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THEORETICAL ANALYSIS OF AN AIRPLANE ACCELERATION RESTRICTOR CONTROLLED BY NORMAL ACCELERATION, PITCHING ACCELERATION, AND PITCHING VELOCITY

By Christopher C. Kraft, Jr.

Langley Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

An acceleration restrictor which limits the elevator motion of the airplane has been analyzed by means of an electronic analog computer. The signal used to control the system was, in one case, proportional to normal and pitching acceleration and, in the other case, a function of normal and pitching acceleration and pitching velocity. The mechanical design of the system has not been considered, but the device used to stop the motion of the elevator has been assumed to have several values of lag.

When controlled by an acceleration restrictor that was sensitive to a signal proportional to normal and pitching acceleration, the airplane had a ratio of peak to preset acceleration of about 1.4 when the device used to stop the elevator had approximately zero lag. The ratio was constant for airspeeds ranging from 200 to 1,000 feet per second. Increasing the lag to about 0.02 second caused the ratio of peak to preset acceleration to vary from 1.2 to 1.6 as the airspeed increased from 200 to 1,000 feet per second. When the control signal was a function of normal and pitching acceleration and pitching velocity, the ratio of peak to preset acceleration was 1.1 throughout the speed range for the case of zero lag. Increasing the lag to 0.02 second had little effect on the performance of the system. An acceleration restrictor with a ratio of peak to preset acceleration of 1.1 throughout the speed range would allow the airplane to reach normal accelerations near the limit load factor and still prevent the airplane from ever exceeding this value. The performance of the acceleration restrictor with a 0.05-second lag in the device used to stop the elevator motion would be unsatisfactory for either of the control signals used in this investigation.

INTRODUCTION

In reference 1, the need for acceleration restrictors was indicated and an analysis of several simple devices to limit the maximum maneuvering acceleration of airplanes was presented. One device which appeared promising in that analysis worked on the principle of stopping the elevator motion when a signal proportional to normal and pitching acceleration reached a certain value. In the analysis of reference 1, however, the magnitude of the pitching-acceleration signal was limited to the value obtained by mounting a linear accelerometer in the nose of the airplane at a point assumed to be 3 chords ahead of the center of gravity. The conclusion was reached that larger values of the pitching-acceleration signal would be necessary.

This paper is a continuation of the analysis of reference 1. In the present investigation, the pitching acceleration is assumed to be measured in such a manner that the gain of the pitching-acceleration signal can be varied independently of the normal acceleration. Two types of signals are assumed to control the acceleration restrictor. One signal is proportional to normal and pitching acceleration, and the other is a function of normal and pitching acceleration and pitching velocity. The two signals are defined by the following expressions:

$$n + \frac{K}{g}\ddot{\theta}$$

and

$$n + \begin{Bmatrix} \frac{K}{g}\ddot{\theta} & \text{for} & \ddot{\theta} > 0 \\ 0 & \text{for} & \ddot{\theta} < 0 \end{Bmatrix} + \frac{A}{g}\dot{\theta}\,\frac{\tau D}{1 + \tau D}$$

where the transfer function $\frac{\tau D}{1 + \tau D}$ is used to eliminate the steady-state value of pitching velocity. The gain constants used for the pitching-velocity and acceleration signals are assumed to be constant throughout the speed range of the airplane in order to simplify the system. In the analysis, the elevator is assumed to move at a constant rate except when it is stopped by the action of the acceleration restrictor. The effects of lag in operation of the acceleration restrictor are also studied. The data presented in this paper were obtained by use of a Reeves Electronic Analog Computer which simulated the airplane dynamics and provided an accurate means of obtaining the airplane response. The analysis applies to a particular type of acceleration restrictor, but no consideration has been given to the mechanical design of such a system.

SYMBOLS

A	gain constant for pitching velocity
$\bar{c}$	mean aerodynamic chord of wing
C_m	pitching-moment coefficient, $\frac{M}{\frac{\rho}{2}V^2S\bar{c}}$
C_Z	vertical-force coefficient, $\frac{Z}{\frac{\rho}{2}V^2S}$
D	differential operator, d/ds
F	Froude number, $g\bar{c}/V^2$
g	acceleration due to gravity
h	scale factor
K	gain constant for pitching acceleration
k_y	radius of gyration about Y-axis
K_y	radius-of-gyration factor, $k_y/\bar{c}$
m	airplane mass
M	pitching moment (positive nose up)
n	normal acceleration (positive up), g units
n_p	preset value of normal acceleration
q	pitching velocity, $\frac{V}{\bar{c}}D\theta$
s	distance traveled, $t\frac{V}{\bar{c}}$, chords
S	wing area
t	time
T	time constant
V	true airspeed

Z	vertical force (positive down)
α	angle of attack
δ_e	elevator angle
θ	angle of pitch
μ	relative-density factor, $m/\rho S\bar{c}$
ρ	air density
τ	time lag

Dot over quantity indicates differentiation with respect to time.

Stability derivatives are defined in accordance with the following examples:

$$C_{Z_\alpha} = \frac{\partial C_Z}{\partial \alpha} \qquad C_{m_q} = \frac{\partial C_m}{\partial \frac{q\bar{c}}{2V}} \qquad C_{m_{D\alpha}} = \frac{\partial C_m}{\partial \frac{\dot{\alpha}\bar{c}}{2V}}$$

PROCEDURE

The data presented in this report were obtained by use of a Reeves Electronic Analog Computer which simulated the airplane dynamics. The simulated elevator-angle input to the computer could be applied at a constant rate or held at a fixed value by manual operation of a switch. The simulated rate of elevator motion was 30° per second. This was chosen to represent the maximum control rate that a human pilot would impose on the airplane. An elevator rate of about 30° per second is probably faster than a pilot normally uses in most high-speed maneuvers, and therefore it is more critical from the standpoint of acceleration restriction. The resulting motions of the simulated airplane subsequent to an elevator input were recorded as functions of time. The quantities recorded were elevator angle, normal acceleration, and the quantity used to control the acceleration restrictor.

The computer operator, in performing the simulated maneuvers, started the elevator input and observed on a meter the buildup of the combined signal. When the combined signal reached a given preset value, the elevator input was stopped. If the combined signal then fell below the preset value, the operator started the elevator input again. This process was repeated until the combined signal remained above the preset value.

The time scale on which the machine operated was made very slow compared with real time in order to enable the operator to start and stop the elevator with very little lag in real time. The rate used was such that the time required for the airplane to travel 1 chord length equaled 1 second of machine time. In order to simulate more closely the action of a mechanical brake on the control, several values of lag in brake operation were used in the following manner: When the combined signal reached the preset value, the computer operator would wait a period which was equivalent to the desired time lag before stopping the elevator. Similarly, when the combined signal fell below the preset value, the operator would wait for the given time lag before allowing the elevator to move again.

A combined signal of normal acceleration and pitching acceleration may be considered equivalent to the output of a linear accelerometer located ahead of the center of gravity. The distance between this accelerometer and the center of gravity is a convenient means of expressing the gain constant of the pitching-acceleration signal. In this report, gain constants of 88.1 and 154.7 feet were used.

For the cases in which a signal proportional to pitching velocity is combined with the signals of normal and pitching acceleration, several factors must be considered in choosing a gain constant for the pitching-velocity signal. Since the pitching velocity has a steady-state value, the airplane would be limited to a steady normal acceleration less than the preset acceleration because the control signal in the steady state would be made up of both the normal acceleration and the pitching velocity. In order to eliminate this restriction, the steady-state value of the pitching-velocity signal could be reduced to zero by means of a canceling network. The time constant of this network and the pitching-velocity gain were chosen to provide a reasonably low ratio of peak to preset acceleration without unduly increasing the time required to reach maximum acceleration beyond the time required when the restrictor was controlled by the signal proportional to normal and pitching acceleration. The constants required were determined on the analog computer for an airspeed of 1,000 feet per second and approximately zero lag, and the same constants were used for all the other conditions tested. The signal used to control the acceleration restrictor can be described by the following expression:

$$n + \left\{ \begin{matrix} \frac{K}{g}\ddot{\theta} & \text{for} & \ddot{\theta} > 0 \\ 0 & \text{for} & \ddot{\theta} < 0 \end{matrix} \right\} + \frac{A}{g}\dot{\theta}\,\frac{TD}{1 + TD}$$

A pitching-velocity gain constant A of 644 feet per second in combination with a time constant T of 0.25 second were found to provide near-optimum characteristics. It should be noted that in the control signal described by the foregoing expression the pitching-acceleration signal has been limited to positive values. This limitation was included in the control signal in an effort to improve further the operation of the acceleration restrictor. A preliminary investigation using the signal of normal and pitching acceleration with the pitching acceleration limited to positive values showed a slight improvement in the performance of the system by a slight reduction in the ratio of peak to preset acceleration. In the mechanical design of an acceleration restrictor, however, it may be desirable to forgo this limitation in order to simplify the system. It is felt that this change would have little effect on the performance.

Response data for a fighter-type airplane having the characteristics presented in table I were obtained for various airspeeds, various lags in brake operation, and, in some cases, two values of static margin. The conditions investigated are presented in table II, and the values of lag in brake operation, along with the ratios of peak to preset acceleration obtained, are presented in table III. All the data are for standard sea-level conditions.

The usual equations of longitudinal motion were used to study the characteristics of the airplane. The transfer functions derived from these equations which describe the normal acceleration and pitching velocity of the airplane were identical to those presented in reference 1. The results obtained were analyzed by measuring the peak normal acceleration subsequent to an elevator input. These values of normal acceleration were divided by the preset acceleration (in this report, 6g) and plotted as a function of airspeed. The machine time unit previously discussed was converted to real time by multiplying it by the ratio of the airspeed to the mean aerodynamic chord. The data presented may be made applicable to different elevator rates, preset accelerations, and airspeeds, provided that the airplane is the same as or dynamically similar to the one assumed in the analysis. The methods of converting the data are given in the appendix.

RESULTS AND DISCUSSION

Acceleration Restrictor Sensitive to Combined Signal of Normal and Pitching Acceleration

Several time histories of the response of the airplane incorporating the acceleration restrictor with approximately zero lag are presented in figure 1 for three values of airspeed. When performing the simulated maneuver, the computer operator attempted to stop or start the elevator motion with as little lag as possible when the combined signal of normal and pitching acceleration reached the preset value. (The preset value may differ slightly from 6g because of operator techniques and inaccuracies of the computer.) The mathematical solution of the equations of motion including the operation of the acceleration restrictor would be indeterminate for the case of zero time lag in brake operation. Any practical system, however, would inherently have a finite lag, and the solution for this case represents a condition of very small lag.

A discussion of the sequence of events occurring in the time histories presented is of interest. The elevator moves at a constant rate of 30° per second until the combined value of normal and pitching acceleration reaches the preset value. At that instant, the elevator is stopped. The normal acceleration changes very little in this length of time, the greater part of the combined signal of normal and pitching acceleration resulting from the pitching acceleration. As the combined signal falls below the preset value, the elevator begins to move again at the rate of 30° per second and continues to move until the signal exceeds the preset value; then the elevator is stopped again. This process continues until the signal no longer falls below the preset value, since the normal acceleration has exceeded the preset value. From this time on, the elevator remains fixed. The normal acceleration has increased smoothly until it becomes a maximum a short time after the elevator has reached its maximum position. The combined signal of normal and pitching acceleration becomes equal to the normal acceleration as the steady state is reached, since the pitching acceleration decreases to zero. The effect of the acceleration restrictor is to reduce the rate of elevator motion, the amount of reduction depending upon the forward speed of the airplane. It is this reduction in effective elevator rate that prevents excessive overshoot of the normal acceleration of the airplane.

The results that were obtained from runs made for five different airspeeds, two values of static margin, and two gain constants are presented in figure 2. The figure shows the ratio of peak normal acceleration to preset acceleration as a function of airspeed. Repeat runs were made for each case, and the scatter in the data is a result of operator technique. For either value of static margin and a gain constant of 154.7, the ratio is constant at approximately 1.3 for airspeeds varying from 400 to 1,000 feet per second. When the gain constant is changed to 88.13, the ratio of peak acceleration to preset acceleration is about 1.4 to 1.45 for airspeeds between 400 and 1,000 feet per second and slightly lower at 200 feet per second.

It is desirable to have an acceleration restrictor with a constant ratio of peak acceleration to preset acceleration throughout the speed range. If the ratio increased with increasing airspeed, it would be necessary to set the preset acceleration at a relatively low value to prevent the airplane from exceeding the limit load factor at high speeds. This low value of preset acceleration would result in undue restriction of the maximum acceleration of the airplane at lower airspeeds. At very low airspeeds the ratio of peak acceleration to preset acceleration has little significance since the airplane would stall before reaching the limit load factor. A constant ratio will not, by itself, assure satisfactory performance of the acceleration restrictor. If the value of this ratio exceeded 1.0 by too large an amount, the preset acceleration required to limit the maximum acceleration would be relatively low and would restrict the maneuverability of the airplane where the acceleration was increased gradually.

Time histories of the response of the airplane to elevator inputs for two values of lag in brake operation ($\tau \approx 0.02$ sec and $\tau \approx 0.05$ sec) are presented in figure 3. The lag resulted in a more pronounced sawtooth shape of the curves for elevator angle and for the combined signal of normal and pitching acceleration. The number of oscillations in the curve of the combined signal and the overshoot above and below the preset value was a function of forward speed and time lag. The data presented in figure 4 are the results obtained for two values of lag at a static margin of 10 percent of the mean aerodynamic chord and a gain constant of 154.7. For a lag of about 0.02 second, the ratio of peak acceleration to preset acceleration varied from about 1.3 at 400 feet per second to about 1.55 at 1,000 feet per second. This amount of variation might be tolerable. The ratio for a lag of about 0.05 second varied from 1.25 at 200 feet per second to about 2.2 at 1,000 feet per second. This amount of overshoot would probably be unacceptable since the limitations imposed on the airplane at lower speeds would be too great. It appears, then, that lags in brake operation of about 0.02 second or less would be required for satisfactory performance of an acceleration restrictor of this type in a fighter airplane.

In the design of an acceleration restrictor incorporating the principles studied in this paper, a device allowing the pilot to move the control back toward trim would be necessary. If a brake without such a device were used, the pilot would be unable to return the airplane to 1g flight since the elevator would be permanently locked whenever the acceleration remained beyond the preset value. One way to allow the pilot freedom to move the control stick would be to incorporate a ratchet in the brake mechanism. If limitation of negative acceleration were also desired, a second brake with a ratchet working in the opposite direction would be required. It is unlikely that provision would ever be made for limiting negative acceleration, but, in order to study the results that could be obtained with such an arrangement, several runs were made in which the pull-up maneuver with a preset acceleration of 6g was followed by a push-down maneuver with a preset acceleration of -6g. At all the speeds previously considered, data were obtained for an acceleration restrictor controlled by the combined signal of normal and pitching acceleration, a lag of 0.018 second, and a pitching-acceleration gain constant of 154.7. A typical time history of the case investigated is presented in figure 5. The results indicate that the buildup of normal acceleration in the negative direction was approximately the same as in the positive direction. The provision of acceleration restriction in the negative direction did not interfere with the ability of the operator to effect recovery from a condition of high positive acceleration.

Acceleration Restrictor Sensitive to Normal and Pitching Acceleration and Pitching Velocity

As mentioned previously, an acceleration restrictor which has a ratio of peak to preset acceleration of 1.0 would be ideal. In an attempt to improve the acceleration restrictor just described, a signal proportional to pitching velocity was combined with the signal of normal and pitching acceleration. The signal controlling the acceleration restrictor was then

$$n + \left\{ \begin{array}{ll} \frac{K}{g}\ddot{\theta} & \text{for } \ddot{\theta} > 0 \\ 0 & \text{for } \ddot{\theta} < 0 \end{array} \right\} + \frac{A}{g}\dot{\theta}\frac{TD}{1 + TD}$$

Typical time histories of the response of the airplane to an elevator input are shown in figures 6 and 7 for several values of airspeed and lag in brake operation. At an airspeed of 1,000 feet per second and approximately zero time lag, the time required to reach maximum acceleration was not greatly changed by the addition of the pitching-velocity signal to the signal of normal and pitching acceleration. The increase in response time at lower airspeeds is somewhat greater, but whether this increase is significant remains to be determined.

The ratio of peak to preset acceleration as a function of airspeed for this system is presented in figure 8. The data shown are for a static margin of 10 percent of the mean aerodynamic chord, a pitching-acceleration gain constant of 154.7, a pitching-velocity gain constant of 644, and lags in brake operation of approximately 0, 0.02, and 0.05 second. The ratio of peak to preset acceleration for approximately zero lag was constant at about 1.1 throughout the speed range. The ratio for a lag of 0.02 second was 1.1 up to 600 feet per second, increased to about 1.25 at 800 feet per second, and decreased thereafter to 1.1 at 1,000 feet per second. Proper selection of the pitching-velocity gain constant, based on a time lag of 0.02 second, would probably eliminate this slight increase in the ratio of peak to preset acceleration for the case of 0.02-second lag in brake operation. Increasing the lag to 0.05 second produced a maximum ratio of peak to preset acceleration of 1.6 at a speed of 800 feet per second, and the ratio decreased to about 1.4 at 1,000 feet per second. It appears from these tests, then, that the most satisfactory acceleration restrictor employing a brake to stop the movement of the elevator would be one that is controlled by the combined signal of normal acceleration, pitching acceleration, and pitching velocity. With values of lag in brake operation up to 0.02 second, the ratio of peak to preset acceleration of 1.1 would allow the airplane to reach normal accelerations near the limit load factor and still prevent the airplane from ever exceeding this value. The results for a lag of 0.05 second, however, would unduly limit the maximum acceleration of the airplane. It is doubtful that any acceleration restrictor using a brake to stop the elevator control would have satisfactory performance with a time lag as high as 0.05 second because the control-stick movements would be too coarse.

CONCLUDING REMARKS

An acceleration restrictor that limits the normal acceleration of an airplane by means of a mechanical brake on the elevator control system has been analyzed. Two different signals have been used to control the acceleration restrictor. In one case the device was assumed to be controlled by the combined signal of normal acceleration and a signal proportional to pitching acceleration. In the other case the control signal was a summation of the normal acceleration, a signal proportional to pitching acceleration which was limited to positive values, and a signal proportional to pitching velocity which had been passed through an electronic canceling device to reduce the steady-state pitching-velocity signal to zero. The data were obtained from an electronic analog computer which simulated the dynamic characteristics of the airplane and acceleration restrictor. The mechanical design of the acceleration restrictor has not been considered, but the device used to simulate the action of a brake on the elevator control was assumed to have several values of lag in order to approximate more closely a practical system.

The results for an acceleration restrictor controlled by the combined signal of normal acceleration and a signal proportional to pitching acceleration indicated fair performance for cases of small lag in brake operation. The ratio of peak to preset acceleration for near-zero lag in brake operation was about 1.4 and remained constant with increasing airspeed. Changing the static margin from 10 to 20 percent of the mean aerodynamic chord had little effect on the operation of the system. Increasing the lag to about 0.02 second caused the ratio of peak to preset acceleration to vary from 1.2 to 1.6 as the airspeed increased from 200 to 1,000 feet per second. The performance of the system with a lag in brake operation of 0.05 second was unsatisfactory because of the high ratios of peak to preset acceleration at the higher airspeeds (2.2 at 1,000 feet per second) and the coarseness of control resulting from this lag.

When the signal controlling the acceleration restrictor was a function of normal and pitching acceleration and pitching velocity, for the case of zero lag in brake operation, a ratio of peak to preset acceleration of 1.1 was obtained. The ratio was constant for airspeeds from 200 to 1,000 feet per second and was only slightly affected by a lag in brake operation of 0.02 second. This system would allow the airplane to reach normal accelerations near the limit load factor and still prevent the airplane from ever exceeding this value. Increasing the lag in brake operation to 0.05 second caused the ratio of peak to preset acceleration to vary from 1.1 at 400 feet per second to 1.6 at 800 feet per second. This variation of the ratio, coupled with the coarseness of control, resulted in unsatisfactory characteristics of the acceleration restrictor.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 28, 1954.

APPENDIX

METHOD OF EXTENDING DATA TO DYNAMICALLY SIMILAR AIRPLANES

The data in this report can be extended to include a number of different conditions for dynamically similar airplanes. The following derivation includes the steps necessary for converting the data.

When the equations of motion were nondimensionalized, the following substitutions were made to convert the equations to nondimensional time:

$$s = t\frac{V}{\bar{c}} \qquad D = \frac{d}{ds} = \frac{\bar{c}}{V}\frac{d}{dt}$$

From the equations of motion, the expression for normal acceleration in nondimensional form is

$$D(\alpha - \theta) = nF$$

where the constant F (Froude's number) is

$$F = \frac{g\bar{c}}{V^2}$$

Also, the nondimensional pitching velocity and pitching acceleration are, by definition,

$$D\theta = \frac{\bar{c}}{V}\dot{\theta} \qquad D^2\theta = \frac{\bar{c}^2}{V^2}\ddot{\theta}$$

and the nondimensional rate of elevator deflection is

$$D\delta_e = \frac{\bar{c}}{V}\dot{\delta}_e$$

Equating the nondimensional and dimensional expressions for the combined values of normal acceleration, pitching acceleration, and pitching velocity gives

$$D(\alpha - \theta) + K'D^2\theta + A'D\theta = \frac{F}{g}(ng + K\ddot{\theta} + A\dot{\theta})$$

where K', K, A', and A are the gain constants used to amplify the pitching acceleration and pitching velocity. The prime is used to indicate that the constant is nondimensional. The limitation of the pitching acceleration to positive values and the cancellation of the pitching velocity, which restrict the preceding equation, are neglected in the derivation to follow. The only discrepancy that this simplification would cause would be in the transfer function describing the canceling network, $\frac{TD}{1 + TD}$. The time constant T in this expression would change in the same manner as the expression for time lag. Substituting the dimensional relationships for $D(\alpha - \theta)$, $D^2\theta$, and $D\theta$ gives

$$K'D^2\theta = \frac{F}{g} K\ddot{\theta} \qquad A'D\theta = \frac{F}{g} A\dot{\theta}$$

or

$$K' \frac{\bar{c}^2}{V^2} \ddot{\theta} = K \frac{\bar{c}}{V^2} \ddot{\theta} \qquad A' \frac{\bar{c}}{V} \dot{\theta} = A \frac{\bar{c}}{V^2} \dot{\theta}$$

Then,

$$K' = \frac{K}{\bar{c}} \qquad A' = \frac{A}{V}$$

Also, by definition, let

$$\tau' = \frac{V}{\bar{c}} \tau \qquad n_p' = n_p F$$

where τ' and τ are the nondimensional and dimensional lags and n_p' and n_p are the nondimensional and dimensional preset accelerations.

The first case to be considered is one in which either V or $\bar{c}$ or both are changed while the final value of δ_e is kept constant. This procedure, in effect, results in stretching or shrinking the time scale of the time history, with a resulting change in the ordinates, as shown by subsequent equations. The subscript 1 is used to denote the data in its original form and the subscript 2 denotes the new values to be obtained. The pertinent values would be changed by the following relationships: The time scale would become

$$t_2 = \frac{V_1\bar{c}_2}{V_2\bar{c}_1} t_1$$

the elevator rate,

$$\dot{\delta}_{e_2} = \frac{V_2\bar{c}_1}{V_1\bar{c}_2}\dot{\delta}_{e_1}$$

the normal acceleration,

$$n_2 = \frac{\bar{c}_1{V_2}^2}{\bar{c}_2{V_1}^2}n_1$$

the time lag,

$$\tau_2 = \frac{V_1\bar{c}_2}{V_2\bar{c}_1}\tau_1$$

the preset acceleration,

$$n_{p2} = \frac{\bar{c}_1{V_2}^2}{\bar{c}_2{V_1}^2}n_{p1}$$

and the combined signal of normal acceleration, pitching acceleration, and pitching velocity would become

$$\left(n + \frac{K_2}{g}\ddot{\theta} + \frac{A_2}{g}\dot{\theta}\right)_2 = \frac{\bar{c}_1{V_2}^2}{\bar{c}_2{V_1}^2}\left(n + \frac{K_1}{g}\ddot{\theta} + \frac{A_1}{g}\dot{\theta}\right)_1$$

where

$$K_2 = \frac{\bar{c}_2}{\bar{c}_1}K_1 \qquad A_2 = \frac{V_2}{V_1}A_1$$

For the case where the combined signal of normal and pitching acceleration $n + \frac{K}{g}\ddot{\theta}$ is used, the constant A can be made equal to zero in order to convert the data.

Changing the values of V or $\bar{c}$ changes the preset acceleration as shown by the previous equations. The next case to be considered is one in which the preset acceleration is kept constant or changed to some given value at the same time that the values of V or $\bar{c}$ are changed. The values of V or $\bar{c}$, although changed from the original data, are considered to be the same as in the previous example, and the subscript 2 denotes the new values. Let

$$n_{p_2} = hn_{p_1}$$

where h is equal to 1 in order to keep the preset acceleration constant while changing V or $\bar{c}$. The elevator angle then becomes

$$\delta_{e_2} = h \frac{\bar{c}_2 {V_1}^2}{\bar{c}_1 {V_2}^2} \delta_{e_1}$$

and the elevator rate is changed to

$$\dot{\delta}_{e_2} = h \frac{V_1}{V_2} \dot{\delta}_{e_1}$$

The ratio of the new normal acceleration to the original normal acceleration is

$$\frac{n_2}{n_1} = h$$

The time lag changes by the same factor as when V or $\bar{c}$ is changed, that is,

$$\tau_2 = \frac{V_1 \bar{c}_2}{V_2 \bar{c}_1} \tau_1$$

and the combined signal of normal acceleration, pitching acceleration, and pitching velocity is

$$\left(n + \frac{K_2}{g} \ddot{\theta} + \frac{A_2}{g} \dot{\theta}\right)_2 = h\left(n + \frac{K_1}{g} \ddot{\theta} + \frac{A_1}{g} \dot{\theta}\right)_1$$

where

$$K_2 = \frac{\bar{c}_2}{\bar{c}_1} K_1 \qquad A_2 = \frac{V_2}{V_1} A_1$$

The last case to be considered is one in which the elevator rate is kept constant while the values of n_p, V, and $\bar{c}$ are varied. Here again, for purposes of derivation, the same values of V and $\bar{c}$ are considered as in the previous cases. Since

$$\dot{\delta}_{e_2} = h \frac{V_1}{V_2} \dot{\delta}_{e_1}$$

in the previous example, if the elevator rate is kept constant (that is, $\dot{\delta}_{e_2} = \dot{\delta}_{e_1}$), then

$$h \frac{V_1}{V_2} = 1$$

and therefore

$$h = \frac{V_2}{V_1}$$

This value of h results in the following relationships (where the subscripts 1 and 2 again represent the original data and the new values to be used, respectively):

The new preset acceleration would be

$$n_{p_2} = \frac{V_2}{V_1} n_{p_1}$$

The elevator angle would be

$$\delta_{e_2} = \frac{\bar{c}_2 V_1}{\bar{c}_1 V_2} \delta_{e_1}$$

The normal acceleration would become

$$n_2 = \frac{V_2}{V_1} n_1$$

The new time lag would be

$$\tau_2 = \frac{V_1 \bar{c}_2}{V_2 \bar{c}_1} \tau_1$$

and the combined value of normal acceleration, pitching acceleration, and pitching velocity would be

$$\left(n + \frac{K_2}{g}\ddot{\theta} + \frac{A_2}{g}\dot{\theta}\right)_2 = \frac{V_2}{V_1}\left(n + \frac{K_1}{g}\ddot{\theta} + \frac{A_1}{g}\dot{\theta}\right)_1$$

where

$$K_2 = \frac{\bar{c}_2}{\bar{c}_1} K_1 \qquad A_2 = \frac{V_2}{V_1} A_1$$

REFERENCE

1. Phillips, William H.: Theoretical Analysis of Some Simple Types of Acceleration Restrictors. NACA TN 2574, 1951.

TABLE I

CHARACTERISTICS OF AIRPLANE USED IN CALCULATIONS

Weight, lb	15,000
Wing area, sq ft	300
Horizontal tail area, sq ft	60
Wing mean aerodynamic chord, ft	7
Tail length, ft	20
Radius of gyration about Y-axis, ft	7
μ (sea level)	93.3
K_Y	1.0
C_{Z_α}	-4.77
$C_{Z_{D\alpha}}$	-2.12
C_{Z_q}	-4.24
$C_{Z_{\delta_e}}$	-0.37
$C_{m_{D\alpha}}$	-6.04
C_{m_q}	-12.06
$C_{m_{\delta_e}}$	-1.05

TABLE II

CONDITIONS INVESTIGATED

Acceleration-restrictor control signal	Lag in brake operation, sec	Static margin, percent $\bar{c}$	K	A	Airspeed, ft/sec
$n + \frac{K}{g}\ddot{\theta}$	0	10 and 20	88.13 and 154.7	---	200 to 1,000
	0.02 and 0.05	10	154.7		
$n + \begin{Bmatrix} \frac{K}{g}\ddot{\theta} & \text{for} & \ddot{\theta} > 0 \\ 0 & \text{for} & \ddot{\theta} < 0 \end{Bmatrix} + \frac{A}{g}\ddot{\theta}\frac{TD}{1 + TD}$	0	10	154.7	644	400 to 1,000
	0.02				
	0.05				

TABLE III

VALUES OF LAG IN BRAKE OPERATION

Acceleration restrictor sensitive to

$$n + \frac{K}{g}\ddot{\theta}$$

Static margin, $0.10\bar{c}$; $K = 154.7$

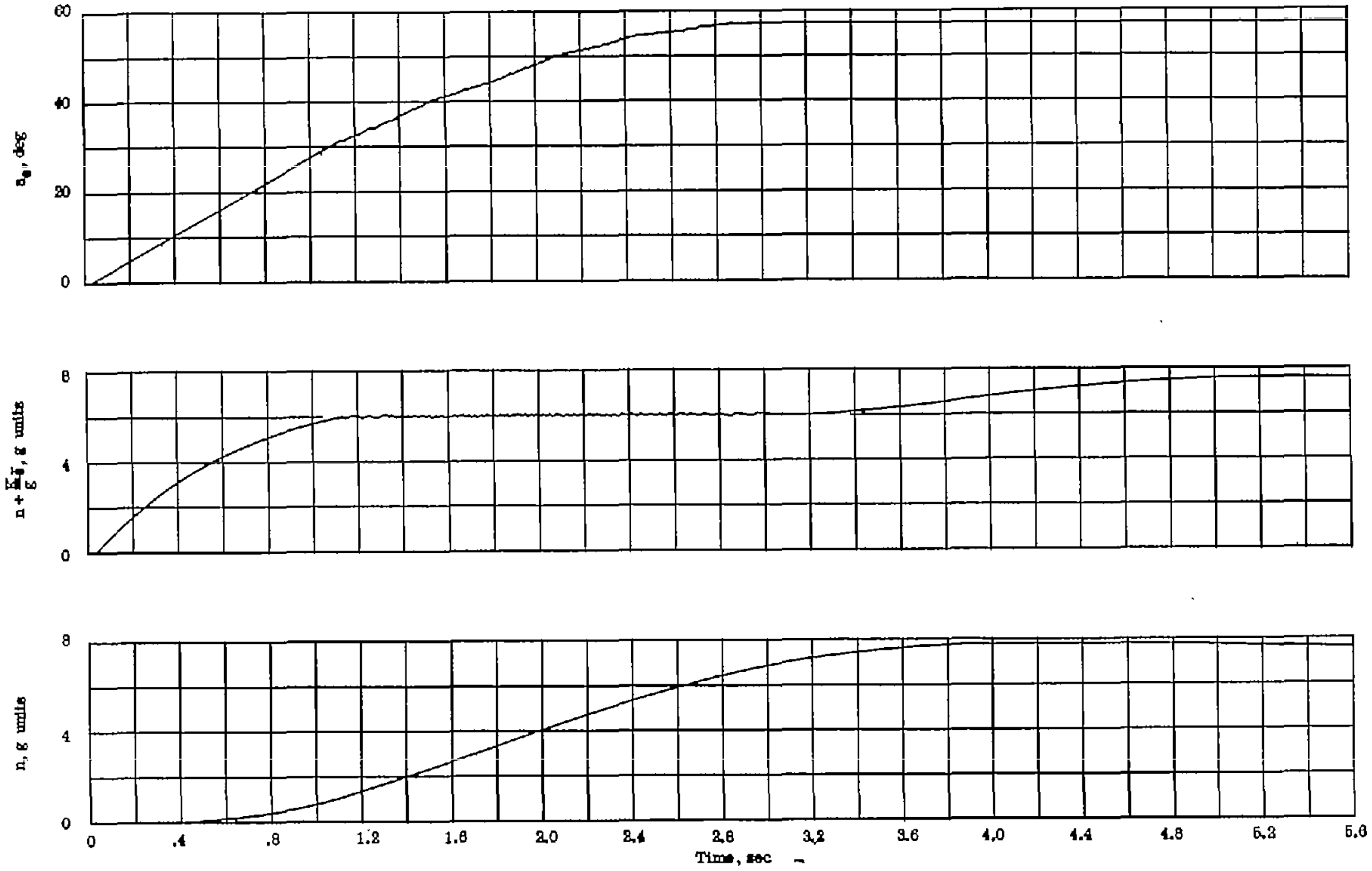
True airspeed, V, ft/sec	Time lag, τ, sec	Ratio of peak acceleration to preset acceleration
200	0.053	1.26
200	.053	1.26
400	.018	1.27
400	.018	1.33
400	.053	1.42
400	.053	1.42
600	.018	1.40
600	.018	1.42
600	.047	1.50
600	.047	1.56
800	.018	1.56
800	.018	1.50
800	.053	1.71
800	.053	1.60
1,000	.021	1.60
1,000	.021	1.52
1,000	.049	2.25
1,000	.049	2.13

Acceleration restrictor sensitive to

$$n + \left\{\begin{matrix}\frac{K}{g}\ddot{\theta} & \text{for} & \ddot{\theta} > 0 \\ 0 & \text{for} & \theta < 0\end{matrix}\right\} + \frac{A}{g}\dot{\theta}\,\frac{TD}{1 + TD}$$

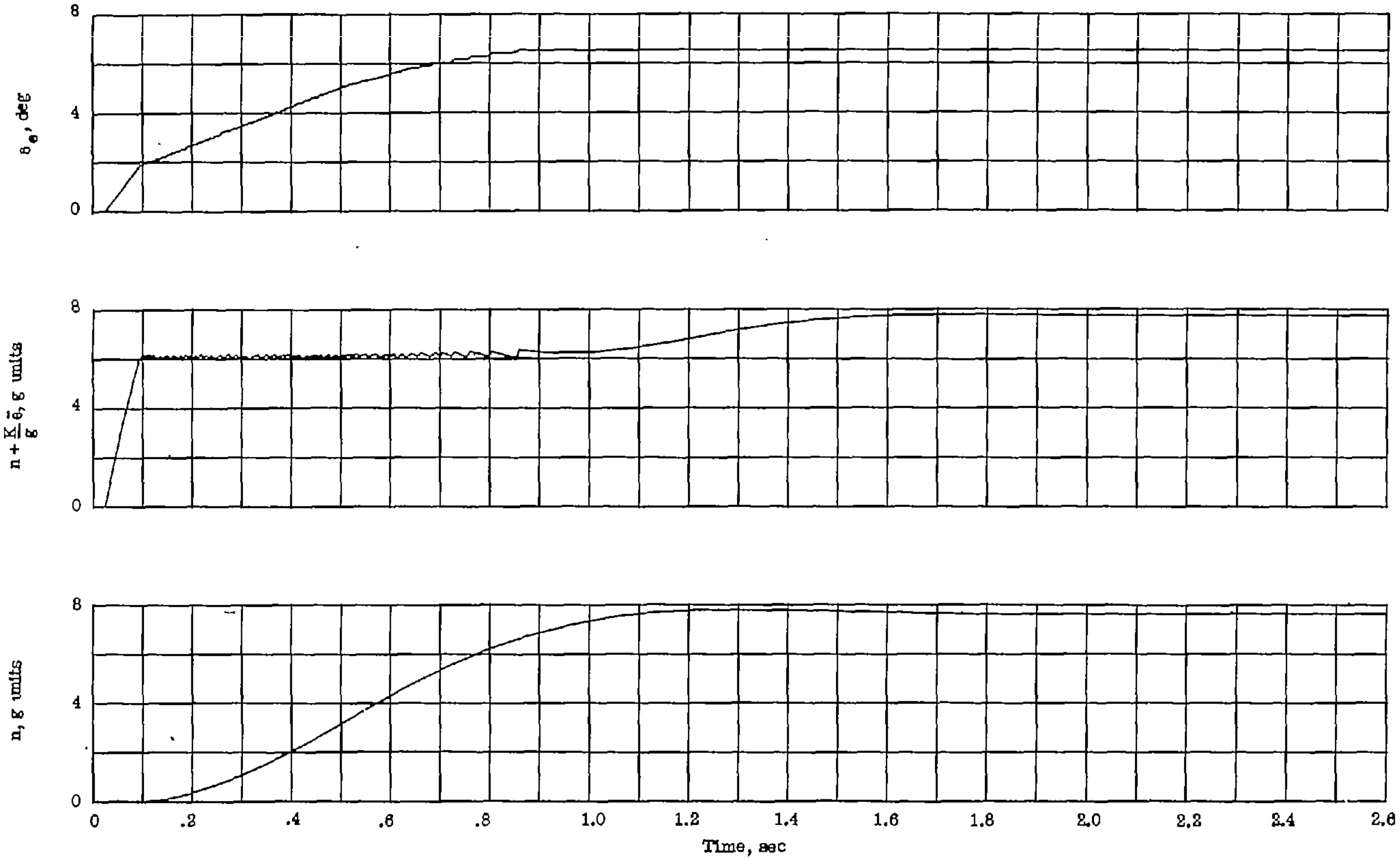
Static margin, $0.10\bar{c}$; $K = 154.7$; $A = 644$

True airspeed, V, ft/sec	Time lag, τ, sec	Ratio of peak acceleration to preset acceleration
400	0.02	1.08
400	.05	1.12
600	.02	1.05
600	.02	1.07
600	.05	1.32
800	.02	1.30
800	.02	1.25
800	.05	1.61
800	.05	1.61
1,000	.02	1.08
1,000	.02	1.13
1,000	.05	1.38



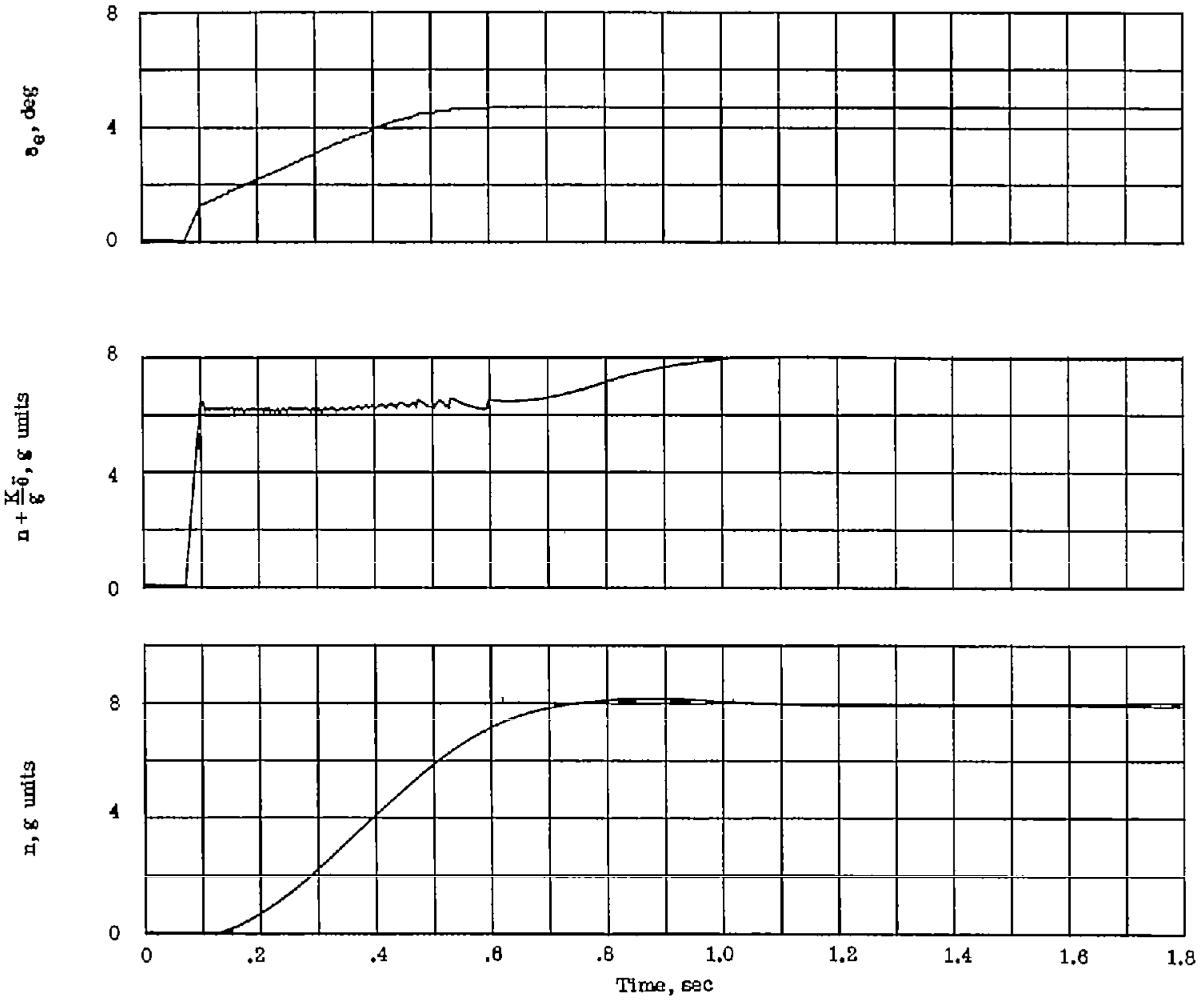
(a) V = 200 feet per second.

Figure 1.- Typical time history of action of acceleration restrictor when controlled by $n + \frac{K}{g}\ddot{\theta}$. K = 154.7; approximately zero lag in brake operation; static margin, $0.10\bar{c}$.



(b) V = 600 feet per second.

Figure 1.- Continued.



(c) V = 1,000 feet per second.

Figure 1.- Concluded.

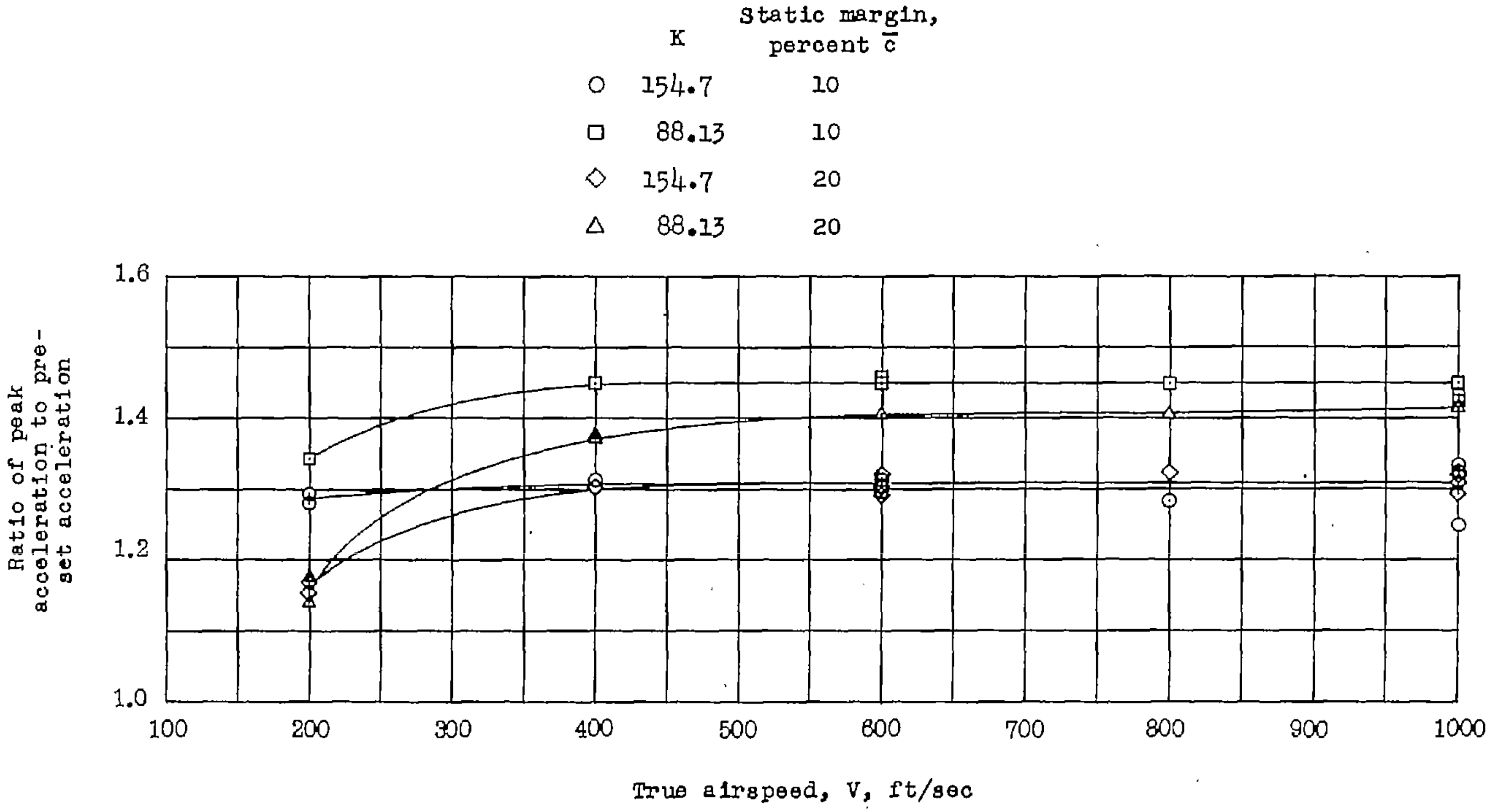
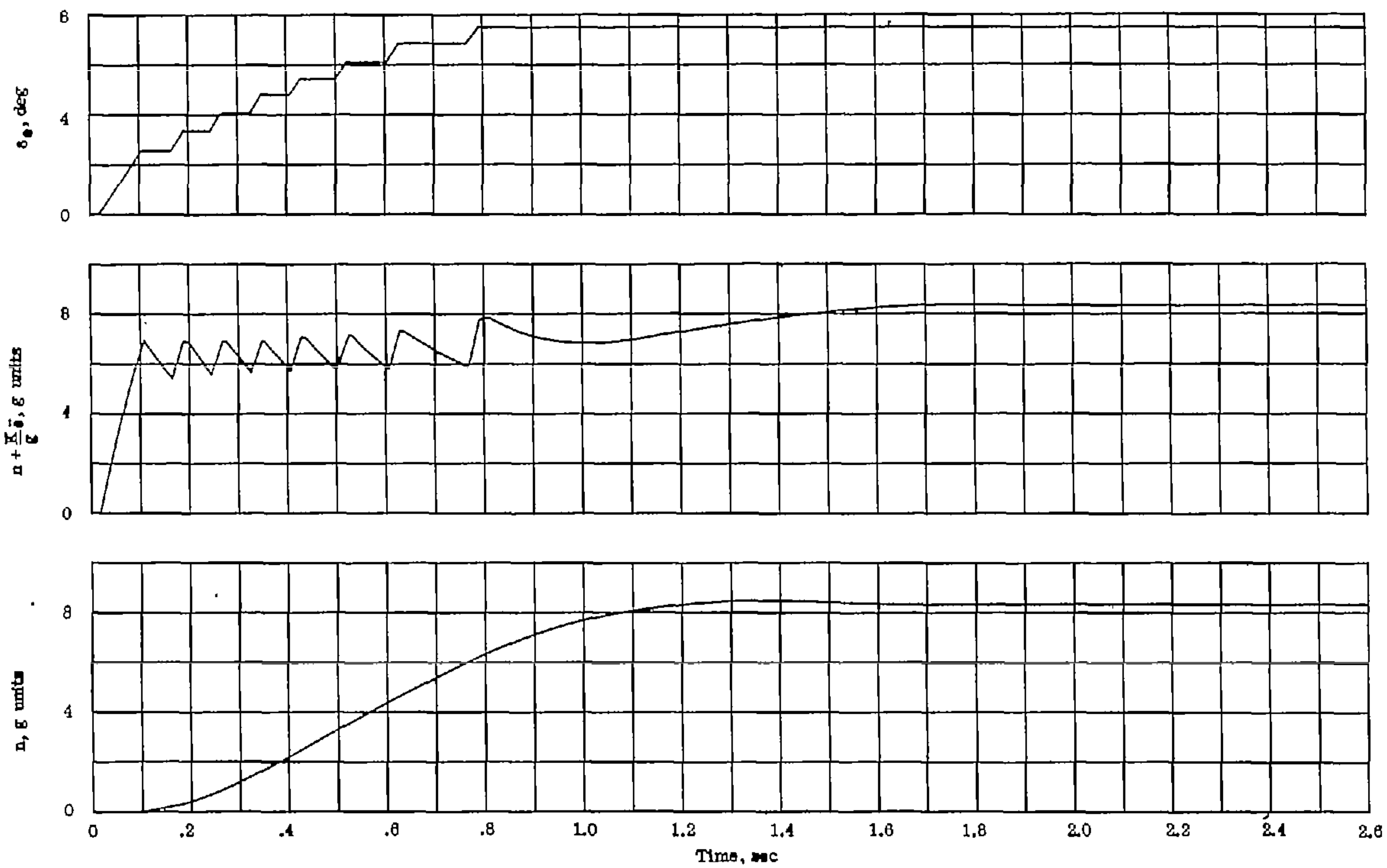
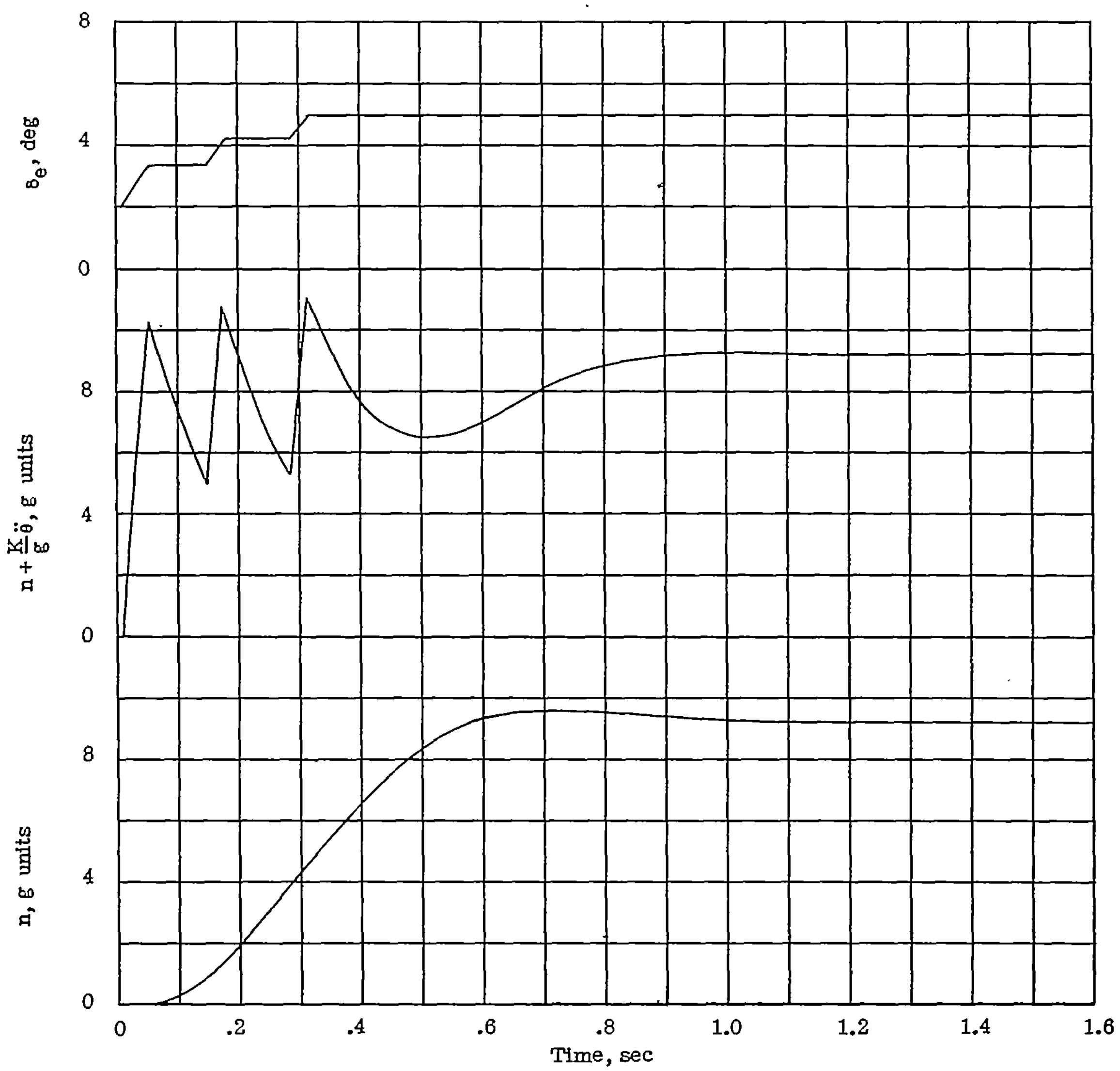


Figure 2.- Ratio of peak acceleration to preset acceleration as a function of true airspeed for two values of K and two values of static margin for approximately zero lag. Acceleration restrictor controlled by $n + \frac{K}{g}\ddot{\theta}$.



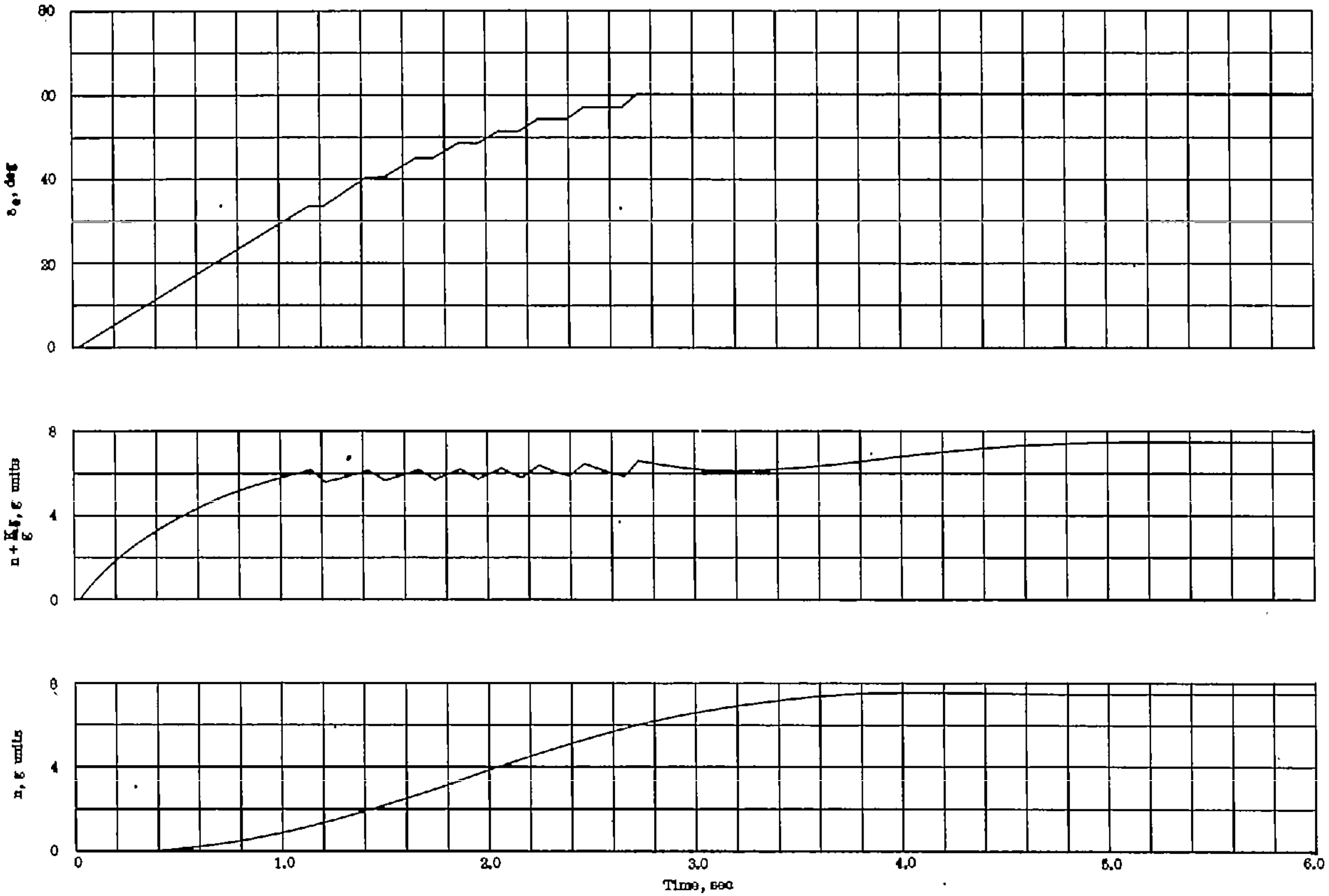
(a) V = 600 feet per second; τ = 0.0175 second.

Figure 3.- Typical time history of action of acceleration restrictor when controlled by $n + \frac{K}{g}\ddot{\theta}$ for two values of lag and several airspeeds. K = 154.7; static margin, $0.10\bar{c}$.



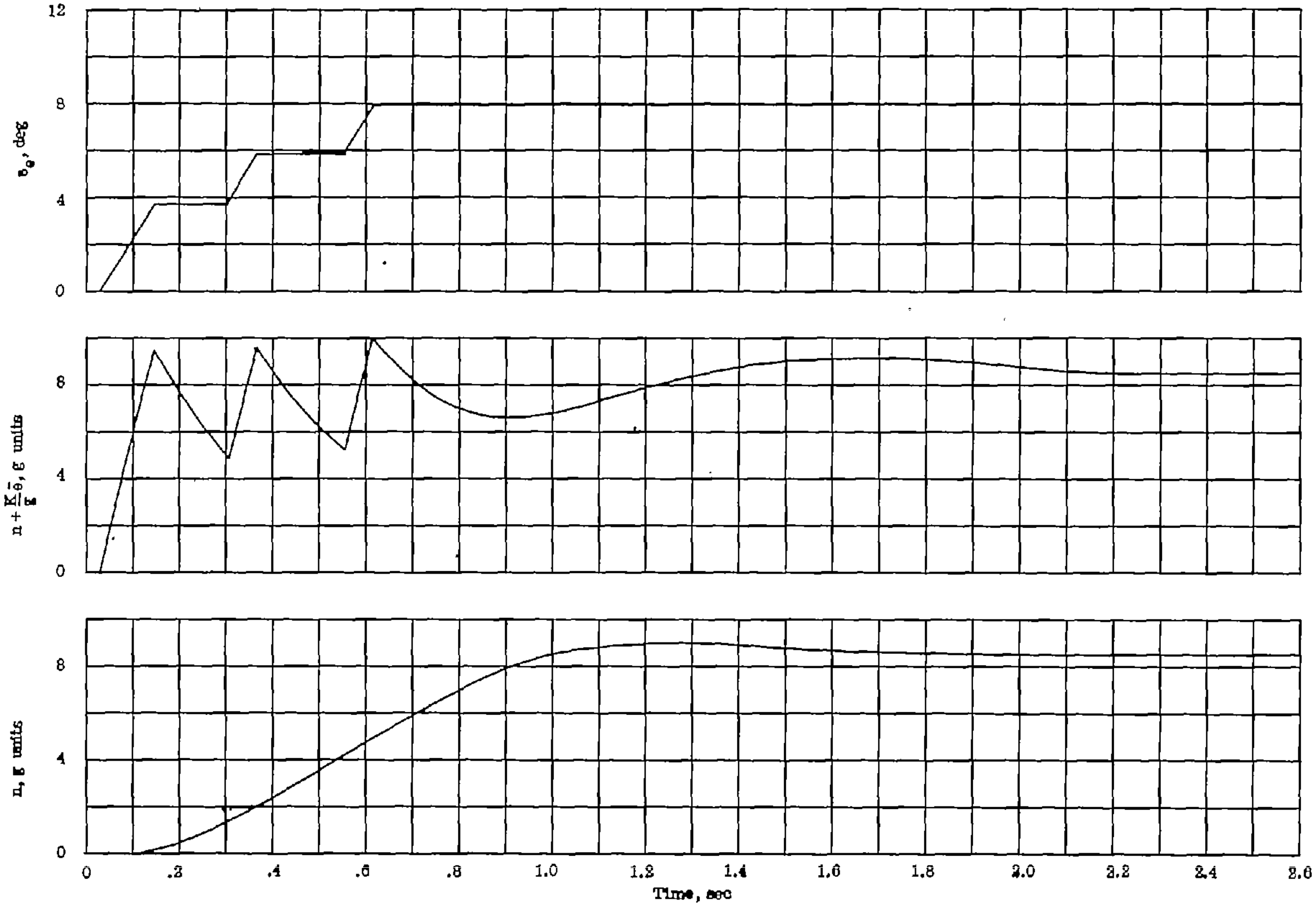
(b) V = 1,000 feet per second; τ = 0.021 second.

Figure 3.- Continued.



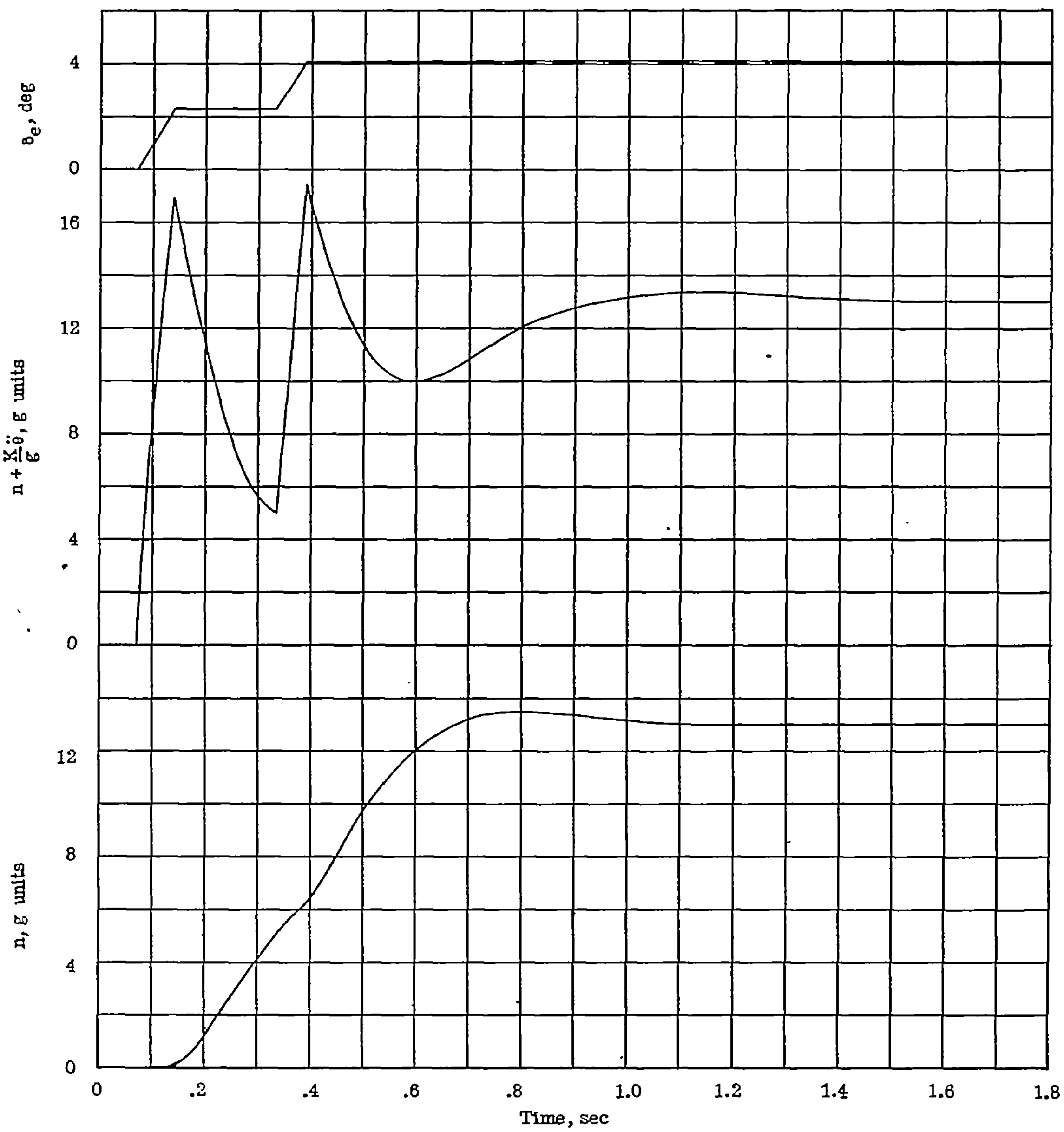
(c) V = 200 feet per second; τ = 0.053 second.

Figure 3.- Continued.



(d) V = 600 feet per second; τ = 0.047 second.

Figure 3.- Continued.



(e) $V = 1{,}000$ feet per second; $\tau = 0.049$ second.

Figure 3.- Concluded.

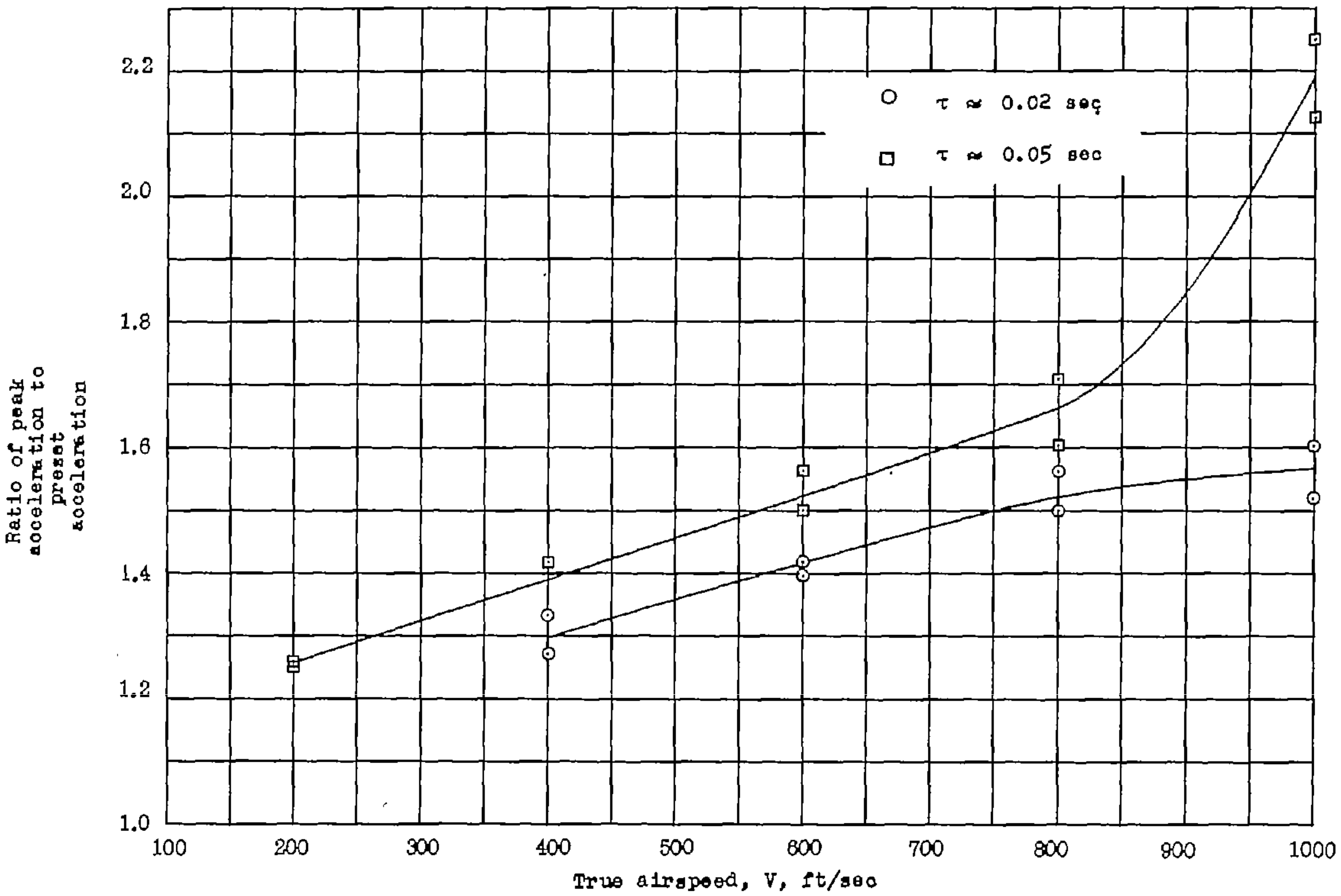


Figure 4.- Ratio of peak acceleration to preset acceleration as a function of true airspeed for two values of lag in brake operation. Acceleration restrictor controlled by $n + \frac{K}{g}\ddot{\theta}$. K = 154.7; static margin, 0.10$\bar{c}$. (See table III for exact values of lag.)

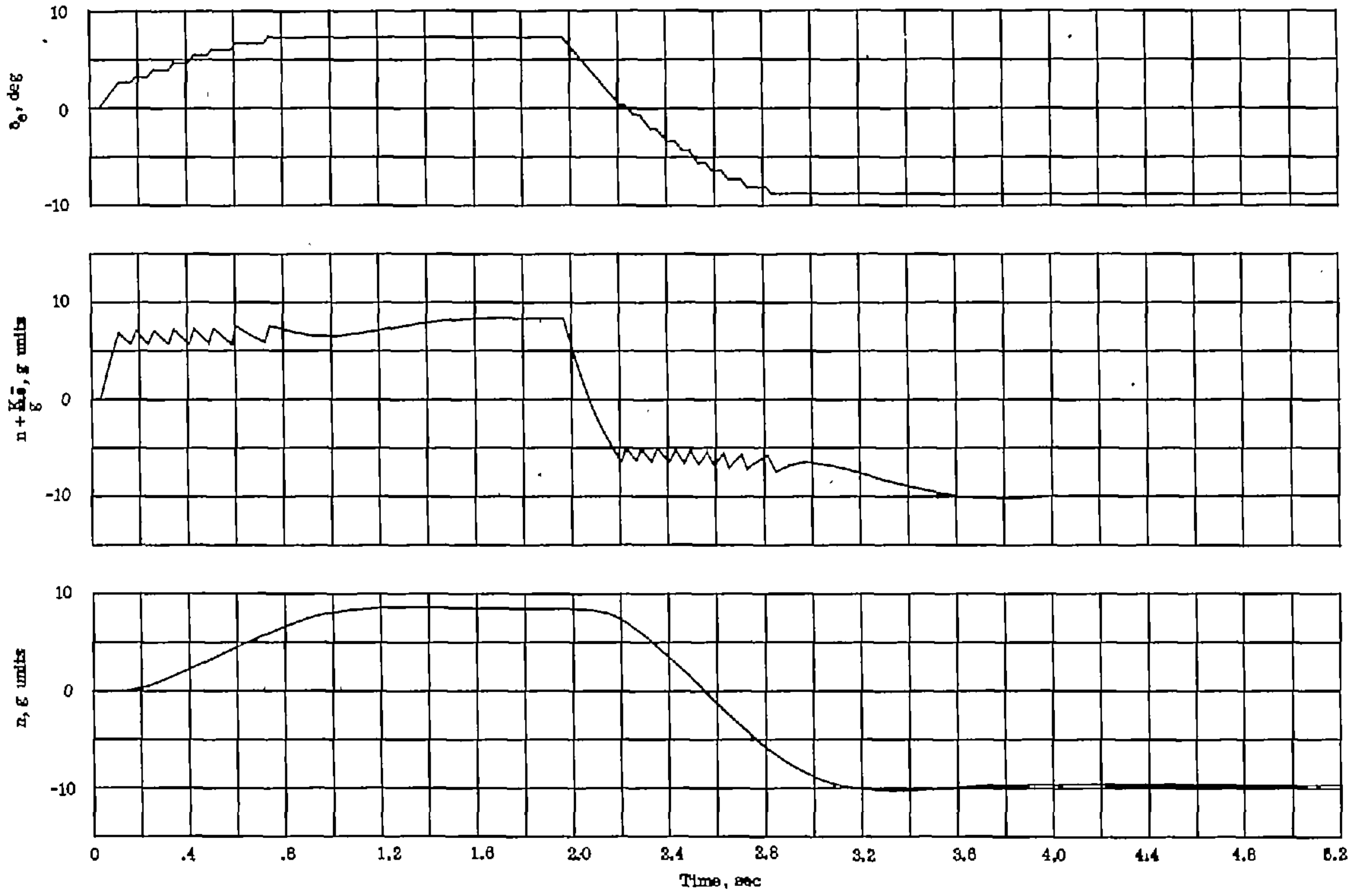
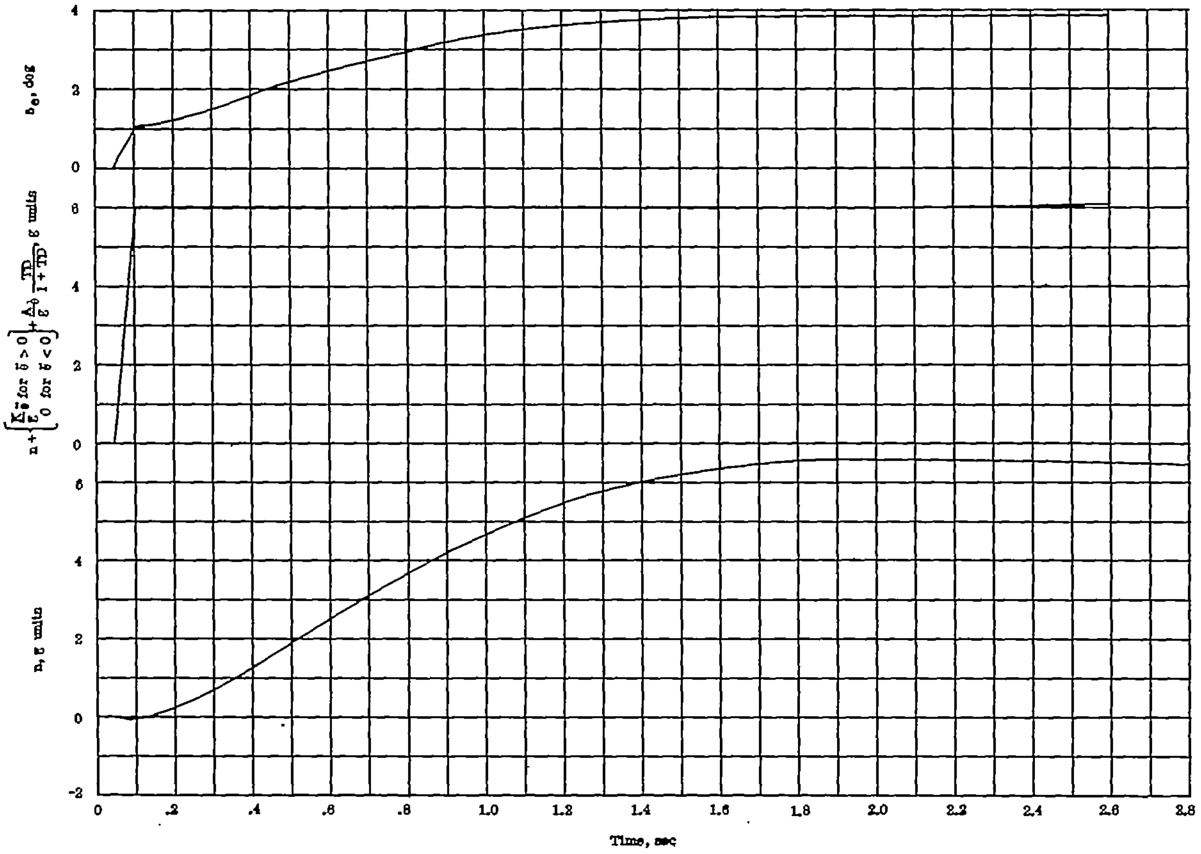
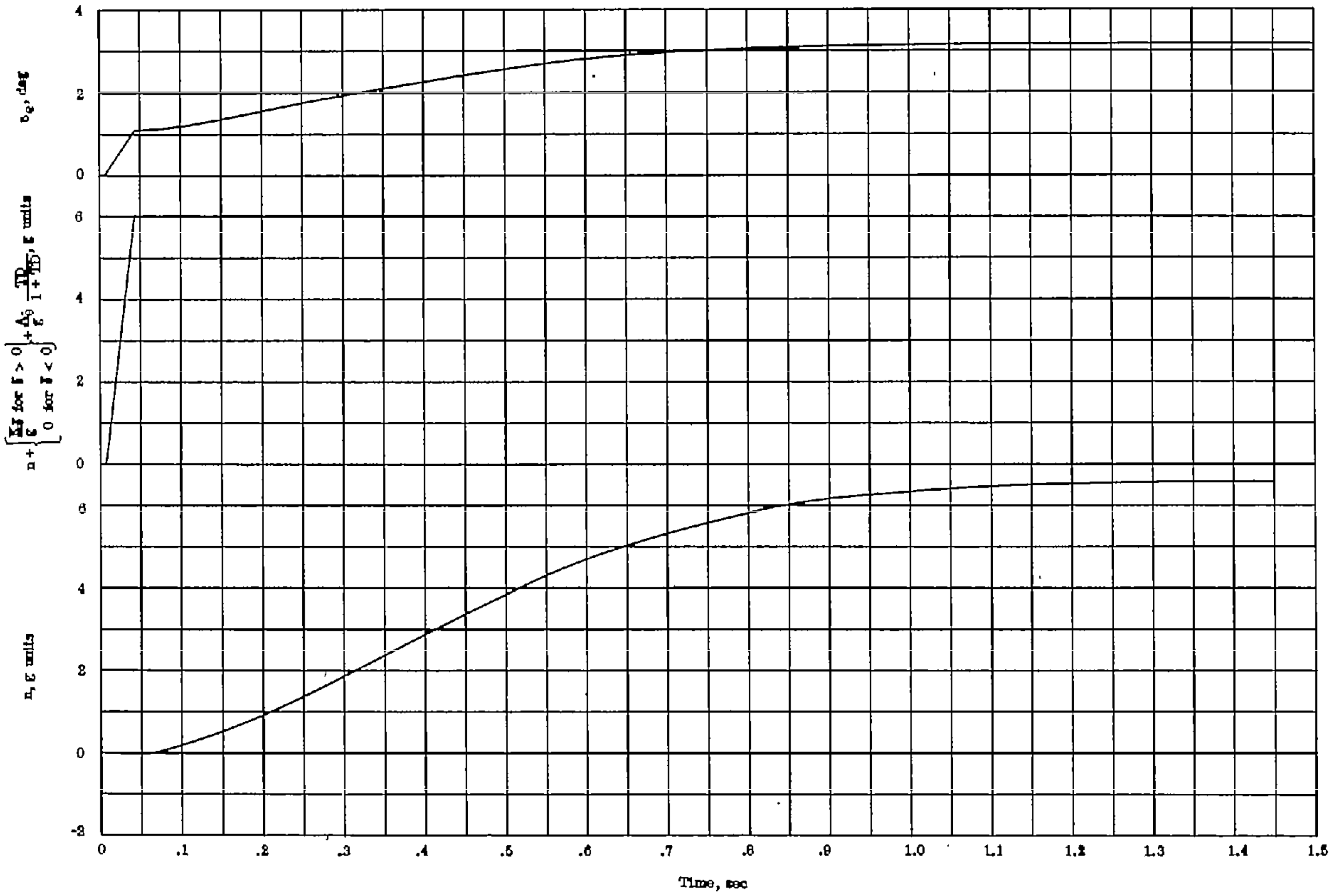


Figure 5.- Typical time history of action of acceleration restrictor in which airplane performed a pull-up maneuver followed by a push-down maneuver. Acceleration restrictor controlled by $n + \frac{K}{g}\ddot{\theta}$. $K = 154.7$; $V = 600$ feet per second; $\tau = 0.018$ second; static margin, $0.10\bar{c}$.



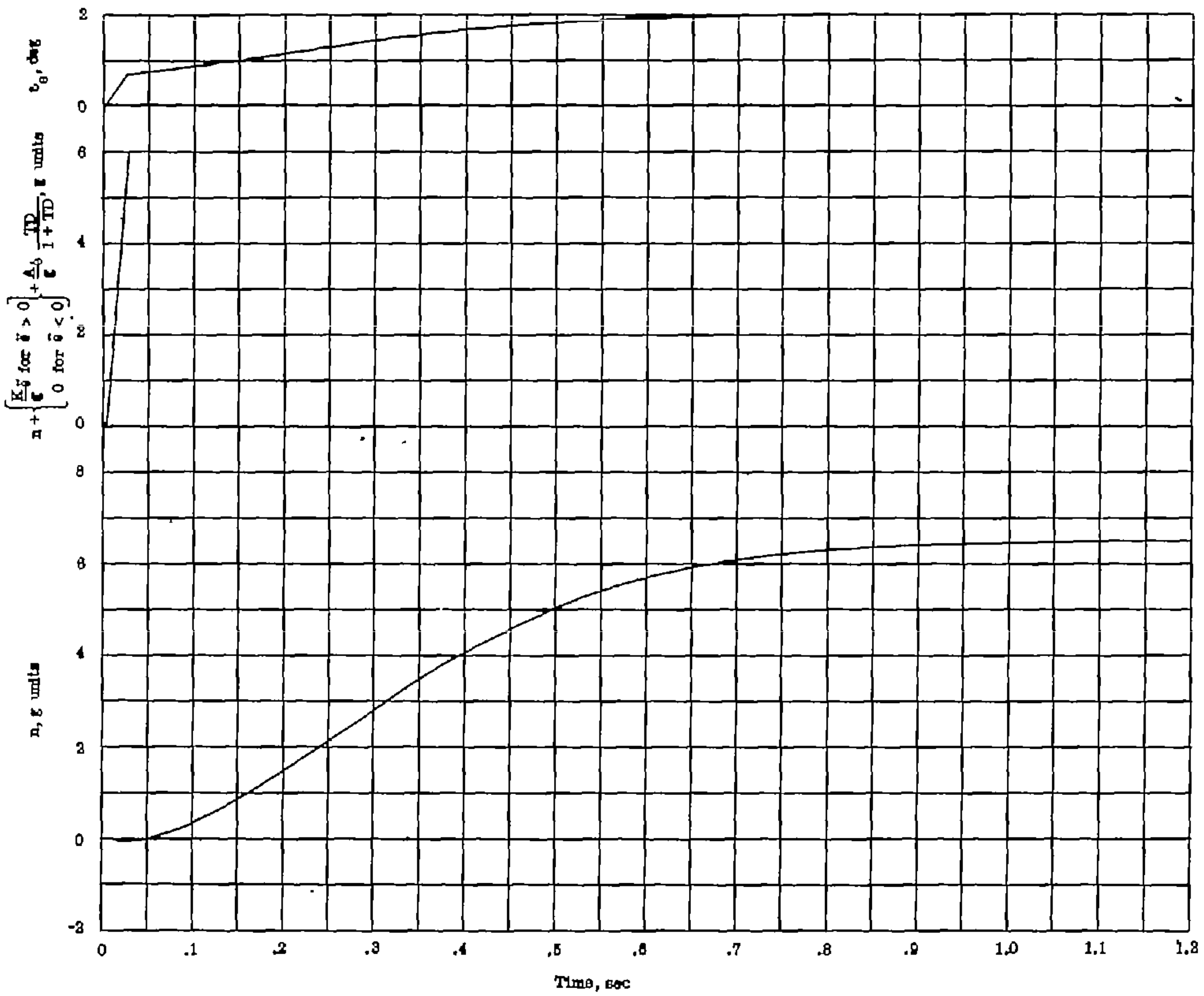
(a) V = 600 feet per second.

Figure 6.- Typical time history of action of acceleration restrictor when controlled by $n + \left\{\begin{matrix}\frac{K}{g}\ddot{\theta} & \text{for} & \ddot{\theta} > 0 \\ 0 & \text{for} & \ddot{\theta} < 0\end{matrix}\right\} + \frac{A}{g}\dot{\theta}\frac{TD}{1 + TD}$. K = 154.7; A = 644; approximately zero lag in brake operation; static margin, $0.10\bar{c}$.



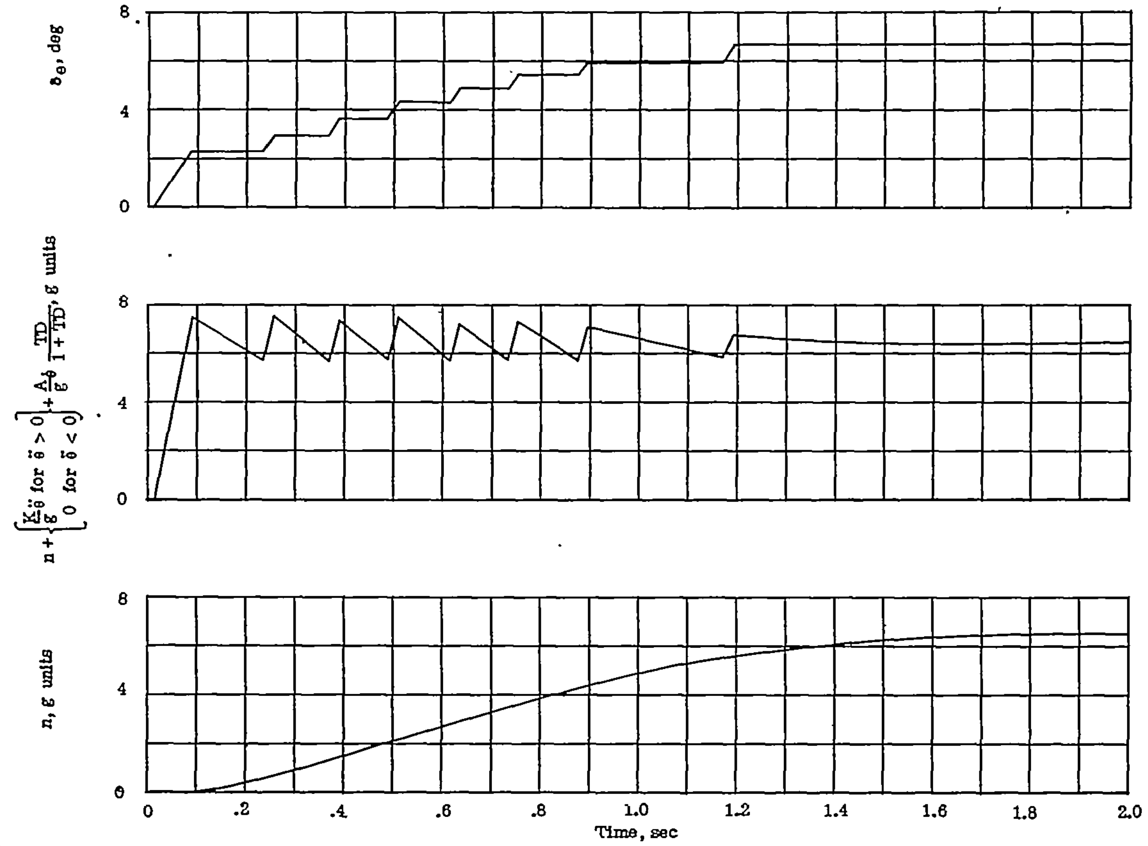
(b) V = 800 feet per second.

Figure 6.- Continued.



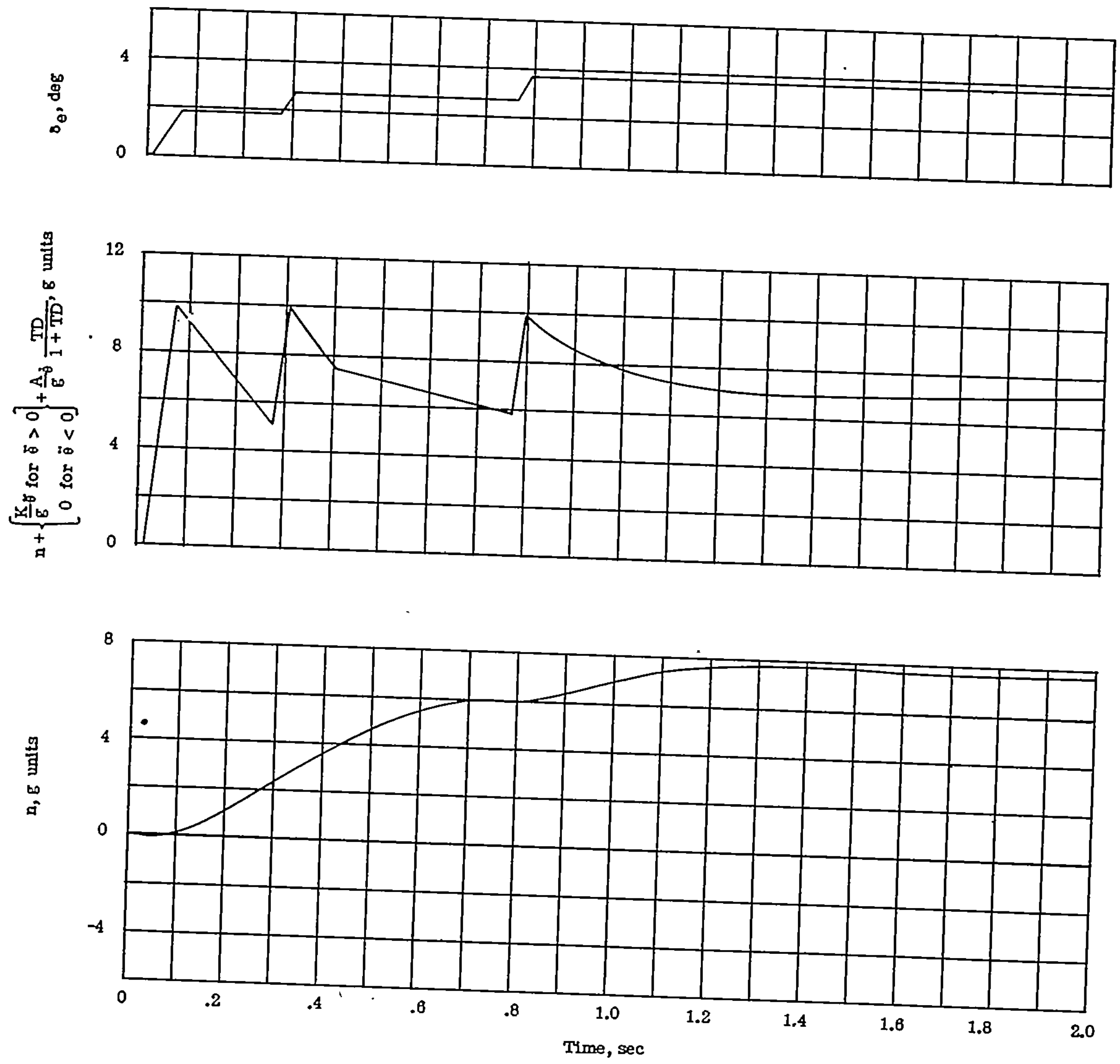
(c) V = 1,000 feet per second.

Figure 6.- Concluded.



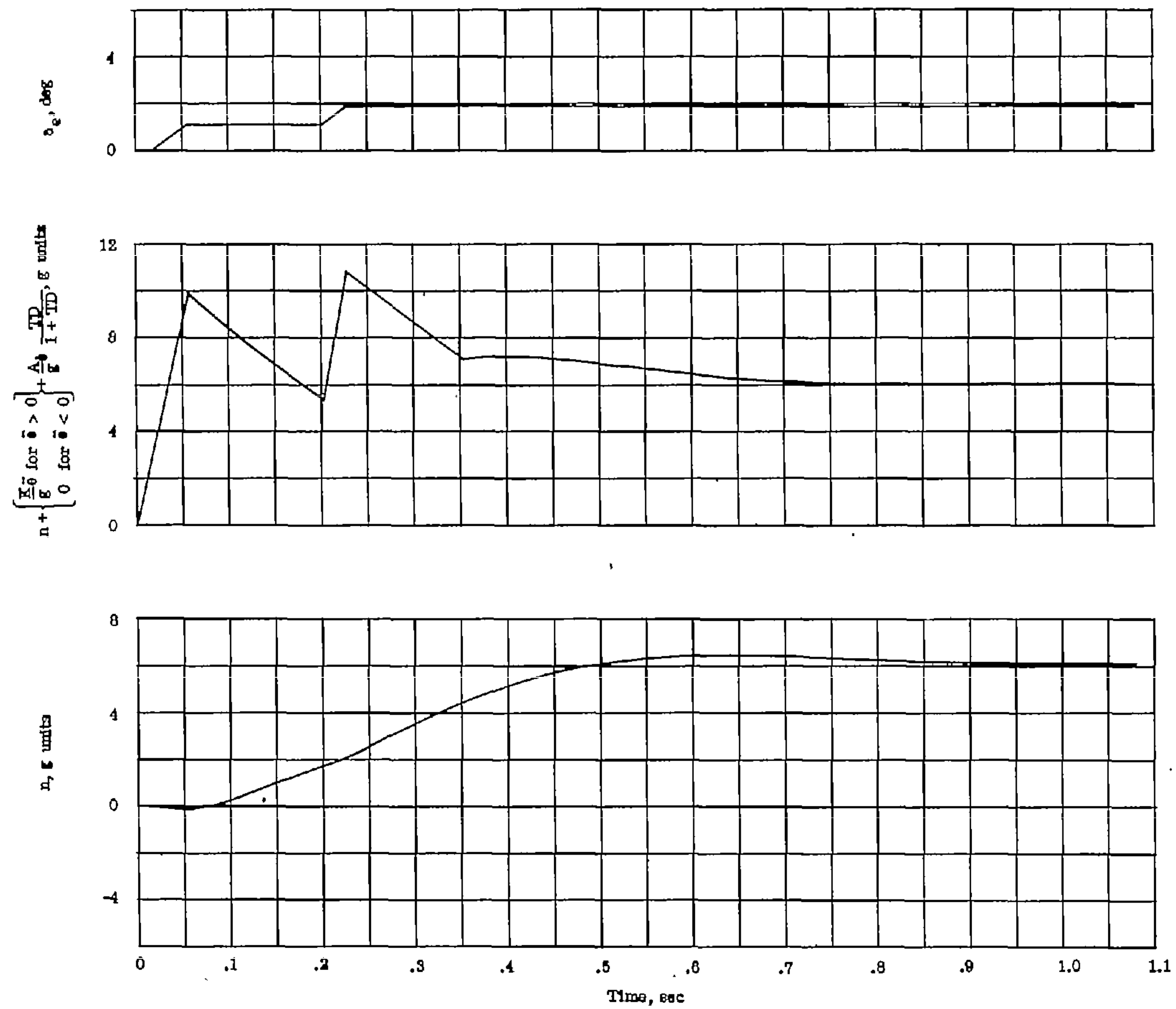
(a) V = 600 feet per second; $\tau = 0.02$ second.

Figure 7.- Typical time history of action of acceleration restrictor when controlled by $n + \begin{Bmatrix} \frac{K}{g}\ddot{\theta} & \text{for} & \ddot{\theta} > 0 \\ 0 & \text{for} & \ddot{\theta} < 0 \end{Bmatrix} + \frac{A}{g}\dot{\theta}\,\frac{\tau D}{1 + \tau D}$ for two values of lag and three airspeeds. $K = 154.7$; $A = 644$; static margin, $0.10\bar{c}$.



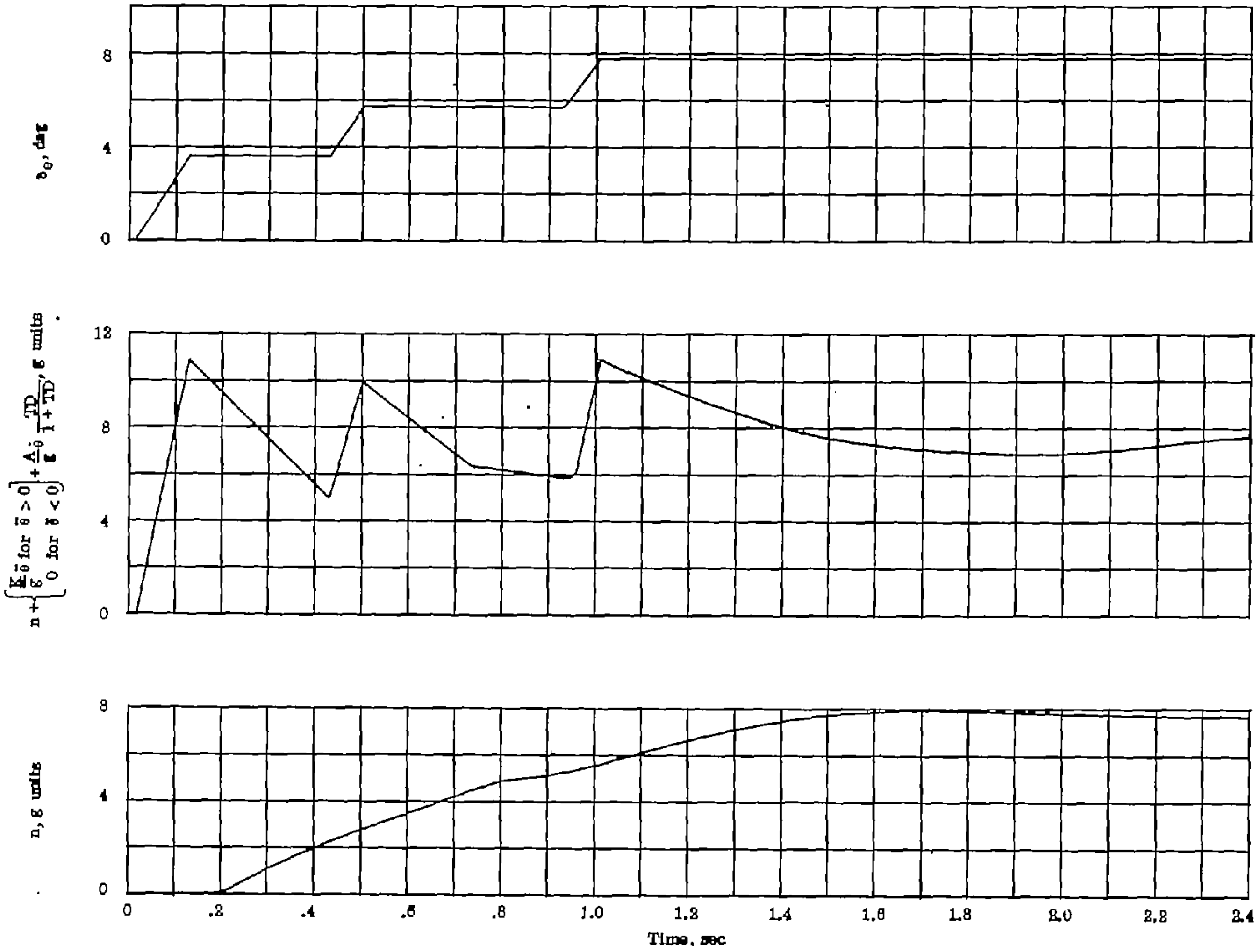
(b) $V = 800$ feet per second; $\tau = 0.02$ second.

Figure 7.- Continued.



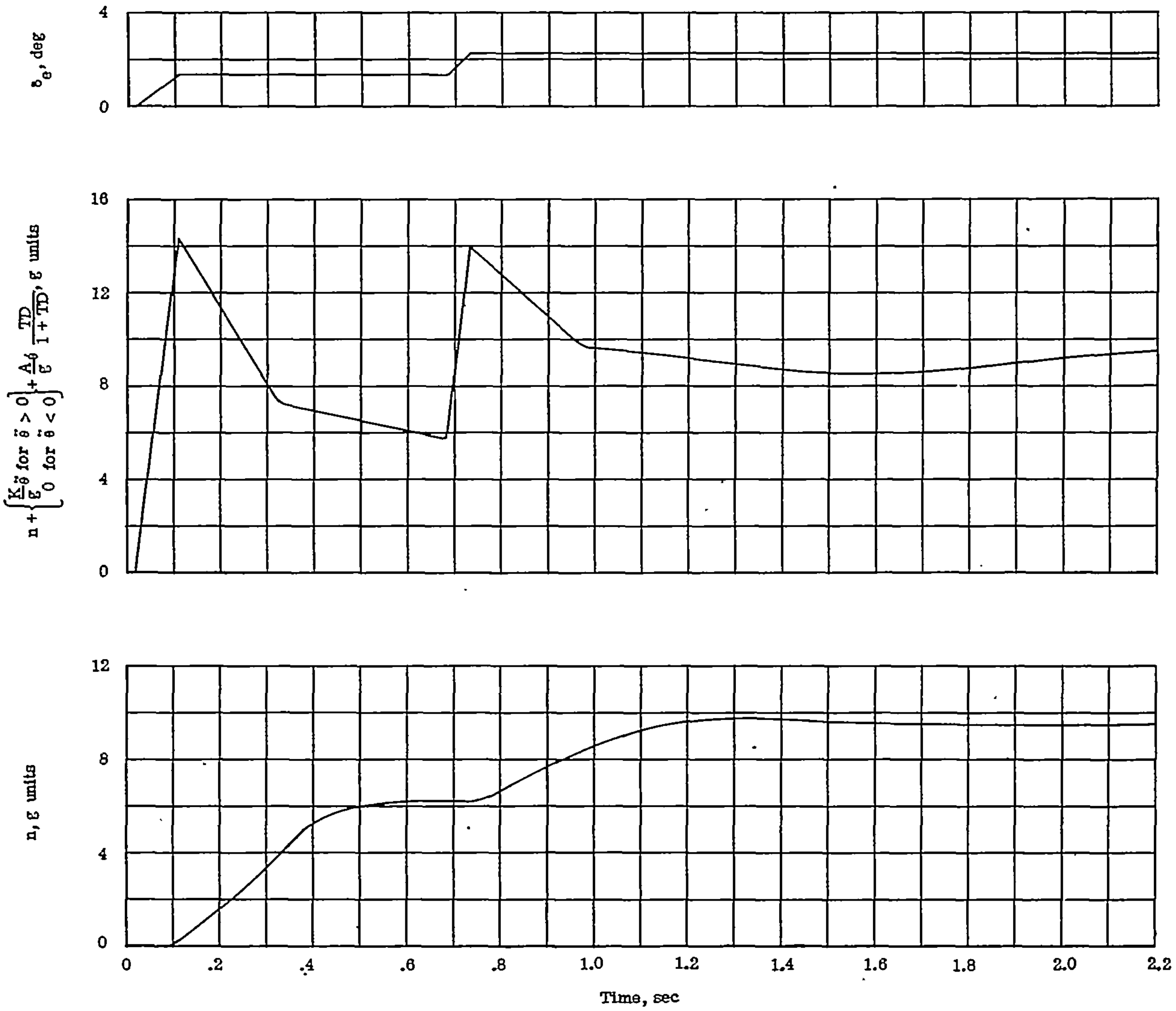
(c) V = 1,000 feet per second; τ = 0.02 second.

Figure 7.- Continued.



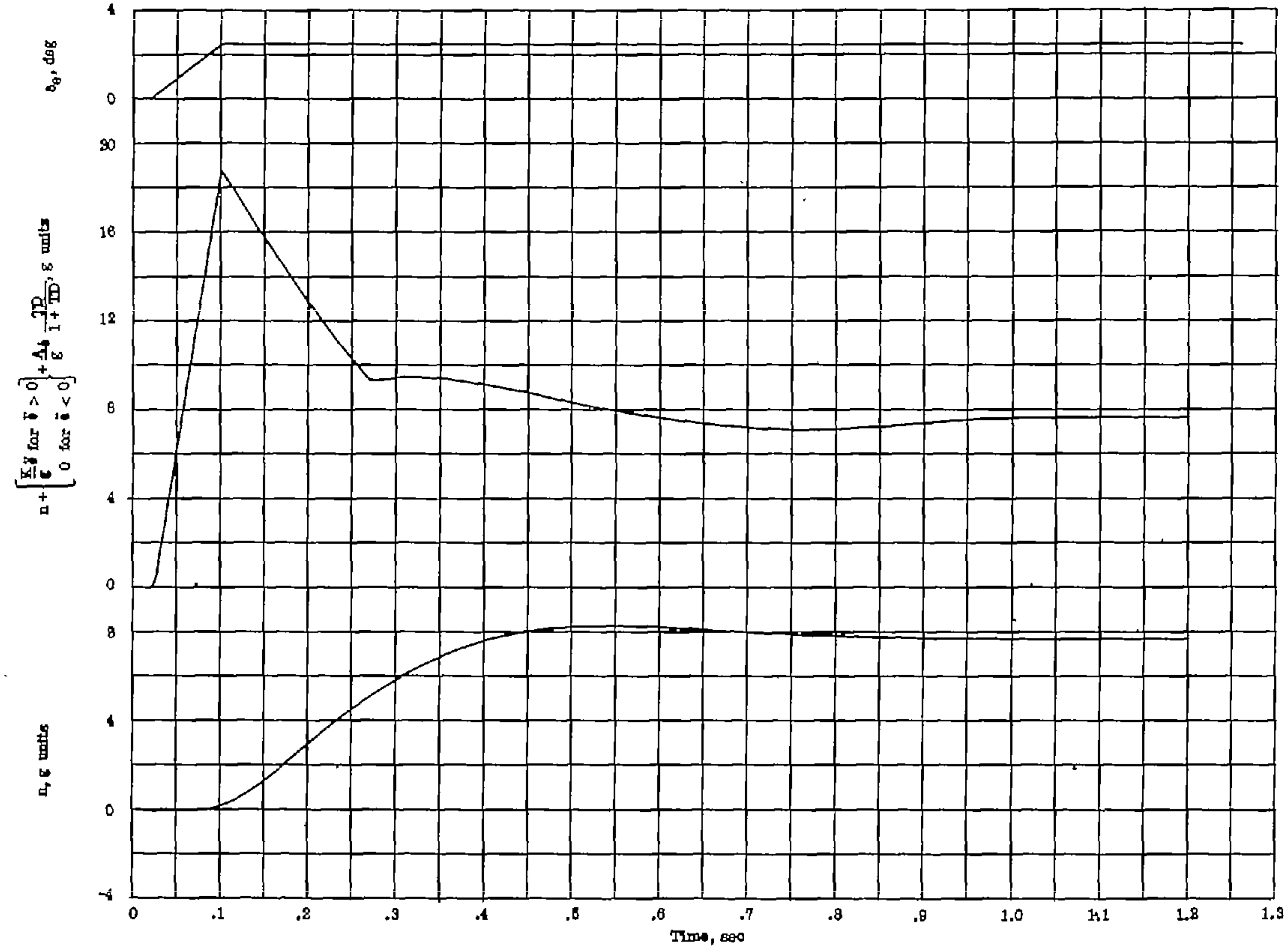
(d) V = 600 feet per second; τ = 0.05 second.

Figure 7.- Continued.



(e) V = 800 feet per second; τ = 0.05 second.

Figure 7.- Continued.



(f) V = 1,000 feet per second; τ = 0.05 second.

Figure 7.- Concluded.

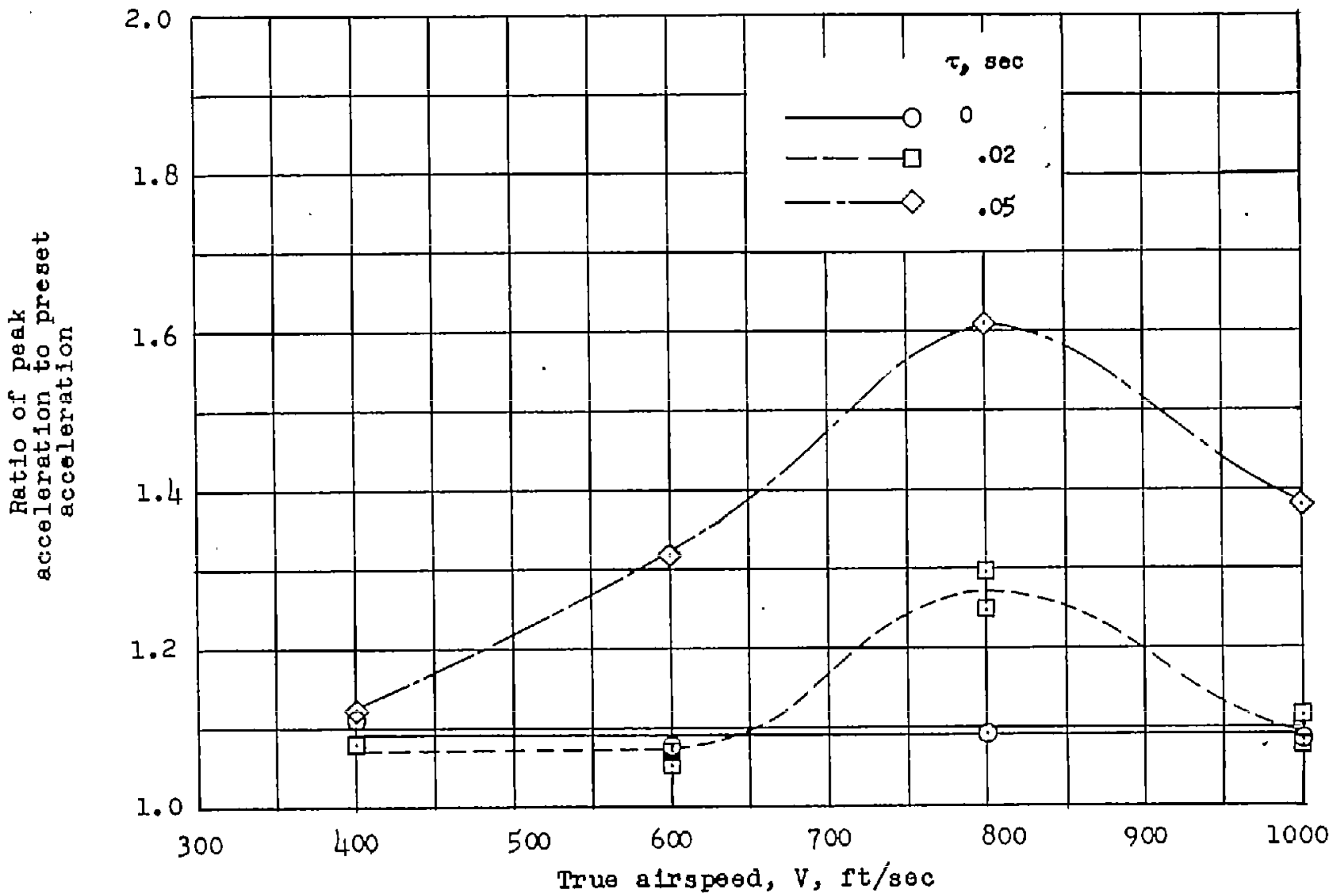


Figure 8.- Ratio of peak to preset acceleration as a function of true airspeed. Acceleration restrictor controlled by $n + \begin{Bmatrix} \frac{K}{g}\ddot{\theta} & \text{for} & \ddot{\theta} > 0 \\ 0 & \text{for} & \ddot{\theta} < 0 \end{Bmatrix} + \frac{A}{g}\dot{\theta}\,\frac{\tau D}{1 + \tau D}$. K = 154.7; A = 644; static margin, 0.10$\bar{c}$.

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