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SIMILAR SOLUTIONS FOR THE COMPRESSIBLE LAMINAR BOUNDARY LAYER WITH HEAT TRANSFER AND PRESSURE GRADIENT

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SUMMARY

Stewartson's transformation is applied to the laminar compressible boundary-layer equations and the requirement of similarity is introduced, resulting in a set of ordinary nonlinear differential equations previously quoted by Stewartson, but unsolved. The requirements of the system are: Prandtl number of 1.0, linear viscosity-temperature relation across the boundary layer, an isothermal surface, and the particular distributions of free-stream velocity consistent with similar solutions. This system admits axial pressure gradients of arbitrary magnitude, heat flux normal to the surface, and arbitrary Mach numbers.

The system of differential equations is transformed to an integral system, with the velocity ratio as the independent variable. For this system, solutions are found for pressure gradients varying from that causing separation to the infinitely favorable gradient and for wall temperatures from absolute zero to twice the free-stream stagnation temperature. Some solutions for separated flows are also presented.

For favorable pressure gradients, the solutions are unique. For adverse pressure gradients, where the solutions are not unique, two solutions of the infinite family of possible solutions are identified as essentially viscid at the outer edge of the boundary layer and the remainder essentially inviscid. For the case of favorable pressure gradients with heated walls, the velocity within a portion of the boundary layer is shown to exceed the local external velocity. The variation of a Reynolds analogy parameter, which indicates the ratio of skin friction to heat transfer, is from zero to 7.4 for a surface of temperature twice the free-stream stagnation temperature, and from zero to 2.8 for a surface held at absolute zero where the value 2 applies to a flat plate.

INTRODUCTION[1]

Factors that affect the development of laminar boundary layers are pressure gradient, Mach number, and heat transfer, plus the properties of the fluid under consideration. Since mathematical complexities preclude solutions of this problem in a completely general fashion, the literature consists largely of solutions treating particular combinations of these factors. For the flow of an ideal gas over a surface without pressure gradient, the remaining factors have been taken into account very completely by Crocco (ref. 2) and Chapman and Rubesin (ref. 3). For small pressure gradients, Low (ref. 4) has, by a perturbation analysis, treated the general problem of the isothermal surface. With the introduction of pressure gradients of arbitrary magnitude, other restrictions become necessary. The assumption of constant fluid properties (density, viscosity, etc.), for example, leads to the greatest simplification - the separation of the momentum and energy equations. With this assumption, for a special case of a decelerating stream, Howarth (ref. 5) has obtained a series solution to the momentum equation. The introduction of a similarity concept (that the velocity or temperature profiles may always be expressed in terms of a single parameter) leads to a power-law free-stream velocity distribution. The momentum equation of this problem was first solved by Falkner and Skan (ref. 6), whose calculations were then improved by Hartree (ref. 7); the energy equation was later treated by Eckert (ref. 8) and others (refs. 9 and 10). For the same problem the restriction of constant fluid properties may be removed by alternatively requiring that the Mach number be essentially zero (ref. 11) or that the Mach number and the heat transfer be limited to small values (ref. 12).

Illingworth (ref. 13) and Stewartson (ref. 14) have demonstrated that, for an insulated surface in a fluid with a Prandtl number of 1.0, any compressible boundary-layer problem may be transformed to a corresponding problem in an incompressible fluid; the earlier solutions thus become applicable to certain compressible problems. For the case of heat flux across the surface, the transformation of Stewartson (ref. 14) with the concept of similarity introduced leads to a set of nonlinear ordinary differential equations previously quoted (ref. 14), but unsolved. Solutions to this set of equations, which are presented herein,

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[1]The principal developments of this paper, which is part of the Doctoral Dissertation of the senior author (ref. 1), were carried out under the stimulus and guidance of Professor Luigi Crocco and the sponsorship of the Daniel and Florence Guggenheim Foundation. The final analysis and the computations were completed at the NACA Lewis laboratory during the Spring of 1954.

are applicable to flows at arbitrary Mach number, pressure gradients of arbitrary magnitude (but of a form consistent with the requirements of similarity), and arbitrary but constant wall temperature.[2]

Since free-stream velocity distributions of the form required by similarity are not generally encountered in practice, the utility of these solutions is principally as follows: (1) the effects of pressure gradient, wall temperature, and Mach number may be viewed qualitatively; (2) the results may be used as a check on any approximate method (such as a Kármán-Pohlhausen method) for reliability; (3) the flow to be solved may be divided intuitively into segments and the solution for each segment may be matched by some arbitrary technique; or (4) the results may be used to construct a new simple method (of the integral type) for the calculation of the laminar compressible boundary layer with heat transfer. This latter analysis has been carried out, utilizing the solutions herein given, and is presented in reference 1.

STEWARTSON'S EQUATIONS

Boundary-Layer Equations

The equations of the steady two-dimensional compressible laminar boundary layer for perfect fluids are:

Continuity:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{1}$$

Momentum:

$$\left.\begin{aligned} \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \\ \frac{\partial p}{\partial y} &= 0 \end{aligned}\right\} \tag{2}$$

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[2]Since this writing, further calculations, which are closely related to the present investigation, have been published by Levy (ref. 15). Solutions to the equations treated herein were obtained in that report. The present investigation includes ranges of variables not treated in ref. 15: for example, favorable pressure gradients applicable to supersonic nozzles and values of adverse pressure gradients including that causing separation. For adverse pressure gradients, the problems of uniqueness and multiple solutions are also considered in some detail. The solutions of ref. 15 were obtained by means of a differential analyzer, whereas the present solutions were obtained by digital calculation and are presented in tabular form.

Energy:

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = u \frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\frac{\mu}{Pr} \frac{\partial h}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

All symbols are defined in appendix A.

The viscosity law to be assumed is

$$\frac{\mu}{\mu_0} = \lambda \frac{t}{t_0} \tag{4}$$

Equation (4) is of the form taken by Chapman and Rubesin (ref. 3), except that the reference conditions (μ_0, t_0) are free-stream stagnation values, since in the presence of pressure gradient the local "external" values are not constant along the outer edge of the boundary layer. The constant λ is used to match the viscosity with the Sutherland value at a desired station. If this station is taken to be the surface, assumed to be at constant temperature, the result is

$$\lambda = \sqrt{\frac{t_w}{t_0}}\left(\frac{t_0 + k_{su}}{t_w + k_{su}}\right) \tag{5}$$

where k_{su} = Sutherland's constant (for air, $k_{su} = 216^\circ$ R). The viscosity law of equations (4) and (5) was demonstrated to be adequate for a flat plate (ref. 3) by comparison with the more exact calculations of reference 2. In the present case no such comparison is available.

Stewartson's Transformation

A slight modification of Stewartson's transformation may be written

$$\left.\begin{aligned} dX &= \lambda \frac{a_e}{a_0} \frac{p_e}{p_0} dx \\ dY &= \frac{\rho}{\rho_0} \frac{a_e}{a_0} dy \\ U &\equiv \psi_Y \\ V &\equiv -\psi_X \end{aligned}\right\} \tag{6}$$

where the stream function is defined by

$$\psi_y = \frac{\rho u}{\rho_0}$$

$$\psi_x = - \frac{\rho v}{\rho_0}$$

The transformed quantities are now represented by upper-case letters (X,Y,U,V), and the subscript e refers to local conditions at the outer edge of the boundary layer (external). The subscript 0 refers to free-stream stagnation values. From the preceding transformation, a useful relation between the transformed and physical velocities is $U = \frac{a_0}{a_e} u$.

If equations (4) and (6) are applied to the boundary-layer equations (1), (2), and (3), and if Pr and c_p are taken to be constant (but it is not yet required that Pr = 1), there result

$$U_X + V_Y = 0 \tag{7}$$

$$UU_X + VU_Y = U_e U_{e_X}(1 + S) + \nu_0 U_{YY} \tag{8}$$

$$US_X + VS_Y = \nu_0 \left\{ \frac{S_{YY}}{Pr} - \frac{1 - Pr}{Pr} \left(\frac{\frac{\gamma - 1}{2} M_e^2}{1 + \frac{\gamma - 1}{2} M_e^2} \right) \left[\left(\frac{U}{U_e} \right)^2 \right]_{YY} \right\} \tag{9}$$

where the enthalpy function S is defined for convenience as

$$S = \frac{h_s}{h_0} - 1 \tag{10}$$

and h_s is the local stagnation enthalpy.

The boundary conditions applicable to the system (7) to (9) are:

$$\left.\begin{aligned} U(X,0) &= 0 \\ V(X,0) &= 0 \\ S(X,0) = S_w \quad &\text{or} \quad \left[\frac{\partial S}{\partial Y}(X,0) = \left(\frac{\partial S}{\partial Y}\right)_w\right] \\ \lim_{Y\to\infty} S &= 0 \\ \lim_{Y\to\infty} U &= U_e(X) \end{aligned}\right\} \tag{11}$$

The solution $S = 0$ and the resultant continuity and momentum equations (7) and (8) make up the extremely useful correlation developed by Stewartson between compressible and incompressible boundary layers on insulated surfaces with $Pr = 1$. Another special case is that of $U_{e_X} = 0$. Then, if $Pr = 1$, the relation $S = S_w\left(1 - \frac{U}{U_e}\right)$ satisfies equation (9); this is Crocco's integral of the energy equation for the flat plate (ref. 2).

Similarity Requirements

When a pressure gradient exists and the surface is not insulated, it is necessary to find a means of solving the system (7) to (9) subject to the boundary conditions (11). To this end, the question will be asked: Under what conditions can this system be reduced to a system of ordinary differential equations by the assumption that the boundary-layer profiles are functions of a similarity variable η and that the wall temperature is constant? This question may be resolved by inserting the following assumed relations into the system (7) to (9) and observing the conditions required for obtaining ordinary differential equations:

$$\left.\begin{aligned} \psi &= AX^aU_e^p\, f(\eta) \\ Y &= BX^bU_e^q\eta \\ S &= S(\eta) \end{aligned}\right\} \tag{12}$$

where $A,B,a,b,p,$ and q are undetermined constants. This procedure has been carried out by Li and Nagamatsu (ref. 16) for $Pr = 1$. In that analysis it was concluded that four classes of similar solutions are possible. It has been pointed out (ref. 17) that three of these four classes can be reduced identically to the case requiring that

$$U_e = CX^m \tag{13}$$

while the remaining case requires that

$$U_e = C_1 \exp\left[C_2 X\right] \tag{14}$$

When equations (12) are used in the form

$$\left.\begin{aligned} \psi &= f(\eta)\sqrt{\frac{2\nu_0 U_e X}{m + 1}} \\ \eta &= Y\sqrt{\frac{m + 1}{2}\frac{U_e}{\nu_0 X}} \end{aligned}\right\} \tag{15}$$

the system of ordinary differential equations corresponding to the power-law velocity distribution of equation (13) may be written

$$\left.\begin{aligned} f''' + ff'' &= \beta(f'^2 - 1 - S) \\ S'' + PrfS' &= (1 - Pr)\left[\frac{(\gamma - 1)M_e^2}{1 + \frac{\gamma - 1}{2} M_e^2}\right](f'f''' + f''^2) \end{aligned}\right\} \tag{16}$$

The pressure-gradient parameter β is defined as $\beta = \frac{2m}{m + 1}$, and the velocity ratio is $U/U_e = u/u_e = f'$, where primes denote differentiation with respect to η.

The boundary conditions are:

$$\left.\begin{aligned} f(0) = f'(0) &= 0 \\ S(0) &= S_w \\ \lim_{\eta \to \infty} f' &= 1 \\ \lim_{\eta \to \infty} S &= 0 \end{aligned}\right\} \tag{17}$$

Since M_e may, in general, be a function of x, the right member of the energy equation is not yet dimensionally consistent with the left member for arbitrary M_e and Pr.

It was shown in reference 17 that, for the exponential case (eq. (14)) with $C_2 > 0$, the system (7) to (9) can be reduced to the ordinary differential equations (16), but with $\beta = 2$. For $C_2 < 0$, the f''' term in equations (16) is replaced by $-f'''$. In this case, with $S = 0$, it can be shown that, because of the sign of the f''' term, no solution is possible in which the velocity ratio approaches its boundary condition smoothly. A question is thus raised as to the validity of any possible solution for $C_2 < 0$ regardless of the value of S. For the remainder of this paper this class will be omitted from consideration.

Corresponding analyses for incompressible flow, including conditions for similarity and the case of the exponential free-stream velocity, have been made by Mangler (ref. 18) and Goldstein (ref. 19), respectively. As previously mentioned, the right member of the energy equation (16) must be zero or a function of η to be consistent with the left member. This may be achieved in the following ways: (1) the external Mach number may be a constant other than zero, (2) the external Mach number may be zero, (3) the Prandtl number may equal 1, (4) the factor $\left[\frac{(\gamma - 1)M_e^2}{1 + \frac{\gamma - 1}{2} M_e^2}\right] \approx 2$ corresponding to hypersonic flow, or (5) the ratio of specific heats γ may equal 1.

The case of constant external Mach number is the flat-plate problem ($\beta = 0$) and, the solution to the momentum equation being known, the energy could be integrated directly. The flat-plate problem has already been solved with great accuracy and completeness by Crocco (ref. 2). If the pressure gradient is small enough, it may be reasonable to consider M_e constant in the energy equation in spite of the gradient, but to retain the pressure-gradient parameter in the momentum equation. However, this problem is treated more completely by the analysis of reference 4.

The case $M_e = 0$ (with arbitrary β) produces the equations of Levy and Seban (ref. 20). In that analysis approximate solutions were obtained by the assumption of simple forms for the velocity and temperature profiles which contained undetermined coefficients. These coefficients were then evaluated by use of the boundary conditions. Because the actual profiles cannot be simply represented, this method is not reliable in some ranges even if the Mach number is nearly zero. Brown and Donoughe (ref. 11) also considered the low Mach number problem with variable fluid properties and $Pr_w = 0.7$. The system of equations encountered in that analysis is much more complicated than the present system because of the power-law viscosity, conductivity, and specific-heat relations used. These refinements do not alter the effects of omitting the viscous-dissipation and compressive-work terms, which may be significant at higher Mach numbers.

The case of hypersonic flow requires the introduction of the effects of displacement thickness upon pressure gradient, such as have been evaluated by Lees and Probstein (ref. 21), for example. This case will not be treated herein.

The possibility of assuming $\gamma = 1$ does not simplify the equations beyond the assumption of Mach number zero. For most gases, the assumption of $\gamma = 1$ is physically unreasonable. Therefore, this case does not appear to warrant further consideration.

If strong pressure gradients and reasonably high Mach numbers are to be considered, it thus appears desirable to restrict the similarity system to $Pr = 1$, with the result that

$$f''' + ff'' = \beta(f'^2 - 1 - S) \tag{18a}$$

$$S'' + fS' = 0 \tag{18b}$$

with the boundary conditions (17). Equations (18) were derived by Stewartson by assigning similarity relations corresponding to (15) to the system (7) to (9) with $Pr = 1$; however, no solution was indicated.

The comparison between assuming that $M_e = 0$ (case (2)) or that $Pr = 1$ (case (3)) may perhaps be indicated by examination of the solutions to the insulated flat-plate problem, which include effects of both Prandtl number and Mach number (ref. 2). If $M_e = 0$, the viscous-dissipation and compressive-work terms are omitted in equation (3). Then the predicted temperature profile is a constant, rather than the correct variation from free-stream static to recovery temperature at the wall. However, if $Pr = 1$ is assumed, a constant stagnation temperature is predicted, rather than the actual slight variation in this quantity. The latter discrepancy is small compared with the former.

METHOD OF SOLUTION

Equations (18) with boundary conditions (17) comprise the system to be solved for the dependent variables $f(\eta)$ and $S(\eta)$. Because of the nonlinearity of the system, its high order (fifth), and its classification as a "two-point boundary-value problem," no standard integration methods will yield results expressible in closed form. Methods applicable to equations of this type may be classified as either (1) forward integrations or (2) integrations by methods of successive approximations.

By "forward integration" is meant the progressive integration of the equations from one (initial) boundary to the other. For this purpose several sets of initial values of the derivatives are assumed.

Then the final boundary values obtained are compared with those specified and, after interpolation of the initial values, this trial-and-error process is repeated until the final boundary conditions are satisfied. The integrations may be carried out by the use of either an analog computer (mechanical or electrical) giving continuous integrals or by digital computations involving finite-difference integration. Although generally applicable, a disadvantage associated with forward integration of nonlinear equations is the lack of any inherent convergence mechanism. Thus, the approach to the correct initial values depends almost entirely on the intuition and experience of the one performing the calculations. This method is particularly troublesome for a problem with more than one dependent variable since evidence for the fitness of a given initial value may be obscured by a poor selection of the corresponding initial value of another dependent variable. Furthermore, when an analog computer is employed the accuracy is generally limited, particularly for nonlinear equations where in certain regions the results tend to be highly sensitive to the chosen initial values. If digital computation is utilized to obtain a desired degree of accuracy, the procedure may become excessively tedious.

Successive approximation methods generally assume an entire function for the dependent variables (satisfying as many of the boundary conditions as possible) rather than only the initial derivatives. Then, by use of the differential equations, a procedure is developed for estimating the error as a function of the independent variable(s). This error is applied to the original choice and the process is repeated until satisfactory convergence occurs. An example of a method of successive approximation is Picard's method.

A difficulty shared by both these methods arises when the range of integration is infinite. Then it is necessary to decide upon a finite value of the independent variable at which the boundary conditions may be approximately satisfied and the degree to which they may be satisfied. This suggests the desirability of changing to an independent variable so that only a finite range of integration is required. In the present problem this change of variables can be achieved by following a method used by Crocco for the solution of the compressible flat-plate boundary layer (ref. 2). The concept is advanced that the velocity is a more suitable independent variable since it is bounded. This concept leads to a set of equations conveniently handled by a method of successive approximations.

Transformation to Velocity Plane

To accomplish the transformation to the velocity ratio f' as the independent variable, the following identity may be used:

$$\frac{d}{d\eta} \equiv f'' \frac{d}{df'} \tag{19}$$

This identity may be applied to f" and f as follows:

$$\left.\begin{aligned} f''' &= f'' \frac{df''}{df'} \\ f &= \int_0^{\eta} f' d\eta = \int_0^{f'} \frac{f' df'}{f''} = \int_0^{f'} \frac{\xi d\xi}{f''(\xi)} \end{aligned}\right\} \tag{20}$$

where the dummy variable of integration is ξ, and $f''(\xi)$ represents the functional relationship between f'' and f', that is, $f''(f')$. The primes continue to denote differentiation with respect to η.

Inserting equations (20) into the momentum equation (18a) results in

$$\frac{df''}{df'} = - \int_0^{f'} \frac{\xi d\xi}{f''(\xi)} + \beta \frac{f'^2 - 1 - S}{f''} \tag{21}$$

which satisfies the following condition at $f' = 0$ required by the momentum equation:

$$f_w''' = - \beta(1 + S_w) \tag{22}$$

Now, if equation (21) is integrated once with respect to f' and if the limits of integration are chosen so that $(f'')_{f'=1} = 0$, the result is

$$f'' = \int_{f'}^{1} d\xi_1 \int_0^{\xi_1} \frac{\xi d\xi}{f''(\xi)} - \beta \int_{f'}^{1} \frac{\xi^2 - 1 - S(\xi)}{f''(\xi)} d\xi \tag{23}$$

By inverting the order of integration (or by integrating by parts) the double integral may be reduced to two single integrals, resulting in:

$$f_{j+1}''(f') = (1-f') \int_0^{f'} \frac{\xi d\xi}{f_j''(\xi)} + \int_{f'}^{1} \frac{(1-\xi)\xi d\xi}{f_j''(\xi)} - \beta \int_{f'}^{1} \frac{\xi^2-1-S_j(\xi)}{f_j''(\xi)} d\xi \tag{24}$$

Equation (24) is the form of the momentum equation as it will be used in this report. The subscript j is the iteration number in the method of successive approximations.

A corresponding form of the energy equation is obtained by writing equation (18b) as

$$\frac{S''}{S'} = - f$$

and integrating with respect to η, to get

$$\ln S' = -\int f \, d\eta + \text{constant} \tag{25}$$

Equation (18a) may be written

$$f \, d\eta = -\frac{f'''}{f''} d\eta + \beta \frac{(f'^2 - 1 - S)}{f''} d\eta$$

$$= -\frac{df''}{f''} + \beta \frac{(f'^2 - 1 - S)}{(f'')^2} df'$$

Substitution of this expression into equation (25) results in .

$$\ln S' = \int \frac{df''}{f''} - \beta \int \frac{\xi^2 - 1 - S(\xi)}{[f''(\xi)]^2} d\xi + \text{constant}$$

or the equivalent expression

$$S' = -C_3 f'' J(f') \tag{26}$$

where

$$J(\xi) = \exp\left[-\beta \int_0^{\xi} \frac{\xi_1^2 - 1 - S(\xi_1)}{\left[f''(\xi_1)\right]^2} d\xi_1\right]$$

If this expression is integrated once again and the boundary conditions $S(0) = S_w$, $(S)_{f'=1} = 0$ are required, the result is

$$\frac{S_{j+1}}{S_w} = \frac{\int_{f'}^{1} J_j(\xi) d\xi}{\int_0^1 J_j(\xi) d\xi} \tag{27}$$

Inspection of equations (24) and (27) indicates that the integrals to be evaluated are singular, or indeterminate, at the upper limit. To evaluate these integrals, closed-form expressions must be obtained for the integrands in this range. This requires knowledge of the solution of the system (18) for large η (near $f' = 1$). This "asymptotic solution" and its development are given in appendix B. The results show that equation (24) can be used in its present form, but that equation (27) must be modified to

$$\frac{S_{j+1}}{S_w} = \frac{\varepsilon J_j(1-\varepsilon) + \int_{f'}^{1-\varepsilon} J_j(\xi) d\xi}{\varepsilon J_j(1-\varepsilon) + \int_{0'}^{1-\varepsilon} J_j(\xi) d\xi} \tag{28}$$

where ε is an arbitrary small quantity ($\varepsilon \ll 1$). In this form the singularity has been removed. Equations (24) and (28) comprise the system used in the present investigation. The convergence of this system is discussed in appendix C, and the method of calculation in appendix D.

PROPERTIES OF SOLUTIONS

In the following sections the solutions obtained in this study are presented and their properties are discussed. The two parameters defining a case are S_w and β. The enthalpy function evaluated at the wall S_w determines the wall temperature through the relation

$$t_w = t_0(1 + S_w) \tag{29}$$

Thus, $S_w = -1$ corresponds to a wall temperature of absolute zero, and $S_w = 1$ corresponds to a wall at twice the free-stream stagnation temperature. The case $S_w = 0$ corresponds to a wall at the free-stream stagnation temperature, which for $Pr = 1$ is the case of an insulated surface.

The pressure-gradient parameter β is related to the exponent m of the velocity distribution in the transformed plane $U_e = CX^m$ through the relation

$$\beta = \frac{2m}{m + 1}$$

For a velocity distribution of this form, m can be represented as

$$m = \left(\frac{u_{e_x}}{u_e}\right)\frac{t_0}{t_e} (a_e p_e)^{-1} \int_0^x a_e p_e \, dx \tag{30}$$

It is apparent that $\beta < 0$ $(m < 0)$ corresponds to an unfavorable gradient; $\beta = 0$ $(m = 0)$ corresponds to flat-plate flow; and $\beta = 2$ $(m = \infty)$ corresponds to an infinitely favorable pressure gradient. Stewartson (ref. 14) has shown that $\beta = 1$ $(m = 1)$ corresponds to flow in the immediate vicinity of a stagnation point for two-dimensional flow, as in the incompressible case. It can be shown that the case of a stagnation point in axisymmetric flow can be transformed to the solution for $\beta = 1/2$ (ref. 22). An approximate method for relating β to more general physical flows is given in reference 1. Values of β of the order of magnitude ± 0.3 correspond to flows over supersonic wings, and a typical nozzle with an exit Mach number of about 2.5 might produce a value of β of about 1.5. In the present investigation, solutions are found for pressure gradients ranging from that causing separation to the infinitely favorable gradient and for wall temperatures from absolute zero to twice free-stream stagnation temperature.[3]

All solutions are presented in tabular and graphic form. Table I shows the values of f, f', f'', S, and S' tabulated against η. From these values and equations (18) the quantities f''' and S'' can be easily calculated. Table II presents a summary of the values of f_w'' (related to wall shear) and S_w' (related to heat transfer) from table I, as well as the Reynolds analogy parameter $C_f Re_w/Nu$, which represents the ratio of skin-friction to heat-transfer effects. Certain other quantities of interest cannot be tabulated in general, but can be easily calculated from the following formulas:

Static-temperature ratio:

$$\frac{t}{t_e} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)(1 + S) - \frac{\gamma - 1}{2} M_e^2 f'^2 \tag{31}$$

or, with the static temperature t referred to the free-stream stagnation temperature t_0,

$$\frac{t}{t_0} = (1 + S) - \left(\frac{\frac{\gamma - 1}{2} M_e^2}{1 + \frac{\gamma - 1}{2} M_e^2}\right) f'^2 \tag{32}$$

Flux density:

$$\frac{\rho u}{\rho_e u_e} = \frac{f'}{\left(1 + \frac{\gamma - 1}{2} M_e^2\right)(1 + S) - \frac{\gamma - 1}{2} M_e^2 f'^2} \tag{33}$$

Uniqueness

For $\beta < 0$, $S_w = 0$, Hartree (ref. 7) first observed that the boundary conditions (17) are not sufficient to determine a unique solution.

[3]It should be noted that all but one of the presented solutions for $S_w = 0$ are those first obtained by Hartree (ref. 7) for the problem of Falkner and Skan (ref. 6). As a further check on the present method, the solutions for $\beta = 1.6$ and 2.0 with $S_w = 0$ were obtained independently in the present investigation; these values agree very well with those of Hartree.

Thus, there is not a unique value of f_w'' for a given β. In studying the uniqueness, it is useful to consider the following expression for velocity ratio (for any S_w) valid for large η:

$$f' = 1 + \left[\alpha_1(\eta - \varkappa)^{-(2\beta+1)} + \frac{\alpha_3}{2}(\eta - \varkappa)^{-1}\right]\exp\left[-\frac{(\eta - \varkappa)^2}{2}\right] + \alpha_2(\eta - \varkappa)^{2\beta} \tag{34}$$

where α_1, α_2, α_3, and $\varkappa$ are integration constants (see appendix B). In case of $S_w = 0$, α_3 is also equal to zero; however, this does not change the uniqueness problem, which is independent of wall temperature. For $\beta > 0$, α_2 is necessarily zero in order to satisfy the boundary condition $\lim_{\eta\to\infty} f' = 1$. For continuity in β, Hartree then selected the asymptotic solution with $\alpha_2 = 0$ for $\beta < 0$.

Another important result of the asymptotic solution is that the integral $\int_0^\infty \left(1 - \frac{\rho u}{\rho_e u_e}\right) d\eta$, related to the displacement thickness, can be shown to become infinite for $\alpha_2 \neq 0$. This result is contrary to the concept of a thin layer outside of which the viscous effects may be neglected. A further effect of the α_2 term on the solution can be observed by examination of the dimensionless quantity f'''/ff'' (suggested by Professors L. Crocco and L. Lees), in which f''' represents the net viscous forces acting on the fluid element and f'' is proportional to the velocity gradient (shearing flow). It can be shown that for $\alpha_2 = 0$

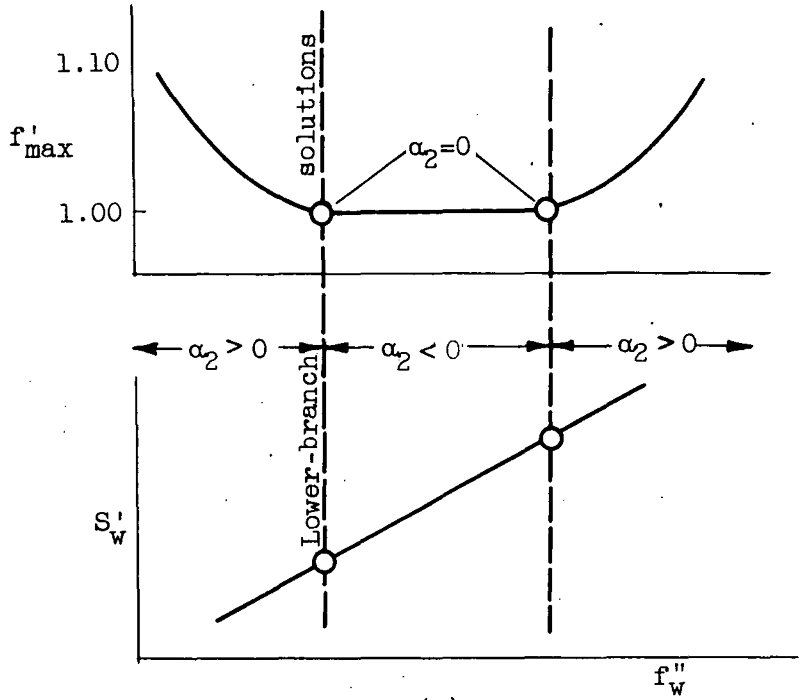
$$\lim_{\eta\to\infty}\left(-\frac{\tilde{f}'''}{\tilde{f}\tilde{f}''}\right) = 1$$

while for $\alpha_2 \neq 0$

$$\lim_{\eta\to\infty}\left(-\frac{\tilde{f}'''}{\tilde{f}\tilde{f}''}\right) = \lim_{\eta\to\infty}\frac{1 - 2\beta}{(\eta - \varkappa)^2} = 0$$

Solutions with $\alpha_2 = 0$ retain the numerator and the denominator of the ratio $-\tilde{f}'''/\tilde{f}\tilde{f}''$ to the same order of magnitude, while if α_2 is different from zero a solution results wherein the magnitude of the net viscous forces in the asymptotic region is small compared with the magnitude of the shearing flow set up by their action. Thus, in order to retain both effects of viscosity to the same order of magnitude, α_2 must be taken equal to zero, as was done by Hartree. The solution thus obtained will be termed the "viscid" solution.

Another feature of solutions with α_2 different from zero is the analytical result that the velocity ratio in the outer portion of the boundary layer may exceed unity. For example, if α_2 is not zero, equation (34) shows that for large η the α_2 term of the velocity ratio expression is dominant, and thus $(f' - 1)^2$ is necessarily of the same sign as α_2. That is, for positive α_2, the velocity ratio approaches unity from above; this phenomenon will be termed "velocity overshoot." Since, for a given β and S_w in this range, each of these various solutions has associated with it a different set of values of f_w'' and S_w', one of these parameters, say f_w'', can be conveniently used in place of α_2 to identify the various solutions. This infinite set of solutions can be represented as in sketch (a) for a typical (cold wall) case.



(a)

It is seen that there are a maximum and a minimum shear (represented by f_w'') and heat transfer (represented by S_w') that can satisfy the equations without incurring velocity overshoot. These distinct solutions (circled points in sketch (a)) correspond to $\alpha_2 = 0$,[4] the viscid solutions; that

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[4]In the evaluation of the singularities of the integrals required for the method of successive approximations, α_2 was taken to be zero. Hence, solutions for $\alpha_2 \neq 0$ were obtained by forward integration (appendix D), although the numerical values of α_2 were not determined.

with the lower shear is designated the "lower-branch" solution. The behavior of the calculated family of solutions is presented in figure 1 for $S_w = -0.8$ and $\beta = -0.325$, -0.3285, and -0.336. For a given value of S_w, as β is decreased the two viscid solutions approach each other. At a value of β to be designated β_{min}, these two solutions become identical and for $\beta < \beta_{min}$, no viscid solution exists. For negative β, only the viscid solutions will be considered in the remainder of this report.

With regard to the physical significance of the double solution, it may be noted that for adverse pressure gradients ($\beta < 0$) a real flow cannot completely reproduce the similar solution because $U_e(0) = \infty$ would be involved. However, a pressure field can, in principle, be applied to a developing boundary layer so that, after a phase of adjustment, the boundary layer would approach one of the similar solutions with $\beta < 0$ and stay quite close to it thereafter. It seems reasonable to believe that, depending on the way the pressure field is applied, one solution or the other corresponding to the same β could be approached after different adjustment phases. This result is exactly what Clauser (ref. 23) has found in his experimental work on similar turbulent boundary-layer flows.

Velocity and Temperature Profiles

The velocity and enthalpy-function profiles obtained from the tabulated solutions are presented as functions of η in figures 2 and 3, respectively. The distance y normal to the surface in the physical plane is related to the similarity variable η through equations (6) and (15), and may be expressed as

$$y = \frac{p_0 a_0}{p_e a_e} \sqrt{\frac{2}{m + 1} \frac{\nu_0 X}{U_e}} \int_0^\eta \frac{t}{t_0} \, d\eta \qquad (35)$$

where t/t_0 is given by equation (32).

Velocity overshoot. - The velocity profiles shown in figure 2 indicate that for a given wall temperature the initial slope decreases as the pressure gradient becomes less favorable. For adverse pressure gradients an inflection point occurs within the boundary layer and moves outward as the gradient becomes more adverse. The velocity ratio varies monotonically from zero to the final value of 1.0 except for the cases of favorable pressure gradients with heated walls. Then the velocity ratio in the outer portion of the boundary layer reaches a maximum value greater than 1.0 before returning to its final value of 1.0 This type of velocity overshoot was also obtained in the investigation of reference 11 for favorable pressure gradients with heated walls and is to be distinguished from that associated with the nonunique inviscid solutions which occur only for adverse pressure gradients. When the wall is heated in a favorable pressure-gradient flow, the density within certain layers of the boundary layer is lowered so that, in spite of the viscous retardation, the flow is accelerated more than the external flow by the external pressure forces. Thus, a velocity greater than the external velocity may be obtained.

This phenomenon can be established by examination of equation (34) and the corresponding asymptotic expression for the enthalpy function (appendix B):

$$S = \alpha_3(\eta - \varkappa)^{-1} \exp\left[-\frac{(\eta - \varkappa)^2}{2}\right] \qquad (36)$$

For favorable pressure gradients, $\alpha_2 = 0$ as previously mentioned. Then, the α_3 term in equation (34) is dominant for large η. Thus, $(f' - 1)$ and α_3 are of the same sign. Hence, for a heated wall (α_3 positive, eq. (36)), the velocity ratio must approach 1.0 from above.

Stagnation-temperature profiles. - Figure 3 shows that for $Pr = 1$, the stagnation temperature varies monotonically across the boundary layer from the wall value to the free-stream value. For favorable pressure gradients with a cold wall, there is small variation with β of this distribution. The variation becomes more pronounced with an increase in wall temperature.

Boundary-layer thickness. - The velocity profiles (fig. 2) indicate that the boundary layer thickens as the wall shear stress diminishes. Also, for a given value of the pressure-gradient parameter β, the boundary layer, when considered in terms of η, thickens as the wall temperature is lowered. However, in the physical plane (in terms of y) because of the relation between y and η (eq. (35)) the trend is just the opposite. This emphasizes the necessity for careful consideration of the relation between the transformed quantities and their physical counterparts.

The thermal boundary layer also thickens as separation is approached. The relative thicknesses of the dynamic and thermal boundary layers may be conveniently observed from a plot of S against f' (fig. 4). Then if a fixed fraction of S_w, say 0.99, is chosen to define the thermal-layer thickness and if the same value of velocity ratio is taken to define the dynamic layer, it can be seen that, regardless of wall temperature, the thermal layer is thicker than the dynamic layer for favorable gradients and thinner for adverse gradients.

For $Pr < 1$ the relative magnitude of the dynamic thickness to the thermal thickness will be decreased, since the Prandtl number represents the ratio of viscous to thermal effects in the fluid.

Shear and Skin Friction

The shear distribution in the boundary layer is presented in figure 5, where f'' is plotted as a function of η. The shear function f'' is related to the shear stress τ through the expression

$$\tau \equiv \mu \frac{\partial u}{\partial y} = \left[\lambda\mu_0 U_e \left(\frac{t_e}{t_0}\right)^{\frac{2\gamma-1}{\gamma-1}} \sqrt{\frac{m+1}{2} \frac{U_e}{\nu_0 X}}\right] f'' \tag{37}$$

For $\beta > 0$ the maximum shear is at the wall, whereas for $\beta < 0$ the point of maximum shear moves increasingly outward as the pressure gradient becomes more adverse.

The quantity that is of primary interest in boundary-layer calculations is the shear stress at the wall τ_w, which can be made dimensionless through the definition of a local skin-friction coefficient, producing the relation

$$C_f \equiv \frac{\tau_w}{\frac{1}{2} \rho_w u_e^2} = f_w''\left[2\lambda(1 + S_w)\right] \sqrt{\frac{m+1}{2} \frac{\nu_0}{U_e X}} \tag{38}$$

The factor $(1 + S_w)$ appears in equation (38) because of the use of ρ_w in the definition of C_f. Although this factor can be easily avoided, it is used later in evaluating a Reynolds number $Re_w = \frac{u_e x}{\nu_w}$ suitable for use in determining the heat transfer. An alternate form for equation (38) is

$$\frac{C_f\sqrt{Re_w}}{2} = f_w'' \sqrt{\frac{m+1}{2} \frac{d \ln X}{d \ln x}} \tag{38a}$$

It should be noted that, in equation (38a), fluid properties are evaluated at the wall temperature. If the skin-friction coefficient and the Reynolds number were to be based on local free-stream fluid properties, rather than on wall values, a factor of $\sqrt{\frac{\mu_w}{\mu_e}\frac{t_e}{t_w}}$ would appear in the right member of equation (38a). When this factor is evaluated using Sutherland's viscosity law, it varies from $\left(\frac{t_w}{t_e}\right)^{1/4}$ to $\left(\frac{t_w}{t_e}\right)^{-1/4}$, depending on the temperature involved.

The quantity f_w'' is presented as a function of β and S_w in figure 6. It can be seen that heating the surface increases the sensitivity of the wall shear to pressure gradient, while cooling the wall has the opposite effect. A suggested physical interpretation for this trend is related to the effect of wall temperature on the mean density of the fluid within the boundary layer. For the heated wall, the boundary-layer density is less than the free-stream density, rendering the boundary-layer fluid more susceptible to free-stream acceleration forces than for the cold wall. Figure 6 shows further that a linear extension of the slope of the curve, f_w'' against β from $\beta = 0$ to large positive β would grossly overemphasize the effects of favorable pressure gradient; while the same linear extension toward negative β would underemphasize the effects of adverse gradient.

In figure 6(b), the two viscid solutions, which occur for adverse pressure gradients for a given β and S_w, are plotted. It is seen that two solutions are given for even the insulated surface ($S_w \doteq 0$), although Hartree reported only one. In this case the lower-branch solution corresponds to negative wall shear stress (separated flow), which was not considered in reference 7. For heated walls ($S_w > 0$) both solutions may be separated near β_{min}, while for cooled walls both solutions may be unseparated in this region. The physical interpretation of these double solutions has been discussed in the section UNIQUENESS.

Heat Transfer

The variation of heat transfer across the boundary layer is plotted in figure 7 in terms of the derivative of the enthalpy function $S' = \frac{dS}{d\eta}$. This quantity is related to the stagnation enthalpy derivative in the physical plane by the expression

$$\frac{\partial}{\partial y}\left(\frac{h_s}{h_0}\right) = \left(\frac{\rho a_e}{\rho_0 a_0}\sqrt{\frac{m+1}{2}\frac{U_e}{\nu_0 X}}\right)S' \tag{39}$$

These curves again indicate the thickening of the thermal layer as separation is approached. Furthermore, as separation is neared, the zone adjacent to the surface where S' is essentially constant spreads rapidly. This is a zone where the heat transfer is primarily by conduction because of the near zero velocities in the neighborhood of the surface.

The values of S' at the surface (S_w') are shown plotted as a function of pressure-gradient parameter β in figure 8 for constant wall temperatures. Two facts are noteworthy: (1) In the region of favorable pressure gradient, S_w' is nearly constant; (2) the heat transfer varies sharply near separation. From these facts the additional conclusion may be drawn that, if a linear extension of these curves is made with the slope at $\beta = 0$, the result will seriously overemphasize the effects of a favorable pressure gradient or heat transfer and underestimate the effects for adverse pressure gradients. A similar influence of pressure gradient on skin friction has already been noted. A comparison of figures 6 and 8 indicates that the effect of pressure gradient on heat transfer is smaller than the corresponding effect upon wall shear.

As with the skin friction, it is convenient to define a dimensionless number from which the heat transfer may be determined. The Nusselt number is

$$\mathrm{Nu} \equiv \frac{x\left(\frac{\partial t}{\partial y}\right)_w}{t_0 - t_w} = \left(-\frac{S_w'}{S_w}\right)\sqrt{\mathrm{Re}_w}\sqrt{\frac{m+1}{2}\frac{d \ln X}{d \ln x}} \tag{40}$$

The quantity $(-S_w'/S_w)$ is plotted in figure 9 for constant wall temperatures as a function of the pressure-gradient parameter β. The Reynolds number Re_w is again defined in terms of wall properties.

<u>Reynolds analogy</u>. - From expressions (38a) and (40), a simple modifed Reynolds analogy parameter is evaluated by

$$\frac{C_f \mathrm{Re}_w}{\mathrm{Nu}} = \frac{2f_w''}{\left(-\frac{S_w'}{S_w}\right)} \tag{41}$$

This quantity is the reciprocal of the usual Reynolds analogy quantity in order to avoid infinite values as separation is approached. It is plotted in figure 10 as a function of the pressure-gradient parameter β. These curves resemble the f_w'' curves (fig. 6) because of the relatively small variation in magnitude of S_w'/S_w compared with that of

f_w''. The variation of $C_f Re_w/Nu$ is from zero to 7.4 for a surface of temperature twice the free-stream stagnation value and from zero to 2.8 for a surface held at a temperature of absolute zero, as shown in figure 10. This indicates the inadequacy of utilizing the flat-plate value of 2.0, as has often been done for estimates of heat transfer. Figure 10 is of particular use in evaluating the heat transfer for a problem when used in conjunction with simple methods for determining C_f, as proposed, for example, in reference 1.

SUMMARY OF RESULTS

From an analysis of the laminar compressible boundary layer based on Stewartson's transformation and including effects of heat transfer and pressure gradient, the following results were obtained:

1. If the condition of similarity is required and the Prandtl number is constant but different from 1.0, the external Mach number must be either zero, constant, or very large. If the Prandtl number is taken as 1.0, the Mach number may be arbitrary. The free-stream velocity distributions consistent with the similarity concept are either power-law or exponential distributions in the transformed coordinates. Since the exponential distribution appears to be limited to favorable gradients and in this range the problem may be reduced to a special case of the power-law distribution, the calculations have been based on the latter class.

2. For flows with favorable pressure gradients, unique solutions were obtained. For flows with adverse pressure gradients, two types of solution were obtained which have been identified as either essentially viscid or inviscid in the outer portions of the boundary layer. The inviscid solution sometimes involved velocity overshoot within the boundary layer. For favorable pressure gradients, the viscid solution is required by the boundary conditions. For adverse pressure gradients there are two viscid solutions; these correspond to the maximum and minimum wall shear, which exclude velocity overshoot.

3. For heated surfaces with favorable pressure gradients a velocity overshoot, which increases with increasingly favorable gradient, results within the boundary layer. This excess velocity is associated with the acceleration of a layer of fluid in the outer portion of the boundary layer, with density less than the external density. Since this layer is subject to the external pressure field and is restrained only slightly by the viscous forces acting on it, it is accelerated more than the external flow.

4. For a Prandtl number of 1.0, when the thicknesses of the dynamic and thermal boundary layers are defined by a fixed fraction

(say 0.99) of the velocity ratio or stagnation-temperature-difference ratio, the thermal boundary layer is thicker than the dynamic layer for favorable pressure gradients and thinner for adverse gradients.

5. The variation of a Reynolds' analogy parameter is from zero to 7.4 for a surface of temperature twice the free-stream stagnation value and from zero to 2.8 for a surface held at a temperature of absolute zero, with the value 2.0 for the flat plate.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, October 15, 1954

APPENDIX A

SYMBOLS

The following symbols are used in this report:

a	sonic velocity
C, C_1, C_2, etc.	arbitrary constants
C_f	local skin-friction coefficient, $C_f = \frac{2\tau_w}{\rho_w u_e^2}$
c_p	specific heat at constant pressure
f	function related to stream function by $f = \psi \sqrt{\frac{m + 1}{2\nu_0 U_e X}}$
g	asymptotic function, $g = \tilde{f}_2'$
h	enthalpy
k	thermal conductivity
k_{su}	Sutherland's constant
M_e	local external Mach number, $M_e = \frac{u_e}{a_e}$
m	exponent from $U_e = CX^m$
Nu	Nusselt number, $Nu = \frac{x\left(\frac{\partial t}{\partial y}\right)_w}{t_0 - t_w}$
Pr	Prandtl number, $Pr = \frac{\mu c_p}{k}$
p	static pressure
Re_w	Reynolds number, $Re_w = \frac{\rho_w u_e x}{\mu_w}$
S	enthalpy function, $S = \frac{h_s}{h_0} - 1$
t	static temperature
U	transformed longitudinal velocity component, $U = \frac{ua_0}{a_e} = \psi_Y$

u	longitudinal velocity component
V	transformed normal velocity component, $V = -\psi_X$
v	normal velocity component
X	transformed longitudinal coordinate, $X = \int_0^x \lambda \frac{p_e}{p_0} \frac{a_e}{a_0} dx$
x	longitudinal coordinate
Y	transformed normal coordinate, $Y = \int_0^y \frac{\rho a_e}{\rho_0 a_0} dy$
y	normal coordinate
α_1, α_2, etc.	integration constants in asymptotic solution
β	pressure gradient parameter, $\beta = \frac{2m}{m + 1}$
β_{min}	minimum value of β corresponding to a viscid solution for a given wall temperature
γ	ratio of specific heats
ε	arbitrary small quantity
η	similarity variable, $\eta = \frac{Y}{X} \sqrt{\frac{m + 1}{2} \frac{U_e X}{\nu_0}}$
$\varkappa$	integration constant in $f_1 = \eta - \varkappa$
λ	$\lambda = \frac{(\mu/\mu_0)}{(t/t_0)} = \left(\frac{t_0 + k_{su}}{t_w + k_{su}}\right)\sqrt{\frac{t_w}{t_0}}$
μ	dynamic viscosity
ν	kinematic viscosity, $\nu = \mu/\rho$
ρ	mass density
τ	shear stress, $\tau = \mu \frac{\partial u}{\partial y}$
ψ	stream function: $\psi_Y = U$, $\psi_X = -V$

Ω oscillation coefficient, eq. (C2)

ω damping coefficient, eq. (C3)

Subscripts:

e local flow outside boundary layer (external)

j result of j^{th} iteration

s stagnation value

w wall or surface value

0 free-stream stagnation value

Other notations:

~ asymptotic quantity

primes denote differentiation with respect to η

APPENDIX B

ASYMPTOTIC SOLUTION

To evaluate the integrals in equations (24) and (27), it is necessary to have closed-form expressions for the integrands concerned, in the range of large η. This requires a solution of the system

$$f''' + ff'' = \beta(f'^2 - 1 - S) \qquad (18a)$$

$$S'' + fS' = 0 \qquad (18b)$$

for large η, which is the asymptotic solution.

The asymptotic solution for f (designated $\tilde{f}$) is assumed to consist of a sum of terms, each smaller than the preceding. Only the first two terms will be discussed herein. The corresponding solution for the enthalpy term $\tilde{S}$ is also obtained.

Let

$$\tilde{f} = \tilde{f}_1 + \tilde{f}_2 \qquad (B1)$$

where

$$\tilde{f}_2 \ll \tilde{f}_1$$

$$\tilde{f}_2' \ll \tilde{f}_1'$$

Now, since $\lim_{\eta \to \infty}(f') = 1$, let

$$\tilde{f}_1 = \eta - \varkappa \qquad (B2)$$

where $\varkappa$ is an undetermined constant. If $\tilde{f}_1$ is inserted into (18), the corresponding enthalpy term S_1 must be identically zero. Inserting equations (B1) and (B2) into equations (18) and dropping higher-order terms result in

$$\left.\begin{aligned} \tilde{f}_2''' + (\eta - \varkappa)\tilde{f}_2'' &= \beta\left[2\tilde{f}_2' - \tilde{S}_2\right] \\ \tilde{S}_2'' + (\eta - \varkappa)\tilde{S}_2' &= 0 \end{aligned}\right\} \qquad (B3)$$

The energy equation can be integrated directly to give

$$\tilde{S}_2' = Ce^{-\frac{(\eta-\varkappa)^2}{2}}$$

which integrates once again to the complementary error function (denoted cerf)

$$\tilde{S}_2 = - C \int_\eta^\infty e^{-\frac{(\eta-\varkappa)^2}{2}} d\eta$$

or

$$\tilde{S}_2 = \alpha_3 \sqrt{\frac{\pi}{2}} \operatorname{cerf}\left(\frac{\eta - \varkappa}{\sqrt{2}}\right) \tag{B4}$$

If equation (B4) is now substituted into the momentum equation of equations (B3), with the notation

$$g(\eta) \equiv \tilde{f}_2'$$

there results

$$g'' + (\eta - \varkappa)g' - 2\beta g = - \alpha_3 \sqrt{\frac{\pi}{2}} \beta \operatorname{cerf}\left(\frac{\eta - \varkappa}{\sqrt{2}}\right) \tag{B5}$$

A particular integral to equation (B5) is

$$g = \frac{\alpha_3}{2} \sqrt{\frac{\pi}{2}} \operatorname{cerf}\left(\frac{\eta - \varkappa}{\sqrt{2}}\right) \tag{B6}$$

The complementary function can be found by noting that the homogeneous part of equation (B5) is Weber's equation. Hartree (ref. 7) gives the general solution for large values of the argument $(\eta - \varkappa)$ which can be written

$$g = \alpha_1 (\eta - \varkappa)^{-(2\beta+1)} \exp\left[- \frac{(\eta - \varkappa)^2}{2}\right] + \alpha_2 (\eta - \varkappa)^{2\beta} \tag{B7}$$

where α_1 and α_2 are undetermined constants.

For $\beta \geqslant 0$ it is clearly necessary to take $\alpha_2 = 0$ if the boundary condition $\lim_{\eta\to\infty} g = 0$ is to be applied. For $\beta < 0$ the boundary condition does not require $\alpha_2 = 0$; this introduces a lack of uniqueness in this range. The significance of $\alpha_2 = 0$ was more fully discussed in the section UNIQUENESS.

Using the first term of the expansion for the complementary error function

$$\operatorname{cerf}\left(\frac{\eta - x}{\sqrt{2}}\right) = \left\{\sqrt{\frac{2}{\pi}}\,(\eta - x)^{-1} \exp\left[-\frac{(\eta - x)^2}{2}\right]\right\}\left[1 - \frac{1}{(\eta - x)^2} + \ldots\right] \tag{B8}$$

and combining the preceding equations result in the following expressions:

$$\tilde{f}' = 1 + \left[\alpha_1(\eta - x)^{-(2\beta+1)} + \frac{\alpha_3}{2}(\eta - x)^{-1}\right]\exp\left[-\frac{(\eta - x)^2}{2}\right] + \alpha_2(\eta - x)^{2\beta} \tag{34}$$

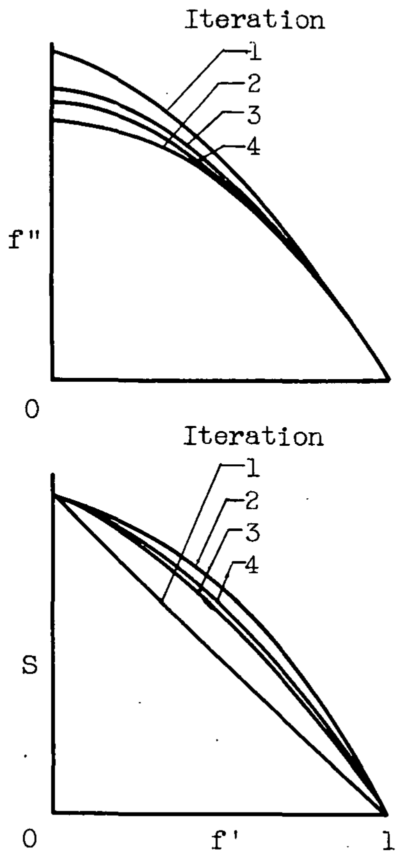
and

$$\tilde{S} = \alpha_3(\eta - x)^{-1}\left[\exp - \frac{(\eta - x)^2}{2}\right] \tag{36}$$

APPENDIX C

CONVERGENCE AND EXTRAPOLATION

The method of successive approximations used in solving equations (24) and (28) is as follows: Two functions $f_j''(f')$ and $S_j(f')$ are assumed and inserted into the right sides of equations (24) and (28). This produces two new functions, $f_{j+1}''(f')$ and $S_{j+1}(f')$ on the left. The question of convergence is the first to consider. In reference 2, Crocco treated a momentum equation which was essentially equation (24) with $\beta = 0$. There it was shown that the result might converge to a pair of functions between which it would oscillate and of which the geometric mean was the proper solution. In practice, the use of the arithmetic mean was demonstrated to be adequate. In the same way in the present case, the property of oscillation cannot be developed analytically; however, it has been found by trial that, if $\frac{f_j'' + f_{j+1}''}{2}$ is used in place of f_{j+1}'' to obtain f_{j+2}'', the oscillation is reduced and a convergence takes place. A typical result is shown in sketch (b).



(b)

When the value for β for which a solution was sought was sufficiently positive, the enthalpy function S also showed a tendency to oscillate. In these cases, applying the same averaging procedure to S again improved the convergence. It was also found that convergence was improved if, in the intial assumed function for $f''(f')$, the slope $\left(\frac{df''}{df'}\right)_w$ was taken so that it satisfied equation (18a); that is,

$$\left(\frac{df''}{df'}\right)_w = \frac{-\beta(1 + S_w)}{f_w''}$$

When an iterative method is used to determine a function, it is always desirable to develop a method of extrapolating the result to correspond to a larger number of iterations than have actually been carried out. This cannot be done in an exact fashion unless a definite law of convergence is established. Recently, an extrapolation method was devised (ref. 24) which required four successive iterants for an arbitrary iterative computing scheme. The development assumed that the remaining error after any iteration consisted essentially of two terms, both of which damped by a factor ω with each iteration. The sign of one of these terms was assumed to change with each iteration. This method extrapolated a function by breaking it into n-1 parts and treating it somewhat like an n-dimensional vector. The method has been demonstrated for Laplace's equation for which it was quite adequate. For nonlinear equations, however, the method is not as suitable.

In reference 1, a method requiring five successive iterants was developed which combined the method of reference 24 and the geometric mean rule. The function to be extrapolated is considered to be made up of a set of numbers F_i, where the subscript i identifies the particular component of the set. Then, the resulting relations for the i^{th} component of the extrapolated function F in terms of the preceding five iterants, $(F_i)_j \cdots (F_i)_{j+4}$, where j is the iteration number, are:

$$F_i = \frac{1}{\Omega_i}\left[\frac{(F_i)_{j+4} - \omega^2 (F_i)_{j+2}}{1 - \omega^2}\right] \tag{C1}$$

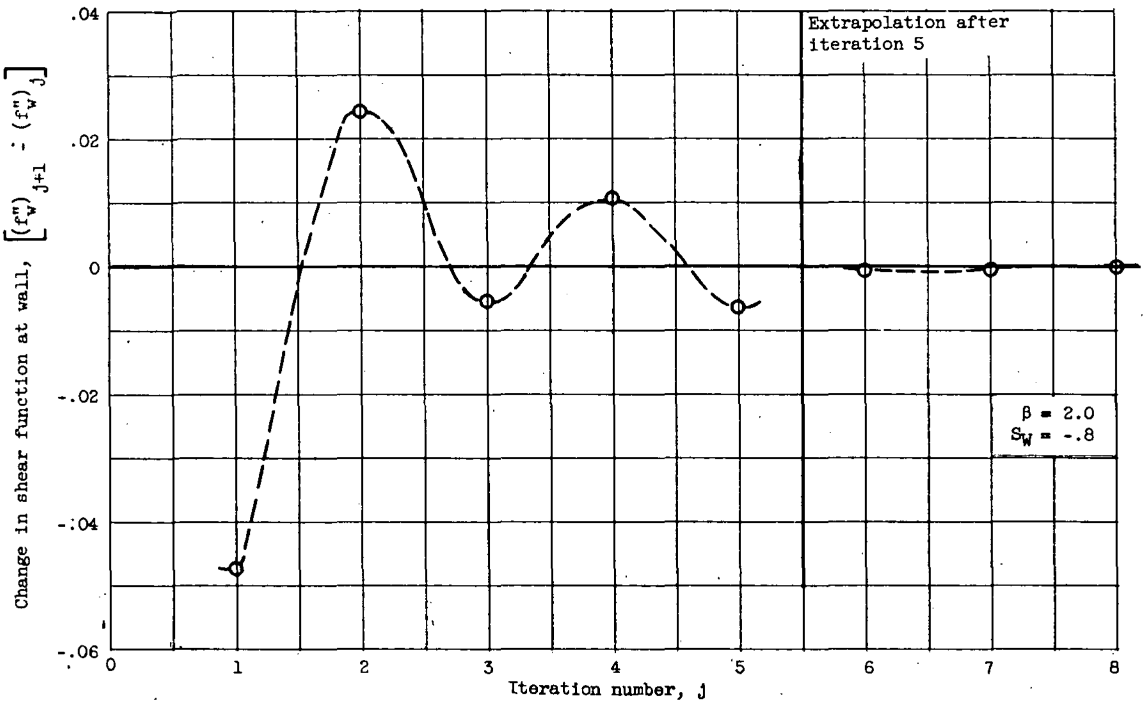
where the oscillation coefficient Ω_i is given by

$$\Omega_i = \sqrt{\frac{(F_i)_{j+4} - \omega^2 (F_i)_{j+2}}{(F_i)_{j+3} - \omega^2 (F_i)_{j+1}}} \tag{C2}$$

and the damping coefficient ω is

$$\omega^2 = \frac{\displaystyle\sum_{i=1}^{n}\left[(F_i)_{j+4} - (F_i)_{j+2}\right]\left[\frac{(F_i)_{j+2} - (F_i)_j}{\left|(F_i)_{j+2} - (F_i)_j\right|}\right]}{\displaystyle\sum_{i=1}^{n}\left|(F_i)_{j+2} - (F_i)_j\right|} \tag{C3}$$

Application of this system was extremely effective. It generally reduced the oscillation remaining after five iterations by a factor of 10. A typical plot of the oscillation of f_w'' is indicated in sketch (c).



(c)

APPENDIX D

CALCULATION PROCEDURE

The successive approximation calculations were carried out by means of IBM Type 604 Calculating Punch machines. The program was coded for fixed-point calculation, with the standard Function-Generating control panel used, plus a control panel especially wired for rapid integration of quotients by a trapezoidal rule. The step size (in f') varied from a maximum value of 0.050 or 0.025 at $f' = 0$ to 0.00001 at $f' = 0.9999$, the total number of intervals being 122 in the former case and 236 in the latter. By doubling and halving the step size for a critical case, the results are judged to contain a maximum error of 0.0002. Comparison with solutions obtained by forward integration, for the same case, confirms this accuracy. A given iteration (utilizing the 0.050 step size) could be carried out in approximately $1\frac{1}{2}$ hours by an experienced machine operator. If the averaging and extrapolation techniques described in appendix C are used, 10 iterations generally would suffice for the accuracy desired. In contrast with forward integration, this number of iterations is not a function of the experience of the person carrying out the calculations.

In the derivation of the integral relations (eqs. (24) and (27)), it was assumed that the velocity ratio varied smoothly and monotonically from zero at the wall to 1.0 at infinity. However, in the range $\beta > 0$ and $S_w > 0$ (favorable pressure gradient and hot wall), the solution involves an increasing velocity ratio to a value greater than 1.0, followed by a smooth decrease to 1.0. Under these unusual circumstances, the method of successive approximation derived herein must be considerably modified if it is to be used at all. For these cases, forward integrations were performed by Dr. Lynn U. Albers.

Equations (18), together with the boundary conditions (17), constitute a nonlinear two-point boundary-value problem. Cases of this boundary-value problem were solved by forward integration, with the IBM Card-Programmed Electronic Calculator (CPC) used to integrate with five-point integration formulas.

For the cases where the solutions are not unique ($\beta < 0$), the solutions were obtained in two patterns: In one pattern, β and S_w were fixed and, for a set of values of f_w'', the quantity S_w' was altered until boundary conditions at infinity were apparently satisfied. In the other pattern, f_w'' and S_w were fixed and, for a set of values of negative β, the quantity S_w' was altered until boundary conditions at infinity were apparently satisfied. An attempt was made with both patterns to include the solution with the minimum value of the maximum velocity ratio f'_{max} within the boundary layer. Except for those cases where no solution existed without velocity overshoot, this minimum value was 1.0.

The details of the integration method used are described very completely by Lynn U. Albers in an appendix to reference 25. The possible error contained in the results is indicated in the footnote to table I. Each trial run of a case required approximately 30 minutes. A person considerably experienced with the method of obtaining solutions by forward integration generally achieved convergence within 12 trials; however, tests indicate that this number is insufficient by a factor of the order of 2 if the person lacks experience.

3278

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TABLE 1. - SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.[1]

$\beta = -0.326$, $S_w = -1.0$					
η	f	f'	f"	S	S'
0	0	0	0	-1.0000	0.2477
.2	.0000	.0001	.0016	-.9505	.2477
.4	.0001	.0009	.0065	-.9009	.2477
.6	.0004	.0029	.0145	-.8514	.2476
.8	.0014	.0069	.0258	-.8019	.2476
1.0	.0034	.0135	.0403	-.7524	.2475
1.2	.0070	.0232	.0580	-.7029	.2472
1.4	.0129	.0369	.0788	-.6535	.2468
1.6	.0220	.0550	.1026	-.6042	.2459
1.8	.0352	.0781	.1290	-.5552	.2445
2.0	.0536	.1067	.1578	-.5064	.2424
2.2	.0783	.1413	.1882	-.4582	.2392
2.4	.1105	.1821	.2196	-.4108	.2348
2.6	.1516	.2291	.2506	-.3645	.2288
2.8	.2026	.2822	.2799	-.3195	.2208
3.0	.2648	.3409	.3056	-.2763	.2108
3.2	.3393	.4042	.3260	-.2353	.1985
3.4	.4267	.4708	.3392	-.1970	.1839
3.6	.5277	.5392	.3434	-.1619	.1672
3.8	.6424	.6075	.3377	-.1303	.1488
4.0	.7706	.6737	.3220	-.1025	.1292
4.2	.9116	.7357	.2969	-.0786	.1092
4.4	1.0645	.7919	.2642	-.0588	.0897
4.6	1.2279	.8410	.2265	-.0427	.0713
4.8	1.4004	.8824	.1867	-.0301	.0548
5.0	1.5803	.9158	.1478	-.0206	.0407
5.2	1.7662	.9417	.1123	-.0137	.0291
5.4	1.9566	.9610	.0818	-.0088	.0201
5.6	2.1502	.9748	.0571	-.0055	.0133
5.8	2.3462	.9843	.0383	-.0033	.0085
6.0	2.5437	.9905	.0245	-.0020	.0052
6.2	2.7422	.9944	.0152	-.0012	.0031
6.4	2.9414	.9967	.0088	-.0007	.0016
6.6	3.1409	.9980	.0050	-.0005	.0009
6.8	3.3405	.9988	.0028	-.0004	.0005

$\beta = -0.3657$, $S_w = -1.0$					
η	f	f'	f"	S	S'
0	0	0	0.0500	-1.0000	0.2958
.2	.0010	.0101	.0522	-.9408	.2958
.4	.0041	.0211	.0586	-.8817	.2956
.6	.0096	.0339	.0693	-.8226	.2952
.8	.0178	.0491	.0841	-.7636	.2944
1.0	.0295	.0678	.1028	-.7049	.2931
1.2	.0452	.0905	.1252	-.6465	.2909
1.4	.0660	.1180	.1509	-.5886	.2877
1.6	.0928	.1510	.1791	-.5314	.2832
1.8	.1268	.1898	.2091	-.4754	.2771
2.0	.1691	.2347	.2395	-.4208	.2690
2.2	.2211	.2856	.2690	-.3679	.2588
2.4	.2837	.3421	.2956	-.3174	.2461
2.6	.3582	.4035	.3174	-.2697	.2308
2.8	.4454	.4686	.3324	-.2252	.2131
3.0	.5458	.5359	.3388	-.1846	.1930
3.2	.6598	.6035	.3354	-.1482	.1711
3.4	.7871	.6694	.3216	-.1162	.1481
3.6	.9273	.7315	.2983	-.0889	.1248
3.8	1.0793	.7881	.2668	-.0663	.1022
4.0	1.2421	.8379	.2299	-.0480	.0810
4.2	1.4140	.8799	.1904	-.0337	.0621
4.4	1.5935	.9140	.1513	-.0229	.0460
4.6	1.7791	.9406	.1154	-.0151	.0328
4.8	1.9693	.9605	.0843	-.0096	.0225
5.0	2.1629	.9747	.0589	-.0059	.0149
5.2	2.3589	.9845	.0395	-.0035	.0095
5.4	2.5565	.9909	.0253	-.0020	.0058
5.6	2.7551	.9949	.0155	-.0011	.0034
5.8	2.9543	.9973	.0092	-.0006	.0019
6.0	3.1539	.9987	.0052	-.0003	.0011
6.2	3.3538	.9995	.0028	-.0001	.0006
6.4	3.5537	.9999	.0014	-.0001	.0003

[1]The accuracy of solutions obtained by the method of successive approximations is believed to be ±0.0002. Solutions by forward integration were obtained in two patterns (appendix D). Where β and S_w were initially fixed, the eigenvalues are believed to be correct to ±0.0002. Where f''_w and S_w were initially fixed, β and S'_w are believed to be correct to ±0.0002 (except in the case of $\beta = 0.2460$, $S_w = -0.4$, where β and S'_w are believed to be correct to ±0.002). The values in the tables are of comparable accuracy except at large η, where the entries may contain errors as large as twice the above amounts.

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = -0.3884,\ S_w = -1.0$					
η	f	f'	f"	S	S'
0	0	0	0.1400	-1.0000	0.3527
.2	.0028	.0282	.1427	-.9294	.3527
.4	.0113	.0574	.1506	-.8589	.3523
.6	.0259	.0887	.1633	-.7886	.3510
.8	.0470	.1230	.1603	-.7186	.3485
1.0	.0754	.1611	.2010	-.6493	.3443
1.2	.1118	.2036	.2243	-.5811	.3379
1.4	.1571	.2509	.2491	-.5143	.3290
1.6	.2125	.3032	.2738	-.4496	.3171
1.8	.2787	.3603	.2963	-.3876	.3020
2.0	.3569	.4215	.3149	-.3291	.2834
2.2	.4476	.4859	.3273	-.2745	.2616
2.4	.5513	.5519	.3318	-.2246	.2368
2.6	.6683	.6180	.3270	-.1800	.2096
2.8	.7964	.6821	.3125	-.1409	.1811
3.0	.9409	.7423	.2888	-.1076	.1522
3.2	1.0950	.7971	.2576	-.0799	.1242
3.4	1.2593	.8450	.2213	-.0577	.0982
3.6	1.4325	.8855	.1827	-.0405	.0750
3.8	1.6130	.9182	.1448	-.0275	.0553
4.0	1.7993	.9436	.1101	-.0181	.0393
4.2	1.9900	.9625	.0802	-.0115	.0269
4.4	2.1839	.9761	.0559	-.0071	.0177
4.6	2.3801	.9853	.0373	-.0043	.0112
4.8	2.5778	.9913	.0239	-.0025	.0068
5.0	2.7765	.9951	.0146	-.0014	.0040
5.2	2.9758	.9974	.0086	-.0008	.0023
5.4	3.1754	.9987	.0048	-.0005	.0012
5.6	3.3752	.9994	.0026	-.0003	.0006
5.8	3.5752	.9998	.0013	-.0002	.0003
6.0	3.7751	1.0000	.0006	-.0002	.0002
6.2	3.9751	1.0000	.0003	-.0001	.0000

$\beta = -0.36,\ S_w = -1.0$					
η	f	f'	f"	S	S'
0	0	0	0.2448	-1.0000	0.0400
.2033	.0050	.050	.2476	-.9186	.0399
.4024	.0199	.100	.2552	-.8391	.0398
.5942	.0438	.150	.2663	-.7628	.0396
.7776	.0759	.200	.2793	-.6905	.0392
.9524	.1151	.250	.2929	-.6225	.0385
1.1193	.1610	.300	.3061	-.5588	.0376
1.2795	.2130	.350	.3178	-.4993	.0365
1.4344	.2711	.400	.3274	-.4436	.0352
1.5855	.3353	.450	.3341	-.3915	.0336
1.7342	.4060	.500	.3375	-.3428	.0318
1.8824	.4838	.550	.3368	-.2970	.0298
2.0316	.5697	.600	.3317	-.2541	.0275
2.1847	.6653	.650	.3215	-.2138	.0251
2.3439	.7728	.700	.3057	-.1760	.0223
2.5134	.8957	.750	.2833	-.1405	.0194
2.6995	1.0400	.800	.2535	-.1074	.0162
2.9129	1.2162	.850	.2146	-.0765	.0127
3.1766	1.4473	.900	.1646	-.0479	.0089
3.2397	1.5043	.910	.1529	-.0425	.0081
3.3078	1.5667	.920	.1406	-.0372	.0073
3.3825	1.6358	.930	.1275	-.0320	.0065
3.4654	1.7133	.940	.1136	-.0270	.0057
3.5596	1.8023	.950	.0988	-.0220	.0048
3.6123	1.8525	.955	.0911	-.0196	.0043
3.6697	1.9075	.960	.0830	-.0172	.0039
3.7332	1.9686	.965	.0746	-.0148	.0034
3.8044	2.0375	.970	.0659	-.0125	.0030
3.8859	2.1168	.975	.0568	-.0103	.0025
3.9822	2.2110	.980	.0472	-.0080	.0020
4.1012	2.3279	.985	.0370	-.0059	.0015
4.2604	2.4851	.990	.0262	-.0038	.0010
4.3003	2.5247	.991	.0239	-.0034	.0009
4.3443	2.5682	.992	.0216	-.0030	.0008
4.3933	2.6169	.993	.0192	-.0026	.0007
4.4489	2.6722	.994	.0167	-.0022	.0006
4.5134	2.7363	.995	.0142	-.0018	.0005
4.5905	2.8130	.996	.0117	-.0014	.0004
4.6872	2.9094	.997	.0090	-.0010	.0003
4.8187	3.0406	.998	.0062	-.0006	.0002
5.0327	3.2543	.999	.0033	-.0003	.0001
6.1247	4.3446	1.000	.0000	.0000	.0000

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = -0.3$, $S_w = -1.0$					
η	f	f'	f''	S	S'
0	0	0	0.3181	-1.0000	0.4262
.1568	.0039	.050	.3196	-.9331	.4261
.3123	.0155	.100	.3236	-.8669	.4255
.4655	.0347	.150	.3292	-.8018	.4239
.6159	.0609	.200	.3359	-.7383	.4209
.7632	.0941	.250	.3427	-.6766	.4163
.9077	.1338	.300	.3491	-.6169	.4095
1.0498	.1800	.350	.3543	-.5593	.4005
1.1902	.2326	.400	.3579	-.5039	.3892
1.3296	.2918	.450	.3591	-.4506	.3752
1.4690	.3581	.500	.3575	-.3994	.3586
1.6098	.4320	.550	.3525	-.3502	.3393
1.7533	.5146	.600	.3436	-.3031	.3170
1.9016	.6072	.650	.3302	-.2580	.2918
2.0572	.7123	.700	.3117	-.2148	.2633
2.2240	.8333	.750	.2871	-.1735	.2315
2.4080	.9761	.800	.2555	-.1342	.1960
2.6202	1.1513	.850	.2155	-.0968	.1565
2.8835	1.3819	.900	.1646	-.0616	.1122
2.9465	1.4389	.910	.1528	-.0549	.1026
3.0147	1.5014	.920	.1404	-.0482	.0928
3.0895	1.5705	.930	.1273	-.0417	.0828
3.1726	1.6483	.940	.1134	-.0352	.0724
3.2670	1.7375	.950	.0986	-.0289	.0617
3.3199	1.7878	.955	.0908	-.0258	.0562
3.3775	1.8430	.960	.0827	-.0227	.0506
3.4412	1.9043	.965	.0744	-.0197	.0449
3.5127	1.9735	.970	.0657	-.0167	.0391
3.5945	2.0531	.975	.0565	-.0137	.0332
3.6912	2.1477	.980	.0470	-.0108	.0271
3.8108	2.2652	.985	.0369	-.0080	.0208
3.9708	2.4232	.990	.0260	-.0052	.0143
4.0109	2.4629	.991	.0238	-.0046	.0129
4.0551	2.5067	.992	.0214	-.0041	.0116
4.1044	2.5556	.993	.0191	-.0035	.0103
4.1603	2.6112	.994	.0167	-.0030	.0089
4.2251	2.6756	.995	.0142	-.0025	.0075
4.3026	2.7528	.996	.0116	-.0020	.0061
4.3997	2.8495	.997	.0090	-.0015	.0046
4.5319	2.9814	.998	.0062	-.0009	.0031
4.7471	3.1963	.999	.0033	-.0004	.0016
5.8542	4.3031	1.000	.0000	-.0000	.0000

$\beta = -0.14$, $S_w = -1.0$					
η	f	f'	f''	S	S'
0	0	0	0.4165	0	0.4554
.1199	.0029	.050	.4170	-.9453	.4554
.2397	.0119	.100	.4179	-.8908	.4551
.3591	.0269	.150	.4190	-.8365	.4541
.4783	.0477	.200	.4199	-.7825	.4522
.5973	.0745	.250	.4202	-.7288	.4490
.7164	.1072	.300	.4194	-.6756	.4443
.8359	.1461	.350	.4173	-.6229	.4377
.9563	.1912	.400	.4133	-.5708	.4290
1.0782	.2431	.450	.4072	-.5192	.4179
1.2023	.3020	.500	.3986	-.4682	.4040
1.3296	.3689	.550	.3869	-.4178	.3872
1.4615	.4448	.600	.3717	-.3681	.3671
1.5995	.5311	.650	.3525	-.3191	.3433
1.7462	.6302	.700	.3287	-.2707	.3153
1.9053	.7457	.750	.2995	-.2232	.2827
2.0829	.8834	.800	.2639	-.1764	.2447
2.2897	1.0543	.850	.2204	-.1304	.2005
2.5484	1.2809	.900	.1669	-.0855	.1483
2.6106	1.3373	.910	.1547	-.0766	.1367
2.6781	1.3991	.920	.1419	-.0678	.1247
2.7523	1.4677	.930	.1284	-.0591	.1121
2.8348	1.5448	.940	.1142	-.0504	.0990
2.9287	1.6336	.950	.0991	-.0417	.0853
2.9813	1.6837	.955	.0912	-.0374	.0782
3.0387	1.7387	.960	.0831	-.0331	.0709
3.1023	1.7999	.965	.0746	-.0289	.0634
3.1738	1.8691	.970	.0658	-.0247	.0556
3.2555	1.9486	.975	.0566	-.0204	.0476
3.3521	2.0430	.980	.0470	-.0163	.0392
3.4717	2.1605	.985	.0369	-.0121	.0305
3.6319	2.3187	.990	.0260	-.0080	.0213
3.6721	2.3585	.991	.0237	-.0072	.0194
3.7163	2.4024	.992	.0214	-.0063	.0174
3.7657	2.4514	.993	.0190	-.0055	.0155
3.8217	2.5071	.994	.0166	-.0047	.0135
3.8867	2.5717	.995	.0142	-.0039	.0114
3.9643	2.6490	.996	.0116	-.0031	.0093
4.0616	2.7459	.997	.0090	-.0023	.0071
4.1939	2.8778	.998	.0062	-.0015	.0049
4.4088	3.0924	.999	.0033	-.0007	.0026
5.4938	4.1772	1.000	.0000	.0000	.0000

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = 0.5,\ S_w = -1.0$					
η	f	f'	f''	S	S'
0	0	0	0.5806	-1.0000	0.4948
.0861	.0021	.050	.5797	-.9574	.4948
.1726	.0086	.100	.5770	-.9147	.4946
.2596	.0195	.150	.5724	-.8718	.4940
.3474	.0349	.200	.5659	-.8285	.4929
.4364	.0549	.250	.5574	-.7848	.4910
.5270	.0798	.300	.5468	-.7406	.4881
.6195	.1099	.350	.5340	-.6957	.4839
.7145	.1456	.400	.5188	-.6501	.4781
.8126	.1873	.450	.5011	-.6037	.4704
.9145	.2358	.500	.4808	-.5564	.4605
1.0211	.2918	.550	.4576	-.5081	.4479
1.1337	.3566	.600	.4310	-.4587	.4319
1.2539	.4318	.650	.4008	-.4081	.4120
1.3842	.5199	.700	.3666	-.3561	.3873
1.5283	.6244	.750	.3277	-.3026	.3567
1.6922	.7516	.800	.2834	-.2474	.3189
1.8866	.9122	.850	.2324	-.1903	.2715
2.1344	1.1293	.900	.1727	-.1307	.2110
2.1947	1.1839	.910	.1595	-.1185	.1968
2.2603	1.2440	.920	.1457	-.1061	.1817
2.3327	1.3109	.930	.1313	-.0936	.1658
2.4135	1.3865	.940	.1163	-.0809	.1486
2.5059	1.4738	.950	.1005	-.0681	.1303
2.5578	1.5233	.955	.0923	-.0616	.1205
2.6146	1.5777	.960	.0839	-.0551	.0719
2.6776	1.6384	.965	.0752	-.0485	.0998
2.7486	1.7071	.970	.0662	-.0418	.0887
2.8300	1.7862	.975	.0568	-.0351	.0769
2.9265	1.8805	.980	.0470	-.0283	.0644
3.0462	1.9981	.985	.0368	-.0215	.0511
3.2069	2.1568	.990	.0259	-.0145	.0366
3.2473	2.1969	.991	.0236	-.0131	.0335
3.2918	2.2410	.992	.0213	-.0117	.0303
3.3415	2.2904	.993	.0189	-.0102	.0271
3.3980	2.3464	.994	.0165	-.0088	.0238
3.4635	2.4116	.995	.0140	-.0074	.0203
3.5418	2.4895	.996	.0115	-.0059	.0168
3.6400	2.5874	.997	.0089	-.0044	.0131
3.7738	2.7209	.998	.0061	-.0030	.0092
3.9916	2.9384	.999	.0032	-.0015	.0049
5.1056	4.0521	1.000	.0000	-.0000	.0000

$\beta = 2.0,\ S_w = -1.0$					
η	f	f'	f''	S	S'
0	0	0	0.7381	-1.0000	0.5203
.0678	.0016	.050	.7359	-.9647	.5203
.1360	.0068	.100	.7293	-.9292	.5201
.2051	.0154	.150	.7188	-.8933	.5198
.2754	.0277	.200	.7045	-.8567	.5190
.3472	.0439	.250	.6869	-.8196	.5177
.4212	.0643	.300	.6660	-.7814	.5157
.4976	.0892	.350	.6420	-.7421	.5128
.5772	.1191	.400	.6150	-.7012	.5086
.6606	.1545	.450	.5852	-.6593	.5029
.7485	.1964	.500	.5525	-.6155	.4953
.8421	.2456	.550	.5170	-.5696	.4852
.9427	.3035	.600	.4786	-.5215	.4721
1.0520	.3719	.650	.4372	-.4707	.4551
1.1726	.4534	.700	.3925	-.4172	.4330
1.3085	.5520	.750	.3444	-.3603	.4045
1.4659	.6742	.800	.2924	-.2996	.3674
1.6562	.8314	.850	.2355	-.2344	.3187
1.9030	1.0477	.900	.1720	-.1639	.2528
1.9636	1.1026	.910	.1583	-.1490	.2369
2.0299	1.1632	.920	.1442	-.1339	.2198
2.1031	1.2310	.930	.1296	-.1185	.2014
2.1852	1.3078	.940	.1144	-.1028	.1814
2.2793	1.3967	.950	.0986	-.0868	.1598
2.3322	1.4471	.955	.0904	-.0786	.1482
2.3903	1.5027	.960	.0821	-.0704	.1360
2.4548	1.5648	.965	.0735	-.0620	.1232
2.5275	1.6351	.970	.0646	-.0536	.1097
2.6109	1.7163	.975	.0554	-.0450	.0954
2.7099	1.8130	.980	.0458	-.0364	.0801
2.8327	1.9338	.985	.0358	-.0275	.0636
2.9977	2.0967	.990	.0252	-.0186	.0456
3.0392	2.1378	.991	.0230	-.0167	.0418
3.0849	2.1831	.992	.0207	-.0149	.0379
3.1359	2.2338	.993	.0184	-.0131	.0338
3.1939	2.2913	.994	.0161	-.0113	.0297
3.2611	2.3581	.995	.0137	-.0094	.0254
3.3414	2.4381	.996	.0112	-.0075	.0209
3.4421	2.5385	.997	.0087	-.0057	.0163
3.5792	2.6753	.998	.0060	-.0038	.0114
3.8023	2.8980	.999	.0032	-.0019	.0061
4.9344	4.0298	1.000	.0000	-.0000	.0000

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = -0.10,\ S_w = -0.8$					
η	f	f'	f"	S	S'
0	0	0	-0.0686	-0.8000	0.0447
.4	-.0053	-.0258	-.0603	-.7821	.0448
.8	-.0202	-.0482	-.0515	-.7642	.0450
1.2	-.0433	-.0670	-.0425	-.7461	.0455
1.6	-.0733	-.0821	-.0330	-.7277	.0466
2.0	-.1085	-.0933	-.0231	-.7087	.0483
2.4	-.1474	-.1005	-.0123	-.6889	.0509
2.8	-.1882	-.1030	-.0003	-.6679	.0544
3.2	-.2291	-.1004	.0136	-.6452	.0591
3.6	-.2678	-.0918	.0301	-.6204	.0653
4.0	-.3016	-.0759	.0501	-.5928	.0732
4.4	-.3273	-.0511	.0746	-.5616	.0830
4.8	-.3410	-.0154	.1048	-.5260	.0950
5.2	-.3379	.0336	.1413	-.4853	.1088
5.6	-.3121	.0984	.1837	-.4388	.1241
6.0	-.2568	.1811	.2301	-.3861	.1392
6.4	-.1647	.2823	.2752	-.3278	.1516
6.8	-.0287	.3999	.3103	-.2656	.1578
7.2	.1566	.5278	.3245	-.2029	.1541
7.6	.3935	.6556	.3091	-.1440	.1383
8.0	.6794	.7709	.2629	-.0937	.1118
8.4	1.0071	.8632	.1964	-.0553	.0799
8.8	1.3662	.9276	.1270	-.0295	.0497
9.2	1.7457	.9665	.0703	-.0145	.0267
9.6	2.1369	.9866	.0332	-.0070	.0123
10.0	2.5335	.9954	.0133	-.0038	.0048
10.4	2.9324	.9987	.0045	-.0026	.0016
10.8	3.3321	.9997	.0013	-.0022	.0005
11.2	3.7321	1.0000	.0003	-.0021	.0001

$\beta = -0.2685,\ S_w = -0.8$					
η	f	f'	f"	S	S'
0	0	0	-0.0500	-0.8000	0.1829
.2	-.0009	-.0089	-.0383	-.7634	.1829
.4	-.0034	-.0152	-.0246	-.7268	.1830
.6	-.0068	-.0186	-.0090	-.6902	.1832
.8	-.0106	-.0187	.0086	-.6535	.1835
1.0	-.0140	-.0150	.0282	-.6168	.1840
1.2	-.0163	-.0072	.0499	-.5799	.1845
1.4	-.0166	.0051	.0737	-.5430	.1851
1.6	-.0139	.0224	.0994	-.5059	.1857
1.8	-.0073	.0450	.1272	-.4687	.1861
2.0	.0044	.0733	.1566	-.4315	.1862
2.2	.0224	.1077	.1872	-.3943	.1857
2.4	.0479	.1483	.2184	-.3572	.1844
2.6	.0822	.1951	.2493	-.3206	.1821
2.8	.1264	.2479	.2786	-.2845	.1783
3.0	.1817	.3063	.3049	-.2493	.1730
3.2	.2492	.3695	.3264	-.2154	.1657
3.4	.3297	.4364	.3413	-.1832	.1564
3.6	.4239	.5055	.3481	-.1530	.1451
3.8	.5320	.5750	.3455	-.1253	.1319
4.0	.6538	.6430	.3330	-.1003	.1172
4.2	.7890	.7076	.3109	-.0784	.1015
4.4	.9365	.7669	.2807	-.0598	.0854
4.6	1.0953	.8195	.2446	-.0442	.0697
4.8	1.2638	.8644	.2052	-.0318	.0551
5.0	1.4405	.9015	.1656	-.0221	.0420
5.2	1.6239	.9309	.1283	-.0148	.0309
5.4	1.8124	.9532	.0954	-.0096	.0219
5.6	2.0047	.9694	.0681	-.0059	.0150
5.8	2.1998	.9808	.0465	-.0035	.0098
6.0	2.3968	.9884	.0305	-.0019	.0062
6.2	2.5950	.9933	.0192	-.0009	.0038
6.4	2.7940	.9963	.0116	-.0003	.0022
6.6	2.9935	.9981	.0067	.0000	.0012
6.8	3.1932	.9991	.0037	.0002	.0007
7.0	3.3931	.9997	.0020	.0003	.0003
7.2	3.5931	1.0000	.0010	.0003	.0002

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = -0.3088,\ S_w = -0.8$					
η	f	f'	f''	S	S'
0	0	0	0	-0.8000	0.2261
.2	.0001	.0013	.0137	-.7548	.2260
.4	.0007	.0057	.0303	-.7096	.2260
.6	.0026	.0136	.0496	-.6644	.2260
.8	.0065	.0257	.0716	-.6192	.2258
1.0	.0132	.0425	.0963	-.5741	.2253
1.2	.0238	.0644	.1234	-.5209	.2245
1.4	.0393	.0920	.1527	-.4843	.2231
1.6	.0610	.1256	.1835	-.4399	.2209
1.8	.0900	.1654	.2151	-.3960	.2176
2.0	.1276	.2116	.2465	-.3529	.2130
2.2	.1750	.2639	.2764	-.3109	.2067
2.4	.2335	.3219	.3031	-.2704	.1984
2.6	.3041	.3848	.3247	-.2317	.1881
2.8	.3877	.4514	.3395	-.1953	.1756
3.0	.4848	.5201	.3458	-.1616	.1609
3.2	.5957	.5890	.3423	-.1311	.1445
3.4	.7203	.6563	.3286	-.1039	.1267
3.6	.8580	.7199	.3055	-.0804	.1082
3.8	1.0079	.7779	.2742	-.0606	.0898
4.0	1.1687	.8291	.2372	-.0444	.0723
4.2	1.3391	.8727	.1975	-.0316	.0563
4.4	1.5173	.9082	.1580	-.0218	.0423
4.6	1.7018	.9360	.1213	-.0145	.0306
4.8	1.8912	.9570	.0893	-.0094	.0214
5.0	2.0842	.9722	.0630	-.0058	.0144
5.2	2.2798	.9827	.0426	-.0035	.0093
5.4	2.4771	.9896	.0276	-.0020	.0058
5.6	2.6754	.9940	.0172	-.0011	.0034
5.8	2.8746	.9967	.0103	-.0006	.0020
6.0	3.0741	.9983	.0059	-.0003	.0011
6.2	3.2738	.9992	.0032	-.0001	.0006
6.4	3.4737	.9996	.0017	-.0000	.0003
6.6	3.6737	.9999	.0009	.0000	.0001
6.8	3.8737	1.0000	.0004	.0000	.0000

$\beta = -0.325,\ S_w = -0.8$					
η	f	f'	f''	S	S'
0	0	0	0.0493	-0.3250	0.2545
.2	.0011	.0113	.0640	-.7491	.2545
.4	.0047	.0258	.0819	-.6982	.2544
.6	.0117	.0442	.1029	-.6474	.2540
.8	.0227	.0672	.1269	-.5966	.2531
1.0	.0389	.0952	.1535	-.5462	.2516
1.2	.0611	.1287	.1821	-.4961	.2491
1.4	.0907	.1681	.2121	-.4466	.2454
1.6	.1288	.2135	.2423	-.3980	.2401
1.8	.1765	.2649	.2714	-.3507	.2329
2.0	.2351	.3219	.2978	-.3050	.2236
2.2	.3056	.3837	.3196	-.2614	.2118
2.4	.3889	.4493	.3349	-.2204	.1977
2.6	.4855	.5172	.3420	-.1825	.1812
2.8	.5958	.5855	.3396	-.1481	.1626
3.0	.7196	.6523	.3272	-.1176	.1426
3.2	.8565	.7157	.3052	-.0911	.1218
3.4	1.0055	.7736	.2749	-.0688	.1012
3.6	1.1656	.8253	.2388	-.0506	.0815
3.8	1.3352	.8691	.1996	-.0361	.0635
4.0	1.5127	.9052	.1604	-.0250	.0477
4.2	1.6967	.9335	.1236	-.0168	.0346
4.4	1.8857	.9549	.0914	-.0110	.0242
4.6	2.0783	.9705	.0648	-.0070	.0163
4.8	2.2735	.9812	.0440	-.0043	.0105
5.0	2.4705	.9884	.0287	-.0027	.0065
5.2	2.6687	.9930	.0179	-.0016	.0039
5.4	2.8676	.9959	.0108	-.0010	.0023
5.6	3.0670	.9975	.0062	-.0007	.0013
5.8	3.2666	.9984	.0030	-.0005	.0006
6.0	3.4664	.9991	.0024	-.0004	.0005

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = -0.3285,\ S_w = -0.8$					
η	f	f'	f"	S	S'
0	0	0	0.0693	-0.8000	0.2644
.2	.0015	.0153	.0842	-.7471	.2644
.4	.0063	.0339	.1024	-.6943	.2642
.6	.0153	.0565	.1238	-.6415	.2636
.8	.0292	.0836	.1482	-.5888	.2625
1.0	.0491	.1159	.1750	-.5365	.2605
1.2	.0760	.1538	.2036	-.4847	.2573
1.4	.1110	.1974	.2331	-.4337	.2525
1.6	.1553	.2470	.2621	-.3838	.2459
1.8	.2101	.3021	.2892	-.3355	.2371
2.0	.2765	.3624	.3125	-.2892	.2259
2.2	.3554	.4267	.3301	-.2453	.2121
2.4	.4474	.4939	.3401	-.2045	.1958
2.6	.5530	.5622	.3412	-.1671	.1772
2.8	.6722	.6297	.3324	-.1337	.1568
3.0	.8047	.6945	.3138	-.1045	.1353
3.2	.9497	.7546	.2863	-.0796	.1136
3.4	1.1061	.8085	.2520	-.0590	.0925
3.6	1.2727	.8552	.2136	-.0425	.0729
3.8	1.4477	.8939	.1741	-.0297	.0556
4.0	1.6297	.9249	.1362	-.0201	.0408
4.2	1.8172	.9487	.1022	-.0131	.0289
4.4	2.0088	.9662	.0735	-.0083	.0197
4.6	2.2033	.9785	.0507	-.0051	.0129
4.8	2.3999	.9868	.0335	-.0030	.0082
5.0	2.5978	.9922	.0212	-.0017	.0050
5.2	2.7967	.9956	.0129	-.0010	.0029
5.4	2.9960	.9976	.0075	-.0005	.0016
5.6	3.1956	.9987	.0042	-.0003	.0009
5.8	3.3954	.9994	.0023	-.0001	.0005
6.0	3.5954	.9997	.0012	-.0001	.0002
6.2	3.7953	.9999	.0006	.0000	.0001
6.4	3.9953	.9999	.0003	.0000	.0000
6.6	4.1953	1.0000	.0001	.0000	.0000
6.8	4.3953	1.0000	.0001	.0000	.0000
7.0	4.5953	1.0000	.0000	.0000	.0000
7.2	4.7953	1.0000	.0000	.0000	.0000

$\beta = -0.3285,\ S_w = -0.8$					
η	f	f'	f"	S	S
0	0	0	0.1100	-0.8000	0.2818
.2	.0023	.0234	.1250	-.7436	.2818
.4	.0096	.0502	.1434	-.6873	.2815
.6	.0226	.0810	.1650	-.6311	.2806
.8	.0423	.1164	.1893	-.5751	.2788
1.0	.0696	.1569	.2156	-.5196	.2758
1.2	.1054	.2027	.2428	-.4649	.2710
1.4	.1510	.2540	.2698	-.4114	.2642
1.6	.2074	.3105	.2948	-.3594	.2549
1.8	.2755	.3716	.3162	-.3096	.2430
2.0	.3563	.4366	.3319	-.2624	.2281
2.2	.4503	.5039	.3401	-.2185	.2105
2.4	.5579	.5720	.3395	-.1784	.1904
2.6	.6791	.6391	.3292	-.1425	.1683
2.8	.8134	.7031	.3093	-.1111	.1450
3.0	.9600	.7622	.2810	-.0845	.1215
3.2	1.1178	.8151	.2463	-.0625	.0987
3.4	1.2855	.8605	.2079	-.0449	.0776
3.6	1.4615	.8981	.1687	-.0313	.0590
3.8	1.6443	.9281	.1314	-.0211	.0432
4.0	1.8323	.9510	.0982	-.0138	.0305
4.2	2.0242	.9678	.0704	-.0087	.0207
4.4	2.2190	.9795	.0483	-.0053	.0136
4.6	2.4158	.9875	.0318	-.0032	.0085
4.8	2.6138	.9926	.0201	-.0018	.0052
5.0	2.8127	.9957	.0121	-.0010	.0030
5.2	3.0121	.9976	.0071	-.0005	.0017
5.4	3.2117	.9987	.0039	-.0003	.0009
5.6	3.4115	.9993	.0021	-.0002	.0005
5.8	3.6114	.9996	.0011	-.0001	.0002
6.0	3.8113	.9998	.0006	-.0001	.0001
6.2	4.0113	.9999	.0002	.0000	.0000

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = -0.325,\ S_w = -0.8$					
η	f	f'	f''	S	S'
0	0	0	0.1353	-0.8000	0.2913
.3379	.0080	.050	.1623	-.7015	.2911
.6210	.0289	.100	.1922	-.6192	.2897
.8632	.0590	.150	.2214	-.5494	.2866
1.0761	.0961	.200	.2487	-.4888	.2819
1.2677	.1391	.250	.2732	-.4353	.2757
1.4439	.1875	.300	.2944	-.3874	.2679
1.6087	.2410	.350	.3120	-.3440	.2587
1.7654	.2997	.400	.3258	-.3042	.2479
1.9165	.3639	.450	.3354	-.2677	.2359
2.0643	.4341	.500	.3406	-.2338	.2224
2.2108	.5110	.550	.3412	-.2023	.2075
2.3582	.5958	.600	.3366	-.1729	.1913
2.5088	.6899	.650	.3265	-.1454	.1737
2.6656	.7958	.700	.3103	-.1197	.1545
2.8327	.9170	.750	.2874	-.0956	.1339
3.0161	1.0592	.800	.2569	-.0731	.1118
3.2268	1.2332	.850	.2172	-.0521	.0878
3.4876	1.4618	.900	.1663	-.0328	.0618
3.5500	1.5182	.910	.1544	-.0291	.0563
3.6175	1.5800	.920	.1419	-.0255	.0507
3.6915	1.6484	.930	.1286	-.0219	.0450
3.7737	1.7253	.940	.1146	-.0185	.0392
3.8672	1.8136	.950	.0996	-.0151	.0332
3.9194	1.8634	.955	.0918	-.0134	.0302
3.9764	1.9180	.960	.0836	-.0118	.0271
4.0394	1.9786	.965	.0752	-.0102	.0239
4.1101	2.0470	.970	.0664	-.0086	.0208
4.1911	2.1258	.975	.0572	-.0071	.0176
4.2867	2.2192	.980	.0475	-.0056	.0143
4.4050	2.3354	.985	.0373	-.0041	.0109
4.5632	2.4917	.990	.0263	-.0026	.0074
4.6029	2.5310	.991	.0240	-.0024	.0067
4.6466	2.5743	.992	.0217	-.0021	.0060
4.6953	2.6228	.993	.0193	-.0018	.0053
4.7507	2.6777	.994	.0168	-.0015	.0046
4.8148	2.7415	.995	.0143	-.0013	.0038
4.8915	2.8178	.996	.0117	-.0010	.0031
4.9876	2.9136	.997	.0091	-.0007	.0024
5.1186	3.0443	.998	.0063	-.0005	.0016
5.3319	3.2573	.999	.0033	-.0002	.0008
6.4255	4.3506	1.000	.0000	-.0000	.0000

$\beta = -0.3,\ S_w = -0.8$					
η	f	f'	f''	S	S'
0	0	0	0.2086	-0.8000	0.3154
.2310	.0056	.050	.2248	-.7271	.3154
.4447	.0215	.100	.2435	-.6597	.3146
.6423	.0461	.150	.2629	-.5978	.3126
.8259	.0782	.200	.2819	-.5407	.3091
.9979	.1168	.250	.2995	-.4879	.3040
1.1605	.1615	.300	.3151	-.4390	.2973
1.3159	.2120	.350	.3280	-.3934	.2888
1.4660	.2682	.400	.3380	-.3507	.2786
1.6124	.3304	.450	.3445	-.3108	.2667
1.7568	.3990	.500	.3472	-.2732	.2530
1.9010	.4747	.550	.3457	-.2378	.2376
2.0468	.5586	.600	.3395	-.2044	.2204
2.1964	.6521	.650	.3282	-.1729	.2013
2.3526	.7576	.700	.3111	-.1431	.1804
2.5194	.8786	.750	.2875	-.1149	.1574
2.7030	1.0210	.800	.2565	-.0883	.1322
2.9142	1.1954	.850	.2166	-.0633	.1046
3.1759	1.4246	.900	.1657	-.0400	.0743
3.2385	1.4813	.910	.1538	-.0356	.0678
3.3063	1.5433	.920	.1413	-.0312	.0612
3.3806	1.6120	.930	.1281	-.0269	.0545
3.4631	1.6892	.940	.1141	-.0227	.0475
3.5569	1.7779	.950	.0992	-.0186	.0404
3.6094	1.8279	.955	.0914	-.0166	.0367
3.6667	1.8827	.960	.0833	-.0146	.0330
3.7299	1.9436	.965	.0749	-.0126	.0293
3.8010	2.0123	.970	.0661	-.0107	.0254
3.8823	2.0915	.975	.0569	-.0088	.0215
3.9784	2.1854	.980	.0473	-.0069	.0175
4.0972	2.3022	.985	.0371	-.0051	.0134
4.2562	2.4592	.990	.0262	-.0033	.0092
4.2961	2.4986	.991	.0239	-.0029	.0083
4.3400	2.5422	.992	.0216	-.0026	.0074
4.3890	2.5908	.993	.0192	-.0022	.0065
4.4446	2.6460	.994	.0167	-.0019	.0056
4.5090	2.7101	.995	.0142	-.0016	.0047
4.5860	2.7868	.996	.0117	-.0012	.0038
4.6827	2.8831	.997	.0090	-.0009	.0029
4.8143	3.0144	.998	.0062	-.0006	.0020
5.0287	3.2285	.999	.0033	-.0003	.0010
6.1270	4.3265	1.000	.0000	-.0000	.0000

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = -0.14$, $S_w = -0.8$

η	f	f'	f''	S	S'
0	0	0	0.3841	-0.8000	0.3590
.1294	.0032	.050	.3882	-.7535	.3590
.2575	.0128	.100	.3926	-.7075	.3590
.3841	.0286	.150	.3970	-.6622	.3580
.5094	.0505	.200	.4009	-.6174	.3560
.6337	.0784	.250	.4039	-.5734	.3530
.7572	.1124	.300	.4057	-.5300	.3490
.8804	.1524	.350	.4058	-.4873	.3440
1.0039	.1987	.400	.4039	-.4453	.3360
1.1283	.2516	.450	.3996	-.4040	.3270
1.2546	.3116	.500	.3925	-.3634	.3160
1.3837	.3795	.550	.3822	-.3235	.3020
1.5170	.4562	.600	.3682	-.2843	.2860
1.6561	.5432	.650	.3500	-.2459	.2670
1.8038	.6429	.700	.3269	-.2081	.2440
1.9636	.7589	.750	.2984	-.1711	.2190
2.1417	.8971	.800	.2633	-.1349	.1890
2.3489	1.0682	.850	.2202	-.0995	.1540
2.6077	1.2950	.900	.1669	-.0650	.1140
2.6699	1.3513	.910	.1547	-.0582	.1050
2.7375	1.4131	.920	.1419	-.0515	.0950
2.8116	1.4817	.930	.1285	-.0448	.0860
2.8941	1.5588	.940	.1143	-.0381	.0760
2.9879	1.6475	.950	.0992	-.0316	.0650
3.0405	1.6976	.955	.0913	-.0283	.0600
3.0979	1.7526	.960	.0831	-.0250	.0540
3.1614	1.8138	.965	.0747	-.0218	.0480
3.2328	1.8828	.970	.0659	-.0186	.0420
3.3145	1.9623	.975	.0567	-.0154	.0360
3.4110	2.0566	.980	.0471	-.0122	.0300
3.5305	2.1741	.985	.0369	-.0091	.0230
3.6905	2.3320	.990	.0260	-.0060	.0160
3.7306	2.3718	.991	.0237	-.0054	.0150
3.7749	2.4158	.992	.0214	-.0047	.0130
3.8243	2.4647	.993	.0191	-.0041	.0120
3.8803	2.5204	.994	.0166	-.0035	.0100
3.9452	2.5849	.995	.0142	-.0029	.0090
4.0228	2.6622	.996	.0116	-.0023	.0070
4.1201	2.7591	.997	.0090	-.0017	.0050
4.2524	2.8912	.998	.0062	-.0011	.0040
4.4678	3.1062	.999	.0033	-.0005	.0020
5.5574	4.1955	1.000	.0000	.0000	.0000

$\beta = 0.5$, $S_w = -0.8$

η	f	f'	f''	S	S'
0	0	0	0.6546	-0.8000	0.403
.0768	.0019	.050	.6464	-.7690	.403
.1547	.0077	.100	.6368	-.7375	.403
.2339	.0177	.150	.6258	-.7056	.403
.3147	.0318	.200	.6133	-.6731	.402
.3971	.0504	.250	.5993	-.6400	.401
.4817	.0737	.300	.5835	-.6063	.399
.5687	.1020	.350	.5659	-.5717	.396
.6586	.1357	.400	.5463	-.5363	.391
.7520	.1755	.450	.5246	-.5000	.386
.8496	.2219	.500	.5004	-.4627	.379
.9524	.2759	.550	.4737	-.4243	.369
1.0614	.3387	.600	.4441	-.3848	.357
1.1784	.4119	.650	.4111	-.3439	.342
1.3058	.4979	.700	.3743	-.3016	.323
1.4472	.6006	.750	.3333	-.2577	.298
1.6086	.7258	.800	.2870	-.2119	.268
1.8010	.8848	.850	.2345	-.1641	.230
2.0469	1.1003	.900	.1737	-.1137	.180
2.1069	1.1546	.910	.1603	-.1032	.169
2.1723	1.2144	.920	.1463	-.0926	.156
2.2444	1.2811	.930	.1318	-.0819	.143
2.3250	1.3565	.940	.1166	-.0710	.128
2.4171	1.4436	.950	.1007	-.0599	.113
2.4690	1.4930	.955	.0925	-.0543	.104
2.5257	1.5473	.960	.0840	-.0486	.096
2.5887	1.6079	.965	.0752	-.0429	.087
2.6596	1.6766	.970	.0662	-.0371	.077
2.7410	1.7557	.975	.0568	-.0312	.067
2.8375	1.8501	.980	.0470	-.0252	.057
2.9573	1.9678	.985	.0367	-.0192	.045
3.1183	2.1268	.990	.0258	-.0130	.032
3.1588	2.1669	.991	.0235	-.0117	.030
3.2034	2.2111	.992	.0212	-.0105	.027
3.2532	2.2606	.993	.0189	-.0092	.024
3.3098	2.3168	.994	.0165	-.0079	.021
3.3755	2.3821	.995	.0140	-.0066	.018
3.4540	2.4603	.996	.0115	-.0053	.015
3.5526	2.5586	.997	.0088	-.0040	.012
3.6869	2.6925	.998	.0061	-.0027	.008
3.9056	2.9109	.999	.0032	-.0013	.004
5.0257	4.0307	1.000	.0000	.0000	.000

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = 1.5,\ S_w = -0.8$					
η	f	f'	f''	S	S'
0	0	0	0.8689	-0.8000	0.4261
.0581	.0014	.050	.8504	-.7752	.4261
.1176	.0059	.100	.8296	-.7498	.4260
.1788	.0136	.150	.8065	-.7238	.4258
.2418	.0246	.200	.7812	-.6970	.4253
.3069	.0393	.250	.7537	-.6693	.4245
.3746	.0580	.300	.7240	-.6406	.4231
.4453	.0810	.350	.6922	-.6108	.4211
.5194	.1088	.400	.6582	-.5797	.4182
.5975	.1421	.450	.6221	-.5471	.4142
.6805	.1815	.500	.5838	-.5130	.4088
.7694	.2283	.550	.5432	-.4770	.4015
.8653	.2836	.600	.5003	-.4389	.3919
.9701	.3491	.650	.4547	-.3985	.3792
1.0864	.4277	.700	.4064	-.3554	.3625
1.2179	.5232	.750	.3549	-.3092	.3406
1.3711	.6420	.800	.2998	-.2592	.3117
1.5572	.7958	.850	.2402	-.2049	.2729
1.7999	1.0085	.900	.1744	-.1452	.2195
1.8597	1.0627	.910	.1603	-.1324	.2063
1.9252	1.1226	.920	.1458	-.1194	.1921
1.9977	1.1897	.930	.1308	-.1061	.1767
2.0790	1.2657	.940	.1154	-.0924	.1599
2.1724	1.3540	.950	.0993	-.0784	.1416
2.2250	1.4041	.955	.0910	-.0712	.1317
2.2827	1.4594	.960	.0825	-.0639	.1213
2.3469	1.5212	.965	.0738	-.0565	.1103
2.4193	1.5912	.970	.0648	-.0489	.0986
2.5025	1.6721	.975	.0555	-.0413	.0861
2.6013	1.7687	.980	.0459	-.0335	.0726
2.7240	1.8893	.985	.0358	-.0255	.0580
2.8891	2.0524	.990	.0252	-.0173	.0419
2.9307	2.0935	.991	.0229	-.0156	.0385
2.9764	2.1389	.992	.0207	-.0139	.0349
3.0276	2.1897	.993	.0184	-.0122	.0313
3.0856	2.2473	.994	.0160	-.0105	.0275
3.1530	2.3144	.995	.0136	-.0088	.0236
3.2336	2.3946	.996	.0112	-.0071	.0195
3.3347	2.4953	.997	.0086	-.0054	.0152
3.4724	2.6327	.998	.0060	-.0036	.0107
3.6964	2.8564	.999	.0032	-.0018	.0058
4.8362	3.9959	1.000	.0000	.0000	.0000

$\beta = 2.0,\ S_w = -0.8$					
η	f	f'	f''	S	S'
0	0	0	0.9480	-0.8000	0.4331
.0533	.0013	.050	.9255	-.7768	.4331
.1081	.0054	.100	.9003	-.7531	.4330
.1645	.0125	.150	.8724	-.7287	.4329
.2229	.0227	.200	.8421	-.7035	.4324
.2834	.0364	.250	.8094	-.6773	.4317
.3466	.0538	.300	.7743	-.6501	.4305
.4128	.0754	.350	.7370	-.6216	.4287
.4826	.1016	.400	.6975	-.5918	.4261
.5565	.1331	.450	.6558	-.5604	.4225
.6355	.1706	.500	.6121	-.5273	.4175
.7205	.2153	.550	.5662	-.4921	.4108
.8129	.2686	.600	.5181	-.4546	.4019
.9144	.3321	.650	.4678	-.4144	.3898
1.0278	.4087	.700	.4152	-.3711	.3738
1.1570	.5025	.750	.3600	-.3242	.3525
1.3086	.6202	.800	.3019	-.2729	.3239
1.4941	.7735	.850	.2401	-.2166	.2848
1.7377	.9870	.900	.1732	-.1539	.2299
1.7980	1.0415	.910	.1590	-.1405	.2163
1.8640	1.1020	.920	.1445	-.1267	.2016
1.9372	1.1697	.930	.1296	-.1125	.1856
2.0193	1.2465	.940	.1142	-.0980	.1680
2.1137	1.3357	.950	.0982	-.0831	.1488
2.1670	1.3864	.955	.0900	-.0754	.1384
2.2254	1.4424	.960	.0815	-.0677	.1274
2.2903	1.5048	.965	.0729	-.0598	.1158
2.3636	1.5758	.970	.0640	-.0518	.1035
2.4477	1.6576	.975	.0549	-.0437	.0904
2.5476	1.7553	.980	.0454	-.0353	.0762
2.6718	1.8773	.985	.0354	-.0269	.0608
2.8387	2.0421	.990	.0249	-.0182	.0438
2.8806	2.0837	.991	.0227	-.0164	.0402
2.9269	2.1295	.992	.0205	-.0146	.0365
2.9785	2.1808	.993	.0182	-.0129	.0326
3.0371	2.2390	.994	.0159	-.0111	.0288
3.1051	2.3066	.995	.0135	-.0093	.0246
3.1864	2.3875	.996	.0111	-.0074	.0203
3.2883	2.4891	.997	.0086	-.0056	.0158
3.4270	2.6274	.998	.0059	-.0037	.0111
3.6526	2.8527	.999	.0031	-.0019	.0060
4.7958	3.9956	1.000	.0000	.0000	.0000

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = -0.2350$, $S_w = -0.4$					
η	f	f'	f''	S	S'
0	0	0	-0.0500	-0.4000	0.1107
.2	-.0008	-.0071	-.0213	-.3779	.1107
.4	-.0025	-.0084	.0085	-.3557	.1107
.6	-.0038	-.0037	.0393	-.3336	.1108
.8	-.0035	-.0073	.0712	-.3114	.1108
1.0	-.0004	.0249	.1041	-.2893	.1109
1.2	.0069	.0490	.1379	-.2671	.1108
1.4	.0197	.0800	.1722	-.2449	.1106
1.6	.0393	.1179	.2067	-.2229	.1099
1.8	.0673	.1627	.2404	-.2010	.1088
2.0	.1048	.2140	.2724	-.1794	.1069
2.2	.1533	.2714	.3013	-.1583	.1042
2.4	.2138	.3342	.3256	-.1378	.1005
2.6	.2873	.4012	.3434	-.1182	.0956
2.8	.3745	.4710	.3533	-.0996	.0895
3.0	.4757	.5419	.3539	-.0825	.0822
3.2	.5912	.6119	.3445	-.0668	.0739
3.4	.7203	.6791	.3252	-.0529	.0649
3.6	.8625	.7414	.2970	-.0409	.0554
3.8	1.0165	.7974	.2621	-.0308	.0459
4.0	1.1809	.8459	.2229	-.0225	.0369
4.2	1.3543	.8865	.1825	-.0160	.0286
4.4	1.5350	.9190	.1436	-.0110	.0214
4.6	1.7214	.9442	.1085	-.0073	.0155
4.8	1.9122	.9628	.0786	-.0048	.0107
5.0	2.1062	.9760	.0547	-.0030	.0072
5.2	2.3024	.9850	.0365	-.0018	.0046
5.4	2.5000	.9910	.0234	-.0011	.0029
5.6	2.6986	.9947	.0144	-.0006	.0017
5.8	2.8978	.9969	.0085	-.0003	.0010
6.0	3.0973	.9982	.0048	-.0002	.0005
6.2	3.2970	.9989	.0027	-.0001	.0003
6.4	3.4969	.9993	.0014	-.0001	.0001
6.6	3.6968	.9995	.0007	-.0001	.0001
6.8	3.8967	.9997	.0003	.0000	.0000

$\beta = -0.2460$, $S_w = -0.4$					
η	f	f'	f''	S	S'
0	0	0	0	-0.4000	0.1249
.2	.0002	.0030	.0301	-.3750	.1249
.4	.0016	.0121	.0615	-.3500	.1249
.6	.0055	.0277	.0940	-.3251	.1248
.8	.0131	.0498	.1276	-.3001	.1246
1.0	.0259	.0787	.1618	-.2752	.1241
1.2	.0451	.1146	.1964	-.2504	.1233
1.4	.0721	.1573	.2305	-.2260	.1218
1.6	.1084	.2067	.2631	-.2018	.1197
1.8	.1552	.2623	.2929	-.1782	.1166
2.0	.2137	.3235	.3184	-.1552	.1124
2.2	.2849	.3893	.3379	-.1333	.1069
2.4	.3696	.4582	.3497	-.1126	.1002
2.6	.4683	.5285	.3524	-.0933	.0922
2.8	.5810	.5985	.3452	-.0758	.0830
3.0	.7075	.6660	.3280	-.0602	.0730
3.2	.8471	.7291	.3017	-.0466	.0625
3.4	.9988	.7861	.2681	-.0352	.0520
3.6	1.1611	.8360	.2298	-.0258	.0419
3.8	1.3326	.8780	.1896	-.0184	.0327
4.0	1.5118	.9119	.1504	-.0127	.0246
4.2	1.6969	.9384	.1146	-.0084	.0178
4.4	1.8867	.9581	.0839	-.0054	.0124
4.6	2.0798	.9723	.0589	-.0034	.0084
4.8	2.2753	.9821	.0397	-.0020	.0054
5.0	2.4724	.9885	.0257	-.0011	.0034
5.2	2.6705	.9926	.0160	-.0006	.0020
5.4	2.8693	.9952	.0096	-.0003	.0012
5.6	3.0685	.9967	.0056	-.0001	.0006
5.8	3.2679	.9975	.0032	.0000	.0003
6.0	3.4675	.9980	.0018	0000	0002

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = -0.2483,\ S_w = -0.4$					
η	f	f'	f''	S	S'
0	0	0	0.0500	-0.4000	0.1360
.2	.0012	.0130	.0805	-.3728	.1360
.4	.0056	.0323	.1122	-.3456	.1359
.6	.0145	.0580	.1450	-.3184	.1357
.8	.0293	.0903	.1786	-.2913	.1351
1.0	.0511	.1294	.2123	-.2644	.1340
1.2	.0815	.1752	.2453	-.2378	.1323
1.4	.1216	.2274	.2765	-.2116	.1296
1.6	.1728	.2856	.3045	-.1860	.1259
1.8	.2362	.3489	.3276	-.1613	.1209
2.0	.3126	.4162	.3441	-.1377	.1145
2.2	.4028	.4860	.3524	-.1156	.1066
2.4	.5071	.5565	.3513	-.0952	.0973
2.6	.6254	.6258	.3402	-.0768	.0869
2.8	.7572	.6919	.3193	-.0605	.0757
3.0	.9018	.7530	.2899	-.0465	.0641
3.2	1.0580	.8075	.2541	-.0348	.0528
3.4	1.2243	.8544	.2146	-.0253	.0420
3.6	1.3992	.8933	.1744	-.0179	.0323
3.8	1.5811	.9243	.1362	-.0123	.0240
4.0	1.7684	.9480	.1021	-.0082	.0172
4.2	1.9598	.9654	.0734	-.0054	.0118
4.4	2.1542	.9778	.0506	-.0034	.0078
4.6	2.3507	.9861	.0335	-.0022	.0050
4.8	2.5485	.9915	.0213	-.0014	.0031
5.0	2.7471	.9949	.0130	-.0009	.0018
5.2	2.9463	.9969	.0076	-.0006	.0010
5.4	3.1458	.9980	.0043	-.0005	.0006
5.6	3.3455	.9987	.0024	-.0004	.0003
5.8	3.5453	.9991	.0013	-.0003	.0001
6.0	3.7451	.9992	.0006	-.0003	.0001
6.2	3.9450	.9993	.0004	-.0003	.0000
6.4	4.1448	.9994	.0000	-.0003	.0000

$\beta = -0.24,\ S_w = -0.4$					
η	f	f'	f''	S	S'
0	0	0	0.1064	-0.4000	0.1473
.3766	.0083	.050	.1628	-.3447	.1460
.6490	.0284	.100	.2056	-.3050	.1452
.8732	.0562	.150	.2409	-.2726	.1438
1.0687	.0903	.200	.2707	-.2447	.1418
1.2453	.1299	.250	.2956	-.2199	.1390
1.4087	.1748	.300	.3160	-.1974	.1356
1.5628	.2248	.350	.3322	-.1768	.1315
1.7106	.2802	.400	.3442	-.1577	.1267
1.8541	.3412	.450	.3518	-.1399	.1212
1.9955	.4083	.500	.3549	-.1232	.1150
2.1366	.4824	.550	.3533	-.1074	.1080
2.2793	.5644	.600	.3466	-.0926	.1002
2.4259	.6561	.650	.3346	-.0785	.0916
2.5793	.7596	.700	.3166	-.0651	.0822
2.7433	.8786	.750	.2921	-.0525	.0719
2.9242	1.0189	.800	.2600	-.0405	.0606
3.1328	1.1912	.850	.2191	-.0292	.0481
3.3918	1.4181	.900	.1672	-.0186	.0343
3.4538	1.4742	.910	.1552	-.0165	.0314
3.5210	1.5357	.920	.1425	-.0145	.0283
3.5947	1.6039	.930	.1291	-.0126	.0252
3.6767	1.6806	.940	.1149	-.0106	.0221
3.7698	1.7686	.950	.0999	-.0087	.0188
3.8220	1.8183	.955	.0920	-.0078	.0171
3.8789	1.8728	.960	.0838	-.0068	.0154
3.9418	1.9333	.965	.0753	-.0059	.0136
4.0125	2.0017	.970	.0664	-.0050	.0119
4.0934	2.0805	.975	.0572	-.0041	.0101
4.1891	2.1740	.980	.0475	-.0033	.0082
4.3074	2.2902	.985	.0372	-.0024	.0063
4.4658	2.4467	.990	.0263	-.0016	.0043
4.5056	2.4861	.991	.0240	-.0014	.0039
4.5493	2.5295	.992	.0216	-.0012	.0035
4.5982	2.5780	.993	.0192	-.0011	.0031
4.6536	2.6330	.994	.0168	-.0009	.0027
4.7179	2.6970	.995	.0143	-.0008	.0022
4.7947	2.7735	.996	.0117	-.0006	.0018
4.8912	2.8696	.997	.0090	-.0004	.0014
5.0226	3.0006	.998	.0062	-.0003	.0009
5.2317	3.2144	.999	.0049	-.0001	.0004
6.5567	4.5414	1.000	.0000	-.0000	.0000

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = -0.2,\ S_w = -0.4$					
η	f	f'	f''	S	S'
0	0	0	0.2182	-0.4000	0.1626
.2161	.0052	.050	.2447	-.3648	.1625
.4108	.0197	.100	.2692	-.3332	.1621
.5892	.0419	.150	.2914	-.3044	.1612
.7551	.0709	.200	.3111	-.2778	.1597
.9115	.1060	.250	.3281	-.2529	.1575
1.0606	.1470	.300	.3422	-.2297	.1546
1.2044	.1937	.350	.3531	-.2077	.1501
1.3444	.2461	.400	.3607	-.1869	.1463
1.4821	.3047	.450	.3646	-.1671	.1409
1.6191	.3697	.500	.3647	-.1482	.1345
1.7569	.4421	.550	.3606	-.1302	.1272
1.8971	.5227	.600	.3519	-.1129	.1189
2.0419	.6132	.650	.3381	-.0964	.1096
2.1939	.7159	.700	.3188	-.0805	.0990
2.3571	.8343	.750	.2932	-.0653	.0873
2.5375	.9742	.800	.2604	-.0508	.0742
2.7460	1.1464	.850	.2190	-.0369	.0595
3.0054	1.3737	.900	.1668	-.0236	.0429
3.0676	1.4300	.910	.1548	-.0211	.0393
3.1350	1.4916	.920	.1421	-.0186	.0356
3.2089	1.5600	.930	.1287	-.0161	.0318
3.2911	1.6369	.940	.1146	-.0136	.0279
3.3846	1.7253	.950	.0995	-.0112	.0239
3.4369	1.7751	.955	.0916	-.0100	.0218
3.4941	1.8298	.960	.0835	-.0088	.0196
3.5572	1.8906	.965	.0750	-.0077	.0175
3.6281	1.9592	.970	.0662	-.0065	.0152
3.7094	2.0382	.975	.0570	-.0054	.0129
3.8054	2.1321	.980	.0473	-.0042	.0106
3.9240	2.2486	.985	.0373	-.0031	.0082
4.0829	2.4056	.990	.0262	-.0020	.0057
4.1228	2.4451	.991	.0239	-.0018	.0051
4.1668	2.4887	.992	.0216	-.0016	.0046
4.2158	2.5373	.993	.0192	-.0014	.0041
4.2714	2.5926	.994	.0167	-.0012	.0035
4.3358	2.6567	.995	.0142	-.0010	.0029
4.4129	2.7334	.996	.0117	-.0008	.0024
4.5096	2.8297	.997	.0090	-.0005	.0019
4.6411	2.9609	.998	.0062	-.0003	.0013
4.8545	3.1740	.999	.0033	-.0001	.0007
5.9279	4.2471	1.000	.0000	.0000	.0000

$\beta = 0.5,\ S_w = -0.4$					
η	f	f'	f''	S	S'
0	0	0	0.7946	-0.4000	0.209
.0637	.0016	.050	.7753	-.3866	.209
.1290	.0065	.100	.7551	-.3730	.209
.1962	.0149	.150	.7339	-.3590	.209
.2654	.0270	.200	.7116	-.3445	.208
.3368	.0431	.250	.6881	-.3296	.208
.4108	.0635	.300	.6632	-.3143	.207
.4878	.0886	.350	.6369	-.2984	.206
.5681	.1187	.400	.6090	-.2819	.204
.6523	.1546	.450	.5793	-.2648	.202
.7411	.1968	.500	.5477	-.2470	.199
.8354	.2464	.550	.5138	-.2284	.195
.9364	.3046	.600	.4774	-.2090	.190
1.0456	.3729	.650	.4381	-.1887	.183
1.1657	.4540	.700	.3956	-.1673	.174
1.3001	.5516	.750	.3493	-.1447	.163
1.4548	.6716	.800	.2984	-.1207	.148
1.6406	.8252	.850	.2417	-.0950	.129
1.8804	1.0353	.900	.1775	-.0672	.103
1.9391	1.0885	.910	.1635	-.0613	.097
2.0032	1.1471	.920	.1490	-.0554	.090
2.0741	1.2127	.930	.1339	-.0492	.083
2.1535	1.2870	.940	.1183	-.0430	.075
2.2445	1.3730	.950	.1020	-.0365	.067
2.2957	1.4218	.955	.0935	-.0332	.062
2.3519	1.4756	.960	.0848	-.0299	.057
2.4143	1.5356	.965	.0759	-.0265	.052
2.4846	1.6037	.970	.0667	-.0230	.047
2.5654	1.6823	.975	.0572	-.0195	.041
2.6614	1.7761	.980	.0472	-.0159	.035
2.7807	1.8933	.985	.0369	-.0122	.028
2.9412	2.0518	.990	.0259	-.0083	.020
2.9816	2.0918	.991	.0236	-.0075	.019
3.0261	2.1360	.992	.0213	-.0068	.017
3.0759	2.1854	.993	.0189	-.0060	.015
3.1324	2.2416	.994	.0165	-.0051	.013
3.1980	2.3068	.995	.0140	-.0043	.012
3.2766	2.3850	.996	.0115	-.0035	.010
3.3752	2.4833	.997	.0088	-.0026	.008
3.5097	2.6174	.998	.0061	-.0018	.005
3.7288	2.8362	.999	.0032	-.0009	.003
4.8386	3.9458	1.000	.0000	.0000	.000

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = 2.0,\ S_w = -0.4$

η	f	f'	f''	S	S'
0	0	0	1.3329	-0.4000	0.230
.0381	.0009	.050	1.2868	-.3912	.230
.0777	.0039	.100	1.2386	-.3820	.230
.1190	.0091	.150	1.1884	-.3725	.230
.1620	.0166	.200	1.1360	-.3626	.230
.2071	.0268	.250	1.0816	-.3523	.230
.2546	.0399	.300	1.0252	-.3413	.229
.3049	.0563	.350	.9668	-.3356	.229
.3583	.0764	.400	.9064	-.3176	.228
.4155	.1007	.450	.8440	-.3046	.227
.4772	.1301	.500	.7797	-.2906	.225
.5443	.1654	.550	.7134	-.2756	.223
.6181	.2079	.600	.6450	-.2592	.220
.7002	.2593	.650	.5746	-.2413	.216
.7932	.3222	.700	.5021	-.2215	.210
.9011	.4005	.750	.4276	-.1992	.202
1.0302	.5007	.800	.3507	-.1738	.191
1.1921	.6345	.850	.2714	-.1442	.174
1.4116	.8269	.900	.1890	-.1087	.148
1.4671	.8772	.910	.1721	-.1007	.141
1.5284	.9333	.920	.1549	-.0922	.134
1.5970	.9967	.930	.1376	-.0833	.125
1.6748	1.0695	.940	.1200	-.0739	.116
1.7651	1.1549	.950	.1021	-.0640	.104
1.8164	1.2038	.955	.0930	-.0587	.098
1.8731	1.2581	.960	.0838	-.0533	.092
1.9365	1.3191	.965	.0744	-.0477	.085
2.0086	1.3888	.970	.0649	-.0419	.077
2.0919	1.4698	.975	.0553	-.0358	.068
2.1914	1.5672	.980	.0454	-.0295	.058
2.3161	1.6897	.985	.0352	-.0229	.048
2.4850	1.8565	.990	.0245	-.0158	.035
2.5276	1.8987	.991	.0223	-.0144	.032
2.5747	1.9454	.992	.0201	-.0129	.030
2.6274	1.9977	.993	.0178	-.0114	.027
2.6873	2.0572	.994	.0155	-.0099	.024
2.7569	2.1264	.995	.0132	-.0083	.020
2.8402	2.2093	.996	.0108	-.0067	.017
2.9449	2.3136	.997	.0083	-.0051	.013
3.0875	2.4559	.998	.0058	-.0034	.009
3.3196	2.6877	.999	.0030	-.0017	.005
4.4979	3.8657	1.000	.0000	.0000	.000

$\beta = -0.1947,\ S_w = 0$

η	f	f'	f''
0	0	0	-0.0500
.2	-.0007	-.0061	-.0111
.4	-.0019	-.0044	.0279
.6	-.0020	.0051	.0669
.8	.0006	.0223	.1058
1.0	.0075	.0474	.1446
1.2	.0201	.0801	.1829
1.4	.0400	.1205	.2203
1.6	.0688	.1681	.2559
1.8	.1077	.2226	.2885
2.0	.1582	.2833	.3169
2.2	.2214	.3490	.3395
2.4	.2981	.4186	.3547
2.6	.3889	.4903	.3610
2.8	.4942	.5623	.3574
3.0	.6138	.6326	.3436
3.2	.7470	.6991	.3202
3.4	.8930	.7601	.2885
3.6	1.0506	.8141	.2510
3.8	1.2182	.8602	.2104
4.0	1.3941	.8983	.1697
4.2	1.5769	.9284	.1316
4.4	1.7650	.9512	.0980
4.6	1.9570	.9679	.0700
4.8	2.1518	.9796	.0480
5.0	2.3486	.9875	.0315
5.2	2.5467	.9926	.0199
5.4	2.7455	.9957	.0121
5.6	2.9448	.9976	.0070
5.8	3.1444	.9986	.0039
6.0	3.3443	.9992	.0021
6.2	3.5442	.9995	.0011
6.4	3.7441	.9997	.0006
6.6	3.9440	.9998	.0002
6.8	4.1440	.9998	.0002

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = -0.1$, $S_w = 1.0$					
η	f	f'	f''	S	S'
0	0	0	-0.1613	1.0000	-0.2076
.2	-.0030	-.0263	-.1217	.9585	-.2076
.4	-.0108	-.0488	-.0831	.9169	-.2079
.6	-.0220	-.0616	-.0455	.8753	-.2086
.8	-.0349	-.0670	-.0086	.8335	-.2093
1.0	-.0483	-.0651	.0276	.7913	-.2115
1.2	-.0605	-.0560	.0635	.7488	-.2138
1.4	-.0702	-.0397	.0990	.7058	-.2167
1.6	-.0759	-.0164	.1344	.6621	-.2199
1.8	-.0763	.0140	.1695	.6178	-.2233
2.0	-.0699	.0514	.2042	.5728	-.2266
2.2	-.0553	.0956	.2378	.5272	-.2295
2.4	-.0312	.1464	.2699	.4811	-.2315
2.6	.0037	.2034	.2992	.4347	-.2322
2.8	.0506	.2658	.3247	.3884	-.2310
3.0	.1104	.3329	.3448	.3425	-.2273
3.2	.1839	.4033	.3582	.2976	-.2208
3.4	.2718	.4756	.3634	.2544	-.2110
3.6	.3742	.5481	.3595	.2135	-.1978
3.8	.4909	.6188	.3461	.1755	-.1815
4.0	.6215	.6859	.3236	.1410	-.1624
4.2	.7649	.7477	.2931	.1106	-.1414
4.4	.9201	.8028	.2568	.0845	-.1196
4.6	1.0855	.8502	.2172	.0628	-.0979
4.8	1.2597	.8896	.1771	.0453	-.0774
5.0	1.4409	.9211	.1390	.0317	-.0591
5.2	1.6277	.9455	.1048	.0215	-.0435
5.4	1.8186	.9635	.0760	.0141	-.0308
5.6	2.0127	.9763	.0529	.0090	-.0210
5.8	2.2089	.9850	.0354	.0056	-.0137
6.0	2.4065	.9908	.0227	.0033	-.0087
6.2	2.6050	.9944	.0140	.0020	-.0052
6.4	2.8041	.9966	.0083	.0012	-.0031
6.6	3.0036	.9978	.0048	.0007	-.0017
6.8	3.2033	.9986	.0026	.0004	-.0009
7.0	3.4030	.9990	.0014	.0003	-.0005
7.2	3.6028	.9992	.0007	.0002	-.0002
7.4	3.8027	.9993	.0004	.0002	-.0001
7.6	4.0025	.9993	.0003	.0002	-.0001
7.8	4.2024	.9994	.0001	.0002	.0000

$\beta = -0.1305$, $S_w = 1.0$					
η	f	f'	f''	S	S'
0	0	0	-0.0500	1.0000	-0.3139
.2	-.0007	-.0048	.0014	.9372	-.3139
.4	-.0013	.0004	.0511	.8744	-.3139
.6	.0002	.0155	.0992	.8117	-.3140
.8	.0056	.0400	.1456	.7489	-.3138
1.0	.0168	.0736	.1900	.6862	-.3132
1.2	.0356	.1159	.2319	.6237	-.3116
1.4	.0637	.1661	.2704	.5616	-.3085
1.6	.1025	.2237	.3046	.5004	-.3035
1.8	.1536	.2876	.3331	.4404	-.2959
2.0	.2179	.3565	.3545	.3822	-.2852
2.2	.2964	.4268	.3672	.3266	-.2709
2.4	.3896	.5028	.3702	.2741	-.2530
2.6	.4975	.5762	.3627	.2256	-.2316
2.8	.6199	.6472	.3449	.1817	-.2072
3.0	.7560	.7135	.3176	.1428	-.1806
3.2	.9049	.7737	.2828	.1095	-.1530
3.4	1.0650	.8263	.2430	.0816	-.1257
3.6	1.2349	.8707	.2012	.0591	-.0999
3.8	1.4128	.9068	.1603	.0414	-.0767
4.0	1.5971	.9351	.1227	.0282	-.0567
4.2	1.7863	.9563	.0903	.0186	-.0404
4.4	1.9792	.9716	.0637	.0118	-.0277
4.6	2.1746	.9822	.0432	.0072	-.0183
4.8	2.3718	.9892	.0281	.0043	-.0116
5.0	2.5701	.9937	.0175	.0025	-.0071
5.2	2.7692	.9964	.0105	.0014	-.0042
5.4	2.9686	.9981	.0061	.0007	-.0024
5.6	3.1684	.9990	.0034	.0004	-.0013
5.8	3.3682	.9995	.0018	.0002	-.0007
6.0	3.5681	.9997	.0009	.0001	-.0003
6.2	3.7681	.9999	.0005	.0000	-.0001
6.4	3.9681	.9999	.0002	.0000	-.0001

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = -0.1295$, $S_w = 1.0$					
η	f	f'	f"	S	S'
0	0	0	0	1.0000	-0.3389
.2	.0003	.0051	.0509	.9322	-.3389
.4	.0027	.0203	.1001	.8645	-.3388
.6	.0091	.0450	.1473	.7967	-.3384
.8	.0213	.0790	.1924	.7291	-.3374
1.0	.0413	.1218	.2347	.6618	-.3354
1.2	.0706	.1727	.2735	.5951	-.3317
1.4	.1108	.2309	.3076	.5293	-.3258
1.6	.1634	.2953	.3358	.4650	-.3170
1.8	.2293	.3647	.3565	.4027	-.3049
2.0	.3094	.4373	.3684	.3433	-.2890
2.2	.4043	.5114	.3702	.2874	-.2691
2.4	.5139	.5847	.3615	.2359	-.2456
2.6	.6380	.6553	.3425	.1894	-.2189
2.8	.7757	.7211	.3141	.1485	-.1901
3.0	.9260	.7804	.2784	.1134	-.1604
3.2	1.0874	.8321	.2382	.0843	-.1312
3.4	1.2583	.8756	.1963	.0608	-.1038
3.6	1.4371	.9107	.1556	.0426	-.0793
3.8	1.6221	.9381	.1185	.0289	-.0584
4.0	1.8118	.9585	.0867	.0189	-.0414
4.2	2.0051	.9731	.0609	.0120	-.0282
4.4	2.2008	.9832	.0410	.0074	-.0185
4.6	2.3981	.9899	.0265	.0044	-.0117
4.8	2.5966	.9941	.0165	.0026	-.0071
5.0	2.7957	.9967	.0098	.0015	-.0042
5.2	2.9952	.9982	.0056	.0009	-.0023
5.4	3.1949	.9991	.0031	.0005	-.0013
5.6	3.3948	.9995	.0017	.0003	-.0007
5.8	3.5947	.9998	.0008	.0002	-.0003
6.0	3.7947	.9999	.0004	.0002	-.0001
6.2	3.9947	1.0000	.0001	.0002	-.0001
6.4	4.1947	1.0000	.0003	.0001	-.0001

$\beta = -0.1$, $S_w = 1.0$					
η	f	f'	f"	S	S'
0	0	0	0.1805	1.0000	-0.4033
.2451	.0057	.050	.2284	.9012	-.4027
.4477	.0207	.100	.2654	.8197	-.4016
.6260	.0429	.150	.2954	.7482	-.3993
.7885	.0712	.200	.3199	.6836	-.3956
.9400	.1053	.250	.3398	.6241	-.3904
1.0837	.1448	.300	.3554	.5684	-.3834
1.2220	.1897	.350	.3670	.5160	-.3747
1.3568	.2402	.400	.3745	.4662	-.3640
1.4896	.2967	.450	.3778	.4186	-.3513
1.6220	.3595	.500	.3769	.3731	-.3364
1.7555	.4296	.550	.3716	.3293	-.3192
1.8918	.5080	.600	.3615	.2872	-.2994
2.0329	.5962	.650	.3463	.2465	-.2770
2.1815	.6966	.700	.3255	.2071	-.2517
2.3416	.8128	.750	.2985	.1691	-.2231
2.5192	.9505	.800	.2642	.1324	-.1908
2.7250	1.1204	.850	.2215	.0969	-.1542
2.9818	1.3455	.900	.1682	.0628	-.1124
3.0436	1.4013	.910	.1560	.0561	-.1033
3.1105	1.4625	.920	.1431	.0496	-.0938
3.1839	1.5304	.930	.1295	.0430	-.0841
3.2656	1.6069	.940	.1152	.0366	-.0739
3.3586	1.6948	.950	.1000	.0302	-.0634
3.4107	1.7444	.955	.0920	.0270	-.0580
3.4676	1.7989	.960	.0838	.0239	-.0524
3.5305	1.8594	.965	.0753	.0208	-.0467
3.6011	1.9278	.970	.0664	.0177	-.0409
3.6821	2.0066	.975	.0571	.0146	-.0348
3.7779	2.1002	.980	.0474	.0116	-.0286
3.8964	2.2166	.985	.0372	.0086	-.0221
4.0551	2.3734	.990	.0262	.0056	-.0154
4.0950	2.4128	.991	.0239	.0050	-.0140
4.1388	2.4563	.992	.0216	.0045	-.0126
4.1878	2.5049	.993	.0192	.0039	-.0111
4.2433	2.5601	.994	.0168	.0033	-.0097
4.3077	2.6241	.995	.0143	.0027	-.0082
4.3847	2.7008	.996	.0117	.0022	-.0066
4.4813	2.7971	.997	.0090	.0016	-.0051
4.6129	2.9283	.998	.0062	.0010	-.0035
4.8269	3.1420	.999	.0033	.0005	-.0018
5.9066	4.2215	1.000	.0000	.0000	.0000

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = 0.3,\ S_w = 1.0$					
η	f	f'	f''	S	S'
0	0	0	0.9829	1.0000	-0.5457
.1	.0048	.0953	.9237	.9454	-.5456
.2	.0189	.1848	.8657	.8909	-.5450
.3	.0416	.2685	.8089	.8365	-.5434
.4	.0724	.3466	.7531	.7823	-.5403
.5	.1107	.4192	.6983	.7285	-.5354
.6	.1560	.4863	.6445	.6752	-.5284
.7	.2078	.5481	.5919	.6229	-.5189
.8	.2655	.6047	.5406	.5716	-.5067
.9	.3286	.6563	.4909	.5216	-.4919
1.0	.3966	.7029	.4430	.4733	-.4744
1.1	.4690	.7449	.3971	.4268	-.4544
1.2	.5454	.7825	.3535	.3825	-.4319
1.3	.6254	.8157	.3124	.3405	-.4074
1.4	.7084	.8450	.2740	.3011	-.3811
1.5	.7942	.8706	.2384	.2643	-.3535
1.6	.8824	.8928	.2058	.2304	-.3251
1.7	.9727	.9119	.1761	.1993	-.2963
1.8	1.0647	.9281	.1494	.1711	-.2676
1.9	1.1582	.9419	.1256	.1458	-.2395
2.0	1.2530	.9534	.1048	.1232	-.2123
2.1	1.3489	.9629	.0865	.1033	-.1864
2.2	1.4456	.9708	.0708	.0859	-.1621
2.3	1.5430	.9771	.0574	.0708	-.1396
2.4	1.6409	.9823	.0460	.0579	-.1190
2.5	1.7394	.9864	.0366	.0470	-.1005
2.6	1.8382	.9897	.0288	.0377	-.0841
2.7	1.9373	.9922	.0224	.0301	-.0696
2.8	2.0366	.9942	.0173	.0238	-.0571
2.9	2.1361	.9957	.0132	.0186	-.0463
3.0	2.2358	.9969	.0100	.0144	-.0372
3.1	2.3355	.9977	.0075	.0111	-.0296
3.2	2.4353	.9984	.0055	.0085	-.0233
3.3	2.5352	.9989	.0040	.0064	-.0182
3.4	2.6351	.9992	.0029	.0048	-.0141
3.5	2.7350	.9995	.0021	.0036	-.0107
3.6	2.8350	.9996	.0015	.0026	-.0081
3.7	2.9349	.9998	.0010	.0019	-.0061
3.8	3.0349	.9998	.0007	.0014	-.0045
3.9	3.1349	.9999	.0005	.0010	-.0033
4.0	3.2349	.9999	.0003	.0007	-.0024
4.1	3.3349	1.0000	.0002	.0005	-.0017
4.2	3.4349	1.0000	.0001	.0004	-.0012
4.3	3.5349	1.0000	.0001	.0003	-.0009
4.4	3.6349	1.0000	.0001	.0002	-.0006
4.5	3.7349	1.0000	.0000	.0001	-.0004
4.6	3.8349	1.0000	.0000	.0001	-.0003
4.7	3.9349	1.0000	.0000	.0001	-.0002

$\beta = 0.5,\ S_w = 1.0$					
η	f	f'	f''	S	S'
0	0	0	1.2351	1.0000	-0.5725
.2	.0234	.2274	1.0409	.8855	-.5716
.4	.0885	.4172	.8591	.7717	-.5656
.6	.1879	.5720	.6918	.6599	-.5505
.8	.3151	.6950	.5414	.5523	-.5237
1.0	.4640	.7898	.4103	.4513	-.4846
1.2	.6295	.8605	.3000	.3592	-.4346
1.4	.8069	.9113	.2108	.2780	-.3767
1.6	.9929	.9462	.1417	.2088	-.3147
1.8	1.1846	.9692	.0908	.1521	-.2532
2.0	1.3800	.9835	.0550	.1072	-.1959
2.2	1.5776	.9920	.0313	.0732	-.1457
2.4	1.7765	.9966	.0165	.0484	-.1041
2.6	1.9761	.9990	.0078	.0310	-.0714
2.8	2.1760	1.0000	.0031	.0193	-.0471
3.0	2.3761	1.0004	.0009	.0117	-.0299
3.2	2.5762	1.0004	-.0001	.0069	-.0182
3.4	2.7762	1.0004	-.0004	.0041	-.0106
3.6	2.9763	1.0003	-.0004	.0025	-.0061
3.8	3.1764	1.0002	-.0003	.0016	-.0032
4.0	3.3764	1.0002	-.0002	.0011	-.0017
4.2	3.5764	1.0001	-.0002	.0008	-.0008
4.4	3.7765	1.0001	-.0001	.0007	-.0004
4.6	3.9765	1.0001	-.0001	.0006	-.0002
4.8	4.1765	1.0001	-.0001	.0006	-.0001
5.0	4.3765	1.0001	-.0001	.0006	.0000
5.2	4.5765	1.0000	-.0001	.0006	.0000
5.4	4.7765	1.0000	-.0001	.0006	.0000
5.6	4.9765	1.0000	-.0001	.0006	.0000

TABLE 1. - Continued. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = 1.0$, $S_w = 1.0$					
η	f	f'	f"	S	S'
0	0	0	1.7368	1.0000	-0.6154
.1	.0084	.1638	1.5403	.9384	-.6152
.2	.0321	.3084	1.3526	.8770	-.6140
.3	.0694	.4347	1.1758	.8157	-.6110
.4	.1185	.5439	1.0114	.7549	-.6053
.5	.1776	.6374	.8603	.6946	-.5965
.6	.2455	.7165	.7232	.6357	-.5840
.7	.3205	.7825	.6004	.5781	-.5677
.8	.4016	.8370	.4915	.5223	-.5476
.9	.4876	.8812	.3965	.4687	-.5238
1.0	.5776	.9167	.3142	.4176	-.4967
1.1	.6707	.9445	.2442	.3694	-.4666
1.2	.7663	.9659	.1857	.3243	-.4343
1.3	.8637	.9820	.1372	.2826	-.4003
1.4	.9625	.9936	.0979	.2444	-.3654
1.5	1.0623	1.0019	.0666	.2096	-.3302
1.6	1.1628	1.0073	.0422	.1783	-.2954
1.7	1.2637	1.0105	.0237	.1505	-.2617
1.8	1.3648	1.0122	.0101	.1259	-.2294
1.9	1.4661	1.0126	.0004	.1045	-.1992
2.0	1.5673	1.0123	-.0061	.0860	-.1711
2.1	1.6685	1.0115	-.0101	.0702	-.1456
2.2	1.7696	1.0104	-.0123	.0568	-.1226
2.3	1.8706	1.0091	-.0131	.0456	-.1022
2.4	1.9714	1.0078	-.0130	.0363	-.0843
2.5	2.0722	1.0065	-.0122	.0286	-.0689
2.6	2.1728	1.0053	-.0111	.0224	-.0557
2.7	2.2732	1.0043	-.0098	.0174	-.0446
2.8	2.3736	1.0034	-.0085	.0134	-.0354
2.9	2.4739	1.0026	-.0071	.0103	-.0277
3.0	2.5741	1.0020	-.0059	.0078	-.0216
3.1	2.6743	1.0014	-.0049	.0059	-.0166
3.2	2.7744	1.0010	-.0040	.0045	-.0126
3.3	2.8745	1.0006	-.0032	.0034	-.0095
3.4	2.9746	1.0003	-.0026	.0026	-.0071
3.5	3.0745	1.0001	-.0020	.0019	-.0053
3.6	3.1746	.9999	-.0016	.0015	-.0038

$\beta = 1.5$, $S_w = 1.0$					
η	f	f'	f"	S	S'
0	0	0	2.1402	1.0000	-0.6425
.1	.0102	.1992	1.8464	.9358	-.6423
.2	.0389	.3699	1.5697	.8716	-.6408
.3	.0833	.5139	1.3150	.8077	-.6370
.4	.1409	.6337	1.0854	.7443	-.6300
.5	.2093	.7318	.8818	.6818	-.6191
.6	.2866	.8109	.7044	.6206	-.6040
.7	.3709	.8736	.5521	.5611	-.5845
.8	.4608	.9221	.4234	.5039	-.5607
.9	.5550	.9590	.3163	.4491	-.5329
1.0	.6523	.9861	.2287	.3974	-.5017
1.1	.7519	1.0053	.1582	.3489	-.4677
1.2	.8531	1.0182	.1026	.3039	-.4316
1.3	.9554	1.0262	.0598	.2626	-.3943
1.4	1.0583	1.0305	.0277	.2251	-.3566
1.5	1.1614	1.0321	.0044	.1913	-.3191
1.6	1.2646	1.0316	-.0117	.1612	-.2827
1.7	1.3677	1.0299	-.0221	.1347	-.2478
1.8	1.4706	1.0274	-.0282	.1116	-.2150
1.9	1.5731	1.0244	-.0310	.0916	-.1847
2.0	1.6754	1.0212	-.0314	.0746	-.1570
2.1	1.7774	1.0181	-.0302	.0601	-.1321
2.2	1.8791	1.0152	-.0279	.0480	-.1100
2.3	1.9805	1.0126	-.0250	.0380	-.0907
2.4	2.0816	1.0102	-.0219	.0298	-.0740
2.5	2.1825	1.0082	-.0188	.0231	-.0598
2.6	2.2832	1.0065	-.0158	.0178	-.0478
2.7	2.3838	1.0050	-.0130	.0135	-.0379
2.8	2.4843	1.0039	-.0106	.0101	-.0297
2.9	2.5846	1.0029	-.0085	.0075	-.0231
3.0	2.6849	1.0022	-.0067	.0055	-.0177
3.1	2.7850	1.0016	-.0052	.0039	-.0135
3.2	2.8852	1.0011	-.0040	.0028	-.0101
3.3	2.9853	1.0007	-.0031	.0019	-.0076
3.4	3.0853	1.0005	-.0023	.0012	-.0056
3.5	3.1854	1.0003	-.0017	.0008	-.0041
3.6	3.2854	1.0001	-.0013	.0004	-.0030
3.7	3.3854	1.0000	-.0009	.0002	-.0021
3.8	3.4854	.9999	-.0007	.0000	-.0015

TABLE 1. - Concluded. SIMILAR SOLUTIONS OF LAMINAR COMPRESSIBLE BOUNDARY-LAYER EQUATIONS.

$\beta = 2.0,\ S_w = 1.0$					
η	f	f'	f"	S	S'
0	0	0	2.4878	1.0000	-0.6613
.1	.0118	.2291	2.0971	.9339	-.6611
.2	.0446	.4204	1.7341	.8678	-.6593
.3	.0947	.5771	1.4072	.8021	-.6548
.4	.1589	.7031	1.1204	.7370	-.6466
.5	.2344	.8025	.8743	.6729	-.6341
.6	.3187	.8793	.6671	.6103	-.6168
.7	.4097	.9372	.4958	.5497	-.5948
.8	.5056	.9795	.3566	.4915	-.5682
.9	.6052	1.0094	.2455	.4362	-.5375
1.0	.7072	1.0294	.1587	.3841	-.5034
1.1	.8108	1.0418	.0921	.3356	-.4666
1.2	.9153	1.0484	.0425	.2909	-.4280
1.3	1.0203	1.0508	.0067	.2500	-.3885
1.4	1.1254	1.0502	-.0180	.2132	-.3490
1.5	1.2303	1.0475	-.0341	.1802	-.3102
1.6	1.3349	1.0436	-.0435	.1511	-.2729
1.7	1.4390	1.0390	-.0478	.1256	-.2375
1.8	1.5426	1.0341	-.0486	.1035	-.2046
1.9	1.6458	1.0293	-.0468	.0846	-.1745
2.0	1.7485	1.0248	-.0434	.0685	-.1473
2.1	1.8508	1.0207	-.0391	.0550	-.1230
2.2	1.9527	1.0170	-.0344	.0438	-.1017
2.3	2.0542	1.0138	-.0296	.0346	-.0832
2.4	2.1554	1.0111	-.0250	.0271	-.0674
2.5	2.2564	1.0088	-.0208	.0210	-.0541
2.6	2.3572	1.0069	-.0170	.0162	-.0429
2.7	2.4578	1.0054	-.0137	.0123	-.0338
2.8	2.5583	1.0041	-.0109	.0094	-.0263
2.9	2.6587	1.0032	-.0086	.0070	-.0202
3.0	2.7589	1.0024	-.0066	.0053	-.0154
3.1	2.8591	1.0018	-.0051	.0039	-.0117
3.2	2.9593	1.0014	-.0038	.0029	-.0087
3.3	3.0594	1.0011	-.0028	.0022	-.0064
3.4	3.1595	1.0008	-.0021	.0016	-.0047
3.5	3.2596	1.0006	-.0015	.0012	-.0034
3.6	3.3596	1.0005	-.0011	.0009	-.0025
3.7	3.4597	1.0004	-.0007	.0007	-.0018
3.8	3.5597	1.0004	-.0005	.0006	-.0012
3.9	3.6598	1.0003	-.0003	.0004	-.0009
4.0	3.7598	1.0003	-.0002	.0004	-.0006

TABLE II. - SUMMARY OF HEAT-TRANSFER AND WALL-SHEAR PARAMETERS.

S_w	β	f''_w	S'_w	$\frac{C_f Re_w}{Nu}$
-1.0	-0.326	0	0.2477	0
	-.3657	.050	.2958	.3381
	-.3884	.140	.3527	.7939
	-.360	.2448	.4001	1.224
	-.30	.3182	.4262	1.493
	-.14	.4166	.4554	1.830
	0	.470	.470	2.000
	.50	.5806	.4948	2.347
	2.00	.7381	.5203	2.837
-0.8	-0.10	-0.0686	0.0447	-2.456
	-.2685	-.050	.1829	-.4374
	-.3088	0	.2261	0
	-.325	.0493	.2545	.3100
	-.3285	.0693	.2644	.4194
	-.3285	.110	.2818	.6245
	-.325	.1354	.2913	.7438
	-.30	.2086	.3155	1.058
	-.14	.3841	.359	1.712
	0	.470	.376	2.000
	.50	.6547	.403	2.599
	1.50	.8689	.4261	3.263
	2.00	.9480	.4331	3.502
-0.4	-0.235	-0.050	0.0447	-0.8949
	-.246	0	.1249	0
	-.2483	.050	.1360	.2941
	-.24	.1064	.1474	.5775
	-.20	.2183	.1626	1.074
	0	.470	.188	2.000
	.50	.7947	.209	3.042
	2.00	1.3329	.2304	4.628
0	-0.1947	-0.050	0	a
	-.1988	0	0	0
	-.16	.1905	0	.9480
	0	.470	0	2.000
	.50	.9277	0	3.436
	1.00	1.2326	0	4.317
	1.60	1.5213	0	5.122
	2.00	1.6870	0	5.565
1.0	-0.10	-0.1613	-0.2076	-1.554
	-.1305	-.050	-.3139	-.3186
	-.1295	0	-.3388	0
	-.10	.1805	-.4033	.8956
	0	.470	-.470	2.000
	.30	.9829	-.5457	3.602
	.50	1.2351	-.5725	4.315
	1.00	1.7368	-.6154	5.644
	1.50	2.1402	-.6425	6.662
	2.00	2.4878	-.661	7.527

[a] This value was not calculated.

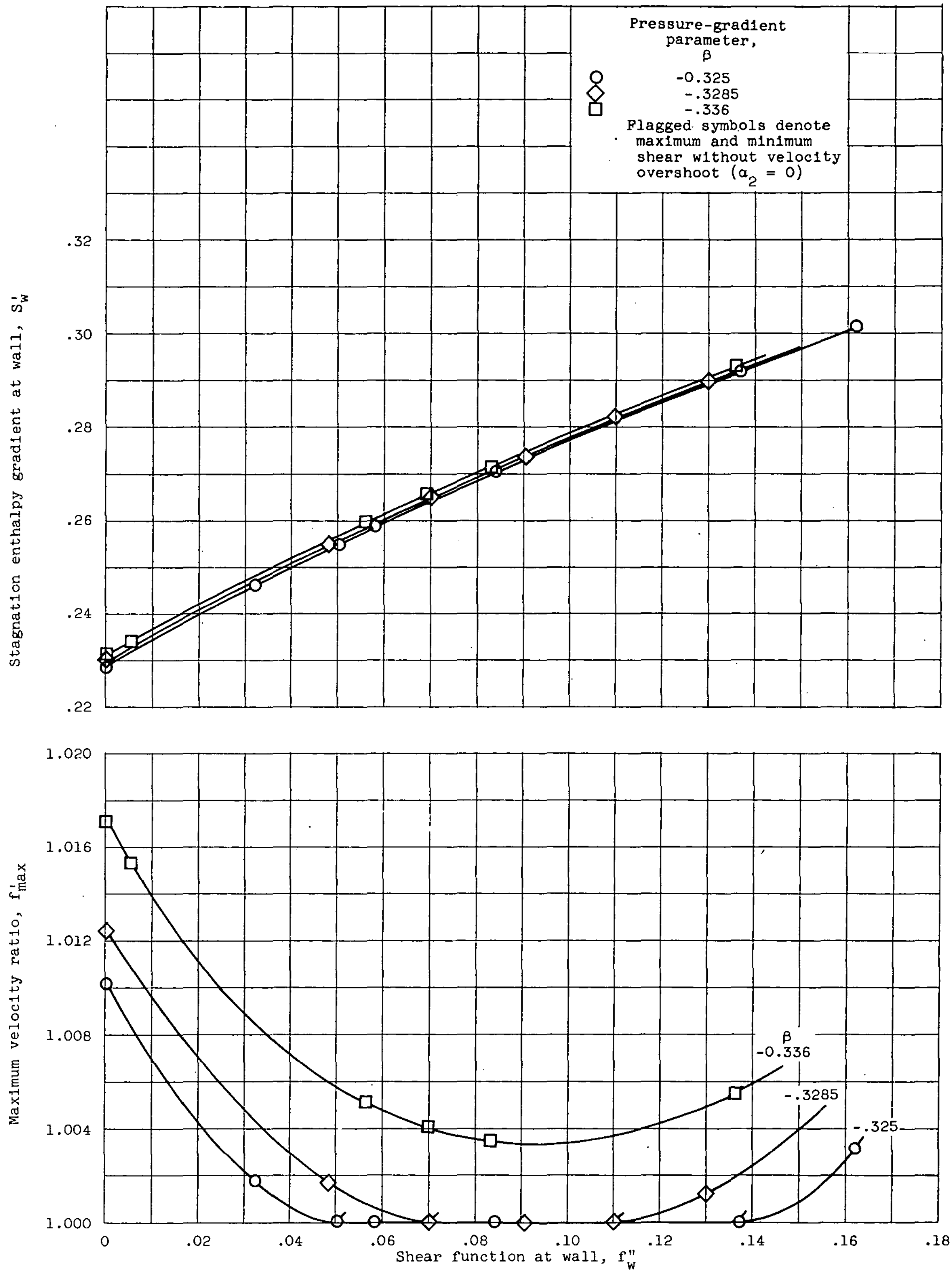
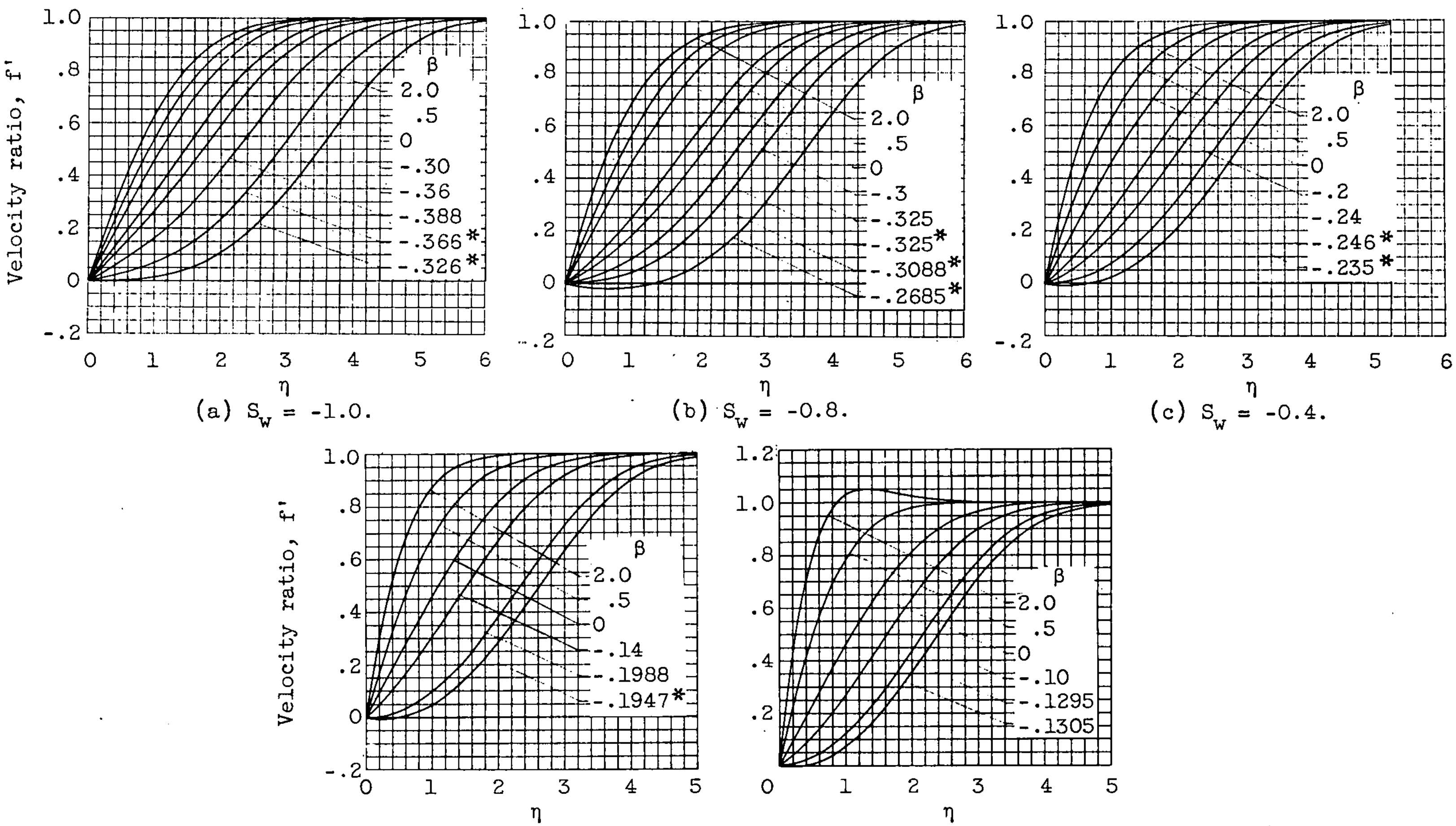


Figure 1. - Family of solutions for adverse pressure gradient. S_w = -0.8.



(a) $S_w = -1.0$.

(b) $S_w = -0.8$.

(c) $S_w = -0.4$.

(d) $S_w = 0$ (Hartree, ref. 7).

(e) $S_w = 1.0$.

Figure 2. - Velocity profiles as function of similarity variable η. (* denotes lower-branch solutions.)

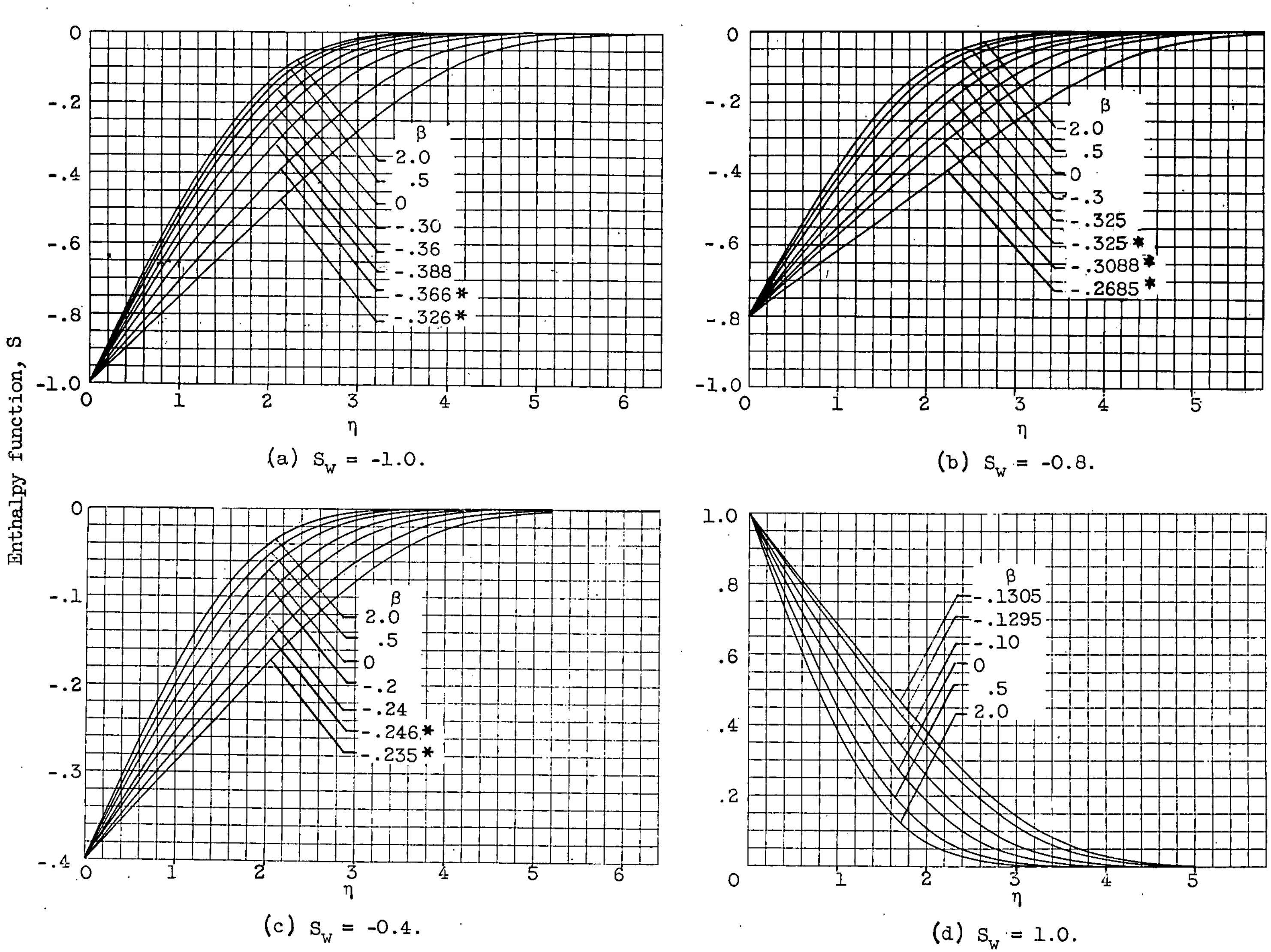


Figure 3. - Enthalpy function profiles. (* denotes lower-branch solutions.)

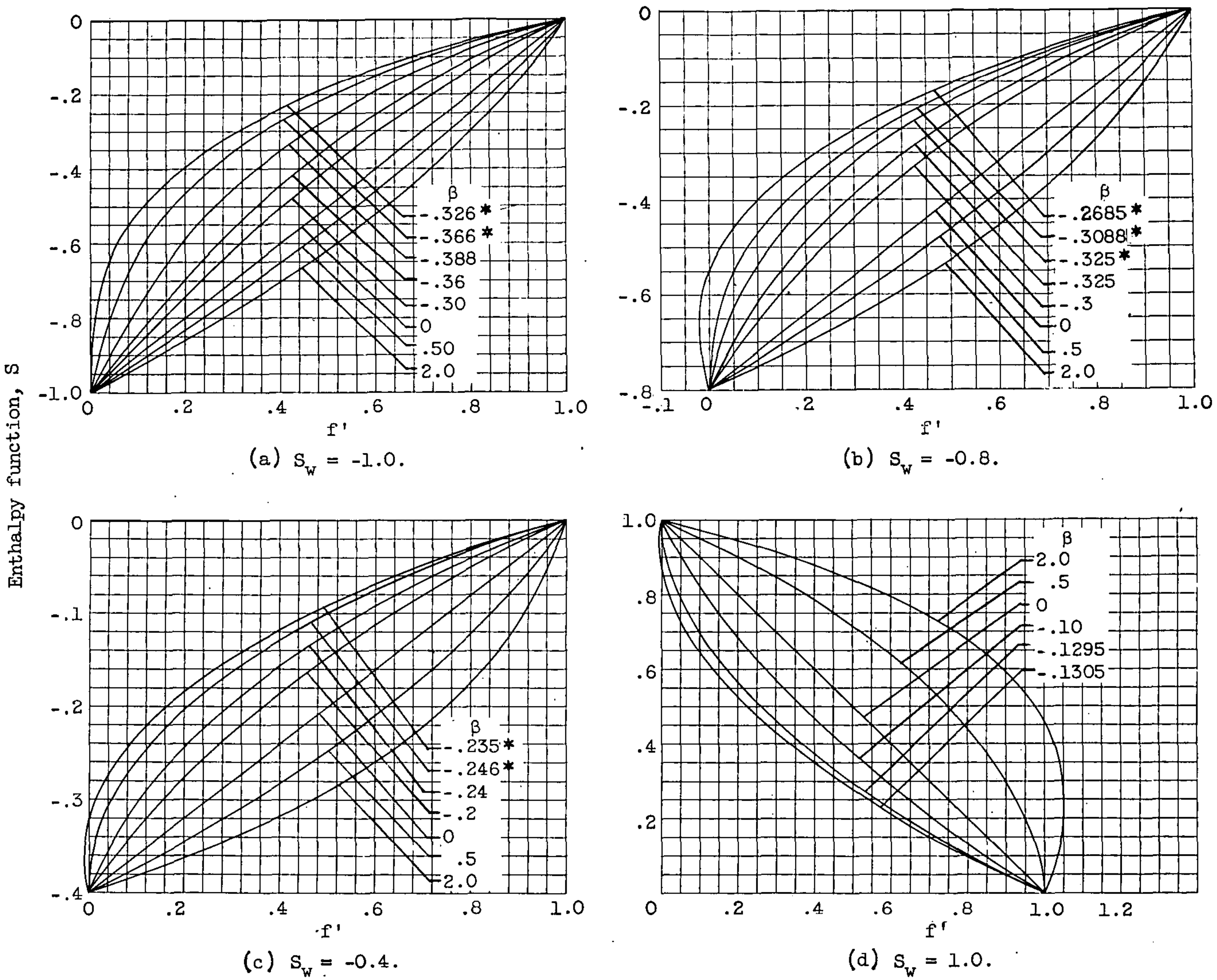
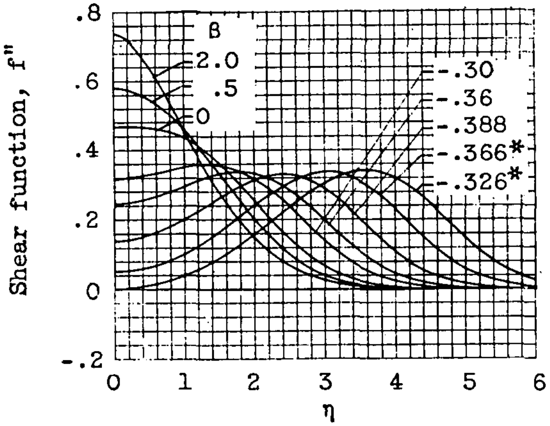
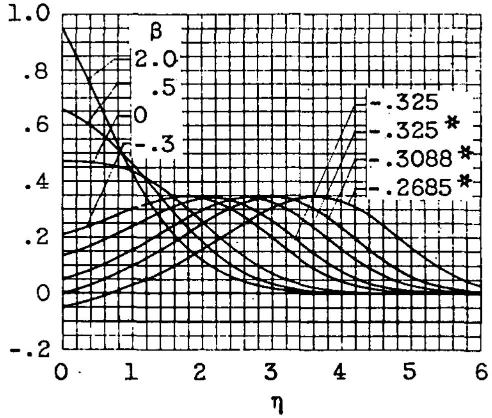


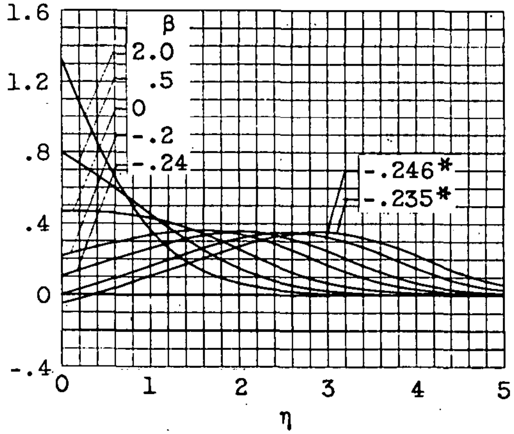
Figure 4. - Enthalpy function representation in velocity plane. (* denotes lower-branch solutions.)



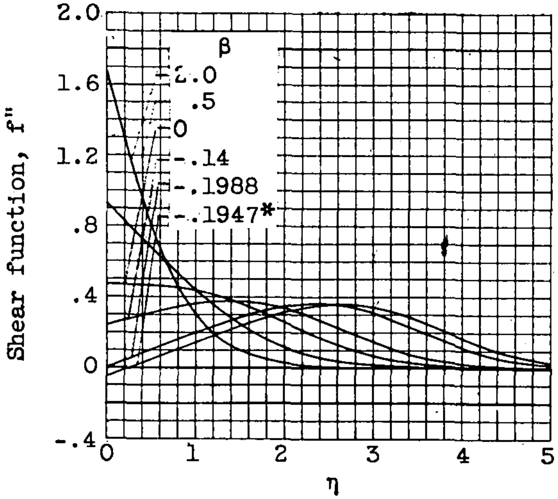
(a) $S_w = -1.0$.



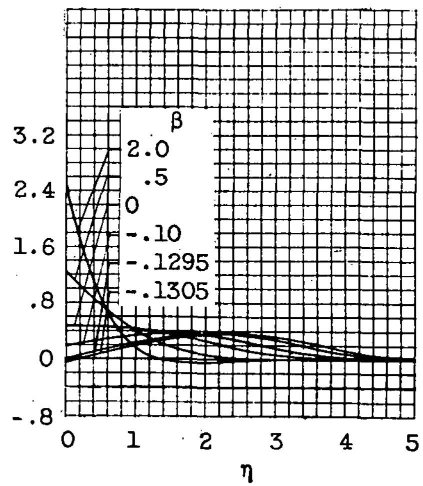
(b) $S_w = -0.8$.



(c) $S_w = -0.4$.

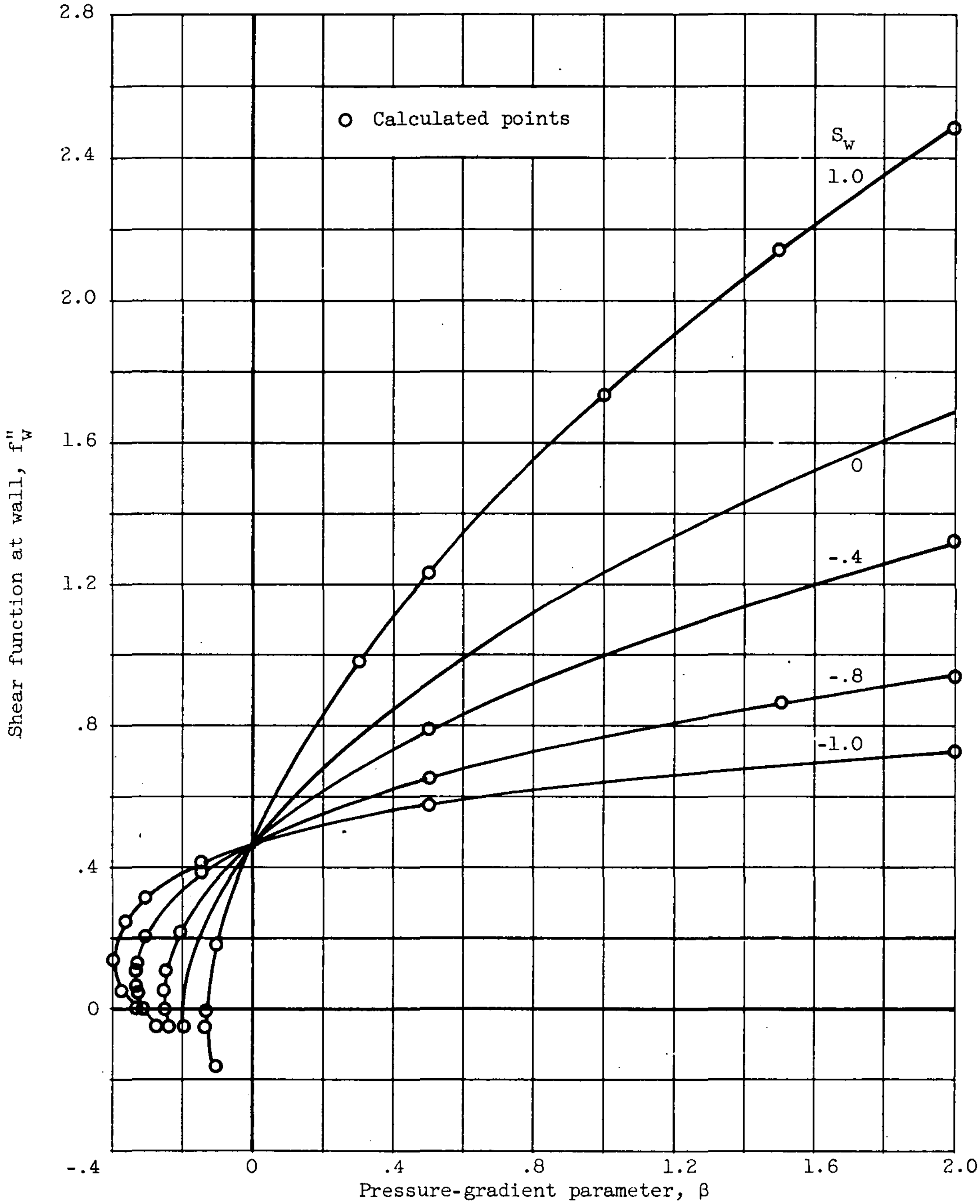


(d) $S_w = 0$.



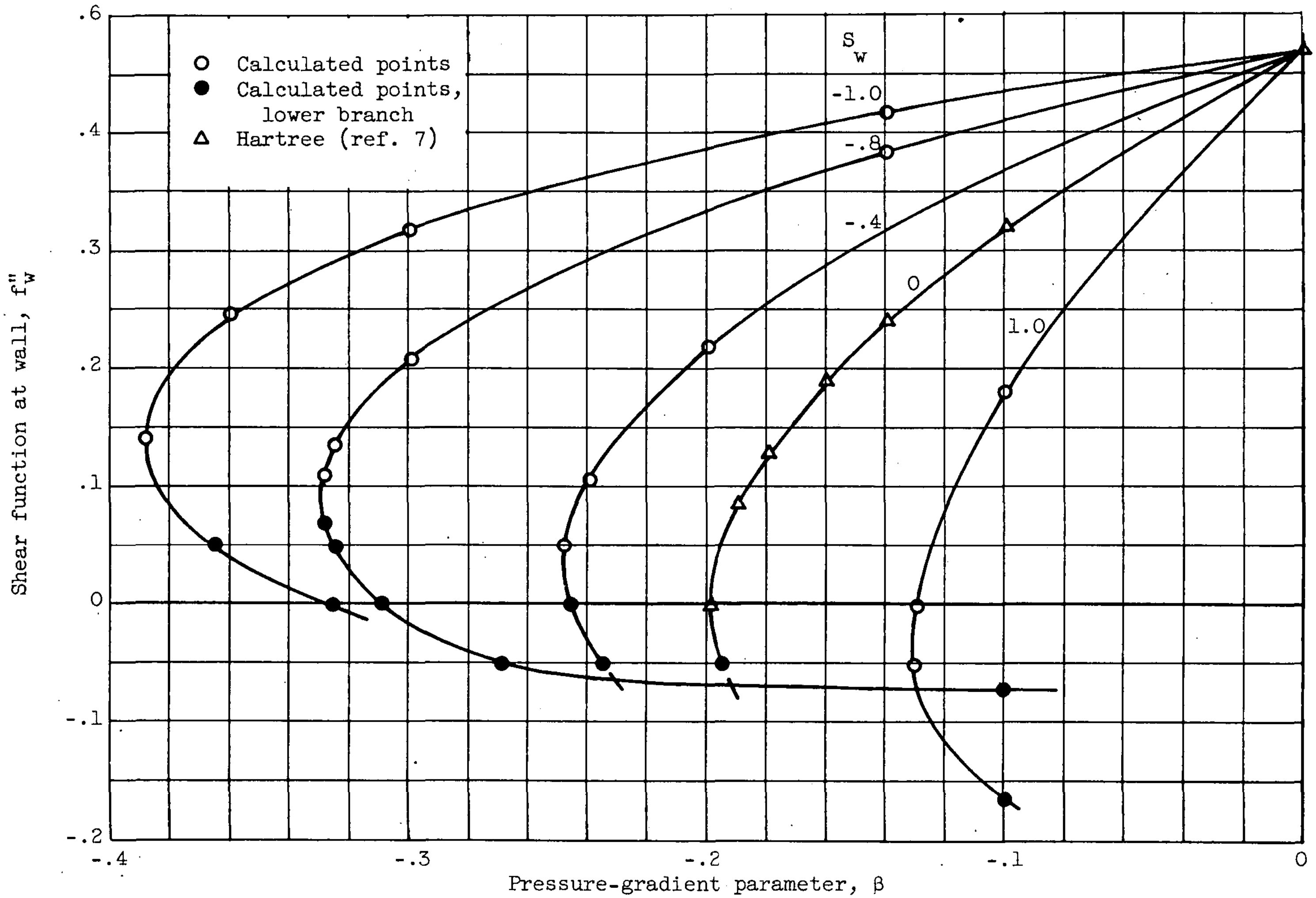
(e) $S_w = 1.0$.

Figure 5. - Shear-function profiles. (* denotes lower-branch solutions.)



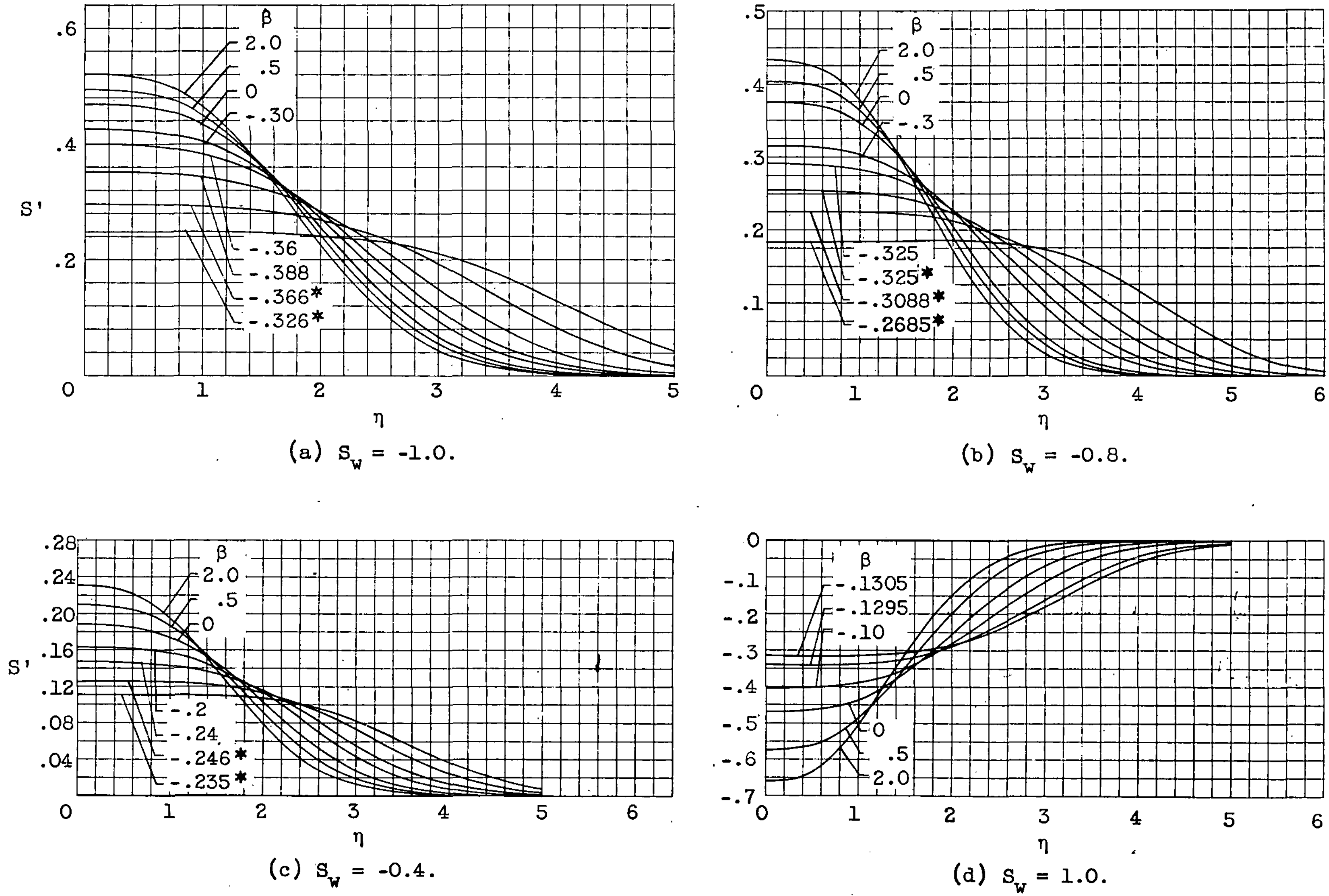
(a) Favorable and adverse pressure gradients.

Figure 6. - Effect of pressure gradient on wall shear.



(b) Adverse pressure-gradient region.

Figure 6. - Concluded. Effect of pressure gradient on wall shear.



(a) $S_w = -1.0$.

(b) $S_w = -0.8$.

(c) $S_w = -0.4$.

(d) $S_w = 1.0$.

Figure 7. - Stagnation enthalpy gradient across boundary layer. (* denotes lower-branch solutions.)

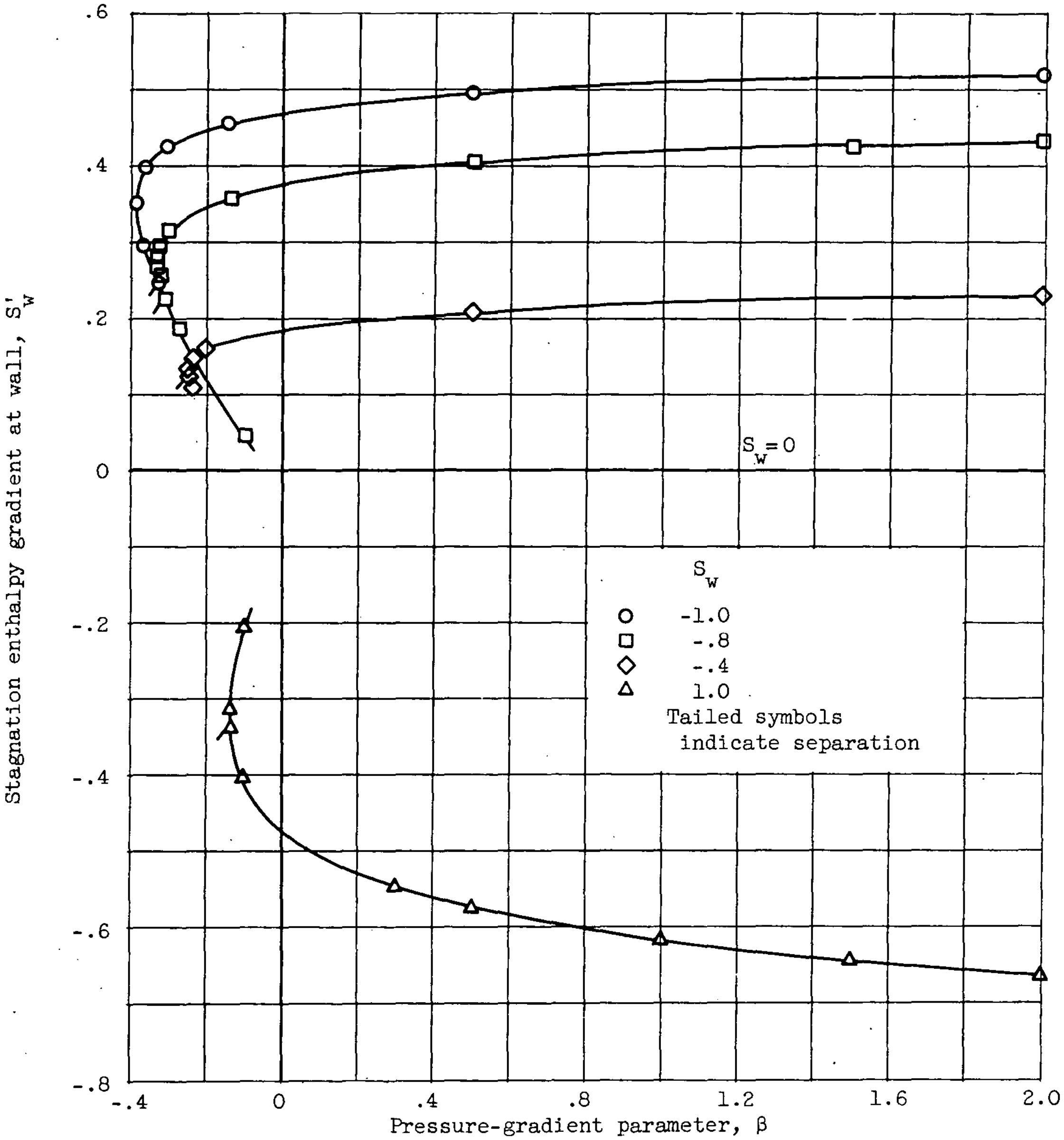


Figure 8. - Variation of heat transfer with pressure gradient.

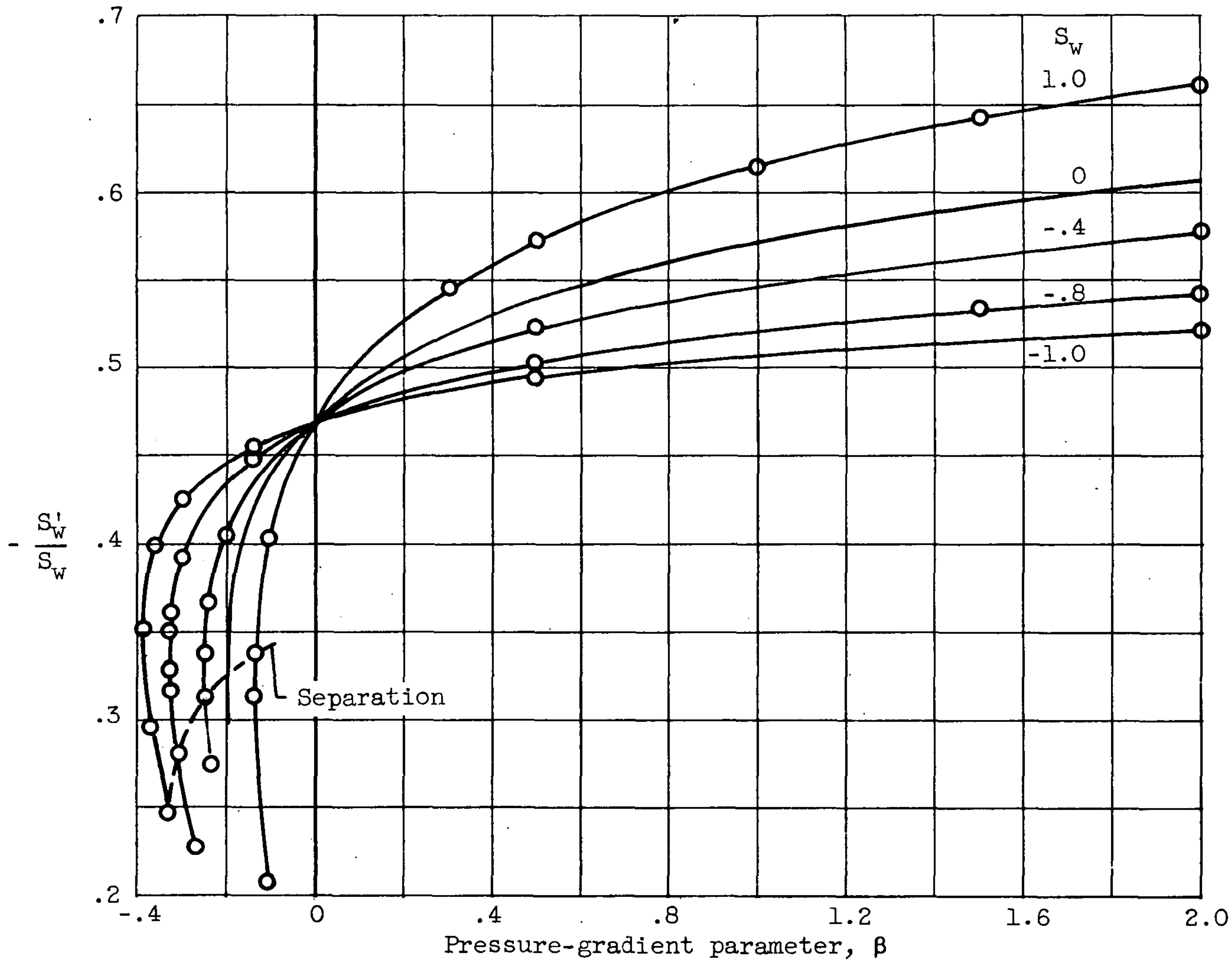


Figure 9. - Variation of $\frac{S_w'}{S_w}$ with pressure gradient.

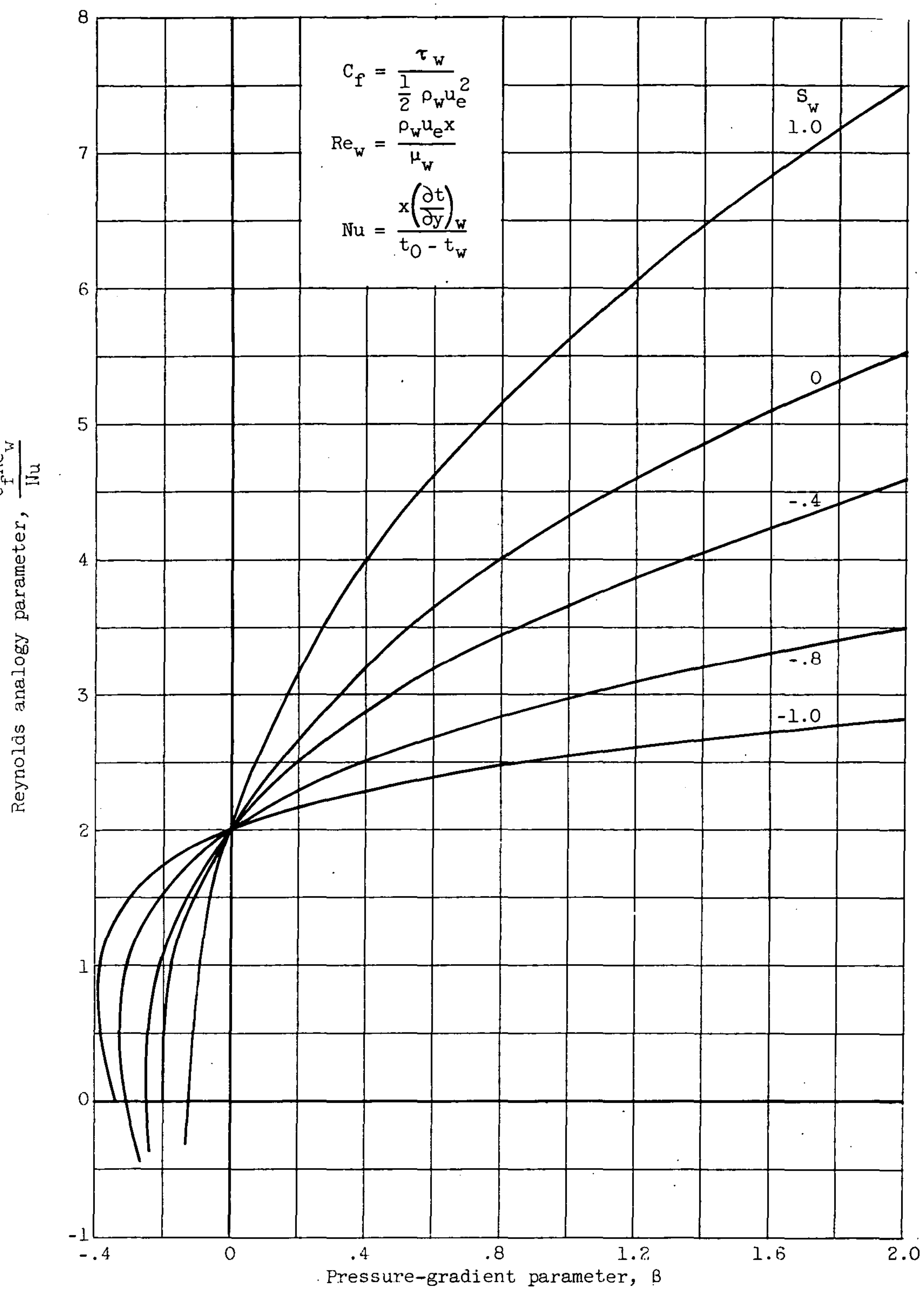


Figure 10. - Variation of Reynolds analogy parameter with pressure gradient.

NACA-Langley - 2-17-55 - 1000