# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

## TECHNICAL NOTE 3386

## SOME CONSIDERATIONS ON TWO-DIMENSIONAL THIN AIRFOILS DEFORMING IN SUPERSONIC FLOW

By Eugene Migotsky

Ames Aeronautical Laboratory
Moffett Field, Calif.

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## SUMMARY

The aerodynamic characteristics of indicially cambered two-dimensional airfoils in supersonic flow are determined theoretically. These indicial functions are used to determine the power required to sustain a general time-varying chordwise deformation. The harmonically oscillating parabolic mode is considered in detail and stability boundaries are presented for this case. Also, the thickness distribution of a beam having a parabolic fundamental bending mode, in vacuo, is determined.

## INTRODUCTION

Until recently, little consideration had been given to the aeroelastic phenomena associated with chordwise deformations of wings. Thus, while spanwise elastic modes were considered in most analyses of aeroelastic phenomena, it was generally assumed that the displacement of a spanwise station was restricted to translations and rotations. In recent years more attention has been given to the problems associated with chordwise deformations. One of these problems is the chordwise bending of a two-dimensional airfoil in a supersonic stream.

Biot (ref. 1) appears to have been the first to consider the static instability of a two-dimensional airfoil in a supersonic stream. Miles, in reference 2, investigated the static and dynamic stability of a cantilevered (chordwise) airfoil section in supersonic flow by including low-frequency aerodynamic forces. In the present paper we consider the dynamic behavior of a two-dimensional airfoil in a supersonic stream, without the restriction of low frequency in the determination of the air forces.

The aerodynamic characteristics of airfoils which are indicially cambered in a relatively arbitrary manner are determined, by linear theory, from known, or easily obtained, characteristics of an indicially plunged airfoil.[1] These indicial characteristics are then used to obtain

[1]In work performed independently and concurrently, Lomax, Fuller, and Sluder (ref. 3) have obtained similar results for the rectangular wing, but by a different method.

the rate of energy input (power) required to sustain a general time-varying chordwise deformation.

The particular case of the harmonically oscillating parabolic arc was chosen for detailed consideration. In addition, the thickness distribution of a beam having a quadratic normal bending mode, in vacuo, was determined.

## SYMBOLS

$a_o$ — half-amplitude of oscillation of leading edge

$A_o$ — section mean-power coefficient (See eq. (21).)

$A_h, A_{\dot{h}}$ — functions defined by equation (15)

$c$ — chord of wing

$c_o$ — speed of sound in free stream

$c_l$ — section lift coefficient, $\frac{\text{section lift}}{q_o c^2}$

$c_m$ — section pitching-moment coefficient, $\frac{1}{q_o c^2} \times$ section moment measured about leading edge

$c_{l_\alpha}$ — indicial section lift coefficient due to angle-of-attack change only

$c_{l_q}$ — indicial section lift coefficient due to pitching on a wing rotating about its leading edge

$c_{m_\alpha}$ — indicial section pitching-moment coefficient due to angle-of-attack change only, measured about the leading edge and considered positive when the trailing edge is forced down

$c_{m_q}$ — indicial section pitching-moment coefficient due to pitching on a wing rotating about its leading edge, measured about the leading edge and considered positive when the trailing edge is forced down

$c_{m,n}$ — generalized indicial force coefficient (See eq. (6).)

$E$ — Young's modulus

$E_1$ — $\frac{E}{1 - \nu^2}$

| | |
|---|---|
| $f_o$ | Schwartz function, $f_{o_R} + if_{o_I} = \frac{1}{\bar{\omega}}\int_0^{\bar{\omega}} e^{-iu} J_o\left(\frac{u}{M}\right) du$ |
| F,G | functions defined by equation (18) |
| $F_{c_0}, F_{c_1}$ | functions defined by equation (A8) |
| $G_{s_0}, G_{s_1}, G_{s_2}$ | functions defined by equation (A9) |
| h | function giving timewise dependence of vertical position of wing |
| $h_o$ | half-amplitude of oscillation of midchord point measured relative to leading edge of wing (sketch (c)) |
| I | moment of inertia of cross-sectional area of beam with respect to neutral axis |
| $J_n$ | Bessel function of the first kind of order n |
| k | reduced frequency, $\frac{\omega c}{2V_o}$ |
| M | flight Mach number |
| p | static pressure |
| $\Delta p$ | discontinuity in pressure across $z = 0$ plane |
| P | loading coefficient, $\frac{\Delta p}{q_o}$ |
| $P_n$ | loading coefficient corresponding to a normal velocity distribution equal to $\frac{(x + Mt_1)^n}{c}$ |
| $P_h, P_{\dot{h}}$ | loading coefficients corresponding to unit steps in h(t) and $\dot{h}(t)$, respectively |
| q | $\frac{\dot{\theta} c}{V_o}$ |
| $q_o$ | dynamic pressure, $\frac{1}{2} \rho_o V_o^2$ |
| t | time |
| $t_a$ | time to reach steady state |
| $t_o$ | $\frac{c_o t}{c}$ |

| | |
|---|---|
| $t_1$ | $c_o t$ |
| $T$ | maximum thickness of airfoil section |
| $V_o$ | flight velocity |
| $w_u$ | perturbation velocity component in $z$ direction on upper surface of $z = 0$ plane |
| $W$ | energy required to sustain the motion |
| $x,z$ | Cartesian coordinates |
| $\alpha$ | angle of attack, radians |
| $\Gamma$ | Gamma function |
| $\zeta$ | function giving chordwise dependence of vertical position of wing section |
| $\eta$ | thickness of beam |
| $\theta$ | angle of pitch, radians |
| $\mu$ | mass per unit length |
| $\nu$ | Poisson's ratio |
| $\xi$ | distance from leading edge of wing section |
| $\rho_o$ | density of undisturbed air |
| $\rho_w$ | density of wing material |
| $\tau$ | variable of integration |
| $\varphi$ | chord lengths traveled, $\frac{V_o t}{c}$ |
| $\varphi_a$ | chord lengths traveled to reach steady state |
| $\Phi$ | perturbation velocity potential |
| $\omega$ | frequency of oscillation, radians/sec |
| $\bar{\omega}$ | reduced frequency parameter, $\frac{2kM^2}{M^2 - 1}$ |
| $(\dot{\ }),(\ ')$ | differentiation with respect to $t$ and $\xi$, respectively |

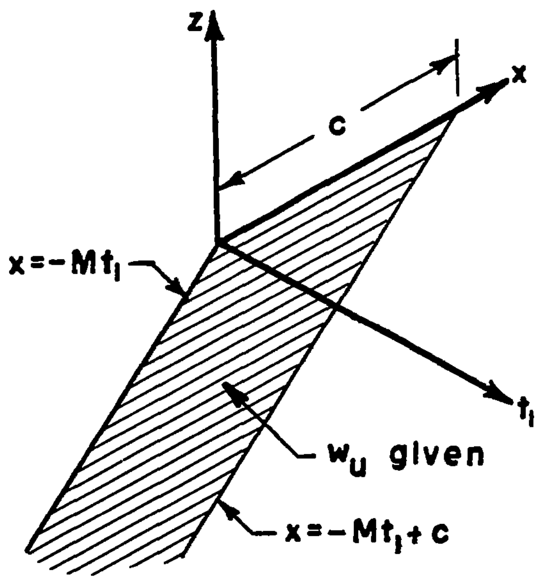
## AERODYNAMIC CHARACTERISTICS OF INDICIALLY CAMBERED AIRFOILS

The problem considered in this section is the determination of the indicial functions (pressure, lift, and moment coefficients) for a thin, two-dimensional airfoil that starts from rest and suddenly moves with a constant, supersonic, forward velocity and simultaneously attains a chordwise variation of normal velocity. The basic, linearized, partial differential equation for this problem, in terms of the perturbation potential $\Phi$, may be written

$$\Phi_{xx} + \Phi_{zz} = \Phi_{t_1 t_1} \tag{1}$$

when the fluid at infinity is at rest with respect to the $xz$ coordinate system. The boundary condition to be satisfied is that the normal velocity $w_u$ be a specified function of distance from the leading edge of the airfoil $(x + Mt_1)$ in the region traced out by the airfoil in the $xzt_1$ space (sketch (a)). The loading coefficient $P$, in terms of the perturbation potential, is then given by the relation

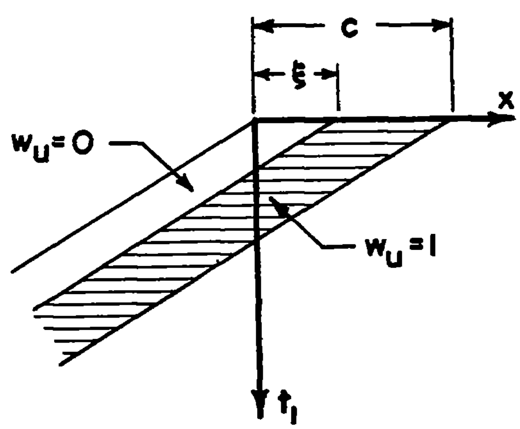
$$P = \frac{4}{V_o M}\frac{\partial\Phi}{\partial t_1} \tag{2}$$




Sketch (a)

### Loading Coefficient

The solution to this boundary-value problem has been obtained for the plunging airfoil, that is, $w_u$ = constant (see ref. 4). At supersonic speeds, this solution applies as well to the case shown in sketch (b) where $w_u = 0$ for $0 < x < -Mt_1 + \xi$. Thus, if $P_o(x,t_1)$ is the loading coefficient for $w_u = 1$ in the shaded region of sketch (a), and if $P(x,\xi,t_1)$ is the loading coefficient for $w_u$ as in sketch (b), then




Sketch (b)

$$\left.\begin{aligned} P(x,\xi,t_1) &= P_o(x - \xi,t_1) && \text{for } -Mt_1 + \xi < x \leq c \\ &= 0 && \text{for } \quad 0 \leq x - Mt_1 + \xi \end{aligned}\right\} \tag{3}$$

Since the theory is linear, the loading coefficient for an airfoil with arbitrary $w_u(\xi)$ can be obtained by superposition of elementary solutions of the form given in equation (3). Note that in this superposition it is necessary that $w$ be a function of $\xi$ only. The superposition gives

$$P(x,t_1) = P_o(x,t_1)\, w_u\Big|_{x=-Mt_1} + \int_0^{x+Mt_1} w'(\xi)\, P_o(x - \xi,t_1)\, d\xi \tag{4a}$$

which, upon introducing $\sigma = x - \xi$, may be written in an alternate form of the Duhamel integral

$$P(x,t_1) = P_o(x,t_1)\, w_u\Big|_{x=-Mt_1} + \int_{-Mt_1}^{x} w'(x - \sigma)\, P_o(\sigma,t_1)\, d\sigma \tag{4b}$$

## Generalized Indicial Force Coefficients

In characterizing the aerodynamic properties of an airfoil that is only pitching and plunging, it is generally sufficient to give only the usual lift and moment derivatives (i.e., $c_{l_\alpha}$, $c_{m_q}$, etc.). For an airfoil undergoing more complex chordwise deformations, it is found that more information about the distribution of loading on the airfoil is needed. This additional information is conveniently represented in the form of nondimensional generalized force coefficients which are defined as follows: Let the normal velocity of the airfoil be given in the form

$$w_u = \sum_n a_n\left(\frac{\xi}{c}\right)^n = \sum_n a_n\left(\frac{x + Mt_1}{c}\right)^n \tag{5}$$

and let $P_n$ be the loading coefficient corresponding to a normal velocity distribution equal to $\left(\frac{x + Mt_1}{c}\right)^n$, that is, for $a_n = 1$, $a_m = 0$ for $m \neq n$. We define the generalized force coefficient as

$$c_{m,n} = \frac{V_o}{c}\int_{-Mt_1}^{-Mt_1+c}\left(\frac{x + Mt_1}{c}\right)^m P_n(x,t_1)\, dx \tag{6}$$

For the special cases of m and n equal to 0 and 1, equation (6) reduces to

$$\left.\begin{aligned} c_{o,o} &= -c_{l_\alpha} \\ c_{o,1} &= -c_{l_q} \\ c_{1,o} &= c_{m_\alpha} \\ c_{1,1} &= c_{m_q} \end{aligned}\right\} \qquad (7)$$

Thus, the coefficient $c_{m,n}$ may be considered to be a generalization of the usual lift and moment derivatives.

Inasmuch as the loading corresponding to an arbitrary normal-velocity distribution can be expressed as a function of the uniform-downwash loading, it follows that the force coefficients for arbitrary n can be obtained from those corresponding to n = 0. To this end we substitute equation (4) into equation (6) to obtain, for $n > 0$ (so that $w_u\Big|_{x=-Mt_1} = 0$)

$$c_{m,n} = \frac{V_o}{c}\int_{-Mt_1}^{-Mt_1+c}\left(\frac{x + Mt_1}{c}\right)^m dx \int_{-Mt_1}^{x} \frac{n(x - \sigma)^{n-1} P_o(\sigma,t_1)\, d\sigma}{c^n}$$

Interchanging the order of integration, we have

$$c_{m,n} = \frac{V_o n}{c^{m+n+1}}\int_{-Mt_1}^{-Mt_1+c} P_o(\sigma,t_1)\, d\sigma \int_{\sigma}^{-Mt_1+c} (x + \sigma)^{n-1} (x + Mt_1)^m\, dx$$

Denoting the inner integral by $I(\sigma)$ and expanding $(x - \sigma)^{n-1}$ by the binomial expansion we obtain

$$I(\sigma) = c^{n+m}\left[\sum_{r=o}^{n-1}\binom{n - 1}{r}\frac{(-1)^r}{m + n - r}\left(\frac{\sigma + Mt_1}{c}\right)^r + \right.$$

$$\left.(-1)^n\left(\frac{\sigma + Mt_1}{c}\right)^{m+n}\sum_{r=o}^{n-1}\binom{n - 1}{r}\frac{(-1)^r}{m + r + 1}\right]$$

The second integration then gives the desired recursion relation for $n > 0$

$$c_{m,n}(t_1) = n\left[\sum_{r=0}^{n-1}\binom{n-1}{r}\left(\frac{(-1)^r}{m+n-r}\right)c_{r,o} + (-1)^n c_{m+n,o}\sum_{r=0}^{n-1}\binom{n-1}{r}\frac{(-1)^r}{m+r+1}\right] \quad (8)$$

which relates the coefficients for arbitrary $n$ to those for $n = 0$.[2] The coefficients $c_{m,o}$ for indicial plunging are given in the Appendix for $m$ between 0 and 4, and, in integral form, for arbitrary $m$.

## POWER REQUIRED TO SUSTAIN A CHORDWISE DEFORMATION

### General Considerations

The dynamic stability of a linear oscillating system is determined by the rate of energy flux into the system. For an airfoil deforming in an air stream, this energy arises from the aerodynamic forces acting upon the airfoil; if power is required to sustain the motion against the lift forces, the motion is stable, conversely, if power must be extracted from the system in order to sustain the motion, the motion is unstable.

Let us assume that the position of the mean line $z$, at any instant of time is given by (in an axis system attached to the airfoil)

$$z = h(t)\,\zeta(\xi) \quad (9)$$

Then, the power required to sustain this motion against the lift forces

$$\dot{W} = -\int_0^c \Delta p(\xi,t)\,\frac{\partial z}{\partial t}\,d\xi$$

[2]An equivalent recurrence relation was obtained in reference 3 for the more difficult case of the rectangular wing.

may be written

$$\dot{W} = -\dot{h}(t)\int_0^c \Delta p(\xi,t)\ \zeta(\xi)\ d\xi \tag{10}$$

The normal velocity distribution across the airfoil, which determines the loading on the airfoil, is given by

$$w_u = V_o \frac{\partial z}{\partial \xi} + \frac{\partial z}{\partial t} \tag{11}$$

or

$$w_u = V_o\zeta'(\xi)\ h(t) + \zeta(\xi)\ \dot{h}(t) \tag{12}$$

This downwash distribution may be considered as the sum of two contributions; the first arising from the instantaneous position of the airfoil and the latter resulting from the instantaneous normal velocity of the airfoil relative to an observer moving with the airfoil. Now the loadings resulting from each of the terms in equation (12) may be obtained, by the Duhamel integral, from the loadings corresponding to unit steps in $h(t)$ and $\dot{h}(t)$. Thus, the loading coefficient for a motion as described by equation (9) is

$$P(\xi,t) = \frac{\partial}{\partial t}\int_0^t P_h(\xi,\tau)\ h(t-\tau)\ d\tau + \frac{\partial}{\partial t}\int_0^t P_{\dot{h}}(\xi,\tau)\ \dot{h}(t-\tau)\ d\tau \tag{13}$$

where $P_h$ is the loading coefficient corresponding to a unit step in $h(t)$, that is, $w_u = V_o\zeta'(\xi)$, and $P_{\dot{h}}$ is the loading corresponding to a unit step in $\dot{h}(t)$, that is, $w_u = \zeta(\xi)$.

Substituting equation (13) into equation (10), and interchanging the order of integration, we obtain the following expression for the power required to sustain the motion against the lift forces

$$\dot{W} = -q_o\dot{h}(t)\left[\int_0^t \dot{h}(t-\tau)\ A_h(\tau)\ d\tau + h(o)\ A_h(t) + \frac{c}{V_o}\int_0^t \ddot{h}(t-\tau)\ A_{\dot{h}}(\tau)\ d\tau + \frac{c}{V_o}\ \dot{h}(o)\ A_{\dot{h}}(t)\right] \tag{14}$$

where

$$\left.\begin{aligned} A_h(t) &= \int_0^c P_h(\xi,t)\ \zeta(\xi)\ d\xi \\ A_{\dot{h}}(t) &= \frac{V_o}{c}\int_0^c P_{\dot{h}}(\xi,t)\ \zeta(\xi)\ d\xi \end{aligned}\right\} \tag{15}$$

The indicial loading coefficients $P_h$ and $P_{\dot{h}}$ can be obtained from equation (4) and the problem of obtaining the power required to sustain a given motion is reduced to a series of quadratures. It should be noted that if the function $\zeta(\xi)$ is a polynomial, the functions $A_h$ and $A_{\dot{h}}$ can then be obtained in terms of the generalized force coefficients $c_{m,n}$. This will be illustrated later where $\zeta(\xi)$ is a quadratic function.

## Application to Sinusoidal Oscillations

We assume now that $h(t)$ is given by

$$h(t) = h_o \sin \omega t \tag{16}$$

Substituting into equation (14) gives

$$\begin{aligned} \dot{W} = -\frac{q_o\omega^2 h_o^2}{2}\Bigg[&\int_0^t A_h \cos \omega t\ d\tau + \frac{\omega c}{V_o}\int_0^t A_{\dot{h}} \sin \omega t\ d\tau + \frac{2c}{V_o} A_{\dot{h}} \cos \omega t + \\ &\sin 2\omega t\left(\int_0^t A_h \sin \omega\tau\ d\tau - \frac{\omega c}{V_o}\int_0^t A_{\dot{h}} \cos \omega\tau\ d\tau\right) + \\ &\cos 2\omega t\left(\int_0^t A_h \cos \omega\tau\ d\tau + \frac{\omega c}{V_o}\int_0^t A_{\dot{h}} \sin \omega\tau\ d\tau\right)\Bigg] \end{aligned} \tag{17}$$

Introducing the functions $F(\tau)$ and $G(\tau)$ such that for $0 \leq \tau \leq t$:

$$\left.\begin{aligned} A_h(\tau) &= A_h(t) + F(\tau) \\ A_{\dot{h}}(\tau) &= A_{\dot{h}}(t) + G(\tau) \end{aligned}\right\} \tag{18}$$

and substituting into equation (17) gives, after integrating and simplifying,

$$\dot{W} = -\frac{q_o h_o^2 \omega^2}{2}\left[\int_0^t F(\tau)\cos\omega\tau\,d\tau + \frac{cA_{\dot{h}}(t)}{V_o} + \frac{\omega c}{V_o}\int_0^t G(\tau)\sin\omega\tau\,d\tau + \right.$$

$$\cos 2\omega t\left(\int_0^t F(\tau)\cos\omega\tau\,d\tau + \frac{cA_{\dot{h}}(t)}{V_o} + \frac{\omega c}{V_o}\int_0^t G(\tau)\sin\omega\tau\,d\tau\right) +$$

$$\left.\sin 2\omega t\left(\frac{A_h(t)}{\omega} + \int_0^t F(\tau)\sin\omega\tau\,d\tau - \frac{\omega c}{V_o}\int_0^t G(\tau)\cos\omega\tau\,d\tau\right)\right] \quad (19)$$

Since the functions $F(\tau)$ and $G(\tau)$ vanish for $\tau \geqq t_a$, where $t_a$ is the time to reach steady state, the integrals in equation (19) become constants. Thus, the power required to sustain the oscillations, after the transients die out, reduces to the form

$$\dot{W} = -\frac{q_o h_o^2 \omega^2}{2}(A + A\cos 2\omega t + B\sin 2\omega t) \quad (20)$$

Upon substitution of the nondimensional frequency $k = \frac{\omega c}{2V_o}$ and the chord lengths traveled $\varphi = \frac{V_o t}{c}$, equation (20) becomes

$$\dot{W} = \frac{2h_o^2 k^2 V_o q_o}{c}(A_o + A_o\cos 4k\varphi + B_o\sin 4k\varphi) \quad (21)$$

where

$$\left.\begin{aligned} A_o &= -\int_0^{\varphi_a} F(\varphi)\cos 2k\varphi\,d\varphi - A_{\dot{h}}(\varphi_a) - 2k\int_0^{\varphi_a} G(\varphi)\sin 2k\varphi\,d\varphi \\ B_o &= -\frac{A_h(\varphi_a)}{2k} - \int_0^{\varphi_a} F(\varphi)\sin 2k\varphi\,d\varphi + k\int_0^{\varphi_a} G(\varphi)\cos 2k\varphi\,d\varphi \end{aligned}\right\} \quad (22)$$

and

$$F(\varphi) = A_h(\varphi) - A_h(\varphi_a)$$

$$G(\varphi) = A_{\dot{h}}(\varphi) - A_{\dot{h}}(\varphi_a)$$

Thus $\dot{W}$ is a function which oscillates about a mean value with twice the frequency of the oscillation of the motion. For considerations of dynamic stability, the constant $A_o$ is the significant parameter. A positive value of $A_o$ indicates that, on the average, there must be a steady influx of energy into the system in order to sustain the motion, that is, the system is stable, conversely, a negative value of $A_o$ means instability.

## QUADRATIC CAMBER DEFORMATIONS

We shall consider, in this section, the application of the preceding analysis to the case of an airfoil which is assumed to oscillate in such a manner that, at any instant of time, its mean line is a simple quadratic function. A question which naturally arises is: Does this assumed quadratic function correspond to a normal vibratory mode of a beam whose thickness distribution is a reasonable approximation to those used in airfoils? In order to help answer this question, we shall first consider the problem of determining the thickness distribution of a beam which has a prescribed vibratory mode shape;[3] in particular, we shall prescribe the assumed quadratic function.

### Determination of Beam Having a Quadratic Normal Bending Mode

The differential equation for the transverse vibrations of a simple beam of nonuniform thickness is given by

$$\frac{\partial^2}{\partial \xi^2}\left(E_1 I \frac{\partial^2 z}{\partial \xi^2}\right) = - \mu \frac{\partial^2 z}{\partial t^2} \tag{23}$$

The boundary conditions for a free-free beam are given by the vanishing of the shear and bending moment at the ends of the beam

$$\left(E_1 I \frac{\partial^2 z}{\partial \xi^2}\right) = \frac{\partial}{\partial \xi}\left(E_1 I \frac{\partial^2 z}{\partial \xi^2}\right) = 0 \qquad \text{at} \quad x = o \quad \text{and} \quad x = c \tag{24}$$

Specifying the mode shape to be

$$z = h_o \sin \omega t \left[\frac{4}{c^2}\left(c\xi - \xi^2\right) - \frac{a_o}{h_o}\right] \tag{25}$$

[3]Mathematically, this problem is the inverse of the usual vibrating-beam problem in which the beam thickness distribution is specified, together with certain boundary conditions, and the normal modes are to be determined. Here, in the inverse problem, a mode shape is specified and the thickness distribution is to be obtained.

and substituting into equation (23) we obtain

$$\left(-\frac{8E_1}{c^2}\right)\frac{d^2I}{d\xi^2} = \mu\omega^2\left[\frac{4}{c^2}\left(c\xi - \xi^2\right) - \frac{a_o}{h_o}\right] \tag{26}$$

Since, for a solid beam of rectangular cross section with thickness $\eta$ and width $b$

$$I = \frac{b\eta^3}{12} \qquad \text{and} \qquad \mu = \rho_w b\eta$$

equation (26) may be written in the form

$$2\left(\frac{d\eta}{d\xi}\right)^2 + \eta\frac{d^2\eta}{d\xi^2} = \left(-\frac{\rho_w c^2\omega^2}{2E_1}\right)\left(\frac{4\xi}{c} - \frac{4\xi^2}{c^2} - \frac{a_o}{h_o}\right) \tag{27}$$

Similarly, the boundary conditions (24) reduce to

$$\eta(o) = \eta(c) = 0 \tag{28}$$

Assuming a solution of the form

$$\eta = b_o + b_1\xi + b_2\xi^2 \tag{29}$$

substituting into equations (27) and (28), and solving for the coefficients $b_o$, $b_1$, and $b_2$, we find that

$$\frac{a_o}{h_o} = \frac{4}{5}$$

$$b_o = 0$$

$$b_1 = c\omega\sqrt{\mu/5E_1}$$

$$b_2 = -\omega\sqrt{\mu/5E_1}$$

or that

$$\eta = \omega\sqrt{\rho_w/5E_1}\left(c\xi - \xi^2\right) \tag{30}$$

Upon substitution of the maximum thickness T, the thickness distribution may also be written as

$$\eta = \frac{4T}{c^2}\left(c\xi - \xi^2\right) \tag{31}$$

From equations (30) and (31), it follows that the frequency of the assumed mode shape (eq. (25)) must be

$$\omega = \frac{4}{c}\left(\frac{T}{c}\right)\sqrt{5E_1/\rho_w} \tag{32}$$

It can be shown further, by applying the Stodola iteration method (see ref. 5, p. 313) that the normal bending mode given by equation (25) with $\frac{a_o}{h_o} = \frac{4}{5}$ is the fundamental mode. Thus, we have shown that a solid airfoil with a parabolic thickness distribution has, for its fundamental chordwise bending mode, one of the assumed quadratic functions.

## Average Power Required to Sustain the Quadratic Camber Oscillation

For the particular case of the harmonically oscillating parabolic arc (see sketch (c)) we have

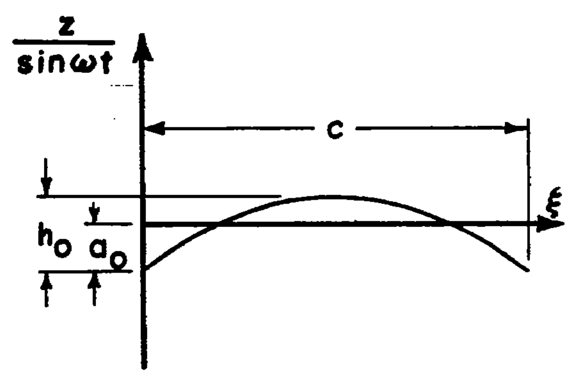
$$\left.\begin{aligned} h(t) &= h_o \sin \omega t \\ \zeta(\xi) &= -4\frac{\xi^2}{c^2} + 4\frac{\xi}{c} - \frac{a_o}{h_o} \end{aligned}\right\} \tag{33}$$

The indicial loading coefficient $P_h$, corresponding to a downwash variation

$$w_u = V_o\zeta'(\xi) = V_o\left[-\left(\frac{8\xi}{c^2}\right) + \left(\frac{4}{c}\right)\right],$$

may be written as

$$P_h = -\frac{8V_o}{c}P_1 + \frac{4V_o}{c}P_o \tag{34}$$




Sketch (c)

Similarly, the indicial loading coefficient $P_{\dot{h}}$, corresponding to a downwash distribution $w_u = \zeta(\xi) = -4(\xi^2/c^2) + 4(\xi/c) - (a_o/h_o)$, becomes

$$P_{\dot{h}} = -4P_2 + 4P_1 - \frac{a_o}{h_o} P_o \tag{35}$$

Substituting equations (33)-(35) into equation (15), comparing the terms obtained with equation (6), and using recurrence relation (8), we obtain

$$A_h = 16\left(-\frac{1}{3} c_{o,o} + c_{1,o} - \frac{2}{3} c_{3,o}\right) + 4 \frac{a_o}{h_o} (c_{o,o} - 2c_{1,o}) \tag{36}$$

and

$$A_{\dot{h}} = 16\left(\frac{1}{3} c_{1,o} - \frac{1}{2} c_{2,o} + c_{4,o}\right) + 4 \frac{a_o}{h_o} (-2c_{1,o} + 2c_{2,o}) + \left(\frac{a_o}{h_o}\right) c_{o,o} \tag{37}$$

Substituting the values of $c_{m,o}$ from the Appendix, we obtain $A_h$ and $A_{\dot{h}}$ as functions of Mach number $M$ and nondimensional time $t_o$; these functions are presented in equations (A7) and (A8) of the Appendix, and are plotted in figure 1 against $\varphi$, the chord lengths traveled, for several Mach numbers and two values of $a_o/h_o$.

The mean power coefficient $A_o$ is obtained by substituting the expressions for $A_h$ and $A_{\dot{h}}$ from the Appendix into equations (22) and (23) and integrating. The method of integration is very briefly outlined in the Appendix and the final results for the integrals appearing in equation (22) are given in equations (A9) and (A10) of the Appendix. The functions $F_{c_o}, F_{c_1}, G_{s_o}, G_{s_1}, G_{s_2}$ which occur in equations (A9) and (A10) are plotted in figure 2 against $\bar{\omega}$ for several Mach numbers. The mean power coefficient $A_o$, as a function of $\bar{\omega}$, is presented in figure 3 for several Mach numbers and two values of $a_o/h_o$. It will be noted from figure 3 that regions of instability $(A_o < 0)$ exist at the lower supersonic Mach numbers.

Whenever regions of both stability and instability exist, it is usually convenient to delineate these regions by stability boundaries which separate the two regions. To this end we note that $A_o$ may be written in the form

$$A_o = \left[\frac{32}{15\sqrt{M^2 - 1}} + F_{c_0}(M,\bar{\omega}) + G_{s_0}(M,\bar{\omega})\right] + \left(\frac{a_o}{h_o}\right)\left[-\frac{16}{3\sqrt{M^2 - 1}} + F_{c_1}(M,\bar{\omega}) + G_{s_1}(M,\bar{\omega})\right] + \left(\frac{a_o}{h_o}\right)^2\left[\frac{4}{\sqrt{M^2 - 1}} + G_{s_2}(M,\bar{\omega})\right] \tag{38}$$

that is, a quadratic function of the parameter $a_o/h_o$ which determines the location of the fixed points of the oscillation. Thus, for given $M$ and $\bar{\omega}$, at most two roots exist for $A_o = 0$. For the limiting case of low frequency, a simple expression may be obtained for the values of $a_o/h_o$ corresponding to the stability boundary; this solution is

$$\left(\frac{a_o}{h_o}\right) = \frac{1}{M^2 - 1}\left(\frac{2M^2 - 3}{3} \pm \sqrt{\frac{-4M^4 + 12M^2 - 3}{45}}\right) \tag{39}$$

The stability boundaries for the harmonically oscillating parabolic arc are presented in figure 4 with $a_o/h_o$ plotted against $\bar{\omega}$ for three Mach numbers at which the boundaries exist. These boundaries are presented in a different form, obtained by cross-plotting the curves of figure 4, in figure 5 with $a_o/h_o$ plotted against Mach number for several values of the reduced frequency $k$.

It will be noted that the motion is always stable for Mach numbers greater than 1.65 regardless of the location of the nodal points and of the frequency of oscillation. In addition, the stability boundaries also disappear for all values of Mach number and $a_o/h_o$ when $k$ is greater than 0.65. These limits on Mach number and frequency, when applied to a solid steel airfoil with quadratic thickness distribution and with the assumption that the mode shape is not affected by the air forces, show that the motion can be unstable only if the thickness ratio is less than 1.5 percent. It should be noted that this limit on thickness ratio is valid only to cases where a one-mode analysis applies. It has been found (ref. 2) that a two-mode approximation yields another stability boundary which is associated with the coalescing of the frequencies of the two modes when the ratio of aerodynamic to elastic forces becomes large. In such cases, a more complete analysis would be required.

## CONCLUSIONS

A recurrence relation for the generalized force coefficients for indicially cambered two-dimensional airfoils in a supersonic flow has been determined theoretically from the response of an indicially plunged airfoil. The rate of energy input required to sustain a general time-dependent chordwise deformation has been obtained in terms of these indicial functions. The particular case of a parabolic chordwise bending mode was considered in detail, and stability boundaries were obtained which show that this mode is unstable only for Mach numbers less than 1.65 and reduced frequencies less than 0.65. In addition, it has been shown that a beam with parabolic thickness distribution has a quadratic fundamental chordwise bending mode in vacuo.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., Oct. 11, 1954

# APPENDIX

## GENERALIZED INDICIAL FORCE COEFFICIENTS

The generalized indicial force coefficients for a plunging two-dimensional airfoil traveling at supersonic speeds may be obtained by integrating equation (6) for $n = 0$. The results of the integration are given in the following expressions.

$0 \leq t_o \leq \frac{1}{M + 1}$

$$c_{0,0} = -\frac{4}{M} \tag{A1a}$$

$$c_{1,0} = -\frac{2}{M} + \frac{t_o^2}{M} \tag{A2a}$$

$$c_{2,0} = -\frac{4}{3M} + \frac{4t_o^3}{3} \tag{A3a}$$

$$c_{3,0} = -\frac{1}{M} + \left(\frac{3}{8M} + \frac{3M}{2}\right) t_o^4 \tag{A4a}$$

$$c_{4,0} = -\frac{4}{5M} + \frac{4}{5}\left(\frac{3}{2} + 2M^2\right) t_o^5 \tag{A5a}$$

$$c_{m,0} = -\frac{4}{(m + 1)M} - \frac{4t_o^{m+1}}{\pi(m + 1)M} \int_0^{\pi} \cos u(M - \cos u)^m du \tag{A6a}$$

$\frac{1}{M + 1} \leq t_o \leq \frac{1}{M - 1}$

$$c_{0,0} = -\frac{4}{\pi}\left[\frac{1}{M} \arccos \frac{Mt_o - 1}{t_o} + \frac{1}{\sqrt{M^2 - 1}} \arccos(t_o + M - t_oM^2) + \frac{1}{M} \sqrt{t_o^2 - (1 - Mt_o)^2}\right] \tag{A1b}$$

$$c_{1,0} = -\frac{2}{\pi}\left[\frac{1}{M}\left(1 - \frac{t_o^2}{2}\right) \arccos \frac{Mt_o - 1}{t_o} + \frac{1}{\sqrt{M^2 - 1}} \arccos(t_o + M - t_oM^2) + \left(\frac{1}{2M} + \frac{t_o^2}{2}\right) \sqrt{t_o^2 - (1 - Mt_o)^2}\right] \tag{A2b}$$

$$c_{2,0} = -\frac{4}{3\pi}\left\{\left(\frac{1}{M} - t_o^3\right) \operatorname{arccos} \frac{Mt_o - 1}{t_o} + \frac{1}{\sqrt{M^2 - 1}} \operatorname{arccos} (t_o + M - t_oM^2) + \left[\frac{1}{3M} + \frac{2t_o}{3M} + \left(\frac{2}{3M} + \frac{M}{3}\right) t_o^2\right] \sqrt{t_o^2 - (1 - Mt_o)^2}\right\} \tag{A3b}$$

$$c_{3,0} = -\frac{1}{\pi}\left\{\left[\frac{1}{M} - \left(\frac{3}{8M} + \frac{3M}{2}\right) t_o^4\right] \operatorname{arccos} \frac{Mt_o - 1}{t_o} + \frac{1}{\sqrt{M^2 - 1}} \operatorname{arccos} (t_o + M - t_oM^2) + \left[\frac{1}{4M} + \frac{t_o}{4} + \left(\frac{M}{4} + \frac{3}{8M}\right) t_o^2 + \left(\frac{13}{8} + \frac{M^2}{4}\right) t_o^3\right] \sqrt{t_o^2 - (1 - Mt_o)^2}\right\} \tag{A4b}$$

$$c_{4,0} = -\frac{4}{5\pi}\left\{\left[\frac{1}{M} - \left(\frac{3}{2} + 2M^2\right) t_o^5\right] \operatorname{arccos} \frac{Mt_o - 1}{t_o} + \frac{1}{\sqrt{M^2 - 1}} \operatorname{arccos}(t_o + M - t_oM^2) + \left[\frac{1}{5M} + \frac{t_o}{5} + \left(\frac{M}{5} + \frac{4}{15M}\right) t_o^2 + \left(\frac{29}{30} + \frac{M^2}{5}\right) t_o^3 + \left(\frac{8}{15M} + \frac{83M}{30} + \frac{M^3}{5}\right) t_o^4\right] \sqrt{t_o^2 - (1 - Mt_o)^2}\right\} \tag{A5b}$$

$$c_{m,0} = -\frac{4}{(m + 1)\pi}\left[\frac{1}{M} \operatorname{arccos} \frac{Mt_o - 1}{t_o} + \frac{1}{\sqrt{M^2 - 1}} \operatorname{arccos}(t_o + M - t_oM^2) + \frac{t_o^{m+1}}{M}\int_0^{\operatorname{arccos}\left(\frac{Mt_o - 1}{t_o}\right)} \cos u(M - \cos u)^m du\right] \tag{A6b}$$

$t_o \geqq \frac{1}{M - 1}$

$$c_{0,0} = -\frac{4}{\sqrt{M^2 - 1}} \tag{A1c}$$

$$c_{1,0} = -\frac{2}{\sqrt{M^2 - 1}} \tag{A2c}$$

$$c_{2,o} = -\frac{4}{3\sqrt{M^2 - 1}} \tag{A3c}$$

$$c_{3,o} = -\frac{1}{\sqrt{M^2 - 1}} \tag{A4c}$$

$$c_{4,o} = -\frac{4}{5\sqrt{M^2 - 1}} \tag{A5c}$$

$$c_{m,o} = -\frac{4}{(m + 1)\sqrt{M^2 - 1}} \tag{A6c}$$

Functions Associated With Quadratic Camber Deformations

The indicial coefficients $A_h$ and $A_{\dot{h}}$, for the quadratic camber deformation, may be obtained by substituting equations (A1) through (A5) into equations (36) and (37) to obtain:

$0 \leq t_o \leq \frac{1}{M + 1}$

$$A_h = 16\left[\frac{t_o^2}{M} - \left(\frac{1}{4M} + M\right)t_o^4\right] + \frac{a_o}{h_o}\left(-\frac{8t_o^2}{M}\right) \tag{A7a}$$

$$A_{\dot{h}} = 16\left[-\frac{2}{15M} + \frac{t_o^2}{3M} - \frac{2t_o^3}{3} + \left(\frac{1}{5} + \frac{4M^2}{15}\right)t_o^5\right] + \left(\frac{a_o}{h_o}\right)\left(\frac{16}{3M} - \frac{8t_o^2}{M} + \frac{32t_o^3}{3}\right) + \left(\frac{a_o}{h_o}\right)^2\left(-\frac{4}{M}\right) \tag{A8a}$$

$\frac{1}{M + 1} \leq t_o \leq \frac{1}{M - 1}$

$$A_h = \frac{1}{\pi}\left[\frac{16t_o^2}{M} - 16\left(\frac{1}{4M} + M\right)t_o^4 + \frac{a_o}{h_o}\left(-\frac{8t_o^2}{M}\right)\right]\arccos\frac{Mt_o - 1}{t_o} + \frac{1}{\pi}\left\{\frac{4}{3M}\left[6 - 10Mt_o + (2M^2 + 3)t_o^2 + (13M + 2M^3)t_o^3\right] + \left(\frac{a_o}{h_o}\right)\left[-\frac{8}{M} + 8t_o\right]\right\}\sqrt{t_o^2 - (1 - Mt_o)^2} \tag{A7b}$$

$$A_{\dot{h}} = \frac{1}{\pi}\left\{16\left[-\frac{2}{15M} + \frac{t_o^2}{3M} - \frac{2t_o^3}{3} + \left(\frac{1}{5} + \frac{4M^2}{15}\right)t_o^5\right] + \left(\frac{a_o}{h_o}\right)\left[\frac{16}{3M} - \frac{8t_o^2}{M} + \frac{32t_o^3}{3}\right] + \left(\frac{a_o}{h_o}\right)^2\left(-\frac{4}{M}\right)\right\} \arccos\frac{Mt_o - 1}{t_o} +$$

$$\frac{1}{\pi\sqrt{M^2 - 1}}\left[-\frac{32}{15} + \frac{16}{3}\left(\frac{a_o}{h_o}\right) - 4\left(\frac{a_o}{h_o}\right)^2\right]\arccos(t_o + M - t_oM^2) +$$

$$\frac{1}{\pi}\left\{\frac{16}{225}\left[-\frac{31}{M} - 31t_o + \left(\frac{92}{M} + 44M\right)t_o^2 - (29 + 6M^2)t_o^3 - \left(\frac{16}{M} + 83M + 6M^3\right)t_o^4\right] + \left(\frac{a_o}{h_o}\right)\left[\frac{40}{9M} + \frac{40t_o}{9} - \left(\frac{64}{9M} + \frac{32M}{9}\right)t_o^2\right] + \left(\frac{a_o}{h_o}\right)^2\left(-\frac{4}{M}\right)\right\}\sqrt{t_o^2 - (1 - Mt_o)^2} \qquad \text{(A8b)}$$

$t_o \geqq \dfrac{1}{M - 1}$

$$A_h = 0 \qquad \text{(A7c)}$$

$$A_{\dot{h}} = \frac{1}{\sqrt{M^2 - 1}}\left[-\frac{32}{15} + \frac{16}{3}\left(\frac{a_o}{h_o}\right) - 4\left(\frac{a_o}{h_o}\right)^2\right] \qquad \text{(A8c)}$$

The integrals appearing in equation (22) for the mean power coefficient may be evaluated by integrating by parts, making the transformation $\varphi = \frac{M}{M^2 - 1}u + \frac{M^2}{M^2 - 1}$, and reducing the many integrals to the Poisson integral representation of the Bessel functions (ref. 6)

$$J_n(z) = \frac{2\left(\frac{z}{2}\right)^n}{\sqrt{\pi}\,\Gamma\left(n + \frac{1}{2}\right)}\int_0^1 (\cos zu)(1 - u^2)^{n-\frac{1}{2}}\,du$$

By means of successive application of the Bessel function recurrence relation

$$J_{n-1}(z) + J_{n+1}(z) = \frac{2n}{z}J_n(z)$$

the sine and cosine integrals in equation (22) can be reduced, after considerable manipulation, to the forms

$$\int_{0}^{\frac{M}{M-1}} F(\varphi)\cos 2k\varphi \, d\varphi = -\left[F_{c_0}(M,\bar{\omega}) + \left(\frac{a_o}{h_o}\right) F_{c_1}(M,\bar{\omega})\right] \tag{A9}$$

where

$$F_{c_0}(M,\bar{\omega}) = 16\left\{f_{o_R}(M,\bar{\omega})\left[\frac{2M^2}{(M^2-1)^{5/2}(\bar{\omega})^2} + \frac{6M^4(4M^2+1)}{(M^2-1)^{9/2}(\bar{\omega})^4}\right] + J_0\left(\frac{\bar{\omega}}{M}\right)\cos\bar{\omega}\left[-\frac{6M^4(4M^2+1)}{(M^2-1)^{9/2}(\bar{\omega})^4}\right] + J_0\left(\frac{\bar{\omega}}{M}\right)\sin\bar{\omega}\left[\frac{-10M^3}{(M^2-1)^{7/2}(\bar{\omega})^3}\right] + J_1\left(\frac{\bar{\omega}}{M}\right)\cos\bar{\omega}\left[\frac{2M^3(2M^2+3)}{(M^2-1)^{7/2}(\bar{\omega})^3}\right] + J_1\left(\frac{\bar{\omega}}{M}\right)\sin\bar{\omega}\left[\frac{-2M^5(2M^2+13)}{(M^2-1)^{9/2}(\bar{\omega})^4}\right]\right\}$$

$$F_{c_1}(M,\bar{\omega}) = 16\left\{f_{o_R}(M,\bar{\omega})\left[\frac{-M^2}{(M^2-1)^{5/2}(\bar{\omega})^2}\right] + J_0\left(\frac{\bar{\omega}}{M}\right)\cos\bar{\omega}\left[\frac{M^2}{(M^2-1)^{5/2}(\bar{\omega})^2}\right] + J_1\left(\frac{\bar{\omega}}{M}\right)\sin\bar{\omega}\left[\frac{M^3}{(M^2-1)^{5/2}(\bar{\omega})^2}\right]\right.$$

and

$$\int_{0}^{\frac{M}{M-1}} G(\varphi)\sin 2k\varphi \, d\varphi = -\frac{1}{k}\left[G_{s_0}(M,\bar{\omega}) + \left(\frac{a_o}{h_o}\right) G_{s_1}(M,\bar{\omega}) + \left(\frac{a_o}{h_o}\right)^2 G_{s_2}(M,\bar{\omega})\right] \tag{A10}$$

where

$$G_{s_0}(M,\bar{\omega}) = 32\left\{-\frac{1}{15\sqrt{M^2-1}} + f_{o_R}(M,\bar{\omega})\left[-\frac{2M^2}{(M^2-1)^{5/2}(\bar{\omega})^2} - \frac{12M^4+16M^6}{(M^2-1)^{9/2}(\bar{\omega})^4}\right] + f_{o_I}(M,\bar{\omega})\left[-\frac{\bar{\omega}\sqrt{M^2-1}}{15M^2} - \frac{1}{3(M^2-1)^{3/2}(\bar{\omega})}\right] + J_o\left(\frac{\bar{\omega}}{M}\right)\cos\bar{\omega}\left[\frac{1}{15\sqrt{M^2-1}} + \frac{2M^2}{15(M^2-1)^{5/2}(\bar{\omega})^2} + \frac{12M^4+16M^6}{(M^2-1)^{9/2}(\bar{\omega})^4}\right] + J_o\left(\frac{\bar{\omega}}{M}\right)\sin\bar{\omega}\left[\frac{1}{15(M^2-1)^{3/2}(\bar{\omega})} + \frac{4M^2(27M^2+8)}{15(M^2-1)^{7/2}(\bar{\omega})^3}\right] + J_1\left(\frac{\bar{\omega}}{M}\right)\cos\bar{\omega}\left[-\frac{M}{15(M^2-1)^{3/2}(\bar{\omega})} - \frac{4M^3(6M^2+29)}{15(M^2-1)^{7/2}(\bar{\omega})^3}\right] + J_1\left(\frac{\bar{\omega}}{M}\right)\sin\bar{\omega}\left[\frac{1}{15M\sqrt{M^2-1}} - \frac{2M(M^2-2)}{15(M^2-1)^{5/2}(\bar{\omega})^2} + \frac{4M^3(6M^4+83M^2+16)}{15(M^2-1)^{9/2}(\bar{\omega})^4}\right]\right\}$$

$$G_{s_1}(M,\bar{\omega}) = 16\left\{\frac{1}{3\sqrt{M^2-1}} + f_{o_R}(M,\bar{\omega})\left[\frac{4M^2}{(M^2-1)^{5/2}(\bar{\omega})^2}\right] + f_{o_I}(M,\bar{\omega})\left[\frac{1}{(M^2-1)^{3/2}(\bar{\omega})} + \frac{\bar{\omega}\sqrt{M^2-1}}{3M^2}\right] + J_o\left(\frac{\bar{\omega}}{M}\right)\cos\bar{\omega}\left[-\frac{1}{3\sqrt{M^2-1}} - \frac{4M^2}{(M^2-1)^{5/2}(\bar{\omega})^2}\right] + J_o\left(\frac{\bar{\omega}}{M}\right)\sin\bar{\omega}\left[-\frac{1}{3(M^2-1)^{3/2}\bar{\omega}}\right] + J_1\left(\frac{\bar{\omega}}{M}\right)\cos\bar{\omega}\left[\frac{M}{3(M^2-1)^{3/2}\bar{\omega}}\right] + J_1\left(\frac{\bar{\omega}}{M}\right)\sin\bar{\omega}\left[-\frac{1}{3M\sqrt{M^2-1}} - \frac{4(M^3+2M)}{3(M^2-1)^{5/2}(\bar{\omega})^2}\right]\right\}$$

$$G_{s_2}(M,\bar{\omega}) = 4\left\{-\frac{1}{\sqrt{M^2-1}} + f_{o_I}(M,\bar{\omega})\left[-\frac{\sqrt{M^2-1}(\bar{\omega})}{M^2}\right] + J_o\left(\frac{\bar{\omega}}{M}\right)\cos\bar{\omega}\left(\frac{1}{\sqrt{M^2-1}}\right) + J_1\left(\frac{\bar{\omega}}{M}\right)\sin\bar{\omega}\left(\frac{1}{M\sqrt{M^2-1}}\right)\right\}$$

In the limit as frequency goes to zero these integrals become

$$\lim_{k \to 0} \int_0^{\frac{M}{M-1}} F(\varphi)\cos 2k\varphi \, d\varphi = \frac{32}{15(M^2 - 1)^{3/2}} - \left(\frac{a_0}{h_0}\right)\left[\frac{8}{3(M^2 - 1)^{3/2}}\right] \qquad \text{(A11)}$$

and

$$\lim_{k \to 0} \left[k \int_0^{\frac{M}{M-1}} G(\varphi)\sin 2k\varphi \, d\varphi\right] = 0 \qquad \text{(A12)}$$

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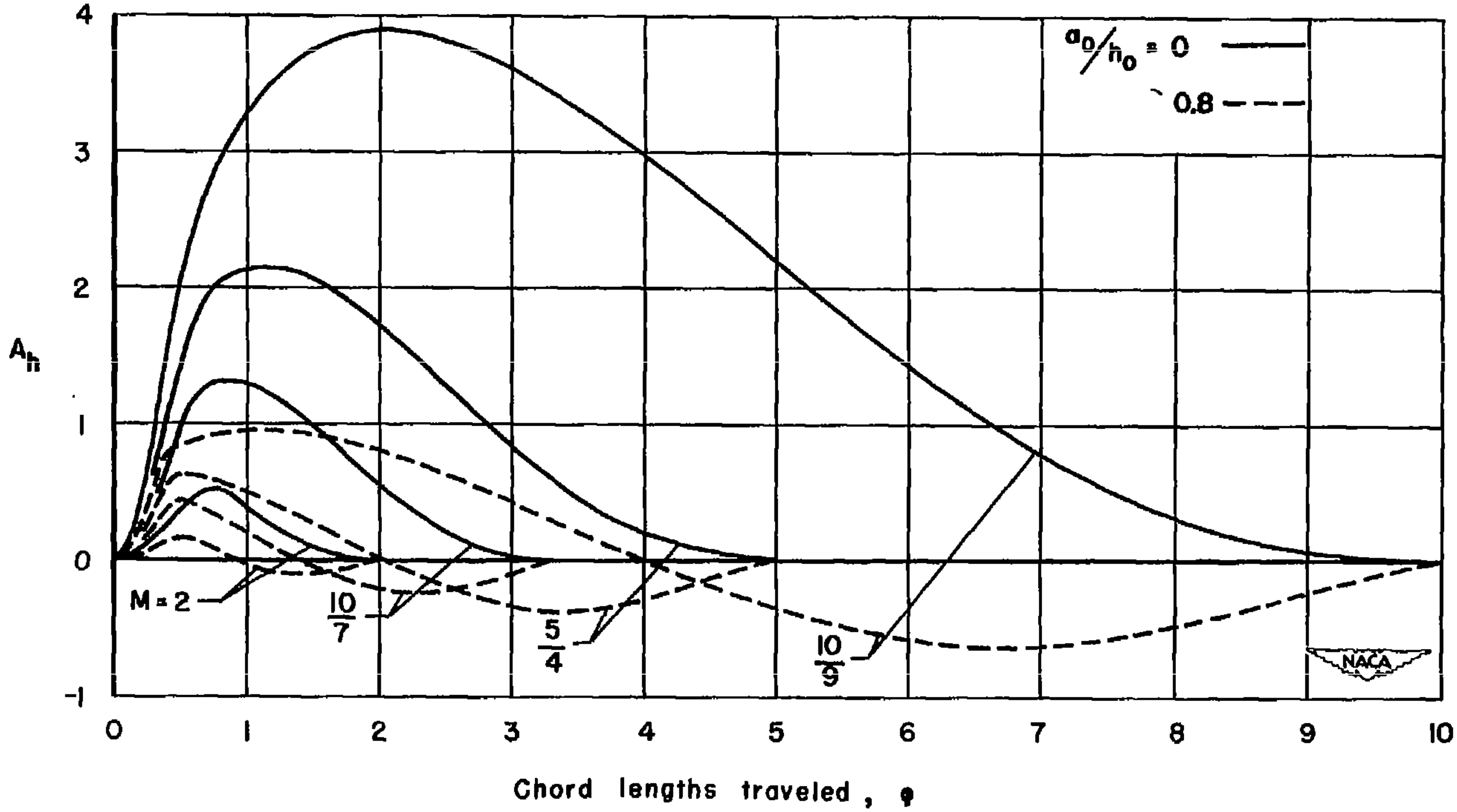
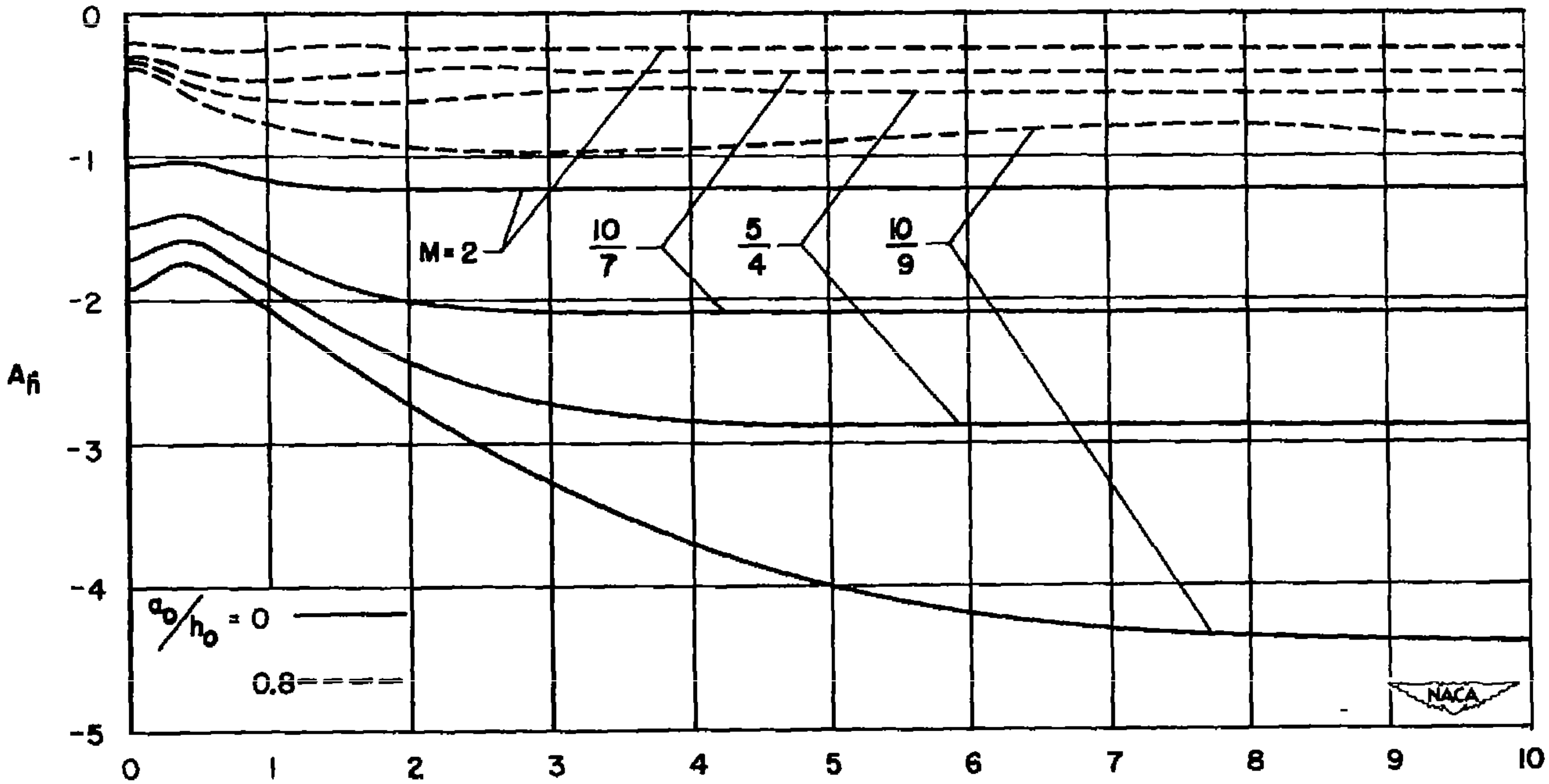


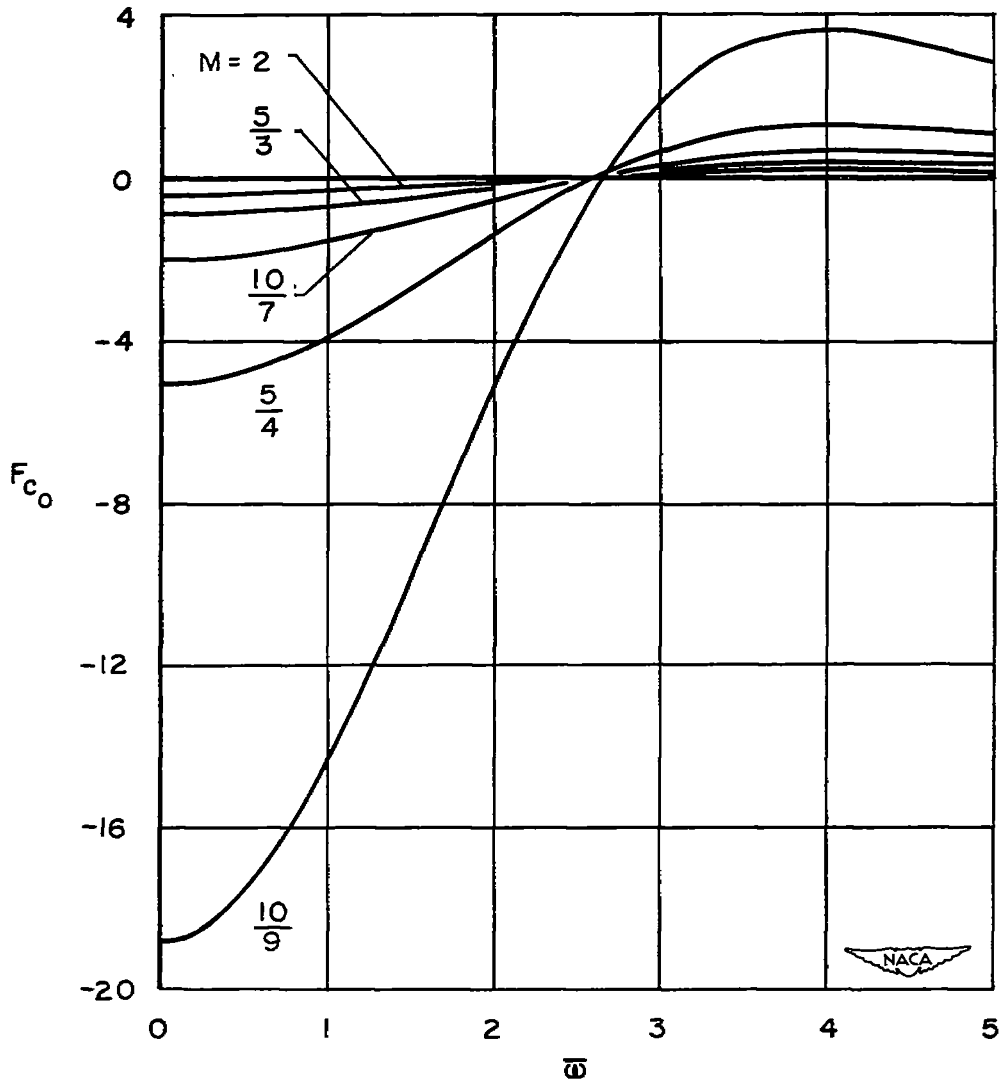

(a) $A_h$

Figure 1.- The functions $A_h$ (eq. (36)) and $A_{\dot{h}}$ (eq. (37)) for indicial quadratic camber.



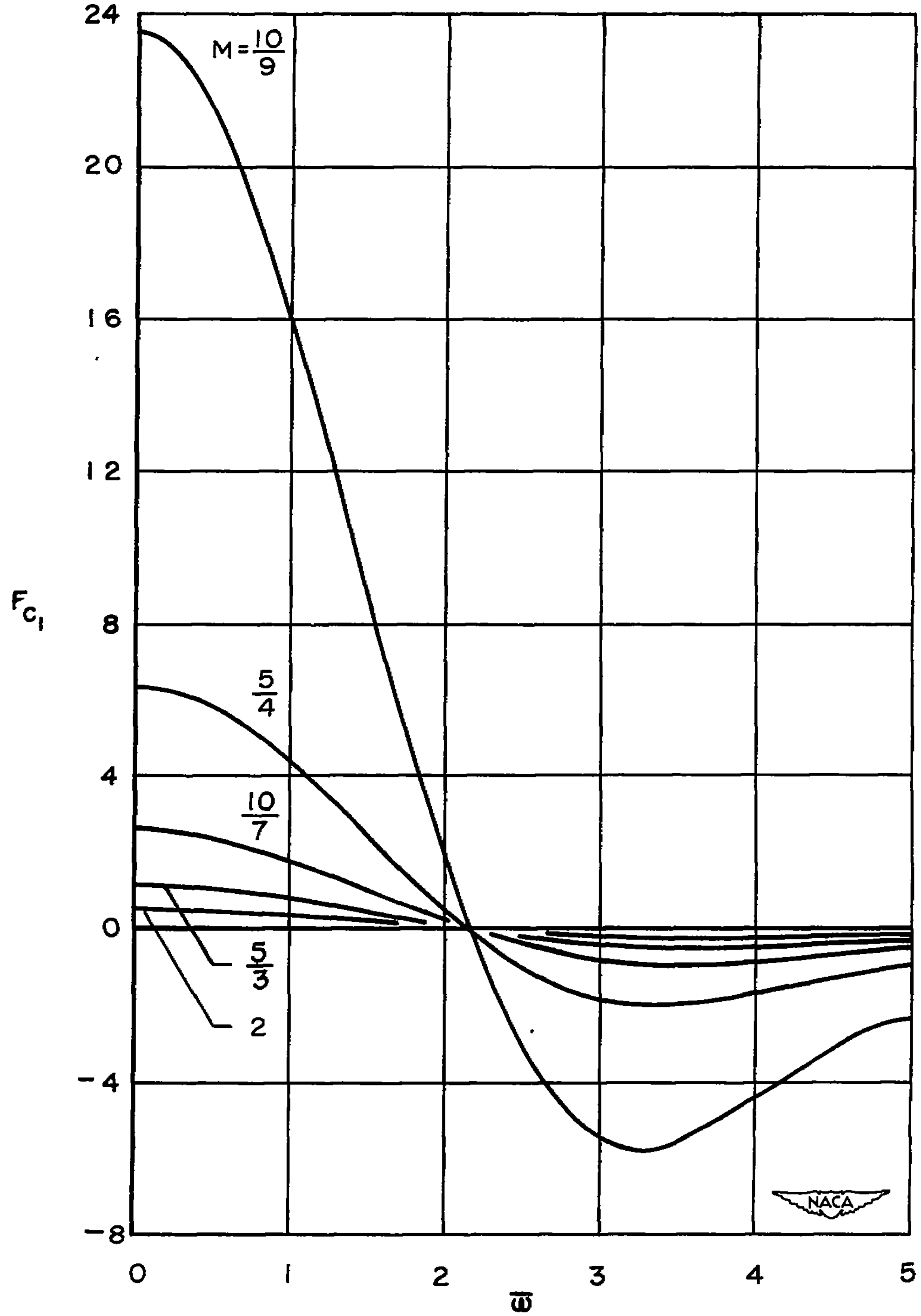

(b) $A_{\dot{h}}$

Figure 1.- Concluded



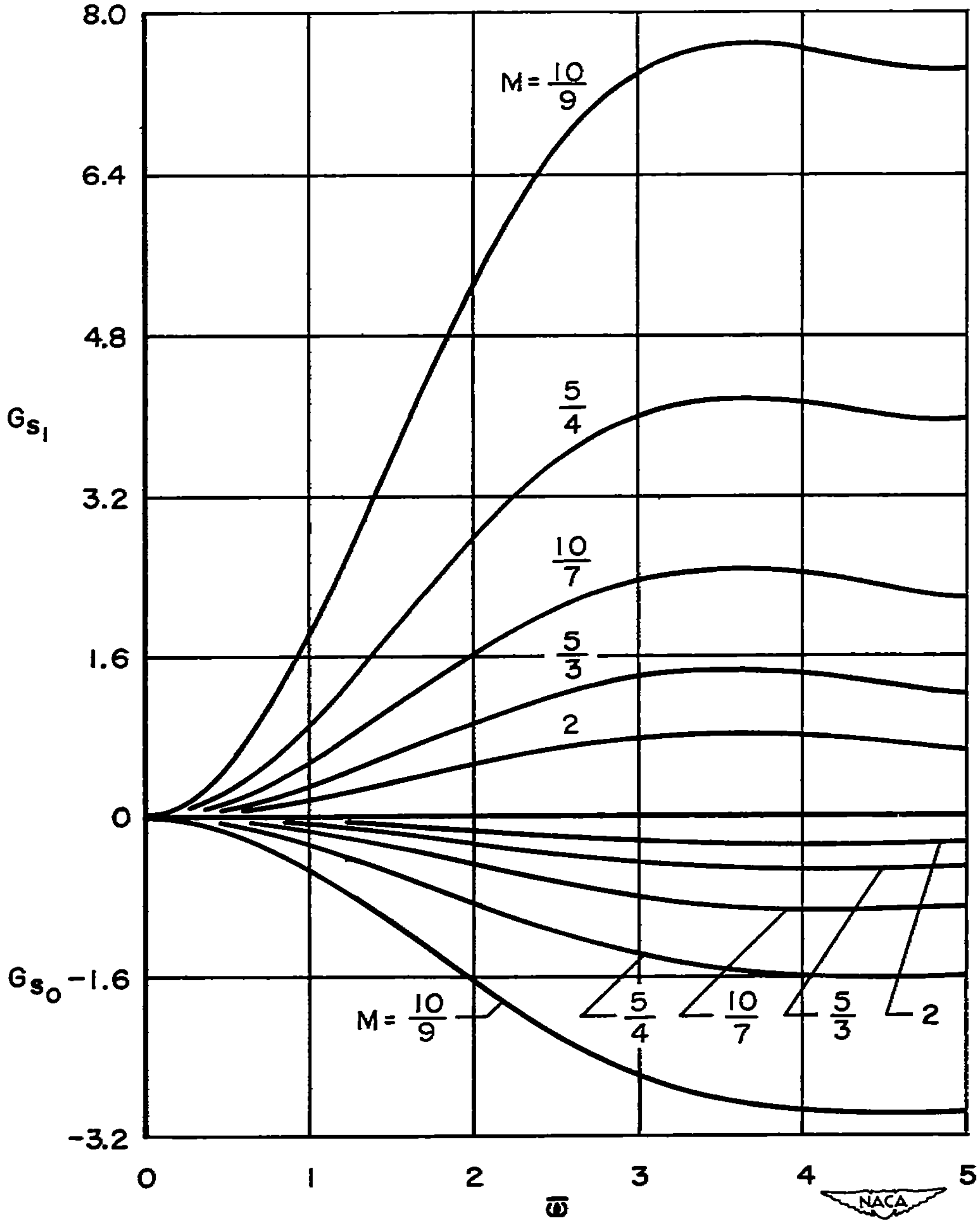

(a) $F_{c_0}$

Figure 2.- The functions $F_{c_0}$, $F_{c_1}$, $G_{s_0}$, $G_{s_1}$, and $G_{s_2}$ for the quadratic camber oscillation.



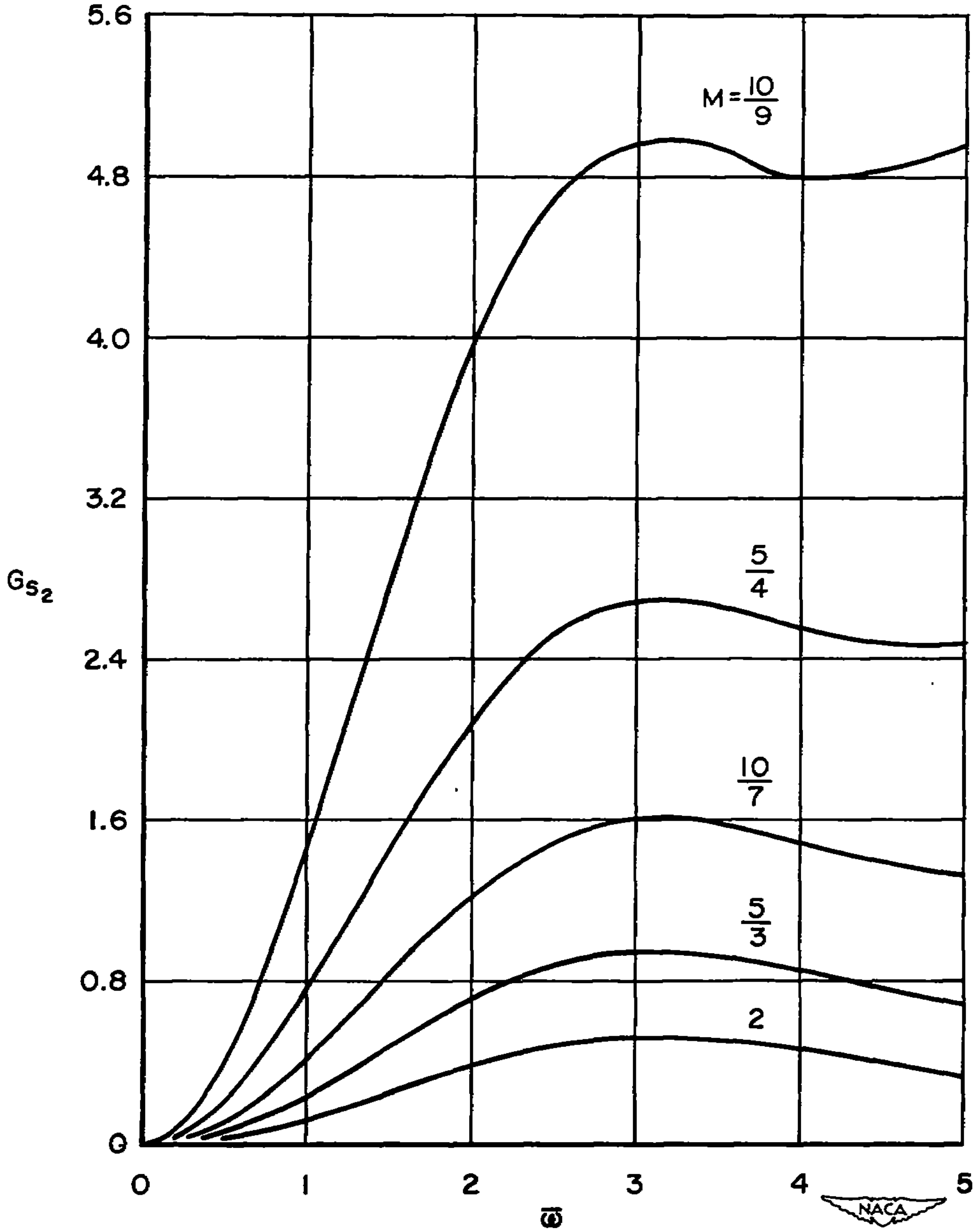

(b) $F_{c_1}$

Figure 2.- Continued.



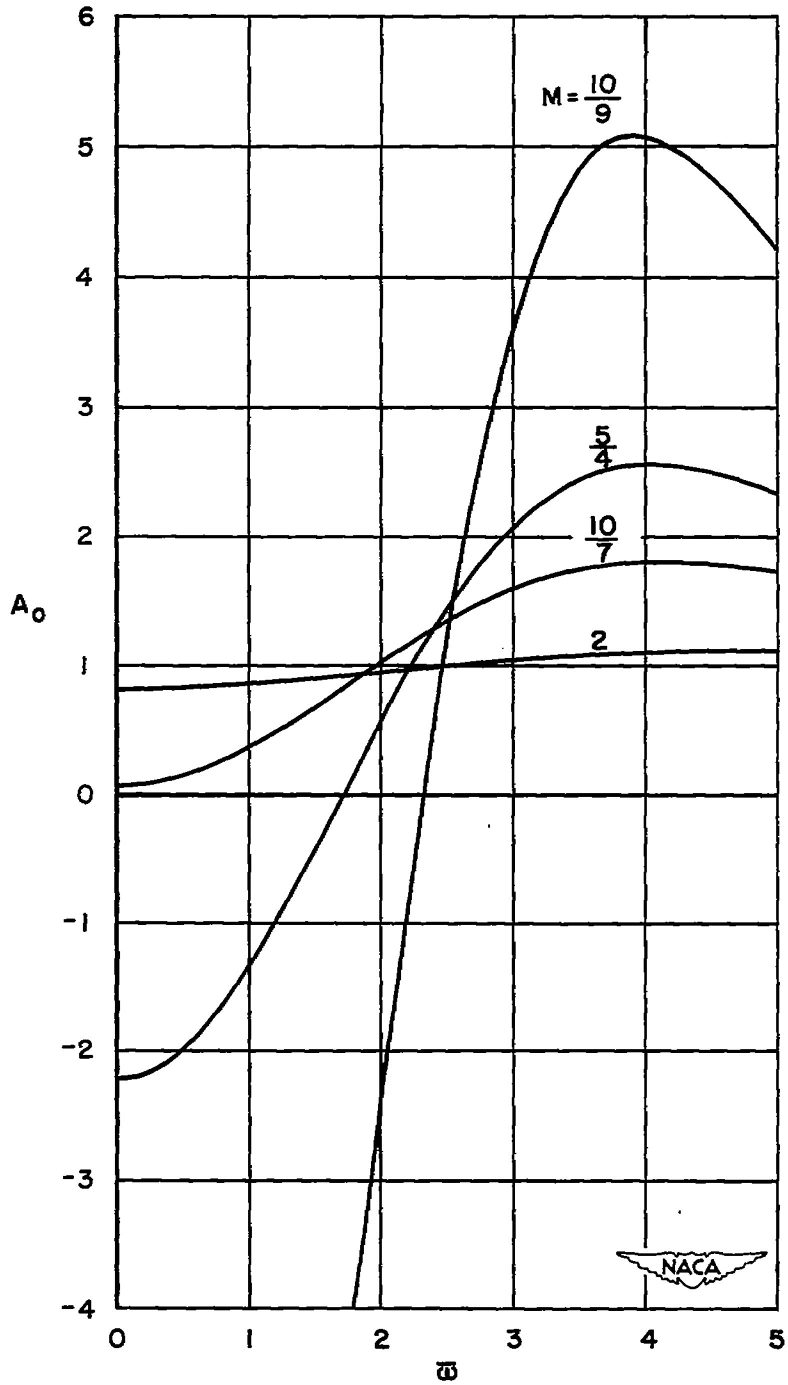

(c) $G_{s_0}$ and $G_{s_1}$

Figure 2.- Continued.



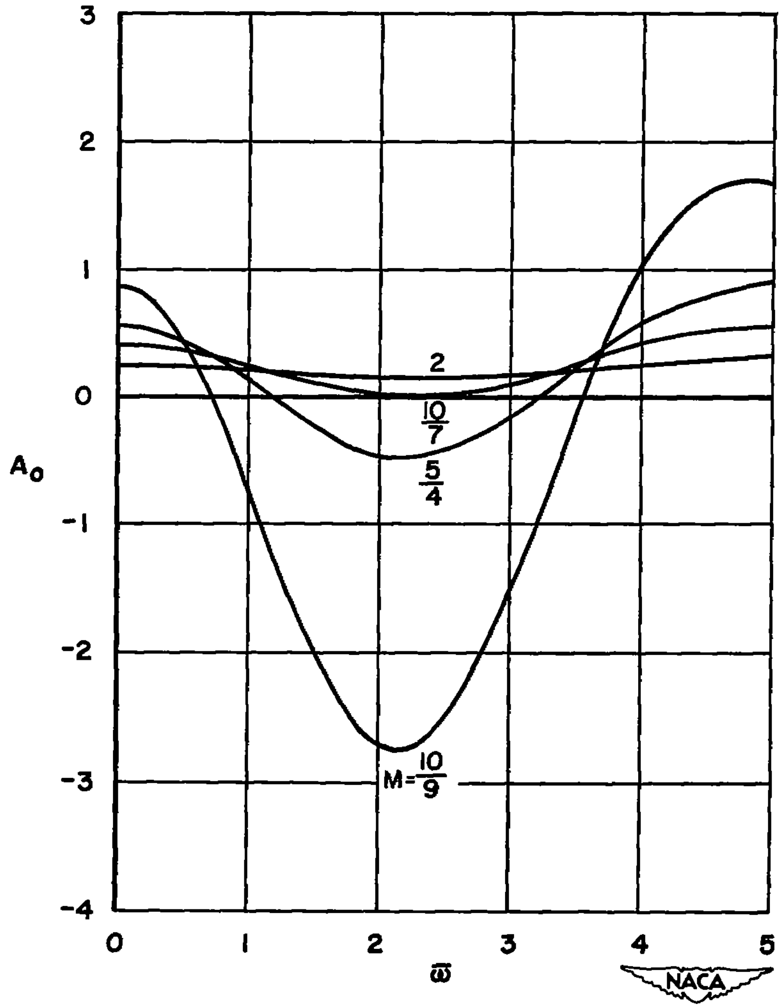

(d) $G_{s_2}$

Figure 2.- Concluded.




(a) $\frac{a_o}{h_o} = 0$

Figure 3.- The mean power coefficient $A_o$ as a function of the reduced frequency parameter $\bar{\omega}$.




(b) $\frac{a_o}{h_o} = 0.8$

Figure 3.- Concluded.

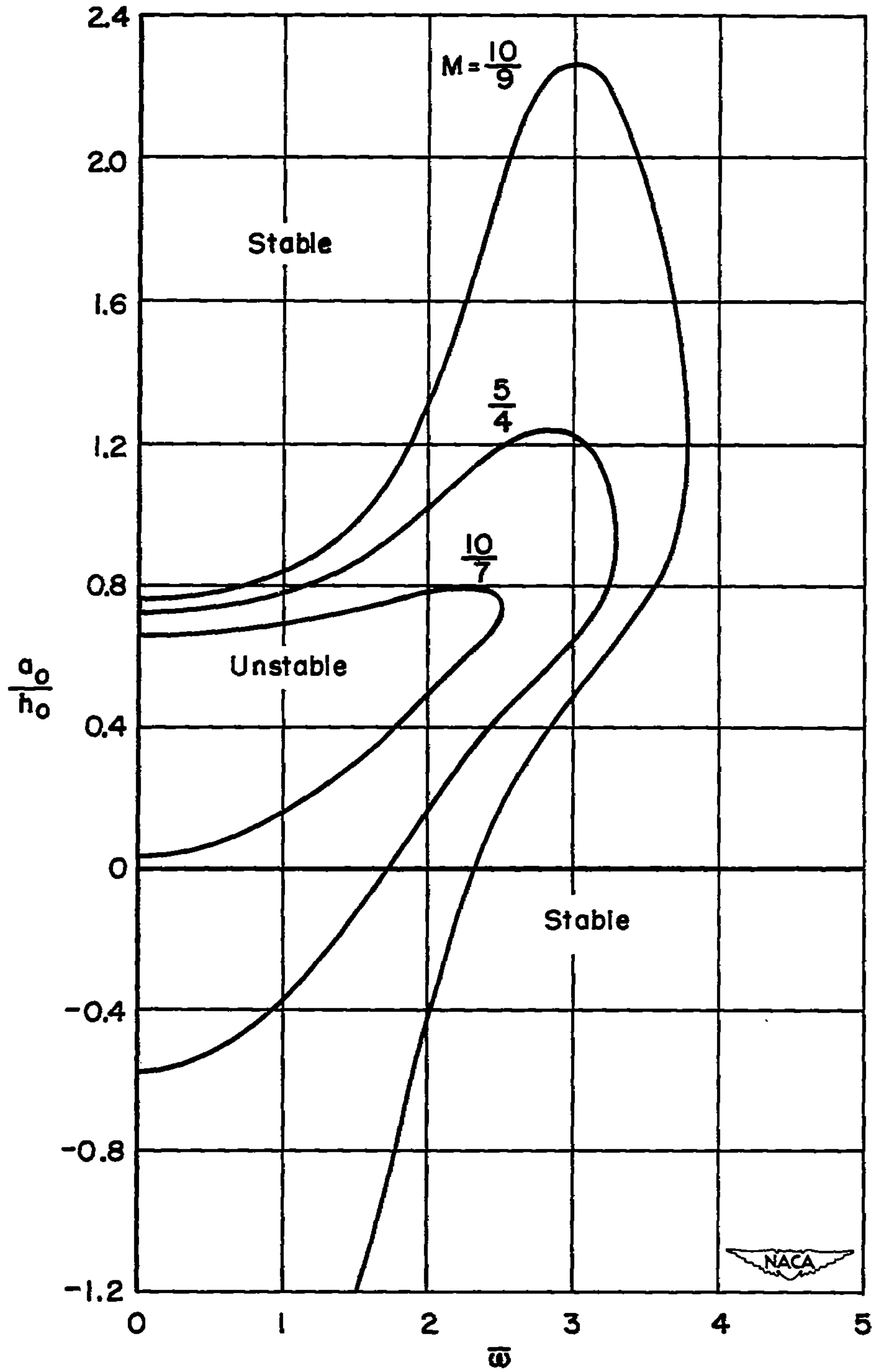



Figure 4.- Stability boundaries for the quadratic camber oscillation at several values of Mach number.

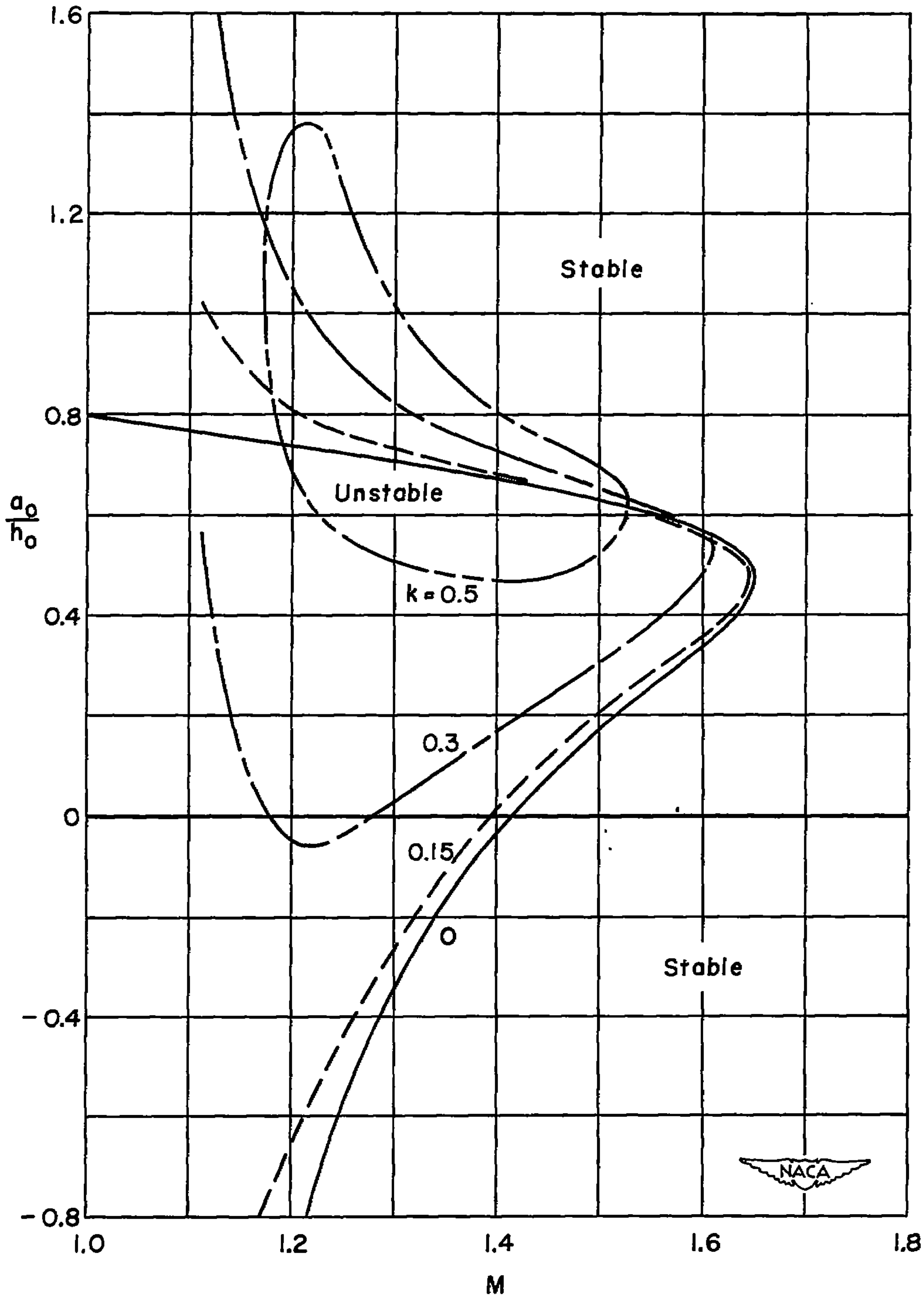



Figure 5.- Stability boundaries for the quadratic camber oscillation at several values of reduced frequency.

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