NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3397

AN EVALUATION OF NON-NEWTONIAN FLOW IN PIPE LINES

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Washington
February 1955

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SUMMARY

An analysis is presented of a method for determining pressure losses due to the flow of non-Newtonian materials in pipe lines by using basic flow data obtained from measurements of flow curves, which are rate-of-shear - shearing-stress curves. The advantages of properly designed rotational viscometers over capillary viscometers for measuring these flow curves and interpreting them to obtain the basic flow parameters are discussed. Dimensionless parameters are calculated from these basic flow data and are used to construct a generalized friction diagram to describe the flow characteristics of Newtonian and non-Newtonian materials in pipe lines.

Reported flow and pressure data are analyzed. Flow-pressure losses due to the flow of non-Newtonian materials in pipe lines, as calculated by the use of reported basic flow data and the generalized friction diagram, are compared with those obtained directly from reported pressure measurements in the pipe line. The importance of considering the effect of temperature on the flow parameters of non-Newtonian materials, when making flow-pressure-loss calculations for the flow in pipes, is mentioned. The pipe transition pressure losses expected from non-Newtonian materials are discussed.

INTRODUCTION

A great number of commercial materials, which heretofore have been considered to be Newtonian liquids, in reality show non-Newtonian flow characteristics and thus have to be treated accordingly. Since many of these materials are passed through piping systems in their process of manufacture and application, the problem of the proper design of such a pipe-line system to carry non-Newtonian materials under given applied pressures and at desired flow rates has become of great interest.

Pipe-line resistance to the flow of Newtonian liquids has been well understood for a long time and is frequently represented by a friction diagram, in which a viscous friction factor is plotted as a function of the Reynolds number for pipes of various surface roughness. Such diagrams can be found in the literature (as, for instance, in ref. 1) and extend over laminar and turbulent flow conditions. Hedström (ref. 2) calculated a friction diagram for plastic materials of the Bingham type, based on a mathematical analysis of their flow curves (refs. 2 and 3) and expressed in terms of two basic flow parameters characterizing the nature of the flow.

This paper presents a similar friction diagram, arranged for general use in calculating pressure losses in pipes induced by the resistance of Bingham plastics. This same friction diagram is extended to allow evaluation of pressure losses in pipe lines carrying pseudoplastic and dilatant materials; this diagram is based on an empirical analysis of the flow curves of the materials and is expressed in terms of two empirical flow parameters characterizing the nature of the flow. Experimental flow and pressure-loss data reported by various investigators for Bingham plastics (refs. 4 and 5) and for pseudoplastic materials (refs. 6 to 8) are used to check the validity of this generalized friction diagram.

An attempt is made to interpret the flow curves obtained for Bingham plastics and pseudoplastic and dilatant materials from measurements with a concentric-cylinder rotational viscometer and a capillary viscometer in order to obtain meaningful values for the flow parameters. A short discussion of the flow characteristics of thixotropic materials is given to indicate the extent to which the generalized friction diagram may be used for those materials. The effect of temperature on the flow characteristics of all materials is mentioned. In addition, the pipe transition losses - such as occur in contractions, extensions, bends, orifices, and pipe entrances, which are important for the proper design of a pipe-line system - are discussed for the case of non-Newtonian flow.

The calculation of friction factors by the use of the characteristic flow parameters, which are obtained from non-Newtonian viscometer flow curves, is demonstrated for a pipe line carrying a Bingham plastic and a pseudoplastic material at different flow rates.

BASIC FLOW RELATIONS

Newtonian Materials

A Newtonian material is characterized by one material constant, namely, the Newtonian viscosity μ. In laminar flow, the ratio of the shearing stress S to the rate of shear dv/dr is constant, and the viscosity μ is defined by this constant relation so that

$$\mu = \frac{S}{dv/dr} \tag{1}$$

(All symbols are defined in appendix A.) Thus, the flow curve, which is the rate-of-shear - shearing-stress curve, is a straight line passing through the origin (fig. 1(a)). In turbulent flow, the shearing stress increases faster than the rate of shear (fig. 1(b)), and equation (1) is not valid. In fact, it is difficult to obtain the absolute viscosity of a Newtonian liquid from its turbulent-flow curve.

The viscosity of a Newtonian material can be obtained from any "one-point" flow determination under laminar-flow conditions.

If a concentric-cylinder rotational viscometer is used, equation (1) can be expressed so that

$$\frac{S}{dv/dr} = \frac{T/2\pi R^2 h}{2\omega \Big/ R^2\left(\frac{1}{R_b^2} - \frac{1}{R_c^2}\right)} = \frac{S_R}{G_R} \tag{2}$$

and thus,

$$\mu = \frac{T\left(\frac{1}{R_b^2} - \frac{1}{R_c^2}\right)}{4\pi h\omega} \tag{3}$$

If a capillary viscometer or a pipe is used, equation (1) can be expressed (ref. 9) so that

$$\frac{S}{dv/dr} = \frac{PD/4L}{8v/D} = \frac{S_W}{G_{m,P}} \tag{4}$$

and becomes Poiseuille's equation, where

$$\mu = \frac{PD^2}{32Lv} \tag{5}$$

The viscosity μ has the units of poises if all the other variables are given in c.g.s. units. Equations (2) to (5) are derived under the assumption that slippage flow, end effects, and kinetic energy effects are negligible.

Near the entrance of a capillary or pipe, the flow approaches the fully developed laminar-flow condition of equations (4) and (5) only asymptotically. In accordance with reference 10, this condition is achieved in laminar flow at an approximate entrance length of

$$L_e = 0.029 \, Re[\mu] \, D \tag{6}$$

where $Re[\mu] = Dv\rho/\mu$.

For the description of pipe-line flow characteristics, it has been customary to rewrite equation (5) in a dimensionless form, so as to relate a friction factor φ to the Reynolds number Re. For laminar flow of Newtonian fluids, then,

$$\varphi = \frac{64}{Re[\mu]} = \varphi_0[\mu] \tag{7}$$

where

$$\varphi = \frac{2PD}{L\rho v^2} \tag{8}$$

For turbulent flow of Newtonian liquids in pipes, empirical equations have been established and are given (ref. 1) as a function of the friction factor φ. They are, for smooth pipes

$$\frac{1}{\sqrt{\varphi}} = 2 \log \left(Re[\mu]\sqrt{\varphi}\right) - 0.8 \tag{9}$$

and for rough pipes

$$\frac{1}{\sqrt{\varphi}} = 2 \log \left(\frac{D}{2k}\right) + 1.7 \tag{10}$$

Thus, the flow depends on the viscosity in smooth pipes but is independent of viscosity in rough pipes under turbulent-flow conditions. In laminar flow the pressure drop is a function of the velocity (eq. (5)), while in turbulent flow it is a function of the square of the velocity (eqs. (9) and (10) after substitution of the expressions for $Re[\mu]$ and φ).

Bingham Plastics

Bingham plastics are defined by Bingham's equation (ref. 11), where

$$U = \frac{S - f}{dv/dr} \tag{11}$$

so that Bingham plastics have two characteristic material constants, namely, the plastic viscosity U and the yield value f. The flow curve in laminar flow is a straight line with an intercept on the shearing-stress axis (fig. 2(a)). The yield-value intercept indicates that flow can start only if a shearing stress larger than the yield-value stress is applied.

Experimental measurements at at least two points must be made under laminar-flow conditions to determine the plastic viscosity and yield value of a Bingham material. The instrument used for these determinations must be constructed in such a way as to make stress and shear calculations possible.

In a concentric-cylinder rotational viscometer, equation (11) can be integrated in accordance with Reiner and Riwlin (ref. 12) so that

$$U = \frac{T - T_0}{4\pi h\omega}\left(\frac{1}{R_b^2} - \frac{1}{R_c^2}\right) = \frac{(S - S_0)_R}{G_R} \qquad (12)$$

and

$$f = \frac{T_0\left(\frac{1}{R_b^2} - \frac{1}{R_c^2}\right)}{4\pi h \ln\left(\frac{R_c}{R_b}\right)} = \left(S_0\right)_R \frac{R^2\left(\frac{1}{R_b^2} - \frac{1}{R_c^2}\right)}{2 \ln\left(\frac{R_c}{R_b}\right)} \qquad (13)$$

When plotting flow curves, G is plotted against S. In the case of rotational viscometers, both these values are functions of R (eq. (2)). However, since the slope of the straight-line flow curves is independent of R, the viscosity values μ and U (eqs. (3) and (12)) are not affected by the choice of R. This is different when calculating the yield value from the stress intercept $(S_0)_R$ (fig. 2(b)), since $(S_0)_R$ varies with the value of R (eq. (13)) that is used to calculate S_R and G_R for plotting the flow curves. The quantity T in equations (2) and (12) is an applied torque and thus corresponds to a mean or integrated value of S_R. Mean values of rate of shear and shearing stress are approximated by setting $R = R_{av} = (R_b + R_c)/2$. Thus, for convenience, these values will be called "mean rate of shear" and "mean shearing stress" and are used in plotting the flow curves. Then, $S_R = S_{m,R}$ and $G_R = G_{m,R}$ in equation (12), and $R = R_{av}$ in equation (13).

The shearing stress in a concentric-cylinder rotational viscometer (eq. (2)) at a constant applied torque or mean shearing stress is larger at the inside cylinder and decreases toward the outside cylinder. Therefore, the yield-value stress f is reached at the inside cylinder first, and flow will start at this member when the local stress equals the yield value f, while the material will remain unsheared at the outside member. Hence, there is formed a "plug" whose radial thickness decreases with increasing applied stress and disappears when the local stress at the outside cylinder equals the yield value.

Plug flow is always obtained with a concentric-cylinder viscometer at shearing stresses between $R_b^2 f/R^2$ and $R_c^2 f/R^2$ (fig. 2(b)), but the plug-flow portion of the flow curve can be held small by proper design of the viscometer. The angular velocity at which plug flow disappears is given as

$$\omega_p = \frac{f}{U}\left[\frac{R_c^2}{2R_b^2} - \frac{1}{2} - \ln\left(\frac{R_c}{R_b}\right)\right] \tag{14}$$

Since equations (12) and (13) are valid only if the material is sheared over the complete radial thickness between R_b and R_c, the plastic viscosity and yield value have to be determined from the straight portion of the flow curve (fig. 2(b)) at $\omega >> \omega_p$.

Only very few materials are true Bingham bodies (fig. 2(b)). Most materials of this type will give a flow curve similar to the dashed curve in figure 2(b), which has a larger curved portion than would be produced by plug flow. The larger curved portion can be assumed in some cases to be caused by an elastic behavior of the materials, which is superimposed upon their viscous behavior. Such materials will nevertheless be called Bingham plastics as long as they can be approximated by a straight line so that their flow can be interpreted by the two Bingham flow parameters, namely, U and f. For these materials f might not be rigorously equal to, but rather a function of the yield value as defined by Bingham. Thus, to avoid misunderstanding and to stress its practical value rather than its physical meaning, the use of the term yield-value intercept for f seems more appropriate for such materials. However, a meaningful choice of terms is possible only if the flow measurements permit a detailed analysis of the flow character of the material.

Buckingham (ref. 13) integrated equation (11) for flow in a capillary viscometer and pipe, so that

$$U = \frac{PD^2}{32Lv}\left[1 - \frac{4}{3}\left(\frac{4Lf}{DP}\right) + \frac{1}{3}\left(\frac{4Lf}{DP}\right)^4\right] \tag{15}$$

In deriving equations (12), (13), and (15), slippage flow, end effects, and kinetic energy effects are again neglected.

The shearing stress in a pipe (eq. (4)), $S_P = Pd/4L$, varies along the pipe diameter d. At the wall in a pipe $d = D$ and the shearing stress is $S_W = DP/4L$, while in the center of the pipe S_P is always zero. The ratio $4Lf/DP = c$ is the ratio of the yield-value stress to the shearing stress at the pipe wall, and is equal to the ratio of plug diameter to pipe diameter. Since a plastic material always has a finite yield-value stress and since the shearing stress in the center of the pipe is always zero, plug flow disappears only at an infinite applied shearing stress and the flow curve becomes a straight line only asymptotically (fig. 2(c)). For this reason, it is difficult to obtain U and f from capillary or pipe flow curves; and to calculate them rigorously from equation (15) is even more difficult. Consequently, the two basic flow parameters U and f for Bingham type plastics are best obtained from rotational-viscometer flow curves.

In analogy to Newtonian liquids, an equation for the friction factor for Bingham plastics flowing in pipe lines in laminar flow is

$$\varphi = \frac{64}{\mathrm{Re}[U]\left(1 - \frac{4}{3}c + \frac{1}{3}c^4\right)} \tag{16}$$

if the plastic viscosity U is used in the equation for the Reynolds number Re, so that $\mathrm{Re}[U] = Dv\rho/U$.

In accordance with an analysis by McMillen (ref. 3) and Hedström (ref. 2), equation (16) can be rewritten so that

$$\varphi = \frac{8\mathrm{Pl}}{c\mathrm{Re}[U]} = \frac{64}{\mathrm{Re}[U]} \times \frac{\mathrm{Pl}}{8c} = \varphi_0[U]\,\frac{\mathrm{Pl}}{8c} \tag{17}$$

where $\mathrm{Pl} = fD/Uv$ is a dimensionless number, which will be called "plasticity number," since it increases if the Bingham body is more highly plastic (larger yield value), decreases if the Bingham body becomes more viscous (lower yield value), and is zero for $f = 0$. Hedström (ref. 2) presents a graph which he determined by numerical calculation, giving $\varphi\mathrm{Re}[U]$ as function of Pl. This graph has been used to extend the general friction diagram for the flow of Newtonian liquids in pipes to that of Bingham plastics. Such a generalized friction diagram for Newtonian liquids and Bingham plastics is presented in figure 3. In this diagram the friction factor φ is plotted as a function of the Reynolds number Re, and the plasticity number is used as a parameter.

Pseudoplastic and Dilatant Materials

Flow curves of a pseudoplastic and a dilatant material are shown in figures 4(a) and (b), respectively. Power functions have been shown empirically to fit these flow curves and have been proposed as rheological equations for such materials (ref. 14), so that

$$\frac{dv}{dr} = KS^N \tag{18}$$

Although this power function (eq. (18)) is not a true rheological equation, since it is not derived from any physical concept, it can be regarded as an interpolation formula, and as such it is frequently very useful. In this equation, the quantity N indicates the structural change or the degree of pseudoplasticity and dilatancy and will be called "structure number." For Newtonian liquids, $N = 1$; while $N > 1$ for pseudoplastic, and $N < 1$ for dilatant materials. In many cases the value of N is constant over an appreciable range of rates of shear. The value of N can then be determined from the slope of the straight-line plot of log (dv/dr) against log S. In cases where this plot is nonlinear, $N = \frac{d \log (dv/dr)}{d \log S}$ and varies with the rate of shear and shearing stress. For Newtonian liquids where $N = 1$, equation (18) reverts to equation (1) and $K = 1/\mu$.

Substituting in equation (18) for S the expression of S_R for the rotational viscometer and that of S_P for the capillary viscometer and pipe line, and assuming N to be either constant over the range of rates of shearing obtained within the cross section of the viscometer test section for any one applied torque or pressure, or assuming N to represent a mean value corresponding to the mean rate of shear, equation (18) becomes after integration (ref. 15)

$$\frac{dv}{dr} = \frac{G}{C} = KS^N \tag{19}$$

where C has a different value for a rotational and a capillary viscometer. The rates of shear G_R and G_P (eqs. (2) and (4)) had been obtained under the assumption of constant viscosity and thus constant flow resistance. Since the flow resistance of non-Newtonian materials varies as a function of shearing stress (eq. (18)) and the shearing stress varies within the confines of a rotational and capillary viscometer (eqs. (2) and (4)), the rates of shear produced within the sheared sample of a non-Newtonian material will be different from those of a Newtonian liquid and will not only depend on the instrument used but also on the material being tested. The factor C in equation (19), which depends on the material and the geometrical configuration of the viscometer, relates the rates of shear produced in non-Newtonian materials of the flow types as characterized by equation (18) to those calculated for Newtonian liquids at the corresponding shearing stress. Thus, the factor C will be called "rate-of-shear correlation factor." Then, for non-Newtonian materials having flow characteristics in accordance with equation (18), the rates of shear are G_R/C_R in a concentric-cylinder rotational viscometer for $S = S_R$, and $G_{m,P}/C_P$ in a capillary viscometer or pipe for $S = S_W$.

The experimentally obtained N, if it is not constant but is a function of rate of shear, represents a mean value corresponding to the mean rate of shear only, if its change over the sheared width of the sample is very small. In a rotational viscometer of proper design, the change in shearing stress and thus of rate of shear within the annula between bob and cup is small, so that, even for large values of dN/dS, the change in N over $(R_c - R_b)$ will be small. Thus N, when experimentally obtained with a rotational viscometer, might be considered a mean value at the measured flow condition, even if it changes substantially with a change in flow conditions, and equation (19) is valid. However, in a capillary viscometer or pipe line the change in shearing stress along the diameter is always large, changing from zero to S_w, so that the change of N over D is small only if dN/dS is small. Thus N, when experimentally obtained with a capillary viscometer, is often not a mean value, and equation (19) is then only an approximation. In this case the experimentally obtained values for N and C_p are not any more constants of the material alone but will depend on the dimensions of the viscometer.

The quantity K in equations (18) and (19) is a constant for a given material, but it has no physical meaning and its dimensions change as a function of N and thus are different for each material. To give K physical meaning, equations (18) and (19) might be rewritten by making use of such established flow constants as the relaxation time τ and the shear modulus M, so that

$$\frac{dv}{dr} = \frac{G}{C} = \frac{1}{\tau}\left(\frac{S}{M}\right)^N \tag{20}$$

It would be best to use basic material constants such as τ and M for the flow parameters in the analytic process of establishing an equation for the friction factor as a function of Reynolds number for pseudoplastic and dilatant materials. However, since equation (20) has no theoretical or experimental basis, and since N is not expressed as a function of established material constants, this equation cannot be used to interpret the flow curve in terms of τ and M, even if these two material constants are obtained from independent measurements. Thus, for the present, it seems useful and expedient to employ an empirical flow parameter, namely, the apparent viscosity η. The apparent viscosity η is defined as the ratio of shearing stress to rate of shear, so that

$$\eta = \frac{SC}{G} \tag{21}$$

For a concentric-cylinder rotational viscometer,

$$\eta = \left[\frac{T\left(\frac{1}{R_b^2} - \frac{1}{R_c^2}\right)}{4\pi h\omega}\right] C_R \tag{22}$$

where

$$C_R = \left\{\frac{1 - (R_b/R_c)^{2N}}{N[1 - (R_b/R_c)^2]}\right\}\left[\left(\frac{R}{R_b}\right)^{2(N-1)}\right] \tag{23}$$

and approaches 1 for $R_b/R_c \rightarrow 1$. For a capillary viscometer or pipe,

$$\eta = \frac{PD}{32Lv} C_P \tag{24}$$

where

$$C_P = \frac{4}{N + 3} \tag{25}$$

Thus, two empirical flow parameters can be stipulated for pseudoplastic and dilatant materials, namely the apparent viscosity η (eq. (21)) and the structure number N (eq. (19)). However, for non-Newtonian materials, η is not a constant, but varies continuously with the applied rate of shear (eq. (21)). Frequently N also varies with the rate of shear, but from experiments it was found that in many of these cases N can be approximated by a different constant value over different rate-of-shear intervals. Nevertheless, both empirical flow parameters η and N are meaningful only if they are obtained at the same local rate of shear at which the flow resistance is to be determined. These "local flow parameters" must be obtained from a flow curve extending over a wide range of rates of shear. An approximate local apparent viscosity might be obtained for any specified rate of shear by extrapolation, if it is outside the flow-curve measurement, by using a logarithmic plot of equation (19) to determine N at the highest measured rates of shear. Assuming then N to be constant for still higher rates of shear, S can be extrapolated for any specified rate of shear by using the same plot. Then the local apparent viscosity is calcualted from equation (21).

In a capillary viscometer and in a pipe line, the mean rate of shear $G_{m,P}/C_P$ is equal to $2v(N + 3)/D$, (eqs. (4) and (25)). The mean rate of shear $G_{m,R}/C_R$ in a concentric-cylinder rotational viscometer is

$$\frac{2\omega}{C_R R_{av}^2\left(\frac{1}{R_b^2} - \frac{1}{R_c^2}\right)}$$ (eqs. (2) and (23) for $R = R_{av}$).

A rotational viscometer is the preferred instrument for measuring the flow curves of pseudoplastic and dilatant materials, since, as previously analyzed, the experimentally obtained N represents in most cases a mean value, and also because the rotational-viscometer mean-rate-of-shear correlation factor C_R (at $R = R_{av}$) will in most practical cases approach 1 very closely (see fig. 5), so that the apparent viscosity (eq. (22)) and the flow curve (eq. (19)) are almost independent of N. In a pipe line or capillary viscometer, however, the experimentally obtained N often does not represent a mean value, and the rate-of-shear correlation factor C_P can differ from 1 considerably, as is shown in figure 5. In fact, the factor C_P is very dependent on N, so that the apparent viscosity (eq. (24)) and the flow curve (eq. (19)) depend greatly upon the validity of the determination of N. The apparent viscosity as given in equation (24) is the apparent viscosity only if the N substituted in equation (24) is a mean value. Otherwise, effective apparent viscosities are obtained, which depend on the dimensions of the viscometer. Another disadvantage of using the capillary viscometer for the determination of the flow character of non-Newtonian materials is that the flow curves of a pseudoplastic material (fig. 4(a)) and of a Bingham plastic (fig. 2(c)), when plotting $G_{m,P}$ against S_w, are so much alike as to make it difficult to differentiate between one and the other flow type from a study of such flow curves, and thus to obtain meaningful mean rates of shear and flow parameters for use in scaling the measured results to the flow conditions required for any specific application.

In analogy to Newtonian liquids, an equation for the friction factor φ for pseudoplastic and dilatant materials for laminar flow in pipes is

$$\varphi = \frac{64}{Re[\eta]}\left(\frac{N+3}{4}\right) = \varphi_O[\eta]\left(\frac{N+3}{4}\right) = \frac{\varphi_O[\eta]}{C_P} \tag{26}$$

if the local apparent viscosity $\eta = SC_P/G_{m,P}$ (eq. (21)) is used in the equation for the Reynolds number so that $Re[\eta] = Dv\rho/\eta$.

In the case that the apparent viscosity is measured in a capillary viscometer, $\eta = S_wC_C/G_{m,C}$ and the measured value of N might not represent a mean value, so that C_C can differ from C_P. To obtain the local apparent viscosity, an approximate mean rate of shear $G_{m,P}/C_C$ is substituted for $G_{m,C}/C_C$, and the approximate local apparent viscosity is $\eta = S_wC_C/G_{m,P}$ where $G_{m,P} = 8v/D$. Then the equation for the friction factor (eq. (26)) can be rewritten to read $\varphi = \varphi[\eta_s]C_C/C_P$ where η_s is defined as $\eta_s = S/G_{m,P}$. Thus, for $C_C = C_P$, equation (26) reduces to

$$\varphi = \frac{64}{Re[\eta_s]} = \varphi_0[\eta_s] \tag{27}$$

In this equation, η_s is not a viscosity, and therefore $Re[\eta_s]$ is not a Reynolds number in accordance with its definition, but equation (27) can be useful. However, the condition that $C_C = C_P$ is fulfilled only if the measured N represents a mean value, which it does only under the conditions that dN/dS is small. In all other cases, C_C might differ considerably from C_P, and thus the use of a capillary viscometer for the measurement of η and φ is questionable.

In the case that η is measured in a rotational viscometer, $\eta = S_{m,R}C_R/G_{m,R}$, and the value of N, when measured in a properly designed instrument, most often represents a mean value. Therefore, the mean value of C_P for the pipe can be calculated (fig. 5). To obtain the local apparent viscosity, the mean pipe rate of shear $G_{m,P}/C_P$ is substituted for $G_{m,R}/C_R$, and the local apparent viscosity is $\eta = S_{m,R}C_P/G_{m,P}$. The equation for the friction factor reduces then to equation (27).

Equation (26) is used to extend the friction diagram (fig. 3) to the flow of pseudoplastic and dilatant materials by using the structure number N as parameter, as indicated by the dashed lines. Since equation (26) was derived from an empirical formula and not from a rheological equation, such a diagram should be considered only as a useful approximate method for calculating flow pressure losses of such materials in pipe lines.

Thixotropic Materials

Flow curves of a plastic, a pseudoplastic, and a dilatant material of thixotropic character are shown in figures 6(a), (b), and (c), respectively. Thixotropic materials change in flow resistance with rate of shear and shear duration under isothermal conditions. Flow curves of thixotropic materials are characterized by a hysteresis loop, when the flow curve is obtained with a viscometer that is capable of applying to the same sample an increasing shear rate followed by a decreasing shear rate. In the case of thixotropic behavior, the "down curve" obtained at decreasing rates of shear will be shifted towards smaller stresses compared with those of the "up curve" determined at increasing rates of shear. This indicates that the flow resistance decreases with increasing rates of shear and shear duration. Upon rest, the flow curve will shift back, which indicates that the flow resistance increases to its original value when the material is left unsheared.

During the measurement of the up curve, the structure of the material is continuously broken down, thus changing the flow character of the material, so that each point on the up curve might be considered to represent a point on a different flow curve. According to the literature (ref. 16), each down curve, when measured rapidly, is said to represent a flow curve of the material. Since the down curve shares one point, namely the top point, with the up curve, the down-flow curve describes the flow behavior that characterizes the material at this point of its shear history. It has been found (ref. 16) that the position of this point and thus that of the down curve depends on two factors, namely, the value of the highest rate of shear applied to the sample, which will be called "top rate of shear" of the flow curve, and the time of shear duration before the down curve is commenced.

Most capillary viscometers are incapable of applying more than one rate of shear to the same sample; therefore, they do not produce hysteresis loops and are unsuited for the measurement of thixotropic materials. Rotational viscometers, however, are capable of detecting hysteresis in flow curves and thus are well suited for the measurement of thixotropic materials.

The flow parameters are obtained from the down-flow curves in a similar manner as was done for materials of non-thixotropic character. In the case of thixotropic materials where the down curve characterizes the material as a Bingham plastic (fig. 6(a)), the flow parameters are the plastic viscosity U and the yield-value intercept f, and equations (12), (13), and (17) are applicable. In the case of thixotropic materials where the down curve characterizes the materials as being pseudoplastic or dilatant (figs. 6(b) and (c), respectively), the flow parameters are the local apparent viscosity η and the local structure number N, and equations (18), (21), and (26) are applicable. However, for all thixotropic materials, the flow parameters obtained from a down-flow curve describe the flow condition of the material only if it has been subjected to a specific shear treatment that is identical to that which the sample experienced in the viscometer before the down-curve measurement. For this reason, the flow parameters thus obtained will be called "specific flow parameters." For a meaningful interpretation they should always be marked by the top rate of shear and the shear duration to which the material was subjected before the measurement.

For thixotropic plastic materials, an empirical extrapolation method has been suggested (ref. 16), which permits the calculation of the specific flow parameters U and f for any specified value of top rate of shear and shear duration. To do this, three flow curves have to be obtained, two at two values of top rate of shear, and a third at one of those top rates of shear but after a different time of shear duration. Such an extrapolation method permits the calculation of the specific flow parameters for shear conditions, which might not easily be obtained with viscometers. For pseudoplastic and dilatant materials of thixotropic character, no extrapolation method has been established. Thus, it is necessary to make the measurement of the flow curve under the identical specified conditions of top rate of shear and shear duration to which the material is subjected in the pipe line. Only then is it possible to obtain meaningful flow-pressure-loss calculations from the interpretation of such a flow curve.

The use of such specific flow parameters to obtain flow pressure losses in pipe lines by employing the friction diagram can best be considered a rough approximation method. The flow of thixotropic materials in pipe lines is very complex. In any pipe line the shearing stress changes from zero at the center of the pipe line to a maximum at the pipe wall, with correspondingly large variations in rate of shear and shear duration along the radius. Since the specific flow parameters of most thixotropic materials are greatly affected by changes in rate of shear and shear duration, they will vary substantially along the entire cross section. Thus, in the case of thixotropic materials, it is rather difficult from pipe-line flow-rate measurements to plot a curve of mean rate of shear as a function of mean shearing stress, and to determine meaningful values of mean rate of shear and of mean shear duration at which to measure the specific flow parameters. For this reason it seems that experimental data would be useful to establish mean values of shear conditions as a function of flow rate at which to calculate the specific flow parameters for determining flow-pressure losses of thixotropic materials in pipe lines by the use of the friction diagram.

GENERALIZED FRICTION DIAGRAM

Newtonian Materials

For Newtonian liquids the plasticity number is zero. The lowest solid curve in the friction diagram (fig. 3) is for smooth pipes. At $Re[\mu] \leq 2000$, the flow is laminar and equation (7) is valid. For turbulent flow, equation (9) is valid for smooth pipes and equation (10) for pipes with an extremely rough surface of $D/k \leq 20$. For flow in pipes of intermediate surface roughness, lines between those two curves can be plotted by using equation (10) or can be taken from the literature (ref. 1).

Bingham Plastics

For Bingham plastics, the lines of different plasticity numbers (fig. 3) are representative of the laminar-flow region. As long as the plasticity number and Reynolds number can be calculated, the friction factor can be obtained from the diagram (fig. 3). An evaluation of experimental data reported for smooth pipe lines (refs. 4 and 5) indicates that the flow of a Bingham plastic in a pipe line becomes turbulent at a Reynolds number at which the line of respective plasticity number intersects the turbulent branches of the curves for Newtonian flow. For instance, at a plasticity number Pl of 100, turbulence will start at $Re[U] > 4.7 \times 10^4$ in smooth pipes and accordingly would be expected to start at $Re[U] > 1.4 \times 10^4$ in the extremely rough pipes. The transition Reynolds number for Bingham plastics increases with increasing plasticity numbers.

Pseudoplastic and Dilatant Materials

For pseudoplastic and dilatant materials, the lines of different structure number (fig. 3) are representative of laminar flow. An approximate friction factor can be obtained if the local structure number and the local apparent viscosity (eq. (21)) can be calculated. In analogy to plastic flow, it is assumed that turbulent flow for such materials starts at the Reynolds number $Re[\eta]$ at which the line of respective structure number intercepts or meets the turbulent branches of the curves for Newtonian flow. Experimental data (refs. 7 and 8) indicate the validity of such an assumption. The friction diagram for pseudoplastic and dilatant materials is similar to that for Bingham plastics, except that different flow parameters are used in its interpretation and that the spacing of the lines is different. Since the spacing of the lines is uniform throughout, all friction factors can be obtained just as readily (eq. (26)) by using the lines for the Newtonian friction factor to obtain $\varphi_0[\eta]$ and figure 5 to obtain C_P. However, the onset of turbulence can be determined only by using the lines of respective structure number in the friction diagram.

CALCULATION OF PRESSURE LOSSES IN PIPE LINES

Uniform Pipe Line

For designing pipe-line systems, the flow rate and applied pressure are frequently specified, so that the pipe lines have to be dimensioned to produce a minimum pressure loss for otherwise optimum operational conditions. The pressure loss in "uniform pipe lines" - that is, in pipe lines of constant cross-sectional area - is due entirely to the flow resistance of the materials carried in that pipe. This "flow-pressure loss" can be readily calculated when the friction factor is known (eqs. (7), (17), and (26)).

The relation between the friction factor and the flow-pressure loss is the same for materials of any flow character by the definition of the friction factor (eq. (8)). The equation for the flow-pressure loss (ref. 1) is

$$\Delta P_F = \rho \frac{v^2}{2} \frac{(L - L_e)}{D} \varphi \tag{28}$$

Equation (6) for the entrance length L_e is strictly valid only for laminar flow of Newtonian materials. Under all other flow conditions, equation (6) is at best an approximation, but useful enough for most practical pipe-line considerations.

Nonuniform Pipe Line

Practical pipe-line systems consist by necessity of uniform and nonuniform pipe-line portions. Nonuniformity can be produced in the pipe line by various means, such as contractions, extensions, bends, inserts, orifices, entrances, and others. The pressure loss in the nonuniform portions of the pipe line, which will be called "pipe transitions," is again due to the flow resistance of the material passing through the nonuniformity. However, this "pipe transition loss" cannot be readily calculated, since the flow is nonuniform, so that a very complex flow pattern results. The pipe transition loss is

$$\Delta P_T = \rho \frac{v^2}{2} C_L \tag{29}$$

For Newtonian liquids the pressure-loss coefficients C_L have been determined experimentally for various types of pipe transitions and can be found in the literature (refs. 1 and 10). For convenience, some values of C_L for Newtonian liquids are given in table I.

The pressure losses are additive, so that the total pressure loss in a pipe-line system is

$$\Delta P_F + \Delta P_T = \rho \frac{v^2}{2} \left[C_L + \frac{(L - L_e)}{D} \varphi\right] \tag{30}$$

In practical pipe-line systems of large L/D, when $(L - L_e)\varphi/D >> C_L$, the pipe transition loss can frequently be neglected.

From an analysis of the generalized friction diagram (fig. 3), it seems that the pipe transition losses for the flow of non-Newtonian materials could become rather large, since the friction factor in laminar flow can be a multiple of that for a Newtonian liquid of the same viscosity. In turbulent flow, however, the friction factors would be equal to those of Newtonian liquids. Thus, it would seem that the pipe transition losses of non-Newtonian materials will greatly depend upon whether laminar or turbulent flow prevails in the pipe transitions. In addition, non-Newtonian materials such as Bingham plastics require that the shearing stress exceed the yield-value stress before flow can start. This requirement tends to make the pipe transition losses not only dependent upon the configuration of the pipe transition but also upon the magnitude of the yield-value stress, since plug flow will occur whenever the shearing stress decreases below the yield-value stress, and an increase in the amount of plug flow will increase the pressure losses as long as laminar flow prevails. Since dilatant materials (fig. 4(b)) increase in flow resistance with increasing rates of shear, any pipe transition that contains a constriction will produce a large pipe transition loss, because the material is subjected to a relatively high shear in such a constriction.

Only one reference was found (ref. 3) presenting pipe transition losses of non-Newtonian materials. In this particular reference, the flow of a Bingham plastic was studied. The pipe transition losses reported were for a pipe contraction and were many times larger than would have been expected for a Newtonian material of the same viscosity subjected to the same flow conditions. This trend seems in accord with the preceding analysis.

EVALUATION OF EXPERIMENTAL DATA IN RELATION TO FIGURE 3

Bingham Plastics

Experimental data of the flow-pressure losses of Bingham plastics in smooth pipe lines are reported in references 4 and 5. They are used to check figure 3. Hedström (ref. 2) has used some of the pressure-drop data reported in reference 4 to establish the onset of turbulence for Bingham plastics flowing in pipes. For this reason Hedström has computed the plastic viscosity and yield value of the material used in this reference, which is a cement rock suspension. The same values of U and f were used to obtain the data in figure 7. The other investigator (ref. 5) studied clay in water suspensions at different solid contents. He measured the pressure loss in the pipe line over a length of 20 feet and obtained the two basic flow parameters, U and f, at the same temperature from flow curves produced with a rotational viscometer. The data used in figure 7 include a range of $0.08 \leq U \leq 0.235$ poise, $0 \leq f \leq 276$ dynes per square centimeter, $1.91 \leq D \leq 7.60$ centimeters, $0.01 \leq v \leq 690$ centimeters per second, and $1.01 \leq \rho \leq 1.52$ grams per cubic centimeter, giving $0 \leq Pl \leq 480{,}000$ and $0.74 \leq Re[U] \leq 161{,}000$. The friction factor $\varphi_{experimental}$ obtained from the experimental pressure-drop data measured in the pipe lines is compared in figure 7 with the friction factor $\varphi_{diagram}$, which is calculated by use of the generalized friction diagram and the two basic flow parameters. The agreement of both friction factors up to $\varphi \approx 10^3$ shows that the generalized friction diagram is valid for evaluating the flow in pipe lines of Bingham plastics from their two basic flow parameters.

Above $\varphi \approx 10^3$ the plasticity number was high (Pl > 600) and the flow velocity was in most instances low. In fact, most of the points which are more than 50 percent off were taken at a flow velocity of $v < 0.3$ centimeter per second. Since, as was mentioned before, only very few materials are true Bingham bodies but most show an elastic flow behavior (dashed curve in fig. 2(b)) at the low shearing stresses, their yield-value intercept and plastic viscosity are constant only above a certain shearing stress S^{**} which is larger than the calculated yield-value stress. Since it will be shown later that for values of Pl > 600 the accuracy of the friction factor is independent of that of the plastic viscosity, only the change in yield-value intercept at the low velocities is considered.

At shearing stresses between S^* and S^{**} the yield-value intercept increases continuously with increasing rate of shear until it reaches at S^{**} and $(dv/dr)^{**}$ its maximum constant value of f. Thus, at flow velocities corresponding to rates of shear of less than $(dv/dr)^{**}$ the flow parameter f is larger than the local yield-value intercept at the given local rate of shear. Consequently, the calculated plasticity number, being proportional to f, is also too large and so is the friction factor $\varphi_{diagram}$ obtained from figure 3. More accurate friction factors can be obtained from the friction diagram even for low flow velocities corresponding to a rate of shear of less than $(dv/dr)^{**}$ if the plasticity number is calculated by using the local yield-value intercept, namely, that which is characteristic for the rate of shear prevailing in the pipe line. The local yield-value intercept, which should replace f in the calculation in such cases, can be approximated by using the intercept on the stress axis, which is obtained by constructing a tangent on the flow curve at the point of local rate of shear.

An analysis of the accuracy of the friction-factor determination as a function of the accuracy of the measurements of U and f, and of flow velocity, density, and pipe dimensions, can be made with the assistance of figure 8. Figure 8 is a logarithmic plot of the plasticity number Pl against $\varphi/\varphi_0[U]$. At plasticity numbers of $Pl \geq 600$, this logarithmic relation is linear, so that $\frac{\Delta\varphi}{\varphi} = \frac{1}{\alpha}\left(\frac{\Delta Pl}{Pl}\right) + \frac{\Delta\varphi_0[U]}{\varphi_0[U]}$. Since the line has a slope of $\alpha = 1.0$, $\frac{\Delta\varphi}{\varphi} = \frac{\Delta Pl}{Pl} + \frac{\Delta\varphi_0[U]}{\varphi_0[U]}$. After substitution of the expressions for Pl and $\varphi_0[U]$, it is found that the accuracy of the friction factor depends on the accuracy of measurement of the yield value, flow velocity, and density and that it is independent of the accuracy of measurement of the plastic viscosity and pipe dimensions. Then, the accuracy of the flow-pressure-loss calculations (eq. (28)) at $Pl \geq 600$ depends entirely on the accuracy of the yield-value determination, if the errors in the pipe-line dimensions and the pipe entrance length are assumed negligible.

In the range of $20 \leq Pl \leq 600$, another linear approximation can be made (fig. 8). Since this line has a slope of $\alpha \approx 1.2$, it is found after substitution that the accuracy of the friction factor in this region of plasticity numbers depends on the accuracy of measurement of both flow parameters and of density, flow velocity, and pipe diameter. However, considering the influence of the flow parameters alone, it is noticed that the accuracy of the yield-value determination has to be about five times better than that of the plastic viscosity measurement to produce an equal error in the determination of the friction factor. With decreasing plasticity numbers $Pl < 20$, the accuracy of the friction factor depends less and less on that of the yield value but to an increasing extent on that of the plastic viscosity measurement. Both flow parameters have to be determined to an equal accuracy for an equal error in the friction factor at a plasticity number of about 5, since at this point the tangent on the curve in figure 8 has a slope of $\alpha \approx 2.0$. With further decreasing plasticity numbers, the influence of the yield-value determination on the accuracy of the friction factor decreases rapidly and is insignificant at $Pl \lessapprox 1.0$.

Pseudoplastic Materials

A few papers in the literature (refs. 6 to 8) deal with suggestions for obtaining the pressure loss due to the flow of pseudoplastic materials in pipe lines from flow data obtained in a viscometer. In laminar and turbulent flow, good correlations have been obtained between the pressure loss directly measured and the one calculated by using the Newtonian friction diagram after substituting a local apparent viscosity into the expression for the Reynolds number, where the local apparent viscosity was calculated in accordance with equation (18).

In laminar flow, the use of only the one flow parameter of local apparent viscosity assumes that the term $(N + 3)/4$ in equation (26) is close to 1 or that this term was cancelled out by the use of an appropriate equation for η so that equation (27) was applicable. The former is valid only for structure numbers of N approaching 1. The latter is valid whenever rotational-viscometer flow curves are used to calculate the local apparent viscosity at a given pipe rate of shear and is approximated if the calculation is made from capillary-viscometer measurements. Since the reported pressure-loss correlations were good, even in laminar flow, it is assumed that either one or both of the two conditions were satisfied in the experiments.

For infinitely high rates of shear, approximate values of local apparent viscosity can also be obtained by extrapolation from a plot of inverse rate of shear against local apparent viscosity. By assuming this plot to be linear, extrapolation to infinite rate of shear yields an intercept on the apparent-viscosity axis equal to the local apparent viscosity at infinite rate of shear. In turbulent flow, when the rates of shear are so high as to approximate the flow conditions at infinite rate of shear, such a "residual apparent viscosity" can be used instead of the local apparent viscosity to calculate $Re[\eta]$. This has the advantage that the knowledge of the local mean rate of shear in the pipe is not required. The residual apparent viscosity has been used in references 7 and 8 in the turbulent-flow evaluations.

The accuracy of the friction factor for pseudoplastic and dilatant materials as a function of the accuracy of the measurements of the flow parameters η and N, and of the flow velocity, density, and pipe dimensions can be determined by using equation (26). After substitution of the expression for $\varphi_0[\eta]$ in equation (26), it is found that the accuracy of the friction factor depends on the accuracy of measurements of both flow parameters and of density, flow velocity, and pipe diameter. However, considering the influence of the two flow parameters alone, it is noticed that the accuracy of the determination of the local structure number N can be less than that of the local apparent viscosity η by a factor of $1/[1 + (3/N)]$ to produce an equal error in the determination of the friction factor. Thus, for $N \approx 2$, the measurement of the structure number can be about 2.5 times less accurate than the apparent-viscosity determination to result in the same error in the friction factor and flow-pressure-loss calculations.

TEMPERATURE CONSIDERATIONS

The flow parameters are functions of temperature. Therefore, the flow parameters used for flow-pressure-loss calculations should preferably be measured under the same temperature conditions as prevail in the pipe line where the pressure loss occurs. Frequently, this temperature cannot be easily determined, since it might depend on environmental factors and thus comprise a complete range of temperatures, rather than a single value. In that case it is necessary to measure the flow parameters as a function of temperature. Frequently, measurements of the flow parameters at two temperatures will permit interpolation or extrapolation over the required temperature range with sufficient accuracy to allow calculation of the corresponding friction factors and flow-pressure losses. In Newtonian flow of liquids, temperature increases will, in most cases, cause a decrease in viscosity and thus will effect a decrease in pressure loss in the pipe line. However, in non-Newtonian flow, an increase in pressure loss can be expected more frequently, since temperature increases can affect either one or both of the flow parameters in either direction. They can cause, for instance, an increase in yield value in the case of a Bingham plastic, and thus effect an increase in pressure loss, in spite of a decrease in viscosity.

CONCLUDING REMARKS

The importance of establishing basic flow parameters, which for non-Newtonian materials can be obtained only from flow curves (rate-of-shear - shearing-stress curves), has been discussed. Use has been made of these characteristic flow data for calculating dimensionless parameters, which are presented in a generalized friction diagram. The purpose of this diagram is to assist in the evaluation of pressure losses resulting from the flow of non-Newtonian materials in pipe lines.

A concentric-cylinder rotational viscometer of proper design has been shown to have advantages over the capillary viscometer when measuring the flow of non-Newtonian materials to obtain the flow parameters. In fact, in some cases, the capillary-viscometer flow curves do not permit a valid interpretation of the flow character of the materials.

The two basic flow parameters for Bingham plastics are the plastic viscosity and the yield value. The local apparent viscosity and the structure number are used as characteristic flow parameters for pseudoplastic and dilatant materials. For thixotropic materials, a method of using specific flow parameters corresponding to the type of flow curve and to the nature of the material is suggested.

In the case of Bingham plastics, it has been shown that for highly plastic materials the accuracy of the yield-value measurements alone determines the accuracy of a pressure-loss calculation when made by use of the friction diagram, while the accuracy of measurement of the plastic viscosity and the flow velocity become important only for less plastic materials. In the case of pseudoplastic and dilatant materials, it has been shown that the accuracy of a pressure-loss calculation, when made by use of the friction diagram, depends on the accuracy of measurement of the local apparent viscosity and the flow velocity and only to a lesser degree on the accuracy of the determination of the local structure number.

The validity of the generalized friction diagram for describing flow of Bingham plastics in pipe lines is shown by an evaluation of experimental data of two independent investigators. Reference is made to other reported experimental data, which indicate that the generalized friction diagram is valid also for evaluating the flow of pseudoplastic materials in pipe lines.

Considerations of the effect of temperature on the various flow parameters suggest that temperature can affect pressure losses in pipe lines carrying non-Newtonian materials more adversely than in the case of

Newtonian flow. Thus, non-Newtonian flow parameters must be determined at various temperatures to establish their trend as a function of temperature. The flow parameters corresponding to the approximate range of temperatures prevailing in the pipe line where the pressure loss occurs can then be selected for the calculations of the friction factors and flow-pressure losses.

An analysis of the basic flow data in reference to the friction factors suggests that pipe transition losses of non-Newtonian materials will in most cases, especially where the flow is laminar in the pipe transition, be larger than those expected for Newtonian liquids of the same viscosity and under identical flow conditions. For this reason, transition pressure-loss coefficients for the flow of non-Newtonian materials in pipe transitions are required. Since they cannot be obtained by calculation, they have to be established by experiment.

Numerical examples of friction-factor calculations by the use of flow curves and the generalized friction diagram are given in appendix B for a Bingham plastic and a pseudoplastic material flowing in a pipe line at different flow rates.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, November 26, 1954.

APPENDIX A

SYMBOLS

The following symbols are used in this report. The dimensions of all dimensional quantities are indicated.

C	rate-of-shear correlation factor
C_c	contraction coefficient
C_L	pressure-loss coefficient in a pipe transition
c	ratio of plug to pipe diameter
D	diameter of pipe or capillary, $[L]$
D_o	orifice diameter, orifice slot width, $[L]$
d	any diameter of pipe or capillary, $0 \leq d \leq D$, $[L]$
dv/dr	rate of shear or velocity gradient, $[T^{-1}]$
f	yield value or yield-value intercept, $[ML^{-1}T^{-2}]$
G	rate of shear, assuming constant viscosity, $[T^{-1}]$
h	height of inner cylinder (bob), $[L]$
K	proportionality constant
k	grain diameter of surface roughness in pipe, $[L]$
L	length of capillary or pipe, $[L]$
M	shear modulus, $[ML^{-1}T^{-2}]$
N	structure number
P, ΔP	pressure difference over given length L, $[ML^{-1}T^{-2}]$
ΔP_F	flow-pressure loss, $[ML^{-1}T^{-2}]$
ΔP_T	pressure loss in pipe transition, $[ML^{-1}T^{-2}]$

Pl plasticity number, $Pl = fD/Uv$

R any radius of rotational viscometer, $R_b \leq R \leq R_c$, [L]

R_{av} average radius between R_b and R_c, $R_{av} = \frac{R_c + R_b}{2}$, [L]

R_b radius of inner cylinder (bob), [L]

R_c radius of outer cylinder (cup), [L]

Re Reynolds number $Re = Dv\rho/\text{viscosity}$

r radius of bend of pipe, [L]

S shearing stress, $[ML^{-1}T^{-2}]$

S_0 stress intercept at $G = 0$, $[ML^{-1}T^{-2}]$

T torque, $[ML^2T^{-2}]$

T_0 torque intercept at $\omega = 0$, $[ML^2T^{-2}]$

U plastic viscosity, $[ML^{-1}T^{-1}]$

v mean velocity of flow (calculated from flow rate), $[LT^{-1}]$

α slope of lines in fig. 8

η apparent viscosity, $\eta = S/(dv/dr) = SC/G$, $[ML^{-1}T^{-1}]$

η_s $= S/G_{m,P}$, $[ML^{-1}T^{-1}]$

μ Newtonian viscosity, $[ML^{-1}T^{-1}]$

ρ density, $[ML^{-3}]$

τ relaxation time, [T]

φ friction factor

φ_0 $= 64/Re$

ω angular velocity, $[T^{-1}]$

Terms in brackets:

U refers to plastic viscosity

η refers to apparent and local apparent viscosity

η_s refers to $S/G_{m,P}$

μ refers to Newtonian viscosity

Subscripts:

C capillary only

e entrance

m mean value

P pipe or capillary

p plug flow

R rotational viscometer

w wall of pipe or capillary

APPENDIX B

NUMERICAL EXAMPLES

The friction factors φ are to be determined for the flow of non-Newtonian materials in a smooth pipe line of $D = 2.0$ centimeters and at different flow velocities. The flow velocities v are given in centimeters per second. They are calculated from the specified required total mass-flow rates, the number of available pipe lines, the pipe diameter, and the density of the materials, which is assumed to be 1.15 grams per cubic centimeter.

Bingham Plastics

The dashed curve in figure 9 is an assumed rotational-viscometer flow curve of a Bingham plastic. The two flow parameters U and f are independent of rate of shear. They are used to calculate $Re[U]$ and Pl, since $Re[U] = Dv\rho/U$ and $Pl = fD/Uv$. The friction factor φ is then found from figure 3. The flow parameters are $U = 0.19$ poise and $f = 400$ dynes per square centimeter. Then the following values are obtained:

v	Re[U]	Pl	φ	Remarks	$\varphi_0[U]$
50	605	84	1.38	Laminar	0.105
100	1,210	42	.40	↓	.053
200	2,420	21	.12		.030
300	3,630	14.1	.063		.040
400	4,850	10.5	.037	Turbulent	.039
500	6,050	8.4	.037	↓	.037
1000	12,000	4.25	.030		.030
2000	24,200	2.1	.024		.024

Pseudoplastic Materials

The solid curve in figure 9 is an actual flow curve of a pseudoplastic suspension measured on a concentric-cylinder rotational viscometer with $R_b/R_c = 0.95$. To obtain N, $\log G_{m,R}$ is plotted against $\log S_{m,R}$ in figure 10. Since no straight line results, N is not constant, but varies with rate of shear. The curve can be approximated by segments of linear functions, so that different constant values of N can be postulated for various mean-rate-of-shear intervals, as is indicated in figure 10. For more accurate values of N, the slope of the logarithmic curve has to be determined at the point of specified local rate of shear. The local apparent viscosity is obtained from equation (21). For extrapolations outside the flow-curve range of rate of shear, the local apparent viscosity is obtained from figure 10 and equation (21). It is used to calculate $Re[\eta]$, since $Re[\eta] = Dv\rho/\eta$. The mean pipe rate of shear is $G_{m,P}/C_P$, where $G_{m,P} = 8v/D$. Then, the local values of $G_{m,R}$ at which η are to be obtained are equal to $G_{m,P}C_R/C_P$. The correlation factors C are obtained from figure 5 for the corresponding values of the local structure number N. The friction factor φ is found from figure 3 or equation (26). This procedure permits the calculation of all local parameters. However, in case only the friction factor is to be obtained, equation (27) can be used, which eliminates all determinations of N and also the need for figure 10, unless extrapolation is necessary. The friction factor φ is again found from figure 3. The calculation of φ from equation (27) instead of equation (26) has not only the advantage of greater simplicity but also of higher accuracy, since some uncertainties in N are eliminated. The calculated values are:

v	$G_{m,P}$	N_{local} (a)	C_P (b)	$G_{m,P}/C_P$	C_R (b)	$G_{m,R}$	Corrected N_{local} (a)	η_{local} (a)	$Re[\eta]$	φ, eq. (26) (c)	Remarks	$\varphi_0[\eta]$ (c)	$Re[\eta_s]$	φ, eq. (27) (c)
50	200	4.00	0.57	350	1.0	350	3.00	0.90	128	0.74	Laminar	0.49	88	0.72
100	400	2.26	.76	525	1.0	525	2.26	.72	320	.26	↓	.20	280	.23
200	800	2.26	.76	1060	1.0	1060	1.90	.49	940	.081	↓	.067	815	.078
300	1200	2.26	.76	1580	1.0	1580	1.66	.40	1,730	.043	↓	.037	1,480	.044
400	1600	1.66	.86	1860	1.0	1860	1.66	.37	2,490	.031	Transition	.030	2,300	.030
500	2000	1.66	.86	2330	1.0	2330	1.66	.33	3,500	.040	Turbulent	.040	3,220	.040
1000	4000	1.66	.86	4670	1.0	4670	1.66	.25	9,250	.033	↓	.033	8,700	.033
2000	8000	1.66	.86	9350	1.0	9350	1.66	.18	25,600	.025	↓	.025	23,000	.025

[a]From figure 10.
[b]From figure 5.
[c]From figure 3.

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TABLE I. - PIPE TRANSITION PRESSURE-LOSS COEFFICIENTS

C_L FOR NEWTONIAN FLOW

Abrupt contractions:

D_2/D_1	0.8	0.6	0.4	0.2	0.0
C_L*	.13	.28	.38	.45	.50 (inlet from reservoir)

Abrupt expansions:

$$C_L = \left[1 - \left(\frac{D_1}{D_2}\right)^2\right]^2$$

$$C_L{}^* = 1.0 \left(\text{outlet into reservoir, } \frac{D_1}{D_2} \to 0\right)$$

90° Bends:

r/D	1.0	1.5	2.0	3.0	4.0
C_L*	.40	.32	.27	.22	.20

Valves (open):

	Gate	Globe
C_L*	0.2	10.0

90° Orifices (circular, slot):

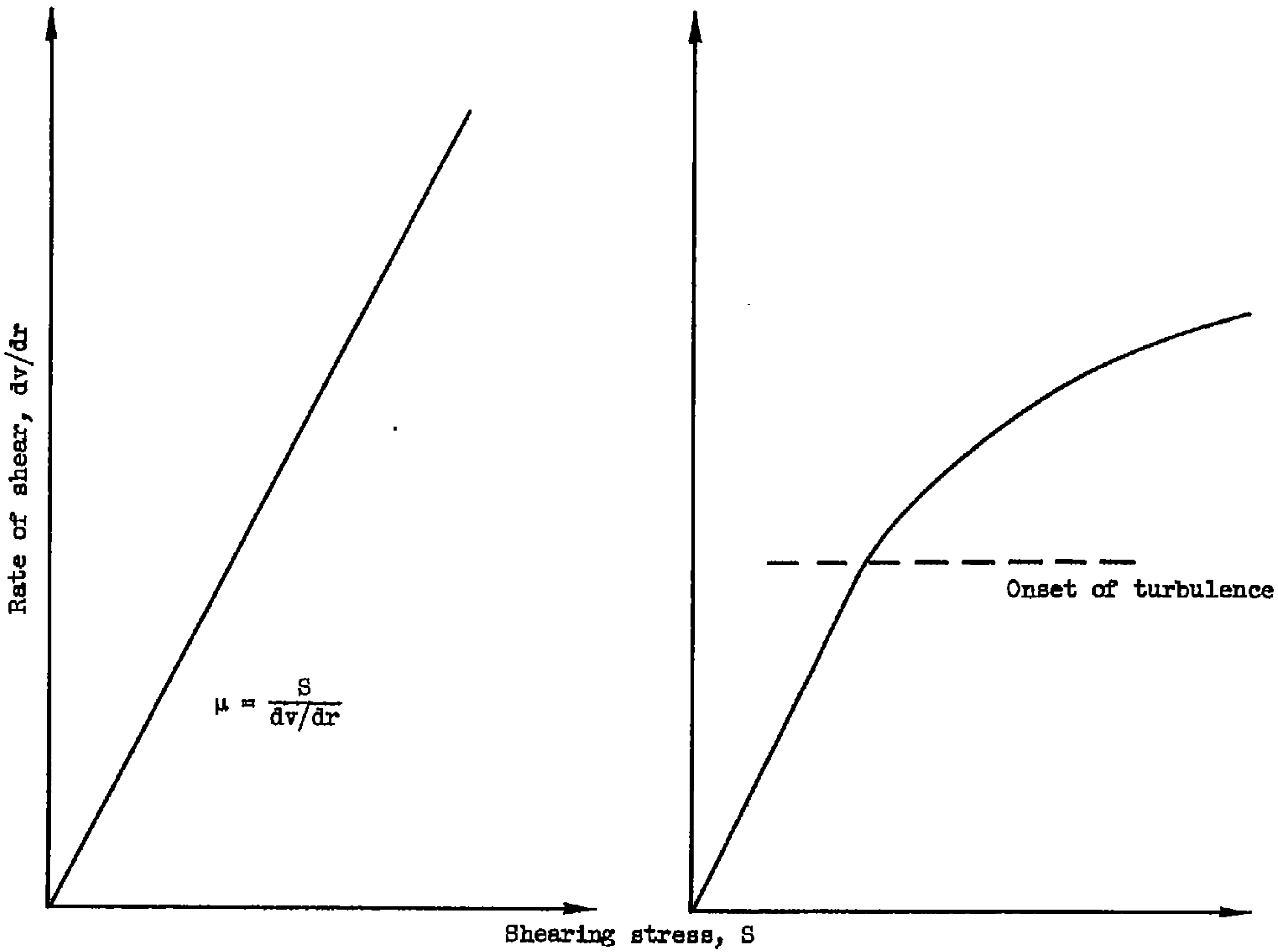
$$C_L = \frac{1}{C_c^2}\left(\frac{D_1}{D_o}\right)^4 - 1$$

D_o/D_1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
C_c*	.611	.612	.616	.622	.631	.644	.662	.687	.722	.781	1.0
C_L	∞	26,500	1640	319	96.0	37.5	16.7	7.9	3.7	1.5	0

Pipe entrance:

C_L** = 2.16 (over entrance length L_e of pipe)

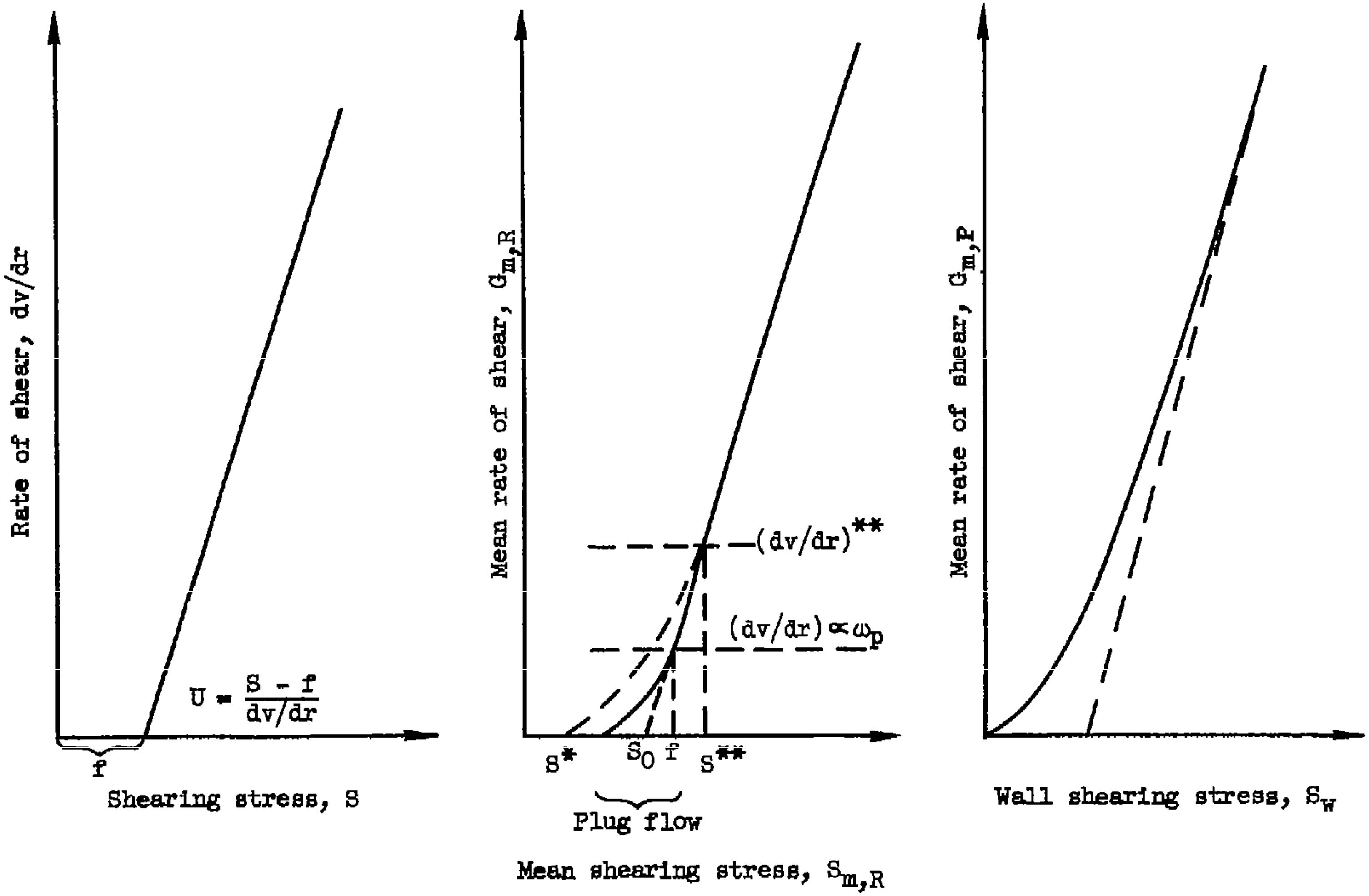
*From ref. 1, pp. 156 and 49.
**From ref. 10, p. 48.



(a) Laminar flow.

(b) Partly turbulent flow.

Figure 1. - Schematic flow curves of a Newtonian liquid.



(a) Ideal.

(b) Rotational viscometer.

(c) Capillary viscometer and pipe line.

Figure 2. - Schematic flow curves of a Bingham plastic.

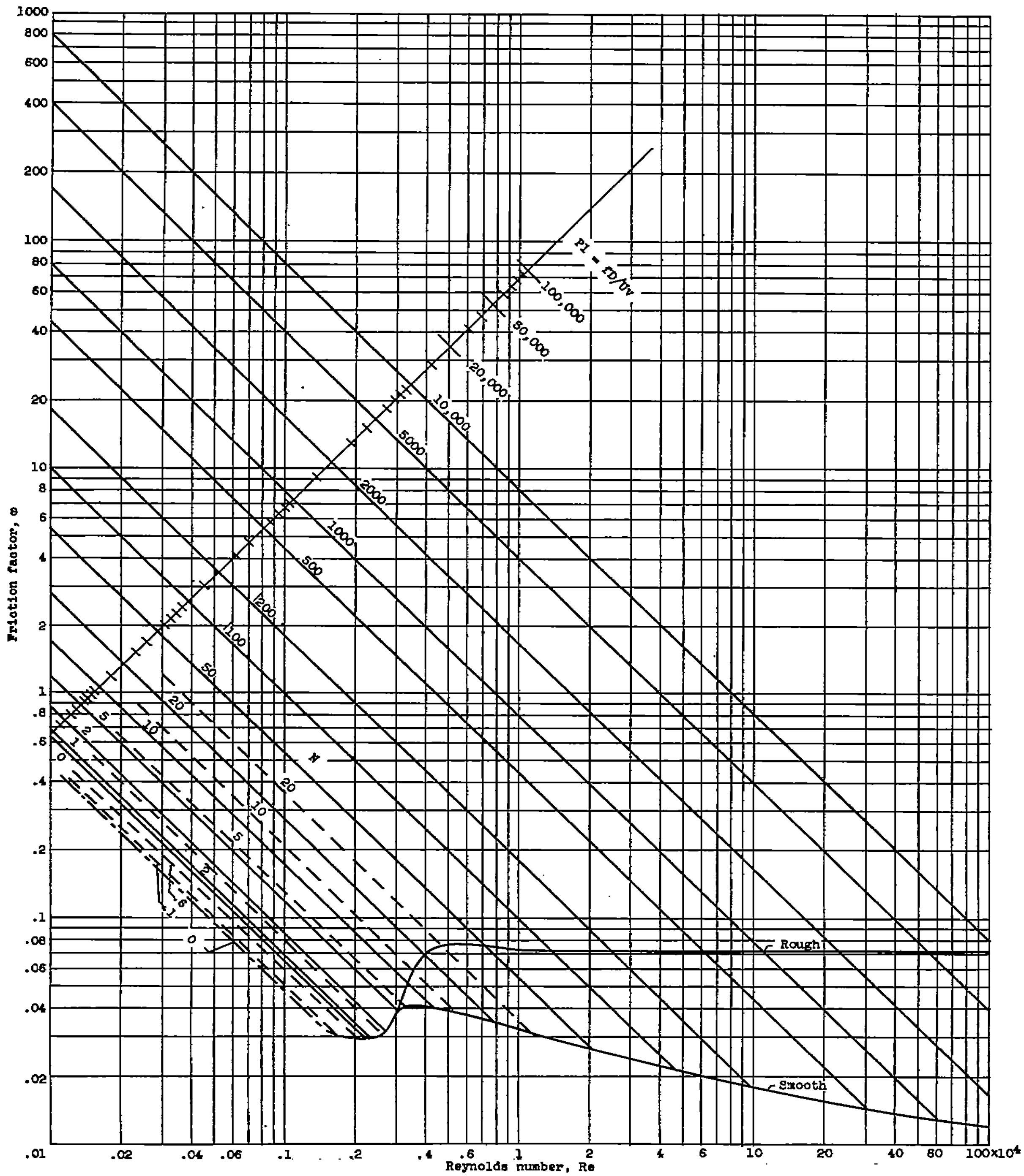
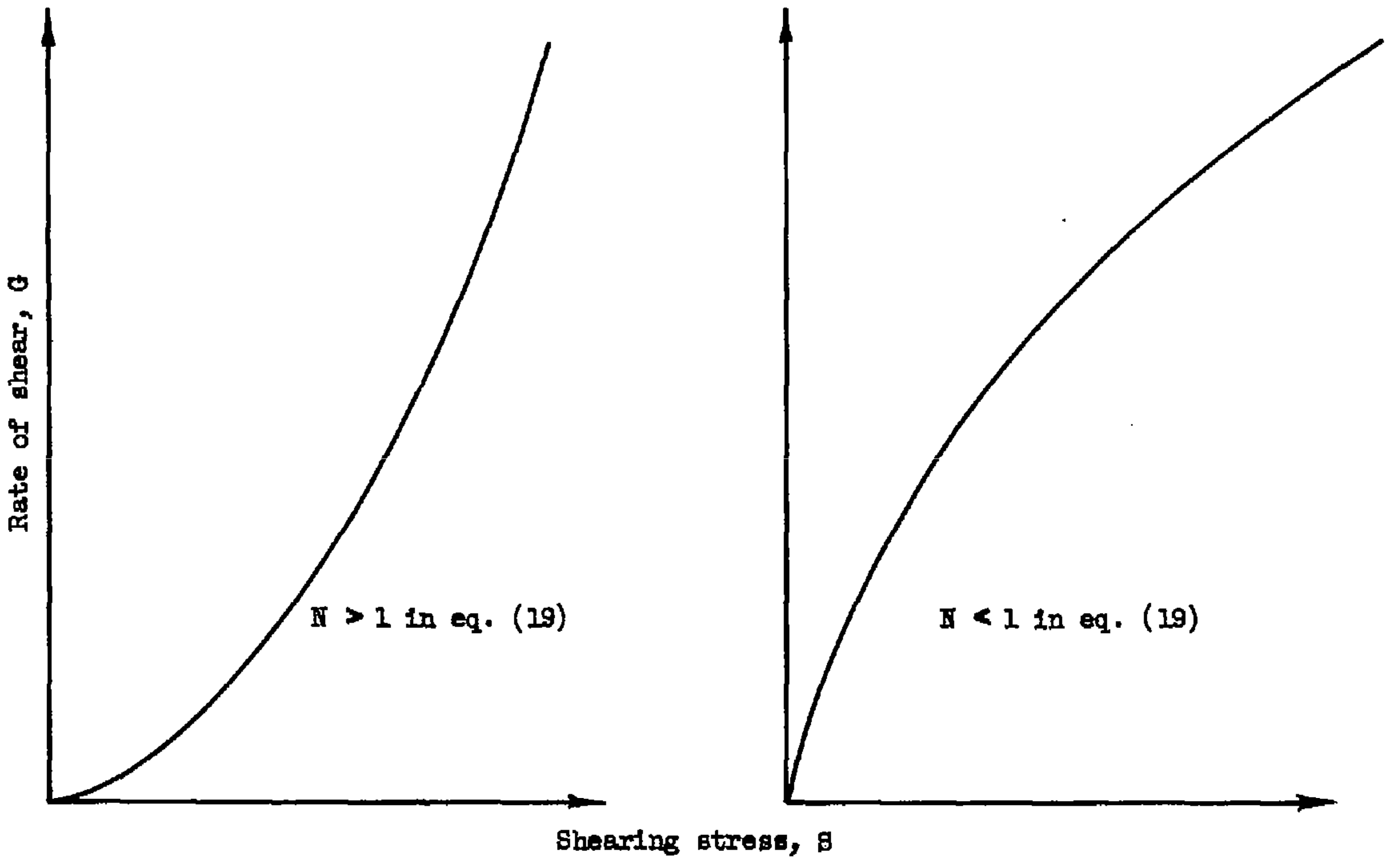


Figure 3. - Generalized friction diagram for Newtonian and non-Newtonian flow in pipe lines.



(a) Pseudoplastic.

(b) Dilatant.

Figure 4. - Schematic flow curves.

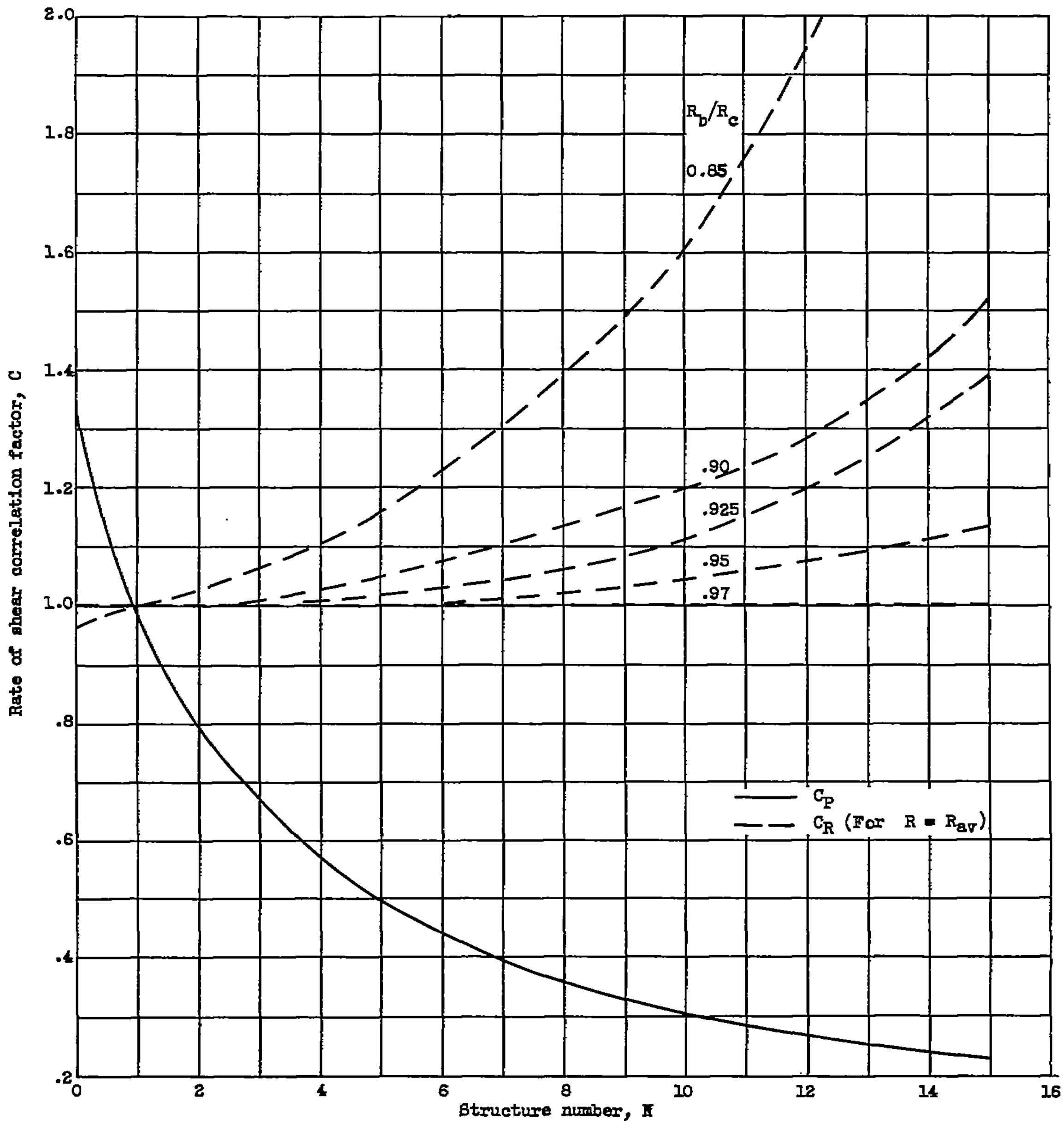
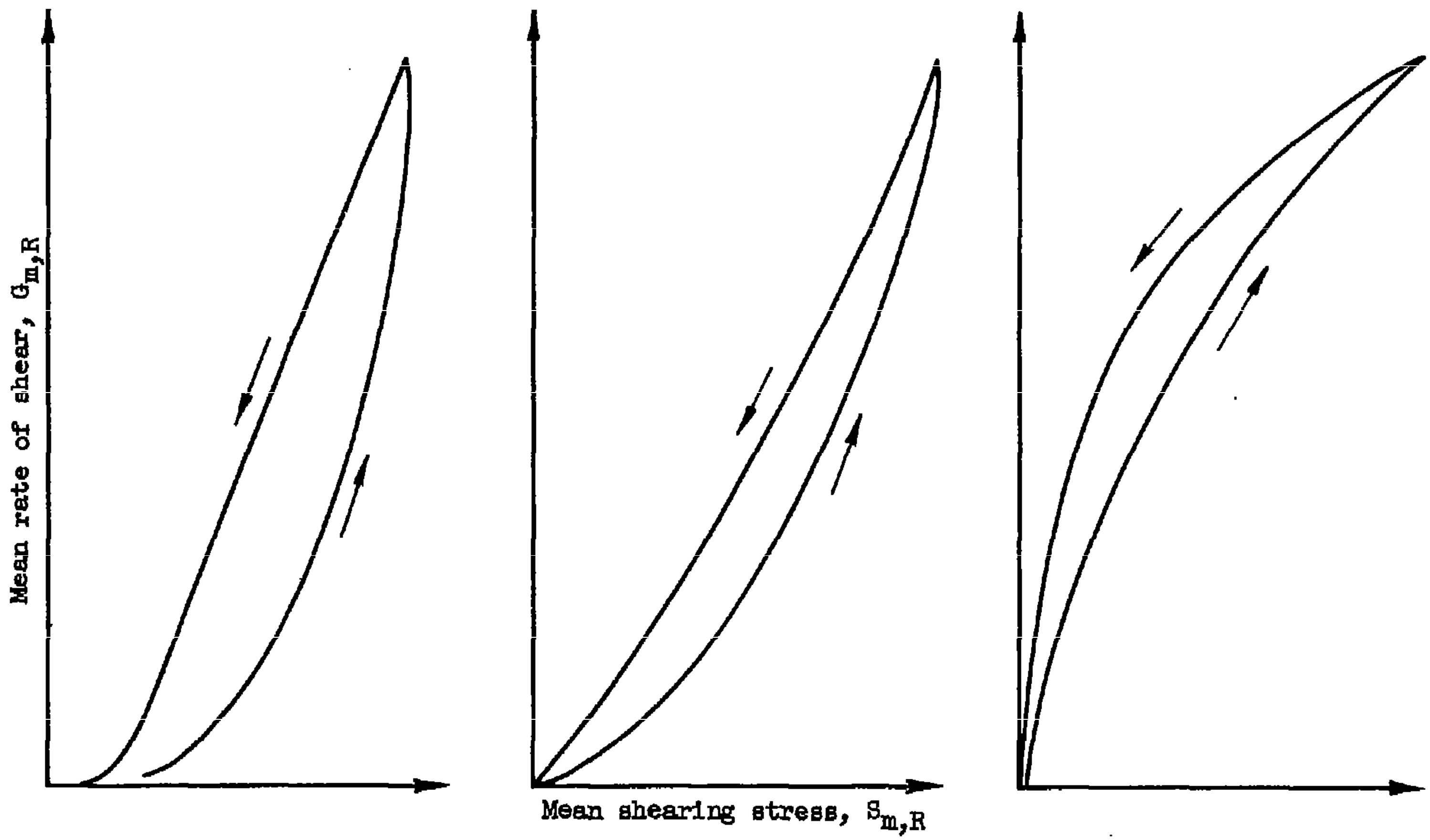


Figure 5. - Mean rate of shear correlation factor as function of structure number for non-Newtonian flow.



(a) Plastic. (b) Pseudoplastic. (c) Dilatant.

Figure 6. - Schematic rotational-viscometer flow curves of thixotropic materials.

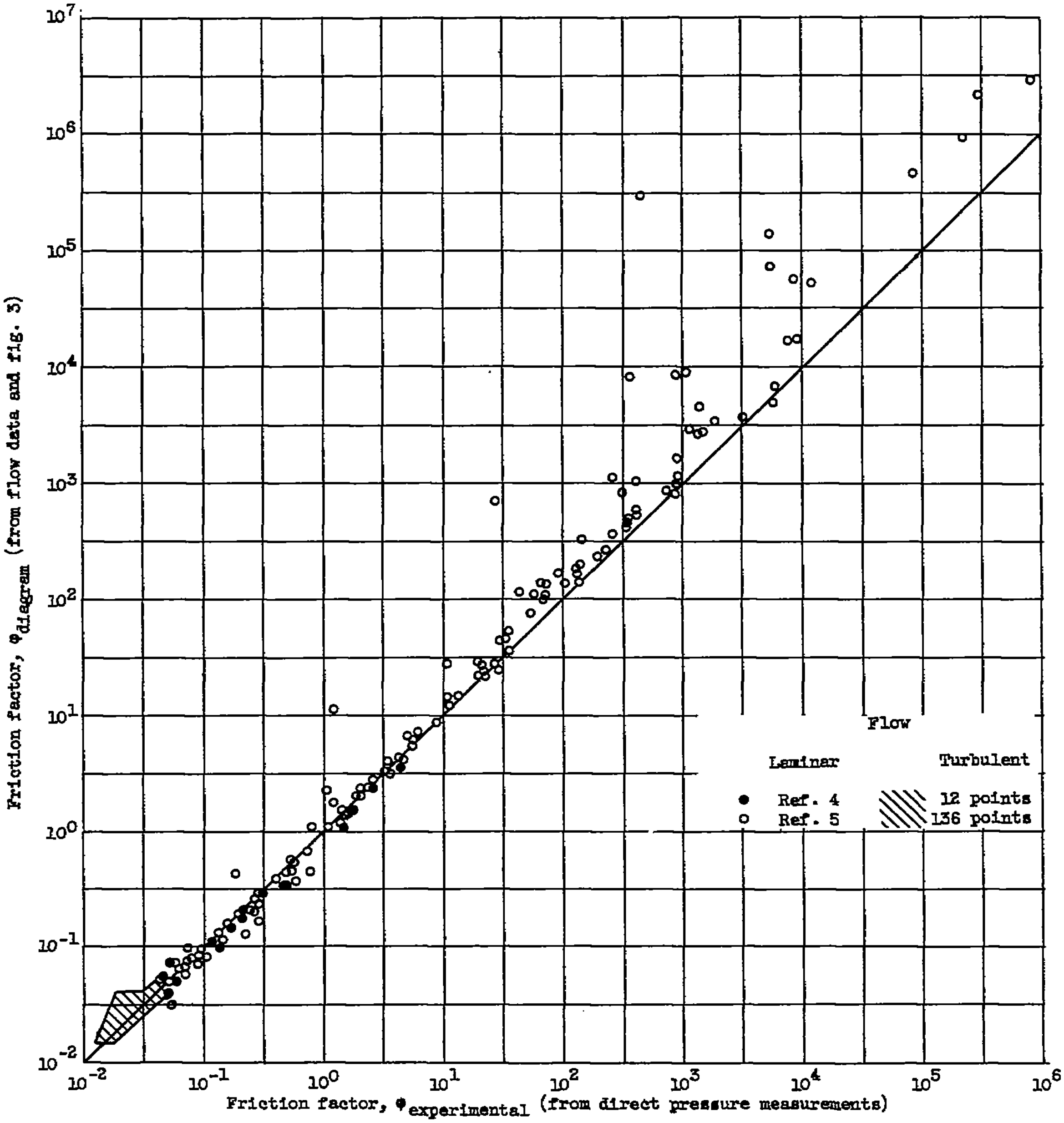


Figure 7. - Friction factors obtained for Bingham plastics from basic flow data and figure 3 compared with those from direct pressure measurements. Pipe diameter, 0.75 to 3.05 inches; plastic viscosity, 0.008 to 0.235 poise; yield value, 0 to 276 dynes per square centimeter.

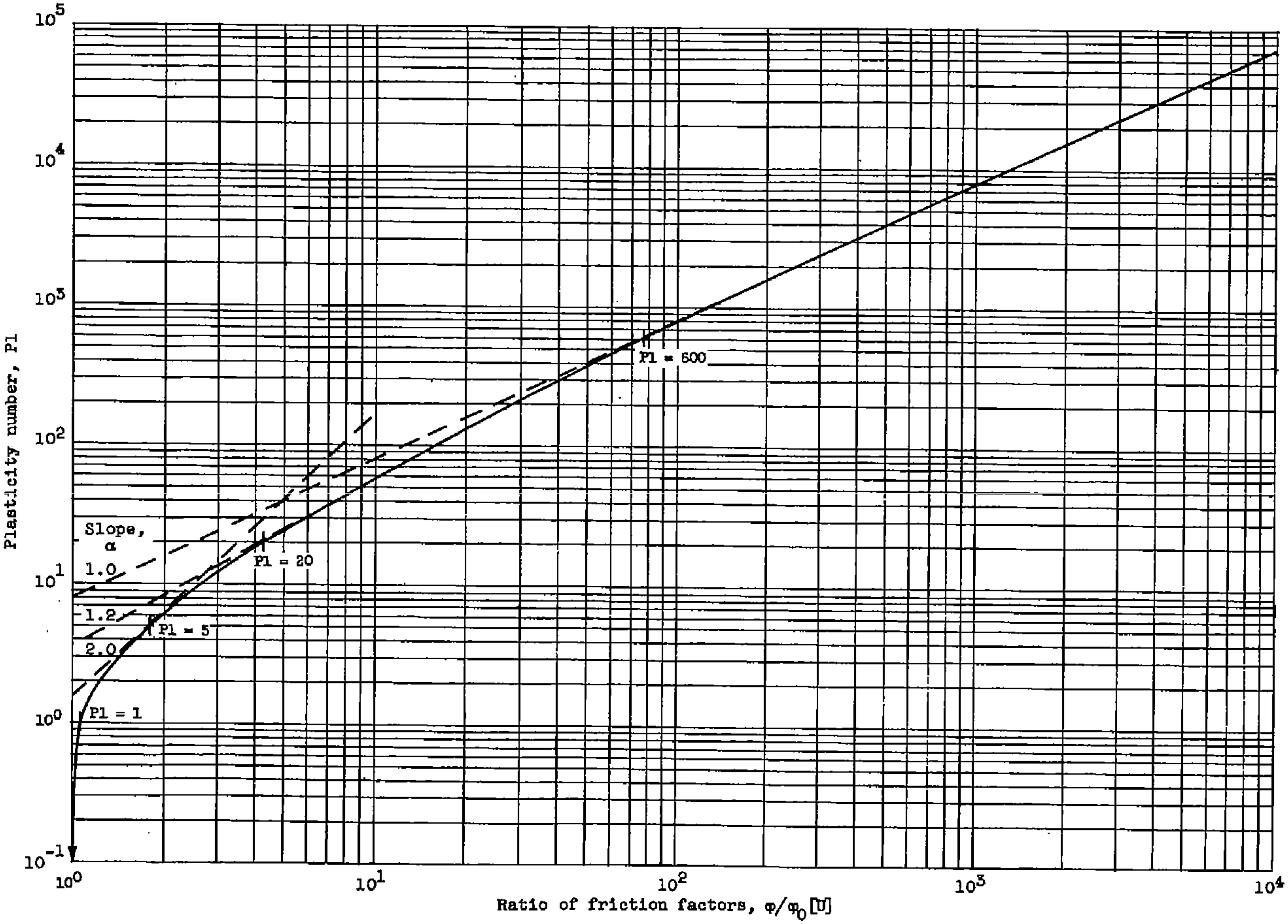


Figure 8. - Plasticity number as function of ratio φ/φ_0 [U] from figure 3 for Bingham plastics.

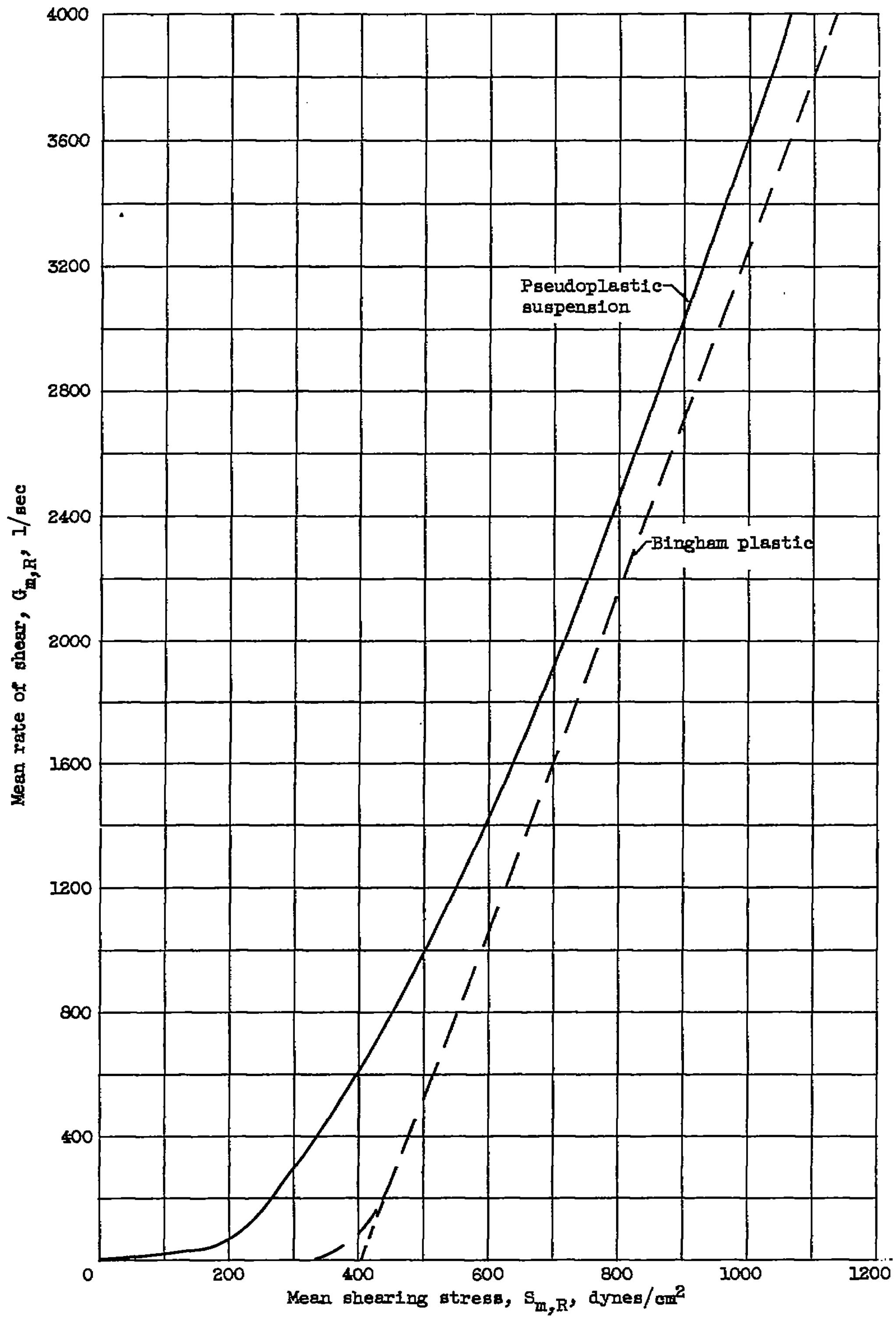


Figure 9. - Rotational-viscometer flow curves.

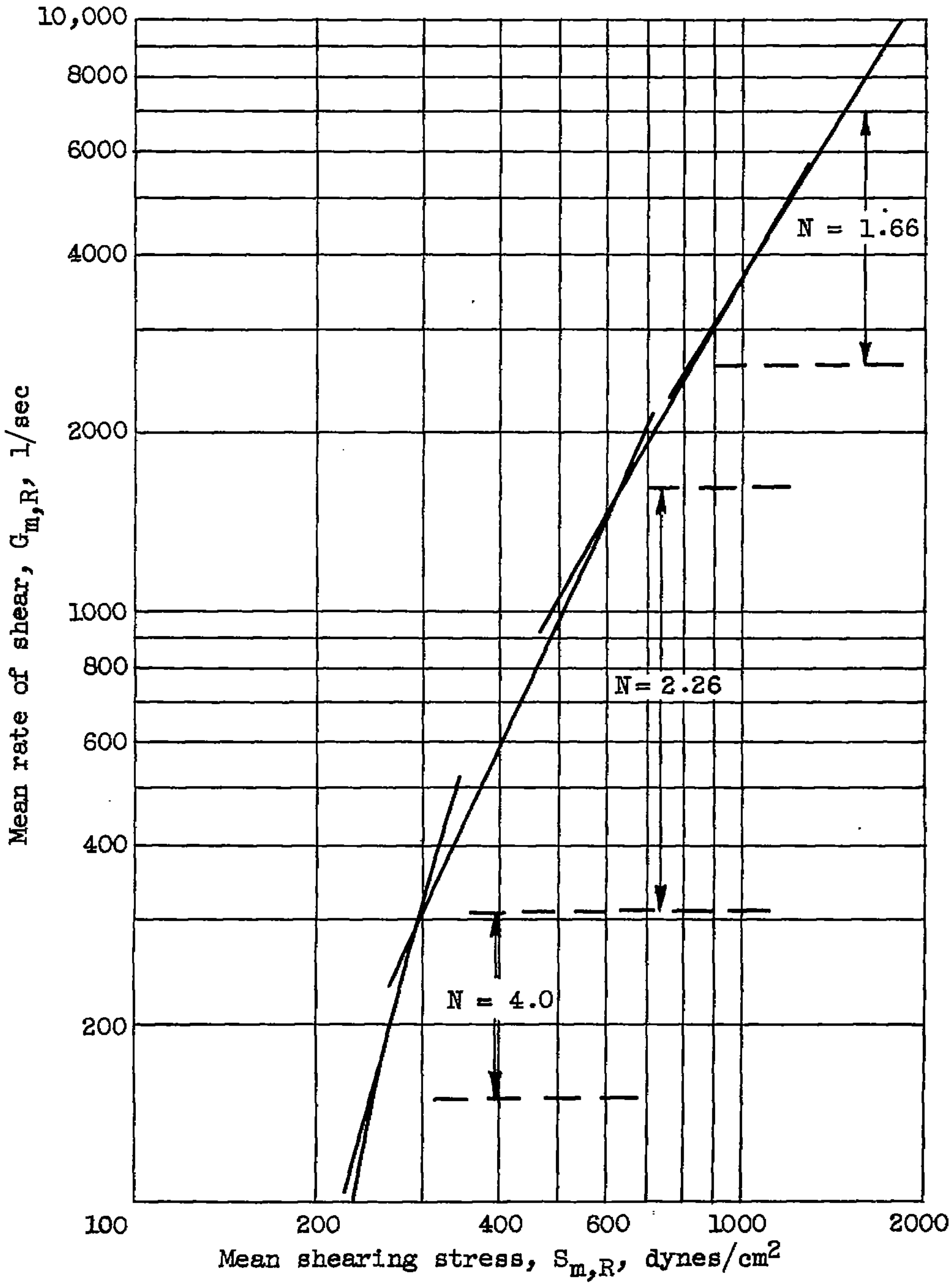


Figure 10. - Logarithmic plot of flow curve of pseudoplastic suspension (fig. 9) in accordance with equation (19).