NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3387

USE OF NONLINEARITIES TO COMPENSATE FOR THE EFFECTS OF A RATE-LIMITED SERVO ON THE RESPONSE OF AN AUTOMATICALLY CONTROLLED AIRCRAFT

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SUMMARY

Hydraulic servos of the type normally used in airplane-autopilot combinations are nonlinear over the greater portion of their operating ranges. One of the most important nonlinearities is the limit on output rate which often results in oscillatory or unstable airplane responses to large input commands and sluggish responses to small error signals. These undesirable effects can generally be compensated for by introducing other nonlinear elements into the system.

This report describes a simple method for determining the nonlinear gains required to give optimum responses for step inputs of all magnitudes. This method is based on the fact that the control surface moves at its maximum rate during practically the entire transient maneuver and thus the servo system can be considered as a simple "on-off" type controller. The method requires a knowledge of the transfer function that describes the airplane response but ignores the dynamics of the servo and requires only simple hand calculations.

INTRODUCTION

The present trend toward the elimination of the human pilot as the primary controller in modern high-speed aircraft has led to severe requirements for autopilot performance. In many applications the autopilot must be capable of controlling the aircraft through violent maneuvers and at the same time must give rapid and precise response to small error signals.

Many of the problems encountered in automatic control of aircraft do not appear in other applications of servomechanisms. For example, the airplane has a complicated dynamic response that varies over its operating range. Furthermore, the autopilot is required to develop large forces or moments and still meet strict requirements with regard to size and weight. For this reason hydraulic servos are generally employed, and oil flow to the cylinder is restricted not only by the limits on size and weight but also by structural considerations. The net result, therefore, is a servo with limited output rate.

Thus, even though its over-all response characteristics are extremely complex, the autopilot for many purposes can be considered as a simple linear system with a limit on the rate of control surface deflection. It is this limit on the control-surface rate that is often a major source of difficulty in designing a system that is stable for large inputs and sufficiently responsive to very small commands. Adjusting the system parameters to give a stable response for large inputs usually results in sluggish responses to small inputs.

One way to compensate for such undesirable effects is to design system parameters as functions of error. Considerable effort has been expended on this general problem of developing nonlinear elements to improve the performance of linear as well as nonlinear servomechanisms. References 1 and 2 are typical examples in which phase plane methods of analysis are used. In this report, however, a different approach was necessary in considering the more complex case that is characteristic of an airplane-autopilot combination. A method is developed for designing appropriate nonlinear functions of error into a rate-limited system to give large gain levels for small errors and low gains for large errors so that satisfactory responses may be obtained with step inputs of any magnitude. The method is illustrated by two examples which consider hypothetical airplane-autopilot combinations with control-surface rate limiting.

NOTATION

θ	pitch angle, deg
θ_i	pitch-angle input, deg
φ	roll angle, deg
φ_i	roll-angle input, deg
ϵ	error, deg
p	Laplace operator
t	time, sec
t_s	control-surface switching time, sec
v_e	servo error, volts
δ_a	aileron angle, deg
δ_e	elevator angle, deg
K_ϵ, K_s	gain constants, volts/deg

$K_{\dot{\theta}}, K_{\dot{\varphi}}$ gain constants, volts/deg/sec

$K_{\epsilon}(\epsilon)$ nonlinear gain function of ϵ, volts/deg

$K_{\dot{\varphi}}(\epsilon)$ nonlinear gain function of ϵ, volts/deg/sec

A dot over symbol indicates derivative with respect to time.

EFFECTS OF SERVO RATE LIMITING ON TYPICAL AIRPLANE AUTOPILOT SYSTEM

In figure 1 is shown a block diagram of the roll-control channel of a typical airplane-autopilot combination. The system consists of a roll-rate feedback for stabilization and a roll-angle signal as an attitude reference. The servo is assumed to have the open-loop transfer function

$$\frac{\delta_a}{v_e} = \frac{50}{p}$$

With aileron position feedback, the closed-loop servo response is

$$\frac{1}{1 + 0.02p}$$

This simplified representation of the servo is usually justified because its response is so much faster than that of the airplane. For this example the airplane roll-angle response is defined as

$$\frac{\varphi}{\delta_a} = \frac{8.1}{p(1 + 0.3p)}$$

Cross-coupling terms have been ignored and the airplane is assumed to have a single degree of freedom in roll. The constants were chosen as typical of a high-speed airplane.

By means of an electronic simulator the gain constants K_{ϵ} and $K_{\dot{\varphi}}$ were adjusted to give the desired response to a step input (a reasonably fast response with little overshoot) such as shown by the solid line of figure 2. Values of the gain constants for this condition are

$$K_\epsilon = 3.33$$

and

$$K_{\dot{\varphi}} = 0.417$$

Also shown for comparison is the more sluggish response obtained when K_ϵ is reduced to 1.67.

When an aileron rate limit of 50° per second is introduced, the solid line of figure 2 is, strictly speaking, descriptive only of responses to inputs of no greater than 0.3° since the maximum rate is attained when v_e is 1 volt (fig. 1). For larger commands the system is nonlinear as indicated in figure 3. Here aileron- and roll-angle responses for step inputs of 2°, 5°, 7.5°, 10°, and 15° are plotted. For an input of 2° the aileron rate is limited for only a short time, and the response differs very little from that shown in figure 2. For larger inputs, however, the response becomes oscillatory and finally unstable for a step command of 15°; in each case the control surface moves at its maximum rate (±50° per sec) until the roll angle has essentially stabilized at the command input. By reducing the gain K_ϵ the stability for large inputs could be improved, but then the response to small inputs would be sluggish, as indicated in figure 2.

To generalize on the control motions shown in figure 3, it can be stated that to correct an initial error in the shortest time, the control surface should move at its maximum rate, reverse direction at precisely the right time, and travel at its maximum rate in the opposite direction. The fact that a response is sluggish indicates that the control surface has changed direction too soon; an oscillatory response means that the surface has traveled too far before changing direction. Thus, with a fixed control-surface rate there is an optimum reversal point, or "switching time" t_s for each input magnitude.

During practically the entire transient response ($\varphi_i > 2.0°$), the operation of the system is essentially the same as an "on-off" or "bang-bang" type of controller commanding a fixed plus-or-minus control-surface rate. The only variable quantity is the time at which the rate changes sign.

It is apparent from figure 3 that the proper control-surface motion can be attained for step inputs of different magnitudes only if the signal to the servo is modified in some fashion. The method described in the following section makes use of this fact in establishing gain level as a suitable nonlinear function of error.

DESCRIPTION OF METHOD

The proposed technique for determining the proper nonlinear gains for compensating the effects of rate limiting can best be described by use of illustrative examples. The first example considers the system of figure 1, while the second example shows how the same basic method can be applied to a more complicated situation.

Example 1

The first step is to determine the optimum switching time, t_s, as a function of φ_i. This is done by calculating the airplane roll responses to a number of constant-rate (50° per sec) triangular inputs as shown in figure 4. These may be calculated readily by evaluating the response to a constant-rate aileron input and using the principle of superposition. Curve (7), for example, shows that if the aileron moves at its maximum rate for 0.35 second and then changes direction, the airplane will attain a maximum roll angle of 39.4° in approximately 0.92 second. Thus, to obtain the quickest response to a step input command of 39.4° without overshoot, the system gain should be adjusted so that the aileron will reverse direction at 0.35 second.

The values of t_s corresponding to the various peak values of φ from figure 4 are plotted as the solid line in figure 5. This curve is labeled "optimum" because it defines the response requiring the least time to reach and remain at zero error. Furthermore, it marks the boundary between an oscillatory and a no-overshoot response. For purposes of comparison, switching times indicated by figure 3 (for K_ϵ of 3.33) are also shown. As expected, the switching times are greater than optimum over most of the range, indicating an oscillatory system. When K_ϵ is reduced to 1.67, figure 5 indicates the system to be sluggish for inputs less than 20° but still oscillatory for larger commands.

After determining the desired switching time as a function of input magnitude, the next step is to find a nonlinear function that will cause the aileron to reverse direction at the proper time. While there are several possible choices, the most obvious is the replacement of K_ϵ with a nonlinear function of error. At the instant the aileron reverses its direction of motion the signal v_e to the servo has just reached zero. Thus at time t_s (from fig. 1) with $K_{\dot{\varphi}} = 0.417$

$$v_e = K_\epsilon \epsilon - 0.417\dot{\varphi} - \delta_a = 0 \tag{1}$$

For a given φ_i, t_s is taken from figure 5; $\dot{\varphi}$ is calculated as shown in figure 6; and δ_a is equal to 50 t_s. As outlined in table I the desired value of $K_\epsilon \epsilon$ for each φ_i is obtained directly from equation (1). The error angle ϵ at t_s is simply $\varphi_i - \varphi$.

In figure 7, K_ϵ, designated as $K_\epsilon(\epsilon)$, is plotted as a function of error and is the gain level necessary for an optimum response. At zero error the curve indicates an infinite gain; however, this value is physically impractical and has no significance because for small errors the system operates primarily in its linear range. Thus with $K_{\dot{\phi}}$ fixed at 0.417, K_ϵ should be restricted to a value of 3.33 as shown by the dotted line so that in its linear range the system will have the response characterized by figure 2.

By use of an electronic analog computer the response of the system with this nonlinear gain function was determined and is shown in figure 8. It can be seen that the system has a rapid and stable response for inputs as large as 60°. It should also be noted that for the larger inputs, two reversal points occur before the system remains within its linear range. While only the first point was considered in the calculation of the nonlinear function, the analog computer results show that the succeeding reversal points (which occur whenever equation (1) is satisfied) are properly timed for a near optimum response.

The same improvement in system response may be obtained by making the damping parameter $K_{\dot{\phi}}$ rather than K_ϵ the nonlinear function of error. The calculations are similar to those for the previous case and are also shown in table I. In this case K_ϵ has a fixed value of 3.33, and at time t_s

$$3.33\epsilon - K_{\dot{\phi}}\dot{\phi} - \delta_a = 0$$

The term $K_{\dot{\phi}}$ is plotted as a nonlinear function of ϵ in figure 9 which indicates an infinitely large negative gain at zero error. As in figure 7 this value is not significant and for small errors $K_{\dot{\phi}}(\epsilon)$ may have the value of 0.417 shown by the dotted line. The corresponding system responses shown in figure 10 are almost identical to those shown in figure 8. Thus, either K_ϵ or $K_{\dot{\phi}}$ may be replaced by an appropriate nonlinear function of error to obtain satisfactory airplane responses to a wide range of step command inputs.

Example 2

This example shows how the basic method may be applied to the more complicated system shown in figure 11 which is the block diagram of the pitch channel of a typical airplane-autopilot combination.

The transfer function of the airplane contains an exceptionally large lead term in the numerator $(1 + 2.2p)$, and hence it was necessary to include the compensating network

$$1 + \frac{1.2}{1 + 2.2p}$$

in the system in order to obtain a satisfactory response even for the linear case with no rate limiting. The system parameters were adjusted to give the response shown in figure 12. In this case

$$K_\epsilon = 10$$

$$K_{\dot{\theta}} = 1.3$$

With an elevator rate limit of 50° per second the response became unstable for input commands greater than 2.5° as shown in figure 13.

The calculations involved in expressing K_ϵ as an optimum nonlinear function of error are the same as described in Example 1, except that at time t_s, when the signal to the servo is zero, the following condition applies

$$v_e = K_\epsilon \epsilon \left(1 + \frac{1.2}{1 + 2.2p}\right) - 1.3\dot{\theta} - \delta_e = 0$$

or

$$K_\epsilon \epsilon + \frac{1.2K_\epsilon \epsilon}{1 + 2.2p} = 1.3\dot{\theta} + \delta_e \tag{2}$$

For each given θ_i, the quantities θ, $\dot{\theta}$, ϵ, and δ_e are calculated at the proper t_s as illustrated in Example 1 (figs. 4 to 6 and table I). Equation (2) may then be written as

$$K_\epsilon \epsilon + v_1 = 1.3\dot{\theta} + \delta_e \tag{3}$$

where

$$v_1 = \frac{1.2K_\epsilon \epsilon}{1 + 2.2p} \tag{4}$$

To solve equation (2) for $K_\epsilon \epsilon$ it is first necessary to evaluate v_1 at the time t_s corresponding to each given value of θ_i. This may be done by writing equation (4) in the form

$$\dot{v}_1 + \frac{1}{2.2} v_1 = \frac{1.2}{2.2} K_\epsilon \epsilon$$

The solution of this differential equation (as shown for example in ref. 3) with zero initial conditions is

$$v_1(t_s) = \frac{1.2}{2.2} e^{-\frac{t_s}{2.2}} \int_0^{t_s} e^{\frac{t}{2.2}} K_\epsilon \epsilon(t)dt \tag{5}$$

However, at this point in the calculations K_ϵ is an unknown function of error and an exact solution is impossible. To obtain a first approximation of $v_1(t_s)$ it may be assumed that K_ϵ remains constant until time t_s. Thus

$$v_1(t_s) \approx \frac{1.2}{2.2} K_\epsilon e^{-\frac{t_s}{2.2}} \int_0^{t_s} e^{\frac{t}{2.2}} \epsilon(t)dt \tag{6}$$

Here $\epsilon(t) = \theta_i - \theta(t)$ where $\theta(t)$ is the known response to a constant-rate elevator input. This integral can be evaluated conveniently by means of graphical or numerical integration to give $v_1(t_s)$ expressed in terms of K_ϵ. This approximate value is then substituted into equation (3) to give a close if not exact value of K_ϵ for each θ_i.

More precise results could be obtained by repeating the solution of equation (5) with actual values of K_ϵ obtained in the first trial. While there may be cases where more than one iteration is required, it was found in the present example that v_1 was small compared to $K_\epsilon \epsilon$ and that a second solution was not necessary. The quantity K_ϵ as a nonlinear function of error is plotted in figure 14 and the corresponding system responses are shown in figure 15. Comparison with figure 13 shows a marked improvement in the performance of the system.

DISCUSSION

The method illustrated in this report was designed to give an optimum response with zero overshoot. By neglecting the lag of the servo it was assumed that the control surface responded instantaneously at a fixed rate when the servo error signal changed sign. However, when the systems were simulated on the analog computer, a representative value of servo lag was included. The small overshoots apparent in figures 8, 10, and 15 are the results of this lag and also of possible inaccuracies in the analog simulation. In any event, the results indicate that the simplified method is valid when considering servos with reasonably small time constants.

In some cases it may be desirable to purposely design a system to have a given overshoot for all input magnitudes. The same basic method is applicable; and in Example 1, if a 10-percent overshoot is desired, the abscissa scale of figure 5 is merely multiplied by 0.90. With reference to curve ⑦ of figure 4, a switching time of 0.35 second would now correspond to an input command of 0.9° of 39.4° or 35.5°, but the response would actually reach a peak value of 39.4°.

It is also possible to expand the basic method to include position limits in addition to rate limits. The procedure is the same except that the triangular inputs, such as shown in figure 4, are cut off at the value corresponding to the position limit. The switching time is still the point at which the control-surface rate becomes negative. For the two examples in this report it was found that the addition of position limits would have a stabilizing influence.

In general, the effects of limiting on a particular system can be shown clearly by plots similar to figure 5. These can be used to good advantage to obtain qualitative and even rough quantitative measures of system performance without resorting to the calculation of complete time responses. After determining the optimum curve for a particular system, it is a simple matter to plot corresponding curves for various fixed values of system gain. As long as the switching time remains below the optimum curve, the step response has no overshoot (if none exists in the linear system with no limiting). If t_s is greater than optimum for a particular input, the response is oscillatory or even unstable, necessitating a decrease in system gain (K_{ϵ}) or an increase in feedback gain ($K_{\dot{\varphi}}$).

In this regard it is interesting to note that figure 9 indicates that with the nonlinear (rate-limited) system the damping parameter ($K_{\dot{\varphi}}$) must increase with increasing error in order to obtain an optimum response. This is in direct contrast to a common practice for improving the response of linear second-order systems (ref. 1) where it is necessary for the damping to decrease with increasing error, thus allowing fast response to large errors while effectively preventing overshoot.

In Example 1, the rolling acceleration $\ddot{\varphi}$ is proportional in the steady state to the control-surface rate $\dot{\delta}_a$. Thus, a limit on $\dot{\delta}_a$ effectively limits the second derivative of the output φ, and may be termed an output acceleration limit. Similarly, a δ_a position limit would restrict $\dot{\varphi}$ and could be specified as an output-velocity limit. In this sense an acceleration limit is generally destabilizing, but for the type of systems considered the addition of a velocity limit tends to improve the stability.

As an extension to the present study a more complete generalization of the effects of acceleration and velocity limits would be of interest. This broader investigation should consider systems of varying degrees of complexity and also the effects of external disturbances other than pure step commands.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., Oct. 15, 1954.

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2. Hopkin, Arthur M.: A Phase Plane Approach to the Compensation of Saturating Servomechanisms. Trans. A.I.E.E., vol. 70, pt. I, 1951, pp. 631-639.

3. Phillips, H. B.: Differential Equations. Third ed., John Wiley & Sons, Inc., 1934.

TABLE I.– CALCULATION OF $K_\epsilon(\epsilon)$ AND $K_{\dot\varphi}(\epsilon)$ FOR EXAMPLE 1

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
φ_1, deg	t_s, sec (fig. 5)	$\dot\varphi$, deg/sec (fig. 6)	$0.417\dot\varphi$	δ_a, deg ($50t_s$)	$K_\epsilon\epsilon$ (4)+(5)	φ, deg (fig. 4)	ϵ, deg (1)-(7)	$K_\epsilon(\epsilon)$ (6)/(8)	$+3.33\epsilon$	$\dot\varphi K_{\dot\varphi}$ (10)-(5)	$K_{\dot\varphi}(\epsilon)$ (11)/(3)
2.5	0.115	7.9	3.29	5.75	9.04	0.30	2.20	4.100	7.3	1.5	0.19
5	.149	12.8	5.34	7.45	12.79	.68	4.32	2.961	14.4	6.9	.54
10	.198	21.6	9.00	9.90	18.90	1.50	8.50	2.224	28.3	18.4	.85
15	.234	29.0	12.08	11.70	23.78	2.42	12.58	1.891	41.9	30.2	1.04
20	.263	35.6	14.83	13.15	27.98	3.36	16.64	1.681	55.5	42.3	1.19
25	.289	42.0	17.50	14.45	31.95	4.36	20.64	1.548	68.8	54.4	1.30
30	.312	47.9	19.96	15.60	35.56	5.37	24.63	1.443	82.1	66.5	1.39
40	.353	58.9	24.54	17.65	42.19	7.56	32.44	1.300	108.2	90.6	1.54
50	.390	69.7	29.04	19.50	48.54	9.95	40.05	1.212	133.5	114.0	1.64
60	.422	79.2	33.00	21.10	54.10	12.34	47.66	1.135	158.9	137.8	1.74
70	.452	88.5	36.88	22.60	59.48	14.82	55.18	1.078	183.9	161.3	1.82
80	.480	97.2	40.50	24.00	64.50	17.45	62.55	1.031	208.5	184.5	1.90
90	.506	105.9	44.13	25.30	69.43	20.00	70.00	.992	233.3	208.0	1.96

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$$K_\epsilon(\epsilon) = \frac{0.417\dot\varphi + \delta_a}{\epsilon}$$

$$K_{\dot\varphi}(\epsilon) = \frac{3.33\epsilon - \delta_a}{\dot\varphi}$$

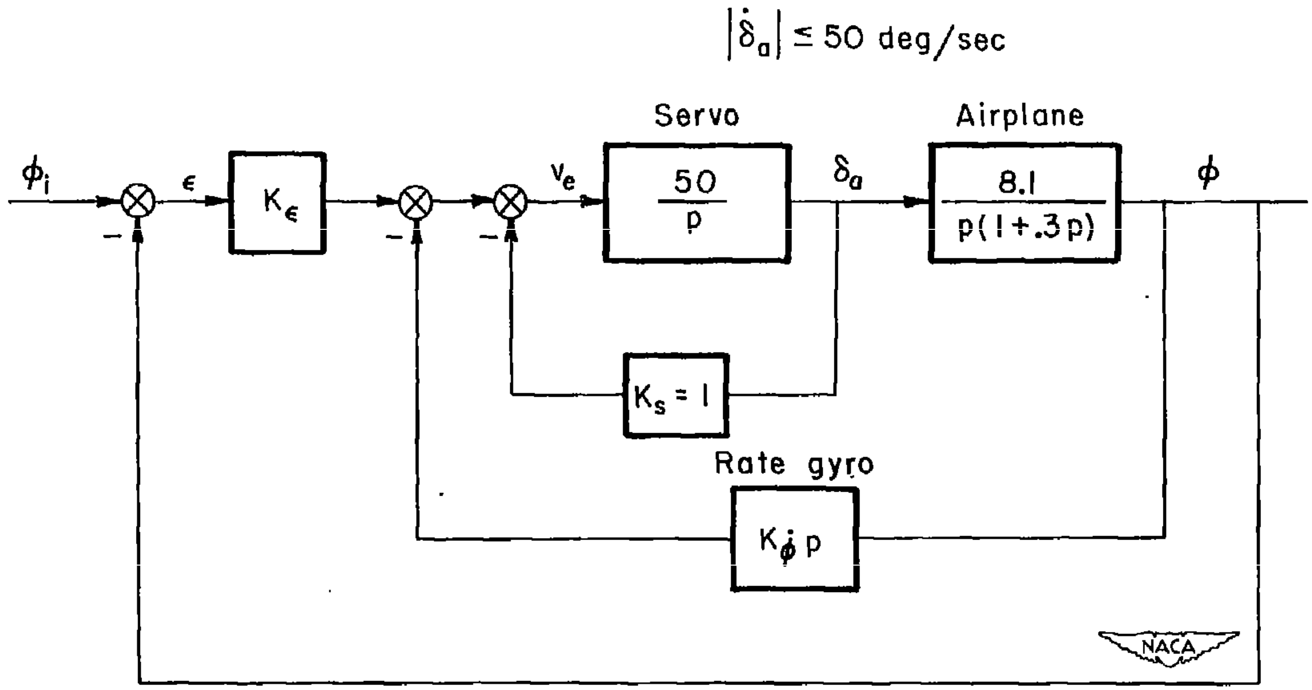


Figure 1.– Block diagram of roll-control system used in Example 1.

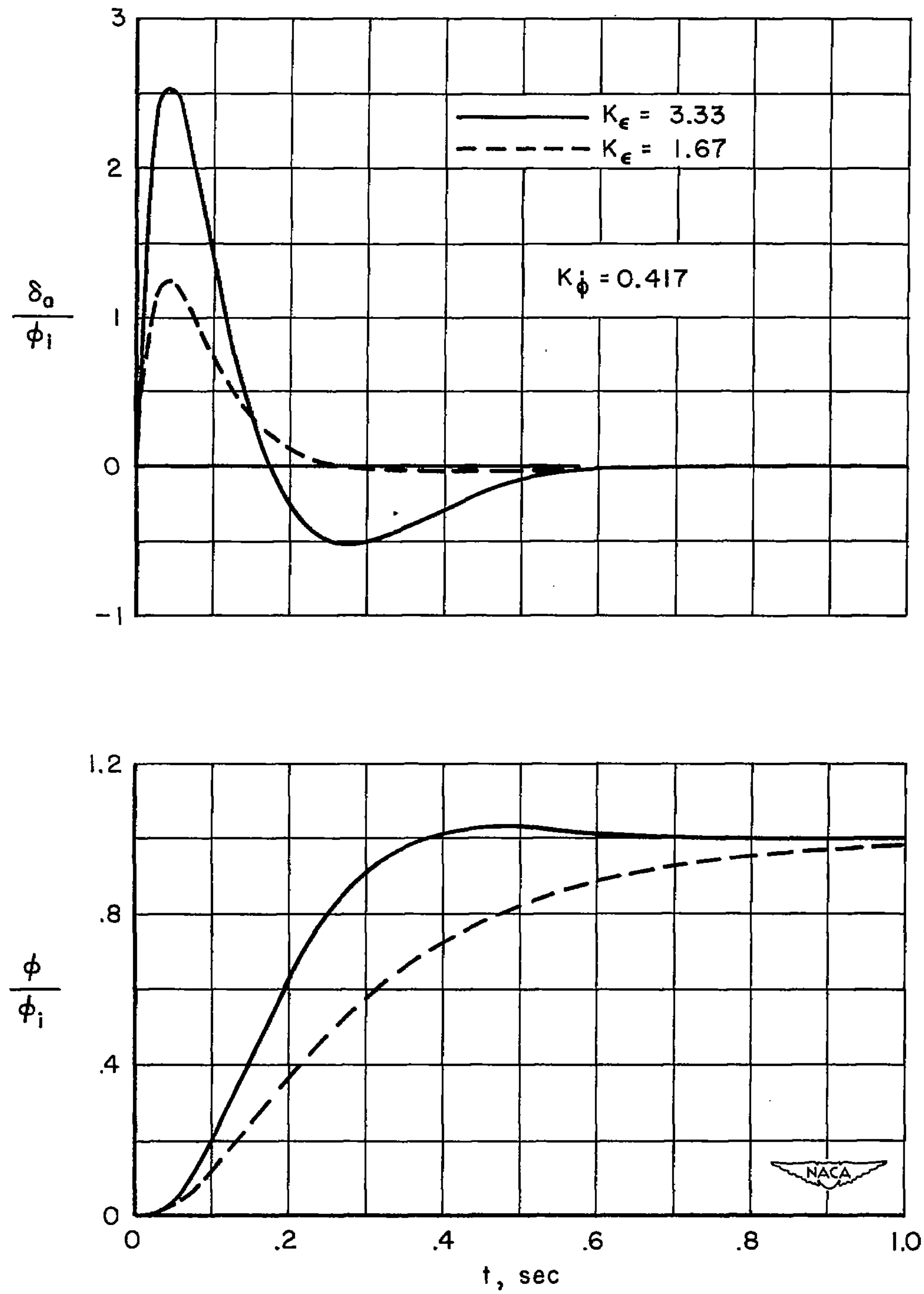


Figure 2.- Aileron and roll-angle response of basic roll-control system with no rate limiting.

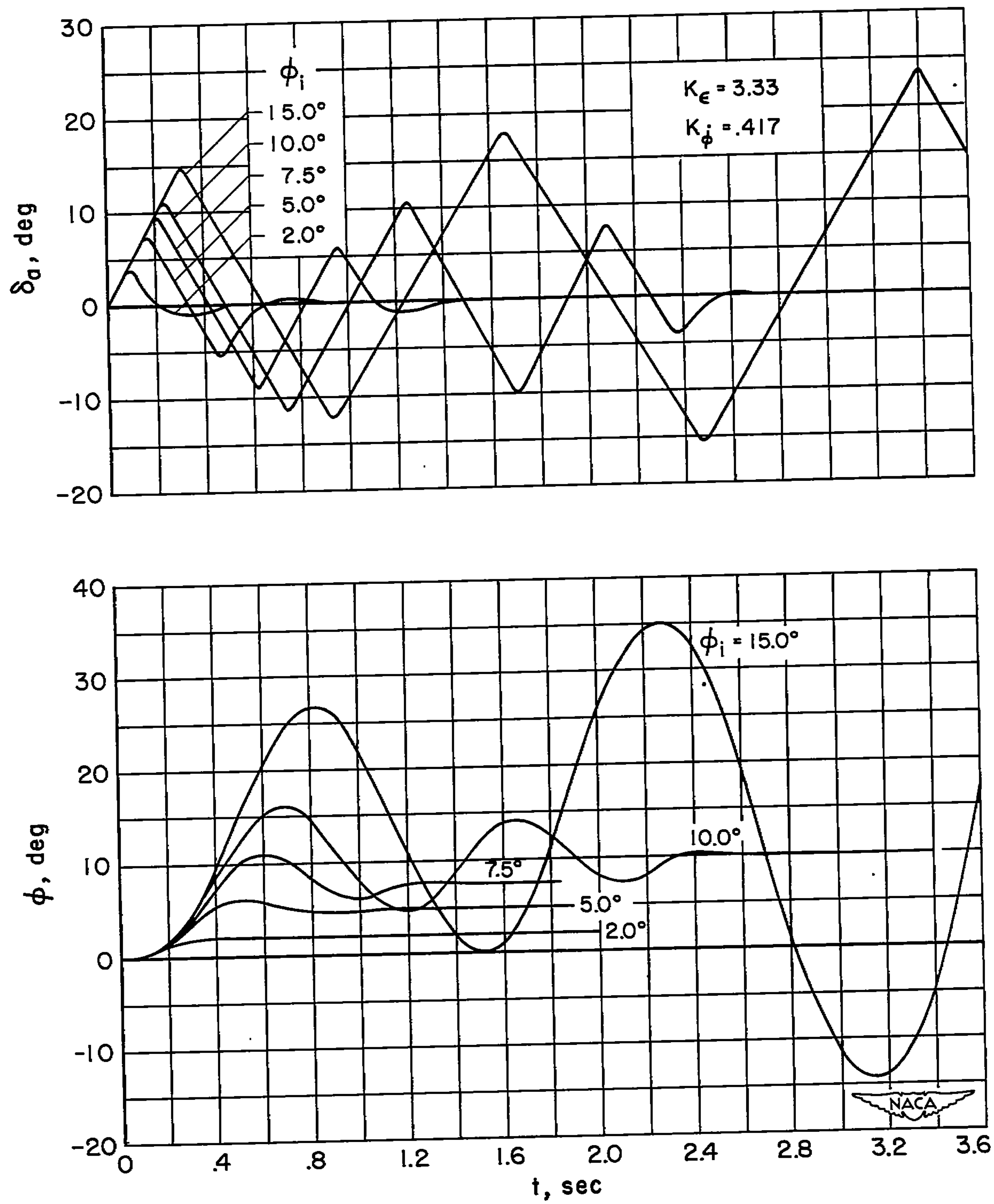


Figure 3.- Response of basic roll-control system to step commands with aileron rate limited to 50° per sec.

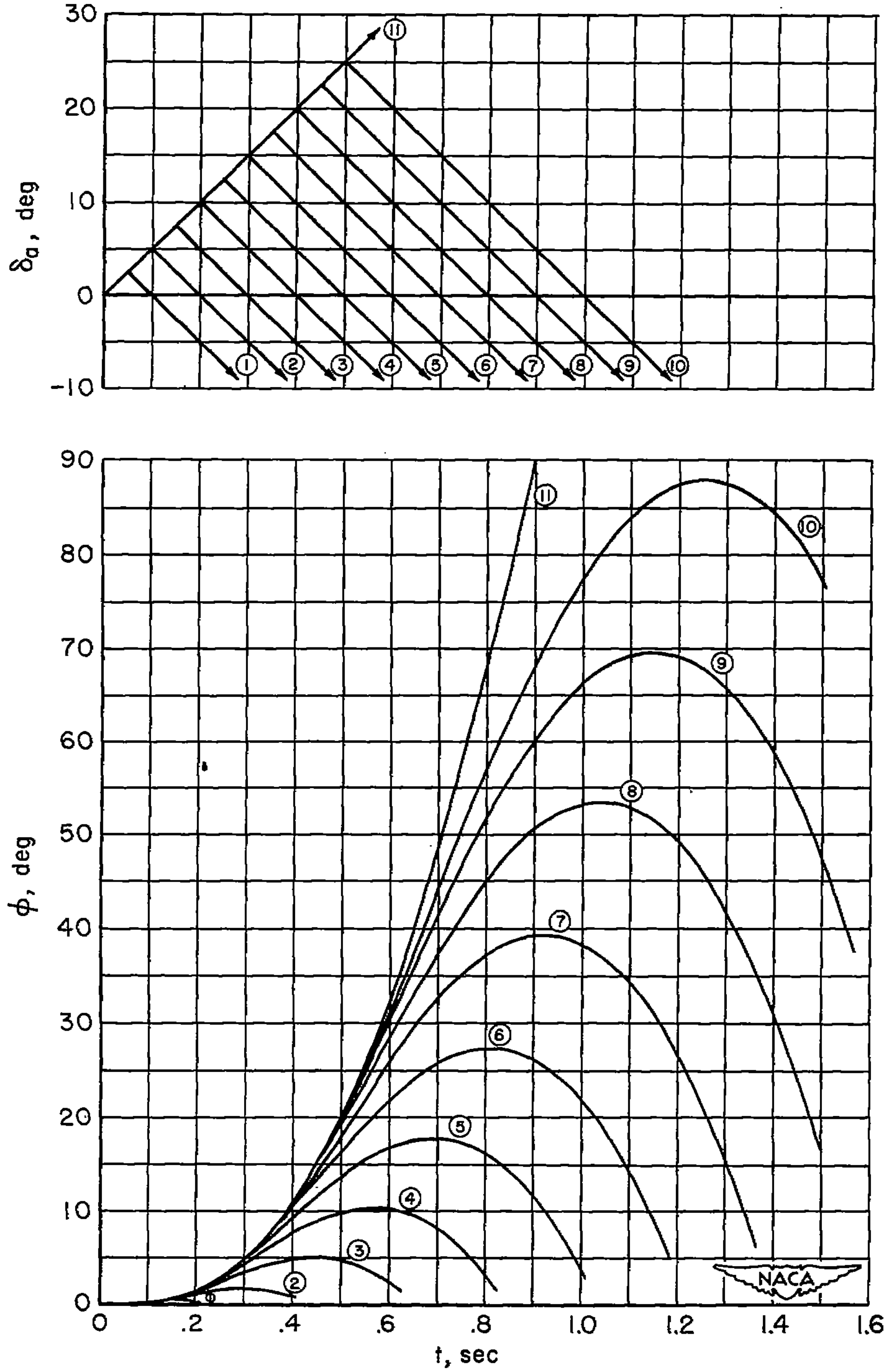


Figure 4.– Roll-angle responses to constant-rate (50° per sec) aileron inputs.

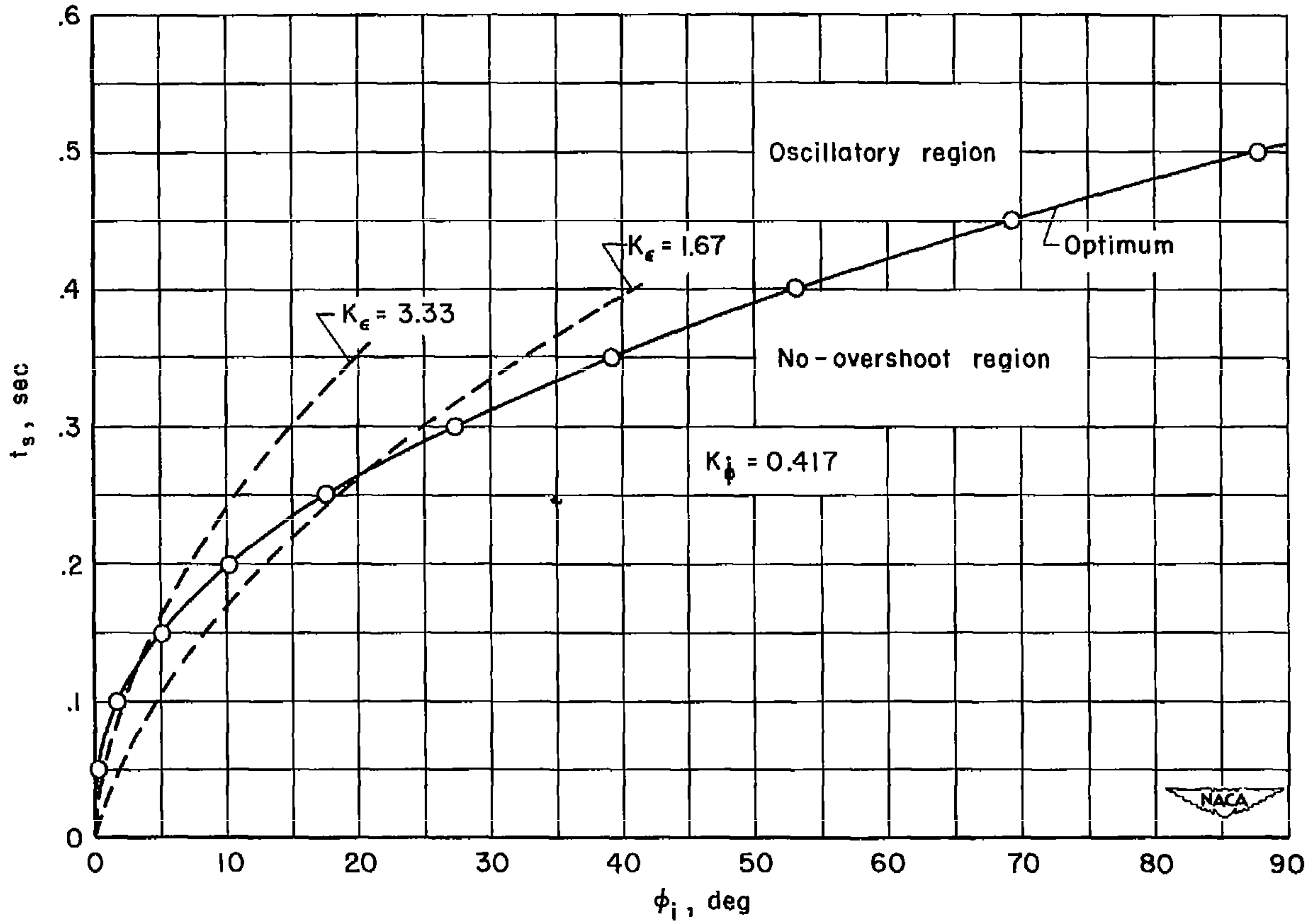


Figure 5.– Aileron switching time as a function of roll-angle command.

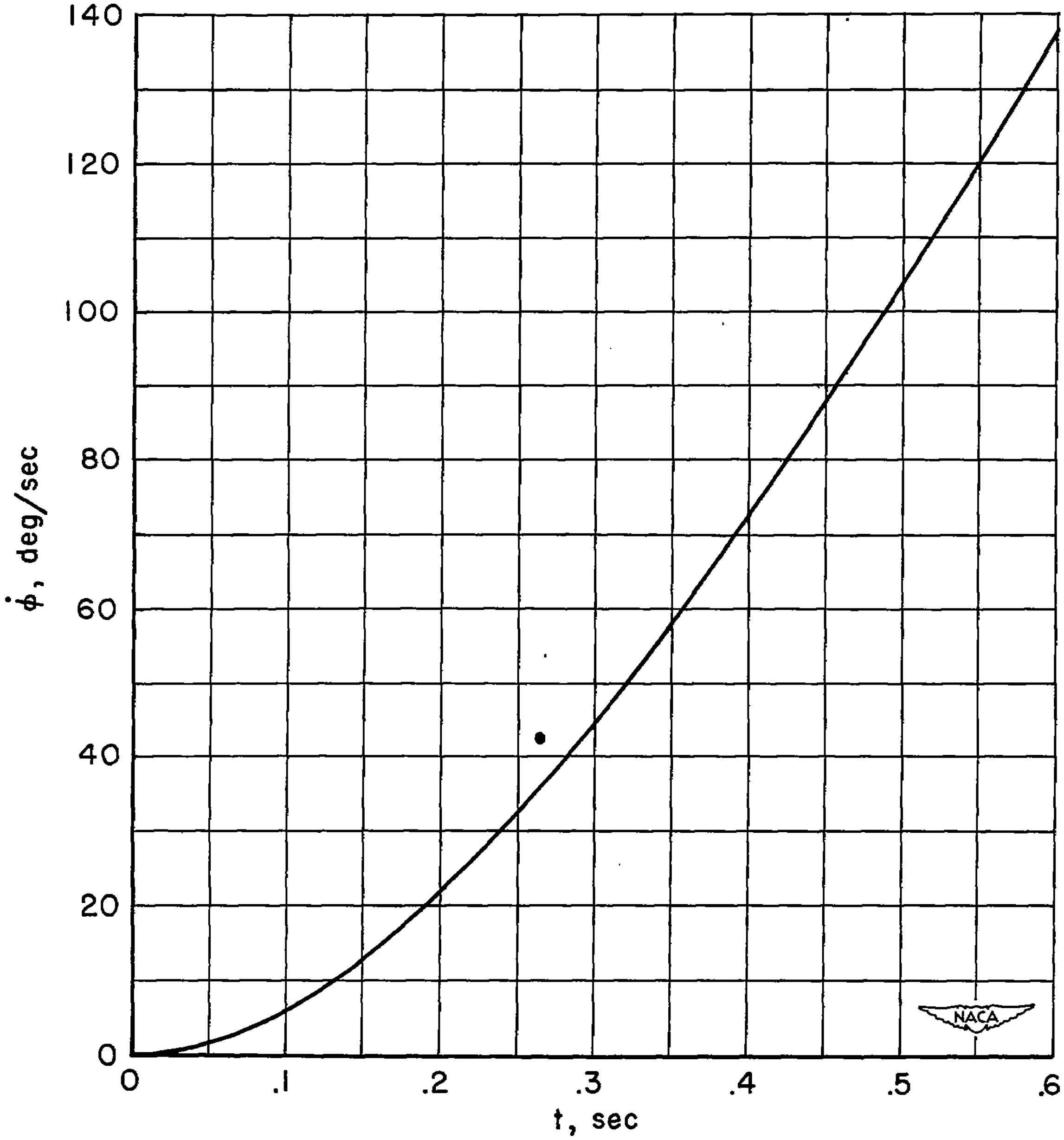


Figure 6.– Rolling-velocity response to a constant rate (50° per sec) aileron input.

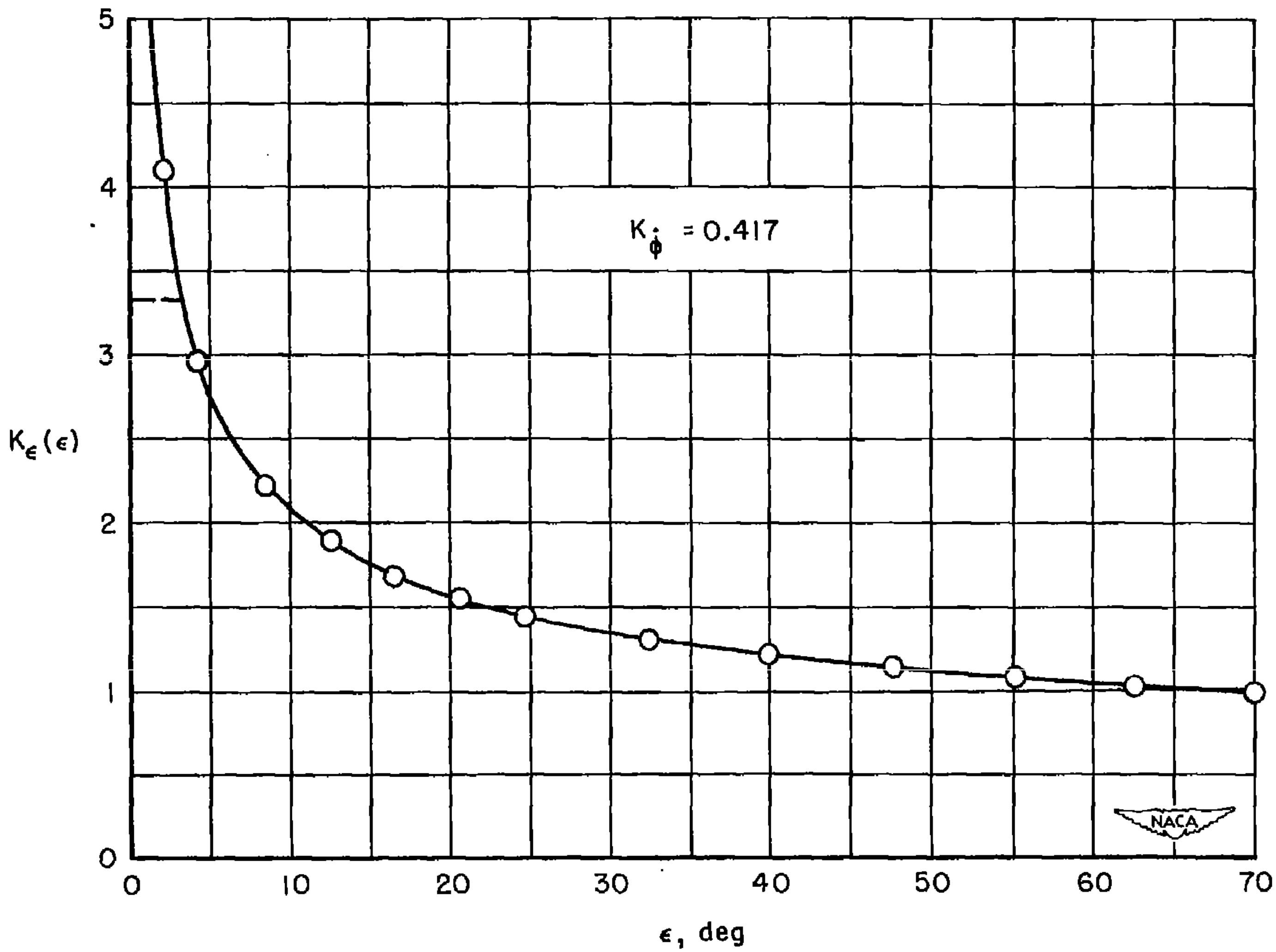
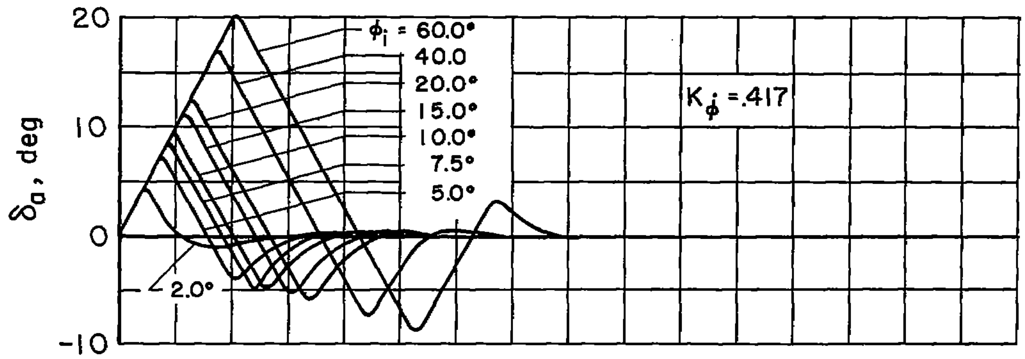


Figure 7.– Optimum nonlinear gain function $K_\epsilon(\epsilon)$ for the roll-control system.



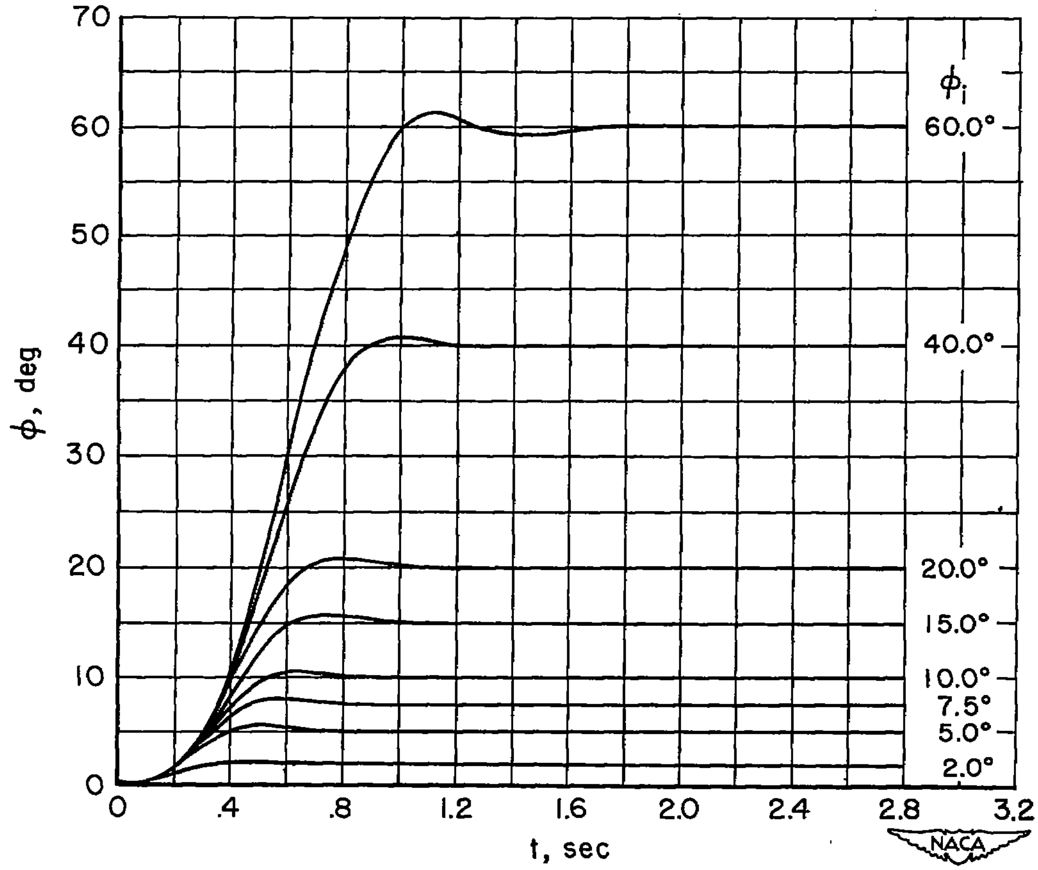


Figure 8.– Response of modified roll-control system to step commands with K_ϵ as a nonlinear function of error.

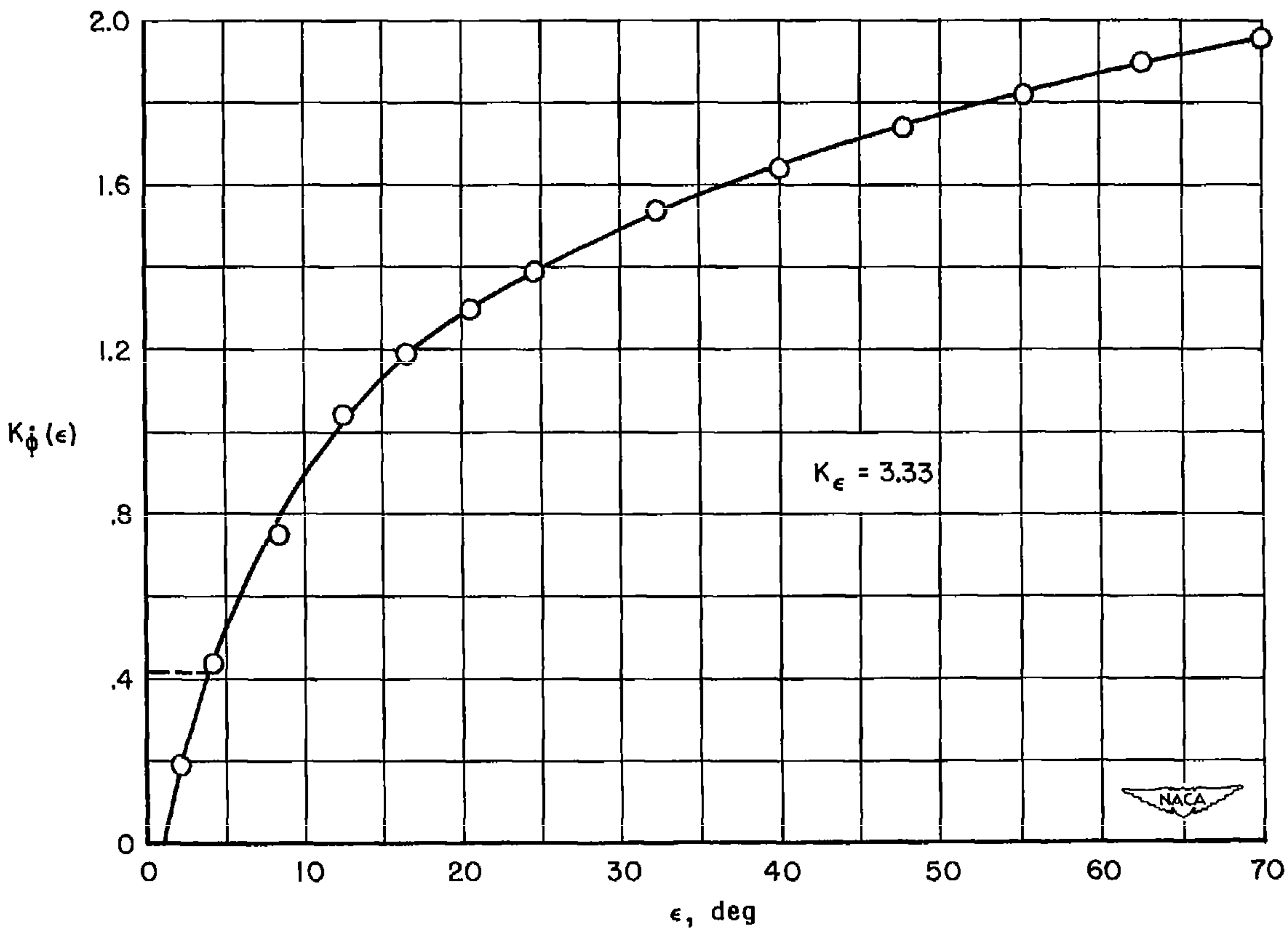


Figure 9.– Optimum nonlinear function $K_{\dot{\phi}}(\epsilon)$ for the roll-control system.

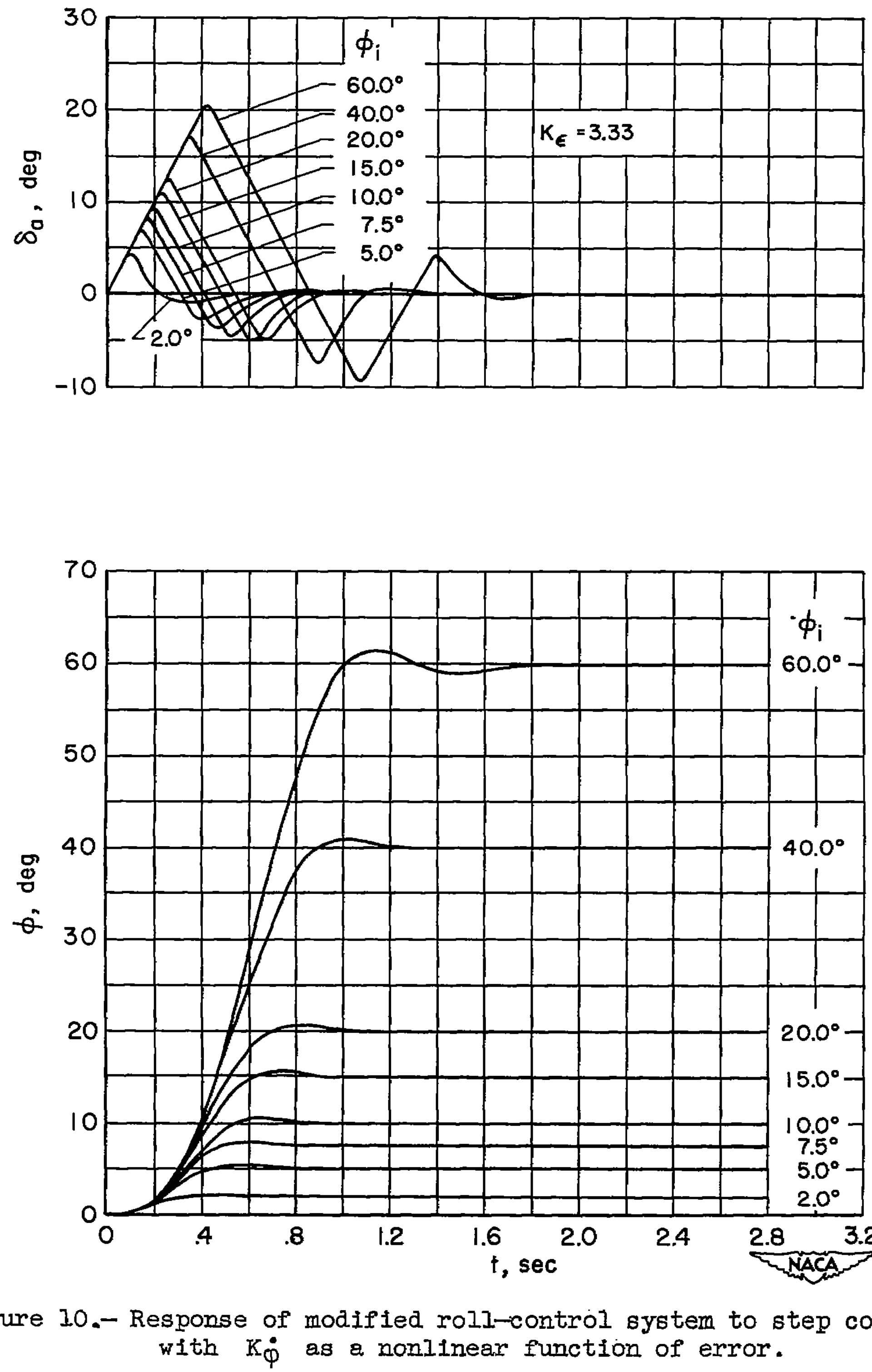


Figure 10.- Response of modified roll-control system to step commands with $K_{\dot{\phi}}$ as a nonlinear function of error.

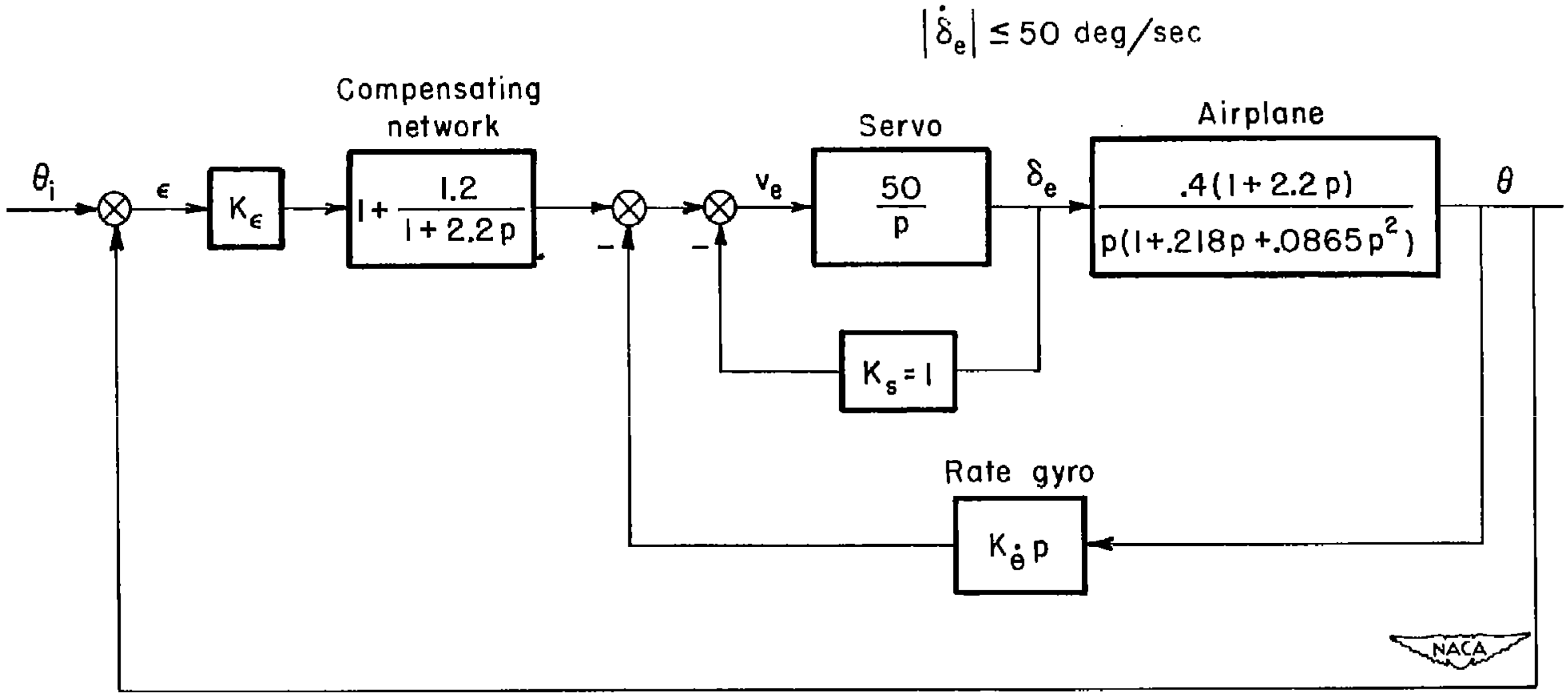


Figure 11.– Block diagram of pitch-control system used in Example 2.

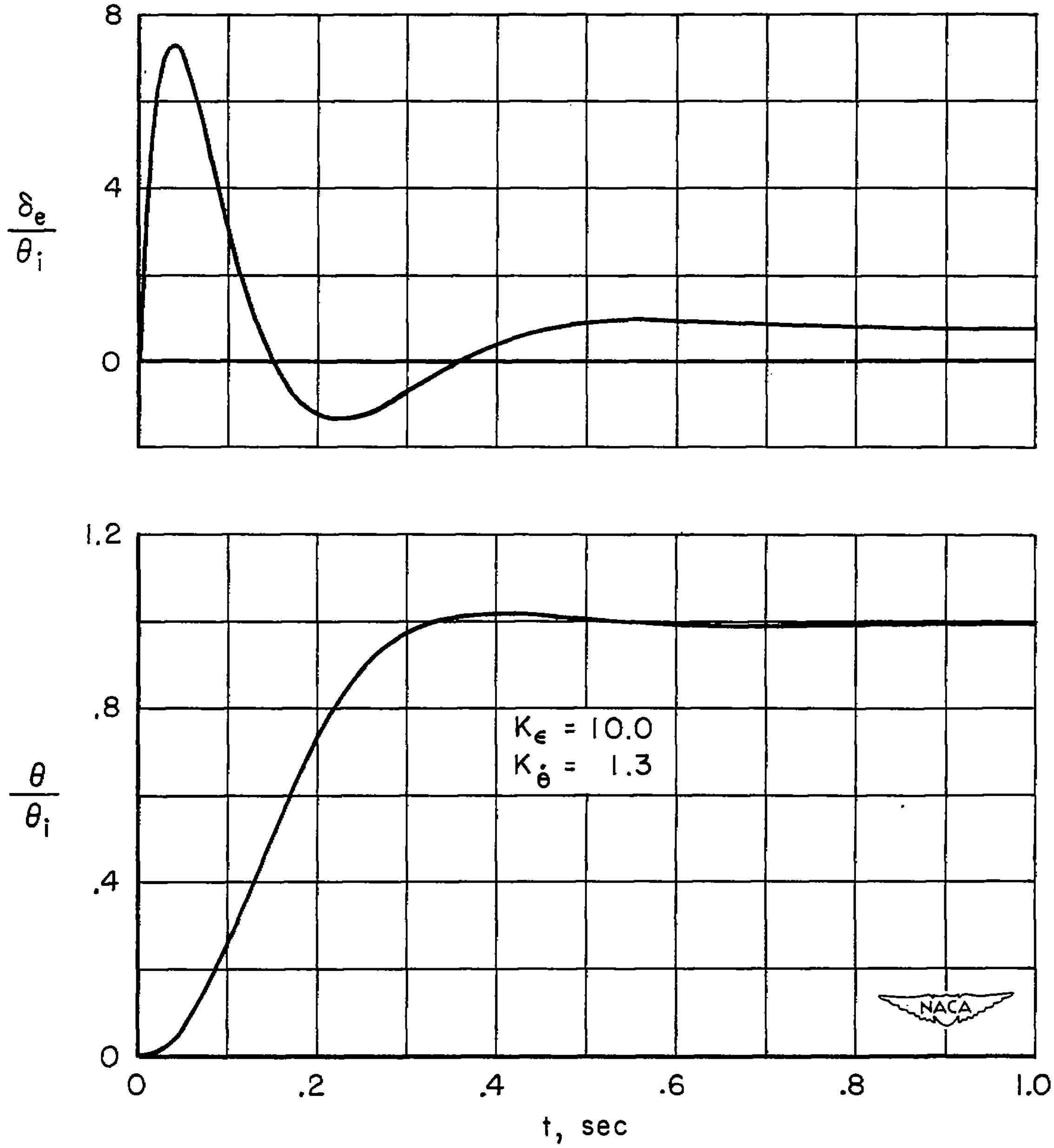


Figure 12.- Elevator- and pitch-angle response of basic pitch-control system with no rate limit.

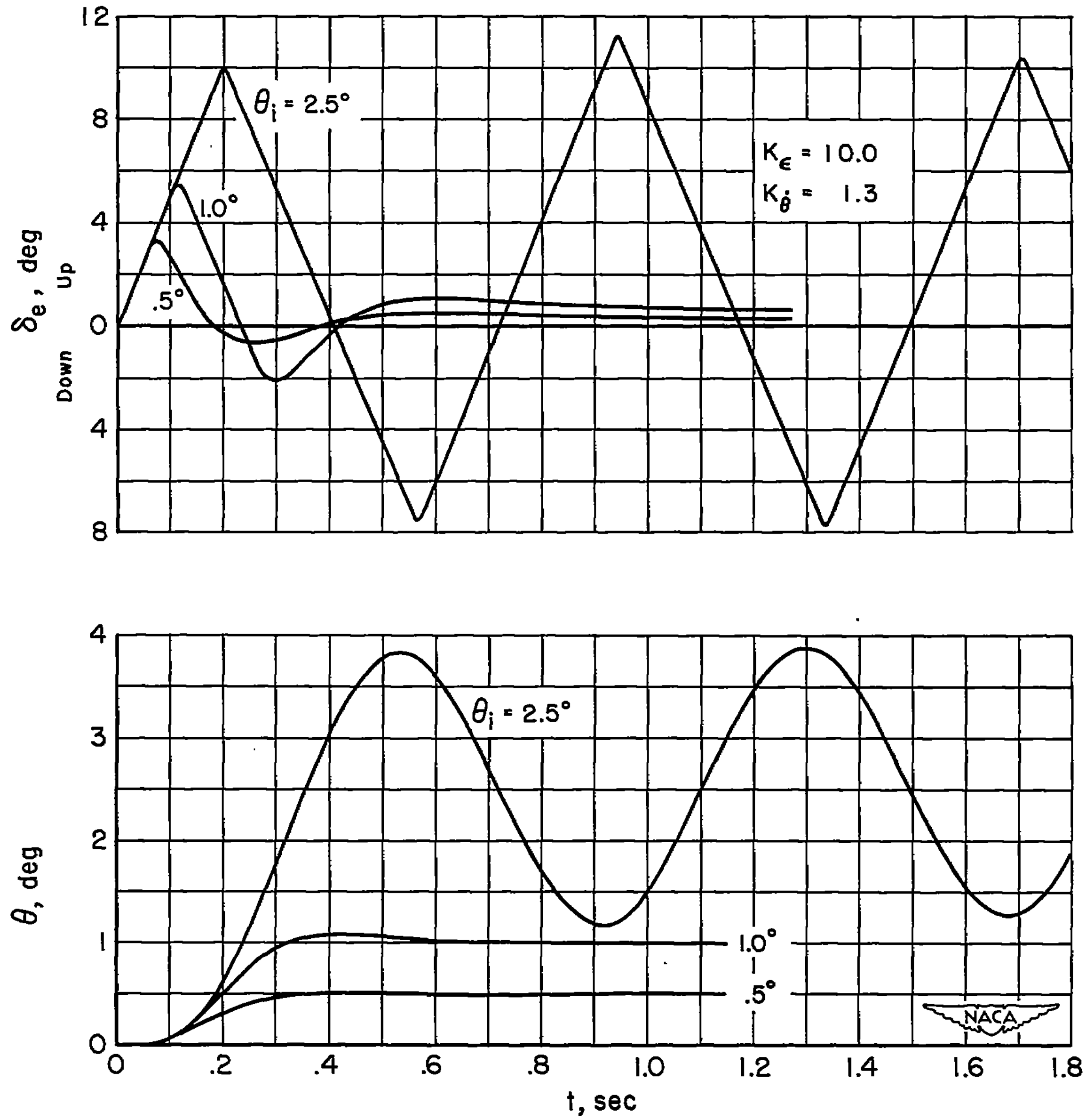


Figure 13.- Response of basic pitch-control system to step commands with elevator rate limit of 50° per sec.

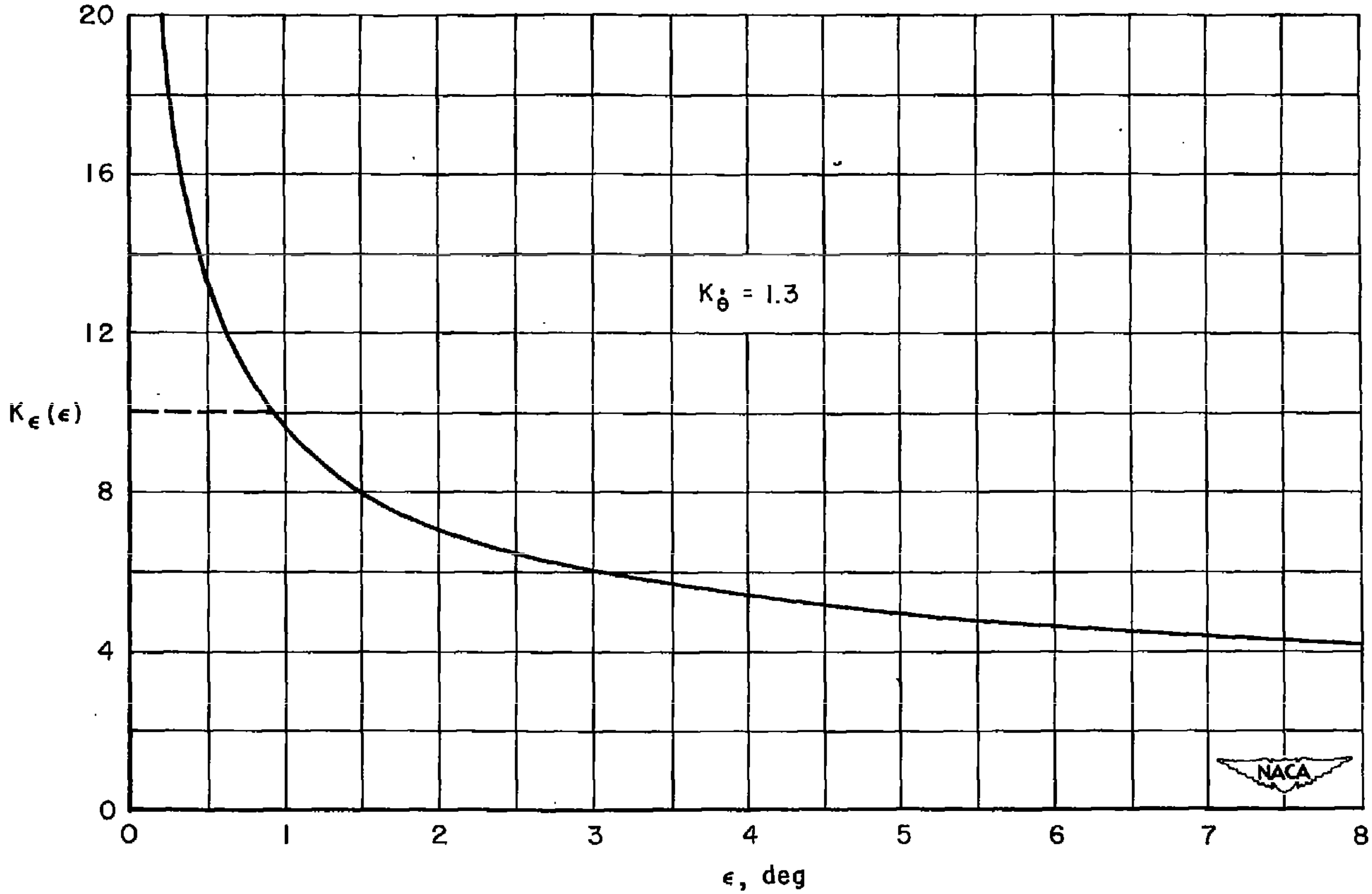


Figure 14.– Optimum nonlinear gain function $K_{\epsilon}(\epsilon)$ for the pitch–control system.

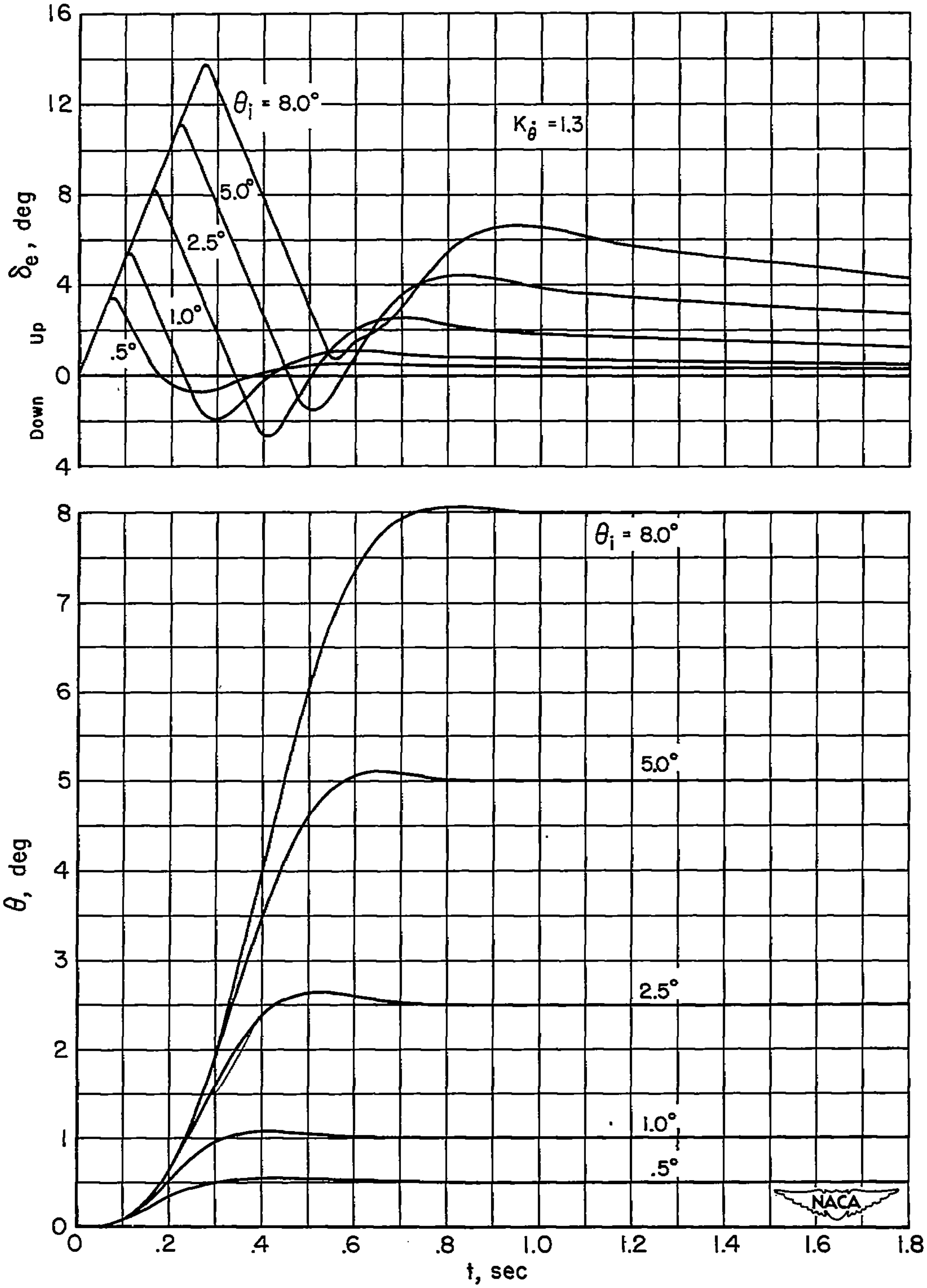


Figure 15.– Response of modified pitch-control system to step commands with K_ϵ as a nonlinear function of error.

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