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TECHNICAL NOTE 3125

A SIMPLE MECHANICAL ANALOGUE FOR STUDYING THE DYNAMIC STABILITY OF AIRCRAFT HAVING NONLINEAR MOMENT CHARACTERISTICS

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SUMMARY

The analogy between a ball rolling over a contoured surface and a pitching and yawing aircraft is developed. Test results from a model representing a missile with linear moment characteristics are presented to verify the analogy. Several examples of the behavior of nonlinear systems are also given. These examples include results from ballistic-range firings as well as analogue tests.

INTRODUCTION

Designers of aircraft are frequently confronted with the problem of predicting the behavior of aircraft having nonlinear pitching- and yawing-moment characteristics. The usual approach to these problems involves tedious calculations or use of elaborate simulators. A simpler approach suitable for some of the problems encountered, particularly those of missiles, is the subject of this paper. The approach uses the analogy between a ball rolling in a suitably shaped bowl and a missile pitching and yawing in flight. This analogue was devised in order to understand a peculiar motion executed by a projectile in the Ames supersonic free-flight range.

SYMBOLS

A,B constants defining variation of z with x and y

a radius of ball, ft

a_g radius of gyration of ball, ft

C,D constants defining variation of C_m and C_n with α and β

C_m pitching-moment coefficient of missile

C_n yawing-moment coefficient of missile

C_σ vector sum of C_m and C_n for axially symmetric missile

F force component acting on ball parallel to bowl surface at point of contact which prevents slippage

g acceleration of gravity, ft/sec^2

I moment of inertia of ball about a diameter, slug-ft^2

K constant, ft/sec^2

$K_{1,2}$ constants, radians/sec^2

L constant defining rolling friction of ball, 1/sec

l body length, ft

$l_{1,2}$ constant defining damping of missile, 1/sec

r $\sqrt{x^2 + y^2}$

W weight of ball, lb

x,y,z coordinate system for bowl description, x and y horizontal and z vertical

X_{cg}/l position of center of gravity as fraction of body length

α angle of attack, radians

β angle of sideslip, radians

δ angle of inclination of bowl surface, radians

ϵ $\tan^{-1} \frac{y}{x}$

θ $\tan^{-1} \frac{\beta}{\alpha}$

σ $\sqrt{\alpha^2 + \beta^2}$

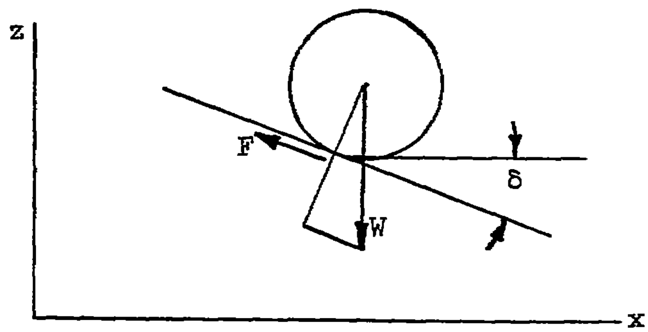
ω rotational velocity of ball, radians/sec

Superscript

$\cdot$ differentiation with respect to time

BASIC DERIVATION

Consider the force system acting on a ball rolling on an inclined surface.



Two forces act on the ball to produce translatory acceleration: the component of the weight force W parallel to the surface at the point of contact, and the tangential force F at the point of contact which prevents slippage.

$$W \sin \delta - F = \frac{W}{g} \frac{\ddot{x}}{\cos \delta} \tag{1}$$

$$Fa = I\dot{\omega}$$

Since slippage does not occur

$$\dot{x} = a\omega \cos \delta$$

$$\ddot{x} = a\dot{\omega} \cos \delta$$

Using this relationship

$$Fa = \frac{I}{a} \frac{\ddot{x}}{\cos \delta}$$

Substituting in equation (1)

$$\sin \delta = \frac{1}{W}\left(\frac{W}{g} + \frac{I}{a^2}\right) \frac{\ddot{x}}{\cos \delta} = \frac{1}{K} \frac{\ddot{x}}{\cos \delta}$$

where

$$\frac{1}{K} = \frac{1}{g}\left(1 + \frac{gI}{Wa^2}\right) = \frac{1}{g}\left[1 + \left(\frac{a_g}{a}\right)^2\right]$$

and a_g is the radius of gyration of the ball.

If $|\delta| < 0.15$ radian,

$$\sin \delta \cos \delta = \sin \delta = \delta = \frac{\partial z}{\partial x}$$

and it follows that

$$\ddot{x} = K \frac{\partial z}{\partial x}$$

The derivation has thus far been limited to motion in the x,z plane. In the general case of motion anywhere on the bowl, the component accelerations parallel to the x and y axes, respectively, are:

$$\ddot{x} = K \frac{\partial z}{\partial x} (x,y)$$

$$\ddot{y} = K \frac{\partial z}{\partial y} (x,y)$$

Similar equations govern the motion of an undamped pitching and yawing missile

$$\ddot{\alpha} = K_1 \, C_m \, (\alpha,\beta)$$

$$\ddot{\beta} = K_2 \, C_n \, (\alpha,\beta)$$

Comparison of these equations shows that α and β are represented in the analogy by x and y and that C_m and C_n are simulated by the bowl slopes, $\partial z/\partial x$ and $\partial z/\partial y$. The ball rolling about the bowl may be visualized as describing the path traversed by the missile tip relative to the center of gravity.

Inserting a friction force proportional to rolling velocity and aerodynamic moments proportional to angular velocities results in

$$\ddot{x} = K \frac{\partial z}{\partial x} (x,y) - L\dot{x}$$

$$\ddot{y} = K \frac{\partial z}{\partial y} (x,y) - L\dot{y}$$

for the ball, and

$$\ddot{\alpha} = K_1 C_m (\alpha,\beta) - l_1\dot{\alpha}$$

$$\ddot{\beta} = K_2 C_n (\alpha,\beta) - l_2\dot{\beta}$$

for the missile. In the case of a missile free to plunge, as well as to pitch, the coefficients contain several terms, but the differential equation is in the same form as that presented.

This, then, is the analogy between a ball rolling on a surface of given contour and a corresponding missile pitching and yawing with the above relations between surface slope and aerodynamic moments.

EXAMPLES

A case representing a missile with linear moment characteristics was tested to verify the analogue, and three nonlinear cases were tested to demonstrate the application. All four cases were chosen axially symmetric to economize on model construction. The feature of axial symmetry suggests use of cylindrical coordinates to define the moment characteristics of the missile and the surface of the bowl.

The moment tending to realign the missile axis with the flight path for the cases tested is given by

$$C_\sigma = C\sigma + D\sigma^s$$

and C_σ is related to C_m and C_n by

$$C_m = C_\sigma \cos\theta$$

$$C_n = C_\sigma \sin\theta$$

The bowl-surface slope is given by

$$\frac{dz}{dr} = 2Ar + 4Br^3$$

and the equation of the bowl surface is then

$$z = z_o + Ar^2 + Br^4$$

Converting the bowl-surface equation to Cartesian coordinates gives

$$z = z_o + A(x^2+y^2) + B(x^2+y^2)^2$$

The slopes of the bowl are then

$$\frac{dz}{dx} = 2Ax + 4Bx^3 + 4Bxy^2$$

$$\frac{dz}{dy} = 2Ay + 4By^3 + 4Byx^2$$

Using the analogy between the bowl and the missile moment characteristics gives

$$C_m = C\alpha + D\alpha^3 + D\alpha\beta^2$$

$$C_n = C\beta + D\beta^3 + D\beta\alpha^2$$

These equations illustrate that C_m, for instance, is a function of both α and β, even for this simple nonlinear system.

The simplification attained by use of cylindrical coordinates may apply to conventional electronic analogue computers as well as to the rolling-ball analogue.[1]

The four cases were obtained by assigning various positive and negative values to the coefficients C and D. The moment equations and the resulting bowl profiles are presented in figure 1. The terms "stable" and "unstable" in this figure refer to conditions near the bowl center. The cubic case has zero stability near the center of the bowl.

CONSTRUCTION OF MODELS

Several techniques of construction were tried. The most accurate bowl, also the least expensive, was cast in hard plaster over a die made of plaster. The first step in making a model was to make the die on a flat work surface by sweeping a steel template about a vertical axis through the center of the die. While this die was hardening, a round frame was made using a piece of plywood for the base and a strip of sheet metal for the side. The hardened male die was greased lightly and the frame, with access holes cut in the base, was placed upside down over it. Plaster was then poured in through the access holes. The final contoured surface was then sanded and sprayed with flat, black lacquer. About eight man-hours of work were required per bowl for the axially symmetric cases built. These bowls were 2 feet in diameter.

TESTING

The bowls were placed directly below a motion-picture camera. Lag screws in the plywood base permitted accurate leveling of the bowls. The balls were all solid and were made of steel, brass, and aluminum and varied in size from 0.25 inch to 1.00 inch in diameter. No elaborate technique was required for starting the balls rolling, but it was found

[1]The equations of motion of the ball and missile may be converted to cylindrical coordinates. The equations for the ball are, assuming axial symmetry

$$\ddot{r} - r\dot{\epsilon}^2 = K\frac{dz}{dr} - L\dot{r}$$

$$2\dot{r}\dot{\epsilon} + r\ddot{\epsilon} = -Lr\dot{\epsilon}$$

The term $2\dot{r}\dot{\epsilon}$ is the Coriolis force. The equivalent equations for the missile are

$$\ddot{\sigma} - \sigma\dot{\theta}^2 = kC_\sigma - l\dot{\sigma}$$

$$2\dot{\sigma}\dot{\theta} + \sigma\ddot{\theta} = -l\sigma\dot{\theta}$$

convenient to roll the balls down a chute rather than roll them by hand because good repeatability of initial conditions was desired. The motion pictures, which were taken at either 8- or 16-frames per second, were projected frame by frame onto a screen for measurement. The camera framing rate was used as the time standard. Oil layers of various thicknesses on the bowls were found to provide good control of the damping. Using balls of various densities also provided some control over damping without oiling the surface. Actually, the damping was measured rather than predetermined.

RESULTS

The results obtained with the linear-case bowl are easily compared with analytic results and serve to check the validity of the analogue. In figure 2 are plotted the data for three runs corresponding to simple pitching motion with progressively heavier damping. This large variation in damping was attained by using a 1-inch-diameter brass ball on the bowl with no oil for the light damping (fig. 2(a)) and a 1/4-inch-diameter aluminum ball with a thin coat of oil for the case of heaviest damping (fig. 2(c)). Superposed on each set of data is a damped sine wave fitted by the method of least squares (ref. 1). That the experimental data fit the theory quite well is clear. The undamped natural period, as predicted on the basis of bowl profile and the inertia characteristics of the solid balls used, is indicated in figure 2.

Several plots of angle of attack versus time for pitching missiles with nonlinear moment curves, obtained with the bowls, are presented in figure 3. The curves for the cubic case (fig. 3(a)) were obtained at different times during the same test and illustrate the extreme dependence of frequency on amplitude of oscillation, a characteristic attributable to the nonlinear moment curve. The large-amplitude curve has the sharp peaks typical of cubic systems. The stable cubic case (fig. 3(b)) exhibits these same characteristics to a lesser extent. The unstable cubic case (fig. 3(c)) exhibits 6 points of inflection per cycle instead of the 2 per cycle shown by all the other cases.

Two plots of motions obtained with the unstable cubic case are shown in figures 4 and 5. The motion of figure 4 gradually degenerates and takes on a form much like that of figure 5. These two motions were not obtained on the same test run.

In the course of experimental firings of a simple body of revolution in the Ames supersonic free-flight range, the pitch-yaw records shown in figure 6 were obtained. In the absence of large gyroscopic forces (the models were fired from a smooth-bore gun), it was difficult to explain this behavior. In attempting to do this, the analogy between the ball and the missile was discovered and the improved intuitive feel for the problem made the explanation clear. The moment curve for this body has not been determined, but the motions observed indicate that the unstable cubic case is a fair representation. The data were obtained using ballistic "yaw cards." The technique consists of deducing the model orientation at any instant from the shape of the hole it punches in a piece of paper. Therefore, the curves are not exactly defined, but the general nature of the motion is illustrated.

LIMITATIONS

The most serious limitation on the use of the analogue is that only pitching and yawing motion may be simulated. The missile which is being simulated must be assumed to be roll stabilized to zero rate of roll. The damping coefficient must be the same for pitch and yaw, which also limits the value of the analogue for airplane-like cases.

It was noted during the test with the linear example that when the ball was executing elliptical motion, the axes of the ellipse precessed slowly. This distortion of the motion is believed to result from second-order momentum effects (arising from the fact that the bowl surface had finite slope) not considered in the derivation. The salient features of the motion are not masked by this effect.

CONCLUDING REMARKS

The use of a ball rolling on a suitably shaped surface to represent the pitching and yawing motion of a missile has been described. Intuitive understanding of some complicated motions is simplified through the use of this device. The use of the analogue for axially symmetric cases suggests that the use of cylindrical coordinates for describing nonlinear moment characteristics may prove helpful in solving problems on conventional analogue computers.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., Dec. 10, 1953

REFERENCE

1. Shinbrot, Marvin: A Least-Squares Curve-Fitting Method With Applications to the Calculation of Stability Coefficients From Transient-Response Data. NACA TN 2341, 1951.

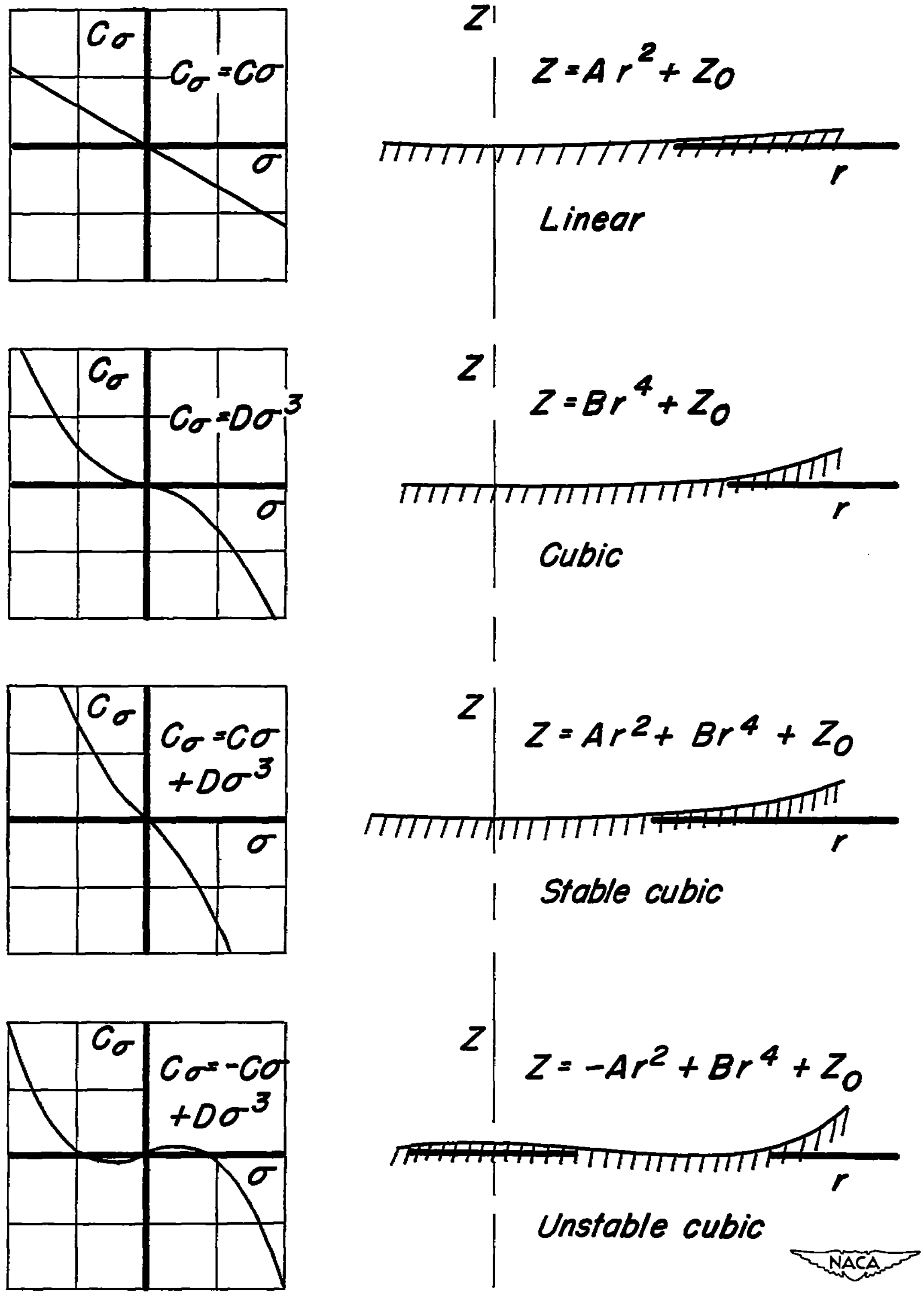
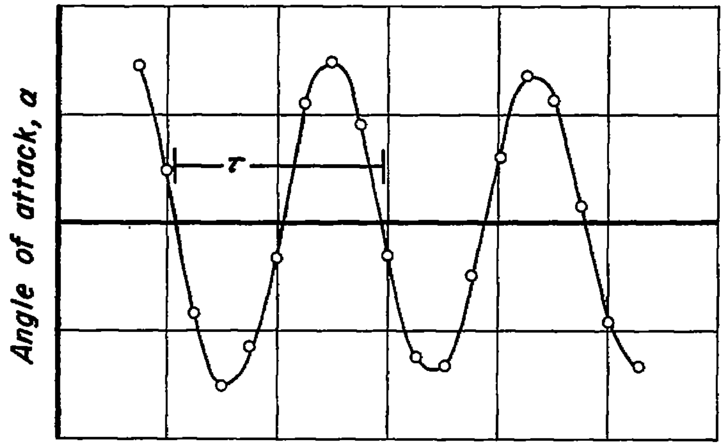
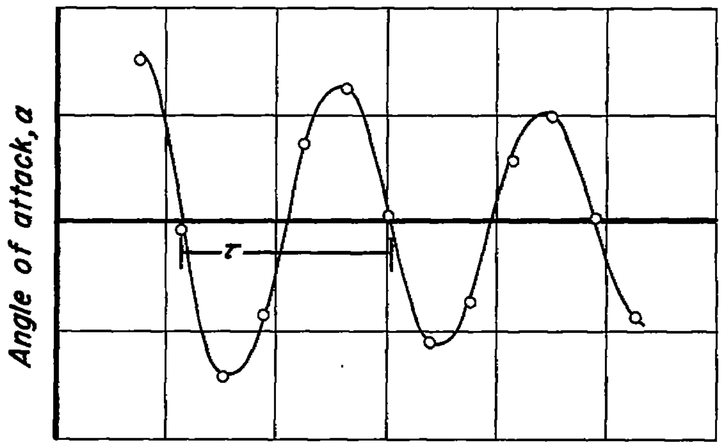


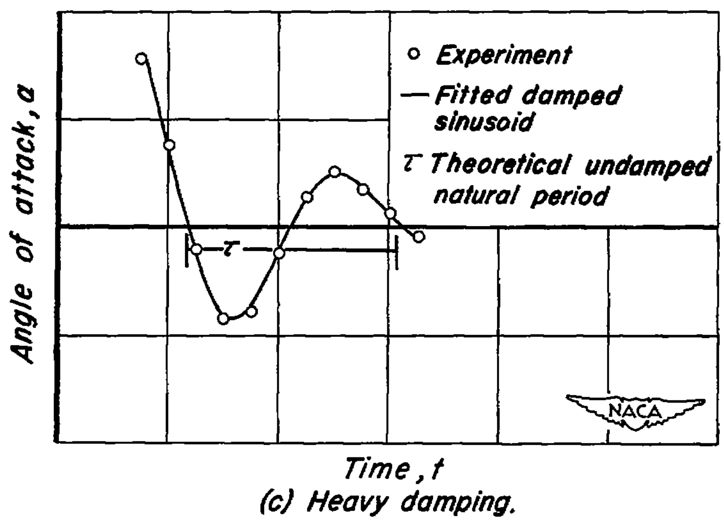
Figure 1.- Moment curves and surface profiles for analogue surfaces tested.



(a) Light damping.

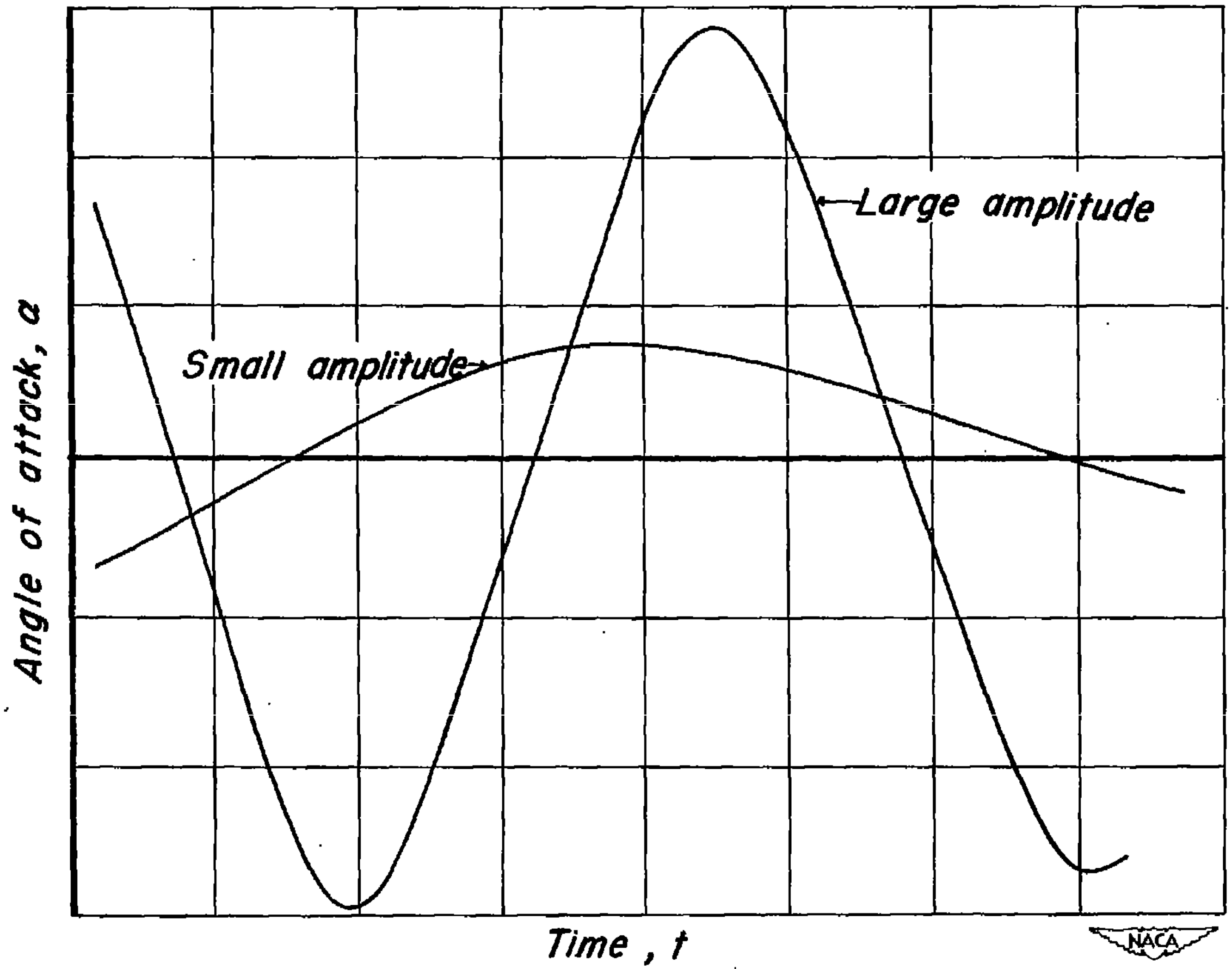


(b) Moderate damping.



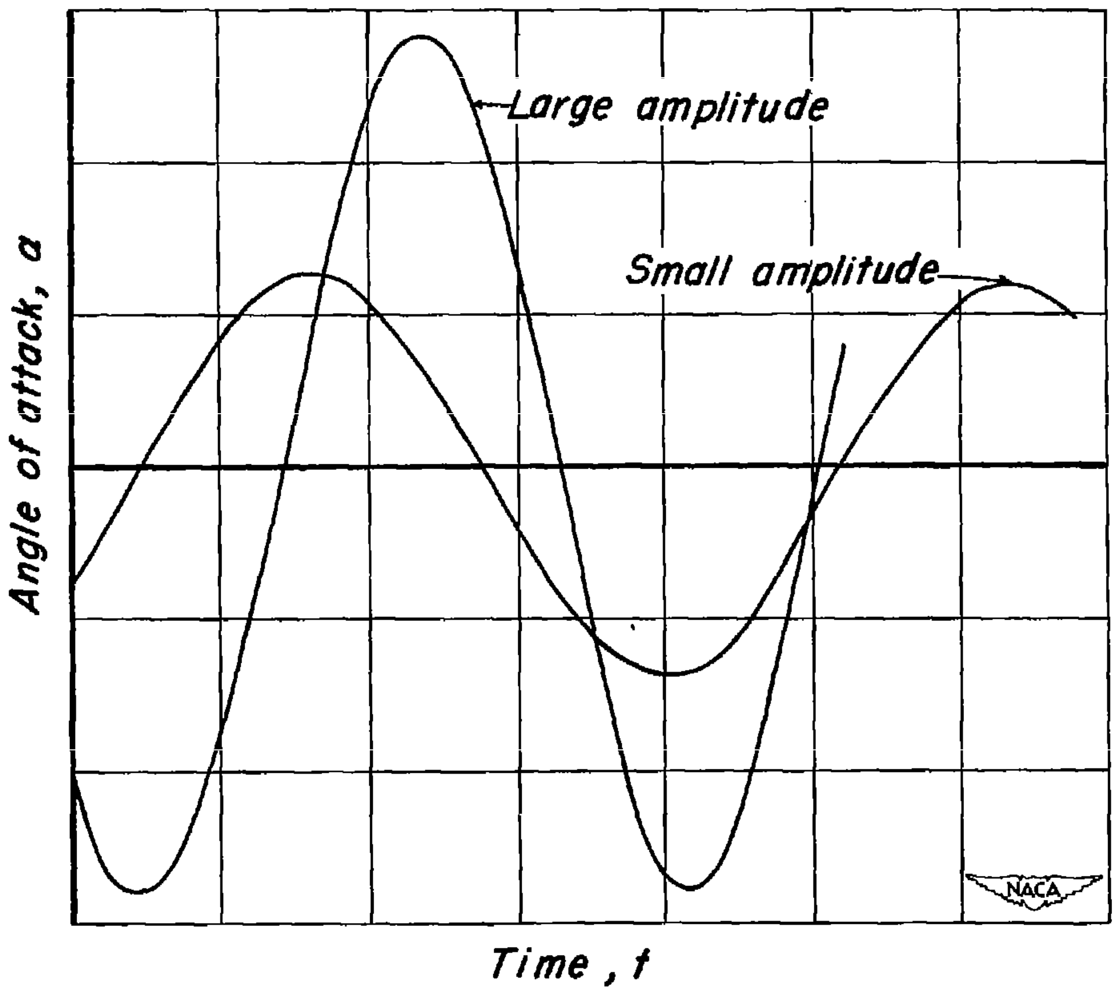
(c) Heavy damping.

Figure 2.– Angle-of-attack history for linear case.



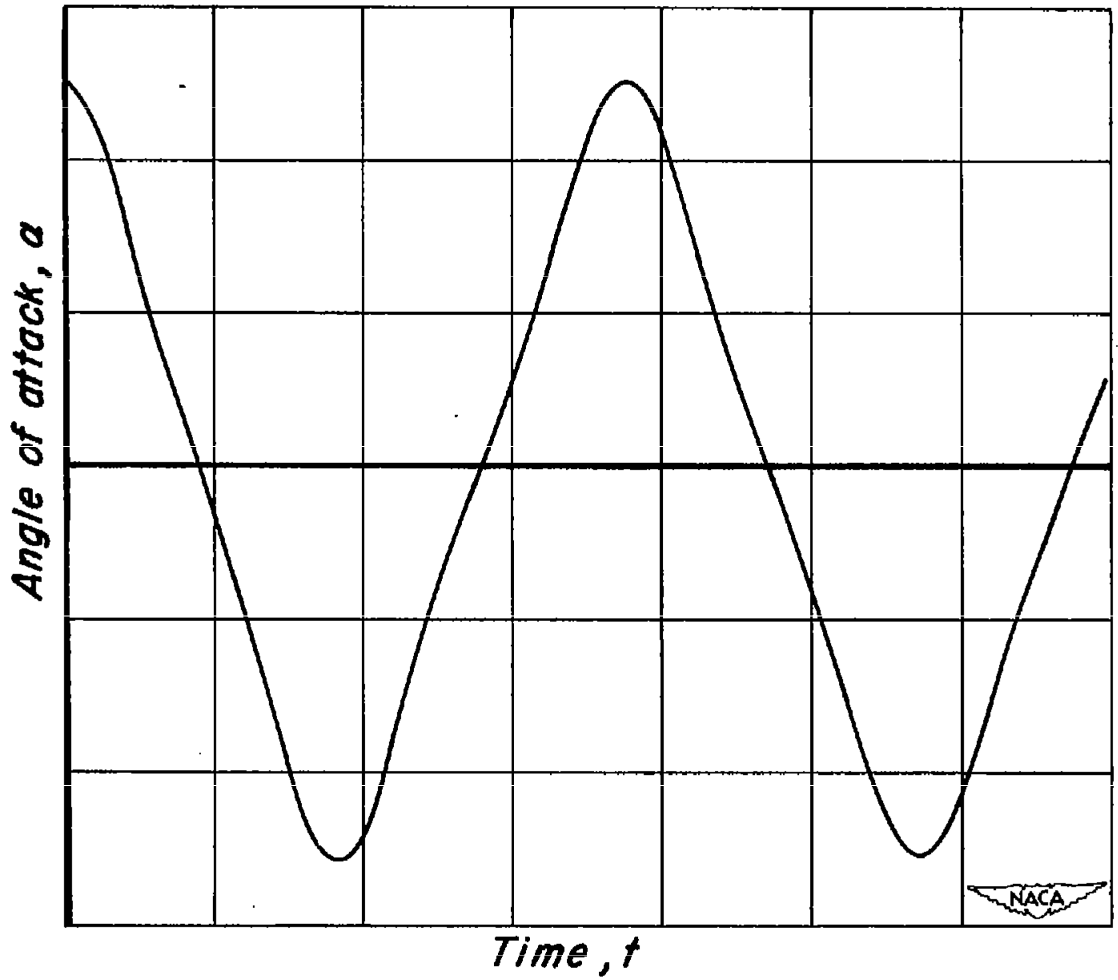
(a) Pure cubic.

Figure 3.– Angle-of-attack histories of nonlinear cases, $\beta = 0$.



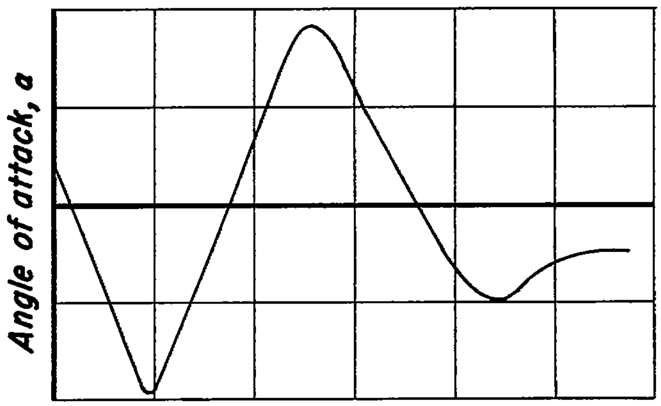
(b) Stable cubic.

Figure 3.— Continued.

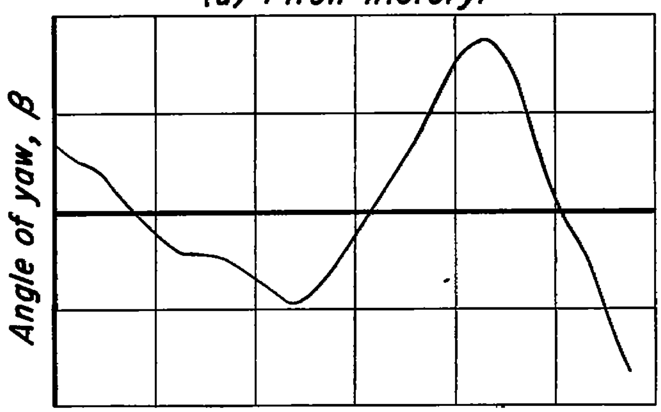


(c) Unstable cubic.

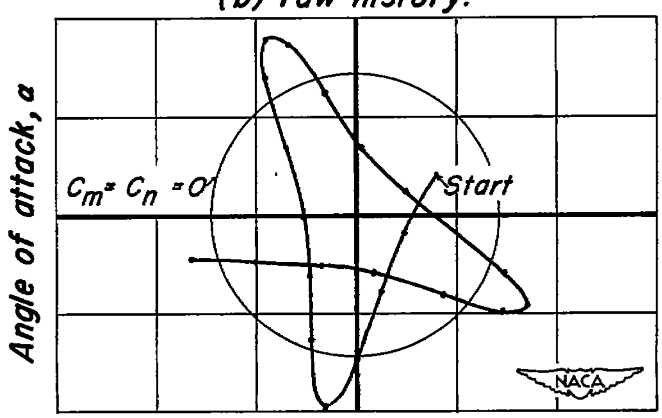
Figure 3.- Concluded.



(a) Pitch history.

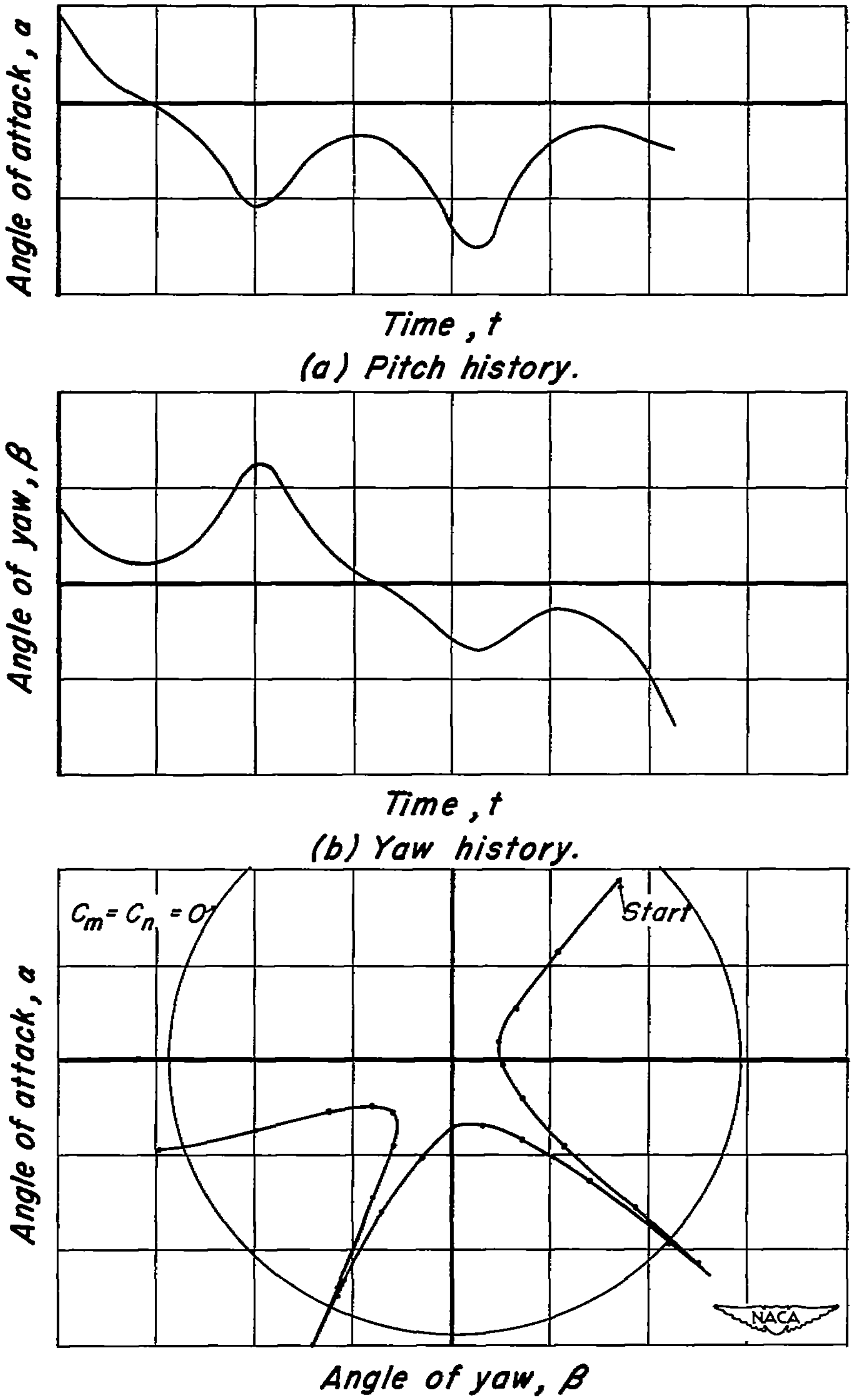


(b) Yaw history.



(c) Combined pitch and yaw histories.

Figure 4.- Pitch and yaw histories for unstable cubic case of large amplitude.



(a) Pitch history.

(b) Yaw history.

(c) Combined pitch and yaw histories.

Figure 5. - Pitch and yaw histories for unstable cubic case of small amplitude.

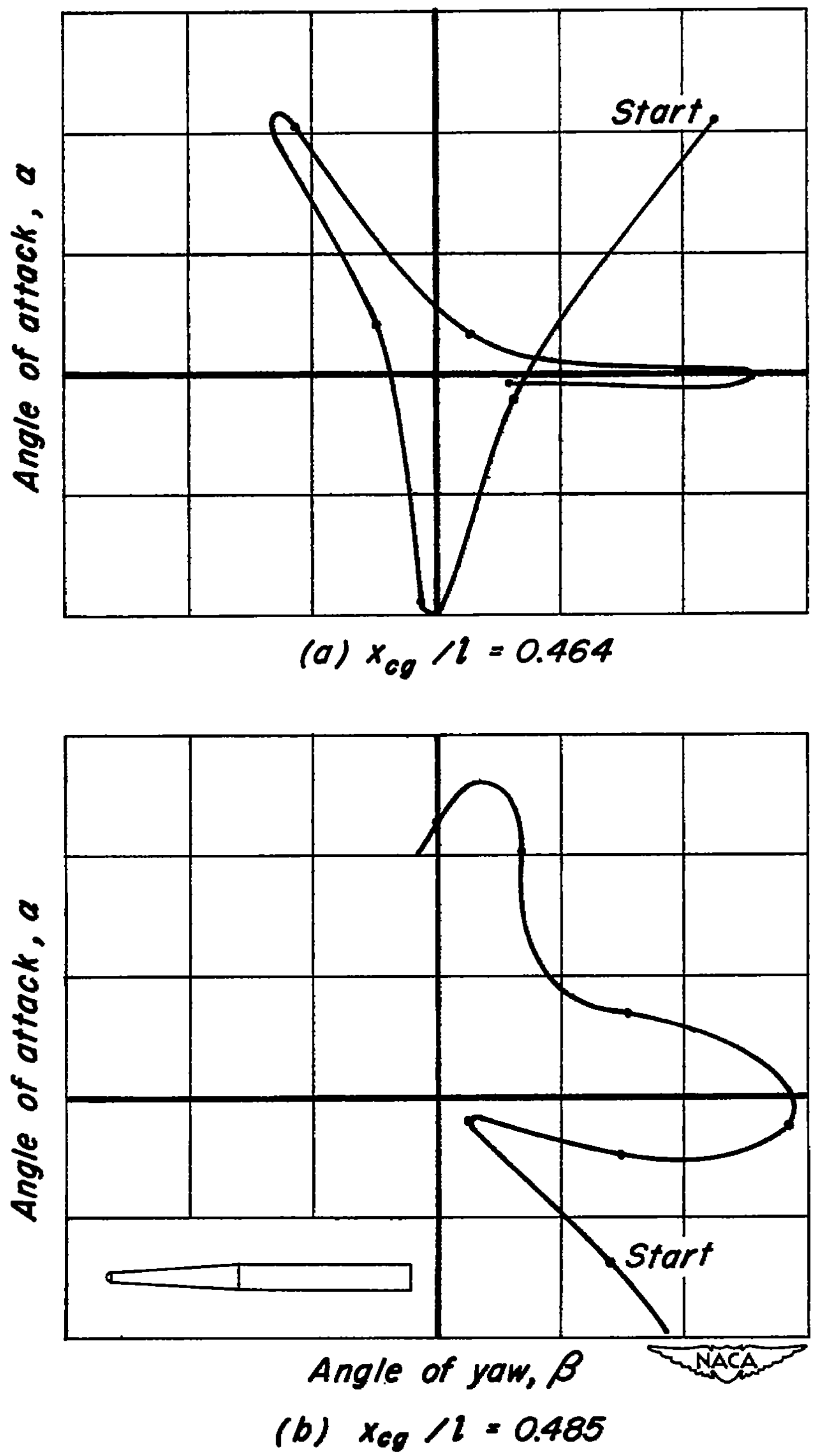


Figure 6.- Experimental pitch-yaw histories for a body of revolution in free flight. Mach number = 4.5.

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