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STRESS ANALYSIS OF CIRCULAR SEMIMONOCOQUE CYLINDERS  
WITH CUTOUTS BY A PERTURBATION LOAD TECHNIQUE

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STRESS ANALYSIS OF CIRCULAR SEMIMONOCOQUE CYLINDERS

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SUMMARY

A method is presented for analyzing the stresses about a cutout in a circular cylinder of semimonocoque construction. The method involves the use of so-called perturbation solutions which are superposed on the stress distribution that would exist in the structure with no cutout in such a way as to give the effects of a cutout. The method can be used for any loading case for which the structure without the cutout can be analyzed and is sufficiently versatile to account for stringer and shear reinforcement about the cutout.

INTRODUCTION

Stresses near a cutout in a semimonocoque shell can be much higher than the stresses in the uniform shell some distance away from the cutout. The stress distribution in the neighborhood of cutouts in circular semimonocoque cylinders is significant in the design of fuselages near large openings such as doors and in the determination of the most efficient reinforcement to be used about these openings.

Some previous investigations relating to the problem of stress analysis of cylindrical semimonocoque shells with cutouts were reported in references 1 to 3. In reference 4, Cicala discussed the limitations of the analyses of references 1 to 3 - particularly the neglect of ring flexibility - and introduced the idea that the effect of a cutout can be reproduced by superposing certain perturbation stress states on the stresses which would occur in the shell without a cutout.

The problem discussed by Cicala in reference 4 is that of a cutout in an infinitely long circular cylinder of semimonocoque construction. Cicala's analysis is somewhat limited because it can be used only for loading conditions which produce stringer stresses longitudinally anti-symmetric about the center line of the cutout (for example, torsion), and it cannot be used to determine the effects of coaming-stringer reinforcement. The present report is an extension of the approach of Cicala

and presents a method of analysis which can be used with more general loading conditions and with either shear or stringer reinforcement about the cutout.

The stress perturbation approach is applied to the stress analysis of sheet-stringer panels with cutouts in reference 5. Three basic unit-perturbation solutions were used as tools in this method of analysis. The analogous perturbation solutions for a circular semimonocoque cylinder have been developed in reference 6. The purpose of the present report is to explain the use of these perturbation solutions for the stress analysis of circular semimonocoque cylinders with cutouts.

### SYMBOLS

A	effective cross-sectional area of stringer
A'	cross-sectional area of additional portion of reinforced stringer
$B = \frac{E}{G} \frac{A}{bt} \frac{R^2}{L^2}$	
b	arc distance between stringers
$C = \frac{AR^6}{IL^3b}$	
E	Young's modulus of elasticity
G	shear modulus of elasticity
I	moment of inertia of ring cross section
i, k	indices for rings and bays
j, l	indices for stringers and panel rows
L	distance between rings
$M_1, M_2$	applied moment and torque, respectively (see fig. 4)
m	total number of stringers in cylinder, $m \geq 3$
P	magnitude of a concentrated perturbation load, lb

$P_{ij}$	load in stringer $j$ at ring station $i$
$\bar{P}_{ij}$	basic stringer load in stringer $j$ at ring station $i$
$P_{ij}(k,l)$	load in stringer $j$ at ring station $i$ due to a unit concentrated perturbation load on stringer $l$ at ring station $k$
$P_{ij}[k,l]$	load in stringer $j$ at ring station $i$ due to a unit shear perturbation load about shear panel $(k,l)$
$Q$	magnitude of a shear perturbation load, lb/in.
$q_{ij}$	shear flow in shear panel $(i,j)$
$\bar{q}_{ij}$	basic shear flow in shear panel $(i,j)$
$q_{ij}(k,l)$	shear flow in shear panel $(i,j)$ due to a unit concentrated perturbation load on stringer $l$ at ring station $k$
$q_{ij}[k,l]$	shear flow in shear panel $(i,j)$ due to a unit shear perturbation load about shear panel $(k,l)$
$R$	radius to middle surface of sheet
$S$	magnitude of a distributed perturbation load, lb
$t$	thickness of sheet
$t'$	thickness of additional portion of a reinforced shear panel, that is, doubler plate

#### BASIC ASSUMPTIONS

A typical structure of the type to be discussed in this report is shown in figure 1. It consists of a thin-walled circular cylinder stiffened by stringers in the longitudinal direction and by rings in the circumferential direction. A cutout is located in a bay far from the ends of the cylinder (the theory is limited to cases where external restraints and free sections are a large distance from the cutout). The cutout which is 1 bay long may remove an arbitrary number of shear panels and portions of stringers.

The loading is considered to be such that buckling does not occur. Some possible loading conditions are shown in figure 1. Any other

loading condition is permissible if the stress distribution in the cylinder without the cutout is known.

The analysis is based on the following assumptions regarding the properties of the structure:

(a) The stringers are uniform and equally spaced around the shell, and the sheet is of constant thickness.

(b) The stringers carry only direct stress, and the sheet takes only shear stress which is constant within each bay; thus, stringer stresses vary linearly between adjacent rings.

(c) The rings have a finite bending stiffness in their own planes, but they do not restrain longitudinal displacements of the stringers. The ring bending is inextensional.

(d) The difference between the radius of the neutral axis of the ring and the radius of the middle surface of the sheet is negligible.

#### PERTURBATION STRESS DISTRIBUTIONS

The tools for the method of analysis to be described are the stress distributions due to three types of loads, called perturbation loads, applied to an infinitely long circular cylinder with no cutout. One perturbation load consists of a concentrated force  $P$  imposed on one stringer of the shell at its intersection with a ring, the force acting in the direction of the stringer. This load is illustrated in figure 2(a) and is called the concentrated perturbation load. A second type, illustrated in figure 2(b), is called the distributed perturbation load and consists of a force  $S$  uniformly distributed along the portion of one stringer which extends between two adjacent rings, the force acting in the direction of the stringer. The third type, shown in figure 2(c), is called the shear perturbation load and consists of uniformly distributed forces per unit length  $Q$  applied along the stringers and rings that border one shear panel of the shell, the forces acting in such a way as to cause pure shear in that panel.

For each of the three perturbation loads, formulas were developed in reference 6 which give stringer loads in every stringer at each ring station and shear flows in each shear panel of the shell. By use of these formulas, tables of influence coefficients can be computed which give stringer loads and shear flows in the neighborhood of each perturbation load due to a unit magnitude of that load. Such tables for a cylinder having 36 stringers and various values of the sheet-stringer parameter  $B$  are presented as tables 1 to 3. The tables presented are

only for the limiting case of rigid rings (ring flexibility parameter  $C = 0$ ). In this report, tables 1 to 3 are used to make sample calculations illustrating the method of analysis for a shell with rigid rings. For a shell with flexible rings, the method of analysis is the same with the exception that tables which include the effect of ring flexibility will have to be used.

The tables are not limited in application to cylinders with 36 stringers. In general, the total stringer area can simply be redistributed into 36 fictitious stringers. The value of the parameter  $B$  is not changed by such a redistribution of stringer area. Then the tables can be thought of as presenting (a) the load which is taken by all of the normal-stress-carrying material up to  $5^\circ$  on either side of the location of a fictitious stringer and (b) the shear flows at points in the sheet halfway between fictitious stringers.

Table 1 contains the values of  $p_{ij}$  and  $q_{ij}L$  due to a concentrated perturbation load  $P = 1$  on stringer  $j = 0$  at ring station  $i = 0$ . Table 2 contains the values of  $p_{ij}$  and  $q_{ij}L$  due to a distributed perturbation load of total magnitude  $S = 1$  on stringer  $j = 0$  between rings  $i = 0$  and  $i = 1$ . Table 3 contains the values of  $p_{ij}/L$  and  $q_{ij}$  due to a shear perturbation load per unit length of magnitude  $Q = 1$  about shear panel  $(0,0)$ . The positive senses of the perturbation loads are the senses shown in figure 2; stringer loads are assumed positive in tension, and shear flow is positive when an element of the sheet is loaded by shears which act in the positive sense of the shear perturbation load. The solutions for arbitrary locations of the perturbation loads are readily obtained from the tables by means of changes of indices.

The application of these perturbation loads and the stress distributions caused by them in the stress analysis of circular semimonocoque cylinders with cutouts is discussed in the following section. The perturbation solutions are exact only for infinitely long cylinders. However, in the solution of a cutout problem, the perturbation loads are applied in self-equilibrating groups in order not to disturb the overall equilibrium of the structure; therefore, the stresses due to the perturbation loads decay rapidly in the longitudinal direction. Consequently, the application of perturbation stress distributions for an infinitely long cylinder to a cylinder of finite length is justified if the vicinity of application of the perturbation loads is far from the ends of the cylinder.

## METHOD OF ANALYSIS

## Structure With No Reinforcement About Cutout

Application of perturbation loads.- Consider, first, a structure like that shown in figure 1 which has no reinforcement about the cutout. The stress distribution in such a shell can be thought of as a superposition of the stresses which would exist in the structure without a cutout and perturbation stress distributions which arise because of the cutout. The structure without a cutout is called herein the basic structure. The stress distribution which would exist in this structure is called herein the basic stress distribution. In the present report the basic stress distribution is assumed to be known. Then the problem of analyzing a structure with a cutout consists of the determination of the perturbation stress distributions to be superposed on the basic stresses in such a manner as to annihilate the effects of that portion of the basic structure which lies within the boundaries of the cutout. Finding the proper magnitudes of these perturbation stresses involves the solution of a system of simultaneous algebraic equations.

At the cutout boundary in the structure with the cutout, two conditions must be satisfied: (a) zero stringer load wherever a stringer is interrupted by the cutout and (b) no external shear forces acting on the portions of stringers and rings which border the cutout. By superposing concentrated and shear perturbation loads on the basic structure, the resultant stresses can be made to satisfy these conditions.

The method of analysis is as follows:

- (1) Find the stress distribution for the basic structure, that is, the cylinder without a cutout.
- (2) Place perturbation loads on the basic structure in the following manner: At each point where a stringer would be interrupted by the cutout, place a concentrated perturbation load; and, about each shear panel which would be removed by the cutout, place a shear perturbation load. For the case of a cutout removing three shear panels and interrupting two stringers, these perturbation loads are shown in figure 3.
- (3) With the use of the tables of influence coefficients, write a set of simultaneous algebraic equations which state the following conditions:
  - (a) At the points where a stringer is to be interrupted by the cutout boundary, the resultant stringer load must vanish on the side of the boundary away from the cutout. This resultant stringer load is composed of the basic stringer load plus the stringer load due to all the perturbation loads.

(b) In each shear panel which is to be removed by the cutout, the basic shear flow plus the shear flow due to all the perturbation loads must be equal to the shear perturbation load applied to the portions of stringers and rings which border that given panel. Thus, the shear flow exerted by the shear panel on the portions of stringers and rings bordering it will exactly cancel the shear perturbation load applied to those same portions of stringers and rings.

(4) Solve the system of equations from step (3) for the magnitudes of the perturbation loads, and superpose the stress distributions due to these loads on the basic distribution. This procedure yields the stress distribution in the structure with cutout.

Upon completion of these four steps, the magnitudes of the perturbation loads on the basic structure have been adjusted so that simultaneous removal of that portion of the basic structure which lies within the cutout boundary and the perturbation loads themselves would not disturb the remainder of the structure. The perturbation loads are in equilibrium with the portion of the basic structure lying within the cutout boundary. The stresses outside the cutout boundary in the basic structure subjected to the actual external loading together with the perturbation loads are precisely the same as the stresses in the structure with the cutout subjected to the external loading alone.

Conditions 3(a) and 3(b) can be expressed mathematically by the following equations, respectively:

$$\sum_k \sum_l P_{kl} p_{ij}(k, l) + \sum_k \sum_l Q_{kl} p_{ij}[k, l] + \bar{p}_{ij} = 0 \quad (1)$$

$$\sum_k \sum_l P_{kl} q_{ij}(k, l) + \sum_k \sum_l Q_{kl} q_{ij}[k, l] + \bar{q}_{ij} = Q_{ij} \quad (2)$$

The unknowns are  $P_{kl}$ , the magnitude of the concentrated perturbation load on stringer  $l$  at station  $k$ , and  $Q_{kl}$ , the magnitude of the shear perturbation load about shear panel  $(k, l)$ . The coefficients  $p_{ij}(k, l)$  and  $q_{ij}(k, l)$  are found in table 1 and the coefficients  $p_{ij}[k, l]$  and  $q_{ij}[k, l]$  are found in table 3. The summations in each case are extended over the appropriate perturbation loads. Equation (1) is written for each  $i, j$  where a stringer is to be interrupted by the



cutout and refers in each case to the stringer load as the point  $i, j$  is approached from within that portion of the structure lying outside the cutout boundary. Equation (2) is written for each  $i, j$  where a shear panel is to be removed by the cutout. The form of equations (1) and (2) is the same regardless of whether the rings are considered rigid or flexible.

This method of analysis may be applied to a cylinder having a cutout more than 1 bay long, but, in such a situation, the effects of removing ring segments from the region within the cutout boundary are neglected. In the rigid-ring case, such effects do not exist if the cut rings remain effectively rigid; in the flexible-ring case, the effects of cutting a ring could, in principle, be taken into account through the introduction of additional types of perturbation loads. It is possible that even with flexible rings the effects of cutting a ring are negligible in certain cases, but this would have to be verified by further investigation.

Sample calculation.- In order to illustrate the method of calculation, the cylinder shown in figure 4 is analyzed. A cutout which removes three shear panels and interrupts two stringers is located in the central bay. The required properties of the cylinder are taken as follows:

$$m = 36$$

$$A = 0.260 \text{ sq in.}$$

$$R = 15 \text{ in.}$$

$$L = 12 \text{ in.}$$

$$t = 0.051 \text{ in.}$$

$$b = R \frac{2\pi}{36} = 2.62 \text{ in.}$$

$$B = \frac{E}{G} \frac{A}{bt} \frac{R^2}{L^2} = 8.05$$

Suppose that the cylinder is loaded with the bending moment  $M_1$  and torque  $M_2$  shown in figure 4.

The perturbation load system for this problem is shown in figure 3. The concentrated perturbation loads are doubly symmetric about the cutout.

The shear perturbation loads are symmetric about panel row  $j = 0$ . Let  $P$  represent the magnitude of each of the concentrated perturbation loads. Let  $Q_0$  represent the magnitude of the shear perturbation load about shear panel (0,0); and let  $Q_1$  represent the magnitude of the shear perturbation loads about shear panels (0,1) and (0,-1).

Equations (1) and (2) are written for this example by use of the tables of influence coefficients for  $B = 8$ . Equation (1) for the stringer load condition in stringer  $j = 1$  at ring station  $i = 1$  is written with the aid of tables 1(a) and 3(a)

$$-0.5000P + 0.0476P + 0.0895P + 0.1192Q_1L - 0.1192Q_0L - 0.0374Q_1L + \bar{P}_{11} = 0$$

where  $\bar{P}_{11}$  is the basic stringer load in stringer  $j = 1$  at station  $i = 1$ . Because of symmetry the same equation results when equation (1) is written for stringer  $j = 1$  at station  $i = 0$  or for stringer  $j = 0$  at ring stations  $i = 0$  or  $i = 1$ . Equation (2) for shear panel (0,0) is

$$-0.2262 \frac{P}{L} + 0.2262 \frac{P}{L} - 0.2262 \frac{P}{L} + 0.2262 \frac{P}{L} + 0.6986Q_0 -$$

$$2(0.0629)Q_1 + \bar{q}_{00} = Q_0$$

where  $\bar{q}_{00}$  is the basic shear flow in shear panel (0,0). For shear panels (0,1) and (0,-1), equation (2) gives

$$-0.2262 \frac{P}{L} + 0.2262 \frac{P}{L} - 0.1368 \frac{P}{L} + 0.1368 \frac{P}{L} + 0.6986Q_1 -$$

$$0.0629Q_0 + 0.0119Q_1 + \bar{q}_{01} = Q_1$$

where  $\bar{q}_{01}$  is the basic shear flow in shear panel (0,1). These three equations in the three unknowns  $P$ ,  $Q_0$ , and  $Q_1$  become

$$\left. \begin{aligned} 0.3629P + 0.1192Q_0L - 0.0818Q_1L &= \bar{P}_{11} \\ 0.3014Q_0L + 0.1258Q_1L &= \bar{q}_{00}L \\ 0.0629Q_0L + 0.2895Q_1L &= \bar{q}_{01}L \end{aligned} \right\} \quad (3)$$

For simplicity, let  $M_1 = M_2 = 100,000$  lb-in. In the present example, the basic stress distribution can be found from engineering beam theory which gives  $\bar{p}_{11} = 370$  pounds and  $\bar{q}_{00} = \bar{q}_{01} = 70.8$  lb/in. When these constants are introduced into the system of equations (3), the solution is

$$P = 1,020 \text{ lb}$$

$$Q_0 L = 1,750 \text{ lb}$$

$$Q_1 L = 2,560 \text{ lb}$$

Stringer loads and shear flows in the neighborhood of the cutout are obtained by superposing the effects of these perturbation loads on the basic stress distribution. For example, with the use of tables 1(a) and 3(a) the stringer load at the intersection of ring  $i = 0$  and stringer  $j = 2$  is given by

$$P(0.0895 + 0.0511) + Q_1 L(0.1192 + 0.0125) + Q_0 L(0.0374) + \bar{p}_{02} = 545 + \bar{p}_{02}$$

The basic stringer load  $\bar{p}_{02}$  equals 358 pounds. Therefore, the load in stringer  $j = 2$  at ring  $i = 0$  is 903 pounds. Other stringer loads at ring  $i = 0$  are shown in figure 5(a). The shear flow in shear panel (-1,1) is given by

$$\frac{1}{L} \left[ P(0.2262 + 0.1368 + 0.0044 - 0.0360) + Q_1 L(0.1357 - 0.0159) + Q_0 L(0.0097) \right] + \bar{q}_{-1,1} = 55.1 + \bar{q}_{-1,1}$$

The basic shear flow  $\bar{q}_{-1,1}$  equals 70.8 lb/in. Thus, the shear flow in panel (-1,1) is 125.9 lb/in. Other shear flows in bay  $i = -1$  are shown in figure 5(b), and in figure 5(c) are presented shear flows in the net section (bay  $i = 0$ ).

## Structure With Reinforcement About Cutout

Shear reinforcement.- The method of analysis is easily extended to problems where shear panels are reinforced in the neighborhood of the cutout. Suppose that some of the shear panels around the cutout are reinforced by the addition of a certain thickness of sheet (i.e., a doubler plate). Then, the procedure consists of adding shear perturbation loads to each of these shear panels in the basic structure. On the doubler plates is placed the same shear perturbation load except with opposite sign. Then, for each reinforced shear panel, an equation is written which states the requirement that the shear stress in the shear panel of the basic structure shall equal the shear stress in the doubler plate used to reinforce that panel. When this condition is satisfied, the loaded doubler plates can conceptually be inserted into the basic structure without disturbing continuity. The shear perturbation loads on the doubler plates cancel the shear perturbation loads on the basic structure.

As an example, consider for simplicity the cylinder shown in figure 4 loaded only with bending moment  $M_1$ . The most highly loaded shear panels are those indicated by the vertical hatching in figure 6. Suppose, now, that these shear panels are reinforced by the addition of plates of thickness  $t'$  to the skin of thickness  $t$  so that the total thickness in these shear panels is  $t + t'$ . The perturbation load system to be placed on the basic structure is shown in figure 7. The four doubler plates of thickness  $t'$  are shown as free bodies in figure 7. The shear perturbation loads applied to them are of the same magnitude as those applied to the basic portions of the reinforced shear panels, but are opposite in sign. The conditions that must be satisfied are:

(a) The stringer load is zero in stringers  $j = 0$  and  $j = 1$  at stations  $i = 0$  and  $i = 1$  as each of these points is approached from the structure outside of the cutout.

(b) The shear flow in shear panels  $(0,-1)$ ,  $(0,0)$ , and  $(0,1)$  cancels any shear perturbation load applied about these panels. (In this example, no shear is developed in the shear panels of bay  $i = 0$  and this condition is automatically satisfied.)

(c) The shear stress in each of the shear panels  $(1,1)$ ,  $(1,-1)$ ,  $(-1,1)$ , and  $(-1,-1)$  in the basic structure must equal the shear stress in the corresponding doubler plate.

Condition (a), which must hold where stringers  $j = 0$  and  $j = 1$  are interrupted by the cutout, is expressed by a single equation because of symmetry:

$$(-0.5000 + 0.0476 + 0.0895)P + (-0.1192 - 0.0374 + 0.0067 - 0.0118)QL + \bar{P}_{11} = 0$$

where  $P$  and  $Q$  are the magnitudes of the concentrated and shear perturbation loads, respectively, and  $\bar{P}_{11}$  is the basic stringer load.

The condition in shear panel (1,1) that the shear stress in the basic portion of the sheet equals the shear stress in the doubler plate (condition (c)) is expressed as

$$\left[ (-0.2262 - 0.1368 - 0.0044 + 0.0360) \frac{P}{L} + (0.6986 - 0.0119 - 0.0068 + 0.0052) Q \right] \frac{1}{t} = -Q \frac{1}{t'}$$

where  $t$  is the thickness of the basic portion of the shear panel and  $t'$  is the thickness of the doubler plate. Because of symmetry, the same equation expresses condition (c) for the other three reinforced shear panels. These equations become

$$\begin{aligned} 0.3629P + 0.1617QL &= \bar{P}_{11} \\ -0.3314P + \left( \frac{t}{t'} + 0.6851 \right) QL &= 0 \end{aligned}$$

For a given value of  $t/t'$  and for a given magnitude of  $M_1$  (so that  $\bar{P}_{11}$  can be computed), this system of equations can be solved for  $P$  and  $Q$ , and the stress distributions due to these perturbation loads can then be superposed on the basic stress distribution to give the stresses about the cutout.

Stringer reinforcement.- The method of analysis is also easily extended to problems where stringers are reinforced in the neighborhood of the cutout. For example, suppose the coaming stringers in the structure shown in figure 4 have reinforcement of constant cross-sectional area extending 1 bay on either side of the cutout. This coaming-stringer reinforcement is illustrated in figure 8. Let the area of the added reinforcing portion of a coaming stringer be  $A'$  so that the total area of the reinforced portion of the stringer is  $A + A'$ . It is assumed that the stringer load is abruptly transmitted into the added portion of the reinforced coaming stringer so that the stress is always given by the force divided by the cross-sectional area.

The perturbation load system to be placed on the basic structure is shown in figure 9. The added reinforcing portions of the coaming stringers are shown as free bodies in figure 9 with the proper perturbation loads applied to them. The conditions that must be satisfied are:

(a) The stringer load is zero in stringers  $j = 0$  and  $j = 1$  at stations  $i = 0$  and  $i = 1$  as each of these points is approached from the structure outside of the cutout.

(b) The shear flow in shear panels  $(0,-1)$ ,  $(0,0)$ , and  $(0,1)$  cancels any shear perturbation load applied about these shear panels. (This condition is automatically satisfied in this example.)

(c) The stress in the basic portions of the coaming stringers  $j = -1$  and  $j = 2$  equals the stress in the added reinforcing portions at stations  $i = 0$  and  $i = 1$ .

(d) In the basic portions of the coaming stringers  $j = -1$  and  $j = 2$  at stations  $i = -1$  and  $i = 2$ , when these points are approached from the side which is reinforced, the stress equals the stress at the ends of the added reinforcing portions of the coaming stringers.

Because of the symmetry in this structure, only three equations are required. The unknowns are  $P_1$  and  $P_2$ , the magnitudes of the concentrated perturbation loads, and  $S$ , the magnitude of the distributed perturbation loads. Condition (a), which must hold where stringer  $j = 1$  is interrupted by the cutout, is expressed with the use of tables 1(a) and 2(a) as follows:

$$(-0.5000 + 0.0476 + 0.0895)P_1 + (-0.0895 - 0.0511 - 0.0490 - 0.0475)P_2 + (-0.0727 - 0.0340 - 0.0629 - 0.0499)S + \bar{p}_{11} = 0$$

The condition that the stringer stress in the basic portion of stringer  $j = 2$  equals the stress in the added reinforcing portion at station  $i = 1$  (condition (c)) is expressed as

$$\left[ (0.0895 + 0.0511)P_1 + (-0.0476 - 0.0330 - 0.0565 - 0.0402)P_2 + (-0.1924 - 0.0195 - 0.0567 - 0.0379)S + \bar{p}_{12} \right] \frac{1}{A} = (P_2 + S) \frac{1}{A'}$$

Finally, the condition that the stress in the basic portion of stringer  $j = 2$ , as the station  $i = 2$  is approached from the reinforced side, equals the stress at the ends of the added reinforcing member (condition (d)) is expressed as follows:

$$\left[ (-0.5000 - 0.0459 - 0.0394)P_2 + (0.1924 + 0.0195 - 0.0499 - 0.0398)S + (-0.0895 - 0.0511 + 0.0490 + 0.0475)P_1 + \bar{p}_{22} \right] \frac{1}{A} = \frac{P_2}{A'}$$

These three equations become

$$0.3629P_1 + 0.2371P_2 + 0.2195S = \bar{p}_{11}$$

$$-0.1406P_1 + \left(\frac{A}{A'} + 0.1773\right)P_2 + \left(\frac{A}{A'} + 0.3065\right)S = \bar{p}_{12}$$

$$0.0441P_1 + \left(\frac{A}{A'} + 0.5853\right)P_2 - 0.1222S = \bar{p}_{22}$$

When  $A/A'$  is known and the magnitude of the external moment  $M_1$  is known so that the basic stringer loads  $\bar{p}_{11}$ ,  $\bar{p}_{12}$ , and  $\bar{p}_{22}$  can be computed, this system of equations can be solved for the unknowns  $P_1$ ,  $P_2$ , and  $S$ . Superposition of the stresses due to these perturbation loads on the basic stress distribution yields the stresses about the cutout.

In this example the basic stringer loads do not vary in the longitudinal direction, and the concentrated and distributed perturbation loads can be applied in pairs, equal in magnitude and opposite in sign, as shown in figure 9. However, in cases where the basic stringer loads do vary longitudinally, for example, when the shell is loaded in shear and bending, the concentrated and distributed perturbation loads in general will not be equal in magnitude. Furthermore, additional distributed perturbation loads will be necessary on the coaming stringers in bay  $i = 0$ . Now the stress conditions which were used in the example alone no longer provide a sufficient number of equations to determine the magnitudes of the perturbation loads. The required supplementary equations are found from the conditions of equilibrium obtained when the added reinforcing portions of the coaming stringers are considered as free bodies.

Comparison of results for reinforced and unreinforced structures.— Some calculated results for the problems of cutouts with reinforcement just discussed are compared with the results for the structure without reinforcement in the following table:

Intersection of ring and stringer	Stringer load, lb, for -		
	Structure without reinforcement	Structure with reinforced coaming stringers, $A' = A$	Structure with shear reinforcement, $t' = t$
(1,2)	501	758	507
(1,3)	422	331	422
(1,4)	359	296	359
(1,5)	303	258	302
(1,6)	244	209	242

Shear panel	Shear flow, lb/in., for -		
	Structure without reinforcement	Structure with reinforced coaming stringers, $A' = A$	Structure with shear reinforcement, $t' = t$
(1,0)	0	0	0
(1,1)	-28.1	-27.3	-30.6
(1,2)	-12.3	-.3	-13.3
(1,3)	-5.6	.4	-5.8
(1,4)	-2.5	.5	-2.3

The reinforced shear panels were assumed to have sheet twice as thick as the uniform sheet; the reinforced portions of the coaming stringers were taken to have twice the area of the uniform stringers. The applied bending moment  $M_1$  was taken as 100,000 lb-in.

The following comparison is noted for these illustrative examples: In the case of coaming-stringer reinforcement, the maximum stringer load is increased, but the maximum stringer stress is decreased (because stringer area is doubled), and the maximum shear flow is not appreciably changed. In the case of shear reinforcement, the maximum shear flow is increased only slightly so that maximum shear stress is considerably reduced, and stringer loads are not appreciably affected.

#### CONCLUDING REMARKS

The method presented in this report facilitates stress analysis of circular semimonocoque cylinders with cutouts. It is most accurate in problems where the cutout is located far from external restraints and



free sections. The loading may be any combination of torsion, bending, shear, axial loading, and, in fact, any loading for which the basic stress distribution is known. Reinforcement about the cutout can be taken into consideration.

The method of analysis is based on the superposition of certain perturbation stress distributions to give the effects of the cutout on the stress distribution which would exist in the cylinder without a cutout. The equations for the three necessary perturbation stress distributions are presented in NACA Technical Note 3199. Perturbation stress distributions in the form of tables of influence coefficients are presented in this report for a 36-stringer shell having rigid rings. The tables can be used for cylinders having any number of stringers by "relumping" the actual stringers into 36 fictitious stringers. These tables are used in the report to make sample calculations which illustrate the analytical procedure. The procedure does not change in the case for which the cylinder has flexible rings except that tables of influence coefficients which include the effects of ring flexibility would have to be used.

Langley Aeronautical Laboratory,  
National Advisory Committee for Aeronautics,  
Langley Field, Va., June 16, 1954.

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TABLE 1.- LOAD DISTRIBUTION DUE TO A UNIT CONCENTRATED PERTURBATION LOAD

ON STRINGER  $j = 0$  AT STATION  $i = 0$ 

$$[C = 0; m = 36]$$

(a)  $B = 8$ 

j	Stringer load, $p_{ij}$ , at station -						
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
0	0.5000	0.0476	0.0565	0.0459	0.0437	0.0426	0.0421
1	0	.0895	.0490	.0457	.0430	.0421	.0416
2	0	.0511	.0475	.0429	.0414	.0406	.0403
3	0	.0330	.0402	.0394	.0387	.0383	.0381
4	0	.0232	.0329	.0349	.0352	.0352	.0352
5	0	.0172	.0266	.0300	.0311	.0315	.0316
6	0	.0130	.0212	.0250	.0266	.0273	.0276
7	0	.0097	.0165	.0202	.0219	.0227	.0231
8	0	.0070	.0123	.0154	.0171	.0180	.0184
9	0	.0047	.0084	.0110	.0124	.0131	.0135
10	0	.0026	.0050	.0067	.0078	.0084	.0087
11	0	.0007	.0018	.0028	.0035	.0039	.0041
12	0	-.0010	-.0010	-.0008	-.0005	-.0003	-.0002
13	0	-.0024	-.0035	-.0039	-.0040	-.0040	-.0040
14	0	-.0036	-.0056	-.0065	-.0070	-.0072	-.0073
15	0	-.0046	-.0072	-.0086	-.0094	-.0098	-.0100
16	0	-.0053	-.0084	-.0102	-.0112	-.0117	-.0120
17	0	-.0057	-.0092	-.0111	-.0122	-.0128	-.0132
18	0	-.0059	-.0094	-.0115	-.0126	-.0132	-.0136

j	Shear flow, $q_{ij}L$ , at station -					
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0	0.2262	-0.0044	0.0053	0.0011	0.0006	0.0002
1	.1368	.0360	.0087	.0038	.0015	.0007
2	.0856	.0396	.0133	.0053	.0023	.0010
3	.0527	.0324	.0141	.0059	.0026	.0012
4	.0294	.0227	.0121	.0056	.0026	.0012
5	.0122	.0133	.0086	.0045	.0022	.0011
6	-.0007	.0052	.0047	.0029	.0016	.0008
7	-.0105	-.0016	.0010	.0012	.0008	.0004
8	-.0175	-.0068	-.0021	-.0005	.0000	.0000
9	-.0222	-.0106	-.0046	-.0019	-.0008	-.0003
10	-.0248	-.0129	-.0064	-.0030	-.0014	-.0006
11	-.0255	-.0140	-.0073	-.0037	-.0018	-.0009
12	-.0246	-.0140	-.0076	-.0040	-.0020	-.0010
13	-.0222	-.0129	-.0072	-.0039	-.0020	-.0010
14	-.0186	-.0110	-.0062	-.0034	-.0018	-.0009
15	-.0140	-.0083	-.0048	-.0026	-.0014	-.0007
16	-.0087	-.0052	-.0030	-.0017	-.0009	-.0005
17	-.0029	-.0018	-.0010	-.0006	-.0003	-.0002

TABLE 1.- LOAD DISTRIBUTION DUE TO A UNIT CONCENTRATED PERTURBATION LOAD

ON STRINGER  $j = 0$  AT STATION  $i = 0$  - Continued

$$[C = 0; m = 36]$$

(b)  $B = 30$ 

j	Stringer load, $p_{ij}$ , at station -						
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
0	0.5000	0.1518	0.0852	0.0636	0.0541	0.0491	0.0463
1	0	.0866	.0711	.0588	.0518	.0478	.0454
2	0	.0374	.0488	.0484	.0462	.0443	.0429
3	0	.0209	.0331	.0380	.0393	.0394	.0392
4	0	.0137	.0237	.0296	.0326	.0341	.0348
5	0	.0098	.0177	.0232	.0267	.0288	.0300
6	0	.0072	.0134	.0181	.0214	.0236	.0251
7	0	.0054	.0100	.0138	.0167	.0188	.0203
8	0	.0038	.0073	.0102	.0125	.0143	.0156
9	0	.0025	.0049	.0069	.0086	.0100	.0111
10	0	.0013	.0027	.0040	.0051	.0061	.0068
11	0	.0003	.0008	.0013	.0019	.0024	.0029
12	0	-.0006	-.0009	-.0010	-.0010	-.0009	-.0007
13	0	-.0014	-.0024	-.0031	-.0035	-.0037	-.0039
14	0	-.0021	-.0037	-.0048	-.0056	-.0062	-.0066
15	0	-.0027	-.0047	-.0062	-.0073	-.0081	-.0087
16	0	-.0030	-.0054	-.0072	-.0085	-.0095	-.0103
17	0	-.0033	-.0058	-.0078	-.0093	-.0104	-.0112
18	0	-.0034	-.0060	-.0080	-.0095	-.0107	-.0116

j	Shear flow, $q_{ij}L$ , at station -					
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0	0.1741	0.0333	0.0108	0.0048	0.0025	0.0014
1	.0875	.0489	.0231	.0118	.0065	.0038
2	.0501	.0375	.0234	.0140	.0084	.0052
3	.0292	.0253	.0186	.0126	.0083	.0054
4	.0155	.0153	.0127	.0096	.0068	.0047
5	.0057	.0074	.0072	.0061	.0047	.0035
6	-.0016	.0012	.0025	.0028	.0025	.0021
7	-.0069	-.0034	-.0013	-.0001	.0004	.0006
8	-.0108	-.0069	-.0042	-.0025	-.0014	-.0007
9	-.0133	-.0092	-.0063	-.0042	-.0027	-.0018
10	-.0146	-.0106	-.0076	-.0053	-.0037	-.0025
11	-.0149	-.0111	-.0081	-.0059	-.0042	-.0030
12	-.0143	-.0108	-.0080	-.0059	-.0043	-.0031
13	-.0128	-.0098	-.0074	-.0055	-.0041	-.0030
14	-.0107	-.0082	-.0063	-.0047	-.0035	-.0026
15	-.0080	-.0062	-.0048	-.0036	-.0027	-.0020
16	-.0050	-.0039	-.0030	-.0023	-.0017	-.0013
17	-.0017	-.0013	-.0010	-.0008	-.0006	-.0004

TABLE 1.- LOAD DISTRIBUTION DUE TO A UNIT CONCENTRATED PERTURBATION LOAD  
ON STRINGER  $j = 0$  AT STATION  $i = 0$  - Continued

$$[C = 0; m = 36]$$

(c)  $B = 100$

j	Stringer load, $p_{ij}$ , at station -						
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
0	0.5000	0.2548	0.1528	0.1062	0.0825	0.0692	0.0611
1	0	.0699	.0800	.0750	.0678	.0615	.0566
2	0	.0241	.0391	.0460	.0483	.0484	.0475
3	0	.0124	.0225	.0297	.0342	.0369	.0383
4	0	.0079	.0149	.0206	.0250	.0283	.0306
5	0	.0055	.0106	.0151	.0189	.0219	.0243
6	0	.0041	.0079	.0113	.0144	.0170	.0191
7	0	.0030	.0058	.0084	.0108	.0129	.0147
8	0	.0021	.0042	.0061	.0079	.0095	.0109
9	0	.0014	.0028	.0040	.0053	.0064	.0075
10	0	.0007	.0015	.0022	.0030	.0037	.0044
11	0	.0002	.0004	.0006	.0009	.0012	.0015
12	0	-.0004	-.0006	-.0008	-.0009	-.0010	-.0010
13	0	-.0008	-.0015	-.0021	-.0025	-.0029	-.0032
14	0	-.0012	-.0023	-.0032	-.0039	-.0045	-.0050
15	0	-.0015	-.0029	-.0040	-.0050	-.0058	-.0065
16	0	-.0018	-.0033	-.0046	-.0058	-.0068	-.0076
17	0	-.0019	-.0036	-.0050	-.0063	-.0074	-.0083
18	0	-.0020	-.0037	-.0052	-.0064	-.0076	-.0085

j	Shear flow, $q_{ij}L$ , at station -					
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0	0.1226	0.0510	0.0233	0.0119	0.0066	0.0041
1	.0527	.0409	.0283	.0191	.0129	.0089
2	.0286	.0259	.0214	.0168	.0129	.0098
3	.0162	.0157	.0142	.0123	.0102	.0083
4	.0083	.0087	.0085	.0079	.0070	.0060
5	.0028	.0036	.0040	.0041	.0040	.0036
6	-.0013	-.0002	.0006	.0011	.0014	.0015
7	-.0043	-.0030	-.0020	-.0013	-.0007	-.0003
8	-.0064	-.0051	-.0040	-.0031	-.0023	-.0018
9	-.0078	-.0064	-.0053	-.0043	-.0035	-.0028
10	-.0085	-.0072	-.0060	-.0050	-.0042	-.0035
11	-.0087	-.0074	-.0063	-.0053	-.0045	-.0038
12	-.0083	-.0071	-.0061	-.0052	-.0044	-.0038
13	-.0074	-.0064	-.0055	-.0048	-.0041	-.0035
14	-.0062	-.0054	-.0047	-.0040	-.0034	-.0030
15	-.0047	-.0041	-.0035	-.0030	-.0026	-.0022
16	-.0029	-.0025	-.0022	-.0019	-.0016	-.0014
17	-.0010	-.0009	-.0007	-.0006	-.0006	-.0005

TABLE 1.- LOAD DISTRIBUTION DUE TO A UNIT CONCENTRATED PERTURBATION LOAD

ON STRINGER  $j = 0$  AT STATION  $i = 0$  - Continued

$$[C = 0; m = 36]$$

(d)  $B = 300$ 

j	Stringer load, $p_{1j}$ , at station -						
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
0	0.5000	0.3354	0.2366	0.1756	0.1370	0.1116	0.0945
1	0	.0506	.0716	.0782	.0781	.0752	.0713
2	0	.0149	.0271	.0359	.0417	.0453	.0472
3	0	.0074	.0142	.0201	.0250	.0290	.0320
4	0	.0046	.0091	.0132	.0168	.0201	.0228
5	0	.0032	.0064	.0094	.0121	.0147	.0170
6	0	.0024	.0047	.0069	.0090	.0110	.0128
7	0	.0018	.0035	.0051	.0067	.0082	.0096
8	0	.0012	.0025	.0036	.0048	.0059	.0070
9	0	.0008	.0016	.0024	.0032	.0039	.0046
10	0	.0004	.0008	.0013	.0017	.0021	.0026
11	0	.0001	.0002	.0003	.0004	.0006	.0007
12	0	-.0002	-.0004	-.0006	-.0007	-.0008	-.0009
13	0	-.0005	-.0010	-.0014	-.0017	-.0021	-.0023
14	0	-.0007	-.0014	-.0020	-.0026	-.0031	-.0035
15	0	-.0009	-.0018	-.0025	-.0032	-.0039	-.0045
16	0	-.0010	-.0020	-.0029	-.0037	-.0045	-.0052
17	0	-.0011	-.0022	-.0031	-.0040	-.0048	-.0056
18	0	-.0012	-.0022	-.0032	-.0041	-.0050	-.0057

j	Shear flow, $q_{1j}L$ , at station -					
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0	0.0823	0.0494	0.0305	0.0193	0.0127	0.0086
1	.0317	.0284	.0239	.0194	.0156	.0125
2	.0168	.0162	.0151	.0136	.0120	.0105
3	.0094	.0094	.0092	.0087	.0081	.0075
4	.0048	.0050	.0051	.0050	.0049	.0047
5	.0015	.0018	.0021	.0022	.0024	.0024
6	-.0009	-.0005	-.0001	.0002	.0004	.0006
7	-.0026	-.0022	-.0018	-.0014	-.0011	-.0009
8	-.0039	-.0034	-.0030	-.0026	-.0022	-.0019
9	-.0047	-.0042	-.0038	-.0034	-.0030	-.0026
10	-.0051	-.0046	-.0042	-.0038	-.0034	-.0031
11	-.0052	-.0047	-.0043	-.0039	-.0036	-.0032
12	-.0049	-.0045	-.0041	-.0038	-.0035	-.0032
13	-.0044	-.0041	-.0037	-.0034	-.0032	-.0029
14	-.0037	-.0034	-.0031	-.0029	-.0026	-.0024
15	-.0028	-.0026	-.0024	-.0022	-.0020	-.0018
16	-.0017	-.0016	-.0015	-.0014	-.0012	-.0011
17	-.0006	-.0005	-.0005	-.0005	-.0004	-.0004



TABLE 2.- LOAD DISTRIBUTION DUE TO A UNIT DISTRIBUTED PERTURBATION LOAD

ON STRINGER  $j = 0$  BETWEEN RINGS  $i = 0$  AND  $i = 1$ 

$$[C = 0; m = 36]$$

(a)  $B = 8$ 

j	Stringer load, $p_{ij}$ , at station -					
	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
0	0.1924	0.0567	0.0499	0.0447	0.0430	0.0423
1	.0727	.0629	.0475	.0441	.0425	.0418
2	.0340	.0499	.0447	.0421	.0410	.0404
3	.0195	.0379	.0398	.0390	.0385	.0382
4	.0128	.0291	.0341	.0351	.0352	.0352
5	.0092	.0226	.0286	.0307	.0313	.0316
6	.0068	.0175	.0234	.0260	.0270	.0274
7	.0051	.0134	.0185	.0211	.0224	.0229
8	.0036	.0098	.0140	.0164	.0176	.0182
9	.0024	.0067	.0098	.0117	.0128	.0134
10	.0013	.0038	.0059	.0073	.0081	.0086
11	.0003	.0013	.0023	.0032	.0037	.0040
12	-.0006	-.0010	-.0009	-.0006	-.0004	-.0002
13	-.0013	-.0030	-.0037	-.0040	-.0040	-.0040
14	-.0020	-.0047	-.0061	-.0068	-.0071	-.0073
15	-.0025	-.0060	-.0080	-.0091	-.0096	-.0099
16	-.0029	-.0070	-.0094	-.0107	-.0114	-.0118
17	-.0031	-.0076	-.0102	-.0117	-.0126	-.0130
18	-.0032	-.0078	-.0105	-.0121	-.0130	-.0134

j	Shear flow, $q_{ij}L$ , at station -					
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0	0.3077	0.0679	0.0034	0.0026	0.0008	0.0004
1	.1623	.0776	.0188	.0060	.0025	.0010
2	.0942	.0617	.0240	.0086	.0036	.0016
3	.0553	.0433	.0221	.0094	.0041	.0018
4	.0296	.0271	.0171	.0085	.0039	.0018
5	.0112	.0137	.0110	.0064	.0033	.0016
6	-.0025	.0030	.0052	.0038	.0022	.0012
7	-.0127	-.0053	.0001	.0012	.0010	.0006
8	-.0200	-.0115	-.0041	-.0012	-.0002	.0000
9	-.0248	-.0158	-.0072	-.0031	-.0013	-.0005
10	-.0273	-.0183	-.0093	-.0045	-.0021	-.0010
11	-.0279	-.0193	-.0104	-.0053	-.0027	-.0013
12	-.0268	-.0189	-.0105	-.0056	-.0029	-.0015
13	-.0240	-.0172	-.0098	-.0054	-.0028	-.0015
14	-.0201	-.0145	-.0084	-.0047	-.0025	-.0013
15	-.0151	-.0109	-.0064	-.0036	-.0020	-.0010
16	-.0094	-.0068	-.0040	-.0023	-.0012	-.0007
17	-.0032	-.0023	-.0014	-.0008	-.0004	-.0002

TABLE 2.- LOAD DISTRIBUTION DUE TO A UNIT DISTRIBUTED PERTURBATION LOAD

ON STRINGER  $j = 0$  BETWEEN RINGS  $i = 0$  AND  $i = 1$  - Continued

$$[C = 0; m = 36]$$

(b)  $B = 30$ 

j	Stringer load, $p_{ij}$ , at station -					
	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
0	0.2850	0.1124	0.0729	0.0583	0.0514	0.0476
1	.0603	.0789	.0644	.0550	.0496	.0465
2	.0214	.0447	.0489	.0473	.0452	.0435
3	.0111	.0278	.0360	.0388	.0394	.0393
4	.0071	.0191	.0270	.0313	.0334	.0345
5	.0050	.0139	.0206	.0251	.0278	.0294
6	.0037	.0104	.0159	.0198	.0226	.0244
7	.0027	.0078	.0120	.0154	.0178	.0196
8	.0019	.0056	.0088	.0114	.0134	.0150
9	.0013	.0037	.0059	.0078	.0094	.0106
10	.0007	.0020	.0034	.0046	.0056	.0065
11	.0001	.0005	.0010	.0016	.0022	.0027
12	-.0004	-.0008	-.0010	-.0010	-.0009	-.0008
13	-.0008	-.0020	-.0028	-.0033	-.0036	-.0038
14	-.0011	-.0029	-.0043	-.0052	-.0059	-.0064
15	-.0014	-.0037	-.0055	-.0068	-.0077	-.0084
16	-.0016	-.0043	-.0063	-.0079	-.0091	-.0099
17	-.0017	-.0046	-.0069	-.0086	-.0099	-.0108
18	-.0018	-.0047	-.0070	-.0088	-.0101	-.0112

j	Shear flow, $q_{ij}L$ , at station -					
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0	0.2150	0.0863	0.0198	0.0073	0.0035	0.0019
1	.0944	.0678	.0343	.0167	.0088	.0050
2	.0516	.0444	.0301	.0183	.0109	.0066
3	.0294	.0277	.0220	.0155	.0103	.0067
4	.0152	.0156	.0141	.0111	.0082	.0057
5	.0052	.0067	.0074	.0067	.0054	.0041
6	-.0022	.0000	.0020	.0027	.0027	.0023
7	-.0076	-.0051	-.0023	-.0006	.0002	.0005
8	-.0115	-.0087	-.0055	-.0033	-.0019	-.0010
9	-.0140	-.0112	-.0077	-.0052	-.0034	-.0022
10	-.0154	-.0125	-.0090	-.0064	-.0045	-.0031
11	-.0156	-.0129	-.0095	-.0070	-.0050	-.0036
12	-.0149	-.0124	-.0094	-.0070	-.0051	-.0037
13	-.0134	-.0112	-.0085	-.0064	-.0048	-.0035
14	-.0112	-.0094	-.0072	-.0055	-.0041	-.0030
15	-.0084	-.0071	-.0054	-.0042	-.0031	-.0023
16	-.0052	-.0044	-.0034	-.0026	-.0020	-.0015
17	-.0018	-.0015	-.0012	-.0009	-.0007	-.0005



TABLE 2.- LOAD DISTRIBUTION DUE TO A UNIT DISTRIBUTED PERTURBATION LOAD  
ON STRINGER  $j = 0$  BETWEEN RINGS  $i = 0$  AND  $i = 1$  - Continued

$$[C = 0; m = 36]$$

(c)  $B = 100$

j	Stringer load, $p_{ij}$ , at station -					
	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
0	0.3600	0.1972	0.1268	0.0932	0.0753	0.0648
1	.0428	.0770	.0779	.0714	.0645	.0589
2	.0127	.0324	.0431	.0474	.0485	.0480
3	.0063	.0177	.0264	.0321	.0357	.0377
4	.0040	.0115	.0179	.0229	.0267	.0295
5	.0028	.0081	.0129	.0170	.0204	.0231
6	.0020	.0060	.0096	.0129	.0157	.0181
7	.0015	.0044	.0072	.0097	.0119	.0138
8	.0011	.0032	.0051	.0070	.0087	.0102
9	.0007	.0021	.0034	.0047	.0059	.0070
10	.0004	.0011	.0019	.0026	.0033	.0040
11	.0001	.0002	.0005	.0008	.0011	.0014
12	-.0002	-.0005	-.0007	-.0009	-.0010	-.0010
13	-.0004	-.0012	-.0018	-.0023	-.0027	-.0030
14	-.0006	-.0018	-.0027	-.0035	-.0042	-.0048
15	-.0008	-.0022	-.0035	-.0045	-.0054	-.0062
16	-.0009	-.0026	-.0040	-.0052	-.0063	-.0072
17	-.0010	-.0028	-.0043	-.0057	-.0068	-.0078
18	-.0010	-.0028	-.0044	-.0058	-.0070	-.0080

j	Shear flow, $q_{ij}L$ , at station -					
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0	0.1400	0.0814	0.0352	0.0168	0.0090	0.0052
1	.0543	.0472	.0343	.0234	.0158	.0108
2	.0289	.0275	.0237	.0191	.0148	.0113
3	.0162	.0161	.0151	.0133	.0112	.0092
4	.0082	.0086	.0087	.0082	.0074	.0065
5	.0026	.0032	.0039	.0041	.0040	.0038
6	-.0015	-.0007	.0002	.0008	.0012	.0014
7	-.0045	-.0036	-.0025	-.0016	-.0010	-.0005
8	-.0066	-.0057	-.0045	-.0035	-.0027	-.0020
9	-.0080	-.0071	-.0058	-.0048	-.0039	-.0031
10	-.0087	-.0078	-.0066	-.0055	-.0046	-.0038
11	-.0089	-.0080	-.0068	-.0058	-.0049	-.0041
12	-.0085	-.0077	-.0066	-.0056	-.0048	-.0041
13	-.0076	-.0069	-.0060	-.0051	-.0044	-.0038
14	-.0064	-.0058	-.0050	-.0043	-.0037	-.0032
15	-.0048	-.0044	-.0038	-.0033	-.0028	-.0024
16	-.0030	-.0027	-.0024	-.0020	-.0018	-.0015
17	-.0010	-.0009	-.0008	-.0007	-.0006	-.0005

TABLE 2.- LOAD DISTRIBUTION DUE TO A UNIT DISTRIBUTED PERTURBATION LOAD  
ON STRINGER  $j = 0$  BETWEEN RINGS  $i = 0$  AND  $i = 1$  - Continued

$$[C = 0; m = 36]$$

(d)  $B = 300$

j	Stringer load, $p_{ij}$ , at station -					
	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
0	0.4108	0.2820	0.2038	0.1549	0.1234	0.1026
1	.0286	.0628	.0757	.0785	.0768	.0733
2	.0076	.0213	.0318	.0390	.0436	.0464
3	.0037	.0109	.0172	.0227	.0271	.0305
4	.0023	.0069	.0111	.0150	.0185	.0215
5	.0016	.0048	.0079	.0108	.0134	.0158
6	.0012	.0036	.0058	.0080	.0100	.0119
7	.0009	.0026	.0043	.0059	.0074	.0089
8	.0006	.0018	.0031	.0042	.0054	.0064
9	.0004	.0012	.0020	.0028	.0035	.0043
10	.0002	.0006	.0010	.0015	.0019	.0024
11	.0000	.0001	.0002	.0003	.0005	.0006
12	-.0001	-.0004	-.0005	-.0007	-.0008	-.0009
13	-.0003	-.0007	-.0012	-.0016	-.0019	-.0022
14	-.0004	-.0011	-.0017	-.0023	-.0028	-.0033
15	-.0005	-.0014	-.0022	-.0029	-.0036	-.0042
16	-.0005	-.0015	-.0025	-.0033	-.0041	-.0048
17	-.0006	-.0017	-.0027	-.0036	-.0044	-.0052
18	-.0006	-.0017	-.0027	-.0037	-.0046	-.0054

j	Shear flow, $q_{ij}L$ , at station -					
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0	0.0892	0.0644	0.0391	0.0244	0.0157	0.0104
1	.0320	.0302	.0262	.0216	.0175	.0140
2	.0168	.0166	.0157	.0144	.0128	.0113
3	.0094	.0094	.0093	.0089	.0084	.0078
4	.0047	.0049	.0050	.0050	.0050	.0048
5	.0014	.0017	.0020	.0022	.0023	.0024
6	-.0010	-.0007	-.0003	.0000	.0003	.0005
7	-.0027	-.0024	-.0020	-.0016	-.0013	-.0010
8	-.0040	-.0036	-.0032	-.0028	-.0024	-.0021
9	-.0048	-.0044	-.0040	-.0036	-.0032	-.0028
10	-.0052	-.0049	-.0044	-.0040	-.0036	-.0032
11	-.0052	-.0049	-.0045	-.0041	-.0037	-.0034
12	-.0050	-.0047	-.0043	-.0040	-.0036	-.0033
13	-.0045	-.0042	-.0039	-.0036	-.0033	-.0030
14	-.0037	-.0035	-.0033	-.0030	-.0028	-.0025
15	-.0028	-.0026	-.0024	-.0023	-.0021	-.0019
16	-.0017	-.0016	-.0015	-.0014	-.0013	-.0012
17	-.0006	-.0006	-.0005	-.0005	-.0004	-.0004



TABLE 3.- LOAD DISTRIBUTION DUE TO A UNIT SHEAR PERTURBATION LOAD  
ABOUT SHEAR PANEL (0,0)

$$[C = 0; m = 36]$$

(a) B = 8

j	Stringer load, $p_{1j}/L$ , at station -					
	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
1	-0.1192	0.0067	-0.0019	-0.0001	-0.0001	0.0000
2	-.0374	-.0118	-.0016	-.0008	-.0003	-.0001
3	-.0125	-.0100	-.0029	-.0010	-.0004	-.0002
4	-.0038	-.0061	-.0029	-.0011	-.0005	-.0002
5	-.0002	-.0031	-.0021	-.0010	-.0005	-.0002
6	.0016	-.0011	-.0012	-.0007	-.0004	-.0002
7	.0026	.0003	-.0005	-.0004	-.0002	-.0001
8	.0032	.0011	.0002	-.0001	-.0001	-.0001
9	.0036	.0017	.0006	.0002	.0000	.0000
10	.0037	.0020	.0009	.0004	.0002	.0001
11	.0037	.0021	.0011	.0005	.0002	.0001
12	.0035	.0021	.0012	.0006	.0003	.0002
13	.0032	.0020	.0011	.0006	.0003	.0002
14	.0028	.0018	.0010	.0006	.0003	.0002
15	.0023	.0014	.0009	.0005	.0003	.0002
16	.0017	.0011	.0007	.0004	.0002	.0001
17	.0010	.0007	.0004	.0002	.0001	.0001
18	.0004	.0002	.0001	.0001	.0000	.0000

j	Shear flow, $q_{1j}$ , at station -					
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0	0.6986	0.1357	0.0068	0.0052	0.0016	0.0008
1	-.0629	.0097	.0154	.0034	.0016	.0007
2	.0119	-.0159	.0052	.0026	.0011	.0005
3	.0370	-.0184	-.0019	.0008	.0005	.0003
4	.0446	-.0162	-.0051	-.0010	-.0001	.0000
5	.0451	-.0134	-.0060	-.0021	-.0007	-.0002
6	.0419	-.0107	-.0058	-.0026	-.0010	-.0004
7	.0366	-.0083	-.0051	-.0026	-.0012	-.0006
8	.0302	-.0062	-.0042	-.0024	-.0012	-.0006
9	.0230	-.0043	-.0031	-.0019	-.0011	-.0006
10	.0156	-.0025	-.0021	-.0014	-.0008	-.0004
11	.0082	-.0010	-.0011	-.0008	-.0006	-.0003
12	.0012	.0004	-.0001	-.0003	-.0002	-.0002
13	-.0052	.0017	.0007	.0002	.0000	.0000
14	-.0108	.0027	.0014	.0007	.0003	.0002
15	-.0153	.0035	.0020	.0011	.0006	.0003
16	-.0187	.0041	.0024	.0013	.0007	.0004
17	-.0207	.0045	.0026	.0015	.0008	.0004
18	-.0214	.0046	.0027	.0016	.0009	.0005

TABLE 3.- LOAD DISTRIBUTION DUE TO A UNIT SHEAR PERTURBATION LOAD  
ABOUT SHEAR PANEL (0,0) - Continued

$$[C = 0; m = 36]$$

(b) B = 30

j	Stringer load, $p_{1j}/L$ , at station -					
	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
1	-0.2242	-0.0331	-0.0081	-0.0029	-0.0013	-0.0007
2	-.0377	-.0329	-.0142	-.0064	-.0032	-.0017
3	-.0082	-.0148	-.0109	-.0064	-.0037	-.0022
4	-.0012	-.0060	-.0062	-.0047	-.0032	-.0021
5	.0013	-.0017	-.0029	-.0028	-.0022	-.0016
6	.0026	.0005	-.0008	-.0012	-.0012	-.0010
7	.0034	.0017	.0006	-.0001	-.0004	-.0004
8	.0039	.0025	.0014	.0007	.0003	.0000
9	.0042	.0029	.0020	.0012	.0008	.0004
10	.0042	.0031	.0023	.0016	.0011	.0007
11	.0041	.0032	.0024	.0017	.0012	.0009
12	.0039	.0031	.0023	.0018	.0013	.0009
13	.0036	.0028	.0022	.0017	.0013	.0009
14	.0031	.0024	.0019	.0015	.0011	.0009
15	.0025	.0020	.0016	.0012	.0010	.0007
16	.0018	.0015	.0012	.0009	.0007	.0006
17	.0011	.0009	.0007	.0006	.0004	.0003
18	.0004	.0003	.0002	.0002	.0002	.0001

j	Shear flow, $q_{1j}$ , at station -					
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0	0.5133	0.1726	0.0395	0.0146	0.0070	0.0038
1	-.0382	-.0186	.0145	.0094	.0054	.0031
2	.0372	-.0233	-.0042	.0016	.0021	.0016
3	.0537	-.0167	-.0081	-.0028	-.0006	.0001
4	.0562	-.0120	-.0079	-.0043	-.0022	-.0010
5	.0535	-.0089	-.0067	-.0044	-.0027	-.0016
6	.0482	-.0067	-.0054	-.0040	-.0028	-.0018
7	.0413	-.0050	-.0042	-.0033	-.0025	-.0018
8	.0336	-.0037	-.0032	-.0026	-.0020	-.0015
9	.0253	-.0024	-.0022	-.0019	-.0016	-.0012
10	.0168	-.0014	-.0013	-.0012	-.0010	-.0009
11	.0085	-.0004	-.0005	-.0006	-.0006	-.0005
12	.0007	.0005	.0002	.0000	-.0001	-.0001
13	-.0064	.0012	.0008	.0005	.0003	.0002
14	-.0126	.0018	.0013	.0010	.0007	.0005
15	-.0176	.0023	.0018	.0013	.0010	.0007
16	-.0212	.0027	.0021	.0016	.0012	.0009
17	-.0235	.0029	.0022	.0017	.0013	.0010
18	-.0243	.0030	.0023	.0018	.0013	.0010

TABLE 3.- LOAD DISTRIBUTION DUE TO A UNIT SHEAR PERTURBATION LOAD

ABOUT SHEAR PANEL (0,0) - Continued

$$[\bar{C} = 0; m = 36]$$

(c) B = 100

j	Stringer load, $p_{1j}/L$ , at station -					
	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
1	-0.3168	-0.1198	-0.0485	-0.0214	-0.0103	-0.0055
2	-.0288	-.0433	-.0335	-.0226	-.0148	-.0097
3	-.0043	-.0127	-.0147	-.0132	-.0107	-.0083
4	.0004	-.0035	-.0057	-.0064	-.0062	-.0054
5	.0022	.0001	-.0015	-.0025	-.0029	-.0029
6	.0032	.0018	.0007	-.0002	-.0008	-.0011
7	.0038	.0028	.0019	.0012	.0006	.0002
8	.0042	.0034	.0027	.0020	.0015	.0010
9	.0045	.0037	.0031	.0025	.0020	.0016
10	.0045	.0039	.0033	.0028	.0023	.0019
11	.0044	.0038	.0033	.0028	.0024	.0020
12	.0041	.0036	.0032	.0027	.0024	.0020
13	.0037	.0033	.0029	.0025	.0022	.0019
14	.0032	.0028	.0025	.0022	.0019	.0017
15	.0026	.0023	.0020	.0018	.0016	.0014
16	.0019	.0017	.0015	.0013	.0012	.0010
17	.0012	.0010	.0009	.0008	.0007	.0006
18	.0004	.0004	.0003	.0003	.0002	.0002

j	Shear flow, $q_{1j}$ , at station -					
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0	0.3632	0.1628	0.0704	0.0337	0.0179	0.0104
1	-.0032	-.0342	-.0009	.0065	.0068	.0056
2	.0546	-.0197	-.0106	-.0043	-.0010	.0005
3	.0632	-.0114	-.0086	-.0058	-.0035	-.0020
4	.0624	-.0075	-.0064	-.0051	-.0038	-.0027
5	.0579	-.0053	-.0048	-.0041	-.0034	-.0027
6	.0515	-.0040	-.0036	-.0032	-.0028	-.0024
7	.0438	-.0029	-.0027	-.0025	-.0022	-.0020
8	.0353	-.0021	-.0020	-.0018	-.0017	-.0015
9	.0264	-.0014	-.0013	-.0013	-.0012	-.0011
10	.0174	-.0008	-.0008	-.0008	-.0007	-.0007
11	.0086	-.0002	-.0002	-.0003	-.0003	-.0003
12	.0004	.0003	.0002	.0001	.0001	.0000
13	-.0071	.0008	.0006	.0005	.0004	.0003
14	-.0135	.0011	.0010	.0008	.0007	.0006
15	-.0188	.0014	.0012	.0011	.0009	.0008
16	-.0226	.0017	.0014	.0012	.0011	.0009
17	-.0250	.0018	.0016	.0013	.0012	.0010
18	-.0258	.0018	.0016	.0014	.0012	.0010

TABLE 3.- LOAD DISTRIBUTION DUE TO A UNIT SHEAR PERTURBATION LOAD

ABOUT SHEAR PANEL (0,0) - Continued

$$[\bar{c} = 0; m = 36]$$

(a) B = 300

j	Stringer load, $p_{ij}/L$ , at station -					
	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
1	-0.3817	-0.2188	-0.1276	-0.0759	-0.0462	-0.0288
2	-.0197	-.0402	-.0427	-.0383	-.0319	-.0256
3	-.0018	-.0084	-.0125	-.0143	-.0145	-.0138
4	.0014	-.0012	-.0033	-.0049	-.0058	-.0063
5	.0027	.0014	.0002	-.0008	-.0016	-.0022
6	.0035	.0027	.0019	.0012	.0006	.0000
7	.0041	.0034	.0029	.0023	.0018	.0014
8	.0044	.0039	.0034	.0030	.0026	.0022
9	.0046	.0042	.0038	.0034	.0030	.0027
10	.0046	.0042	.0039	.0035	.0032	.0029
11	.0045	.0042	.0038	.0035	.0032	.0030
12	.0042	.0039	.0036	.0034	.0031	.0029
13	.0038	.0036	.0033	.0031	.0029	.0026
14	.0033	.0031	.0029	.0027	.0025	.0023
15	.0027	.0025	.0023	.0022	.0020	.0019
16	.0020	.0018	.0017	.0016	.0015	.0014
17	.0012	.0011	.0011	.0010	.0009	.0009
18	.0004	.0004	.0004	.0003	.0003	.0003

j	Shear flow, $q_{ij}$ , at station -					
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0	0.2618	0.1287	0.0782	0.0489	0.0315	0.0209
1	.0253	-.0341	-.0130	-.0028	.0017	.0035
2	.0648	-.0137	-.0105	-.0072	-.0046	-.0027
3	.0684	-.0071	-.0064	-.0054	-.0044	-.0035
4	.0657	-.0045	-.0043	-.0039	-.0034	-.0030
5	.0602	-.0032	-.0031	-.0029	-.0027	-.0024
6	.0532	-.0024	-.0023	-.0022	-.0020	-.0019
7	.0450	-.0017	-.0017	-.0016	-.0015	-.0015
8	.0362	-.0012	-.0012	-.0012	-.0011	-.0011
9	.0270	-.0008	-.0008	-.0008	-.0008	-.0007
10	.0177	-.0004	-.0004	-.0004	-.0004	-.0004
11	.0087	-.0001	-.0001	-.0001	-.0001	-.0002
12	.0002	.0002	.0002	.0002	.0001	.0001
13	-.0074	.0005	.0004	.0004	.0003	.0003
14	-.0140	.0007	.0006	.0006	.0005	.0005
15	-.0194	.0009	.0008	.0007	.0007	.0006
16	-.0234	.0010	.0009	.0009	.0008	.0007
17	-.0258	.0011	.0010	.0009	.0008	.0008
18	-.0266	.0011	.0010	.0010	.0009	.0008

TABLE 3.- LOAD DISTRIBUTION DUE TO A UNIT SHEAR PERTURBATION LOAD

ABOUT SHEAR PANEL (0,0) - Concluded

$$[C = 0; m = 36]$$

(e)  $B = 1,000$ 

j	Stringer load, $p_{ij}/L$ , at station -					
	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
1	-0.4300	-0.3169	-0.2346	-0.1745	-0.1305	-0.0981
2	-.0118	-.0297	-.0388	-.0424	-.0425	-.0406
3	-.0002	-.0043	-.0077	-.0104	-.0124	-.0136
4	.0020	.0005	-.0009	-.0022	-.0033	-.0042
5	.0030	.0023	.0015	.0008	.0002	-.0004
6	.0037	.0032	.0028	.0023	.0019	.0015
7	.0042	.0039	.0035	.0032	.0029	.0026
8	.0045	.0042	.0040	.0037	.0035	.0032
9	.0047	.0045	.0042	.0040	.0038	.0036
10	.0047	.0045	.0043	.0041	.0039	.0037
11	.0046	.0044	.0042	.0040	.0038	.0037
12	.0043	.0041	.0040	.0038	.0036	.0035
13	.0039	.0038	.0036	.0035	.0033	.0032
14	.0034	.0032	.0031	.0030	.0029	.0028
15	.0027	.0026	.0025	.0024	.0024	.0023
16	.0020	.0019	.0019	.0018	.0017	.0017
17	.0012	.0012	.0011	.0011	.0011	.0010
18	.0004	.0004	.0004	.0004	.0004	.0004

j	Shear flow, $q_{ij}$ , at station -					
	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0	0.1879	0.0870	0.0658	0.0502	0.0386	0.0299
1	.0479	-.0261	-.0165	-.0099	-.0055	-.0025
2	.0715	-.0082	-.0074	-.0064	-.0054	-.0044
3	.0718	-.0040	-.0039	-.0037	-.0034	-.0031
4	.0678	-.0025	-.0025	-.0024	-.0023	-.0022
5	.0617	-.0018	-.0018	-.0017	-.0017	-.0016
6	.0542	-.0013	-.0013	-.0013	-.0012	-.0012
7	.0458	-.0010	-.0010	-.0009	-.0009	-.0009
8	.0367	-.0007	-.0007	-.0007	-.0007	-.0006
9	.0273	-.0004	-.0004	-.0004	-.0004	-.0004
10	.0179	-.0002	-.0002	-.0002	-.0002	-.0002
11	.0088	.0000	.0000	.0000	-.0001	-.0001
12	.0001	.0001	.0001	.0001	.0001	.0001
13	-.0076	.0003	.0003	.0002	.0002	.0002
14	-.0144	.0004	.0004	.0004	.0003	.0003
15	-.0198	.0005	.0005	.0005	.0004	.0004
16	-.0238	.0006	.0006	.0005	.0005	.0005
17	-.0263	.0006	.0006	.0006	.0005	.0005
18	-.0271	.0006	.0006	.0006	.0006	.0005



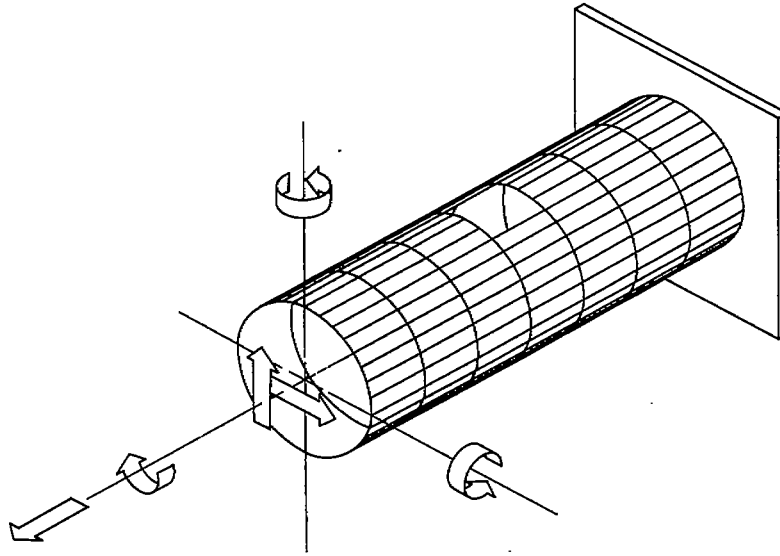


Figure 1.- Circular semimonocoque cylinder with cutout.

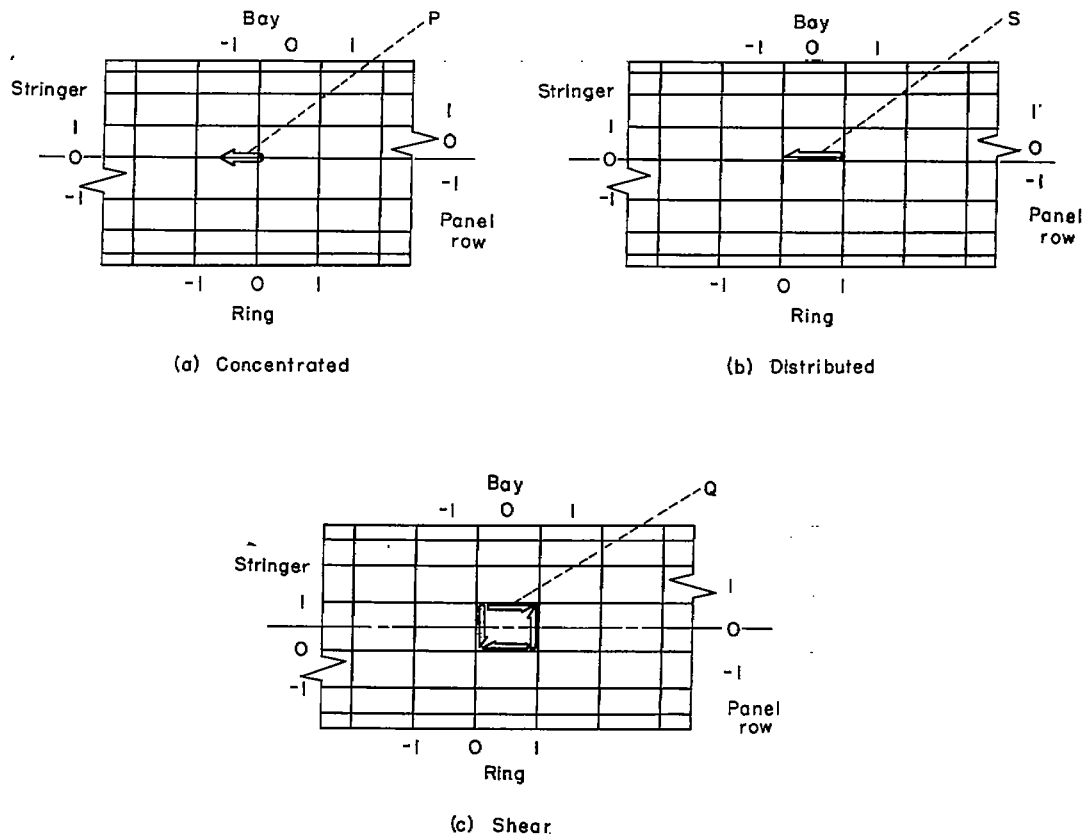


Figure 2.- Perturbation loads.

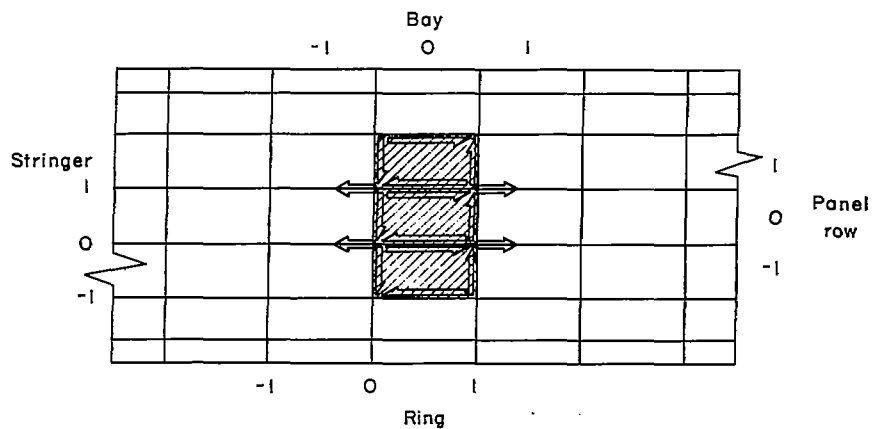


Figure 3.- Application of perturbation loads.

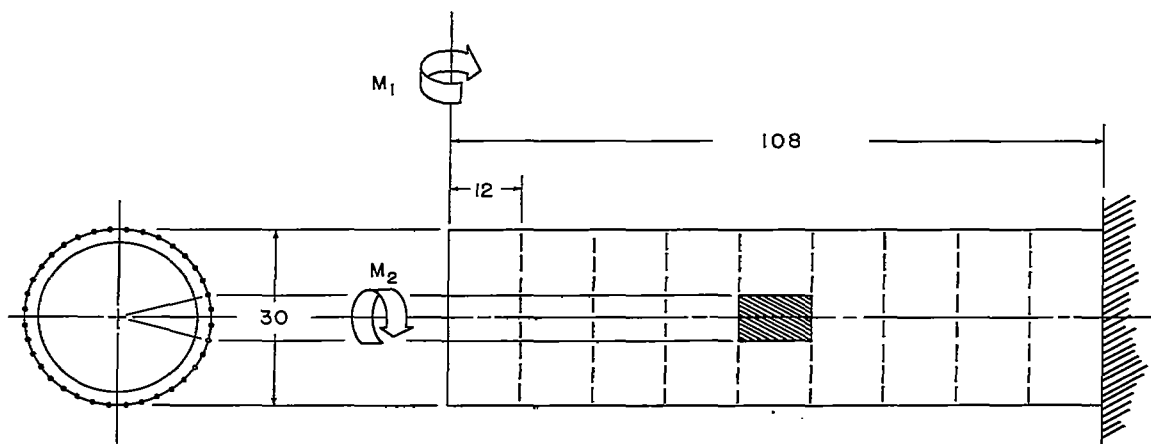


Figure 4.- Circular cylinder with cutout used in sample calculation.

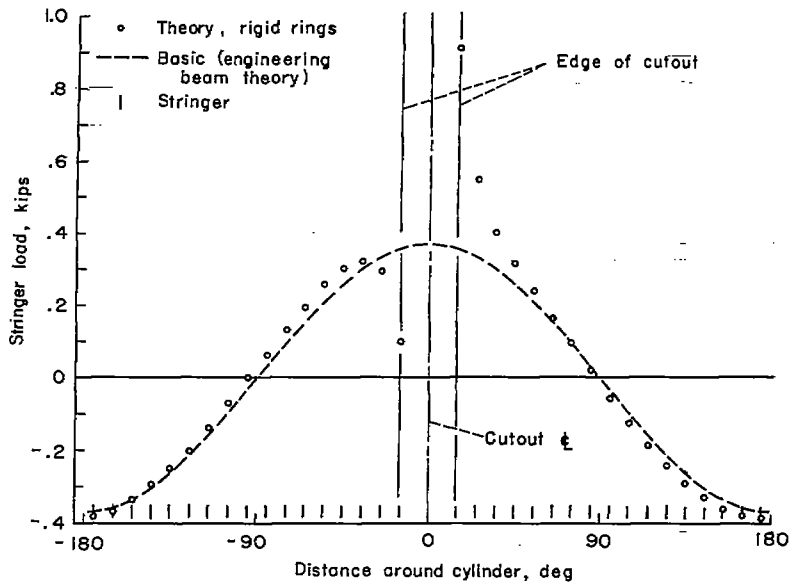
(a) Stringer loads at ring bordering cutout (ring  $l=0$ ).

Figure 5.- Results of sample calculation.

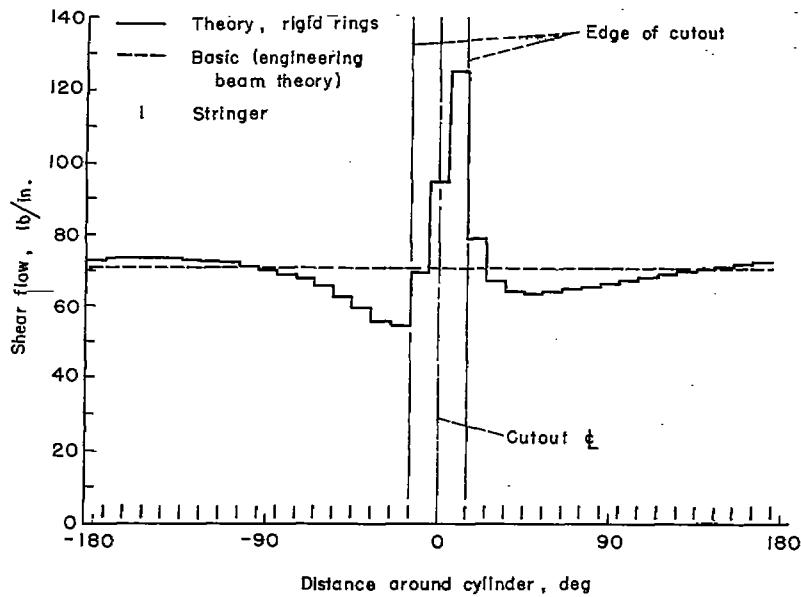
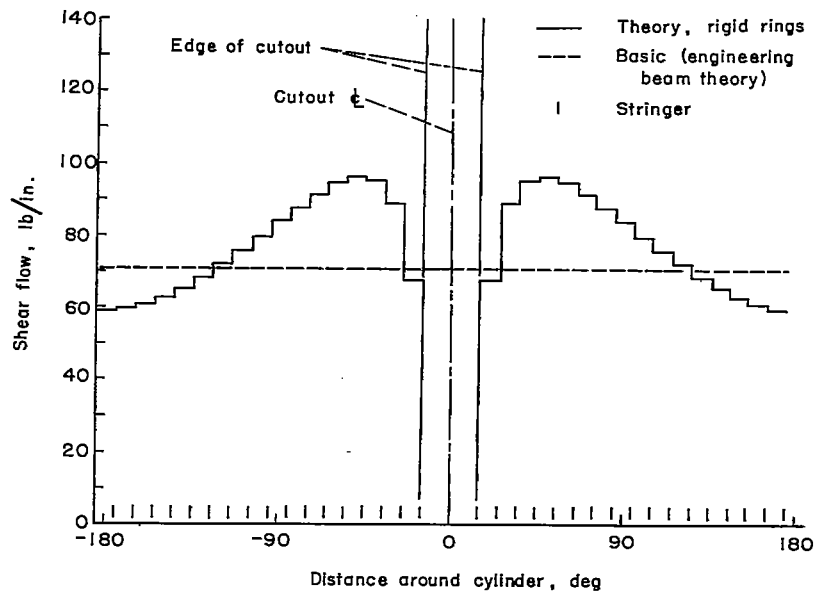
(b) Shear flow in bay adjacent to cutout (bay  $l=-1$ ).

Figure 5.- Continued.



(c) Shear flow in net section (bay  $l=0$ ).

Figure 5.- Concluded.

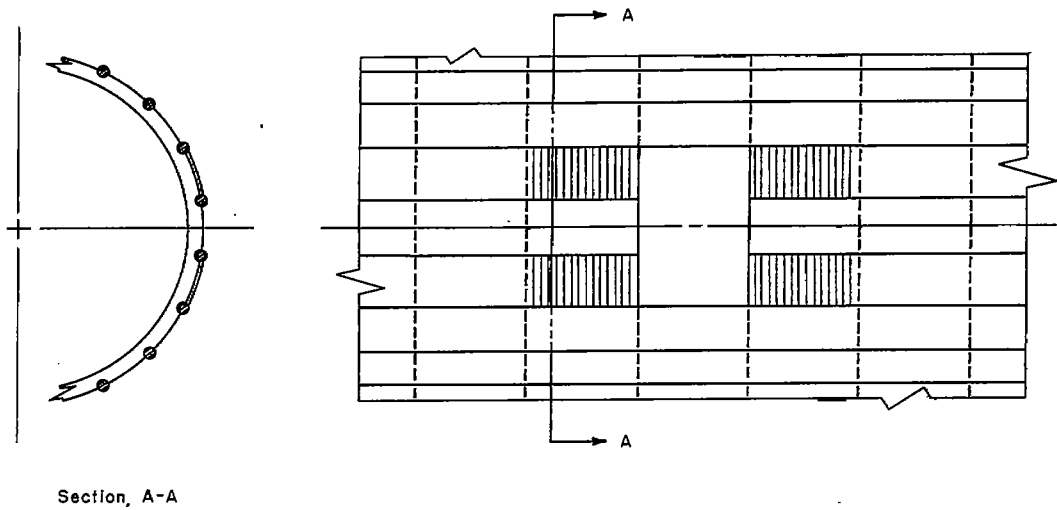


Figure 6.- Cutout with shear reinforcement.

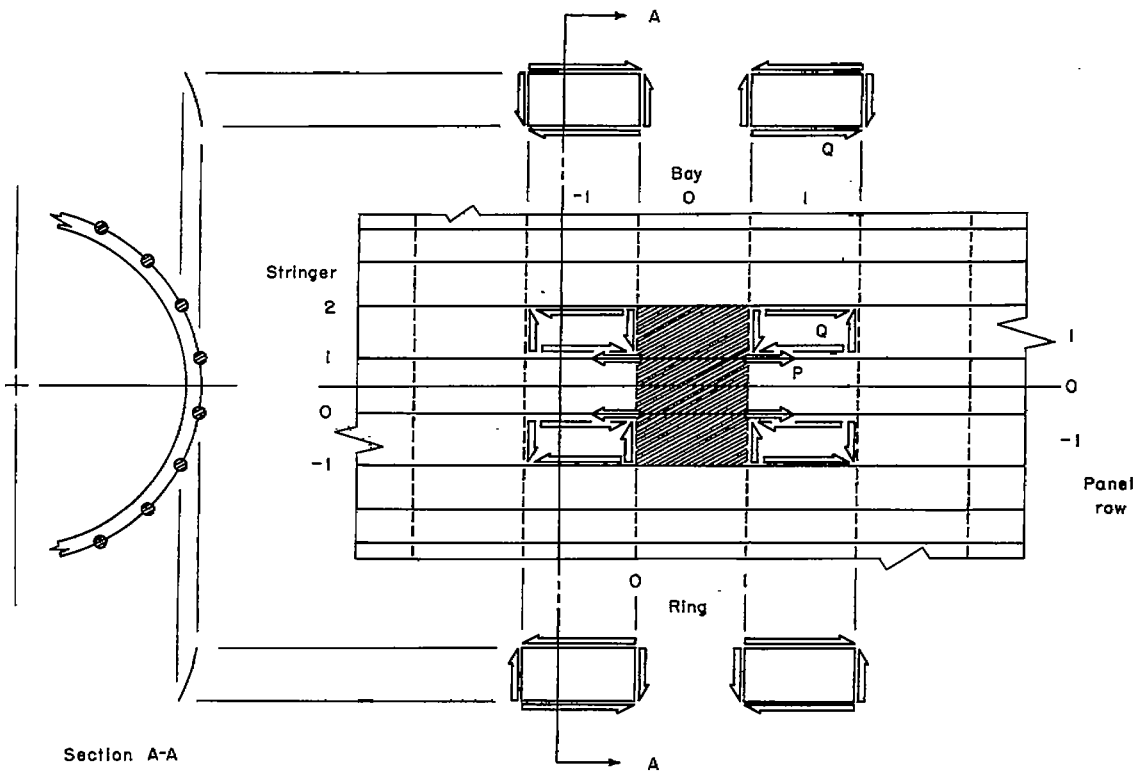
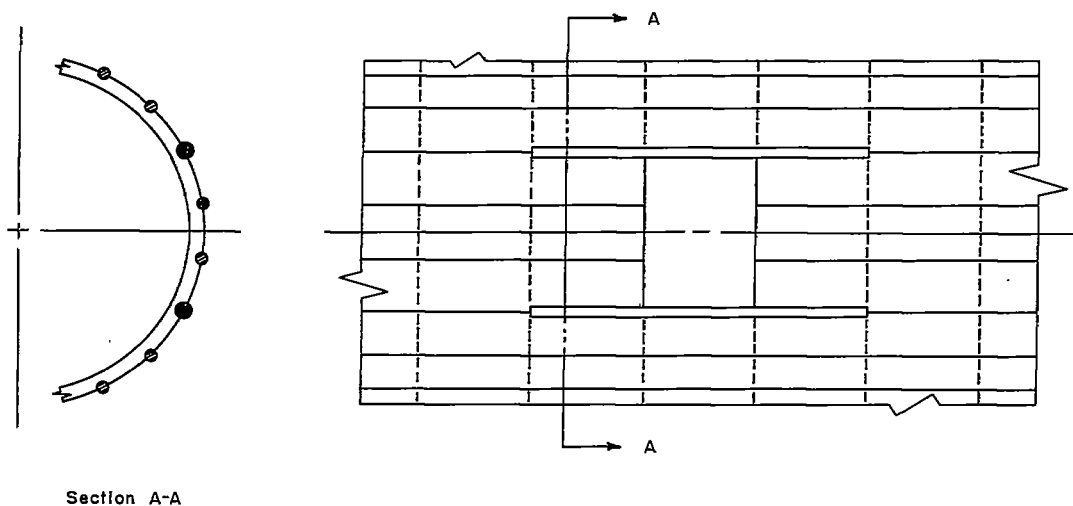
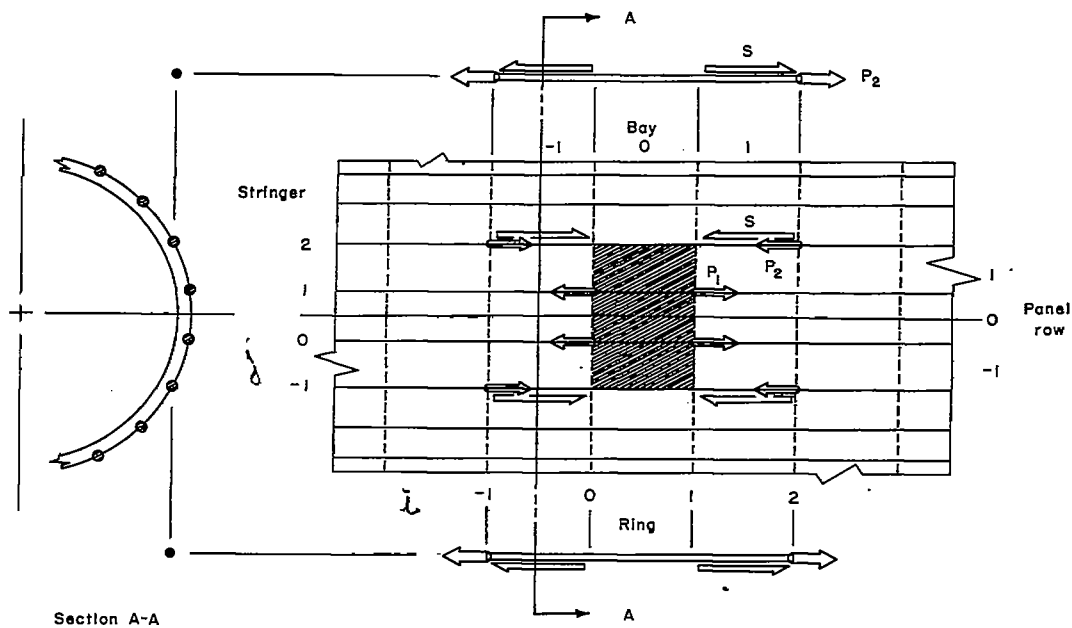


Figure 7.- Perturbation load system for a problem of shear reinforcement.



Section A-A

Figure 8.- Cutout with reinforced coaming stringers.



Section A-A

Figure 9.- Perturbation load system for a problem of coaming-stringer reinforcement.