NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3401

LAMINAR BOUNDARY LAYER BEHIND SHOCK ADVANCING INTO STATIONARY FLUID

By Harold Mirels

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

Washington
March 1955

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3401

LAMINAR BOUNDARY LAYER BEHIND SHOCK ADVANCING INTO STATIONARY FLUID

By Harold Mirels

SUMMARY

A study was made of the laminar compressible boundary layer induced by a shock wave advancing into a stationary fluid bounded by a wall. For weak shock waves, the boundary layer is identical with that which occurs when an infinite wall is impulsively set into uniform motion (Rayleigh problem). A numerical solution was required for strong shocks.

Velocity and temperature profiles, recovery factors, and skin-friction and heat-transfer coefficients are tabulated for a wide range of shock strengths.

INTRODUCTION

If a shock wave advances into a stationary fluid bounded by a wall, a boundary-layer flow is established along the wall behind the shock. This boundary layer is often important in studies of phenomena involving nonstationary shock waves. In a shock tube, for example, this boundary layer acts to attenuate the strength of the shock which propagates through the low-pressure side of the tube (refs. 1 and 2). If the shock tube is used as an aerodynamic wind tunnel, the test time available may depend, for long shock tubes, on the time it takes the boundary layer to introduce nonuniformities in the test section.

Another example of a shock-generated boundary layer occurs when a combustible mixture is ignited within a tube. In this case, a shock wave, followed by a flame front, is observed, as discussed in references 3 to 5. The shock wave is particularly strong when ignition occurs at a closed end. For long tubes, the progress of the flame front will be related to the boundary-layer development behind the shock. Since flame speed is increased by fluid turbulence, a transition from laminar to turbulent flow will accelerate the flame. Thus, the boundary layer may play a role in the acceleration of a low-speed flame to a detonation wave in a long tube (ref. 4).

The boundary layer behind the shock was studied in references 1 and 2. Both papers were primarily concerned with shock-wave attenuation. In reference 1, an approximate solution for the boundary layer was estimated by referring to the flow induced within a circular cylinder that is impulsively set into uniform translation. The validity of this analogy was not established. In reference 2, the correct boundary-layer equations were considered. These were integrated with a REAC (Reeves Electronic Analog Computer). Values of skin-friction and heat-transfer coefficients were presented. However, no velocity or temperature profiles were reported. Because of the growing interest in phenomena related to these shock-induced boundary layers, it was felt that a more detailed and more accurate study of this boundary-layer problem was warranted. Such a study was conducted at the NACA Lewis laboratory and the results are presented herein.[1]

In the following sections, the laminar compressible boundary layer behind a shock wave advancing into a stationary fluid, bounded by a wall, is analyzed. For weak shocks, an analytical perturbation solution is presented. Numerical results for velocity and temperature profiles and heat-transfer and skin-friction coefficients are tabulated, covering the range from weak to strong shocks. The numerical results are correct to four decimal places.

ANALYSIS

Coordinate Systems

A shock wave of constant strength is considered to move, parallel to a wall, into a stationary fluid. Let $(\bar{x},\bar{y})$ be a coordinate system fixed in respect to the wall and let $\bar{u}$ and $\bar{v}$ be velocities parallel to the $\bar{x}$ and $\bar{y}$ coordinates, respectively, as indicated in figure 1(a). The flow is unsteady in this coordinate system. Let (x,y) represent a coordinate system moving with the shock wave (fig. 1(b)). The velocities parallel to the x and y coordinates are denoted by u and v. In this coordinate system, the flow is steady.

Assume that, at time $t = 0$, the two coordinate systems coincide. If $\bar{u}_s$ is the velocity of the shock wave relative to the wall, then $\bar{x}$ and x are related by $x = \bar{x} - \bar{u}_s t$. Similarly, the axial velocities are related by $u = \bar{u} - \bar{u}_s$. Note that the wall moves with velocity $u_w = -\bar{u}_s$ in the steady coordinate system.

---

[1]The writer's present interest in the shock-induced boundary-layer problem was stimulated by a private communication from Prof. N. Rott of Cornell University, who is studying heat-transfer problems associated with shock tubes.

Boundary-Layer Equations

The Prandtl boundary-layer equations apply for the flow in the vicinity of the wall (except at the base of the shock, where the boundary-layer assumptions break down). Assuming the flow to be laminar and $dp/dx = 0$, the equations of motion are, for $x > 0$,

$$\left.\begin{aligned}
&\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \text{ (continuity)}\\
&u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y}\right) \text{ (momentum)}\\
&\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \text{ (energy)}\\
&p = \rho R T \text{ (state)}
\end{aligned}\right\} \qquad (1)$$

The additional symbols are defined in appendix A. The boundary conditions for $x > 0$ are

$$\left.\begin{aligned}
&u(x,0) = -\bar{u}_s \qquad && u(x,\infty) = u_e\\
&v(x,0) = 0 && T(x,\infty) = T_e\\
&T(x,0) = T_w
\end{aligned}\right\} \qquad (2)$$

These are the usual boundary conditions, except that the fluid at the wall moves with velocity $u(x,0) \equiv u_w = -\bar{u}_s$ in order to satisfy the condition of zero slip at the wall. It will be assumed that the wall temperature T_w is constant. The magnitudes of u_e, T_e, and $\bar{u}_s$ depend on the shock strength and can be found from the normal shock relations quoted in appendix B. The ratio u_w/u_e increases from a value of 1 for a very weak shock to a value of $(\gamma + 1)/(\gamma - 1)$ for a very strong shock wave. Thus, in the steady coordinate system, the u velocities in the boundary layer have a maximum at the wall and decrease monotonically to the value in the free stream (as indicated in fig. 1(b)).

Transformation. - Equations (1) and (2) are transformed to a system of ordinary differential equations with the methods of references 6 and 7.

From the continuity equation, a stream function ψ exists such that $\partial\psi/\partial y = \rho u/\rho_w$, $-\partial\psi/\partial x = \rho v/\rho_w$. Following reference 6, a similarity parameter η is defined according to the relation

$$\eta = \sqrt{\frac{1}{2} \frac{u_e}{x\nu_w}} \int_0^y \frac{T_w}{T} dy \tag{3}$$

and the stream function is written as

$$\psi = \sqrt{2u_e x\nu_w}\, f(\eta) \tag{4}$$

Note that $f' = u/u_e$. As in reference 7, the viscosity is assumed to vary linearly with temperature. If the viscosity is referenced to the wall value,

$$\mu = \left(\frac{\mu_w}{T_w}\right) T \tag{5}$$

Substitution of equations (3), (4), and (5) into the momentum equation yields

$$f''' + ff'' = 0 \tag{6}$$

with the boundary conditions

$$\left.\begin{aligned} f(0) &= 0 \\ f'(0) &= u_w/u_e \\ f'(\infty) &= 1 \end{aligned}\right\} \tag{7}$$

Equation (6) is the familiar Blasius differential equation. However, the tangential velocity boundary condition at the wall $(f'(0) = u_w/u_e)$ is different from the zero value usually encountered in studies of viscous flow past a semi-infinite plate. Numerical integration is required except for the limiting case of a weak shock, $[(u_w/u_e) - 1] \ll 1$, for which an analytical perturbation solution is possible.

For T a function of η only, the energy equation becomes

$$T'' + \sigma f T' = -\sigma(\gamma - 1) M_e^2 (f'')^2 \tag{8}$$

assuming that the Prandtl number σ is constant. Since equation (8) is linear, the general solution for T can be expressed as the linear superposition of the solution for zero heat transfer plus the effect of heat transfer. That is, T/T_e can be expressed in the form

$$\frac{T}{T_e} = 1 + \frac{\gamma - 1}{2}\left[\left(\frac{u_w}{u_e} - 1\right) M_e\right]^2 r(\eta) + \left\{\left(\frac{T_w}{T_e} - 1\right) - \frac{\gamma - 1}{2}\left[\left(\frac{u_w}{u_e} - 1\right) M_e\right]^2 r(0)\right\} s(\eta) \tag{9}$$

where $r(\eta)$ satisfies

$$\left.\begin{aligned} r'' + \sigma f r' &= \frac{-2\sigma}{\left(\frac{u_w}{u_e} - 1\right)^2}(f'')^2 \\ r(\infty) &= r'(0) = 0 \end{aligned}\right\} \tag{10}$$

and $s(\eta)$ satisfies

$$\left.\begin{aligned} s'' + \sigma f s' &= 0 \\ s(0) = 1;\ s(\infty) &= 0 \end{aligned}\right\} \tag{11}$$

Note that $[(u_w/u_e) - 1] M_e \equiv |\bar{u}_e|/a_e$ is the Mach number of the external flow relative to the wall. For an insulated wall, the coefficient of s (in eq. (9)) equals zero, so that the wall temperature is

$$\frac{T_{w,i}}{T_e} = 1 + \frac{\gamma - 1}{2}\left[\left(\frac{u_w}{u_e} - 1\right) M_e\right]^2 r(0) \tag{12}$$

Thus, $r(0)$ is a recovery factor based on the Mach number of the external flow relative to the wall. Equation (9), in terms of $T_{w,i}$, is

$$\frac{T}{T_e} = 1 + \frac{\gamma - 1}{2}\left[\left(\frac{u_w}{u_e} - 1\right) M_e\right]^2 r(\eta) + \left(\frac{T_w}{T_e} - \frac{T_{w,i}}{T_e}\right) s(\eta) \tag{12a}$$

Equations (10) and (11) can be expressed in quadrature form

$$r = \frac{2\sigma}{\left(\frac{u_w}{u_e} - 1\right)^2} \int_\eta^\infty [f''(\xi)]^\sigma \, d\xi \int_0^\xi [f''(\theta)]^{2-\sigma} \, d\theta \tag{13}$$

$$s = \int_\eta^\infty [f''(\xi)]^\sigma \, d\xi \bigg/ \int_0^\infty [f''(\xi)]^\sigma \, d\xi \tag{14}$$

For $\sigma = 1$, these equations can be integrated to yield

$$r = 1 - \left(\frac{\frac{u_w}{u_e} - f'}{\frac{u_w}{u_e} - 1}\right)^2 \tag{15}$$

$$s = \frac{f' - 1}{\frac{u_w}{u_e} - 1} \tag{16}$$

The solutions for other Prandtl numbers are discussed later.

An alternate system of equations is described in appendix C.

Relation between y and η. - For x constant, the relation between y and η is, from equation (3),

$$y\sqrt{\frac{1}{2}\frac{u_e}{x\nu_w}} = \int_0^\eta \frac{T}{T_w} \, d\eta \tag{17}$$

Substitution of equation (12a) into equation (17) yields

$$y\sqrt{\frac{1}{2}\frac{u_e}{x\nu_w}} = \frac{T_e}{T_w}\left\{\eta + \frac{\gamma - 1}{2}\left[\left(\frac{u_w}{u_e} - 1\right) M_e\right]^2 \int_0^\eta r \, d\eta + \frac{T_w - T_{w,i}}{T_e} \int_0^\eta s \, d\eta\right\} \tag{18}$$

For $\sigma = 1$, equations (15) and (16) can be substituted into equation (18) with the following result:

$$y\sqrt{\frac{1}{2}\frac{u_e}{x\nu_w}} = \frac{T_e}{T_w}\left\{\eta + \frac{\frac{T_w}{T_e} - 1}{\frac{u_w}{u_e} - 1}(f - \eta) + \right.$$

$$\left.\frac{\gamma - 1}{2} M_e^2 \left[\frac{u_w}{u_e}(f - \eta) + f(1 - f') + f''(0) - f''\right]\right\} \tag{19}$$

If the wall is insulated, equation (19) becomes

$$y\sqrt{\frac{1}{2}\frac{u_e}{x\nu_w}} = \frac{\eta + \frac{\gamma - 1}{2} M_e^2 \left[(f - \eta)\left(\frac{2u_w}{u_e} - 1\right) + f(1 - f') + f''(0) - f''\right]}{1 + \frac{\gamma - 1}{2}\left[\left(\frac{u_w}{u_e} - 1\right) M_e\right]^2} \tag{20}$$

Equations (18) to (20) are useful for obtaining velocity and temperature profiles in terms of y rather than η.

Perturbation Solution for $\left(\frac{u_w}{u_e} - 1\right) \ll 1$

If the shock wave is weak, $(u_w/u_e) - 1$ is small, and a perturbation analysis in terms of this parameter is possible. Let

$$f = \sum_{n=0}^{\infty} f_n \left(\frac{u_w}{u_e} - 1\right)^n \tag{21a}$$

$$r = \sum_{n=0}^{\infty} r_n \left(\frac{u_w}{u_e} - 1\right)^n \tag{21b}$$

$$s = \sum_{n=0}^{\infty} s_n \left(\frac{u_w}{u_e} - 1\right)^n \tag{21c}$$

Substituting equation (21a) into equations (6) and (7) and equating coefficients of $[(u_w/u_e) - 1]^0$ and $[(u_w/u_e) - 1]^1$ yield, respectively,

$$\left.\begin{aligned} f_0''' + f_0 f_0'' &= 0 \\ f_0(0) = 0;\; f_0'(0) &= f_0'(\infty) = 1 \end{aligned}\right\} \qquad (22)$$

$$\left.\begin{aligned} f_1''' + f_0 f_1'' + f_1 f_0'' &= 0 \\ f_1(0) = 0;\; f_1'(0) = 1;\; f_1'(\infty) &= 0 \end{aligned}\right\} \qquad (23)$$

Integration of equations (22) and (23) gives

$$\left.\begin{aligned} f_0 &= \eta \\ f_1 &= \eta \operatorname{erfc}\left(\frac{\eta}{\sqrt{2}}\right) + \sqrt{\frac{2}{\pi}}\left[1 - \exp\left(-\frac{\eta^2}{2}\right)\right] \end{aligned}\right\} \qquad (24)$$

so that

$$\left.\begin{aligned} f &= \eta + \left\{\eta \operatorname{erfc}\left(\frac{\eta}{\sqrt{2}}\right) + \sqrt{\frac{2}{\pi}}\left[1 - \exp\left(-\frac{\eta^2}{2}\right)\right]\right\}\left(\frac{u_w}{u_e} - 1\right) + O\left(\frac{u_w}{u_e} - 1\right)^2 \\ f' &= 1 + \left[\operatorname{erfc}\left(\frac{\eta}{\sqrt{2}}\right)\right]\left(\frac{u_w}{u_e} - 1\right) + O\left(\frac{u_w}{u_e} - 1\right)^2 \\ f'' &= -\sqrt{\frac{2}{\pi}}\left(\frac{u_w}{u_e} - 1\right) e^{-\eta^2/2} + O\left(\frac{u_w}{u_e} - 1\right)^2 \end{aligned}\right\} \qquad (25)$$

Substituting the preceding solution for f into equations (10) and (11) and equating coefficients of $\left(\frac{u_w}{u_e} - 1\right)^0$ yield

$$\left.\begin{aligned} r_0'' + \sigma\eta r_0' &= -\frac{4}{\pi}\,\sigma\, e^{-\eta^2} \\ r_0'(0) = r_0(\infty) &= 0 \end{aligned}\right\} \qquad (26)$$

$$\left.\begin{aligned} s_0'' + \sigma\eta s_0' &= 0 \\ s_0(0) = 1;\; s_0(\infty) &= 0 \end{aligned}\right\} \qquad (27)$$

Integration of equation (26) yields (from results of ref. 8)

$$r = \frac{4}{\pi}\left(\frac{\sigma}{2 - \sigma}\right)^{1/2} \int_{\sin^{-1}\sqrt{\frac{\sigma}{2}}}^{\pi/2} \exp\left(\frac{-\sigma\eta^2}{2} \frac{1}{\sin^2\theta}\right) d\theta + O\left(\frac{u_w}{u_e} - 1\right) \tag{28}$$

The integral of equation (27) is

$$s = \operatorname{erfc}\left(\sqrt{\frac{\sigma}{2}}\,\eta\right) + O\left(\frac{u_w}{u_e} - 1\right) \tag{29}$$

From equations (28) and (29),

$$r(0) = \frac{4}{\pi}\left(\frac{\sigma}{2 - \sigma}\right)^{1/2}\left(\frac{\pi}{2} - \sin^{-1}\sqrt{\frac{\sigma}{2}}\right) + O\left(\frac{u_w}{u_e} - 1\right) \tag{30}$$

$$s'(0) = -\sqrt{\frac{2\sigma}{\pi}} + O\left(\frac{u_w}{u_e} - 1\right) \tag{31}$$

where $r(0)$ is the recovery factor and $s'(0)$ is used to calculate heat transfer. Equations (22) to (31) (neglecting the higher-order terms) apply if an infinite plate is started, impulsively, with velocity $\left[(u_w/u_e) - 1\right]$. The latter, often termed the "Rayleigh problem," has been much discussed. The zero-order solution indicated by equations (28) and (30) was obtained in reference 8 in a study of the aerodynamic heating associated with the Rayleigh problem.

In reference 1, the boundary layer behind a shock wave advancing into a stationary fluid was estimated by analogy with the Rayleigh problem. The work of the present section shows that this approach is exact for weak shocks.

Numerical Solution

For other than weak shock waves, a numerical solution of equations (6), (10), and (11) is required. This was obtained by Lynn U. Albers with the use of an IBM card-programmed electronic calculator. The integration technique is described in appendix B of reference 9. The results, correct to four decimal places, are tabulated in table I. Values of f, f', and f'' are given for u_w/u_e of 1.5, 2, 3, 4, 5, and 6, while values of r,

r', s, s', $\int_0^\eta r\,d\eta$, and $\int_0^\eta s\,d\eta$ are presented for u_w/u_e of 2, 4, and 6 (with $\sigma = 0.72$). The data of table I plus the perturbation solution of the previous section define solutions covering the range from very weak shocks to very strong shocks.

RESULTS AND DISCUSSION

In the steady coordinate system, the boundary-layer similarity parameter is $\eta = \sqrt{\frac{1}{2}\frac{u_e}{\nu_w x}}\int_0^y \frac{T_w}{T}\,dy$. Transforming to the unsteady coordinate system according to the relation $x = \bar{x} + u_w t$ and considering station $\bar{x} = 0$ gives $\eta = \sqrt{\frac{1}{\nu_w t}}\sqrt{\frac{u_e}{2u_w}}\int_0^y \frac{T_w}{T}\,dy$. Thus, the boundary layer behind the shock wave has features of a Blasius type flow or a Rayleigh type flow, depending on whether the observer is stationary with respect to the shock wave, or wall, respectively.

The Rayleigh viewpoint is used herein to correlate and discuss the numerical data. That is, attention is fixed at station $\bar{x} = 0$, and the boundary-layer development for $t > 0$ is considered. The velocity which characterizes the boundary-layer development is $u_w - u_e$. Similarly, a characteristic length is $(u_w - u_e)\,t$, which is the distance a particle in the free stream moves relative to the wall in time t. The form of the Reynolds number used herein can then be defined as

$$Re \equiv \frac{(u_w - u_e)^2\,t}{\nu_w} \tag{32}$$

Boundary-Layer Profiles

The parameter $[(u_w/u_e) - f']/[(u_w/u_e) - 1]$ varies from a value of zero at the wall to a value of 1 at the edge of the boundary layer. A boundary-layer thickness δ may be defined as the value of y corresponding to $[(u_w/u_e) - f']/[(u_w/u_e) - 1] = 0.99$. Values of the boundary-layer-thickness parameter $\delta/\sqrt{\nu_e t}$ have been computed assuming $\gamma = 1.4$ and an insulated plate, for σ of 0.72 and 1.0 (using eqs. (18) and (20)). These are tabulated in table II. It is seen that $(\delta/\sqrt{\nu_e t})_i$ increases with increasing u_w/u_e. As expected, the values for $\sigma = 1$ are greater than those for $\sigma = 0.72$ (for $u_w/u_e \neq 1$). This is due to the fact that the larger recovery factor of the former leads to higher temperatures near the wall and therefore a larger value of y corresponding to a given η.

Velocity profiles for u_w/u_e of 1 (limiting case of very weak shocks) and 6 are plotted in figure 2. Curves for intermediate values of u_w/u_e lie smoothly between the curves in the figure. No marked departure from an error function type velocity profile is indicated.

Skin Friction and Heat Transfer

The shear stress at the wall is given by

$$\tau_w = \mu_w\left(\frac{\partial u}{\partial y}\right)_w = \mu_w u_e \sqrt{\frac{1}{2}\frac{u_e}{x\nu_w}}\, f''(0)$$

Because of the coordinate system used, τ_w is negative. If $(u_w - u_e)$ is used as a reference velocity, a positive local friction coefficient can be defined as $c_f = -\tau_w\Big/\frac{1}{2}\rho_w(u_w - u_e)^2$. Then, using the Reynolds number as defined by equation (32),

$$c_f\sqrt{Re} = \frac{-\sqrt{2}\, f''(0)}{\sqrt{\frac{u_w}{u_e}\left(\frac{u_w}{u_e} - 1\right)}} \tag{33}$$

Values of $c_f\sqrt{Re}$ are tabulated in table II. These vary from the Rayleigh value of 1.128 at $u_w/u_e = 1$ to 0.935 at $u_w/u_e = 6$. The corresponding value of $c_f\sqrt{Re}$ for incompressible flow past a semi-infinite plate (Blasius problem) is 0.664.

The heat transferred into the fluid from a unit area of the wall, per unit time, is

$$q = -k_w\left(\frac{\partial T}{\partial y}\right)_w = -k_w\sqrt{\frac{1}{2}\frac{u_e}{x\nu_w}}\,(T_w - T_{w,i})\, s'(0) \tag{34}$$

Defining a heat-transfer coefficient h by $h = (T_w - T_{w,i})/q$ and a Nusselt number by the relation $Nu = h(u_w - u_e)\, t/k_w$ permits the Nusselt number to be written as

$$\mathrm{Nu} = \frac{-s'(0)}{\sqrt{\frac{1}{2}\frac{u_w}{u_e}}} \sqrt{\mathrm{Re}}$$

The relation between skin friction and heat transfer can be expressed in terms of a Reynolds analogy parameter $\sqrt{\sigma}\ c_f\mathrm{Re}/\mathrm{Nu}$. The factor $\sqrt{\sigma}$ is included, since, for $u_w/u_e = 1$, $\sqrt{\sigma}\ c_f\mathrm{Re}/\mathrm{Nu}$ is a constant. For $\sigma = 1$, the parameter equals 2 at all values of u_w/u_e; while for $\sigma = 0.72$, the parameter increases from 2 at $u_w/u_e = 1$ to 2.07 at $u_w/u_e = 6$.

The recovery factor for $u_w/u_e = 1$ is given by equation (30). Evaluating this equation for $\sigma = 0.72$ gives $r(0) = 0.885$. The recovery factor for $\sigma = 0.72$ increases with increasing u_w/u_e to a value of 0.920 at $u_w/u_e = 6$. These compare with the value 0.845 for flow past a semi-infinite plate at $\sigma = 0.72$.

Thus, for the range of u_w/u_e investigated, the numerical results for skin friction, heat transfer, and recovery factor (in terms of the parameters defined herein) depart relatively less from Rayleigh ($u_w/u_e = 1$) values than from the Blasius values for equivalent flows past a semi-infinite flat plate.

CONCLUDING REMARKS

The laminar boundary layer behind a shock wave advancing into a stationary fluid, bounded by a wall, has been determined. Various boundary-layer parameters have been tabulated for several shock strengths.

With increasing Reynolds numbers, the laminar boundary layer behind the shock will become unstable, and transition to turbulent flow will ultimately occur. A theoretical study of the stability of this laminar boundary layer would be of interest. Shock-tube experiments might provide criteria defining the transition to turbulent boundary-layer flow as well as the characteristics of the turbulent boundary layer. At present, little is known about the structure of such turbulent boundary layers, and it is felt that some experimental data should be available before an analytical study is attempted.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, December 10, 1954

APPENDIX A

SYMBOLS

The following symbols are used in this report:

a	speed of sound
c_f	local skin-friction coefficient, $-\tau_w \Big/ \left[\frac{1}{2}\rho_w(u_w - u_e)^2\right]$
c_p	specific heat at constant pressure
erf(x)	error function, $(2/\sqrt{\pi})\int_0^x e^{-y^2}\,dy$
erfc(x)	complementary error function, 1 - erf(x)
f	function of η defined by eq. (4)
h	heat-transfer coefficient, $(T_w - T_{w,i})/q$
k	thermal conductivity
M	Mach number
M_e	u_e/a_e
Nu	Nusselt number, $h(u_w - u_e)t/k_w$
p	pressure
q	local rate of heat transfer
R	gas constant
Re	Reynolds number, $(u_w - u_e)^2 t/\nu_w$
r	function defined by eq. (10)
r(0)	recovery factor
s	function defined by eq. (11)
T	static temperature (abs)
t	time
u,v	velocities parallel to x and y coordinates, respectively

$\bar{u},\bar{v}$	velocities parallel to $\bar{x}$ and $\bar{y}$ coordinates, respectively
$\bar{u}_s$	velocity of shock wave relative to wall
x,y	coordinates parallel to and normal to wall, respectively, and moving with shock wave (fig. 1(b))
$\bar{x},\bar{y}$	coordinates parallel to and normal to wall, respectively, and stationary with respect to wall (fig. 1(a))
γ	ratio of specific heats
δ	value of y for which $(u_w/u_e - u/u_e)/(u_w/u_e - 1) = 0.99$ (i.e., boundary-layer thickness)
η	variable defined by eq. (3)
η_δ	value of η at $y = \delta$
μ	coefficient of viscosity
ν	kinematic viscosity
ρ	mass density
σ	Prandtl number, $\mu c_p/k$
τ_w	local shearing stress at wall
ψ	stream function

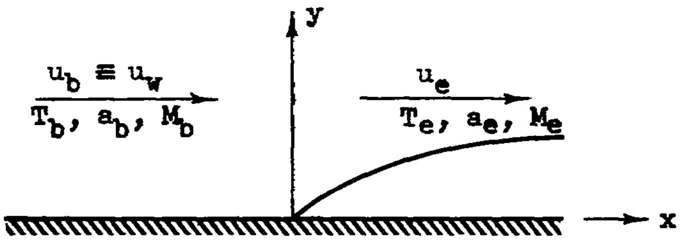
Subscripts:

b	undisturbed flow in front of shock wave (appendix B)
e	flow external to boundary layer
i	insulated-wall problem
w	conditions at wall ($y = \bar{y} = 0$)

APPENDIX B

SHOCK RELATIONS

For convenience, some useful shock-wave relations are noted herein:



Consider flow in a steady coordinate system. Let subscript b designate undisturbed flow in front of the shock wave; and subscript e, the flow behind the shock wave and external to the boundary layer. Note that $u_b = u_w$, so that $M_b \equiv u_b/a_b = u_w/a_b$. Then,

$$\left.\begin{aligned} \frac{u_w}{u_e} &= \frac{(\gamma+1)M_b^2}{(\gamma-1)M_b^2 + 2} \\ &= \frac{6M_b^2}{M_b^2 + 5} \qquad \text{for } \gamma = 1.4 \end{aligned}\right\} \tag{B1}$$

$$\left.\begin{aligned} M_e^2 &= \frac{2}{(\gamma+1)\dfrac{u_w}{u_e} - (\gamma-1)} \\ &= \frac{5}{6\dfrac{u_w}{u_e} - 1} \qquad \text{for } \gamma = 1.4 \end{aligned}\right\} \tag{B2}$$

$$\left.\begin{aligned} \frac{T_e}{T_b} &= \frac{(\gamma+1)\dfrac{u_w}{u_e} - (\gamma-1)}{\dfrac{u_w}{u_e}\left[(\gamma+1) - (\gamma-1)\dfrac{u_w}{u_e}\right]} \\ &= \frac{6\dfrac{u_w}{u_e} - 1}{\dfrac{u_w}{u_e}\left(6 - \dfrac{u_w}{u_e}\right)} \qquad \text{for } \gamma = 1.4 \end{aligned}\right\} \tag{B3}$$

APPENDIX C

ALTERNATE FORMULATION OF BOUNDARY-LAYER PROBLEM

The transformation based on equation (3) leads to a system of equations identical to that which arises in boundary-layer studies of the flow past a semi-infinite flat plate, except for the non-zero velocity boundary condition at the wall. This sytem was convenient for numerical computation, since a variety of flat-plate boundary-layer problems had previously been solved at the NACA Lewis laboratory, and the card-programming for the IBM electronic calculator was already established.

An alternate system can be obtained by normalizing the momentum-equation boundary conditions. That is, the parameter u_w/u_e appears in the differential equation rather than in the boundary conditions. Such a system leads more directly to the correct physical parameters of the problem and is described by the following equations.

Define ζ and $g(\zeta)$ according to the relations

$$\zeta \equiv \sqrt{\frac{u_w}{u_e}}\,\eta \tag{C1}$$

$$g(\zeta) = \frac{1}{\frac{u_w}{u_e} - 1}\left(\frac{u_w}{u_e}\zeta - \sqrt{\frac{u_w}{u_e}}\,f\right) \tag{C2}$$

Using equations (C1) and (C2), equations (6), (7), (10), and (11) become

$$\left.\begin{aligned} &\frac{d^3g}{d\zeta^3} + \frac{d^2g}{d\zeta^2}\left(\zeta - \frac{\frac{u_w}{u_e} - 1}{\frac{u_w}{u_e}}\,g\right) = 0 \\ &g(0) = \left(\frac{dg}{d\zeta}\right)_{\zeta=0} = 0; \qquad \left(\frac{dg}{d\zeta}\right)_{\zeta=\infty} = 1 \end{aligned}\right\} \tag{C3}$$

$$\left.\begin{aligned} &\frac{d^2r}{d\zeta^2} + \sigma\left(\zeta - \frac{\frac{u_w}{u_e} - 1}{\frac{u_w}{u_e}}\,g\right)\frac{dr}{d\zeta} = -2\sigma\left(\frac{u_w}{u_e} - 1\right)\left(\frac{d^2g}{d\zeta^2}\right)^2 \\ &r(\infty) = \left(\frac{dr}{d\zeta}\right)_{\zeta=0} = 0 \end{aligned}\right\} \tag{C4}$$

$$\left.\begin{aligned}&\frac{d^2 s}{d\zeta^2} + \sigma\left(\zeta - \frac{\frac{u_w}{u_e} - 1}{\frac{u_w}{u_e}} g\right)\frac{ds}{d\zeta} = 0\\&s(0) = 1; \qquad s(\infty) = 0\end{aligned}\right\} \tag{C5}$$

Note that, for $\bar{x} = 0$, $\zeta = 1/\sqrt{2\nu_w t}$, which is the correct parameter for the Rayleigh problem. Also $dg/d\zeta = [(u_w/u_e) - f']/[(u_w/u_e) - 1]$, which is a normalized velocity. Finally, the reduction of the system to a Rayleigh problem, for $u_w/u_e = 1$, is more apparent from equations (C3) to (C5) than from equations (6) to (11).

REFERENCES

1. Donaldson, Coleman duP., and Sullivan, Roger D.: The Effect of Wall Friction on the Strength of Shock Waves in Tubes and Hydraulic Jumps in Channels. NACA TN 1942, 1949.

2. Hollyer, Robert N., Jr.: A Study of Attenuation in the Shock Tube. Eng. Res. Inst., Univ. of Michigan, July 1, 1953. (U.S. Navy Dept., office Naval Res. Contract No. N6-ONR-232-TO IV, Proj. M720-4.)

3. Payman, W., and Titman, H.: Explosion Waves and Shock Waves. III - The Initiation of Detonation in Mixtures of Ethylene and Oxygen and of Carbon Monoxide and Oxygen. Proc. Roy. Soc. (London), ser. A, vol. 152, no. A876, Nov. 1, 1935, pp. 418-445.

4. Schmidt, Ernst H. W., Steinicke, H., und Neubert, U.: Aufnahmen der Verbrennung von Gasgemischen in Rohren mit dem Eigenlicht der Flamme und bei Schlierenbeleuchtung. VDI-Forsch, Bd. 17, 1951, p. 431.

5. Courant, R., and Friedricks, K. O.: Supersonic Flow and Shock Waves. Interscience Pub., Inc., 1948, p. 224.

6. Howarth, L.: Concerning the Effect of Compressibility on Laminar Boundary Layers and Their Separation. Proc. Roy. Soc. (London), ser. A, vol. 194, no. A1036, July 1948, pp. 16-42.

7. Chapman, Dean R., and Rubesin, Morris W.: Temperature and Velocity Profiles in the Compressible Laminar Boundary Layer with Arbitrary Distribution of Surface Temperature. Jour. Aero. Sci., vol. 16, no. 9, Sept. 1949, pp. 547-565.

8. Emmons, H. W.: Note on Aerodynamic Heating. Quart. Appl. Math., vol. VIII, no. 4, Jan. 1951, pp. 402-405.

9. Low, George M.: The Compressible Laminar Boundary Layer with Heat Transfer and Small Pressure Gradient. NACA TN 3028, 1953.

TABLE I. - LAMINAR BOUNDARY LAYER BEHIND SHOCK WAVE

(a) Solution of momentum equation for u_w/u_e = 1.5, 3.0 and 5.0

η	u_w/u_e = 1.5			u_w/u_e = 3.0			u_w/u_e = 5.0		
	f	f'	-f"	f	f'	-f"	f	f'	-f"
0.0	0.0000	1.5000	0.4578	0.0000	3.0000	2.3973	0.0000	5.0000	5.9726
.1	.1477	1.4543	.4544	.2880	2.7614	2.3620	.4703	4.4075	5.8309
.2	.2909	1.4093	.4445	.5525	2.5296	2.2649	.8824	3.8419	5.4470
.3	.4296	1.3656	.4288	.7944	2.3101	2.1170	1.2402	3.3237	4.8964
.4	.5641	1.3238	.4080	1.0151	2.1074	1.9336	1.5492	2.8656	4.2573
.5	.6945	1.2842	.3831	1.2165	1.9241	1.7291	1.8155	2.4729	3.5968
.6	.8210	1.2472	.3551	1.4006	1.7618	1.5168	2.0459	2.1452	2.9644
.7	.9440	1.2138	.3251	1.5696	1.6207	1.3073	2.2466	1.8780	2.3913
.8	1.0637	1.1822	.2941	1.7254	1.5000	1.1087	2.4233	1.6645	1.8930
.9	1.1806	1.1544	.2628	1.8702	1.3984	.9261	2.5810	1.4968	1.4737
1.0	1.2947	1.1297	.2382	2.0057	1.3141	.7689	2.7239	1.3672	1.1303
1.1	1.4066	1.1079	.2089	2.1335	1.2451	.6803	2.8555	1.2685	.8551
1.2	1.5164	1.0890	.1755	2.2551	1.1894	.4980	2.9784	1.1942	.6387
1.3	1.6245	1.0728	.1498	2.3718	1.1449	.3952	3.0950	1.1391	.4714
1.4	1.7310	1.0590	.1267	2.4844	1.1097	.3100	3.2067	1.0986	.3440
1.5	1.8363	1.0474	.1060	2.5940	1.0823	.2405	3.3151	1.0692	.2483
1.6	1.9406	1.0377	.0877	2.7011	1.0612	.1845	3.4209	1.0481	.1773
1.7	2.0439	1.0297	.0719	2.8064	1.0451	.1401	3.5249	1.0331	.1253
1.8	2.1466	1.0233	.0583	2.9102	1.0329	.1053	3.6276	1.0226	.0876
1.9	2.2486	1.0180	.0468	3.0131	1.0237	.0783	3.7295	1.0153	.0606
2.0	2.3502	1.0138	.0372	3.1151	1.0170	.0576	3.8308	1.0102	.0415
2.1	2.4514	1.0105	.0292	3.2165	1.0120	.0420	3.9316	1.0068	.0282
2.2	2.5523	1.0079	.0228	3.3175	1.0085	.0303	4.0322	1.0044	.0189
2.3	2.6530	1.0059	.0176	3.4182	1.0059	.0216	4.1325	1.0029	.0126
2.4	2.7535	1.0044	.0134	3.5187	1.0041	.0153	4.2328	1.0019	.0083
2.5	2.8539	1.0032	.0101	3.6191	1.0028	.0107	4.3329	1.0012	.0054
2.6	2.9542	1.0023	.0076	3.7193	1.0019	.0074	4.4330	1.0008	.0035
2.7	3.0544	1.0017	.0056	3.8195	1.0013	.0051	4.5331	1.0005	.0022
2.8	3.1545	1.0012	.0041	3.9196	1.0008	.0035	4.6331	1.0003	.0014
2.9	3.2546	1.0008	.0030	4.0196	1.0005	.0025	4.7331	1.0002	.0009
3.0	3.3547	1.0006	.0021	4.1197	1.0004	.0015	4.8331	1.0001	.0005
3.1	3.4548	1.0004	.0015	4.2197	1.0002	.0010	4.9332	1.0001	.0003
3.2	3.5548	1.0003	.0011	4.3197	1.0001	.0007	5.0332	1.0000	.0002
3.3	3.6548	1.0002	.0007	4.4197	1.0001	.0004	5.1332	1.0000	.0001
3.4	3.7548	1.0001	.0005	4.5197	1.0001	.0003	5.2332	1.0000	.0001
3.5	3.8549	1.0001	.0004	4.6197	1.0000	.0002	5.3332	1.0000	.0000
3.6	3.9549	1.0001	.0002	4.7197	1.0000	.0001			
3.7	4.0549	1.0000	.0002	4.8197	1.0000	.0001			
3.8	4.1549	1.0000	.0001	4.9197	1.0000	.0000			
3.9	4.2549	1.0000	.0001						
4.0	4.3549	1.0000	.0000						
4.1									
4.2									
4.3									
4.4									

TABLE I. - Continued. LAMINAR BOUNDARY LAYER BEHIND SHOCK WAVE

(b) Solution of momentum and energy equation for $u_w/u_e = 2.0$

η	f	f'	-f"	Prandtl number, σ = 0.72: $\int_0^\eta r\,d\eta$	r	-r'	$\int_0^\eta s\,d\eta$	s	-s'
0.0	0.0000	2.0000	1.0191	0.0000	0.8997	0.0000	0.0000	1.0000	0.8512
.1	.1949	1.8984	1.0091	.0897	.8922	.1479	.0958	.9151	.8452
.2	.3798	1.7988	.9804	.1780	.8704	.2861	.1831	.8313	.8279
.3	.5548	1.7029	.9356	.2634	.8356	.4068	.2621	.7498	.8004
.4	.7205	1.6121	.8777	.3447	.7898	.5043	.3331	.6715	.7645
.5	.8774	1.5276	.8103	.4210	.7356	.5758	.3965	.5978	.7217
.6	1.0263	1.4503	.7367	.4916	.6785	.6209	.4527	.5274	.6739
.7	1.1677	1.3804	.6601	.5560	.6122	.6412	.5082	.4625	.6227
.8	1.3026	1.3183	.5834	.6141	.5480	.6398	.5454	.4089	.5697
.9	1.4316	1.2637	.5088	.6657	.4849	.6206	.5829	.3486	.5162
1.0	1.5556	1.2164	.4382	.7111	.4244	.5878	.6153	.2996	.4636
1.1	1.6751	1.1759	.3728	.7507	.3676	.5455	.6430	.2558	.4127
1.2	1.7910	1.1416	.3135	.7848	.3155	.4974	.6666	.2170	.3643
1.3	1.9036	1.1129	.2606	.8139	.2683	.4465	.6866	.1829	.3189
1.4	2.0137	1.0893	.2142	.8386	.2262	.3953	.7034	.1531	.2769
1.5	2.1216	1.0699	.1742	.8594	.1892	.3456	.7174	.1274	.2386
1.6	2.2278	1.0542	.1402	.8766	.1570	.2988	.7290	.1053	.2040
1.7	2.3326	1.0417	.1116	.8909	.1293	.2556	.7385	.0864	.1731
1.8	2.4362	1.0317	.0879	.9026	.1057	.2166	.7463	.0705	.1458
1.9	2.5390	1.0239	.0686	.9122	.0858	.1819	.7527	.0572	.1219
2.0	2.6411	1.0179	.0529	.9199	.0692	.1514	.7578	.0460	.1012
2.1	2.7426	1.0133	.0404	.9261	.0554	.1250	.7620	.0368	.0833
2.2	2.8438	1.0097	.0306	.9310	.0441	.1024	.7653	.0293	.0682
2.3	2.9446	1.0071	.0229	.9350	.0348	.0832	.7679	.0231	.0553
2.4	3.0452	1.0051	.0170	.9381	.0273	.0671	.7699	.0181	.0446
2.5	3.1456	1.0036	.0124	.9405	.0213	.0538	.7715	.0141	.0357
2.6	3.2459	1.0026	.0090	.9424	.0165	.0427	.7728	.0109	.0284
2.7	3.3462	1.0018	.0065	.9438	.0127	.0337	.7737	.0084	.0224
2.8	3.4463	1.0012	.0046	.9449	.0097	.0264	.7745	.0064	.0175
2.9	3.5464	1.0008	.0033	.9458	.0073	.0205	.7750	.0049	.0136
3.0	3.6465	1.0006	.0023	.9464	.0055	.0158	.7755	.0037	.0105
3.1	3.7465	1.0004	.0016	.9469	.0041	.0121	.7758	.0028	.0081
3.2	3.8466	1.0003	.0011	.9473	.0031	.0092	.7760	.0020	.0061
3.3	3.9466	1.0002	.0007	.9475	.0023	.0070	.7762	.0015	.0046
3.4	4.0466	1.0001	.0005	.9477	.0017	.0052	.7763	.0011	.0035
3.5	4.1466	1.0001	.0003	.9479	.0012	.0039	.7764	.0008	.0026
3.6	4.2466	1.0000	.0002	.9480	.0009	.0029	.7765	.0006	.0019
3.7	4.3466	1.0000	.0001	.9480	.0006	.0021	.7765	.0004	.0014
3.8	4.4466	1.0000	.0001	.9481	.0005	.0016	.7766	.0003	.0010
3.9	4.5466	1.0000	.0001	.9481	.0003	.0011	.7766	.0002	.0007
4.0	4.6466	1.0000	.0000	.9482	.0002	.0008	.7766	.0001	.0005
4.1				.9482	.0002	.0006	.7766	.0001	.0004
4.2				.9482	.0001	.0004	.7766	.0001	.0003
4.3				.9482	.0001	.0003	.7766	.0001	.0002
4.4				.9482	.0001	.0002	.7766	.0000	.0001
4.5				.9482	.0000	.0001	.7767	.0000	.0001
4.6				.9482	.0000	.0001	.7767	.0000	.0001
4.7				.9482	.0000	.0001	.7767	.0000	.0000
4.8				.9482	.0000	.0000	.7767	.0000	.0000
4.9									

TABLE I. - Continued. LAMINAR BOUNDARY LAYER BEHIND SHOCK WAVE

(c) Solution of momentum and energy equation for $u_w/u_e = 4.0$

η	f	f'	-f"	Prandtl number, $\sigma = 0.72$					
				$\int_0^\eta r\,d\eta$	r	-r'	$\int_0^\eta s\,d\eta$	s	-s'
0.0	0.0000	4.0000	4.0623	0.0000	0.9129	0.0000	0.0000	1.0000	1.1156
.1	.3798	3.5964	3.9845	.0909	.8998	.2582	.0945	.8892	1.1005
.2	.7198	3.2077	3.7702	.1791	.8623	.4847	.1779	.7811	1.0576
.3	1.0222	2.8457	3.4546	.2626	.8047	.6579	.2509	.6784	.9931
.4	1.2901	2.5188	3.0766	.3396	.7328	.7690	.3139	.5830	.9136
.5	1.5273	2.2313	2.5717	.4089	.6529	.8208	.3677	.4960	.8253
.6	1.7377	1.9844	2.2688	.4701	.5705	.8209	.4134	.4180	.7337
.7	1.9255	1.7768	1.8887	.5231	.4900	.7838	.4517	.3492	.6429
.8	2.0943	1.6055	1.5446	.5683	.4145	.7218	.4835	.2893	.5563
.9	2.2476	1.4664	1.2430	.6062	.3461	.6458	.5098	.2378	.4757
1.0	2.3885	1.3554	.9858	.6378	.2855	.5646	.5313	.1939	.4026
1.1	2.5195	1.2679	.7712	.6636	.2331	.4844	.5488	.1570	.3374
1.2	2.6427	1.1998	.5957	.6846	.1885	.4089	.5629	.1262	.2801
1.3	2.7600	1.1476	.4547	.7016	.1511	.3406	.5749	.1007	.2306
1.4	2.8727	1.1079	.3431	.7151	.1201	.2804	.5832	.0798	.1883
1.5	2.9819	1.0781	.2560	.7258	.0948	.2283	.5903	.0620	.1545
1.6	3.0885	1.0560	.1890	.7342	.0742	.1842	.5959	.0491	.1226
1.7	3.1933	1.0398	.1380	.7407	.0577	.1473	.6003	.0382	.0978
1.8	3.2967	1.0280	.0998	.7458	.0445	.1168	.6036	.0294	.0774
1.9	3.3990	1.0195	.0714	.7497	.0341	.0919	.6062	.0226	.0608
2.0	3.5006	1.0135	.0506	.7527	.0260	.0717	.6082	.0172	.0474
2.1	3.6018	1.0092	.0355	.7550	.0196	.0556	.6097	.0130	.0367
2.2	3.7025	1.0062	.0246	.7567	.0147	.0427	.6108	.0097	.0282
2.3	3.8030	1.0042	.0169	.7580	.0110	.0326	.6116	.0073	.0216
2.4	3.9034	1.0028	.0115	.7589	.0082	.0247	.6123	.0054	.0163
2.5	4.0036	1.0018	.0077	.7596	.0060	.0186	.6127	.0040	.0123
2.6	4.1038	1.0012	.0052	.7602	.0044	.0139	.6131	.0029	.0092
2.7	4.2039	1.0008	.0034	.7605	.0032	.0103	.6133	.0021	.0068
2.8	4.3039	1.0005	.0022	.7606	.0023	.0076	.6135	.0015	.0050
2.9	4.4040	1.0003	.0014	.7610	.0016	.0055	.6136	.0011	.0037
3.0	4.5040	1.0002	.0009	.7611	.0012	.0040	.6137	.0008	.0027
3.1	4.6040	1.0001	.0006	.7612	.0008	.0029	.6138	.0005	.0019
3.2	4.7040	1.0001	.0004	.7613	.0006	.0021	.6138	.0004	.0014
3.3	4.8040	1.0001	.0002	.7613	.0004	.0015	.6139	.0003	.0010
3.4	4.9040	1.0000	.0001	.7614	.0003	.0010	.6139	.0002	.0007
3.5	5.0040	1.0000	.0001	.7614	.0002	.0007	.6139	.0001	.0005
3.6	5.1040	1.0000	.0001	.7614	.0001	.0005	.6139	.0001	.0003
3.7	5.2040	1.0000	.0000	.7614	.0001	.0003	.6139	.0001	.0002
3.8				.7614	.0001	.0002	.6139	.0000	.0002
3.9				.7614	.0000	.0002	.6139	.0000	.0001
4.0				.7614	.0000	.0001	.6139	.0000	.0001
4.1				.7615	.0000	.0001	.6139	.0000	.0000
4.2				.7615	.0000	.0000	.6139	.0000	.0000
4.3									
4.4									

TABLE I. - Concluded. LAMINAR BOUNDARY LAYER BEHIND SHOCK WAVE

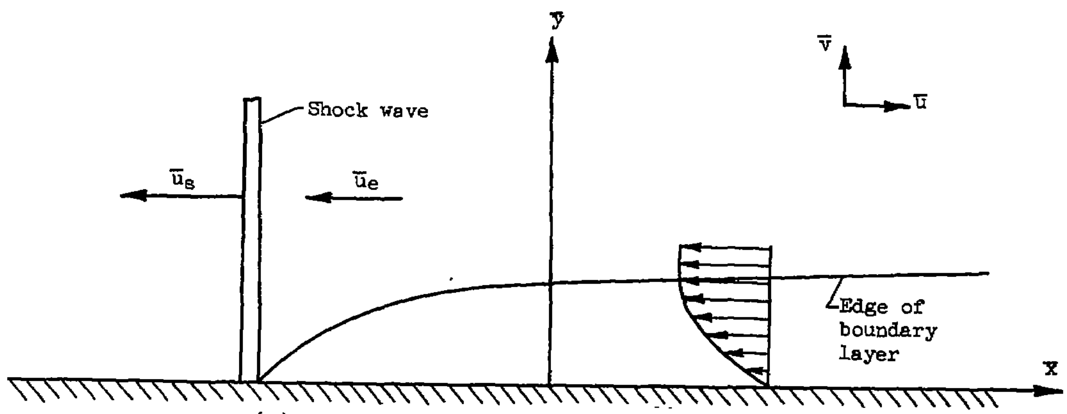
(d) Solution of momentum and energy equation for $u_w/u_e = 6.0$

η	f	f'	-f"	Prandtl number, σ = 0.72 $\int_0^\eta r\,d\eta$	r	-r'	$\int_0^\eta s\,d\eta$	s	-s'
0.0	0.0000	6.0000	8.1009	0.0000	0.9195	0.0000	0.0000	1.0000	1.3262
.1	.5597	5.1977	7.8781	.0913	.9009	.3658	.0934	.8683	1.2991
.2	1.0410	4.4383	7.2620	.1791	.8486	.6668	.1738	.7417	1.2258
.3	1.4499	3.7534	6.4078	.2602	.7710	.8671	.2420	.6242	1.1202
.4	1.7947	3.1603	5.4454	.3328	.6787	.9619	.2991	.5183	.9963
.5	2.0852	2.6643	4.4831	.3958	.5816	.9681	.3461	.4252	.8662
.6	2.3307	2.2613	3.5937	.4492	.4871	.9118	.3845	.3450	.7387
.7	2.5402	1.9418	2.8161	.4935	.4004	.8187	.4155	.2771	.6197
.8	2.7214	1.6938	2.1642	.5296	.3239	.7094	.4403	.2206	.5127
.9	2.8809	1.5048	1.6353	.5586	.2586	.5983	.4600	.1742	.4190
1.0	3.0240	1.3631	1.2171	.5817	.2040	.4941	.4755	.1364	.3388
1.1	3.1548	1.2583	.8935	.5998	.1594	.4012	.4875	.1060	.2712
1.2	3.2766	1.1818	.6478	.6138	.1234	.3212	.4969	.0818	.2151
1.3	3.3918	1.1266	.4641	.6247	.0947	.2542	.5040	.0626	.1692
1.4	3.5024	1.0873	.3288	.6330	.0721	.1991	.5095	.0476	.1320
1.5	3.6097	1.0597	.2304	.6393	.0545	.1545	.5137	.0360	.1022
1.6	3.7146	1.0403	.1597	.6440	.0409	.1189	.5168	.0270	.0785
1.7	3.8180	1.0270	.1096	.6476	.0305	.0908	.5192	.0201	.0599
1.8	3.9202	1.0179	.0744	.6502	.0226	.0687	.5209	.0149	.0453
1.9	4.0217	1.0118	.0500	.6521	.0166	.0517	.5222	.0109	.0340
2.0	4.1226	1.0077	0.0333	.6536	.0121	.0385	.5231	.0080	.0254
2.1	4.2232	1.0049	.0219	.6546	.0088	.0285	.5238	.0058	.0188
2.2	4.3236	1.0031	.0143	.6554	.0063	.0210	.5243	.0042	.0138
2.3	4.4239	1.0020	.0092	.6559	.0045	.0153	.5246	.0030	.0101
2.4	4.5240	1.0012	.0059	.6563	.0032	.0111	.5249	.0021	.0073
2.5	4.6241	1.0008	.0037	.6565	.0023	.0080	.5251	.0015	.0053
2.6	4.7242	1.0005	.0023	.6567	.0016	.0057	.5252	.0010	.0038
2.7	4.8242	1.0003	.0015	.6569	.0011	.0040	.5253	.0007	.0027
2.8	4.9243	1.0002	.0009	.6570	.0008	.0028	.5253	.0005	.0019
2.9	5.0243	1.0001	.0005	.6570	.0005	.0020	.5254	.0003	.0013
3.0	5.1243	1.0001	.0003	.6571	.0004	.0014	.5254	.0002	.0009
3.1	5.2243	1.0000	.0002	.6571	.0002	.0010	.5254	.0002	.0006
3.2	5.3243	1.0000	.0001	.6571	.0002	.0007	.5254	.0001	.0004
3.3	5.4243	1.0000	.0001	.6571	.0001	.0004	.5255	.0001	.0003
3.4	5.5243	1.0000	.0000	.6571	.0001	.0003	.5255	.0000	.0002
3.5				.6572	.0000	.0002	.5255	.0000	.0001
3.6				.6572	.0000	.0001	.5255	.0000	.0001
3.7				.6572	.0000	.0001	.5255	.0000	.0001
3.8				.6572	.0000	.0001	.5255	.0000	.0000
3.9				.6572	.0000	.0000	.5255	.0000	.0000

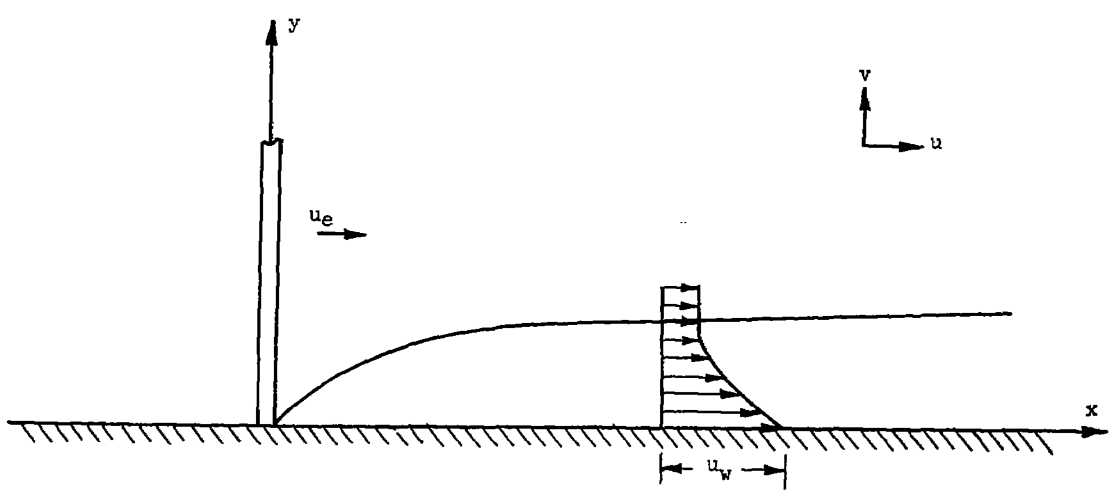
TABLE II. - SKIN-FRICTION AND HEAT-TRANSFER COEFFICIENTS

[$\gamma = 1.4$]

$\frac{u_w}{u_e}$	$c_f\sqrt{Re}$	Prandtl number, σ			
		0.72		1.0	
		$(\delta/\sqrt{\nu_e t})_i$	$\frac{\sqrt{\sigma} c_f Re}{Nu}$	$(\delta/\sqrt{\nu_e t})_i$	$\frac{\sqrt{\sigma} c_f Re}{Nu}$
1.0	1.128	3.64	2.0	3.64	2.0
1.5	1.057	----	----	4.33	2.0
2.0	1.019	4.55	2.032	4.91	2.0
3.0	.979	----	-----	5.91	2.0
4.0	.958	5.86	2.060	6.80	2.0
5.0	.944	----	-----	7.63	2.0
6.0	.935	6.94	2.074	8.42	2.0



(a) Coordinate system fixed with respect to wall.



(b) Coordinate system fixed with respect to shock wave.

Figure 1. - Coordinate systems used to study boundary layer behind shock advancing into stationary fluid.

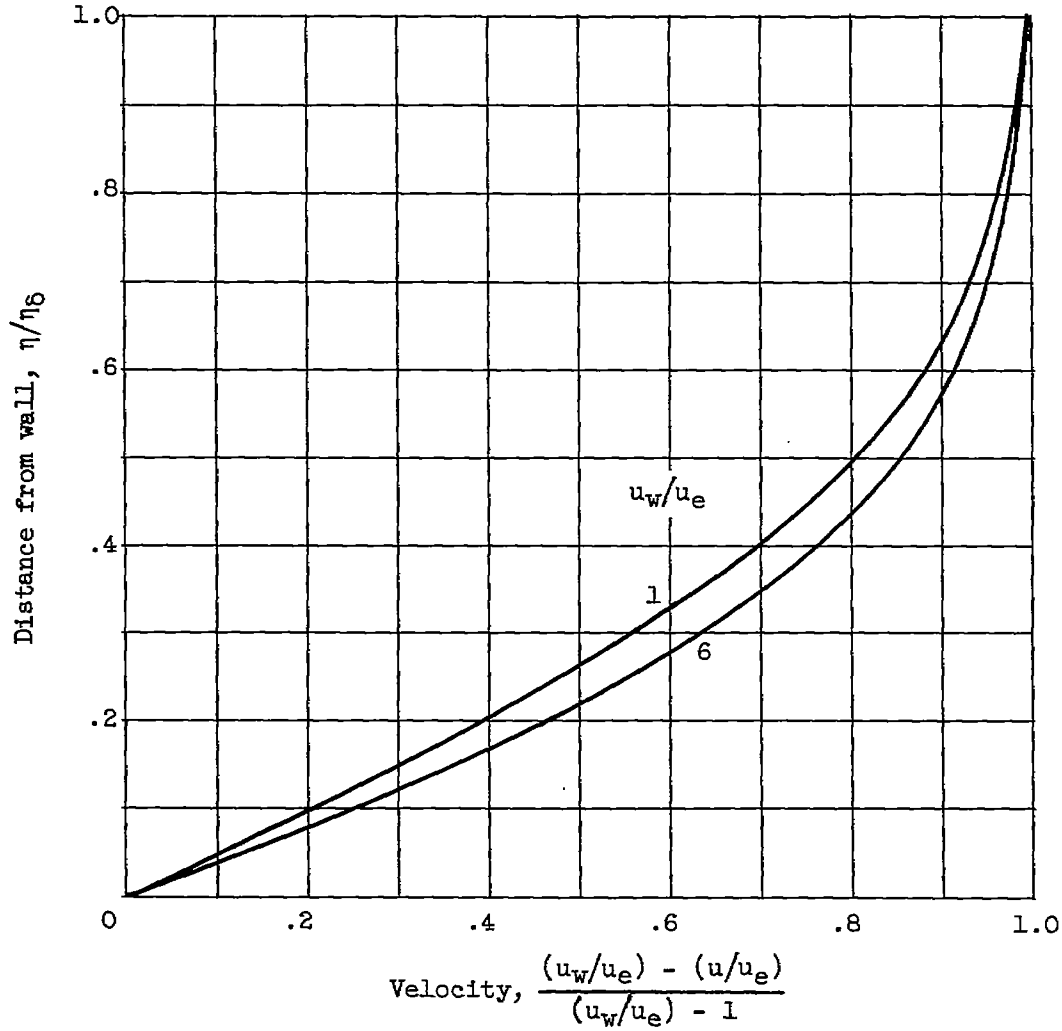


Figure 2. - Boundary-layer velocity profile. ($u_w/u_e = 1$ refers to limiting case of very weak shocks.)