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THE COMPRESSIBLE LAMINAR BOUNDARY LAYER WITH FLUID INJECTION

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SUMMARY

A solution of the equations of the compressible laminar boundary layer including the effects of transpiration cooling is presented. The analysis applies to the flow over an isothermal porous plate with a velocity of fluid injection proportional to the reciprocal of the square root of the distance from the leading edge.

The effect of several flow parameters on coolant-flow rates is discussed with the aid of representative examples. A stability analysis indicates that, although transpiration cooling requires a lower surface temperature for stable flow than does internal wall cooling, this lower temperature can be obtained with a smaller expenditure of coolant.

INTRODUCTION

One of the many problems encountered in high-speed flight is the aerodynamic heating of aircraft surfaces. Structural damage due to heating can be avoided either by utilizing high-temperature materials in aircraft construction, or by one of several methods of surface cooling.

In the present paper the method of transpiration cooling is studied. In this method, the coolant is passed through a porous skin and enters the airstream with a finite velocity normal to the surface. One of the advantages of transpiration cooling is that the coolant can be brought to the highest available temperature at the outside surface and, therefore, can be utilized with the maximum possible effectiveness. Transpiration cooling also decreases aerodynamic heating by altering the velocity and temperature profiles in a manner that reduces the rate of heat transfer at the surface.

In the past, exact solutions of the transpiration-cooling problem have been obtained only for low-speed flow (e.g., refs. 1 and 2). For high-speed flow, the problem has been solved either empirically (ref. 3) or by integral methods (e.g., ref. 4). The assumptions made in these analyses have often been quite restrictive. The purpose of this report, prepared at the NACA Lewis laboratory, is to present a solution to the problem that is free of many of the limitations of previous methods. This is accomplished by means of a direct solution of the boundary-layer differential equations.

Because the velocity and temperature profiles obtained by this method are quite accurate, they can be used in a stability analysis. It is by now well-known that surface cooling tends to stabilize the laminar boundary layer. However, the normal velocity associated with transpiration cooling has a destablizing effect. Stability calculations are included in this paper in order to determine how these opposing factors combine.

SYMBOLS

The following symbols are used in this report:

C constant of proportionality in viscosity-temperature relation

C_F average skin-friction coefficient

c_f local skin-friction coefficient

c_p specific heat at constant pressure

f dimensionless stream function

H heat absorbed by coolant

k thermal conductivity

M Mach number

Pr Prandtl number, $\mu c_p/k$

p static pressure

q local rate of heat transfer

R gas constant

Re Reynolds number, $\rho_\infty x u_\infty/\mu_\infty$

r temperature function defined by eq. (14)

S Sutherland's constant ($S = 216^\circ$ R)

s temperature function defined by eq. (14)

t static temperature

u velocity in x-direction

v velocity in y-direction

x distance along surface measured from leading edge

y distance from surface measured perpendicular to surface

α asymptotic value appearing in eq. (22)

β relative mass-flow parameter defined by eq. (28)

γ ratio of specific heats

δ^* displacement thickness

η characteristic variable

μ coefficient of viscosity

ν kinematic viscosity, μ/ρ

ρ mass density

τ shearing stress

Subscripts:

aw adiabatic wall

c conditions of coolant

w conditions at wall or surface

x,y,η partial differentiation with respect to x, y, or η

0 conditions for equivalent plate cooled by internal wall cooling system

∞ conditions at $y = \infty$

The symbol I preceding a variable indicates integration from zero to η.

GOVERNING EQUATIONS

Differential equations and boundary conditions. - The steady laminar flow of a viscous compressible fluid in a boundary layer on a flat plate is governed by the momentum equation

$$uu_x + vu_y = \frac{1}{\rho} (\mu u_y)_y \tag{1}$$

the equation of continuity

$$(\rho u)_x + (\rho v)_y = 0 \tag{2}$$

the energy equation

$$\rho c_p(ut_x + vt_y) = (kt_y)_y + \mu(u_y)^2 \tag{3}$$

and the equation of state

$$\rho Rt = p = \text{constant} \tag{4}$$

These equations are subject to the following boundary conditions:

$$\left.\begin{aligned} u &= 0 \\ v &= v(x) \\ t &= t_w \end{aligned}\right\} \text{at } y = 0$$

$$\left.\begin{aligned} u &= u_\infty \\ t &= t_\infty \end{aligned}\right\} \text{at } y = \infty$$

The present problem differs from that solved by Chapman and Rubesin (ref. 5) only in the existence of a finite normal component of velocity at the surface.

Assumptions and limitations. - The following assumptions are introduced in order to simplify the problem and, in particular, in order to transform the partial to ordinary differential equations:

(1) The normal velocity at the wall is proportional to the $1/\sqrt{x}$. As shown in references 1 and 6, this assumption leads to "similar" velocity and temperature profiles and a uniform plate temperature. (This problem is mathematically no more complicated if a variable wall temperature is selected. However, only the assumption of a constant wall temperature is compatible with the specified similar solution.)

(2) The viscosity and temperature are linearly related as follows:

$$\frac{\mu}{\mu_\infty} = C \frac{t}{t_\infty} \tag{5}$$

It has been shown in the past (ref. 5) that this relation provides reasonable accuracy at moderate Mach numbers when the constant C is determined by

$$C = \sqrt{\frac{t_w}{t_\infty}}\left[\frac{t_\infty + S}{t_w + S}\right] \tag{6}$$

where S is Sutherland's constant ($S = 216^\circ$ R for air).

(3) The Prandtl number and specific heat are constant. A Prandtl number of 0.72 was used in all numerical solutions.

Solution of equations. - The method of solution of the governing equations is analogous to the methods described by Chapman and Rubesin (ref. 5) and by Howarth (ref. 7), and hence only an outline of the solution is presented herein. The important steps in the analysis consist of the introduction of a similarity variable

$$\eta = \frac{1}{2}\sqrt{\frac{u_\infty}{\nu_\infty x C}} \int_0^y \frac{t_\infty}{t} \, dy \tag{7}$$

and the definition of a modified stream function $f(\eta)$, which is related to the velocity by

$$\frac{u}{u_\infty} = \frac{1}{2} f'(\eta) \tag{8}$$

(The prime denotes total differentiation with respect to η.) The continuity equation then yields the following expression for v:

$$\frac{v}{u_\infty} = -\frac{1}{2}\frac{\rho_\infty}{\rho}\sqrt{\frac{\nu_\infty C}{u_\infty x}}\,(f - \eta f') \tag{9}$$

and the momentum equation becomes

$$ff'' + f''' = 0 \tag{10}$$

$$\left.\begin{aligned} f'(0) &= 0 \\ f(0) &= -\frac{2}{\sqrt{C}} \frac{\rho_w v_w}{\rho_\infty u_\infty} \sqrt{\frac{u_\infty x}{\nu_\infty}} = \text{constant} \\ f'(\infty) &= 2 \end{aligned}\right\} \tag{11}$$

The boundary condition $f(0)$ is obtained from equation (9) and the assumption that $v \sim 1/\sqrt{x}$. Physically, $f(0)$ is proportional to the mass flow of the fluid injected through the porous plate.

The energy equation, in terms of the similarity variable η becomes:

$$t'' + Pr\, ft' = -\frac{\gamma - 1}{4} M_\infty^2\, t_\infty\, Pr(f'')^2 \tag{12}$$

$$\left.\begin{aligned} t(0) &= t_w \\ t(\infty) &= t_\infty \end{aligned}\right\} \tag{13}$$

In order to eliminate M as a parameter in the solution of equation (12), this equation is split into two linearly related parts in the following manner:

$$\frac{t}{t_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2\, r(\eta) + \left(\frac{t_w}{t_\infty} - \frac{t_{aw}}{t_\infty}\right) s(\eta) \tag{14}$$

where t_{aw} is the adiabatic wall temperature, given by

$$\frac{t_{aw}}{t_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2\, r(0) \tag{15}$$

and where r and s satisfy

$$r'' + Pr\, fr' = -\frac{Pr}{2} (f'')^2 \tag{16}$$

$$r'(0) = 0 \qquad r(\infty) = 0$$

and

$$s'' + Pr\, fs' = 0 \tag{17}$$

$$s(0) = 1 \qquad s(\infty) = 0$$

Equation (10) is the well-known Blasius equation with a nonvanishing initial value. Numerical solutions of this equation for many values of $f(0)$ have recently been presented in reference 8. Solutions of the energy equations can be given in terms of quadratures (ref. 5), but it was found more convenient and accurate to solve the differential equations numerically. The numerical calculations were made on an IBM card-programmed calculator under the supervision of Lynn U. Albers. Solutions were found by the method outlined in appendix B of reference 9 for $f(0) = -0.5$, -0.75, and -1.0. Because only the surface boundary values are needed in many calculations, they are listed separately in table I; functions for all values of η can be found in table II. Results for the case without fluid injection ($f(0) = 0$) are those of reference 9, and are included so that a consistent set of transpiration-cooling and internal-wall-cooling solutions can be compared. Solutions of the momentum equation for $f(0) \neq 0$ were taken from reference 8.

BOUNDARY-LAYER CHARACTERISTICS

Velocity and temperature profiles. - Velocity and temperature profiles are obtained from the following relations:

$$\frac{u}{u_\infty} = \frac{1}{2} f'(\eta) \tag{8}$$

and

$$\frac{t}{t_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 r(\eta) + \left(\frac{t_w}{t_\infty} - \frac{t_{aw}}{t_\infty}\right) s(\eta) \tag{14}$$

The relation between y and η is

$$y = 2\sqrt{\frac{\nu_\infty xC}{u_\infty}}\left[\eta + \frac{\gamma - 1}{2} M_\infty^2 Ir(\eta) + \left(\frac{t_w}{t_\infty} - \frac{t_{aw}}{t_\infty}\right) Is(\eta)\right] \tag{18}$$

(The symbol I preceding a function indicates integration of that function from zero to η.)

Because the function $f'(\eta)$ is independent of Mach number, the effect of compressibility is on the velocity profile merely one of thickening (or thinning) the boundary layer as described by equation (18). For zero heat transfer (i.e., $t_w = t_{aw}$), the boundary-layer thickness increases as the Mach number is increased. This increase is more pronounced when the wall is hot ($t_w > t_{aw}$), and conversely; if the surface is very cold, the boundary layer may actually become thinner as the Mach number increases. Although these trends apply for all values of the blowing parameter $f(0)$, they are accentuated at high rates of fluid injection, because under those conditions both Ir and Is increase.

The function $r(0)$(see table I) changes only slightly as a result of fluid injection; consequently, the adiabatic wall temperature is quite insensitive to the rate of blowing. The temperature profile, under conditions of large heat transfer, is altered considerably as a result of the effects of porous cooling on the function $s(\eta)$.

Skin friction and heat transfer. - The local and average skin-friction coefficients are obtained as follows:

$$c_f \equiv \frac{\tau_w}{\frac{1}{2}\rho_\infty u_\infty^2} = \frac{1}{2} f''(0)\sqrt{\frac{C}{Re}} \tag{19}$$

and

$$C_F \equiv \frac{1}{\frac{1}{2}\rho_\infty u_\infty^2 x}\int_0^x \tau_w \, dx = f''(0)\sqrt{\frac{C}{Re}} \tag{20}$$

The effect of compressibility on skin friction is brought about by the proportionality factor C and, hence, is only a function of wall temperature, and not of the rate of blowing. Regardless of Mach number, however, skin friction decreases at high rates of fluid injection as a result of the decrease in $f''(0)$.

The local rate of heat transfer is given by

$$q \equiv -k\left(\frac{\partial t}{\partial y}\right)_w = \frac{-c_p t_\infty}{2Pr}\sqrt{\frac{\mu_\infty \rho_\infty u_\infty C}{x}}\left(\frac{t_w}{t_\infty} - \frac{t_{aw}}{t_\infty}\right) s'(0) \tag{21}$$

The main effect of compressibility on heat transfer is taken into account by the adiabatic wall temperature t_{aw} (see eq. (15)), which appears in the temperature potential $t_w - t_{aw}$. Fluid injection at all Mach numbers result in an appreciable decrease in wall heat transfer caused by the decrease in the function $s'(0)$.

Boundary-layer displacement thickness. - The displacement thickness δ^* can be expressed as follows:

$$\delta^* \equiv \int_0^\infty \left(1 - \frac{\rho u}{\rho_\infty u_\infty}\right) dy = 2\sqrt{\frac{\nu_\infty x C}{u_\infty}}\left\{\alpha + \frac{\gamma - 1}{2} M_\infty^2 Ir(\infty) + \left(\frac{t_w}{t_\infty} - \frac{t_{aw}}{t_\infty}\right) Is(\infty)\right\} \tag{22}$$

where the asymptotic values α, $Ir(\infty)$ and $Is(\infty)$ are listed in table III. The effects of fluid injection and compressibility on δ^* are similar to those on the velocity profile, as discussed earlier.

COOLANT MASS-FLOW REQUIREMENTS

Heat balance. - In general, the amount of coolant mass flow is not arbitrary but depends on the properties of the coolant, the mainstream conditions, and the surface temperature. The value of $f(0)$ required to obtain a given surface temperature is obtained from a heat balance which states that the heat removed from the airstream equals the heat absorbed by the coolant. The heat absorbed by the coolant is proportional to its enthalpy change H and to its mass flow $\rho_w v_w$; the heat removed from the airstream is determined from Fourier's law. Therefore,

$$\rho_w v_w H = k\left(\frac{\partial t}{\partial y}\right)_w \tag{23}$$

Through the use of equations (11) and (21), this heat balance becomes

$$\frac{c_p(t_w - t_{aw})}{H} = \frac{-Pr\ f(0)}{s'(0)} \tag{24}$$

The term H in this expression, for an evaporating liquid, becomes

$$H = c_{p,l}(t_v - t_c) + H_v + c_{p,v}(t_w - t_v) \tag{25}$$

where the subscripts l and v refer to the liquid and vapor states, H_v is the latent heat of vaporization of the coolant, and t_v is the temperature of vaporization. When the coolant is a gas, equation (25) becomes

$$H = c_{p,c}(t_w - t_c) \tag{26}$$

Equation (24) yields the value of $f(0)$ necessary to obtain a given wall temperature. The coolant mass-flow requirements follow from equation (11), which can be written as

$$\rho_w v_w = -\frac{\rho_\infty u_\infty f(0)}{2}\sqrt{\frac{C}{Re}} \tag{27}$$

Relative mass-flow rates. - The boundary-layer analysis presented in this paper is exact only if the coolant and the flow over the plate are the same gas. This is the case because it was assumed that the velocity and temperature fields are unaffected by the mass transfer of the injected fluid. Results of calculations based on the present analysis are therefore exact if the coolant is air or liquid air that evaporates fully before it enters the boundary layer. In some applications it may be desirable, however, to use a different coolant, such as water. Although it is not possible to predict the accuracy of calculations made for coolants other than air or liquid air, it is expected that results should be reasonable so long as either the rate of injection is small, or the property values of the coolant are similar to those of the main flow. The rate of coolant injection can be described in terms of the ratio of the amount of fluid injected up to a point L to the fluid present in the boundary layer at L:

$$\beta = \frac{\int_0^L \rho_w v_w \, dx}{\int_0^\delta \rho u \, dy} \tag{28}$$

With the aid of equations (18), (22), and (27), equation (28) becomes

$$\beta = \frac{-f(0)}{2(\eta_\delta - \alpha)} \tag{29}$$

In general, the value of η at the edge of the boundary layer η_δ is undefined. For the present calculations η_δ was taken at the point where $u/u_\infty = 0.99$.

SURFACE COOLING WITH FLUID INJECTION

Several examples were calculated in order to determine the relative importance of some of the parameters appearing in the heat balance (eq. (24)). The coolants used in these examples are air, liquid air, and water; their properties are listed in the following table:

Coolant	t_c, °F	t_v, °F	$c_{p,v}$, $\frac{\text{Btu}}{(\text{lb})(^\circ\text{R})}$	H_v, $\frac{\text{Btu}}{\text{lb}}$
Air	40	----	0.24	----
Liquid air	-320	-320	.24	88
Water	40	40	.50	1071

Thus, it was assumed that the coolant evaporates at a temperature equal to t_c, the maximum possible error due to this assumption being ±1.3 percent.

Effect of Mach number on surface temperature. - The effect of Mach number on surface temperature is shown in figure 1 as a function of the rate of fluid injection. Results are plotted for air, liquid air, and water in figures 1(a), (b), and (c), respectively. The ambient air temperature for all calculations was assumed to be -67.6° F. It is noted that, for large rates of blowing, the wall temperature is less dependent on Mach number than for small rates of blowing.

Effect of coolant on surface temperature. - A comparison of air, liquid air, and water as coolants can be made by examining figures 1 (a) to (c). In order to maintain a given surface temperature, relatively large amounts of air, smaller amounts of liquid air, and still smaller amounts of water are required. This is true because air cools only by the action of its heat capacity, whereas water and liquid air cool also by the process of evaporation. These trends are not limited to the case of transpiration cooling alone, but apply for all cooling methods wherein evaporation is possible.

Calculations were also made for liquid nitrogen as a coolant, but results are not included in the figures, because they are almost identical to those for liquid air.

Comparison of transpiration cooling and internal wall cooling. - In order to assess properly cooling by fluid injection, a comparison was made with internal wall cooling. In internal wall cooling, the coolant is passed along the inside of the skin and absorbs heat from the skin. In transpiration cooling, the coolant not only absorbs heat from the porous skin as it passes through it but also enters the boundary layer with a finite velocity that leads to a lowered heat-transfer coefficient. In figure 2, the rate of cooling q necessary to obtain a given surface temperature by use of transpiration cooling is compared with the rate of cooling q_0 that is required to obtain the same temperature with internal cooling. The ratio q/q_0 has a relatively low value when the rate of blowing is high, because the function $s'(0)$ decreases rapidly as $-f(0)$ increases. Hence, when the coolant is air, which requires large values of $-f(0)$, q/q_0 is small; when the coolant is water, q/q_0 is never much less than unity, because the flow rates are fairly low. Note that the foregoing comparison was made under the assumptions that the coolant attains the wall temperature in both cooling systems and, when water is the coolant, that evaporation takes place in both systems.

BOUNDARY-LAYER STABILITY

It has been shown by Lees and Lin (ref. 10) and others that the withdrawal of heat at the surface has a stabilizing effect on the laminar boundary layer. In particular it was found that if, in supersonic flow, sufficient heat is withdrawn at the surface the boundary layer is stable to two-dimensional disturbances for all Reynolds numbers. Calculations of the limits of this "complete" stability, though not directly related to transition, should yield some indication of the effects of cooling on transition.

It is also known that fluid injection has a destabilizing effect on the boundary layer. Stability calculations were therefore made to determine just how the two opposing factors of cooling and blowing combine to affect stability. These calculations were made with the aid of the improved viscous solutions of the stability equations as derived by Dunn and Lin (ref. 11). The results apply at moderate supersonic speeds and indicate the complete stability limits for two-dimensional disturbances.[1] An outline of the method of calculation can be found in the appendix of reference 11.

The stability curves for several rates of fluid injection are shown in figure 3. Each of the curves delineates the region of complete stability for a given rate of blowing; if the wall temperature is below the curve, the flow is stable, and conversely. As anticipated, the surface must be cooled to a lower temperature in order to attain complete stability as the rate of fluid injection is increased. This lower temperature can, however, be reached with a smaller expenditure of coolant as a consequence of the lower heat-transfer coefficient. This fact can be deduced from figure 4, where the heat-transfer rate q required for complete stability with transpiration cooling is compared with the heat transfer rate q_0 needed to stabilize with internal cooling. The fact that q/q_0 is zero at high Mach numbers is physically insignificant, because there the temperature required for stability must be equal to absolute zero (see fig. 3).

Results of a comparison of heat-transfer rates for complete stability with transpiration cooling and with internal wall cooling are shown in figure 5. The values plotted in this figure are obtained from figure 4 for the appropriate value of $f(0)$, as determined from equation (24).

DISCUSSION OF EXAMPLES

The foregoing examples have served to show that transpiration cooling is considerably more effective (on a weight-of-coolant basis) than internal wall cooling if large coolant-flow rates are required.

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[1]Three-dimensional disturbances, as discussed in reference 11, are not included in the present analysis.

When the flow rates are small, as is the case when water is used as a coolant, transpiration cooling is only slightly more effective than internal wall cooling (similar results for low-speed flow are presented in ref. 12).

The curves of relative heat-transfer rates (figs. 2 and 5), on which these conclusions are based, apply only if the same amount of heat is absorbed by the coolant in both systems. However, in some practical applications it may not be feasible to allow the coolant to boil in the internal wall cooling system; this would be the case when a uniform temperature is required for structural integrity. Then, if water is a coolant, each pound of water would carry away approximately five times as much heat in the transpiration-cooling system (where boiling is permitted) than in the internal-wall-cooling system. Under these conditions, of course, transpiration cooling would have a large advantage over internal wall cooling.

It appears, therefore, that a transpiration-cooling system using water as a coolant may be of practical importance. As mentioned earlier, the results of this analysis can only be approximate if the coolant is not the same gas as the main flow. However, the required rate of water injection is so small that it is believed that the results are quite accurate (e.g., when $M_\infty = 4$, and $t_w/t_\infty = 2$, eq. (29) shows that only 3 percent of the boundary layer consists of water vapor).

If it is assumed that the stability curves correspond qualitatively to transition curves, then it is interesting to reexamine the examples discussed in the section entitled SURFACE COOLING WITH FLUID INJECTION with respect to the stability calculations. A composite curve of the stability and the cooling curves for $M_\infty = 4$ is presented in figure 6. It is evident that air at normal temperatures cannot be used to stabilize the boundary layer at this Mach number. Both water and liquid air can, however, be used, although the weight flow of liquid air is considerably greater than that of water.

CONCLUDING REMARKS

A solution of the equations of high-speed flow over a porous flat plate with continuous fluid injection has been presented. This solution was used to calculate several examples in which air, liquid air, and water were assumed to be the coolant. From these examples, the following conclusions were reached:

1. Of the three coolants considered, water is the most effective, on a weight basis.

2. Transpiration cooling is more effective than internal wall cooling. The relative advantage of transpiration cooling increases as the coolant-flow rate is increased.

3. Although the surface must be cooled to a lower temperature than that required with internal wall cooling in order to achieve complete stability with transpiration cooling, this lower temperature can be obtained with a smaller expenditure of coolant.

4. Because of the low temperatures required for stability, it appears unlikely that unliquified air can be used as a coolant to produce laminar stability.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
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TABLE I. - INITIAL VALUES

f(0)	f"(0)	r(0)	s'(0)
0	1.3282	0.8477	-0.5912
- .50	.6580	.7987	- .3324
- .75	.3745	.7680	- .2127
-1.00	.1421	.7332	- .1010

TABLE II. - SOLUTIONS OF MOMENTUM AND ENERGY EQUATIONS

(a) f(0) = 0

η	f	f'	f"	Ir	r	r'	Is	s	s'
0	0.0000	0.0000	1.3282	0.0000	0.8477	0.0000	0.0000	1.0	-0.5912
.1	.0066	.1328	1.3279	.0847	.8445	-.0635	.0970	.9409	-.5911
.2	.0266	.2655	1.3259	.1687	.8350	-.1268	.1882	.8818	-.5905
.3	.0597	.3979	1.3203	.2514	.8192	-.1893	.2734	.8228	-.5887
.4	.1061	.5294	1.3095	.3323	.7972	-.2503	.3528	.7641	-.5853
.5	.1656	.6596	1.2920	.4107	.7692	-.3086	.4262	.7058	-.5796
.6	.2379	.7875	1.2663	.4860	.7356	-.3626	.4939	.6483	-.5713
.7	.3230	.9125	1.2314	.5576	.6969	-.4110	.5559	.5917	-.5599
.8	.4203	1.0335	1.1866	.6252	.6536	-.4522	.6123	.5364	-.5452
.9	.5295	1.1495	1.1317	.6882	.6067	-.4846	.6632	.4827	-.5269
1.0	.6500	1.2595	1.0670	.7464	.5570	-.5071	.7089	.4311	-.5050
1.1	.7812	1.3626	.9934	.7996	.5057	-.5189	.7496	.3819	-.4797
1.2	.9223	1.4579	.9124	.8476	.4536	-.5198	.7854	.3353	-.4512
1.3	1.0725	1.5449	.8259	.8903	.4020	-.5101	.8167	.2917	-.4199
1.4	1.2310	1.6230	.7361	.9260	.3519	-.4906	.8438	.2513	-.3865
1.5	1.3968	1.6921	.6455	.9608	.3042	-.4627	.8671	.2144	-.3516
1.6	1.5691	1.7522	.5566	.9890	.2596	-.4282	.8868	.1811	-.3161
1.7	1.7469	1.8035	.4715	1.0128	.2187	-.3889	.9034	.1512	-.2805
1.8	1.9295	1.8467	.3924	1.0328	.1819	-.3470	.9172	.1249	-.2458
1.9	2.1160	1.8822	.3205	1.0494	.1494	-.3042	.9285	.1020	-.2125
2.0	2.3057	1.9110	.2569	1.0629	.1211	-.2622	.9377	.0824	-.1812
2.1	2.4980	1.9339	.2021	1.0737	.0969	-.2222	.9451	.0657	-.1524
2.2	2.6924	1.9517	.1559	1.0824	.0765	-.1854	.9509	.0518	-.1264
2.3	2.8882	1.9654	.1179	1.0891	.0597	-.1522	.9556	.0403	-.1034
2.4	3.0853	1.9756	.0875	1.0944	.0459	-.1231	.9591	.0310	-.0834
2.5	3.2833	1.9831	.0636	1.0984	.0349	-.0981	.9618	.0236	-.0663
2.6	3.4819	1.9885	.0454	1.1014	.0262	-.0770	.9639	.0176	-.0520
2.7	3.6809	1.9923	.0317	1.1037	.0194	-.0595	.9654	.0131	-.0401
2.8	3.8803	1.9950	.0217	1.1054	.0142	-.0454	.9665	.0095	-.0306
2.9	4.0799	1.9967	.0146	1.1066	.0102	-.0341	.9673	.0069	-.0230
3.0	4.2796	1.9979	.0096	1.1075	.0073	-.0252	.9679	.0049	-.0170
3.1	4.4794	1.9987	.0062	1.1081	.0051	-.0184	.9683	.0035	-.0124
3.2	4.6793	1.9992	.0039	1.1085	.0035	-.0132	.9686	.0024	-.0089
3.3	4.8793	1.9995	.0024	1.1088	.0024	-.0094	.9688	.0016	-.0063
3.4	5.0793	1.9997	.0015	1.1090	.0016	-.0066	.9690	.0011	-.0044
3.5	5.2792	1.9998	.0009	1.1091	.0011	-.0045	.9690	.0007	-.0030
3.6	5.4792	1.9999	.0005	1.1092	.0007	-.0031	.9691	.0005	-.0021
3.7	5.6792	2.0000	.0003	1.1093	.0004	-.0020	.9692	.0003	-.0013
3.8	5.8792	2.0000	.0002	1.1093	.0003	-.0013	.9692	.0002	-.0009
3.9	6.0792	2.0000	.0001	1.1093	.0002	-.0009	.9692	.0001	-.0006
4.0	6.2792	2.0000	.0000	1.1093	.0001	-.0006	.9692	.0001	-.0004
4.1	6.4792	2.0000	.0000	1.1093	.0001	-.0003	.9692	.0000	-.0002
4.2	6.6792	2.0000	.0000	1.1093	.0000	-.0002	.9692	.0000	-.0001
4.3	6.8792	2.0000	.0000	1.1093	.0000	-.0001	.9692	.0000	-.0001
4.4	7.0792	2.0000	.0000	1.1094	.0000	-.0001	.9692	.0000	-.0000
4.5	7.2792	2.0000	.0000	1.1094	.0000	-.0001	.9692	.0000	-.0000

TABLE II. - Continued. SOLUTIONS OF MOMENTUM AND ENERGY EQUATIONS

(b) f(0) = - 0.50

η	f	f'	f"	Ir	r	r'	Is	s	s'
0	-0.50000	0.00000	0.65796	0.0000	0.7987	0.0000	0.0000	1.0000	-0.3324
.1	-.49665	.06747	.69162	.0798	.7975	-.0167	.0983	.9659	-.3446
.2	-.48639	.13837	.72651	.1595	.7953	-.0357	.1932	.9311	-.3570
.3	-.46886	.21279	.76210	.2388	.7907	-.0573	.2845	.8947	-.3695
.4	-.44371	.29079	.79773	.3176	.7837	-.0814	.3721	.8572	-.3819
.5	-.41059	.37231	.83260	.3955	.7743	-.1083	.4559	.8184	-.3938
.6	-.36914	.45725	.86576	.4723	.7620	-.1377	.5357	.7784	-.4050
.7	-.31903	.54537	.89614	.5478	.7467	-.1695	.6115	.7374	-.4152
.8	-.25997	.63634	.92253	.6216	.7280	-.2031	.6832	.6954	-.4240
.9	-.19168	.72970	.94367	.6933	.7060	-.2381	.7506	.6527	-.4310
1.0	-.11397	.82486	.95828	.7626	.6804	-.2735	.8137	.6093	-.4358
1.1	-.02667	.92110	.96512	.8293	.6513	-.3083	.8725	.5656	-.4380
1.2	.07026	1.01759	.96310	.8928	.6188	-.3413	.9268	.5218	-.4373
1.3	.17682	1.11340	.95135	.9529	.5831	-.3712	.9768	.4782	-.4335
1.4	.29288	1.20752	.92934	1.0093	.5447	-.3966	1.0225	.4352	-.4262
1.5	.41823	1.29892	.89694	1.0618	.5040	-.4163	1.0639	.3931	-.4155
1.6	.55254	1.38657	.85451	1.1101	.4617	-.4291	1.1012	.3523	-.4012
1.7	.69539	1.46951	.80288	1.1541	.4184	-.4345	1.1344	.3130	-.3836
1.8	.84626	1.54688	.74336	1.1938	.3751	-.4320	1.1638	.2756	-.3629
1.9	1.00456	1.61798	.67770	1.2291	.3323	-.4218	1.1896	.2405	-.3395
2.0	1.16963	1.68228	.60792	1.2603	.2909	-.4043	1.2120	.2078	-.3140
2.1	1.34078	1.73950	.53624	1.2874	.2516	-.3807	1.2313	.1777	-.2869
2.2	1.51729	1.78954	.46485	1.3107	.2150	-.3520	1.2476	.1505	-.2588
2.3	1.69845	1.83255	.39582	1.3305	.1814	-.3198	1.2614	.1260	-.2305
2.4	1.88357	1.86884	.33093	1.3471	.1511	-.2856	1.2729	.1043	-.2026
2.5	2.07201	1.89891	.27155	1.3608	.1243	-.2507	1.2824	.0854	-.1758
2.6	2.26317	1.92337	.21864	1.3720	.1009	-.2164	1.2901	.0691	-.1504
2.7	2.45652	1.94287	.17268	1.3811	.0809	-.1839	1.2963	.0553	-.1269
2.8	2.65160	1.95814	.13376	1.3883	.0641	-.1537	1.3012	.0437	-.1055
2.9	2.84803	1.96985	.10161	1.3940	.0501	-.1266	1.3051	.0341	-.0866
3.0	3.04548	1.97866	.07568	1.3984	.0386	-.1026	1.3081	.0263	-.0700
3.1	3.24369	1.98517	.05526	1.4018	.0294	-.0820	1.3122	.0200	-.0558
3.2	3.44245	1.98987	.03956	1.4044	.0221	-.0645	1.3135	.0150	-.0439
3.3	3.64162	1.99321	.02776	1.4063	.0164	-.0500	1.3144	.0112	-.0340
3.4	3.84106	1.99553	.01910	1.4077	.0120	-.0382	1.3151	.0082	-.0260
3.5	4.04070	1.99711	.01288	1.4087	.0087	-.0288	1.3156	.0059	-.0196
3.6	4.24046	1.99817	.00851	1.4095	.0062	-.0214	1.3160	.0042	-.0145
3.7	4.44032	1.99886	.00552	1.4100	.0044	-.0156	1.3162	.0030	-.0106
3.8	4.64023	1.99930	.00350	1.4104	.0030	-.0113	1.3164	.0021	-.0077
3.9	4.84017	1.99958	.00218	1.4106	.0021	-.0080	1.3165	.0014	-.0054
4.0	5.04014	1.99975	.00133	1.4108	.0014	-.0056	1.3166	.0010	-.0038
4.1	5.24012	1.99986	.00080	1.4109	.0009	-.0039	1.3167	.0006	-.0026
4.2	5.44011	1.99992	.00047	1.4110	.0006	-.0026	1.3167	.0004	-.0018
4.3	5.64010	1.99995	.00027	1.4110	.0004	-.0018	1.3167	.0003	-.0012
4.4	5.84010	1.99998	.00015	1.4111	.0003	-.0012	1.3167	.0002	-.0008
4.5	6.04010	1.99999	.00008	1.4111	.0002	-.0008	1.3167	.0001	-.0005
4.6	6.24010	1.99999	.00005	1.4111	.0001	-.0005	1.3167	.0001	-.0003
4.7	6.44010	2.00000	.00002	1.4111	.0001	-.0003	1.3167	.0000	-.0002
4.8	6.64010	2.00000	.00001	1.4111	.0000	-.0002	1.3167	.0000	-.0001
4.9	6.84010	2.00000	.00001	1.4111	.0000	-.0001	1.3167	.0000	-.0001
5.0	7.04010	2.00000	.00000	1.4111	.0000	-.0001	1.3167	.0000	-.0000
5.1	7.24010	2.00000	.00000	1.4111	.0000	-.0001	1.3167	.0000	-.0000
5.2	7.44010	2.00000	.00000	1.4111	.0000	-.0000	1.3167	.0000	-.0000

TABLE II. - Continued. SOLUTIONS OF MOMENTUM AND ENERGY EQUATIONS

(c) f(0) = - 0.75

η	f	f'	f"	Ir	r	r'	Is	s	s'
0	-0.75000	0.00000	0.37446	0.0000	0.7680	0.0000	0.0000	1.0000	-0.2127
.1	-.74808	.03888	.40359	.0768	.7676	-.0056	.0989	.9779	-.2245
.2	-.74212	.08079	.43483	.1536	.7669	-.0124	.1956	.9551	-.2369
.3	-.73181	.12592	.46810	.2302	.7652	-.0206	.2899	.9307	-.2498
.4	-.71682	.17447	.59329	.3066	.7627	-.0304	.3817	.9051	-.2632
.5	-.69680	.22663	.54017	.3827	.7591	-.0420	.4709	.8781	-.2769
.6	-.67137	.28255	.57844	.4584	.7542	-.0557	.5573	.8497	-.2909
.7	-.64016	.34235	.61768	.5336	.7479	-.0716	.6408	.8199	-.3050
.8	-.60277	.40610	.65732	.6080	.7398	-.0898	.7212	.7887	-.3190
.9	-.55881	.47381	.69666	.6815	.7298	-.1105	.7985	.7561	-.3326
1.0	-.50788	.54540	.73487	.7539	.7176	-.1336	.8724	.7222	-.3456
1.1	-.44960	.62071	.77096	.8250	.7030	-.1591	.9429	.6870	-.3578
1.2	-.38362	.69948	.80381	.8945	.6808	-.1866	1.0098	.6507	-.3687
1.3	-.30960	.78133	.83222	.9621	.6656	-.2157	1.0730	.6133	-.3780
1.4	-.22727	.86574	.85492	1.0276	.6426	-.2458	1.1324	.5752	-.3854
1.5	-.13639	.95208	.87067	1.0906	.6165	-.2761	1.1880	.5363	-.3905
1.6	-.03681	1.02960	.87830	1.1508	.5874	-.3055	1.2397	.4971	-.3930
1.7	.07154	1.12744	.87685	1.2080	.5554	-.3328	1.2874	.4578	-.3925
1.8	.18865	1.21464	.86557	1.2618	.5209	-.3570	1.3312	.4188	-.3889
1.9	.31441	1.30021	.84413	1.3121	.4842	-.3767	1.3712	.3802	-.3819
2.0	.44861	1.38313	.81259	1.3587	.4458	-.3909	1.4073	.3425	-.3716
2.1	.59092	1.46241	.77149	1.4013	.4062	-.3987	1.4397	.3060	-.3579
2.2	.74094	1.53714	.72183	1.4399	.3663	-.3997	1.4685	.2710	-.3412
2.3	.89817	1.60654	.66507	1.4746	.3265	-.3936	1.4940	.2378	-.3217
2.4	1.06205	1.66998	.60301	1.5053	.2878	-.3808	1.5162	.2067	-.2998
2.5	1.23195	1.72703	.53769	1.5322	.2506	-.3619	1.5354	.1779	-.2760
2.6	1.40723	1.77748	.47124	1.5555	.2156	-.3378	1.5518	.1516	-.2510
2.7	1.58723	1.82131	.40573	1.5755	.1831	-.3099	1.5658	.1278	-.2254
2.8	1.77128	1.85872	.34302	1.5923	.1537	-.2793	1.5775	.1065	-.1997
2.9	1.95877	1.89006	.28467	1.6064	.1273	-.2475	1.5872	.0878	-.1746
3.0	2.14911	1.91583	.23182	1.6179	.1042	-.2157	1.5951	.0715	-.1506
3.1	2.34177	1.93663	.18520	1.6273	.0841	-.1849	1.6016	.0576	-.1281
3.2	2.53629	1.95309	.14512	1.6409	.0671	-.1560	1.6108	.0459	-.1075
3.3	2.73226	1.96587	.11151	1.6456	.0528	-.1296	1.6140	.0361	-.0889
3.4	2.92936	1.97560	.08402	1.6492	.0411	-.1060	1.6164	.0280	-.0725
3.5	3.12730	1.98286	.06207	1.6520	.0315	-.0854	1.6183	.0215	-.0583
3.6	3.32587	1.98817	.04496	1.6541	.0239	-.0678	1.6197	.0163	-.0462
3.7	3.52489	1.99198	.03192	1.6557	.0179	-.0530	1.6208	.0122	-.0361
3.8	3.72423	1.99466	.02222	1.6568	.0132	-.0409	1.6216	.0090	-.0278
3.9	3.92379	1.99651	.01516	1.6576	.0096	-.0310	1.6221	.0065	-.0211
4.0	4.12351	1.99776	.01014	1.6583	.0069	-.0232	1.6225	.0047	-.0158
4.1	4.32333	1.99859	.00665	1.6587	.0049	-.0172	1.6228	.0033	-.0117
4.2	4.52322	1.99913	.00427	1.6590	.0035	-.0125	1.6230	.0023	-.0085
4.3	4.72315	1.99947	.00269	1.6592	.0024	-.0089	1.6231	.0016	-.0061
4.4	4.92311	1.99969	.00166	1.6594	.0016	-.0063	1.6232	.0011	-.0043
4.5	5.12308	1.99982	.00101	1.6595	.0011	-.0044	1.6233	.0007	-.0030
4.6	5.32307	1.99989	.00060	1.6596	.0007	-.0030	1.6233	.0005	-.0021
4.7	5.52306	1.99994	.00035	1.6596	.0005	-.0020	1.6233	.0003	-.0014
4.8	5.72306	1.99997	.00020	1.6597	.0003	-.0014	1.6234	.0002	-.0009
4.9	5.92306	1.99998	.00011	1.6597	.0002	-.0009	1.6234	.0001	-.0006
5.0	6.12305	1.99999	.00006	1.6597	.0001	-.0006	1.6234	.0001	-.0004
5.1	6.32305	2.00000	.00003	1.6598	.0001	-.0004	1.6234	.0001	-.0003
5.2	6.52305	2.00000	.00002	1.6598	.0001	-.0002	1.6234	.0000	-.0002
5.3	6.72305	2.00000	.00001	1.6598	.0000	-.0001	1.6234	.0000	-.0001
5.4	6.92305	2.00000	.00000	1.6598	.0000	-.0001	1.6234	.0000	-.0001
5.5	7.12305	2.00000	.00000	1.6598	.0000	-.0000	1.6234	.0000	-.0001

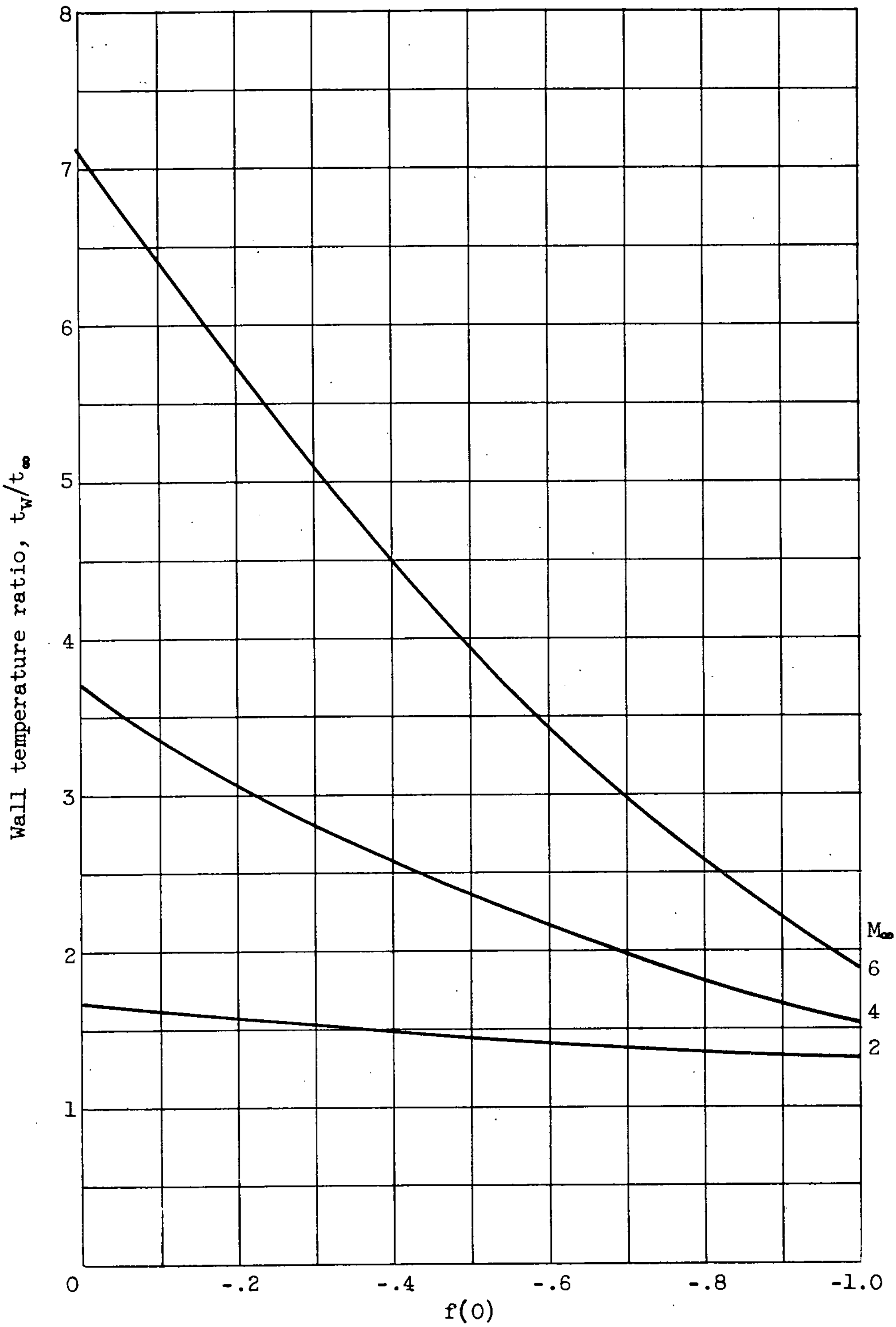
TABLE II. - Concluded. SOLUTIONS OF MOMENTUM AND ENERGY EQUATIONS

(d) f(0) = - 1.00

η	f	f'	f"	Ir	r	r'	Is	s	s'
0	-1.00000	0.00000	0.14208	0.0000	0.7332	-0.0000	0.0000	1.0000	-0.1010
.1	-.99927	.01494	.15701	.0733	.7331	-.0008	.0995	.9894	-.1086
.2	-.99696	.03145	.17350	.1466	.7330	-.0019	.1979	.9783	-.1166
.3	-.99292	.04970	.19165	.2199	.7328	-.0033	.2951	.9662	-.1253
.4	-.98696	.06984	.21160	.2932	.7323	-.0050	.3911	.9532	-.1346
.5	-.97888	.09208	.23346	.3664	.7317	-.0073	.4857	.9392	-.1444
.6	-.96846	.11660	.25734	.4395	.7309	-.0100	.5789	.9243	-.1549
.7	-.95547	.14362	.28333	.5125	.7297	-.0135	.6705	.9082	-.1660
.8	-.93965	.17334	.31149	.5854	.7282	-.0177	.7605	.8910	-.1778
.9	-.92071	.20599	.34187	.6582	.7261	-.0229	.8487	.8727	-.1901
1.0	-.89835	.24179	.37443	.7306	.7235	-.0292	.9350	.8530	-.2030
1.1	-.87224	.28095	.40910	.8028	.7203	-.0368	1.0193	.8320	-.2163
1.2	-.84204	.32367	.44573	.8747	.7161	-.0459	1.1014	.8097	-.2301
1.3	-.80738	.37015	.48407	.9460	.7110	-.0567	1.1812	.7860	-.2442
1.4	-.76788	.42053	.52376	1.0168	.7047	-.0694	1.2585	.7609	-.2585
1.5	-.72314	.47493	.56433	1.0869	.6970	-.0842	1.3333	.7343	-.2727
1.6	-.67276	.53341	.60515	1.1562	.6878	-.1012	1.4053	.7063	-.2868
1.7	-.61632	.59595	.64547	1.2244	.6767	-.1204	1.4745	.6770	-.3004
1.8	-.55343	.66345	.68439	1.2915	.6637	-.1418	1.5407	.6463	-.3134
1.9	-.48371	.73274	.72086	1.3571	.6483	-.1653	1.6037	.6143	-.3253
2.0	-.40677	.80651	.75372	1.4211	.6305	-.1906	1.6635	.5813	-.3359
2.1	-.32230	.88332	.78176	1.4831	.6102	-.2172	1.7200	.5472	-.3449
2.2	-.23002	.96265	.80370	1.5430	.5871	-.2444	1.1493	.5124	-.3518
2.3	-.12971	1.04382	.81834	1.6004	.5613	-.2714	1.8224	.4769	-.3564
2.4	-.02122	1.12604	.82460	1.6552	.5328	-.2973	1.8683	.4411	-.3584
2.5	.09550	1.20843	.82160	1.7069	.5019	-.3209	1.9106	.4053	-.3575
2.6	.22044	1.29003	.80877	1.7555	.4688	-.3411	1.9494	.3698	-.3535
2.7	.35345	1.36985	.78595	1.8006	.4338	-.3568	1.9846	.3348	-.3463
2.8	.49432	1.44690	.75338	1.8422	.3976	-.3672	2.0164	.3006	-.3359
2.9	.64271	1.52023	.71179	1.8801	.3606	-.3714	2.0448	.2677	-.3225
3.0	.79821	1.58899	.66235	1.9143	.3235	-.3693	2.0700	.2362	-.3062
3.1	.96033	1.65249	.60663	1.9448	.2870	-.3607	2.0921	.2065	-.2874
3.2	1.12851	1.71017	.54649	1.9717	.2516	-.3462	2.1113	.1788	-.2666
3.3	1.30216	1.76170	.48397	1.9952	.2179	-.3263	2.1279	.1533	-.2443
3.4	1.48064	1.80695	.42112	2.0154	.1864	-.3023	2.1421	.1300	-.2211
3.5	1.66334	1.84598	.35988	2.0326	.1576	-.2751	2.1540	.1091	-.1974
3.6	1.84964	1.87904	.30191	2.0470	.1315	-.2462	2.1639	.0905	-.1740
3.7	2.03896	1.90652	.24857	2.0590	.1083	-.2166	2.1722	.0742	-.1513
3.8	2.23078	1.92894	.20079	2.0688	.0882	-.1874	2.1789	.0602	-.1297
3.9	2.42460	1.94688	.15910	2.0767	.0708	-.1596	2.1843	.0482	-.1097
4.0	2.62003	1.96097	.12363	2.0830	.0562	-.1337	2.1886	.0382	-.0915
4.1	2.81669	1.97181	.09421	2.0880	.0440	-.1103	2.1920	.0299	-.0752
4.2	3.01430	1.97999	.07039	2.0919	.0340	-.0897	2.1946	.0231	-.0610
4.3	3.21262	1.98605	.05156	2.0949	.0260	-.0718	2.1966	.0176	-.0488
4.4	3.41145	1.99045	.03702	2.0971	.0196	-.0566	2.1982	.0141	-.0384
4.5	3.61066	1.99357	.02606	2.0988	.0146	-.0440	2.1993	.0099	-.0298
4.6	3.81014	1.99575	.01798	2.1001	.0107	-.0337	2.2002	.0072	-.0228
4.7	4.00979	1.99725	.01217	2.1010	.0078	-.0254	2.2008	.0053	-.0172
4.8	4.20957	1.99824	.00807	2.1017	.0055	-.0189	2.2012	.0038	-.0128
4.9	4.40943	1.99890	.00524	2.1021	.0039	-.0139	2.2015	.0027	-.0094
5.0	4.60934	1.99932	.00334	2.1025	.0027	-.0100	2.2018	.0018	-.0068
5.1	4.80929	1.99959	.00209	2.1027	.0019	-.0072	2.2019	.0013	-.0048
5.2	5.00926	1.99976	.00128	2.1029	.0013	-.0050	2.2020	.0009	-.0034
5.3	5.20924	1.99986	.00077	2.1030	.0009	-.0035	2.2021	.0006	-.0024
5.4	5.40923	1.99991	.00045	2.1031	.0006	-.0024	2.2021	.0004	-.0016
5.5	5.60922	1.99995	.00026	2.1031	.0004	-.0016	2.2022	.0002	-.0011
5.6	5.80922	1.99997	.00015	2.1031	.0002	-.0011	2.2022	.0002	-.0007
5.7	6.00921	1.99998	.00008	2.1031	.0001	-.0007	2.2022	.0001	-.0005
5.8	6.20921	1.99999	.00004	2.1031	.0001	-.0004	2.2022	.0001	-.0003
5.9	6.40921	1.99999	.00002	2.1031	.0001	-.0003	2.2022	.0000	-.0002
6.0	6.60921	1.99999	.00001	2.1031	.0000	-.0002	2.2022	.0000	-.0001
6.1	6.80921	1.99999	.00001	2.1032	.0000	-.0001	2.2022	.0000	-.0001
6.2	7.00921	1.99999	.00000	2.1032	.0000	-.0001	2.2022	.0000	-.0001
6.3	7.20921	1.99999	.00000	2.1032	.0000	-.0000	2.2022	.0000	-.0001

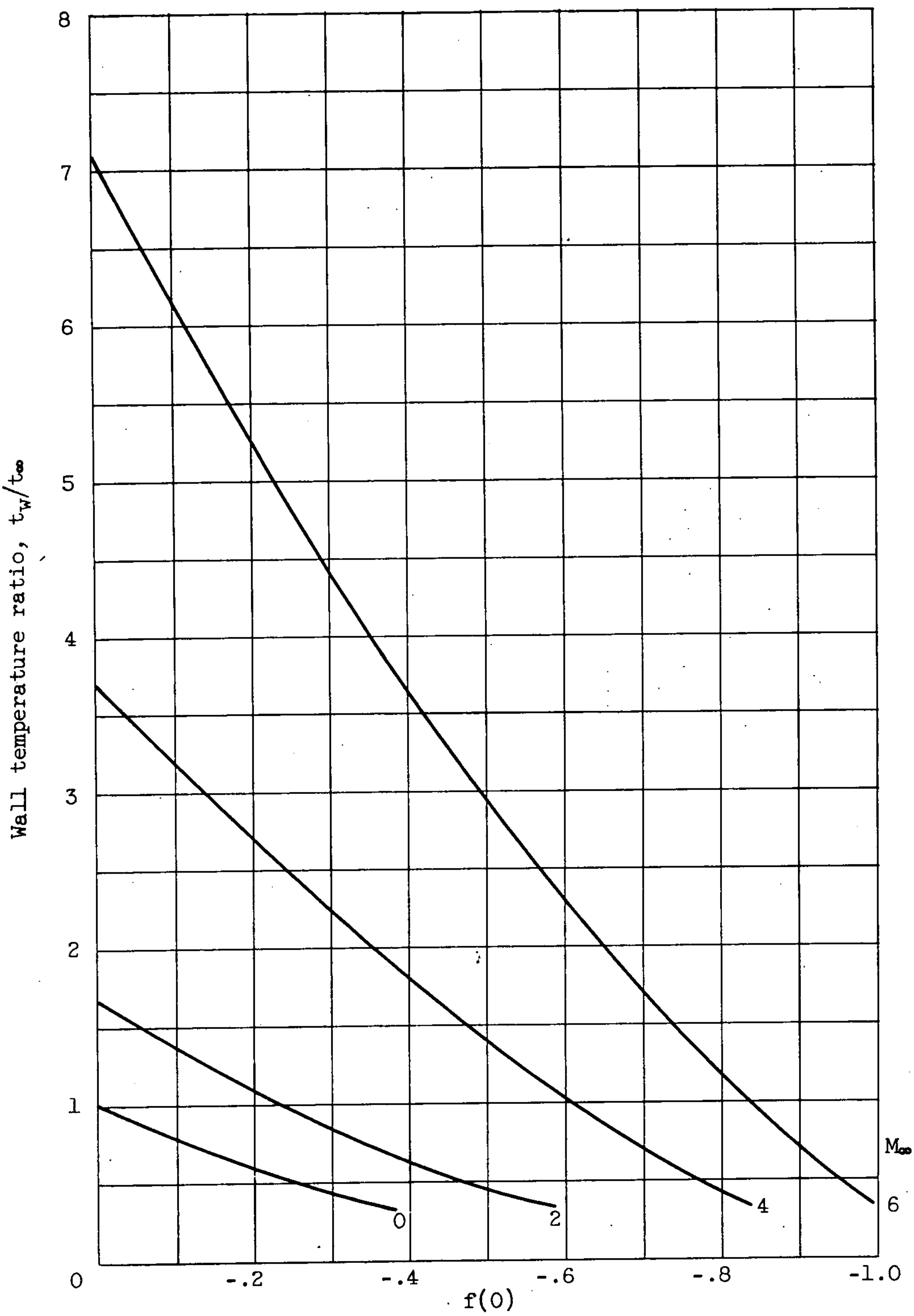
TABLE III. - VALUES APPEARING IN
EXPRESSION FOR DISPLACEMENT THICKNESS

$f(0)$	α	$I_r(\infty)$	$I_s(\infty)$
0	0.8604	1.1094	0.9692
- .50	1.2300	1.4111	1.3167
- .75	1.5635	1.6598	1.6234
-1.00	2.1954	2.1032	2.2022



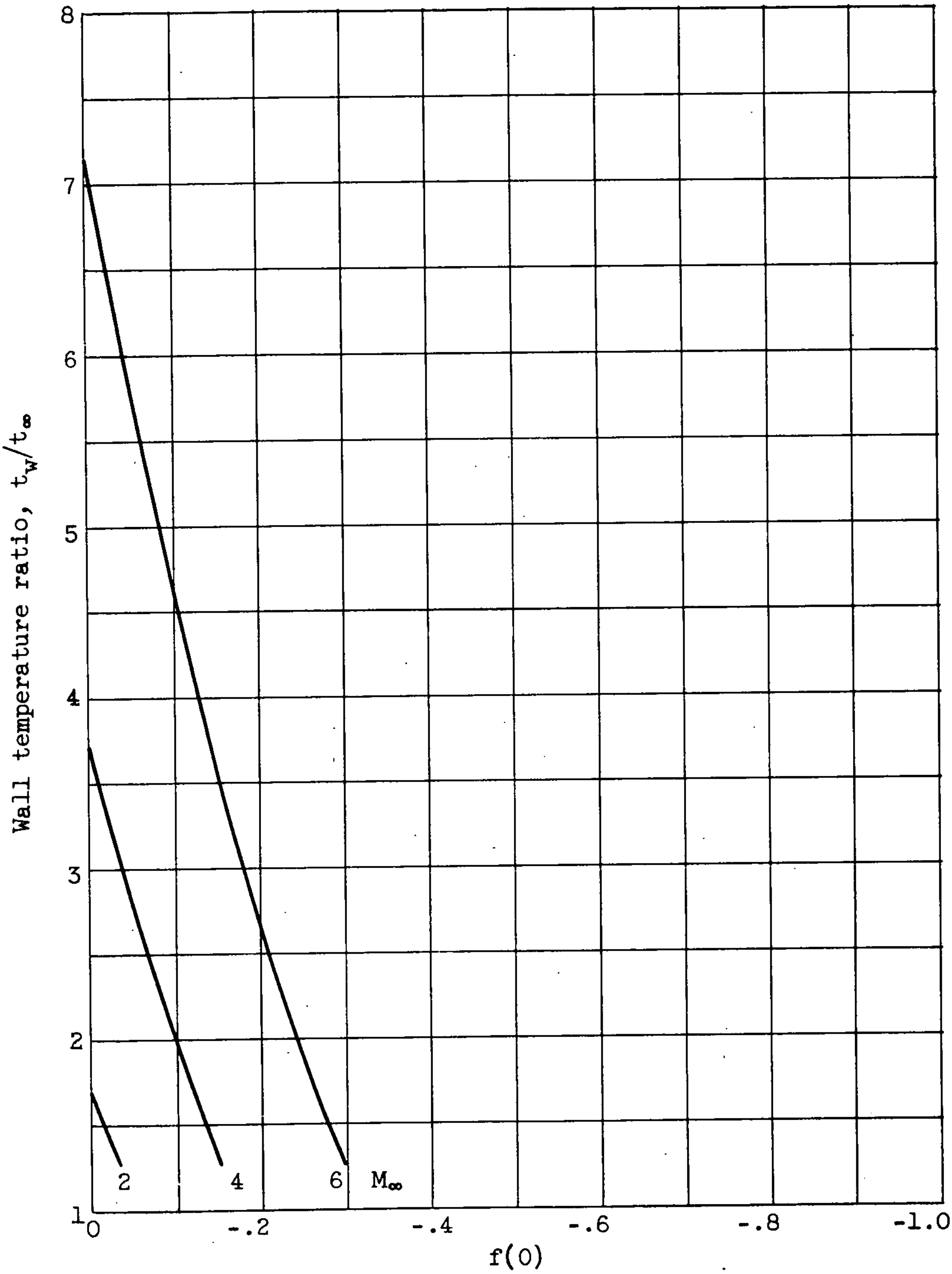
(a) Coolant, air; t_c, 40° F.

Figure 1. - Effect of Mach number on surface temperature.



(b) Coolant, liquid air; t_c, -320° F; t_∞, -67.6° F.

Figure 1. - Continued. Effect of Mach number on surface temperature.



(c) Coolant, water; t_c, 40° F; t_∞, -67.6° F.

Figure 1. - Concluded. Effect of Mach number on surface temperature.

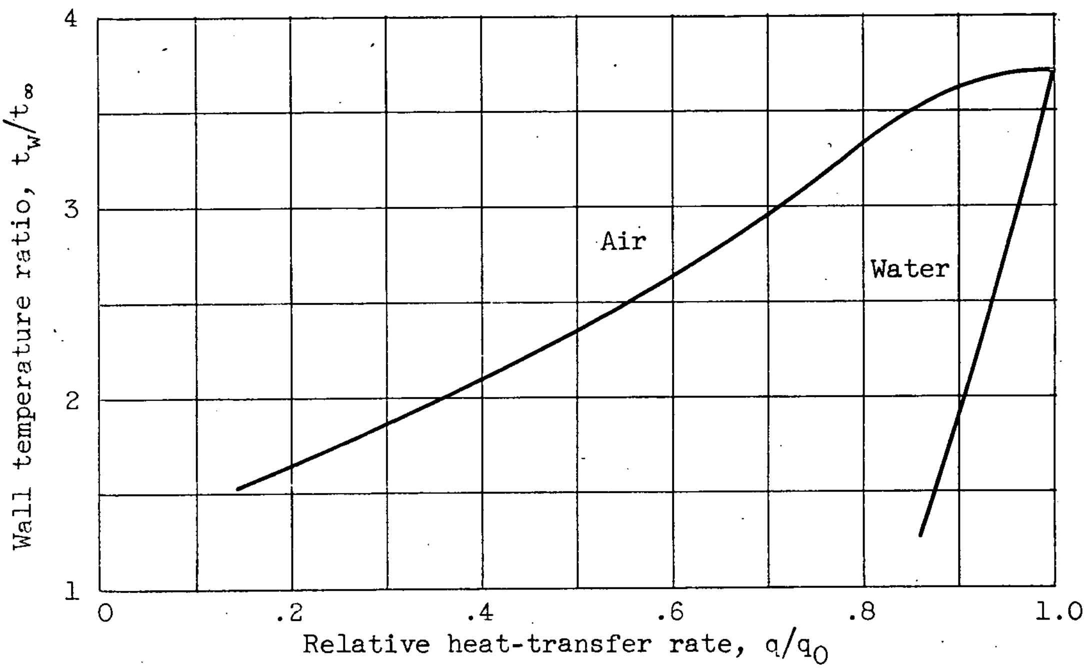


Figure 2. - Relative heat-transfer rates required to maintain given surface temperature. q ~ transpiration cooling; q_0 ~ internal cooling. M_∞, 4; t_c, 40° F; t_∞, -67.6° F.

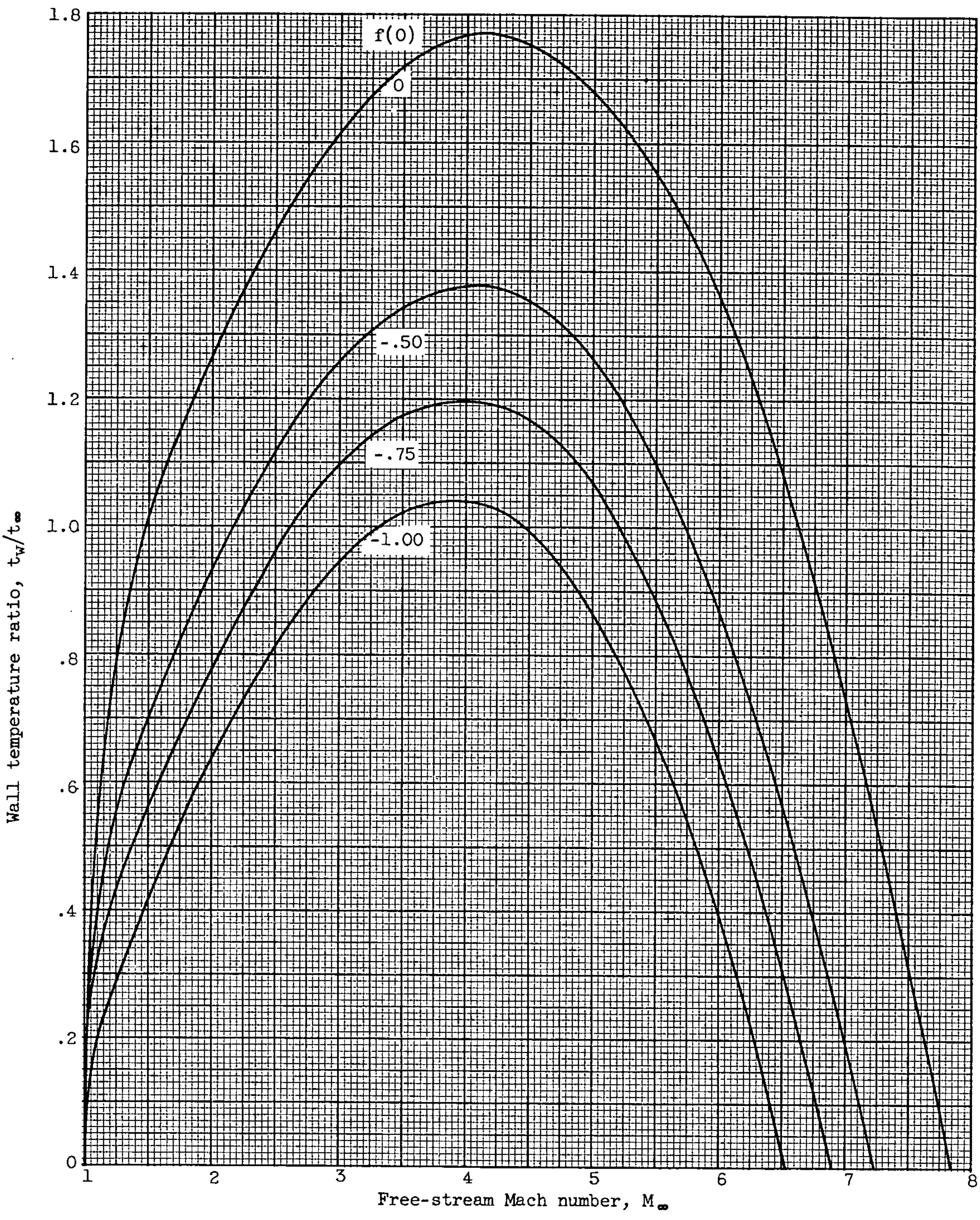


Figure 3. - Limiting wall temperature required for complete stabilization of boundary layer.

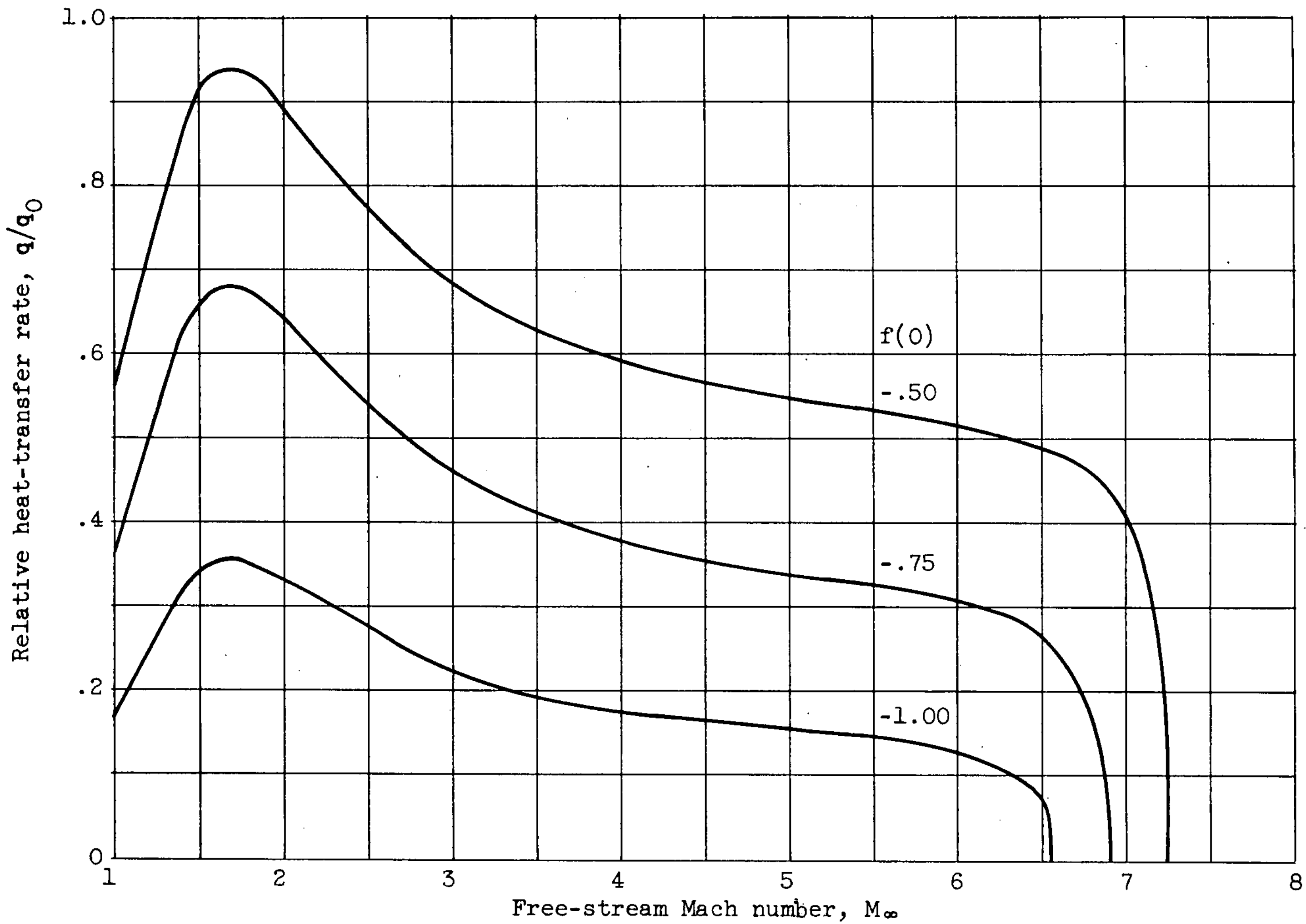


Figure 4. - Comparison of cooling requirements for complete stability for several rates of fluid injection. q ~ transpiration cooling for stability; q_0 ~ internal cooling for stability.

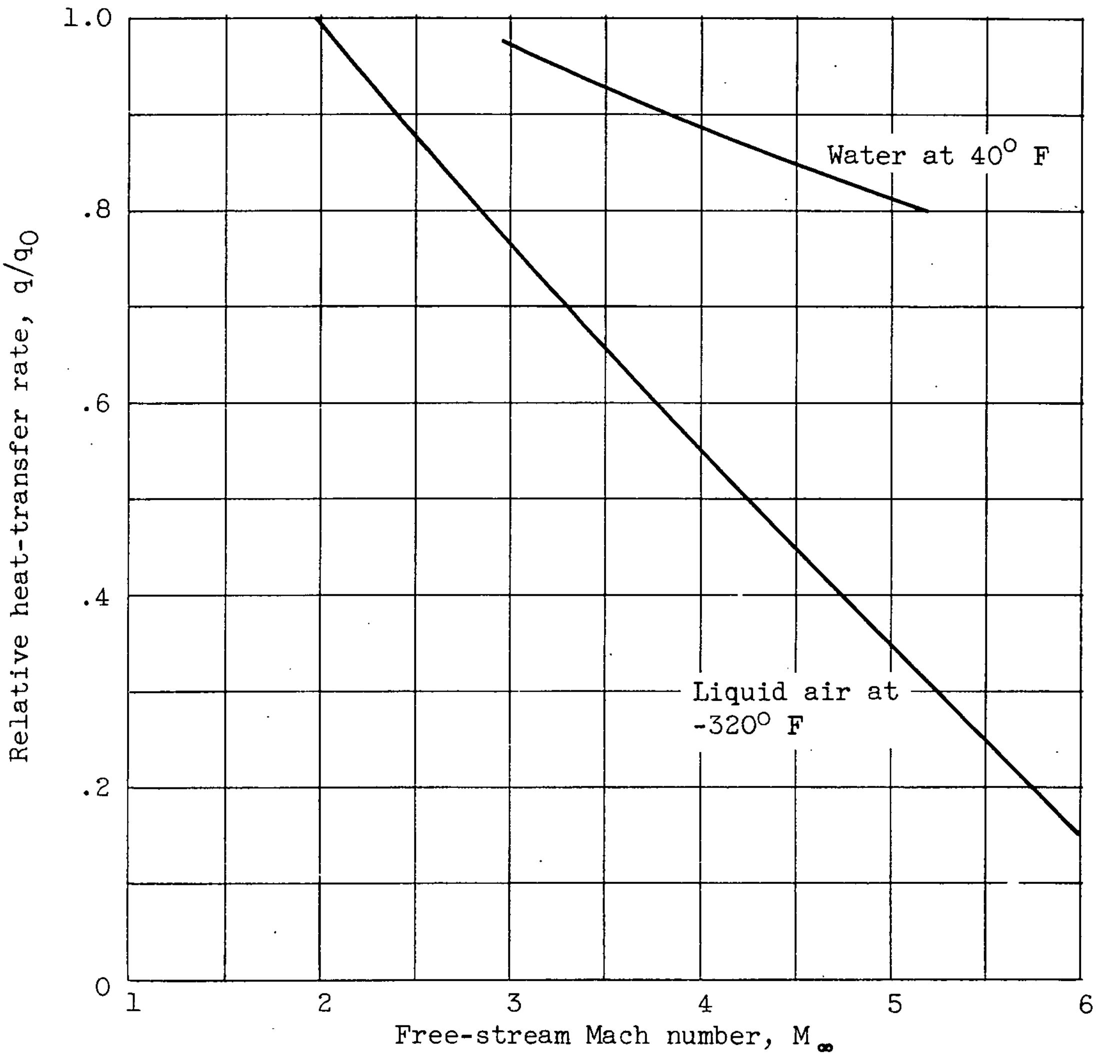


Figure 5. - Relative heat-transfer rates required for complete stabilization of boundary layer. q ~ transpiration cooling; q_0 ~ internal cooling.

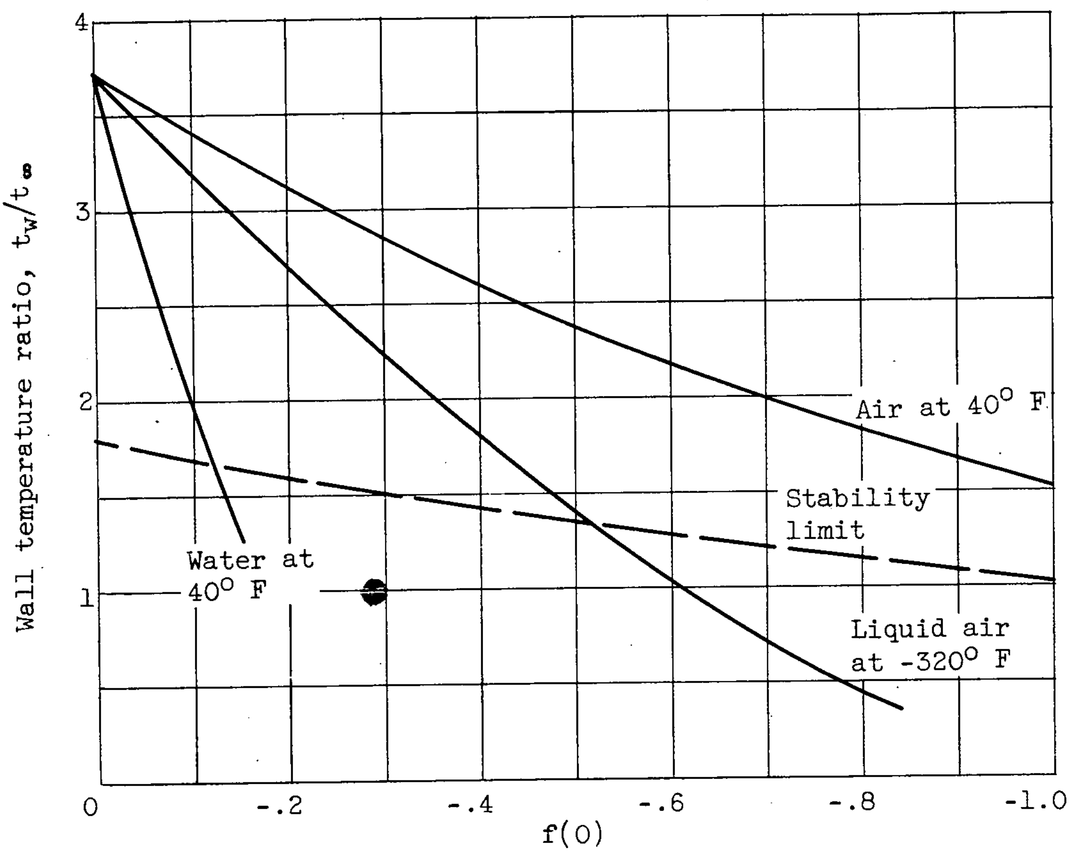


Figure 6. - Comparison of several coolants and stability limit. M_∞, 4; t_∞, -67.6° F.

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