NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2688

THREE-DIMENSIONAL SUPERSONIC NOZZLES AND INLETS OF ARBITRARY EXIT CROSS SECTION

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SUMMARY

A method is presented for obtaining three-dimensional unsymmetric supersonic nozzles and inlets from known axisymmetric flows. Streamlines bounding the desired exit shape are traced through the known basic flow solution to give the required unsymmetric wall contours. Several examples are given.

INTRODUCTION

Because of the great complexity of the method of characteristics in three dimensions (see, for example, references 1 and 2), the accurate theoretical design of general three-dimensional supersonic nozzles and diffusers has not been feasible. The designer is thus limited to two-dimensional and axisymmetric flows. In many cases this may be a serious disadvantage. In the case, particularly, of a hypersonic wind tunnel, it would be very helpful if a three-dimensional expansion could be used to avoid the usual thin slit which would appear as the throat of such a nozzle if it were two dimensional. Furthermore, such a nozzle might have better secondary flow characteristics than the corresponding two-dimensional nozzle in which transverse pressure gradients may cause cross flow in, and local build-up of, the boundary layer. However, the usual three-dimensional nozzle is axially symmetric and thus precludes the use of flat schlieren windows. In the design of supersonic inlets, considerations of space, as well as of permissible flow turning, make the use of shapes other than two-dimensional or axisymmetric ones desirable.

A simple method for obtaining three-dimensional nozzles from axisymmetrical ones has been developed at the NACA Lewis laboratory and is presented herein. The procedure is exact within the limitations of the method of characteristics.

METHOD

In any inviscid fluid flow, the streamsheets, by definition, form surfaces across which there is no flow and hence may be replaced by solid boundaries. This fact will herein be applied to find unsymmetric nozzles and inlets from axially symmetric flows. Two salient features of an axisymmetric flow are that the streamlines lie in planes through the streamwise axis and that the flow in any one such plane is the same as that in any other.

In order to find the nozzle of a desired shape, the axisymmetric nozzle having the desired length and Mach number characteristics is computed and then, choosing the desired cross section at some station, for example, the exit, the streamlines which pass through the periphery of that section are found. The streamlines can be determined either by constructing them from the known local flow directions or by the better method of integrating the axisymmetric stream function (appendix). The streamsheet formed by these streamlines will constitute the walls of the desired inlet or nozzle.

If the initial and final flows are uniform and axial, it can readily be shown that the cross-section shapes at the inlet and exit will be similar. For any arbitrary streamline, the ratio of the radii at the initial and final stations is given by the one-dimensional continuity equation for isentropic flow:

$$\left(\frac{r_i}{r_f}\right)^2 = \frac{M_f}{M_i}\left(\frac{1 + \frac{\gamma-1}{2} M_i^2}{1 + \frac{\gamma-1}{2} M_f^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = K^2 \tag{1}$$

where M and r are Mach number and radius, respectively, and the subscripts i and f refer to the initial and final stations. This ratio depends only on the initial and final Mach numbers and hence is independent of the initial streamline. Likewise each streamline continues in its own axial plane throughout the nozzle. Thus, the initial and final shapes will be similar, all dimensions being changed by the ratio K of equation (1). In most applications, the condition that the flow be uniform and axial at the inlet and exit is desirable. In the region between the two ends, the cross-section shape will generally vary, although this variation has been found small in the flows computed up to the present time. It may be expected that the variation of shape increases with the rapidity of Mach number change between the inlet and exit.

EXAMPLES

Two axisymmetric nozzles were designed by the method of characteristics (see, for example, reference 3). The solution was started by a series expansion near the throat, using the method of reference 4.

These nozzles were designed to expand from sonic velocity to a Mach number of 2.98 and 5.70, respectively, and to have uniform axial inlet and exit flow. From these calculations, the resulting streamlines were found by integrating along a characteristic to find the stream function. The details of this computation are described in the appendix. In figures 1 and 2 are given the results of this calculation. With the use of these results, nozzles or inlets of any shape are readily formed. The details for the case of the Mach number 2.98 nozzle of figure 3 will be given. The exit section was selected as a square centered on the axis and having a side of length 14.5. The stream function was then found (at the exit) from equation (6) of the appendix for the periphery of the square. Throughout the length of the nozzle on the wall, ψ will be constant for constant values of the cylindrical angle. Thus, for the example considered, at the corner of the section, $\psi = 13.10$. Then, from figure 1 at $x = 30$, for instance, for this value of ψ, r is equal to 16.20. In this manner the shape of the nozzle may be found. The result is shown in figure 3. This illustrates how little the cross section varies from the initial shape for this particular basic flow. In figure 4, there is shown a similar nozzle designed for $M = 5.70$. In this case it is seen that the nozzle shape shows approximately the same variations as in the case of the lower Mach number. Of course, if exit cross sections with only two flat walls are required, the other two could have circular-arc cross sections. Such a configuration would probably be easier to fabricate than the square nozzles already discussed but would still have flat side walls suitable for schlieren windows.

If it is desired to have perfect flow at the exit, such as is required in a wind tunnel nozzle, and still to have constant cross-section shape, the method of reference 5 might be used to modify the cross sections already found. This possibility was pointed out by Ferri at the February 1952 meeting of the American Physical Society. On the other hand, if it is not essential to have perfect flow at the exit, sufficient accuracy might be obtained by the use of constant cross-section shape through the duct.

Another use of the method would be for the design of diffuser inlets. Such a configuration is shown in figure 5. In this case the inlet and exit sections are circles tangent to the axis of the basic flow. Since it was designed to diffuse to $M = 1$, the inlet would have to be perforated in order for the shock to be swallowed, but not as much as in the case of the usual axisymmetric perforated inlet, because some external compression is obtained by cutting the diffuser back along the Mach cone A (fig. 5). For the same reason, if the inlet were designed for only the Kantrowitz contraction ratio (reference 6) so that no perforations were needed, the permissible contraction ratio based on the free-stream tube swallowed would be increased.

To illustrate this point, rough computations were made for two convergent-divergent diffusers, one symmetric and the other not. A conventional convergent-divergent diffuser designed for a Mach number of 2.98 has a permissible contraction ratio of 1.39 and a pressure recovery of 44.9 percent when the shock occurs at the throat Mach number of 2.63. The comparable diffuser designed by the present method would be one which appears as in figure 5, except that the area ratio from section B to the throat would be the Kantrowitz ratio corresponding to the average Mach number at section B (fig. 5). To get some numbers, this average Mach number may be assumed to be 2.70. The permissible contraction ratio for this Mach number is 1.35 giving an over-all contraction ratio for the diffuser of 1.76 and a pressure recovery of 54.8 percent when the shock occurs at the throat Mach number of 2.38. These two inlets have the same mass flow and entrance area, but the unsymmetric one has about 10 percent higher theoretical pressure recovery than the usual convergent-divergent diffuser.

The configuration shown in figure 5 could also represent a convergent-divergent exit jet. Such a jet might be useful in fairing the exit to the tail structure or aft portion of an airplane.

CONCLUDING REMARKS

A method has been presented for obtaining theoretical wall contours of supersonic nozzles and inlets of arbitrary cross-section shape from known axisymmetric or two-dimensional flows. If the basic flow is axisymmetric and uniform at the throat and the exit, the desired nozzle will have similar cross sections at these stations. For the examples computed, the wall contour cross sections between the throat and exit did not deviate drastically from similarity with the initial shape.

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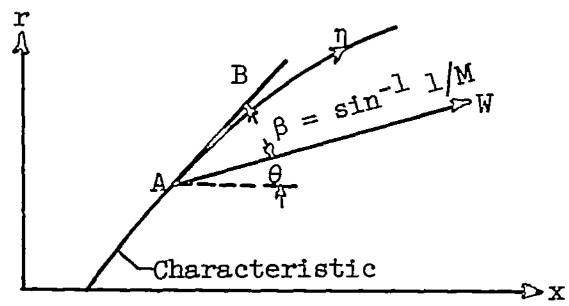
APPENDIX

DETERMINATION OF STREAMLINES IN AXISYMMETRIC FLOWS FROM KNOWN VELOCITIES ON A CHARACTERISTIC NET

The continuity equation in axially symmetric flow may be written as

$$\frac{\partial}{\partial x}(\rho W r \cos\theta) + \frac{\partial}{\partial r}(\rho W r \sin\theta) = 0$$

where ρ is the density, W the speed, and the remaining quantities are defined by the sketch.



Hence, there exists a stream function, $\psi(x,r)$, such that

$$\frac{\partial\psi}{\partial x} = -\frac{\rho W r \sin\theta}{\rho_0} \qquad \frac{\partial\psi}{\partial r} = \frac{\rho W r \cos\theta}{\rho_0} \tag{2}$$

where ρ_0 is the stagnation density. Then, along the η-characteristic

$$\frac{\partial\psi}{\partial\eta} = \frac{\partial\psi}{\partial x}\frac{\partial x}{\partial\eta} + \frac{\partial\psi}{\partial r}\frac{\partial r}{\partial\eta}$$

or, from the sketch

$$\frac{\partial\psi}{\partial\eta} = \frac{\partial\psi}{\partial x}\cos(\beta+\theta) + \frac{\partial\psi}{\partial r}\sin(\beta+\theta) \tag{3}$$

If equations (2) are substituted into (3), there results, after some manipulation,

$$\frac{\partial\psi}{\partial\eta} = \frac{\rho}{\rho_0}\frac{Wr}{M} \tag{4}$$

From the sketch,

$$d\eta = \frac{dr}{\sin(\beta+\theta)}$$

Therefore,

$$\psi_B - \psi_A = \int_A^B \frac{\rho W}{\rho_0 M \sin(\beta + \theta)} r\, dr$$

If the interval AB is small, say the distance between two points on the characteristic net, then it may be assumed that the quantity $\frac{\rho W}{\rho_0 M \sin(\beta + \theta)}$ takes its average. Experience shows that this will be a particularly good approximation along characteristics proceeding downstream away from the axis. Then

$$\psi_B - \psi_A = \left[\left(\frac{\rho W}{4\rho_0 M \sin(\beta + \theta)}\right)_A + \left(\frac{\rho W}{4\rho_0 M \sin(\beta + \theta)}\right)_B\right] (r_B^2 - r_A^2) \quad (5)$$

At the exit, $\theta = 0$ and $M =$ constant. In this case, from the second of equations (2), if the axis is taken as the $\psi = 0$ streamline,

$$\psi_B = \left(\frac{\rho W}{2\rho_0}\right)_E r_B^2 \quad (6)$$

Equations (5) and (6) are as accurate as the characteristic diagram. In the applications of this report, W has been made nondimensional by dividing by the maximum speed.

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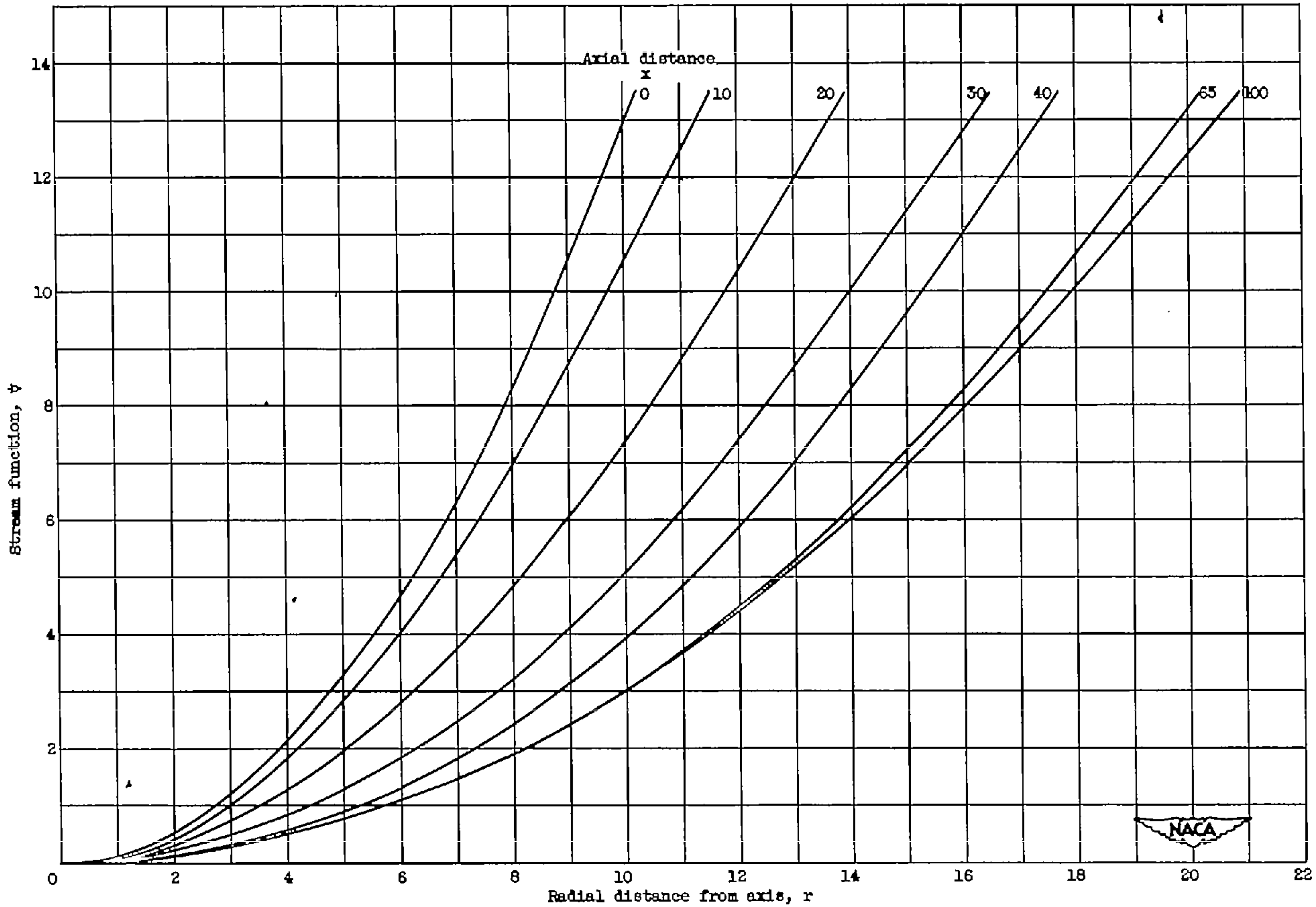


Figure 1. - Variation of stream function with position for Mach number of 2.98.

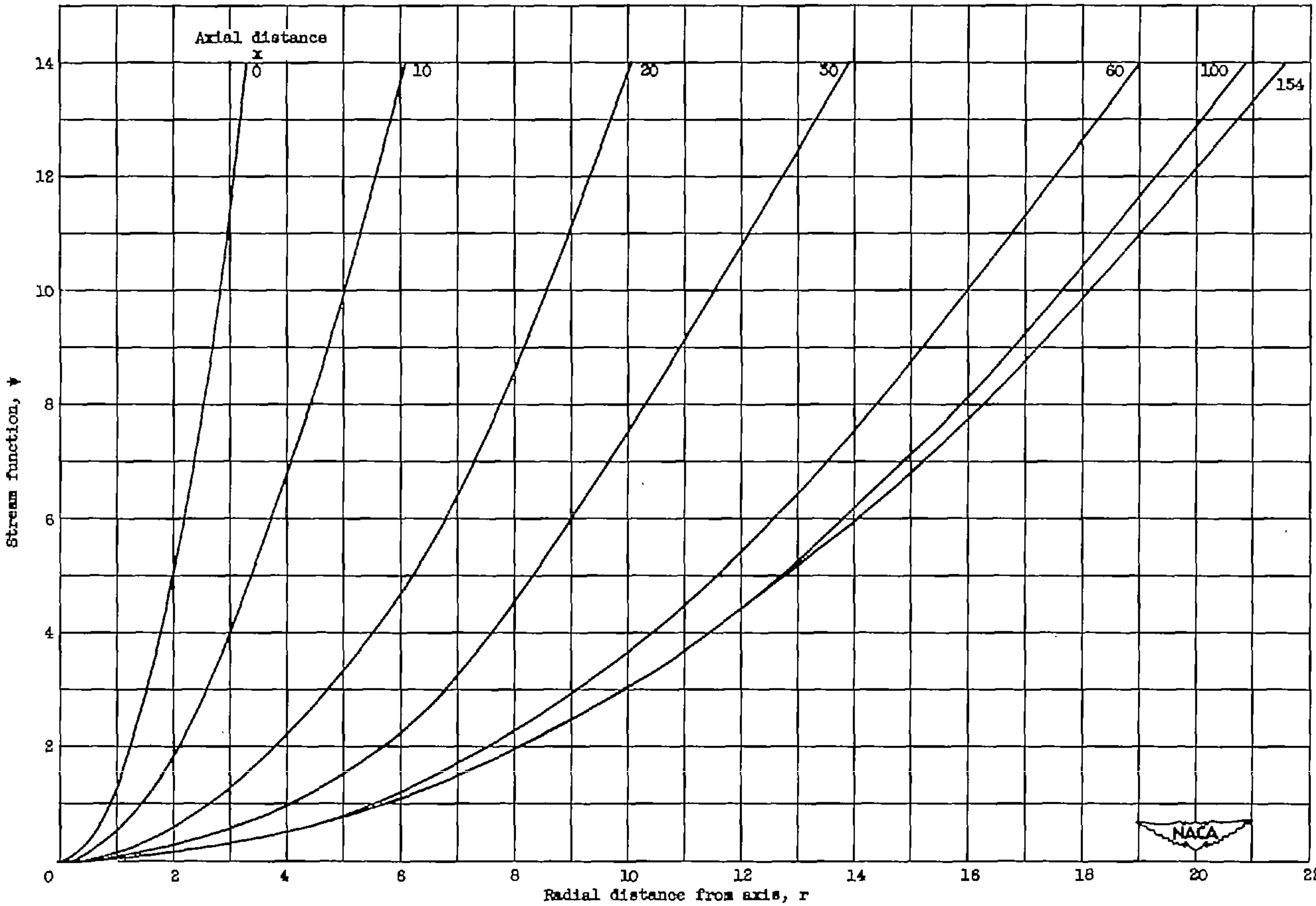


Figure 2. - Variation of stream function with position for Mach number of 5.70.

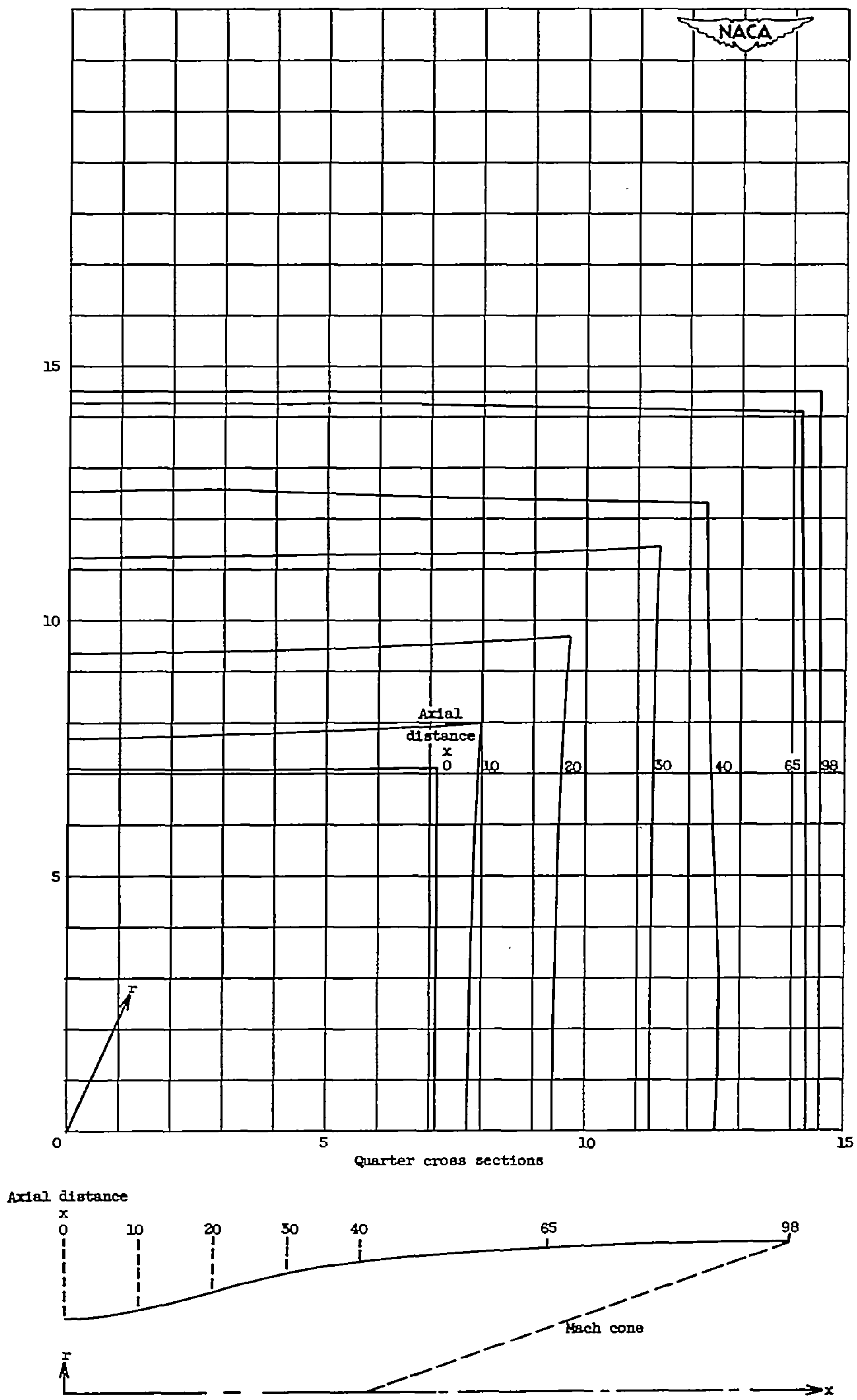


Figure 3. - Square nozzle for Mach number of 2.98.

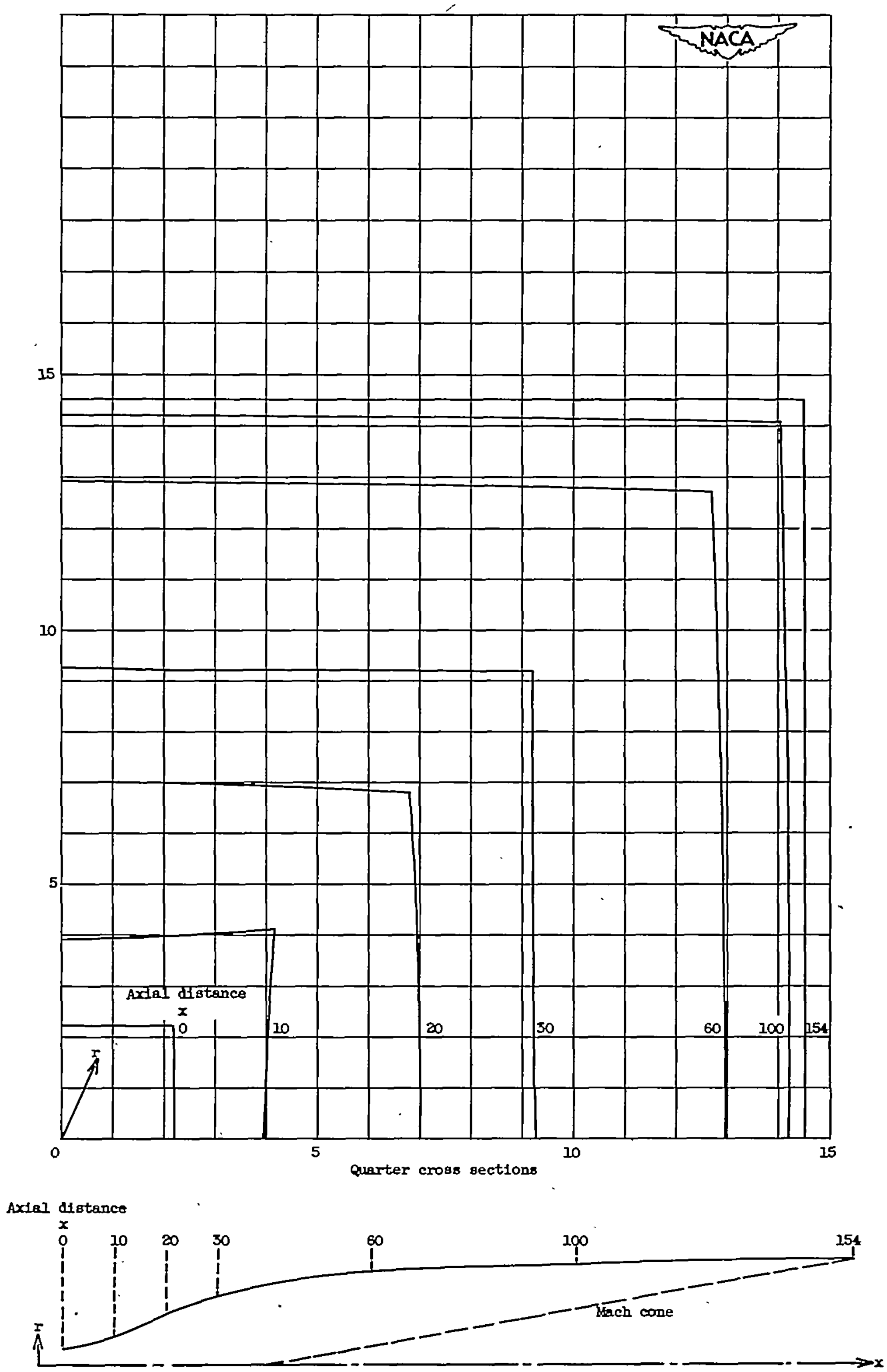
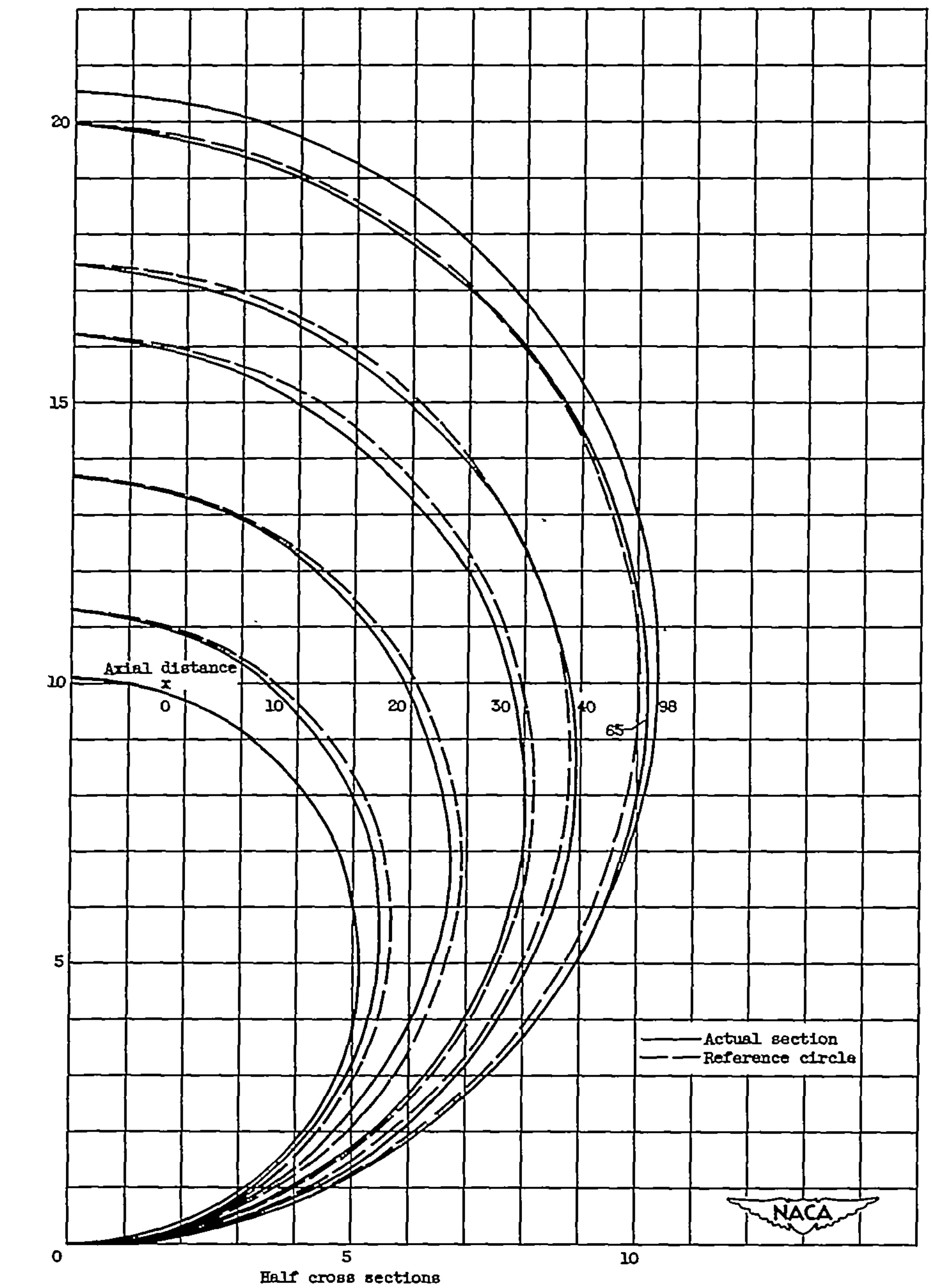


Figure 4. - Square nozzle for Mach number of 5.70.



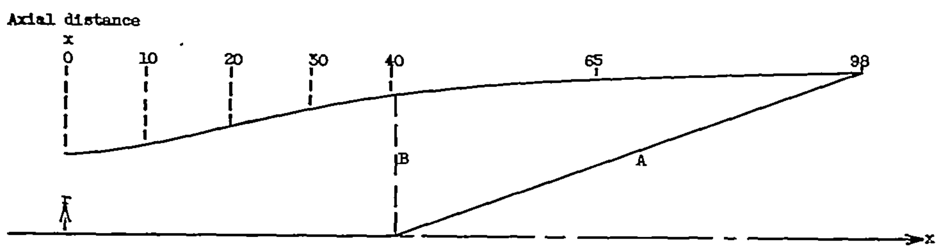


Figure 5. - Unsymmetric circular inlet or nozzle for Mach number of 2.98.