When the boundary layer is laminar, solutions of the boundary-layer equations resulting from wedge-type flow (flow for which the main-stream velocity is proportional to a power of the distance from the stagnation point) have been reported for a permeable wall with a constant wall temperature and for an impermeable wall with variable wall temperature. (These solutions will be discussed later in the INTRODUCTION). The simultaneous effects of a variable temperature and a permeable wall on the heat transfer apparently have not been obtained heretofore. These effects are analyzed herein by solution of the laminar-boundary-layer equations with constant property values and wedge-type flow.

Solutions for wedge-type flow can be used directly as a first approximation for calculating local heat-transfer coefficients to bodies of arbitrary cross section such as turbine blades (refs. 1 and 2), airfoils (ref. 2), and cylinders (ref. 3). When the need arises for more accurate heat-transfer predictions, a second or better approximation which utilizes the solutions for wedge-type flow is presented in references 4 to 6.

In references 7 to 9 exact solutions of the laminar-boundary-layer equations are presented for wedge-type flow with a constant wall temperature under conditions of variable property values, transpiration cooling, and small Mach numbers. Experimental velocity distributions for an isothermal, porous flat plate are available in reference 10. References 5 and 7 to 9 summarize previous analyses of wedge-type flow with constant wall temperature. Consequently, only the investigations which include the effects of variable wall temperature will be noted herein. Such calculations contained in the references which follow were made only for the impermeable or solid wall.

Exact solutions of the energy equation for a variable wall temperature with wedge-type flow were first presented by Fage and Falkner (ref. 11). These solutions were obtained for conditions of constant property values, a Prandtl number of 0.77, and a linear velocity increase normal to the wall; heat produced by friction and compression were neglected. Calculations given by Schuh (ref. 12) for constant property values and a Prandtl number of 0.7 employ the exact velocity distributions of Hartree (ref. 13); frictional and compression heating were again neglected. Chapman and Rubesin give results for zero pressure gradient (the flat-plate case or zero wedge-opening angle) for a Prandtl number of 0.72 and an arbitrary surface-temperature variation; these results include frictional heating (ref. 14). Heat-transfer results are reported by Levy (ref. 15) for wedge-type flow and a range of Prandtl numbers appropriate for gases and liquids (Prandtl numbers from 0.7 to 20); frictional and compression heating are partially accounted for.

Approximate solutions for the heat-transfer rate with an arbitrary distribution of main-stream velocity and wall temperature are obtained by Lighthill (ref. 16). These solutions are discussed and utilized in references 17 to 20. In reference 16 the formulas are of the nature of an asymptotic formula for large Prandtl number and it is shown that the approximate asymptotic formulas are not too much in error even for a Prandtl number of 0.7. A different method of solution for large Prandtl number is given by references 21 and 22. For either large Prandtl number or large wall-temperature variations, asymptotic solutions are found in reference 23; extensions, corrections, and simplifications are contained in references 24 to 27.

The previous literature indicates quite pronounced effects of a variable wall temperature on heat transfer. Current interest in transpiration cooling led to an investigation of such effects for porous surfaces. This investigation was conducted at the NACA Lewis laboratory and the results are presented herein. Solutions of the laminar-boundary-layer equations with constant property values are given for ranges of pressure-gradient parameters, dimensionless flow rates through the porous wall, and dimensionless wall-temperature gradients. Velocity and temperature distributions and their derivatives are tabulated. For each case, nondimensional forms of heat-transfer and friction coefficients, and various dimensionless boundary-layer thicknesses are also tabulated.

The numerical calculations were made on the IBM Card Programmed Calculator under the supervision of Lynn U. Albers.

SYMBOLS

The following symbols are used in this report:

B,C	constants of proportionality
C_f	$\dfrac{\tau_w}{\dfrac{\rho U_\infty^2}{2}}$
c_p	specific heat at constant pressure
Eu	Euler number, $\dfrac{-x\dfrac{dp}{dx}}{\rho U_\infty^2}$; $U_\infty = Cx^{Eu}$
f	dimensionless stream function
f', f'', f'''	first, second, and third derivatives of f with respect to η
H	local heat-transfer coefficient at x
k	thermal conductivity
Nu	local Nusselt number, Hx/k

n	temperature gradient parameter, $\left[x/(T_w-T_\infty)\right]\ (dT_w/dx)$; $T_w-T_\infty = Bx^n$
Pr	Prandtl number, $c_p\mu/k$
p	static pressure
Re	Reynolds number, $\frac{U_\infty x}{\nu}$
T	temperature
U_∞	fluid velocity at edge of boundary layer
u	fluid velocity in boundary layer parallel to wall
v	fluid velocity in boundary layer normal to wall
x	distance along surface
Y	temperature difference ratio, $(T-T_\infty)/(T_w-T_\infty)$
Y',Y''	first and second derivatives of Y with respect to η
y	distance normal to surface
δ	boundary-layer thickness
δ^*	displacement boundary-layer thickness, $\delta^* = \int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy$
δ_c	convection boundary-layer thickness, $\delta_c = \int_0^\infty \frac{u}{U_\infty} \left(\frac{T - T_\infty}{T_w-T_\infty}\right) dy$
δ_i	momentum boundary-layer thickness, $\delta_i = \int_0^\infty \frac{u}{U_\infty} \left(1 - \frac{u}{U_\infty}\right) dy$
δ_t	thermal boundary-layer thickness, $\delta_t = \int_0^\infty \frac{T - T_\infty}{T_w-T_\infty} dy$

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η	nondimensional boundary-layer coordinate, $y\sqrt{\frac{U_\infty}{\nu x}}$
μ	absolute viscosity of fluid
ν	kinematic viscosity of fluid, μ/ρ
ρ	density of fluid
τ	shear stress
ψ	stream function

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Subscripts:

1	location along plate (see fig. 3)
w	wall
∞	main stream, outside boundary layer

ANALYSIS

Laminar-Boundary-Layer Equations

The equations of the laminar boundary layer for steady-state flow of a fluid with constant properties may be written:

Momentum:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho} \frac{\partial p}{\partial x} \tag{1}$$

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

Energy:

If the temperature differences between the wall and the main stream are assumed large as compared with temperature changes caused by compression and frictional heating and appendix A is used, the energy equation may be written:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\nu}{Pr} \frac{\partial^2 T}{\partial y^2} \tag{3}$$

The boundary conditions are for $y = 0$:

$$\left.\begin{array}{l} u = 0;\ v = v_w;\ T = T_w \\ \text{and for } y \to \infty: \\ u \to U_\infty;\ \dfrac{\partial u}{\partial y} \to 0;\ \dfrac{\partial^2 u}{\partial y^2} \to 0;\ \dots\ \dfrac{\partial^m u}{\partial y^m} \to 0 \\ T \to T_\infty;\ \dfrac{\partial T}{\partial y} \to 0;\ \dfrac{\partial^2 T}{\partial y^2} \to 0;\ \dots\ \dfrac{\partial^m T}{\partial y^m} \to 0 \end{array}\right\} \tag{4}$$

In order to reduce the number of calculations and increase the flexibility of the results, dimensionless parameters for pressure and wall-temperature variations are introduced.

Parameters for Pressure and Wall-Temperature Gradients

For wedge-type flow, the main-stream-velocity variation is given by

$$U_\infty = Cx^{Eu} \tag{5}$$

where Eu is a constant for a given wedge. Differentiation of equation (5) with Eu constant and use of Bernoulli's equation yield

$$Eu = \frac{x}{U_\infty}\frac{dU_\infty}{dx} = \frac{-x}{\rho U_\infty^{\ 2}}\frac{dp}{dx} \tag{6}$$

Equation (6) shows that the Euler number is a dimensionless measure of the main-stream pressure gradient.

A similar procedure may be employed in the determination of the wall-temperature-gradient parameter. It is assumed that the difference between the wall and the stream temperature is proportional to a power of the distance from the leading edge, that is

$$T_w - T_\infty = Bx^n \tag{7}$$

where n and T_∞ are constant. This relation is used in references 11, 12, 14, and 15. Differentiation of equation (7) gives

$$n = \frac{x}{(T_w - T_\infty)}\frac{dT_w}{dx} \tag{8}$$

Equation (8) offers a formula for calculation of the wall-temperature-gradient parameter similar to equation (6) which has been used (e.g., refs. 2 and 4) to calculate the pressure-gradient parameter.

Transformation to Ordinary Differential Equations

The transformation from partial to ordinary differential equations is accomplished by the following changes in variables:

$$\left.\begin{aligned} \eta &= y\sqrt{\frac{U_\infty}{\nu x}} \\ Y &= \frac{T - T_\infty}{T_w - T_\infty} \\ f &= \frac{\psi}{\sqrt{\nu x U_\infty}} \end{aligned}\right\} \qquad (9)$$

where η is the dimensionless independent variable of Blasius and f and Y are dimensionless dependent variables representing stream function and temperature, respectively.

The continuity equation (2) is satisfied by the stream function ψ since

$$u = \frac{\partial\psi}{\partial y} \quad \text{and} \quad v = \frac{-\partial\psi}{\partial x} \qquad (10)$$

Transformation of the momentum equation (1) and the energy equation (3) into ordinary differential equations is accomplished by use of equations (5) through (10). The momentum equation becomes

$$f''' = Eu(f')^2 - \frac{(Eu+1)}{2} ff'' - Eu \qquad (11)$$

and the energy equation becomes

$$Y'' = \frac{-(Eu+1)}{2} \Pr fY' + n \Pr f'Y \qquad (12)$$

with the boundary conditions for $\eta = 0$:

$$\left.\begin{array}{l} f = f_w;\ f' = 0;\ \text{and}\ \ Y = 1 \\ \text{and for}\ \ \eta \to \infty: \\ f' \to 1;\ f'' \to 0;\ f''' \to 0;\ .\ .\ .\ ;\ f^m \to 0 \\ Y \to 0;\ Y' \to 0;\ Y'' \to 0;\ .\ .\ .\ ;\ Y^m \to 0 \end{array}\right\} \tag{13}$$

From equations (9) and (10) there results

$$-\,v = \frac{Eu+1}{2}\sqrt{\frac{U_\infty \nu}{x}}\, f + \frac{Eu-1}{2}\,\frac{y}{x}\, U_\infty f' \tag{14}$$

Use of the boundary conditions at the wall ($\eta = 0$) gives the following explicit expression for f_w (a dimensionless measure of the coolant flow through the porous wall) in terms of the velocity v_w out of the porous wall:

$$f_w = \frac{-2}{(Eu+1)}\,\frac{v_w}{U_\infty}\sqrt{Re} \tag{15}$$

For numerical solution of equation (11), f_w is usually assumed to be a constant. Use of equations (15) and (5) shows that this constancy dictates $v_w \propto x^{\frac{Eu-1}{2}}$. In the absence of conduction and radiation, a constant f_w yields a constant wall temperature (ref. 28). Only conduction along the wall, or radiation, or both may lead to a variation in wall temperature if f_w is constant.

It should be noted that equations (11) and (12) can be made identical to those employed by previous investigators (refs. 11, 12, and 15) and that the inclusion of transpiration cooling into the investigation results only in a change in one of the boundary conditions (eq. (13)) at the wall; that is, at $\eta = 0$, f now equals f_w which may be nonzero.

For the case where the heat transferred to the plate from the boundary layer is zero, a boundary condition $Y'(0) = 0$ is utilized in solution of equation (12). Under this condition, two solutions are possible. The mathematically trivial solution $Y = 0$ has been published by Chapman and Rubesin (ref. 14) and results in $T = T_\infty$ everywhere so that the wall temperature is constant. The other solution is obtained by determination of the value of n which satisfies equation (12) when $Y'(0) = 0$. Attention in this report is confined to the latter solution for each combination of the parameters considered.

Formulas for Boundary-Layer Thicknesses, Heat Transfer, and Friction

From equations (9) and (10) the boundary-layer velocity distribution is expressible as follows:

$$u/U_\infty = f' \tag{16}$$

Use of equations (9) and (16) in the definitions of the various boundary-layer-thicknesses as given in the SYMBOLS yields the following dimensionless formulas for these thicknesses:

Displacement thickness:

$$\frac{\delta^* \sqrt{Re}}{x} = \int_0^\infty (1 - f')\, d\eta \tag{17}$$

Momentum thickness:

$$\frac{\delta_i \sqrt{Re}}{x} = \int_0^\infty f'(1 - f')\, d\eta \tag{18}$$

Convection thickness:

The convection thickness defined in the SYMBOLS and in reference 29, (pp. 118, 119) becomes, by use of equations (9) and (16)

$$\frac{\delta_c \sqrt{Re}}{x} = \int_0^\infty f'\, Y\, d\eta \tag{19}$$

Thermal thickness:

$$\frac{\delta_t \sqrt{Re}}{x} = \int_0^\infty Y\, d\eta \tag{20}$$

A balance between the heat transfer by convection $H(T_\infty - T_W)$ and the heat transfer by conduction $k(\partial T/\partial y)_w$ along with equations (9) and (10) and the definitions of Nu and Re yield

$$Nu/\sqrt{Re} = -Y'(0) \tag{21}$$

an expression for the dimensionless local heat-transfer coefficient to the surface.

The shear stress τ is given by

$$\tau = \mu \frac{\partial u}{\partial y}$$

A friction coefficient C_f is defined as

$$C_f = \frac{\tau_w}{\frac{\rho U_\infty^2}{2}}$$

so that by use of equations (9) and (16)

$$\frac{C_f}{2}\sqrt{Re} = f_w'' \tag{23}$$

an expression for the dimensionless skin friction.

NUMERICAL CALCULATION

The numerical solutions of equations (11) and (12) were obtained for a Prandtl number of 0.7 (appropriate for air), pressure variations represented by values of Eu of 0, 1/2, and 1, flow rates through the porous wall represented by values of f_w of 0, -1/2, and -1, and wall-temperature variations represented by values of n from the value corresponding to a zero boundary-layer temperature gradient at the wall to unity.

For the case of constant property values considered herein, equations (11) and (12) are independent; consequently, equation (11) is solved previous to solution of equation (12). Since the f solutions given by reference 8 were obtained by desk computation, the punch cards required for solution of equation (12) on the IBM calculator were not available. In order to obtain the punch cards it was found more expedient to solve the f problem anew.

Equation (11) together with the boundary conditions, equation (13), constitutes a nonlinear boundary-value problem with parameters f_w and Eu. It was solved by an iterative method using punched cards on the IBM Card Programmed Calculator. Each step of the iterative method required an estimation of $f''(0)$ and a subsequent integration, using five-point formulas, of the resulting initial value problem. As soon as values of f and its derivatives were considered correct to four decimal places, results were punched for use in the related Y problem. (The f solutions obtained herein are in good agreement with those tabulated in ref. 8.)

Equations (12) and (13) constitute a linear boundary-value problem with parameters n and Eu (when Pr is fixed) and input data f and f'. Being linear, the problem should be solvable by combining any two independent solutions. In practice, however, it is necessary to combine two solutions near the final one to obtain a result valid to four decimal places. Hence, four trials were necessary for each solution of a Y problem.

The integration technique used for both problems is described in detail in the appendix of reference 30 and more concisely in appendix B of reference 31. The accuracy of results is believed to be within one in the fourth decimal place.

RESULTS AND DISCUSSIONS

The results of the calculations for each of the 29 cases investigated are presented in table I. Values of f and its derivatives and of Y and its derivatives are tabulated as functions of η; f' represents the velocity distribution, and Y the temperature distribution through the boundary layer. Table II presents a summary of the principal results, which are obtained from table I and the use of formulas (17) through (23). For the cases where $n = 0$, the results were taken from references 8 and 9. Table II also gives the part number of table I where the velocity and temperature distributions and their derivatives are listed. For the cases where $n = 0$, the distributions and their derivatives may be obtained from reference 8.

In addition to the tables, some of the results are also presented in the form of curves. The graphical presentations are used to indicate the influence of the various parameters on such quantities as velocity and temperature distributions, dimensionless convection, thermal thicknesses, and heat-transfer coefficients. Plots of dimensionless displacement, momentum thicknesses, and friction coefficients may be found in references 7 and 9.

Boundary-Layer Profiles

Figure 1 shows the velocity distribution f' plotted as a function of the dimensionless-boundary-layer coordinate η with the coolant-flow parameter f_w for each of the Euler numbers considered ($Eu = 0$, $1/2$, and 1). The velocity distributions for $Eu = 0$, $f_w = -0.75$, and for $Eu = 1$, $f_w = -3.1905$, and $f_w = -4.3346$ were obtained from reference 32. The variables η and f' used in reference 32 were converted to those given herein.

In figure 1(a) an increase in coolant flow ($|f_w|$ increasing) is seen to thicken the boundary layer for all Eu and also to result in the S-shape velocity profile which is undesirable from the stability viewpoint. It is noted in reference 33 that the velocity gradient at the wall becomes zero ($f''(0) = 0$) for $Eu = 0$ when $f_w = -1.23849$. Although calculations for the f_w values that result in $f''(0) = 0$ have not been made for other Euler numbers, comparison of figures 1(a), (b), and (c) indicate that the boundary layer with pressure gradient can tolerate much more coolant flow than a flat plate. Calculations for $Eu = 1$ with $f_W = -4.3346$ yield velocity profiles which appear to be quite stable (have no inflection point) as may be seen in figure 1(c). Indeed, it is shown in reference 32 for stagnation-point flow ($Eu = 1$) that coolant emission from the wall regardless of its magnitude never results in a point of inflection inside the boundary layer.

Figure 2 contains plots of the temperature profile Y against the dimensionless boundary-layer coordinate η, with f_w as parameter, for various values of the wall-temperature-gradient parameter n for a flat-plate or zero pressure gradient ($Eu = 0$). Figure 2(a) presents the temperature profiles for the case with zero temperature gradient at the wall, that is, $Y'(0) = 0$. The values of n for this case vary with the parameter f_w, and hence the curves are distinguished as solid-, dashed-, and broken-line curves.

Figure 2(b) presents the temperature distributions for the various values of the parameter f_w for the case of a constant wall temperature, that is, for $n = 0$. The distributions for a Prandtl number of unity are obtained quite simply from the velocity distributions, since for a constant wall temperature and a flat plate with $Pr = 1$, equations (11) and (12) are similar, so that for this case, $Y = 1 - f'$. The distributions so obtained are in good agreement with those reported in reference 28 where the velocity distributions of reference 32 were utilized. The effect of the coolant flow f_w is similar to that shown in figure 2(a); viz, $|f_w|$ increasing forces the temperature boundary layer away from the wall. It is also interesting to note that for $f_w = 0$, the stipulation of $Pr = 1$ yields a larger temperature gradient at the wall than for $Pr = 0.7$; whereas for $f_w = -1.0$, the gradient is less for $Pr = 1$ than for $Pr = 0.7$. As illustrated by the following table of $-Y'(0)$ for $Eu = 0 = n$, when $f_w = -0.5$ the gradient at the wall is about the same for both Prandtl numbers:

f_w	$-Y'(0)$ ($Pr=0.7$) $Eu=0=n$	$-Y'(0)$ ($Pr=1.0$) $Eu=0=n$
0	0.2927	0.3320
-.5	.1662	.1648
-1.0	.0516	.0355

Figure 2(c), obtained from table I, presents curves for the cases where $n = 1.0$. The increased boundary-layer temperature gradients due to the influence of n are apparent by comparing figure 2(c) with figures 2(a) and (b). For $T_w > T_\infty$, the wall temperature increases in flow direction for positive n and decreases for negative n as depicted in figure 3. These changes in the wall temperature are transmitted into the boundary layer with a certain delay due to the heat capacity of the boundary layer, as previously pointed out by Schuh (ref. 12). At a location x_1, the temperatures in the boundary layer T are greater, therefore, for $n > 0$ and smaller for $n < 0$ than for constant wall temperature $n = 0$. This disparity may be noted quantitatively in figure 2 and qualitatively in figure 3.

Figure 4 also shows temperature distributions in the boundary layer but for stagnation-point flow ($Eu = 1.0$). Figure 4(a) is for zero boundary-layer temperature gradient at the wall. Figure 4(b) (note the different scale for the abscissa) presents results for constant wall temperature for $Pr = 0.7$ (ref. 8) and for $Pr = 1.0$ (ref. 28). At the common curve for both Prandtl numbers ($f_w = 0$), the negative of the temperature gradient at $\eta = 0$ is 0.4958 for $Pr = 0.7$ and 0.570 for $Pr = 1.0$. The coolant flows of -3.1905 and -4.3346 both resulted in a zero temperature gradient at $\eta = 0$. Figure 4(c) shows the temperature profiles for $n = 1$. The influence of the wall-temperature variation for $Eu = 1$ is similar to the influence for $Eu = 0$.

Figure 4 (and fig. 2, as well) reveal that increases in $|f_w|$ diminish the temperature gradients in the boundary layer for all values of the wall-temperature-gradient parameter. Increases in the wall-temperature gradient, however, increase the boundary-layer temperature gradient.

These increases due to wall temperature gradient are similar to those encountered in the velocity boundary layer due to main-stream velocity gradient (cf. fig. 1). A positive pressure gradient forces the velocity boundary layer into the wall; the wall-temperature gradient (for positive n) draws the temperature boundary layer into the wall resulting in steeper temperature profiles. Whereas the velocity boundary layer is affected by velocity gradients in the main stream (outer edge of the boundary layer), the temperature boundary layer is influenced not only by the velocity gradient but also by the temperature gradient along the wall (inner edge of the boundary layer).

Heat-Transfer Results

Dimensionless local heat-transfer coefficients are presented in figure 5. (These coefficients are in general agreement with those reported in the literature as discussed in appendix B.) For each Euler number and coolant flow, there is a wall-temperature variation which results in $Y'(0) = 0$. These values of n are given by the intercepts of the various curves with the horizontal axis. A curve to be presented later will illustrate the zero-heat-transfer cases more thoroughly.

For fixed values of the Euler number and the coolant flow, increases in the wall-temperature gradient yield increases in the local heat-transfer coefficient. This behavior is a result of the increased gradients in the temperature profiles due to increased n and was noted in the discussion of figure 2. In all instances the effect of the coolant emission from the wall is to reduce the local heat-transfer coefficients. This reduction is more marked for the flat-plate case ($Eu = 0$) than for the flow with velocity gradient ($Eu \neq 0$). It is seen in figures 5(a) and (c) that for a linear wall-temperature gradient (i.e., $n = 1.0$), a coolant flow represented approximately by $f_w = -0.5$ is required to obtain about the same heat-transfer coefficient as for a solid wall with a constant temperature.

The influence of the pressure-gradient parameter Eu can be determined from the positions of the various curves in figure 5. It can be seen that, in general, as the Euler number increases from 0 to 1, the value of the dimensionless local heat-transfer coefficient $Nu/\sqrt{Re}$ increases considerably for fixed values of the wall-temperature-gradient parameter n and the coolant-flow parameter f_w. Exceptions can be noted, however. For an Euler number of 1 and a cooled wall ($f_w = -0.5$ and -1.0), these curves are essentially the same as the corresponding ones for $Eu = 0.5$. This similarity emphasizes that the primary pressure gradient effects occur as Eu changes from 0 to 0.5. The pressure gradient also influences the impermeable wall only slightly as Eu increases from 0.5 to 1.0.

Comparison of figures 5(a) and (b) for $f_w = 0$ and -0.5 reveals that the effect on the local heat-transfer coefficient of increasing the wall-temperature-gradient parameter from 0 to 1 is from one and a half to twice the effect of the pressure-gradient parameter (e.g., for $f_w = 0 = \mathrm{Eu}$, a change in n from 0 to 1 causes about a 65-percent increase in $Y'(0)$. For $f_w = 0 = n$, a change in Eu from 0 to 0.5 causes about a 40-percent increase in $Y'(0)$.). For the strongly cooled wall ($f_w = -1$), the opposite trend is observed, namely, that the pressure-gradient effects overshadow the effects of the wall-temperature-gradient parameter. Figures 5(b) and (c) indicate that a change in n from 0 to 1 is about twice as influential as the pressure gradient on the local heat-transfer coefficient for an impermeable wall. For a cooled wall, as noted before, the pressure gradient is not influential as Eu changes from 0.5 to 1, whereas an increase in wall-temperature-gradient parameter from 0 to 1 about doubles the value of the heat-transfer coefficient.

Figure 6 presents plots of the ratio of the gas-to-wall heat-transfer coefficient for a variable-wall temperature to that for a constant wall temperature against n for the different Euler numbers with f_w as parameter. These ratios were obtained by dividing the ordinates of figure 5 for various values of n by the ordinate for $n = 0$ for each coolant-flow parameter and Euler number. This method of plotting the results emphasizes the influence of a nonzero wall-temperature gradient on the local heat-transfer coefficient. For each Euler number, the curves represented by the different coolant-flow rates cross at the value $n = 0$. The intercept of each curve with the horizontal axis again gives the value of n for no heat transfer at the wall.

The effect of a variable wall temperature on the local heat-transfer coefficient for an impermeable flat plate with a turbulent boundary layer can be obtained by utilizing reference 34. For the turbulent case, the ratio $H_n/H_{n=0}$ is found to be 1.22, 1.13, and 0.86 for n of 1.0, 0.5, and -0.3, respectively. These values may be compared with the corresponding coordinates ($f_w = 0$) given in figure 6(a) for the laminar boundary layer. This comparison indicates that a wall-temperature variation with a turbulent boundary layer influences the local heat-transfer coefficient about one third as much as a similar variation with a laminar boundary layer.

Dimensionless convection boundary-layer thicknesses are plotted in figure 7 against n with f_w as parameter, for each of the Euler numbers considered. Figures 7(a), (b), and (c) show $\frac{\delta_c\sqrt{Re}}{x}$ for $Eu = 0$, 0.5, and 1.0, respectively. The effect of coolant flow is more marked for $n \geq 0$ than for $n < 0$. In fact, for $Eu = 0$ (fig. 7(a)), when $n = 1$, there are only slight differences in the convection thickness for the different coolant flows. For all Euler numbers and coolant flows, an increase in the wall-temperature gradient results in a decrease in the convection thickness. This behavior is due to the influence of the wall-temperature gradient on the boundary-layer temperature profile. Thus, from figures 2 and 4, an increase in n results in a smaller Y for a given η, Eu, and f_w, which is reflected in the convection thickness, since

$$\frac{\delta_c \sqrt{Re}}{x} = \int_0^x f' \; Y \; d\eta$$

The dimensionless thermal boundary-layer thicknesses presented in figure 8 indicate trends similar to those found for the convection thickness; increases in n result in decreases in the thermal thickness. For given values of the parameters f_w, n, and Eu, the thermal boundary-layer thickness is greater than the convection thickness. This is to be expected since

$$\frac{\delta_t \sqrt{Re}}{x} = \int_0^\infty Y \; d\eta$$

whereas the convection thickness is tempered by the velocity profile as noted in the preceding equation.

It was already pointed out that the intercepts of the various curves with the horizontal axes in figures 5 and 6 give the values of n for which there is a zero temperature gradient at the wall. Figure 9 presents this same information in a more compact form. The value n is plotted against the Euler number with the coolant-flow rate as parameter. The values for $f_w = 0$ have been presented by Levy (ref. 15). The increase in n for increasing $|f_w|$ indicates that a smaller wall-temperature gradient is needed to reduce the gradient at the wall to zero when coolant flow is emitted than for the impermeable plate. Because of the larger local heat-transfer coefficient for increased Euler number, a larger $|n|$ is needed to reduce the temperature gradient at the wall to zero for $Eu > 0$ than for $Eu = 0$.

SUMMARY OF RESULTS

Numerical solutions of the laminar-boundary-layer equations were obtained for a porous wall with a variable temperature and a pressure gradient. The assumptions utilized were constant-property values, negligible temperature changes caused by compression and frictional heating compared with the difference between the wall and the main-stream temperature, constant pressure and wall-temperature-gradient parameters, and a Prandtl number of 0.7. Tabulation was made of the velocity and temperature distributions, their derivatives, and dimensionless forms of the heat-transfer and friction coefficients and boundary-layer thicknesses.

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A summary of the results of this investigation follows:

1. The temperature distributions indicated that increased temperature gradients throughout the boundary layer resulted from increases in the wall-temperature-gradient parameter. Correspondingly, the local heat-transfer coefficients also increased.

2. Coolant-flow emission acted in a fashion similar to reducing the wall-temperature gradient, that is, increasing the coolant flow decreased the local heat-transfer coefficient. To obtain about the same local heat-transfer coefficient for a linear wall-temperature variation as for an impermeable wall with constant temperature, it was necessary to supply a coolant flow represented by $f_w = -0.5$.

3. Wall-temperature variations which result in zero-boundary-layer temperature gradient at the wall were obtained. As the pressure gradient was increased, larger wall-temperature variation was required to obtain a zero temperature gradient at the wall. Flow through the porous wall reduced the wall-temperature variation needed to yield a zero temperature gradient for all pressure gradients.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, July 15, 1954

APPENDIX A

ALLOWABLE MAGNITUDE FOR WALL-TEMPERATURE VARIATION

It is noted in reference 14 and also in the discussion of reference 19 that equation (3) is valid when $\delta/x \ll 1$ and

$$(\partial T/\partial x)_w \leq (T_w - T_\infty)/\delta \tag{A1}$$

By use of equation (7) and from reference 14, $\delta/x \approx 6/\sqrt{Re}$, equation (A1) becomes

$$n \leq \frac{\sqrt{Re}}{6} \tag{A2}$$

For the flow of air, Re is on the order of 10^4 away from stagnation point. Thus, n may be quite high and still allow equation (3), which neglects the effect of conduction within the fluid in the streamwise direction, to be used.

APPENDIX B

COMPARISON OF PRESENT RESULTS WITH RESULTS FROM PREVIOUS INVESTIGATIONS

The following table shows values of the negative of the boundary-layer-temperature gradient at the impermeable wall (local heat-transfer coefficient) for the present results and the results previously reported in the literature for Pr = 0.7. For each investigation, the relation between -Y'(0) and the notation employed in the reference is given.

$\left[-Y'(0) \text{ for } f_w = 0 \text{ and } Pr = 0.7\right]$

Eu	n	Pohlhausen (ref. 35)	Eckert (ref. 4)	Schuh (ref. 12)	Levy (ref. 15)	Present
0	0	0.2925	0.2927	0.293	0.2874	0.2927
	.5				.4023	.4059
	1.0			.407	.4770	.4803
1.0	-.5			.318		.3228
	0		.4959	.496	.4879	.4958
	.5				.6094	.6159
	1.0			.707	.7033	.7090
-Y'(0) =		$\frac{\alpha}{2}$	$\sqrt{\frac{m+1}{2}}\,A$	$-\sqrt{\frac{m+1}{2}}\left(\frac{d\theta_v}{dz}\right)_0$	$-\sqrt{\frac{m+1}{2}}\left(\frac{d\theta}{d\eta}\right)_{n=0}$	-Y'(0)

Examination of the table reveals a check for Eu = 0, n = 0 between the present results and those reported by Pohlhausen, Eckert, and Schuh. At Eu = 1 and n = 0, the present results are in agreement with those presented by Eckert and Schuh. As already pointed out by Levy (ref. 15) his results are subjected to an error of the order of 1 to 2 percent. If the present results are assumed correct, this small error is seen to hold true for n = 0 with both the flat-plate and stagnation-point flow. For n $\neq$ 0, there is better agreement between Levy's results and the present results.

Levy (ref. 15) also noted the validity of Schuh's results (ref. 12) for stagnation-point flow and the discrepancy for flat-plate flow with a variable wall temperature. The validity for Eu = 1 and Eu = 0, n = 0 as well as the discrepancy for Eu = 0, n = 1 is apparent from the table.

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TABLE I. - VELOCITY AND TEMPERATURE DISTRIBUTIONS FOR WEDGE FLOW WITH A VARIABLE TEMPERATURE ALONG THE POROUS WALL

	$f_w = 0.0$; Eu = 0.0												
					(1)			(2)			(3)		
	$\frac{\delta^* \sqrt{Re}}{x} = 1.7215$ $\frac{\delta_1 \sqrt{Re}}{x} = 0.6652$				n = -0.5 $\frac{\delta_c \sqrt{Re}}{x} = 1.8610$ $\frac{\delta_t \sqrt{Re}}{x} = 3.4167$			n = 0.5 $\frac{\delta_c \sqrt{Re}}{x} = 0.5782$ $\frac{\delta_t \sqrt{Re}}{x} = 1.5617$			n = 1.0 $\frac{\delta_c \sqrt{Re}}{x} = 0.4582$ $\frac{\delta_t \sqrt{Re}}{x} = 1.3627$		
	f	f'	f"	f'"	Y	Y'	Y"	Y	Y'	Y"	Y	Y'	Y"
0	0	0	0.3320	0	1.0000	0	0	1.0000	-0.4059	0	1.0000	-0.4803	0
.2	.0066	.0664	.3319	-.0011	.9998	-.0023	-.0232	.9190	-.4036	.0223	.9042	-.4759	.0431
.4	.0266	.1327	.3314	-.0044	.9987	-.0093	-.0463	.8388	-.3971	.0427	.8102	-.4635	.0796
.6	.0597	.1989	.3300	-.0099	.9958	-.0208	-.0689	.7604	-.3867	.0610	.7193	-.4445	.1094
.8	.1061	.2647	.3273	-.0174	.9901	-.0367	-.0903	.6844	-.3728	.0772	.6327	-.4201	.1328
1.0	.1655	.3297	.3230	-.0267	.9808	-.0568	-.1099	.6114	-.3560	.0912	.5515	-.3918	.1500
1.2	.2379	.3937	.3165	-.0377	.9671	-.0805	-.1266	.5422	-.3365	.1027	.4762	-.3605	.1613
1.4	.3229	.4562	.3078	-.0497	.9484	-.1072	-.1393	.4770	-.3150	.1118	.4074	-.3276	.1671
1.6	.4202	.5167	.2966	-.0623	.9241	-.1359	-.1471	.4162	-.2920	.1182	.3452	-.2940	.1681
1.8	.5294	.5747	.2829	-.0749	.8940	-.1656	-.1491	.3602	-.2679	.1221	.2898	-.2607	.1649
2.0	.6499	.6297	.2667	-.0867	.8579	-.1951	-.1447	.3091	-.2433	.1235	.2409	-.2283	.1581
2.2	.7811	.6812	.2483	-.0970	.8161	-.2230	-.1336	.2629	-.2187	.1225	.1983	-.1976	.1486
2.4	.9221	.7289	.2281	-.1052	.7689	-.2481	-.1161	.2216	-.1945	.1193	.1617	-.1690	.1370
2.6	1.0723	.7723	.2064	-.1107	.7171	-.2691	-.0928	.1851	-.1711	.1142	.1306	-.1429	.1242
2.8	1.2308	.8114	.1840	-.1132	.6616	-.2850	-.0651	.1531	-.1489	.1076	.1044	-.1194	.1107
3.0	1.3966	.8459	.1614	-.1127	.6035	-.2950	-.0345	.1254	-.1281	.0998	.0826	-.0986	.0971
3.2	1.5688	.8760	.1391	-.1091	.5440	-.2987	-.0028	.1017	-.1090	.0911	.0648	-.0805	.0839
3.4	1.7467	.9016	.1179	-.1030	.4844	-.2961	.0281	.0817	-.0917	.0819	.0503	-.0650	.0714
3.6	1.9292	.9232	.0981	-.0946	.4259	-.2876	.0566	.0649	-.0763	.0725	.0386	-.0518	.0600
3.8	2.1157	.9410	.0801	-.0848	.3697	-.2738	.0809	.0510	-.0627	.0633	.0294	-.0409	.0496
4.0	2.3054	.9554	.0642	-.0740	.3167	-.2556	.1003	.0397	-.0510	.0544	.0221	-.0319	.0405
4.2	2.4977	.9668	.0505	-.0631	.2677	-.2341	.1140	.0305	-.0409	.0461	.0165	-.0246	.0327
4.4	2.6920	.9758	.0390	-.0525	.2232	-.2104	.1220	.0232	-.0325	.0385	.0122	-.0188	.0260
4.6	2.8878	.9826	.0295	-.0426	.1836	-.1857	.1246	.0174	-.0255	.0317	.0089	-.0141	.0204
4.8	3.0849	.9877	.0219	-.0337	.1489	-.1609	.1222	.0129	-.0197	.0258	.0065	-.0105	.0158
5.0	3.2828	.9914	.0159	-.0261	.1191	-.1370	.1161	.0095	-.0151	.0206	.0047	-.0077	.0121
5.2	3.4814	.9941	.0113	-.0197	.0940	-.1146	.1071	.0068	-.0114	.0163	.0033	-.0056	.0092
5.4	3.6804	.9960	.0079	-.0146	.0732	-.0943	.0960	.0048	-.0085	.0127	.0024	-.0040	.0068
5.6	3.8798	.9974	.0054	-.0105	.0562	-.0763	.0840	.0034	-.0063	.0097	.0017	-.0028	.0051
5.8	4.0793	.9983	.0037	-.0075	.0425	-.0607	.0719	.0023	-.0046	.0074	.0012	-.0020	.0037
6.0	4.2791	.9989	.0024	-.0051	.0317	-.0475	.0600	.0015	-.0033	.0055	.0009	-.0014	.0026
6.2	4.4789	.9993	.0016	-.0035	.0233	-.0366	.0493	.0009	-.0024	.0041	.0007	-.0009	.0019
6.4	4.6787	.9995	.0010	-.0023	.0169	-.0277	.0395	.0005	-.0017	.0029	.0005	-.0006	.0013
6.6	4.8787	.9997	.0006	-.0015	.0121	-.0207	.0311	.0002	-.0012	.0021	.0004	-.0004	.0009
6.8	5.0786	.9998	.0004	-.0009	.0085	-.0152	.0240	.0000	-.0008	.0015	.0004	-.0002	.0006
7.0	5.2786	.9998	.0002	-.0006	.0059	-.0110	.0182				.0003	-.0001	.0004
7.2	5.4785	.9998	.0001	-.0003	.0040	-.0078	.0136				.0003	.0000	.0003
7.4	5.6785	.9999	.0001	-.0002	.0027	-.0055	.0100				.0003	.0000	.0002
7.6	5.8785	.9999	.0000	.0000	.0018	-.0038	.0072				.0002	.0000	.0001
7.8					.0012	-.0026	.0051						
8.0					.0008	-.0017	.0037						
8.2					.0005	-.0011	.0023						
8.4					.0003	-.0007	.0015						
8.6					.0002	-.0005	.0009						
8.8					.0001	-.0003	.0007						
9.0					.0001	-.0002	.0005						
9.2					.0001	-.0001	.0003						
9.4					.0001	.0000	.0003						

TABLE I. - Continued. VELOCITY AND TEMPERATURE DISTRIBUTIONS FOR WEDGE FLOW WITH A VARIABLE TEMPERATURE ALONG THE POROUS WALL

	$f_w = 0.0$; Eu = 0.5												
					(4)			(5)			(6)		
	$\frac{\delta^* \sqrt{Re}}{x} = 0.8542$ $\frac{\delta_1 \sqrt{Re}}{x} = 0.3773$				$n = -0.75$ $\frac{\delta_c \sqrt{Re}}{x} = 1.6075$ $\frac{\delta_t \sqrt{Re}}{x} = 2.4034$			$n = -0.5$ $\frac{\delta_c \sqrt{Re}}{x} = 0.6204$ $\frac{\delta_t \sqrt{Re}}{x} = 1.1861$			$n = 1.0$ $\frac{\delta_c \sqrt{Re}}{x} = 0.5181$ $\frac{\delta_t \sqrt{Re}}{x} = 1.0515$		
η	f	f'	f"	f'''	Y	Y'	Y"	Y	Y'	Y"	Y	Y'	Y"
0	0	0	0.89975	-0.5000	1.0000	0	0	1.0000	-0.5426	0	1.0000	-0.6350	0
.1	.0044	.0875	.2498	-.4990	.9999	-.0023	-.0459	.9458	-.5411	.0302	.9366	-.6320	.0588
.2	.0173	.1700	.8000	-.4960	.9994	-.0091	-.0891	.8919	-.5367	.0579	.8738	-.6235	.1096
.3	.0382	.2475	.7507	-.4909	.9980	-.0200	-.1293	.8385	-.5296	.0833	.8121	-.6103	.1529
.4	.0667	.3201	.7019	-.4839	.9953	-.0348	-.1661	.7860	-.5201	.1063	.7519	-.5931	.1892
.5	.1021	.3879	.6540	-.4748	.9909	-.0531	-.1990	.7346	-.5084	.1270	.6935	-.5727	.2190
.6	.1441	.4509	.6070	-.4639	.9845	-.0745	-.2274	.6844	-.4948	.1454	.6374	-.5495	.2428
.7	.1921	.5093	.5612	-.4512	.9759	-.0985	-.2510	.6357	-.4794	.1617	.5837	-.5243	.2610
.8	.2458	.5632	.5168	-.4367	.9648	-.1246	-.2692	.5886	-.4625	.1757	.5326	-.4975	.2742
.9	.3046	.6128	.4740	-.4206	.9510	-.1522	-.2816	.5433	-.4443	.1876	.4842	-.4696	.2828
1.0	.3682	.6581	.4328	-.4030	.9344	-.1807	-.2879	.4998	-.4250	.1973	.4387	-.4411	.2874
1.1	.4361	.6994	.3934	-.3841	.9149	-.2095	-.2879	.4583	-.4049	.2049	.3960	-.4123	.2883
1.2	.5080	.7368	.3560	-.3642	.8925	-.2380	-.2817	.4188	-.3841	.2105	.3562	-.3835	.2860
1.3	.5834	.7706	.3206	-.3433	.8673	-.2656	-.2696	.3815	-.3629	.2140	.3193	-.3552	.2810
1.4	.6620	.8010	.2873	-.3218	.8394	-.2917	-.2516	.3463	-.3414	.2157	.2852	-.3274	.2737
1.5	.7435	.8282	.2562	-.2999	.8090	-.3157	-.2286	.3132	-.3198	.2156	.2538	-.3005	.2644
1.6	.8275	.8523	.2274	-.2779	.7763	-.3372	-.2009	.2823	-.2983	.2138	.2250	-.2746	.2536
1.7	.9138	.8737	.2007	-.2558	.7416	-.3558	-.1695	.2535	-.2771	.2105	.1988	-.2498	.2415
1.8	1.0022	.8925	.1762	-.2341	.7052	-.3711	-.1352	.2269	-.2563	.2057	.1750	-.2263	.2284
1.9	1.0923	.9090	.1538	-.2129	.6675	-.3828	-.0990	.2022	-.2360	.1997	.1535	-.2042	.2148
2.0	1.1839	.9234	.1336	-.1923	.6288	-.3908	-.0619	.1796	-.2164	.1925	.1342	-.1834	.2007
2.1	1.2769	.9358	.1153	-.1726	.5895	-.3951	-.0247	.1589	-.1975	.1845	.1168	-.1640	.1865
2.2	1.3710	.9465	.0990	-.1539	.5499	-.3957	.0116	.1401	-.1795	.1756	.1013	-.1461	.1723
2.3	1.4661	.9557	.0845	-.1363	.5105	-.3928	.0462	.1230	-.1624	.1662	.0875	-.1296	.1583
2.4	1.5621	.9635	.0717	-.1199	.4715	-.3865	.0785	.1076	-.1463	.1563	.0754	-.1144	.1447
2.5	1.6588	.9701	.0605	-.1048	.4333	-.3772	.1078	.0937	-.1312	.1461	.0646	-.1006	.1315
2.6	1.7561	.9756	.0507	-.0909	.3962	-.3651	.1337	.0813	-.1171	.1357	.0552	-.0881	.1189
2.7	1.8539	.9803	.0423	-.0783	.3604	-.3506	.1557	.0703	-.1040	.1254	.0470	-.0768	.1070
2.8	1.9521	.9841	.0350	-.0670	.3262	-.3341	.1739	.0605	-.0920	.1151	.0398	-.0667	.0957
2.9	2.0507	.9873	.0288	-.0570	.2937	-.3160	.1880	.0518	-.0810	.1051	.0336	-.0576	.0853
3.0	2.1495	.9899	.0236	-.0481	.2631	-.2967	.1981	.0442	-.0710	.0954	.0282	-.0496	.0755
3.1	2.2486	.9920	.0192	-.0403	.2344	-.2765	.2043	.0376	-.0619	.0861	.0236	-.0425	.0666
3.2	2.3479	.9938	.0155	-.0335	.2078	-.2559	.2070	.0318	-.0537	.0773	.0197	-.0363	.0584
3.3	2.4474	.9952	.0125	-.0277	.1832	-.2352	.2065	.0268	-.0464	.0690	.0163	-.0308	.0510
3.4	2.5470	.9963	.0099	-.0227	.1607	-.2147	.2030	.0225	-.0399	.0612	.0135	-.0260	.0442
3.5	2.6466	.9972	.0079	-.0185	.1402	-.1947	.1971	.0188	-.0342	.0540	.0111	-.0219	.0382
3.6	2.7464	.9979	.0062	-.0149	.1217	-.1754	.1891	.0157	-.0291	.0474	.0091	-.0184	.0329
3.7	2.8462	.9984	.0049	-.0120	.1051	-.1570	.1795	.0130	-.0247	.0414	.0074	-.0153	.0281
3.8	2.9461	.9989	.0038	-.0096	.0903	-.1396	.1685	.0107	-.0208	.0359	.0060	-.0127	.0239
3.9	3.0460	.9992	.0030	-.0076	.0772	-.1233	.1567	.0088	-.0175	.0310	.0049	-.0105	.0202
4.0	3.1459	.9995	.0023	-.0059	.0656	-.1082	.1445	.0072	-.0146	.0266	.0039	-.0087	.0171
4.1	3.2459	.9997	.0018	-.0046	.0555	-.0944	.1318	.0059	-.0121	.0227	.0031	-.0071	.0143
4.2	3.3458	.9998	.0014	-.0036	.0467	-.0818	.1192	.0048	-.0100	.0193	.0025	-.0058	.0119
4.3	3.4458	.9999	.0010	-.0027	.0391	-.0705	.1070	.0038	-.0083	.0163	.0020	-.0047	.0099
4.4	3.5458	1.0000	.0008	-.0021	.0326	-.0604	.0953	.0031	-.0068	.0137	.0015	-.0038	.0082
4.5	3.6458	1.0001	.0006	-.0016	.0270	-.0514	.0842	.0025	-.0054	.0112	.0012	-.0030	.0066
4.6	3.7458	1.0001	.0005	-.0012	.0223	-.0435	.0738	.0020	-.0044	.0093	.0009	-.0024	.0054
4.7	3.8459	1.0002	.0004	-.0009	.0183	-.0366	.0643	.0016	-.0034	.0074	.0007	-.0018	.0042
4.8	3.9459	1.0002	.0003	-.0007	.0149	-.0306	.0556	.0012	-.0028	.0062	.0005	-.0015	.0034
4.9	4.0459	1.0002	.0002	-.0005	.0121	-.0254	.0476	.0010	-.0021	.0048	.0004	-.0011	.0026
5.0					.0098	-.0210	.0406	.0008	-.0018	.0042	.0003	-.0010	.0023
5.1					.0079	-.0173	.0345	.0007	-.0013	.0031	.0002	-.0007	.0016
5.2					.0063	-.0141	.0289	.0005	-.0012	.0028	.0001	-.0006	.0015
5.3					.0050	-.0115	.0242	.0004	-.0007	.0018	.0001	-.0004	.0009
5.4					.0040	-.0093	.0201	.0004	-.0005	.0014	.0000	-.0003	.0007
5.5					.0032	-.0075	.0166	.0003	-.0004	.0011	.0000	-.0002	.0005
5.6					.0025	-.0060	.0136	.0003	-.0003	.0008	.0000	-.0002	.0004
5.7					.0020	-.0048	.0111	.0003	-.0002	.0007	.0000	-.0001	.0003
5.8					.0016	-.0038	.0091	.0002	-.0001	.0005	.0000	-.0001	.0002
5.9					.0013	-.0030	.0072	.0002	-.0001	.0004			
6.0					.0010	-.0024	.0060	.0002	-.0001	.0002			
6.1					.0008	-.0019	.0048						
6.2					.0006	-.0015	.0039						
6.3					.0005	-.0011	.0031						
6.4					.0004	-.0008	.0021						
6.5					.0003	-.0006	.0016						
6.6					.0003	-.0005	.0013						
6.7					.0003	-.0004	.0010						
6.8					.0003	-.0003	.0007						
6.9					.0003	-.0002	.0004						
7.0					.0003	-.0002	.0004						

TABLE I. - Continued. VELOCITY AND TEMPERATURE DISTRIBUTIONS FOR WEDGE FLOW WITH A VARIABLE TEMPERATURE ALONG THE POROUS WALL

	$f_w = 0.0$; Eu = 1.0															
	$\frac{\delta^* \sqrt{Re}}{x} = 0.6477$ $\frac{\delta_1 \sqrt{Re}}{x} = 0.2921$				(7) n = -1.0 $\frac{\delta_c \sqrt{Re}}{x} = 1.4086$ $\frac{\delta_t \sqrt{Re}}{x} = 2.0175$			(8) n = -0.5 $\frac{\delta_c \sqrt{Re}}{x} = 0.9185$ $\frac{\delta_t \sqrt{Re}}{x} = 1.4398$			(9) n = 0.5 $\frac{\delta_c \sqrt{Re}}{x} = 0.5861$ $\frac{\delta_t \sqrt{Re}}{x} = 1.0361$			(10) n = 1.0 $\frac{\delta_c \sqrt{Re}}{x} = 0.5061$ $\frac{\delta_t \sqrt{Re}}{x} = 0.9356$		
η	f	f'	f"	f'"	Y	Y'	Y"	Y	Y'	Y"	Y	Y'	Y"	Y	Y'	Y"
0	0	0	1.2326	-1.0000	1.0000	0	0	1.0000	-0.3228	0	1.0000	-0.6159	0	1.0000	-0.7090	0
.1	.0060	.1183	1.1328	-.9928	.9999	-.0042	-.0828	.9677	-.3248	-.0387	.9385	-.6138	.0414	.9292	-.7049	.0799
.2	.0233	.2266	1.0345	-.9728	.9989	-.0163	-.1581	.9349	-.3302	-.0688	.8774	-.6078	.0795	.8593	-.6934	.1476
.3	.0510	.3252	.9386	-.9421	.9964	-.0355	-.2255	.9015	-.3383	-.0905	.8170	-.5980	.1144	.7908	-.6757	.2042
.4	.0881	.4145	.8463	-.9027	.9916	-.0611	-.2839	.8672	-.3481	-.1043	.7579	-.5850	.1460	.7243	-.6529	.2504
.5	.1336	.4947	.7583	-.8566	.9840	-.0920	-.3321	.8319	-.3589	-.1105	.7001	-.5690	.1744	.6603	-.6260	.2872
.6	.1867	.5663	.6752	-.8054	.9731	-.1272	-.3691	.7954	-.3699	-.1093	.6442	-.5502	.1996	.5992	-.5958	.3154
.7	.2466	.6299	.5974	-.7506	.9585	-.1655	-.3940	.7579	-.3805	-.1014	.5902	-.5291	.2214	.5412	-.5632	.3358
.8	.3124	.6859	.5251	-.6935	.9399	-.2056	-.4063	.7194	-.3900	-.0874	.5384	-.5061	.2399	.4866	-.5289	.3493
.9	.3835	.7351	.4587	-.6356	.9173	-.2463	-.4059	.6800	-.3978	-.0681	.4890	-.4813	.2550	.4355	-.4935	.3566
1.0	.4592	.7779	.3980	-.5777	.8907	-.2863	-.3930	.6399	-.4035	-.0445	.4422	-.4552	.2667	.3879	-.4577	.3584
1.1	.5389	.8149	.3431	-.5209	.8601	-.3245	-.3682	.5993	-.4066	-.0175	.3980	-.4280	.2750	.3439	-.4220	.3554
1.2	.6220	.8467	.2938	.4659	.8259	-.3596	-.3329	.5586	-.4069	.0116	.3566	-.4003	.2800	.3035	-.3868	.3483
1.3	.7081	.8738	.2499	-.4134	.7883	-.3907	-.2885	.5181	-.4043	.0419	.3180	-.3722	.2817	.2666	-.3525	.3378
1.4	.7967	.8968	.2110	-.3638	.7479	-.4170	-.2369	.4779	-.3985	.0722	.2822	-.3440	.2804	.2330	-.3193	.3243
1.5	.8873	.9162	.1770	-.3176	.7051	-.4379	-.1802	.4384	-.3898	.1016	.2492	-.3162	.2763	.2026	-.2877	.3086
1.6	.9798	.9324	.1474	-.2751	.6605	-.4529	-.1205	.4000	-.3783	.1289	.2189	-.2889	.2696	.1754	-.2577	.2912
1.7	1.0737	.9458	.1218	-.2363	.6147	-.4619	-.0598	.3629	-.3642	.1536	.1914	-.2623	.2605	.1511	-.2295	.2725
1.8	1.1689	.9569	.1000	-.2013	.5683	-.4649	-.0003	.3273	-.3477	.1749	.1664	-.2368	.2495	.1294	-.2032	.2530
1.9	1.2650	.9659	.0815	-.1701	.5219	-.4621	.0563	.2934	-.3293	.1924	.1440	-.2125	.2368	.1104	-.1789	.2330
2.0	1.3620	.9733	.0659	-.1425	.4761	-.4538	.1082	.2615	-.3094	.2059	.1239	-.1895	.2229	.0936	-.1566	.2131
2.1	1.4596	.9792	.0528	-.1184	.4313	-.4406	.1546	.2316	-.2883	.2152	.1060	-.1680	.2079	.0790	-.1363	.1934
2.2	1.5578	.9839	.0421	-.0975	.3881	-.4231	.1941	.2038	-.2665	.2204	.0902	-.1479	.1924	.0663	-.1179	.1742
2.3	1.6564	.9877	.0333	-.0796	.3468	-.4020	.2263	.1783	-.2443	.2217	.0764	-.1295	.1765	.0553	-.1014	.1558
2.4	1.7553	.9906	.0261	-.0645	.3078	-.3781	.2512	.1550	-.2222	.2193	.0643	-.1126	.1607	.0459	-.0867	.1384
2.5	1.8545	.9929	.0203	-.0517	.2713	-.3521	.2685	.1338	-.2006	.2138	.0538	-.0973	.1451	.0379	-.0737	.1220
2.6	1.9539	.9947	.0157	-.0412	.2375	-.3247	.2787	.1148	-.1796	.2056	.0448	-.0836	.1299	.0312	-.0623	.1068
2.7	2.0534	.9961	.0120	-.0324	.2064	-.2966	.2824	.0979	-.1595	.1951	.0370	-.0713	.1154	.0254	-.0523	.0929
2.8	2.1531	.9971	.0091	-.0254	.1782	-.2684	.2801	.0829	-.1406	.1830	.0305	-.0605	.1018	.0207	-.0436	.0802
2.9	2.2528	.9979	.0069	-.0196	.1528	-.2407	.2729	.0697	-.1230	.1695	.0249	-.0509	.0890	.0167	-.0362	.0687
3.0	2.3527	.9985	.0051	-.0151	.1301	-.2139	.2614	.0583	-.1067	.1554	.0202	-.0426	.0773	.0134	-.0298	.0585
3.1	2.4525	.9990	.0038	-.0115	.1100	-.1885	.2467	.0483	-.0919	.1409	.0163	-.0355	.0666	.0107	-.0245	.0495
3.2	2.5524	.9993	.0028	-.0086	.0924	-.1647	.2297	.0398	-.0785	.1264	.0131	-.0293	.0569	.0085	-.0199	.0415
3.3	2.6524	.9995	.0021	-.0064	.0770	-.1427	.2110	.0326	-.0666	.1123	.0104	-.0240	.0483	.0067	-.0161	.0346
3.4	2.7523	.9997	.0015	-.0048	.0638	-.1226	.1916	.0265	-.0561	.0987	.0083	-.0196	.0407	.0052	-.0130	.0286
3.5	2.8523	.9998	.0011	-.0035	.0525	-.1044	.1717	.0213	-.0468	.0860	.0065	-.0159	.0340	.0040	-.0104	.0235
3.6	2.9523	.9999	.0008	-.0025	.0429	-.0882	.1523	.0171	-.0388	.0743	.0051	-.0128	.0282	.0031	-.0082	.0192
3.7	3.0523	1.0000	.0006	-.0018	.0348	-.0739	.1335	.0135	-.0319	.0635	.0039	-.0102	.0232	.0024	-.0065	.0156
3.8	3.1523	1.0001	.0004	-.0013	.0280	-.0615	.1161	.0106	-.0261	.0538	.0030	-.0081	.0190	.0018	-.0051	.0125
3.9	3.2523	1.0001	.0003	-.0009	.0224	-.0507	.0997	.0083	-.0211	.0452	.0023	-.0064	.0154	.0014	-.0040	.0100
4.0	3.3523	1.0001	.0003	-.0006	.0178	-.0415	.0849	.0064	-.0166	.0368	.0017	-.0049	.0120	.0010	-.0030	.0077
4.1	3.4523	1.0002	.0002	-.0004	.0140	-.0337	.0716	.0048	-.0133	.0304	.0013	-.0038	.0096	.0007	-.0023	.0061
4.2					.0110	-.0271	.0597	.0037	-.0100	.0236	.0009	-.0027	.0071	.0005	-.0016	.0044
4.3					.0086	-.0217	.0495	.0026	-.0081	.0199	.0006	-.0022	.0059	.0003	-.0013	.0036
4.4					.0067	-.0172	.0405	.0019	-.0059	.0147	.0004	-.0015	.0041	.0002	-.0009	.0025
4.5					.0052	-.0136	.0328	.0014	-.0052	.0134	.0003	-.0014	.0038	.0001	-.0008	.0023
4.6					.0040	-.0106	.0265	.0009	-.0032	.0084	.0002	-.0008	.0021	.0001	-.0004	.0012
4.7					.0031	-.0082	.0211	.0006	-.0020	.0054	.0001	-.0004	.0012	.0000	-.0002	.0006
4.8					.0024	-.0063	.0166	.0004	-.0012	.0034	.0000	-.0002	.0006	.0000	-.0001	.0003
4.9					.0018	-.0048	.0130	.0003	-.0009	.0025	.0000	-.0001	.0004	.0000	-.0001	.0002
5.0					.0014	-.0036	.0100	.0002	-.0007	.0019	.0000	-.0001	.0003	.0000	.0000	.0001
5.1					.0011	-.0027	.0076	.0001	-.0004	.0013	.0000	-.0001	.0002	.0000	.0000	.0001
5.2					.0009	-.0020	.0058	.0001	-.0004	.0011	.0000	-.0001	.0002	.0000	.0000	.0001
5.3					.0007	-.0015	.0044	.0000	-.0002	.0006	.0000	.0000	.0001	.0000	.0000	.0000
5.4					.0006	-.0009	.0026	.0000	-.0002	.0003	.0000	.0000	.0001			
5.5					.0005	-.0007	.0020	.0000	.0000	.0001	.0000	.0000	.0000			
5.6					.0004	-.0005	.0011									
5.7					.0004	-.0004	.0011									
5.8					.0004	-.0003	.0008									
5.9					.0004	-.0002	.0004									
6.0					.0004	-.0002	.0004									

TABLE I. - Continued. VELOCITY AND TEMPERATURE DISTRIBUTIONS FOR WEDGE FLOW WITH A VARIABLE TEMPERATURE ALONG THE POROUS WALL

	$f_w = -0.5$; Eu = 0.0												
					(11)			(12)			(13)		
	$\frac{\delta^* \sqrt{Re}}{x} = 2.4595$ $\frac{\delta_1 \sqrt{Re}}{x} = 0.8288$				n = -0.3702 $\frac{\delta_c \sqrt{Re}}{x} = 1.9212$ $\frac{\delta_t \sqrt{Re}}{x} = 4.1340$			n = 0.5 $\frac{\delta_c \sqrt{Re}}{x} = 0.6231$ $\frac{\delta_t \sqrt{Re}}{x} = 2.0470$			n = 1.0 $\frac{\delta_c \sqrt{Re}}{x} = 0.4724$ $\frac{\delta_t \sqrt{Re}}{x} = 1.7627$		
η	f	f'	f"	f'''	Y	Y'	Y"	Y	Y'	Y"	Y	Y'	Y"
0	-0.5000	0	0.1645	0.0411	1.0000	0	0	1.0000	-0.2611	-0.0457	1.0000	-0.3211	-0.0562
.2	-.4967	.0337	.1729	.0429	1.0000	-.0007	-.0081	.9469	-.2695	-.0367	.9348	-.3306	-.0374
.4	-.4864	.0692	.1816	.0442	.9996	-.0034	-.0185	.8924	-.2757	-.0253	.8680	-.3358	-.0151
.6	-.4689	.1064	.1905	.0447	.9985	-.0081	-.0289	.8368	-.2796	-.0147	.8007	-.3369	.0044
.8	-.4437	.1454	.1995	.0442	.9962	-.0149	-.0389	.7806	-.2815	-.0040	.7335	-.3342	.0228
1.0	-.4106	.1862	.2082	.0427	.9924	-.0238	-.0513	.7243	-.2809	.0068	.6673	-.3272	.0399
1.2	-.3691	.2287	.2165	.0400	.9865	-.0359	-.0631	.6684	-.2785	.0175	.6028	-.3178	.0554
1.4	-.3190	.2727	.2241	.0357	.9780	-.0490	-.0746	.6131	-.2740	.0279	.5404	-.3053	.0691
1.6	-.2599	.3182	.2307	.0300	.9666	-.0653	-.0857	.5589	-.2674	.0379	.4808	-.2903	.0807
1.8	-.1916	.3649	.2359	.0226	.9517	-.0835	-.0956	.5062	-.2589	.0473	.4244	-.2732	.0901
2.0	-.1139	.4125	.2396	.0136	.9331	-.1035	-.1039	.4555	-.2485	.0558	.3717	-.2544	.0972
2.2	-.0266	.4606	.2413	.0032	.9102	-.1249	-.1099	.4069	-.2366	.0634	.3228	-.2345	.1019
2.4	.0703	.5088	.2408	-.0085	.8830	-.1472	-.1129	.3609	-.2233	.0698	.2779	-.2138	.1043
2.6	.1769	.5567	.2378	-.0270	.8513	-.1698	-.1124	.3177	-.2088	.0748	.2372	-.1929	.1044
2.8	.2930	.6038	.2323	-.0340	.8152	-.1919	-.1079	.2775	-.1934	.0785	.2007	-.1722	.1025
3.0	.4183	.6495	.2242	-.0469	.7747	-.2127	-.0993	.2404	-.1775	.0806	.1683	-.1520	.0988
3.2	.5526	.6933	.2136	-.0590	.7302	-.2313	-.0865	.2065	-.1613	.0813	.1399	-.1328	.0936
3.4	.6955	.7348	.2007	-.0698	.6823	-.2470	-.0699	.1759	-.1451	.0805	.1151	-.1147	.0871
3.6	.8464	.7735	.1858	-.0787	.6317	-.2591	-.0499	.1485	-.1291	.0784	.0939	-.0980	.0799
3.8	1.0047	.8091	.1694	-.0851	.5790	-.2668	-.0276	.1242	-.1138	.0752	.0759	-.0828	.0721
4.0	1.1698	.8412	.1520	-.0889	.5252	-.2700	-.0040	.1029	-.0991	.0709	.0607	-.0692	.0641
4.2	1.3409	.8698	.1341	-.0899	.4713	-.2685	.0197	.0845	-.0855	.0658	.0481	-.0571	.0561
4.4	1.5175	.8948	.1162	-.0882	.4182	-.2622	.0423	.0686	-.0729	.0602	.0377	-.0467	.0484
4.6	1.6986	.9163	.0990	-.0840	.3667	-.2517	.0625	.0552	-.0614	.0542	.0293	-.0377	.0412
4.8	1.8838	.9345	.0827	-.0779	.3177	-.2375	.0796	.0440	-.0512	.0481	.0226	-.0302	.0346
5.0	2.0722	.9495	.0679	-.0703	.2719	-.2202	.0927	.0347	-.0421	.0421	.0172	-.0238	.0287
5.2	2.2634	.9618	.0546	-.0618	.2298	-.2006	.1016	.0271	-.0343	.0363	.0129	-.0186	.0235
5.4	2.4568	.9715	.0432	-.0530	.1917	-.1798	.1063	.0209	-.0276	.0308	.0097	-.0144	.0189
5.6	2.6519	.9791	.0334	-.0443	.1579	-.1584	.1069	.0159	-.0219	.0258	.0071	-.0110	.0151
5.8	2.8483	.9850	.0254	-.0362	.1284	-.1372	.1040	.0120	-.0172	.0213	.0052	-.0083	.0119
6.0	3.0458	.9894	.0189	-.0288	.1030	-.1170	.0983	.0090	-.0134	.0174	.0038	-.0062	.0092
6.2	3.2440	.9926	.0138	-.0224	.0815	-.0981	.0904	.0066	-.0103	.0139	.0027	-.0046	.0071
6.4	3.4428	.9950	.0099	-.0170	.0636	-.0809	.0811	.0048	-.0078	.0110	.0019	-.0034	.0054
6.6	3.6419	.9967	.0069	-.0126	.0490	-.0657	.0711	.0035	-.0058	.0086	.0013	-.0024	.0040
6.8	3.8414	.9978	.0048	-.0092	.0372	-.0525	.0609	.0025	-.0043	.0066	.0009	-.0017	.0030
7.0	4.0410	.9986	.0032	-.0065	.0279	-.0413	.0512	.0018	-.0031	.0050	.0006	-.0012	.0022
7.2	4.2408	.9991	.0021	-.0045	.0206	-.0320	.0421	.0012	-.0023	.0038	.0004	-.0009	.0016
7.4	4.4407	.9995	.0014	-.0031	.0150	-.0244	.0340	.0008	-.0016	.0028	.0003	-.0006	.0011
7.6	4.6406	.9997	.0009	-.0020	.0107	-.0183	.0269	.0006	-.0011	.0020	.0002	-.0004	.0008
7.8	4.8405	.9998	.0005	-.0013	.0076	-.0135	.0210	.0004	-.0008	.0015	.0001	-.0003	.0005
8.0	5.0405	.9999	.0003	-.0008	.0052	-.0098	.0160	.0003	-.0005	.0010	.0001	-.0002	.0004
8.2	5.2405	1.0000	.0002	-.0005	.0036	-.0071	.0120	.0002	-.0004	.0007	.0000	-.0001	.0002
8.4	5.4405	1.0000	.0001	-.0003	.0024	-.0050	.0089	.0001	-.0002	.0005	.0000	-.0001	.0002
8.6	5.6405	1.0000	.0001	-.0002	.0015	-.0035	.0064	.0001	-.0002	.0003	.0000	-.0001	.0001
8.8	5.8405	1.0000	.0000	-.0001	.0010	-.0024	.0046	.0000	-.0001	.0002	.0000	.0000	.0001
9.0					.0006	-.0016	.0032	.0000	-.0001	.0001	.0000	.0000	.0000
9.2					.0003	-.0011	.0022	.0000	.0000	.0001			
9.4					.0001	-.0007	.0015	.0000	.0000	.0001			
9.6					.0000	-.0004	.0010	.0000	.0000	.0000			
9.8					.0000	-.0003	.0007						
10.0					.0000	-.0001	.0004						

TABLE I. - Continued. VELOCITY AND TEMPERATURE DISTRIBUTIONS FOR WEDGE FLOW WITH A VARIABLE TEMPERATURE ALONG THE POROUS WALL

	$f_w = -0.5$; Eu = 0.5															
					(14)			(15)			(16)			(17)		
	$\frac{\delta^* \sqrt{Re}}{x} = 1.0342$ $\frac{\delta_1 \sqrt{Re}}{x} = 0.4440$				n = -0.5356 $\frac{\delta_c \sqrt{Re}}{x} = 1.7434$ $\frac{\delta_t \sqrt{Re}}{x} = 2.7060$			n = -0.5 $\frac{\delta_c \sqrt{Re}}{x} = 1.6506$ $\frac{\delta_t \sqrt{Re}}{x} = 2.5929$			n = 0.5 $\frac{\delta_c \sqrt{Re}}{x} = 0.7382$ $\frac{\delta_t \sqrt{Re}}{x} = 1.4450$			n = 1.0 $\frac{\delta_c \sqrt{Re}}{x} = 0.5988$ $\frac{\delta_t \sqrt{Re}}{x} = 1.2578$		
η	f	f'	f"	f'''	Y	Y'	Y"	Y	Y'	Y"	Y	Y'	Y"	Y	Y'	Y"
0	-0.5000	0	0.6974	-0.2385	1.0000	0	0	1.0000	-0.0272	-0.0071	1.0000	-0.3834	-0.1006	1.0000	-0.4711	-0.1237
.2	-.4864	.1346	.6480	-.2545	.9997	-.0052	-.0518	.9941	-.0335	-.0554	.9216	-.3993	-.0586	.9039	-.4872	-.0393
.4	-.4468	.2590	.5958	-.2668	.9972	-.0206	-.1017	.9860	-.0492	-.1008	.8408	-.4070	-.0193	.8062	-.4877	.0317
.6	-.3835	.3728	.5416	-.2747	.9908	-.0456	-.1477	.9739	-.0735	-.1419	.7593	-.4072	.0171	.7097	-.4753	.0896
.8	-.2985	.4756	.4863	-.2781	.9784	-.0790	-.1838	.9561	-.1054	-.1757	.6784	-.4004	.0503	.6167	-.4527	.1344
1.0	-.1940	.5673	.4307	-.2764	.9588	-.1179	-.2159	.9313	-.1431	-.1996	.5995	-.3874	.0797	.5291	-.4224	.1671
1.2	-.0723	.6479	.3760	-.2697	.9306	-.1652	-.2323	.8986	-.1843	-.2108	.5238	-.3689	.1048	.4481	-.3866	.1885
1.4	.0645	.7178	.3232	-.2580	.8930	-.2099	-.2332	.8575	-.2264	-.2077	.4523	-.3458	.1253	.3747	-.3476	.2001
1.6	.2142	.7774	.2731	-.2417	.8464	-.2562	-.2179	.8082	-.2664	-.1899	.3857	-.3191	.1408	.3092	-.3072	.2028
1.8	.3748	.8273	.2268	-.2215	.7910	-.2967	-.1870	.7513	-.3014	-.1583	.3248	-.2898	.1511	.2518	-.2670	.1984
2.0	.5445	.8684	.1847	-.1984	.7282	-.3299	-.1428	.6881	-.3289	-.1151	.2699	-.2590	.1560	.2023	-.2282	.1882
2.2	.7216	.9015	.1475	-.1735	.6597	-.3532	-.0892	.6204	-.3469	-.0643	.2212	-.2277	.1561	.1603	-.1920	.1739
2.4	.9047	.9277	.1154	-.1479	.5877	-.3653	-.0309	.5500	-.3544	-.0103	.1788	-.1969	.1516	.1253	-.1589	.1568
2.6	1.0923	.9480	.0883	-.1230	.5144	-.3657	.0269	.4793	-.3512	.0424	.1424	-.1673	.1431	.0965	-.1293	.1382
2.8	1.2835	.9634	.0661	-.0995	.4422	-.3550	.0795	.4102	-.3380	.0894	.1117	-.1398	.1319	.0733	-.1036	.1192
3.0	1.4774	.9748	.0483	-.0785	.3731	-.3346	.1232	.3447	-.3161	.1276	.0863	-.1147	.1184	.0548	-.0816	.1007
3.2	1.6732	.9830	.0345	-.0602	.3088	-.3065	.1555	.2842	-.2877	.1549	.0656	-.0925	.1039	.0404	-.0632	.0833
3.4	1.8704	.9888	.0240	-.0449	.2508	-.2733	.1754	.2299	-.2549	.1707	.0491	-.0732	.0889	.0293	-.0482	.0676
3.6	2.0686	.9928	.0164	-.0326	.1997	-.2372	.1833	.1824	-.2201	.1756	.0362	-.0569	.0744	.0209	-.0361	.0537
3.8	2.2675	.9955	.0108	-.0230	.1560	-.2006	.1806	.1419	-.1853	.1712	.0262	-.0434	.0608	.0147	-.0265	.0418
4.0	2.4668	.9972	.0070	-.0157	.1194	-.1655	.1696	.1082	-.1521	.1592	.0186	-.0325	.0486	.0102	-.0192	.0320
4.2	2.6663	.9984	.0044	-.0105	.0896	-.1331	.1528	.0808	-.1219	.1424	.0130	-.0239	.0380	.0069	-.0136	.0239
4.4	2.8661	.9991	.0027	-.0068	.0659	-.1045	.1326	.0592	-.0953	.1227	.0090	-.0172	.0290	.0046	-.0095	.0175
4.6	3.0659	.9995	.0016	-.0043	.0475	-.0801	.1112	.0424	-.0728	.1024	.0060	-.0122	.0217	.0030	-.0065	.0126
4.8	3.2659	.9997	.0010	-.0026	.0335	-.0600	.0903	.0298	-.0543	.0827	.0040	-.0084	.0158	.0020	-.0044	.0089
5.0	3.4658	.9999	.0006	-.0016	.0232	-.0439	.0711	.0205	-.0396	.0649	.0026	-.0057	.0113	.0012	-.0029	.0061
5.2	3.6658	1.0000	.0003	-.0009	.0158	-.0314	.0545	.0137	-.0282	.0495	.0017	-.0038	.0079	.0008	-.0019	.0042
5.4	3.8658	1.0000	.0002	-.0005	.0105	-.0219	.0405	.0090	-.0196	.0366	.0010	-.0025	.0054	.0005	-.0012	.0028
5.6	4.0658	1.0000	.0001	-.0002	.0068	-.0150	.0294	.0057	-.0134	.0266	.0007	-.0016	.0036	.0003	-.0007	.0017
5.8	4.2658	1.0000	.0001	-.0002	.0044	-.0100	.0208	.0035	-.0089	.0187	.0004	-.0010	.0023	.0002	-.0005	.0013
6.0	4.4658	1.0000	.0000	.0000	.0027	-.0065	.0143	.0021	-.0058	.0129	.0002	-.0006	.0015	.0001	-.0003	.0008
6.2					.0017	-.0042	.0096	.0012	-.0037	.0087	.0001	-.0004	.0010	.0000	-.0002	.0005
6.4					.0010	-.0026	.0064	.0006	-.0023	.0057	.0001	-.0002	.0005	.0000	-.0001	.0003
6.6					.0006	-.0015	.0038	.0002	-.0014	.0036	.0000	-.0001	.0003			
6.8					.0003	-.0011	.0029	.0000	-.0008	.0022						
7.0					.0002	-.0002	.0006	.0000	-.0004	.0013						
								.0000	-.0001	.0007						

TABLE I. - Continued. VELOCITY AND TEMPERATURE DISTRIBUTIONS FOR WEDGE FLOW WITH A VARIABLE TEMPERATURE ALONG THE POROUS WALL

η	$f_w = -0.5$; Eu = 1.0												
					(18)			(19)			(20)		
	$\frac{\delta^* \sqrt{Re}}{x} = 0.7805$ $\frac{\delta_i \sqrt{Re}}{x} = 0.3439$				n = -0.6789 $\frac{\delta_c \sqrt{Re}}{x} = 1.5535$ $\frac{\delta_t \sqrt{Re}}{x} = 2.2880$			n = 0.5 $\frac{\delta_c \sqrt{Re}}{x} = 0.7265$ $\frac{\delta_t \sqrt{Re}}{x} = 1.2921$			n = 1.0 $\frac{\delta_c \sqrt{Re}}{x} = 0.6089$ $\frac{\delta_t \sqrt{Re}}{x} = 1.1433$		
	f	f'	f"	f'''	Y	Y'	Y"	Y	Y'	Y"	Y	Y'	Y"
0	-0.5000	0	0.9692	-0.5154	1.0000	0	0	1.0000	-0.4132	-0.1446	1.0000	-0.5030	-0.1760
.1	-.4952	.0943	.9165	-.5372	.9999	-.0023	-.0456	.9580	-.4262	-.1161	.9489	-.5176	-.1168
.2	-.4813	.1832	.8621	-.5515	.9994	-.0091	-.0901	.9149	-.4364	-.0884	.8967	-.5265	-.0624
.3	-.4588	.2667	.8065	-.5589	.9980	-.0203	-.1330	.8708	-.4439	-.0613	.8438	-.5302	-.0128
.4	-.4282	.3445	.7505	-.5600	.9952	-.0355	-.1695	.8262	-.4487	-.0349	.7908	-.5292	.0321
.5	-.3901	.4168	.6947	-.5553	.9908	-.0537	-.2109	.7812	-.4509	-.0092	.7381	-.5240	.0723
.6	-.3450	.4835	.6396	-.5456	.9842	-.0780	-.2450	.7361	-.4506	.0157	.6861	-.5149	.1079
.7	-.2935	.5447	.5857	-.5313	.9752	-.1026	-.2735	.6911	-.4478	.0398	.6352	-.5026	.1390
.8	-.2362	.6007	.5335	-.5132	.9635	-.1317	-.2968	.6466	-.4426	.0627	.5857	-.4873	.1657
.9	-.1736	.6515	.4832	-.4917	.9488	-.1623	-.3135	.6027	-.4353	.0845	.5378	-.4696	.1882
1.0	-.1061	.6974	.4352	-.4675	.9310	-.1941	-.3230	.5596	-.4258	.1050	.4919	-.4498	.2067
1.1	-.0342	.7386	.3898	-.4411	.9100	-.2266	-.3249	.5176	-.4143	.1239	.4479	-.4283	.2213
1.2	.0415	.7754	.3470	-.4131	.8857	-.2588	-.3189	.4768	-.4011	.1411	.4062	-.4056	.2323
1.3	.1207	.8081	.3072	-.3840	.8582	-.2901	-.3051	.4374	-.3862	.1564	.3668	-.3820	.2398
1.4	.2030	.8370	.2703	-.3543	.8277	-.3196	-.2838	.3996	-.3699	.1696	.3298	-.3578	.2441
1.5	.2880	.8623	.2363	-.3245	.7944	-.3466	-.2557	.3635	-.3523	.1807	.2953	-.3333	.2454
1.6	.3753	.8843	.2053	-.2950	.7585	-.3705	-.2214	.3292	-.3338	.1896	.2632	-.3088	.2441
1.7	.4647	.9034	.1773	-.2662	.7204	-.3908	-.1822	.2968	-.3145	.1961	.2335	-.2845	.2403
1.8	.5559	.9199	.1521	-.2383	.6805	-.4069	-.1392	.2663	-.2946	.2004	.2063	-.2608	.2343
1.9	.6486	.9340	.1296	-.2118	.6392	-.4185	-.0937	.2378	-.2745	.2024	.1813	-.2377	.2265
2.0	.7427	.9459	.1097	-.1867	.5970	-.4256	-.0471	.2114	-.2542	.2022	.1587	-.2156	.2171
2.1	.8378	.9560	.0922	-.1633	.5542	-.4279	-.0008	.1870	-.2341	.1999	.1382	-.1944	.2065
2.2	.9338	.9644	.0769	-.1418	.5115	-.4258	.0439	.1646	-.2143	.1956	.1198	-.1743	.1948
2.3	1.0306	.9714	.0638	-.1220	.4692	-.4193	.0858	.1441	-.1950	.1897	.1033	-.1554	.1824
2.4	1.1280	.9772	.0525	-.1042	.4278	-.4087	.1241	.1255	-.1764	.1823	.0886	-.1378	.1695
2.5	1.2260	.9820	.0429	-.0883	.3876	-.3946	.1578	.1088	-.1586	.1735	.0757	-.1215	.1563
2.6	1.3244	.9858	.0348	-.0741	.3490	-.3774	.1863	.0938	-.1418	.1638	.0643	-.1066	.1432
2.7	1.4232	.9890	.0280	-.0617	.3122	-.3575	.2094	.0804	-.1259	.1533	.0543	-.0929	.1302
2.8	1.5222	.9915	.0224	-.0510	.2775	-.3357	.2269	.0686	-.1111	.1422	.0457	-.0805	.1175
2.9	1.6214	.9935	.0177	-.0417	.2451	-.3123	.2388	.0581	-.0975	.1309	.0382	-.0694	.1053
3.0	1.7209	.9951	.0140	-.0339	.2151	-.2881	.2453	.0490	-.0850	.1194	.0318	-.0594	.0937
3.1	1.8204	.9963	.0109	-.0273	.1875	-.2634	.2469	.0411	-.0736	.1081	.0263	-.0506	.0828
3.2	1.9201	.9973	.0085	-.0217	.1624	-.2388	.2441	.0343	-.0633	.0971	.0216	-.0429	.0727
3.3	2.0199	.9980	.0065	-.0172	.1397	-.2147	.2374	.0284	-.0542	.0865	.0177	-.0361	.0633
3.4	2.1197	.9986	.0050	-.0135	.1194	-.1915	.2274	.0234	-.0460	.0765	.0144	-.0302	.0548
3.5	2.2196	.9990	.0038	-.0104	.1014	-.1693	.2150	.0192	-.0388	.0671	.0116	-.0251	.0471
3.6	2.3195	.9993	.0029	-.0080	.0855	-.1486	.2006	.0156	-.0326	.0584	.0093	-.0207	.0402
3.7	2.4195	.9996	.0022	-.0061	.0717	-.1293	.1849	.0126	-.0271	.0504	.0074	-.0170	.0340
3.8	2.5194	.9998	.0017	-.0046	.0596	-.1116	.1685	.0102	-.0225	.0432	.0059	-.0139	.0286
3.9	2.6194	.9999	.0013	-.0034	.0493	-.0956	.1518	.0081	-.0185	.0367	.0046	-.0113	.0239
4.0	2.7194	1.0000	.0010	-.0025	.0405	-.0812	.1354	.0064	-.0151	.0310	.0036	-.0091	.0198
4.1	2.8194	1.0000	.0008	-.0018	.0330	-.0685	.1195	.0051	-.0123	.0260	.0028	-.0073	.0163
4.2	2.9194	1.0000	.0006	-.0013	.0267	-.0573	.1044	.0040	-.0099	.0216	.0022	-.0058	.0134
4.3	3.0195	1.0000	.0005	-.0009	.0215	-.0476	.0903	.0031	-.0079	.0178	.0016	-.0046	.0109
4.4	3.1195	1.0000	.0004	-.0007	.0171	-.0392	.0774	.0024	-.0063	.0146	.0012	-.0036	.0088
4.5	3.2195	1.0000	.0003	-.0004	.0136	-.0320	.0658	.0018	-.0050	.0119	.0009	-.0028	.0070
4.6	3.3194				.0107	-.0260	.0553	.0014	-.0039	.0096	.0007	-.0022	.0056
4.7	3.4194				.0084	-.0209	.0461	.0010	-.0031	.0077	.0005	-.0017	.0044
4.8	3.5194				.0065	-.0167	.0381	.0008	-.0024	.0061	.0003	-.0013	.0035
4.9	3.6194				.0050	-.0133	.0313	.0006	-.0018	.0048	.0002	-.0010	.0027
5.0	3.7194				.0038	-.0104	.0254	.0004	-.0014	.0038	.0001	-.0008	.0021
5.1	3.8194				.0029	-.0082	.0205	.0003	-.0011	.0029	.0001	-.0006	.0016
5.2	3.9194				.0022	-.0063	.0163	.0002	-.0008	.0023	.0000	-.0004	.0012
5.3	4.0194				.0016	-.0049	.0129	.0001	-.0006	.0017	.0000	-.0003	.0009
5.4	4.1194				.0012	-.0037	.0102	.0001	-.0005	.0013	.0000	-.0003	.0007
5.5	4.2194				.0009	-.0028	.0079	.0000	-.0003	.0010	.0000	-.0002	.0005
5.6	4.3194				.0006	-.0021	.0061	.0000	-.0002	.0008			
5.7	4.4194				.0004	-.0016	.0047	.0000	-.0002	.0006			
5.8	4.5194				.0003	-.0012	.0036	.0000	-.0001	.0004			
5.9					.0002	-.0009	.0027	.0000	-.0001	.0003			
6.0					.0001	-.0006	.0020	.0000	-.0001	.0002			
6.1					.0001	-.0004	.0015	.0000	-.0001	.0002			
6.2					.0000	-.0003	.0012	.0000	.0000	.0001			
6.3					.0000	-.0002	.0007						

TABLE I. - Continued. VELOCITY AND TEMPERATURE DISTRIBUTIONS FOR WEDGE FLOW WITH A VARIABLE TEMPERATURE ALONG THE POROUS WALL

	$f_w = -1$; Eu = 0												
					(21)			(22)			(23)		
	$\frac{\delta^* \sqrt{Re}}{x} = 4.3923$ $\frac{\delta_1 \sqrt{Re}}{x} = 1.0717$				n = -0.2384 $\frac{\delta_c \sqrt{Re}}{x} = 1.9092$ $\frac{\delta_t \sqrt{Re}}{x} = 5.8598$			n = 0.5 $\frac{\delta_c \sqrt{Re}}{x} = 0.6499$ $\frac{\delta_t \sqrt{Re}}{x} = 3.3380$			n = 1.0 $\frac{\delta_c \sqrt{Re}}{x} = 0.4649$ $\frac{\delta_t \sqrt{Re}}{x} = 2.8709$		
η	f	f'	f''	f'''	Y	Y'	Y''	Y	Y'	Y''	Y	Y'	Y''
0	-1.0000	0	0.0355	0.0178	1.0000	0	0	1.0000	-0.1052	-0.0368	1.0000	-0.1383	-0.0484
.2	-.9993	.0075	.0392	.0196	1.0000	-.0001	-.0013	.9782	-.1125	-.0368	.9714	-.1478	-.0466
.4	-.9970	.0157	.0434	.0216	.9999	-.0005	-.0028	.9550	-.1199	-.0366	.9409	-.1570	-.0444
.6	-.9929	.0248	.0479	.0238	.9998	-.0013	-.0046	.9303	-.1271	-.0361	.9086	-.1656	-.0417
.8	-.9870	.0349	.0529	.0261	.9994	-.0024	-.0063	.9042	-.1343	-.0353	.8747	-.1736	-.0386
1.0	-.9789	.0460	.0583	.0286	.9988	-.0037	-.0090	.8766	-.1412	-.0343	.8392	-.1810	-.0350
1.2	-.9685	.0583	.0643	.0311	.9978	-.0060	-.0118	.8477	-.1480	-.0329	.8024	-.1876	-.0309
1.4	-.9555	.0718	.0708	.0338	.9964	-.0087	-.0148	.8174	-.1544	-.0311	.7643	-.1933	-.0262
1.6	-.9397	.0866	.0778	.0366	.9944	-.0118	-.0183	.7860	-.1604	-.0289	.7251	-.1980	-.0212
1.8	-.9208	.1029	.0854	.0393	.9916	-.0159	-.0222	.7533	-.1659	-.0263	.6851	-.2017	-.0156
2.0	-.8984	.1208	.0936	.0420	.9880	-.0207	-.0265	.7196	-.1709	-.0233	.6445	-.2043	-.0097
2.2	-.8723	.1404	.1022	.0446	.9832	-.0265	-.0311	.6850	-.1752	-.0198	.6035	-.2056	-.0035
2.4	-.8421	.1618	.1114	.0469	.9773	-.0332	-.0362	.6496	-.1788	-.0159	.5624	-.2056	.0031
2.6	-.8075	.1850	.1210	.0488	.9699	-.0410	-.0415	.6136	-.1815	-.0116	.5213	-.2043	.0098
2.8	-.7680	.2102	.1309	.0503	.9608	-.0498	-.0471	.5771	-.1834	-.0068	.4807	-.2017	.0165
3.0	-.7233	.2374	.1410	.0510	.9497	-.0598	-.0528	.5403	-.1842	-.0018	.4407	-.1978	.0232
3.2	-.6729	.2666	.1512	.0509	.9368	-.0711	-.0584	.5035	-.1840	.0036	.4017	-.1925	.0296
3.4	-.6165	.2978	.1613	.0497	.9214	-.0832	-.0638	.4668	-.1828	.0092	.3638	-.1859	.0357
3.6	-.5537	.3311	.1710	.0474	.9035	-.0964	-.0686	.4304	-.1803	.0149	.3274	-.1782	.0413
3.8	-.4840	.3662	.1801	.0436	.8828	-.1106	-.0727	.3947	-.1768	.0206	.2926	-.1694	.0463
4.0	-.4071	.4031	.1884	.0383	.8592	-.1254	-.0757	.3598	-.1721	.0262	.2597	-.1597	.0505
4.2	-.3226	.4415	.1954	.0315	.8326	-.1408	-.0773	.3260	-.1663	.0316	.2288	-.1493	.0538
4.4	-.2304	.4811	.2009	.0231	.8029	-.1562	-.0771	.2934	-.1595	.0365	.2000	-.1383	.0562
4.6	-.1301	.5217	.2045	.0133	.7701	-.1715	-.0749	.2622	-.1517	.0410	.1735	-.1269	.0576
4.8	-.0217	.5628	.2061	.0022	.7343	-.1860	-.0704	.2327	-.1431	.0448	.1493	-.1153	.0579
5.0	.0950	.6040	.2054	-.0098	.6958	-.1995	-.0635	.2050	-.1339	.0478	.1274	-.1038	.0573
5.2	.2199	.6448	.2022	-.0222	.6547	-.2113	-.0542	.1792	-.1241	.0500	.1078	-.0924	.0558
5.4	.3528	.6847	.1965	-.0347	.6114	-.2210	-.0426	.1554	-.1139	.0513	.0904	-.0815	.0534
5.6	.4936	.7232	.1884	-.0465	.5664	-.2282	-.0290	.1336	-.1036	.0517	.0751	-.0711	.0503
5.8	.6420	.7598	.1780	-.0571	.5203	-.2325	-.0138	.1139	-.0933	.0513	.0619	-.0614	.0467
6.0	.7974	.7942	.1656	-.0660	.4737	-.2336	.0024	.0963	-.0832	.0500	.0505	-.0525	.0427
6.2	.9595	.8260	.1517	-.0728	.4271	-.2315	.0188	.0806	-.0734	.0480	.0408	-.0444	.0385
6.4	1.1276	.8548	.1367	-.0771	.3813	-.2261	.0348	.0669	-.0640	.0453	.0327	-.0371	.0342
6.6	1.3012	.8806	.1210	-.0788	.3369	-.2176	.0496	.0550	-.0553	.0421	.0259	-.0307	.0300
6.8	1.4797	.9033	.1053	-.0779	.2944	-.2064	.0625	.0448	-.0472	.0386	.0204	-.0251	.0259
7.0	1.6623	.9228	.0900	-.0748	.2545	-.1928	.0730	.0361	-.0399	.0348	.0158	-.0203	.0220
7.2	1.8486	.9393	.0755	-.0698	.2174	-.1774	.0807	.0288	-.0333	.0310	.0122	-.0162	.0185
7.4	2.0379	.9531	.0622	-.0634	.1836	-.1607	.0854	.0227	-.0275	.0272	.0093	-.0129	.0154
7.6	2.2297	.9643	.0502	-.0560	.1531	-.1434	.0873	.0177	-.0224	.0235	.0070	-.0101	.0126
7.8	2.4234	.9733	.0398	-.0482	.1262	-.1260	.0864	.0137	-.0181	.0200	.0052	-.0078	.0102
8.0	2.6188	.9803	.0309	-.0405	.1027	-.1090	.0831	.0105	-.0141	.0168	.0039	-.0060	.0081
8.2	2.8155	.9858	.0236	-.0332	.0825	-.0929	.0780	.0079	-.0113	.0139	.0028	-.0045	.0064
8.4	3.0131	.9898	.0176	-.0265	.0654	-.0779	.0714	.0059	-.0088	.0113	.0020	-.0034	.0050
8.6	3.2114	.9929	.0129	-.0207	.0513	-.0644	.0637	.0044	-.0068	.0091	.0015	-.0025	.0038
8.8	3.4102	.9951	.0093	-.0158	.0396	-.0524	.0560	.0032	-.0051	.0072	.0010	-.0018	.0029
9.0	3.6093	.9966	.0065	-.0118	.0302	-.0420	.0480	.0023	-.0038	.0057	.0007	-.0013	.0022
9.2	3.8088	.9977	.0045	-.0086	.0227	-.0332	.0404	.0016	-.0029	.0044	.0005	-.0010	.0016
9.4	4.0084	.9985	.0030	-.0061	.0168	-.0258	.0334	.0011	-.0021	.0033	.0003	-.0007	.0012
9.6	4.2082	.9990	.0020	-.0042	.0123	-.0198	.0271	.0008	-.0015	.0025	.0002	-.0005	.0008
9.8	4.4080	.9993	.0013	-.0029	.0089	-.0149	.0215	.0005	-.0011	.0018	.0001	-.0003	.0006
10.0	4.6079	.9995	.0008	-.0019	.0063	-.0111	.0168	.0003	-.0008	.0013	.0001	-.0002	.0004
10.2	4.8078	.9997	.0005	-.0013	.0044	-.0081	.0130	.0002	-.0005	.0010	.0000	-.0002	.0003
10.4	5.0078	.9998	.0003	-.0008	.0030	-.0059	.0098	.0001	-.0004	.0007	.0000	-.0001	.0002
10.6	5.2077	.9998	.0002	-.0005	.0020	-.0042	.0073	.0000	-.0002	.0005	.0000	-.0001	.0001
10.8	5.4077	.9998	.0001	-.0003	.0013	-.0029	.0053	.0000	-.0002	.0003	.0000	-.0001	.0001
11.0					.0008	-.0020	.0038						
11.2					.0004	-.0014	.0027						
11.4					.0002	-.0009	.0019						
11.6					.0001	-.0006	.0013						
11.8					.0001	-.0004	.0009						
12.0					.0000	-.0002	.0006						
12.2					.0000	-.0002	.0004						
12.4					.0000	-.0001	.0002						

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TABLE I. - Continued. VELOCITY AND TEMPERATURE DISTRIBUTIONS FOR WEDGE FLOW WITH A VARIABLE TEMPERATURE ALONG THE POROUS WALL

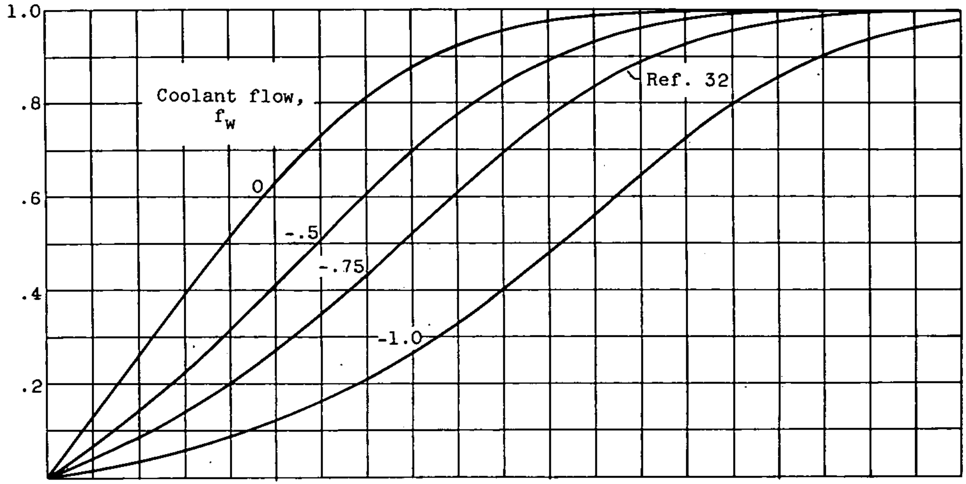
	$f_w = -1$; Eu = 0.5												
					(24)			(25)			(26)		
	$\frac{\delta^* \sqrt{Re}}{x} = 1.2597$ $\frac{\delta_1 \sqrt{Re}}{x} = 0.5236$				$n = -0.3585$ $\frac{\delta_c \sqrt{Re}}{x} = 1.9012$ $\frac{\delta_t \sqrt{Re}}{x} = 3.0745$			$n = 0.5$ $\frac{\delta_c \sqrt{Re}}{x} = 0.8886$ $\frac{\delta_t \sqrt{Re}}{x} = 1.7826$			$n = 1.0$ $\frac{\delta_c \sqrt{Re}}{x} = 0.6997$ $\frac{\delta_t \sqrt{Re}}{x} = 1.5239$		
η	f	f'	f''	f'''	Y	Y'	Y''	Y	Y'	Y''	Y	Y'	Y''
.0	-1.0000	0	0.5345	-0.0992	1.0000	0	0	1.0000	-0.2528	-0.1327	1.0000	-0.3314	-0.1740
.2	-.9894	.1048	.5132	-.1137	.9998	-.0028	-.0277	.9469	-.2770	-.1091	.9306	-.3606	-.1190
.4	-.9584	.2051	.4890	-.1275	.9985	-.0112	-.0570	.8895	-.2964	-.0853	.8565	-.3792	-.0679
.6	-.9078	.3002	.4622	-.1402	.9949	-.0257	-.0872	.8287	-.3111	-.0612	.7796	-.3880	-.0211
.8	-.8387	.3898	.4330	-.1517	.9878	-.0455	-.1086	.7654	-.3209	-.0368	.7018	-.3880	.0207
1.0	-.7522	.4733	.4017	-.1614	.9766	-.0673	-.1426	.7007	-.3258	-.0126	.6249	-.3801	.0569
1.2	-.6498	.5504	.3686	-.1689	.9598	-.1040	-.1680	.6354	-.3259	.0112	.5502	-.3656	.0873
1.4	-.5325	.6207	.3343	-.1739	.9357	-.1350	-.1835	.5706	-.3214	.0341	.4790	-.3457	.1115
1.6	-.4019	.6841	.2993	-.1758	.9049	-.1736	-.1920	.5072	-.3124	.0555	.4123	-.3214	.1296
1.8	-.2594	.7404	.2642	-.1745	.8663	-.2128	-.1899	.4460	-.2993	.0748	.3506	-.2942	.1417
2.0	-.1063	.7898	.2297	-.1698	.8200	-.2490	-.1764	.3877	-.2826	.0914	.2947	-.2652	.1481
2.2	.0561	.8324	.1965	-.1618	.7669	-.2821	-.1519	.3331	-.2630	.1048	.2446	-.2353	.1495
2.4	.2263	.8685	.1652	-.1509	.7076	-.3092	-.1175	.2827	-.2410	.1146	.2005	-.2057	.1464
2.6	.4031	.8986	.1363	-.1375	.6437	-.3286	-.0756	.2368	-.2174	.1205	.1623	-.1770	.1396
2.8	.5853	.9232	.1103	-.1223	.5768	-.3392	-.0294	.1958	-.1930	.1226	.1296	-.1500	.1299
3.0	.7720	.9430	.0875	-.1061	.5087	-.3404	.0176	.1596	-.1686	.1210	.1021	-.1252	.1182
3.2	.9622	.9584	.0679	-.0897	.4412	-.3324	.0618	.1283	-.1448	.1162	.0794	-.1028	.1052
3.4	1.1552	.9703	.0516	-.0739	.3763	-.3161	.1001	.1016	-.1223	.1087	.0608	-.0831	.0917
3.6	1.3502	.9793	.0383	-.0593	.3153	-.2929	.1302	.0793	-.1015	.0991	.0459	-.0661	.0784
3.8	1.5467	.9858	.0277	-.0462	.2594	-.2647	.1508	.0608	-.0828	.0882	.0342	-.0517	.0656
4.0	1.7444	.9905	.0196	-.0351	.2096	-.2333	.1616	.0460	-.0663	.0766	.0251	-.0398	.0539
4.2	1.9428	.9938	.0136	-.0259	.1662	-.2007	.1632	.0342	-.0521	.0650	.0181	-.0301	.0433
4.4	2.1418	.9961	.0091	-.0186	.1278	-.1685	.1575	.0250	-.0402	.0539	.0129	-.0224	.0342
4.6	2.3412	.9975	.0060	-.0130	.0972	-.1382	.1454	.0180	-.0305	.0437	.0091	-.0164	.0264
4.8	2.5408	.9985	.0038	-.0088	.0724	-.1107	.1294	.0127	-.0226	.0346	.0063	-.0117	.0200
5.0	2.7406	.9991	.0024	-.0058	.0527	-.0866	.1112	.0088	-.0165	.0268	.0043	-.0083	.0149
5.2	2.9405	.9995	.0015	-.0037	.0376	-.0663	.0926	.0060	-.0118	.0203	.0029	-.0057	.0108
5.4	3.1404	.9997	.0009	-.0023	.0261	-.0495	.0748	.0040	-.0083	.0151	.0020	-.0039	.0077
5.6	3.3403	.9999	.0005	-.0014	.0176	-.0362	.0588	.0026	-.0057	.0109	.0013	-.0026	.0054
5.8	3.5403	1.0000	.0003	-.0008	.0115	-.0259	.0449	.0016	-.0039	.0078	.0009	-.0017	.0037
6.0	3.7403	1.0000	.0002	-.0005	.0084	-.0181	.0335	.0010	-.0026	.0054	.0006	-.0010	.0025
6.2	3.9403	1.0000	.0001	-.0003	.0042	-.0121	.0242	.0006	-.0017	.0037	.0005	-.0006	.0016
6.4	4.1403	1.0000	.0001	-.0001	.0022	-.0081	.0172	.0003	-.0011	.0024	.0004	-.0004	.0011
6.6	4.3403	1.0000	.0000	-.0001	.0007	-.0052	.0117	.0002	-.0007	.0016	.0003	-.0002	.0007
6.8					.0000	-.0035	.0082	.0000	-.0004	.0010	.0003	-.0001	.0004
7.0					.0000	-.0019	.0047	.0000	-.0003	.0006	.0003	.0000	.0003
7.2					.0000	-.0004	.0012	-.0001	-.0002	.0004	.0003	.0000	.0002
7.4								-.0001	-.0001	.0002	.0003	.0000	.0001

TABLE I. - Concluded. VELOCITY AND TEMPERATURE DISTRIBUTIONS FOR WEDGE FLOW WITH A VARIABLE TEMPERATURE ALONG THE POROUS WALL

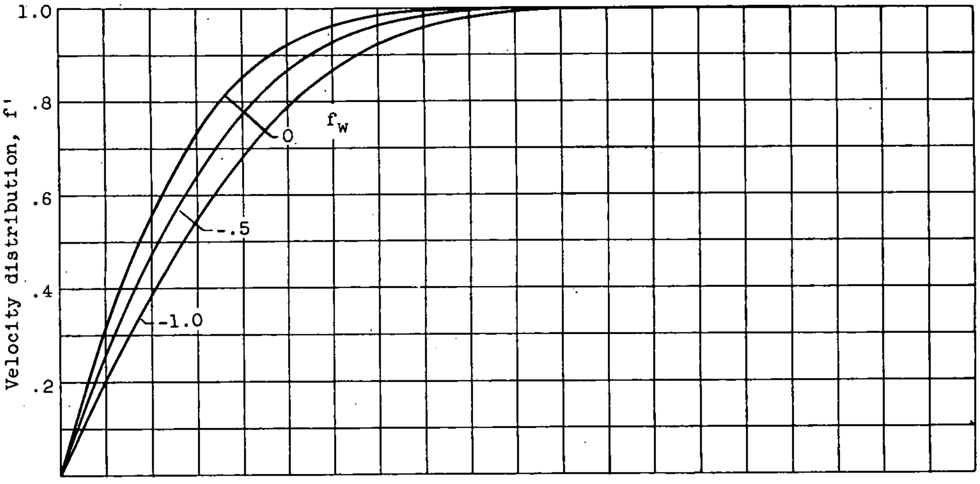
	$f_w = -1$; Eu 1.0												
					(27)			(28)			(29)		
	$\frac{\delta^* \sqrt{Re}}{x} = 0.9448$ $\frac{\delta_1 \sqrt{Re}}{x} = 0.4047$				$n = -0.4235$ $\frac{\delta_c \sqrt{Re}}{x} = 1.7309$ $\frac{\delta_t \sqrt{Re}}{x} = 2.6219$			$n = 0.5$ $\frac{\delta_c \sqrt{Re}}{x} = 0.9099$ $\frac{\delta_t \sqrt{Re}}{x} = 1.6269$			$n = 1.0$ $\frac{\delta_c \sqrt{Re}}{x} = 0.7402$ $\frac{\delta_t \sqrt{Re}}{x} = 1.4109$		
η	f	f'	f''	f'''	Y	Y'	Y''	Y	Y'	Y''	Y	Y'	Y''
0	-1.0000	0	0.7565	-0.2435	1.0000	0	0	1.0000	-0.2553	-0.1787	1.0000	-0.3360	-0.2352
.2	-.9852	.1461	.7034	-.2854	.9997	-.0046	-.0465	.9456	-.2882	-.1504	.9286	-.3758	-.1642
.4	-.9423	.2809	.6431	-.3148	.9975	-.0186	-.0901	.8851	-.3154	-.1210	.8506	-.4019	-.0979
.6	-.8737	.4031	.5783	-.3320	.9917	-.0430	-.1448	.8198	-.3365	-.0902	.7687	-.4153	-.0371
.8	-.7820	.5120	.5111	-.3382	.9801	-.0754	-.1901	.7509	-.3513	-.0577	.6852	-.4172	.0172
1.0	-.6698	.6075	.4437	-.3338	.9609	-.1175	-.2281	.6797	-.3595	-.0240	.6025	-.4089	.0645
1.2	-.5399	.6896	.3782	-.3202	.9326	-.1659	-.2534	.6075	-.3609	.0103	.5222	-.3919	.1040
1.4	-.3948	.7590	.3161	-.2992	.8943	-.2177	-.2614	.5358	-.3555	.0441	.4462	-.3678	.1354
1.6	-.2371	.8164	.2589	-.2721	.8456	-.2691	-.2493	.4658	-.3434	.0761	.3755	-.3383	.1584
1.8	-.0690	.8629	.2075	-.2410	.7869	-.3161	-.2166	.3988	-.3252	.1048	.3111	-.3050	.1732
2.0	.1074	.8998	.1626	-.2078	.7197	-.3545	-.1653	.3361	-.3018	.1286	.2536	-.2696	.1800
2.2	.2904	.9284	.1244	-.1742	.6459	-.3813	-.1003	.2784	-.2742	.1462	.2033	-.2335	.1796
2.4	.4784	.9500	.0929	-.1418	.5681	-.3942	-.0280	.2266	-.2438	.1570	.1602	-.1981	.1729
2.6	.6701	.9660	.0675	-.1121	.4892	-.3925	.0440	.1810	-.2119	.1606	.1239	-.1647	.1610
2.8	.8645	.9774	.0478	-.0860	.4121	-.3770	.1088	.1418	-.1800	.1574	.0941	-.1340	.1455
3.0	1.0608	.9854	.0329	-.0638	.3392	-.3498	.1607	.1089	-.1493	.1484	.0701	-.1066	.1276
3.2	1.2585	.9908	.0220	-.0459	.2727	-.3138	.1964	.0819	-.1209	.1349	.0512	-.0830	.1087
3.4	1.4571	.9944	.0142	-.0319	.2140	-.2724	.2148	.0603	-.0955	.1184	.0366	-.0632	.0899
3.6	1.6562	.9967	.0090	-.0214	.1639	-.2290	.2171	.0435	-.0736	.1005	.0257	-.0470	.0724
3.8	1.8557	.9981	.0055	-.0139	.1224	-.1865	.2061	.0306	-.0553	.0826	.0176	-.0341	.0566
4.0	2.0554	.9990	.0032	-.0087	.0891	-.1472	.1855	.0211	-.0405	.0657	.0119	-.0242	.0431
4.2	2.2553	.9995	.0019	-.0053	.0632	-.1127	.1592	.0142	-.0289	.0506	.0078	-.0167	.0318
4.4	2.4552	.9998	.0011	-.0031	.0436	-.0837	.1309	.0094	-.0201	.0378	.0050	-.0113	.0229
4.6	2.6552	.9999	.0006	-.0017	.0293	-.0603	.1034	.0060	-.0136	.0274	.0032	-.0074	.0160
4.8	2.8552	1.0000	.0003	-.0009	.0192	-.0421	.0786	.0038	-.0090	.0193	.0020	-.0048	.0109
5.0	3.0552	1.0000	.0002	-.0005	.0122	-.0286	.0576	.0023	-.0058	.0132	.0012	-.0030	.0072
5.2	3.2552	1.0000	.0001	-.0002	.0075	-.0188	.0407	.0014	-.0036	.0087	.0008	-.0018	.0046
5.4	3.4552	1.0000	.0001	-.0001	.0044	-.0120	.0278	.0008	-.0022	.0056	.0005	-.0010	.0029
5.6	3.6552	1.0000	.0001	.0000	.0025	-.0075	.0184	.0005	-.0013	.0035	.0003	-.0006	.0018
5.8					.0013	-.0045	.0118	.0003	-.0008	.0021	.0002	-.0003	.0010
6.0					.0006	-.0026	.0073	.0002	-.0004	.0012	.0002	-.0002	.0006
6.2					.0002	-.0015	.0044	.0001	-.0002	.0007	.0002	-.0001	.0003
6.4					.0000	-.0008	.0025	.0001	-.0001	.0004	.0002	.0000	.0002
6.6					.0000	-.0004	.0014	.0001	-.0001	.0002			
6.8					.0000	-.0002	.0008						

TABLE II. - SUMMARY OF HEAT-TRANSFER AND FRICTION PARAMETERS AND BOUNDARY-LAYER THICKNESSES

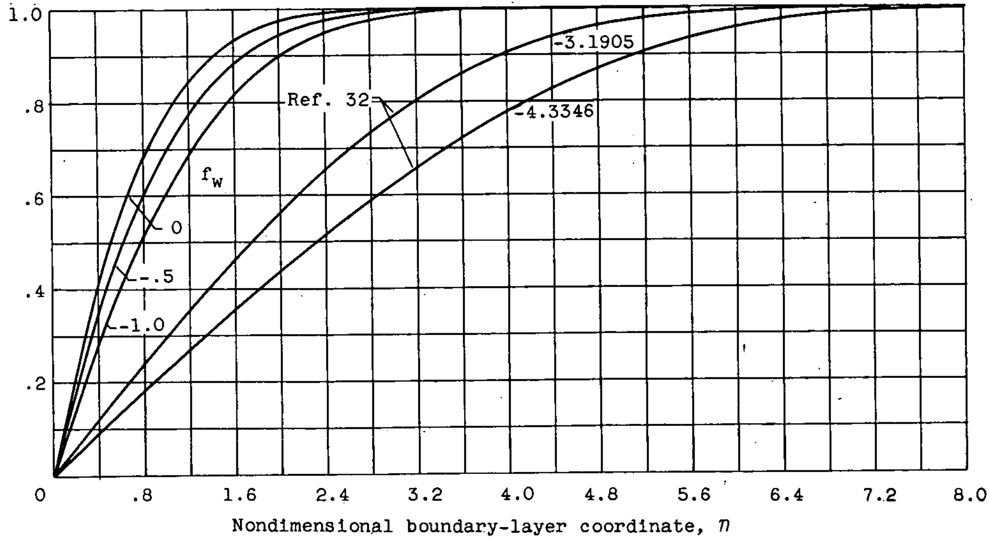
f_w	Eu	n	$Nu/\sqrt{Re}$ $-Y'_w$	$\frac{C_f}{2}\sqrt{Re}$ f''_w	$\delta^*\sqrt{Re}/x$	$\delta_i\sqrt{Re}/x$	$\delta_c\sqrt{Re}/x$	$\delta_t\sqrt{Re}/x$	Table I part no.
0	0	-0.5000	0	0.3320	1.7215	0.6652	1.8610	3.4167	1
		0	0.2927	↓	↓	↓	.8353	1.9590	
		.5000	.4059	↓	↓	↓	.5782	1.5617	2
		1.000	.4803	↓	↓	↓	.4582	1.3627	3
	.5	-0.7500	0	0.89975	0.8542	0.3773	1.6075	2.4034	4
		0	.4162	↓	↓	↓	.7921	1.4067	
		.5000	.5426	↓	↓	↓	.6204	1.1861	5
		1.000	.6350	↓	↓	↓	.5181	1.0515	6
	1.0	-1.000	0	1.2326	0.6477	0.2921	1.4086	2.0175	7
		-.5000	.3228	↓	↓	↓	.9185	1.4398	8
		0	.4958	↓	↓	↓	.7080	1.1867	
		.5000	.6159	↓	↓	↓	.5861	1.0361	9
		1.000	.7090	↓	↓	↓	.5061	0.9356	10
-0.5	0	-0.3702	0	0.1645	2.4595	0.8288	1.9212	4.1340	11
		0	.1661	↓	↓	↓	.9738	2.6495	
		.5000	.2611	↓	↓	↓	.6231	2.0470	12
		1.000	.3211	↓	↓	↓	.4724	1.7627	13
	.5	-0.5356	0	0.6974	1.0342	0.4440	1.7434	2.7060	14
		-.5000	.0272	↓	↓	↓	1.6506	2.5929	15
		0	.2594	↓	↓	↓	.9944	1.7756	
		.5000	.3834	↓	↓	↓	.7382	1.4450	16
		1.0000	.4711	↓	↓	↓	.5988	1.2578	17
	1.0	-0.6789	0	0.9692	0.7805	0.3439	1.5535	2.2880	18
		0	.2934	↓	↓	↓	.9184	1.5296	
		.5000	.4132	↓	↓	↓	.7265	1.2921	19
		1.0000	.5030	↓	↓	↓	.6089	1.1433	20
-1.0	0	-0.2384	0	0.0355	4.3923	1.0717	1.9092	5.8598	21
		0	.0516	↓	↓	↓	1.1499	4.4140	
		.5000	.1052	↓	↓	↓	.6499	3.3380	22
		1.0000	.1383	↓	↓	↓	.4649	2.8709	23
	.5	-0.3585	0	0.5345	1.2597	0.5231	1.9048	3.0781	24
		0	.1392	↓	↓	↓	1.2696	2.2737	
		.5000	.2528	↓	↓	↓	.8886	1.7826	25
		1.0000	.3314	↓	↓	↓	.6997	1.5239	26
	1.0	-0.4235	0	0.7565	0.9448	0.4047	1.7309	2.6219	27
		0	0.1457	↓	↓	↓	1.2077	1.9946	
		.5000	.2553	↓	↓	↓	.9099	1.6269	28
		1.0000	.3360	↓	↓	↓	.7402	1.4109	29



(a) Euler number, 0.

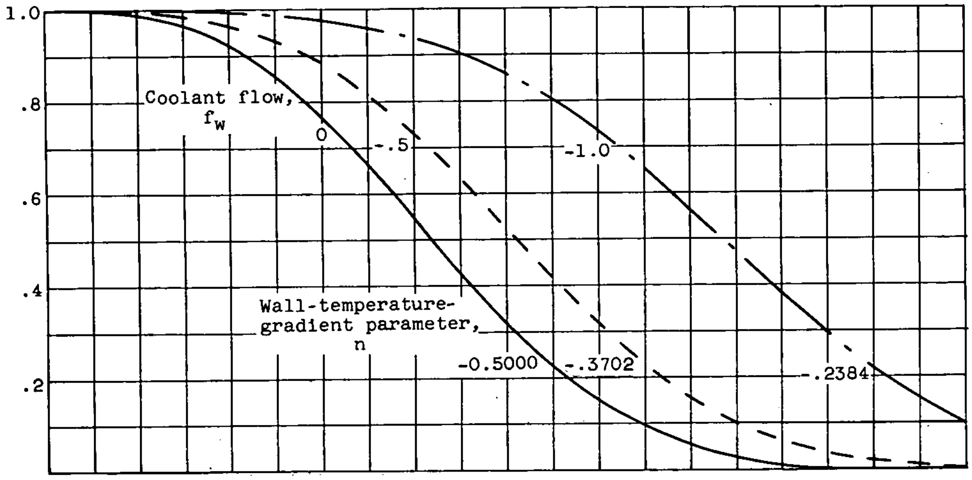


(b) Euler number, 0.5.

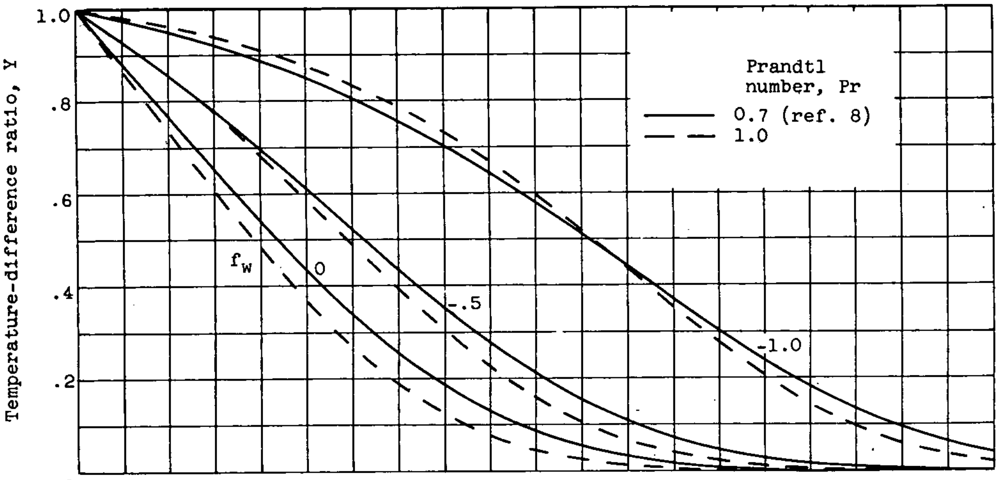


(c) Euler number, 1.0.

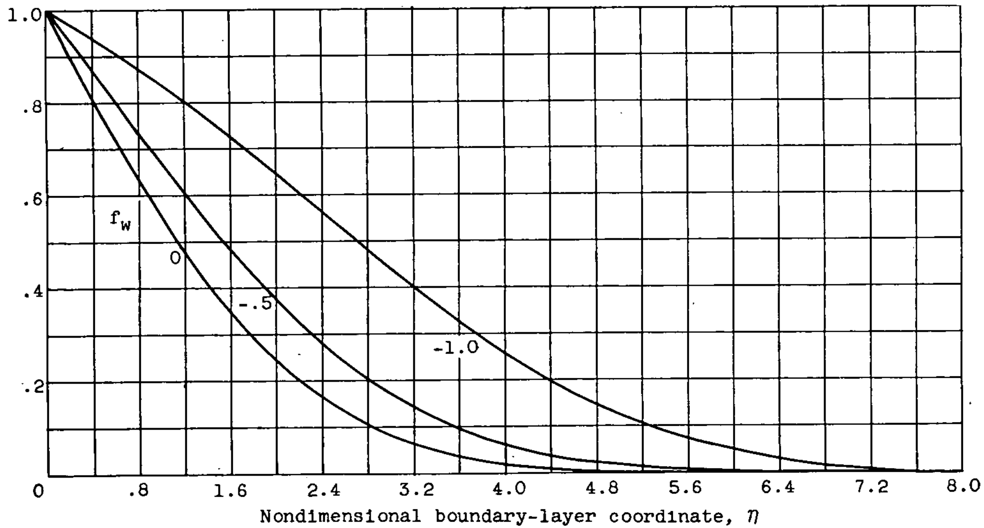
Figure 1. - Velocity distribution in constant-property laminar boundary layer for permeable and impermeable wall.



(a) Wall-temperature-gradient parameter for $Y'(0) = 0$; Prandtl number, 0.7.

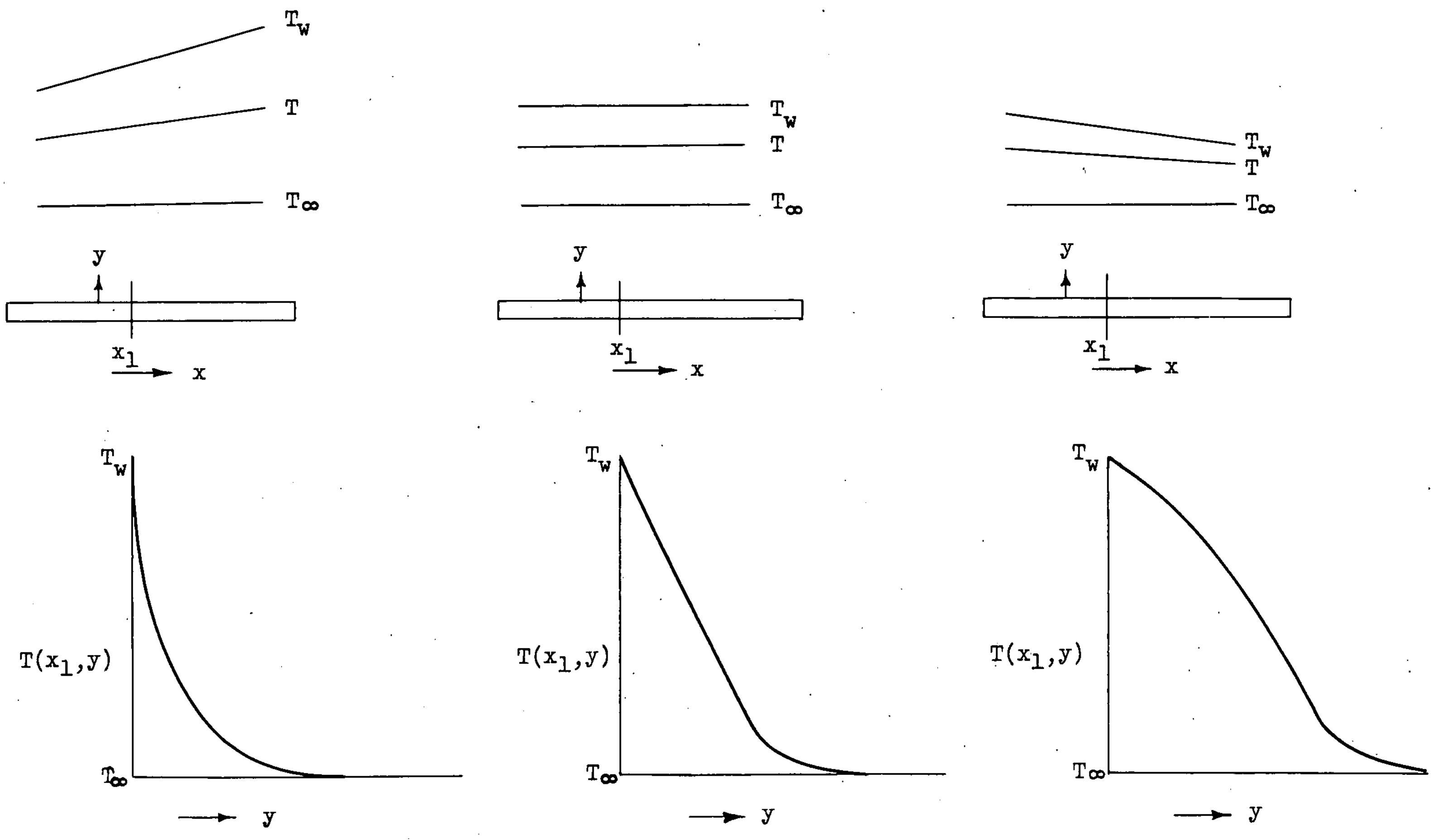


(b) Wall-temperature-gradient parameter, 0.



(c) Wall-temperature-gradient parameter, 1; Prandtl number, 0.7.

Figure 2. - Temperature distribution in constant-property laminar boundary layer for permeable and impermeable wall at variable temperature. Flat-plate flow; Euler number, 0.

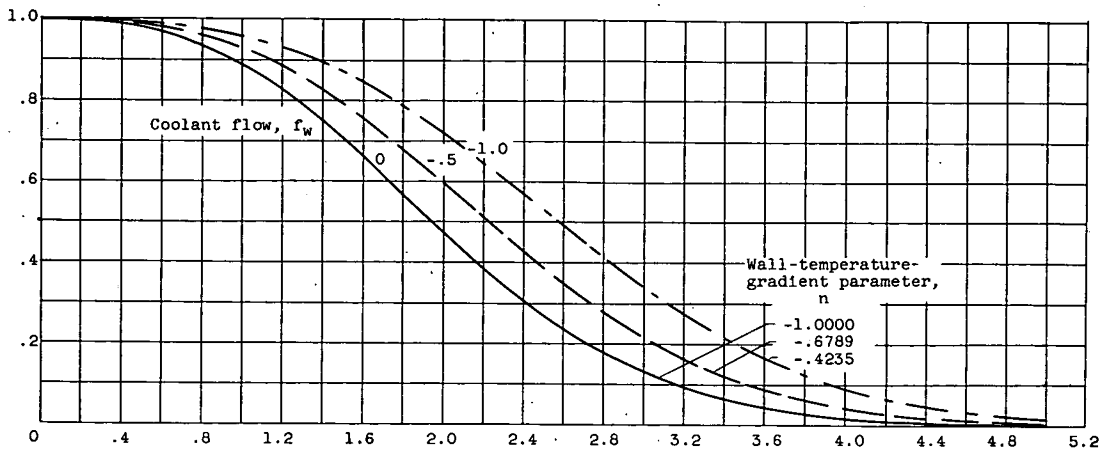


(a) $n > 0$.

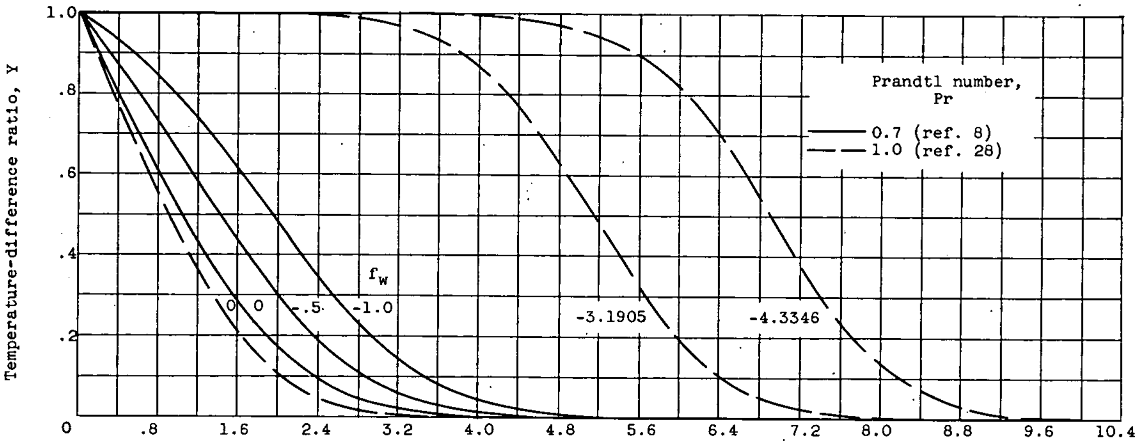
(b) $n = 0$.

(c) $n < 0$.

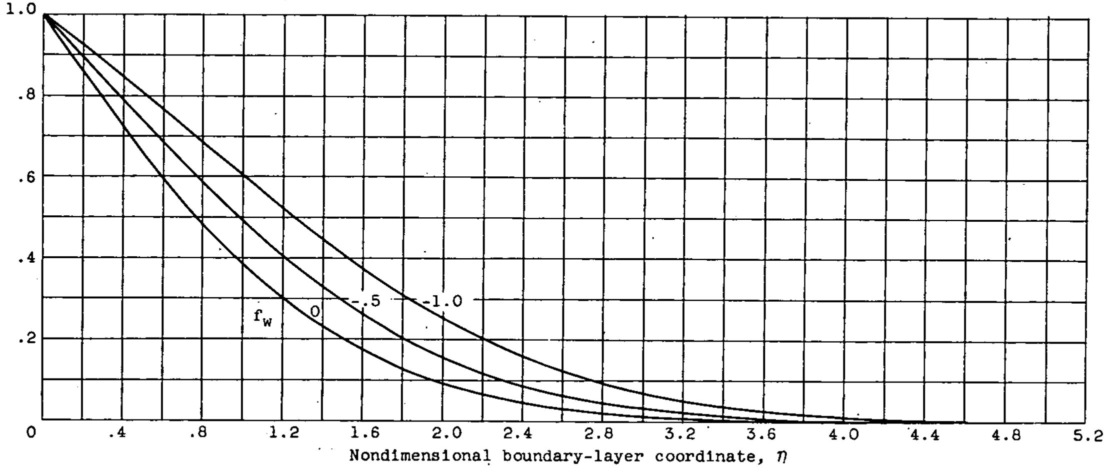
Figure 3. - Flow over impermeable flat plate with variable wall temperature; $n = \frac{x}{T_w - T_\infty} \frac{dT_w}{dx}$.



(a) Wall-temperature-gradient parameter for $Y'(0) = 0$; Prandtl number, 0.7.

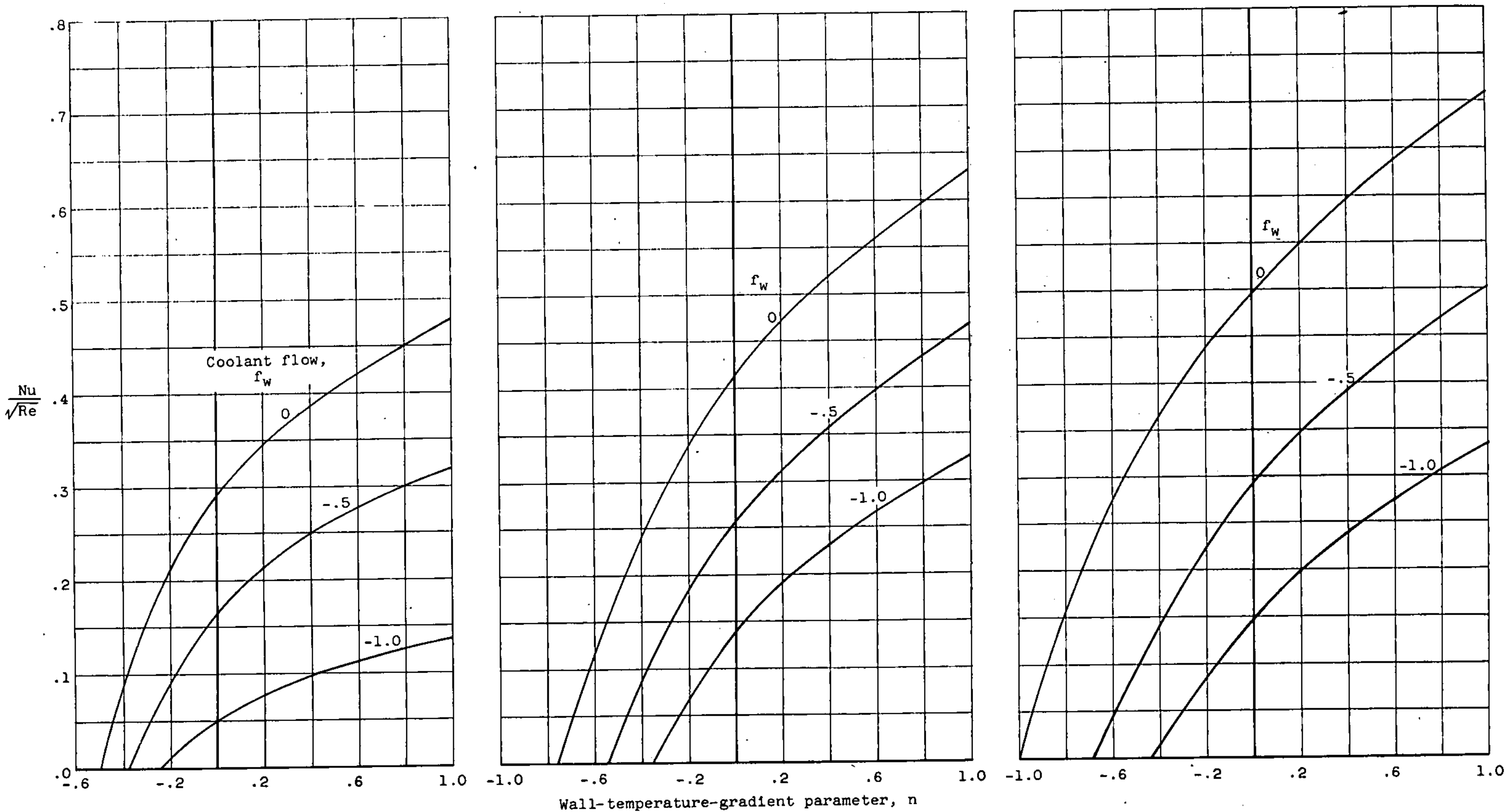


(b) Wall-temperature-gradient parameter, 0.



(c) Wall-temperature-gradient parameter, 1; Prandtl number, 0.7.

Figure 4. - Temperature distributions in constant-property laminar boundary layer for permeable and impermeable wall at variable temperature, Stagnation point flow; Euler number, 1.0.

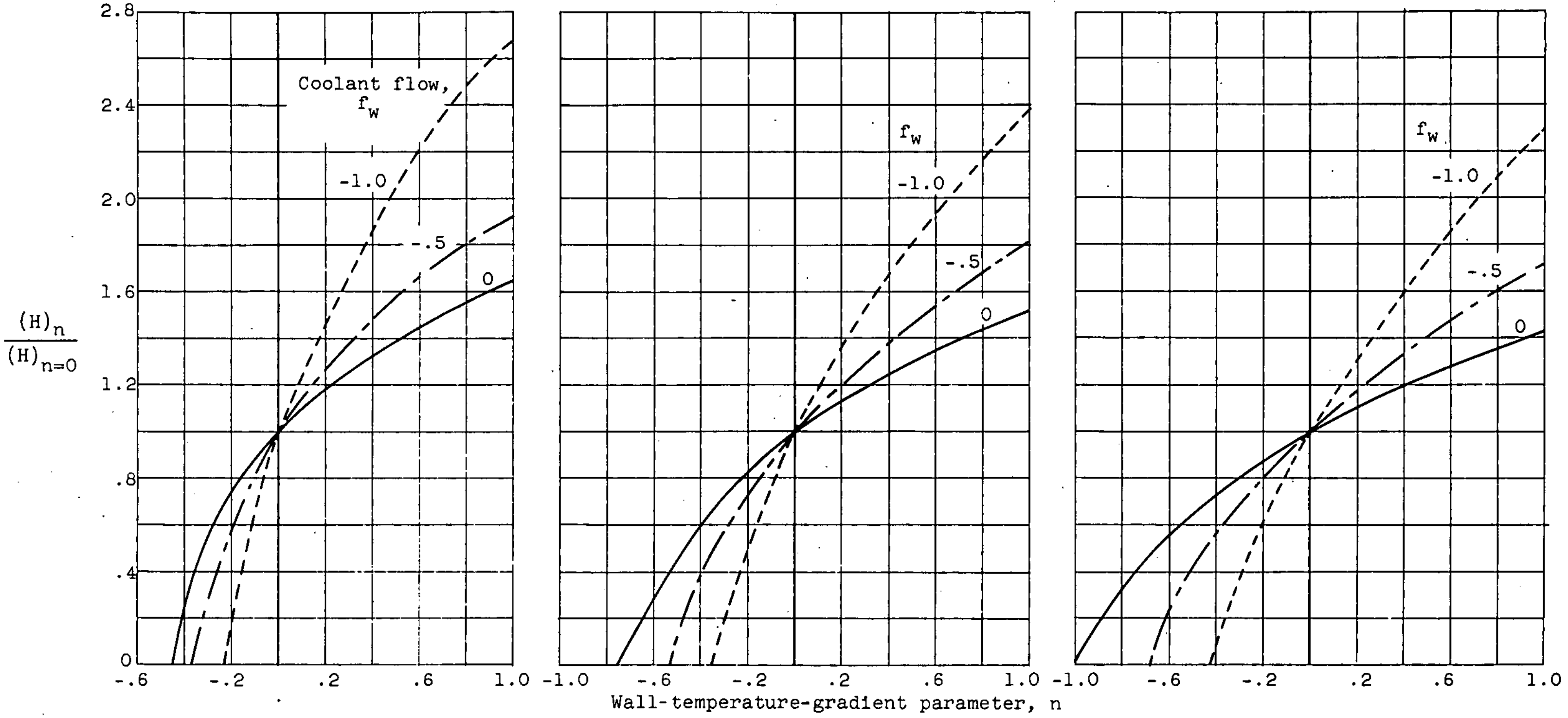


(a) Euler number, 0.

(b) Euler number, 0.5.

(c) Euler number, 1.0.

Figure 5. - Heat transfer through constant-property laminar boundary layer for permeable and impermeable walls at variable temperature; Prandtl number, 0.7.

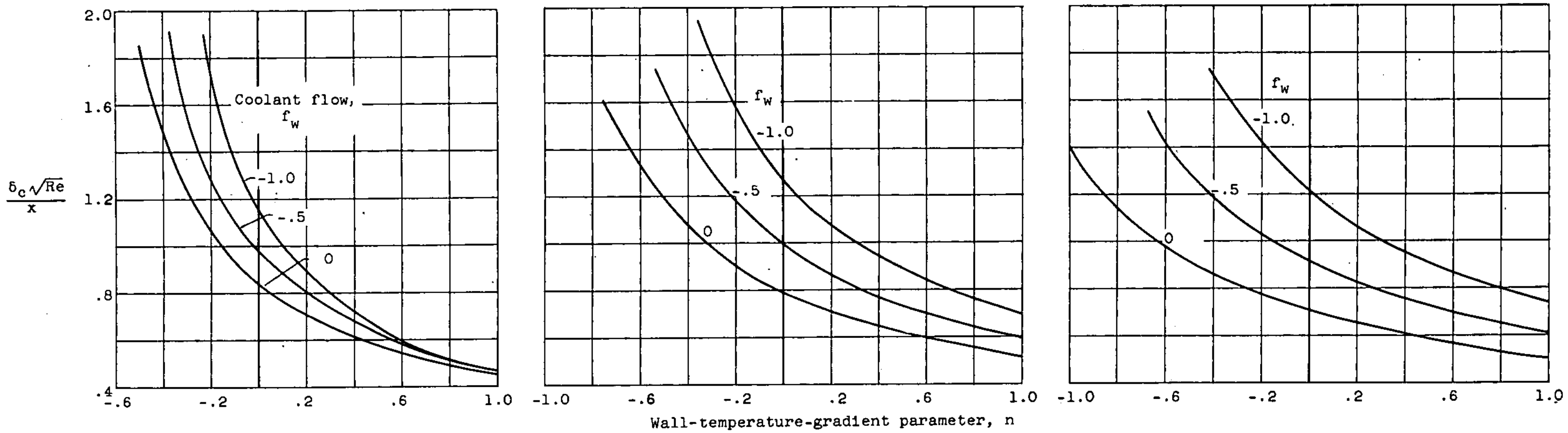


(a) Euler number, 0.

(b) Euler number, 0.5.

(c) Euler number, 1.0.

Figure 6. - Effect of variable wall temperature on local heat-transfer coefficient for laminar boundary layer; Prandtl number, 0.7.



(a) Euler number, 0.

(b) Euler number, 0.5.

(c) Euler number, 1.0.

Figure 7. - Convection thickness of constant-property laminar boundary layer for permeable and impermeable walls at variable temperature; Prandtl number, 0.7.

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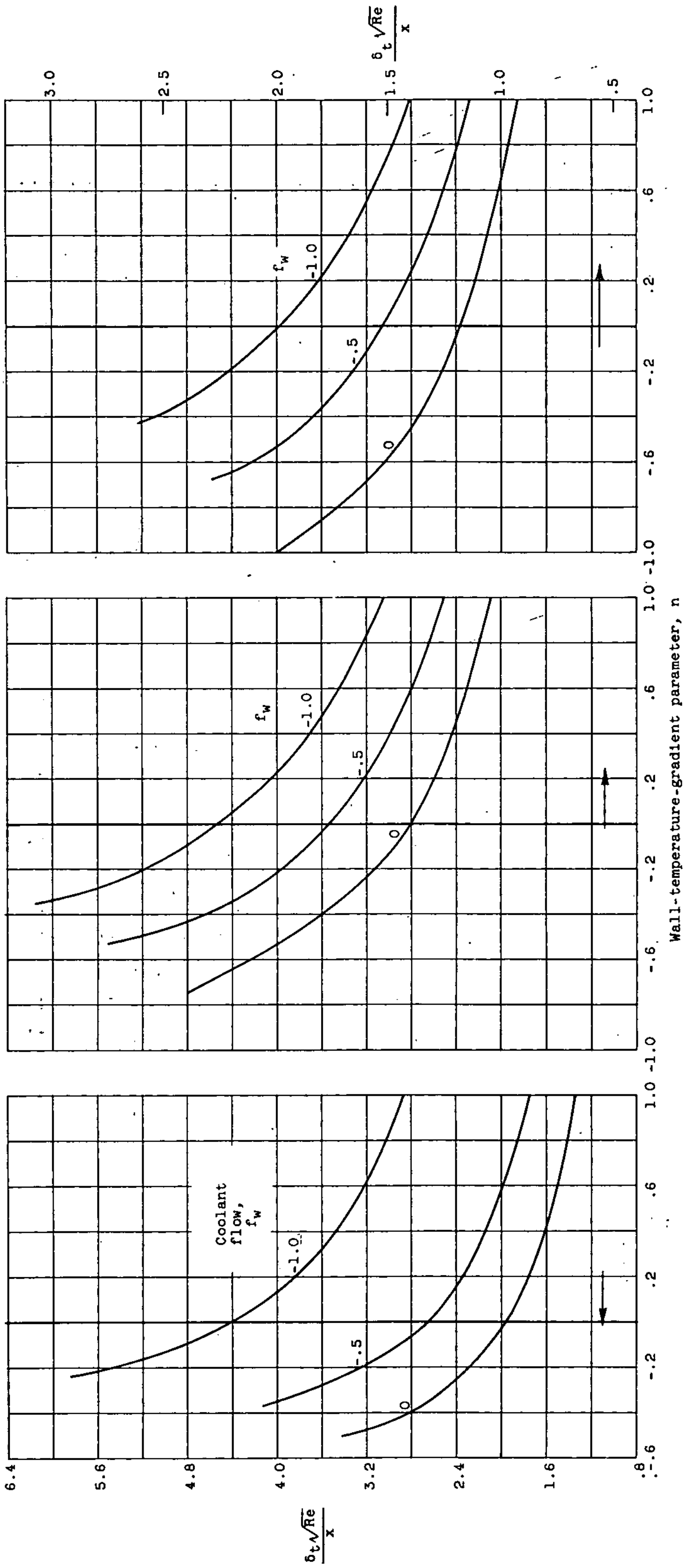

(a) Euler number, 0.

(b) Euler number, 0.5.

(c) Euler number, 1.0.

Figure 8. - Thermal thickness of constant-property laminar boundary layer for permeable and impermeable walls at variable temperature; Prandtl number, 0.7.

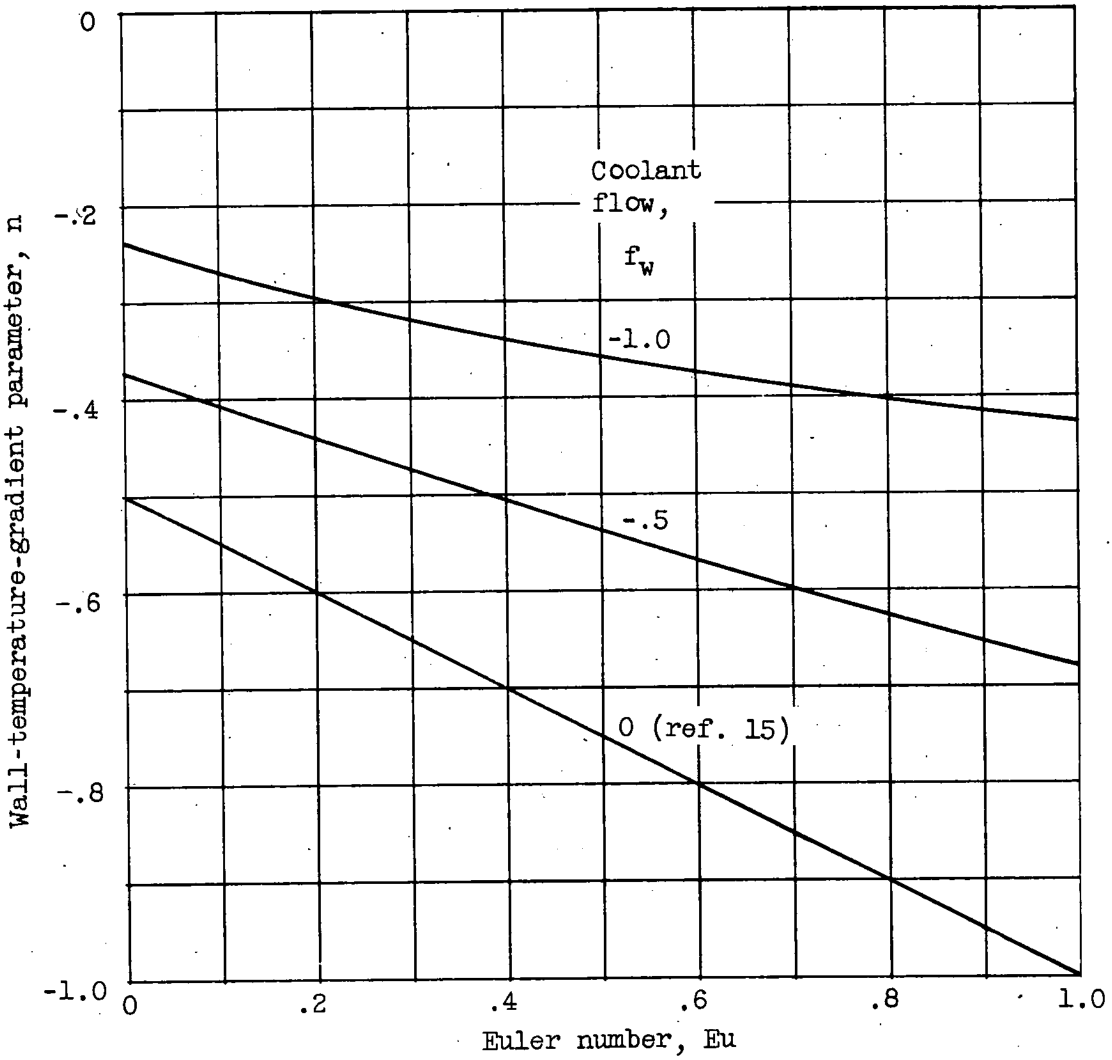

Figure 9. - Value of n for zero temperature gradient at wall ($Y'(0) = 0$); Prandtl number, 0.7.