

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS



# TECHNICAL NOTE 3489

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# CONTRIBUTIONS ON THE MECHANICS OF

#### BOUNDARY-LAYER TRANSITION

### By G. B. Ekhubauer and P. S. Klebanoff

## SUMMARY

The manner in which flow in a boundary layer becomes turbulent was  $^{\prime}$  investigated on a flat plate at wind speeds generally below 100 feet per second. Hot-wire techniques were used, and many of the results are derived from oscillograms of velocity fluctuations in the transition region. Following a presentation of the more familiar aspects of transition, there are presented the very revealing facts discovered while studying the characteristics of artificially produced turbulent spots. These are: (1) Oscillograms of natural transition are identical to oscillograms of spot passage. (2) Transition starts from perturbations in the laminar flow as spots which then grow in accordance with the general concept proposed by Emmons. (3) Turbulence always moves downstream followed by laminar flow. **(4)** The following flow is in a state of calm for a perid during which transition will not occur.

#### INTRODUCTION

The present paper presents the principal results of an experimental investigation performed in the boundary layer of a flat plate in an attempt to supply much-needed information about the process of transition from laminar to turbulent flow. In spite of the fact that a great deal was known about stability and the general circumstances surrounding transition, little was known about the actual mechanics of transition and its immediate cause. Consequently, it has been difficult to explain why flows known to be unstable do not necessarily become turbulent and flows calculated to be stable do not always remain laminar. The engiflows calculated to be stable do not always remain laminar. neer has had disappointing results in his attempts to maintain laminar flow, and he has often been unable to locate the source of the trouble. The theorist has not been able to come to grips with the problem for want of a physical model.

Experiments have failed to agree an a consistent picture of transition. The water-table experiment of Bmnons showed isolated patches of turbulence which suggested to him the thecx'yof transition by

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formation and growth of turbulent spots, as describd in references 1 and 2. Hot-wire probes used in boundary layers in air generally could not confirm this picture. The same was generally true of shortexposure schlieren and shadowgraph observations. For example, in the recent experiments of Evvard, Tucker, and Burgess on a 10° cone at supersonic speeds (ref.  $5)$  evidence for the growth of turbulent spots was seldom seen. While their'results did not preclude amechanism of transition involving turbulent spots, they indicated transition to be abrupt and fluctuating and followed by flow that was predominantly turbulent. One might even suspect that nature has confused the issue by providing more than one transition pattern.

The present investigation employed the same flat plate and wind tunnel as were used for the investigation of laminar-boundary-layer oscillations described in reference 4. The plate, which was a  $1/4$ -inch rolled aluminum sheet with a sharpened leading edge, was 12 feet long and completely spanned the  $\frac{11}{2}$ -foot distance across the test section of the tunnel. The top speed of the tunnel was around 140 feet per second, but most of the work was done-below 100 feet per second in order to place the transition region at a convenient location on the plate. The pressure gradient was adjustable between moderate limits and, unless otherwise specified, was zero.

Except for the measurement of pressures and mean-velocity profiles, for which static- and total-head tubes were used, all measurements were made with hot-wire probes and the associated amplifying and recording equipment. Only hot-wire arrangements sensitive to the longitudinal component of the fluctuations were employed, but in many cases the signals from two wires or two probes were observed simultaneously. While certain mean quantities, such as the root-mean-square value of the fluctuations, were useful, records of the actual wave form of the fluctuations turned out to be by far the more meaningful; hence film recording from a cathcde-ray oscilloscope yielded most of the significant information.

The cases studied included free transition with various amounts of free-stiesm turbulence, transition induced by the so-called trip wire, the turbulence wedge behind a three-dimensional roughness element, and spark-initiated transition and mibsequent growth of the turbulent spot. Only after the last of these was it possible to appreciate the real significance of what had been observed in all previous cases. In short, a transition region was found to be a region of spot growth as had been concluded by Emmons from his water-table observations. The seemingly futile attempts to reach and stuly phencmena at a transition point, all of which turned out to be steps toward a final picture, together with the confirming experiments are described in the following sections.

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# **SYMBOLS**



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## MONSTATIONARY CHARACTER OF TRANSITION

If a hot-wire probe sensitive to the velocity fluctuation  $u$  is placed sufficiently close to the surface in a transition region, the wave form of the output signal appears as shown in figure 1. This figure shows sample records made by photographing the screen of a cathode-ray oscilloscope with a continuously moving film. The film speed in these cases is about  $5$  feet per second and the progression of events is from right to left. Upward displacement of the trace corre-<br>sponds to an increase in velocity at the location of the probe. The sponds to an increase in velocity at the location of the probe. wire length and diameter were, respectively, 0.03 inch and 0.0001 inch. Since the lag was too small to be of concern here, the lag compensation, usually introduced during amplification, was omitted in obtaining these records. The distance of the wire from the surface was about 0.013 inch in these cases.

The top record was made  $5.6$  feet from the leading edge of the plate which was near the beginning of the transition region when the freestream turbulence was low (u'/U<sub>1</sub> = 0.03 percent) and the velocity of the free stream was 80 feet per second. Here the regular laminar-boundarylayer oscillations are followed by a burst of turbulence and this in turn is followed by laminar flow. The flat-topped character of the turbulent section is caused by overloading of the amplifier. The second record was made farther into the tremsition region where the flow was turbulent about 70 percent of the time. In the third and fourth records the position was selected so that the flow was again turbulent about 70 percent of the time, but in these cases the disturbance level was raised, in the third case by increasing the free-stream turbulence to 1.0 percent and in the fourth case by placing a l-mildlmeter wire along the surface 2 feet from the leading edge.

In all cases the flow was laminar for some distance (even downstream from the l-millimeter tire) but contained low-frequency fluctuations, generally consisting of amplified sinusoidal waves when the stream turbulence was low or fluctuations of a less regular nature when the disturbance level was high. This was followed by the transition region, usually around 2 feet in length, in which increasing smounts of turbulence were observed and finally by the completely turbulent regime. Throughout the transition region the mean characteristics of the layer change graduslly from those characterizing lsminar flow to those characterizing fuJJy developed turbulent flow.

Records similar to those shown in figure 1 have often been used in the past as proof that transition occurs suddenly and that the observed gradual change in mean flow through a transition region is due to the varying position of the sudden change. The concept most commonly held was that transition occurred abruptly along an irregular line which

separated the laminar flow ahead frmn turbulent flow behind and that this line surged upstream and downstream. In 1951 Emmons (refs. 1 and 2) introduced the idea of turbulent spots which formed at random and grew while moving downstream until they finally encompassed the entire field. There is a very basic difference between these two points of view and one on which much of the mechanics of transition hangs. The true state of affairs proved to be very illusive, but once the key to the mystery was found the answer was clearly displayed in all records. Figure 1 therefore contains much more significant information about transition than had previously been supposed. This information is the manner in which a turbulent section begins and ends. This feature will be referred to repeatedly in subsequent sections.

From numerous records such as those shown in figure 1 it is possible to determine the fraction of the total time that the flow is turbulent at any point in the transition region. This fraction, defined as an intermittency factor  $\gamma$ , is shown in figure 2 for several cases where conditions leading to transition were varied. The length of the region was different in all of these cases, but the distributions were similar. In order to obtain a common basis of comparison, Gaussian integral curves were fitted to the cases separately and the standard deviation  $\sigma$  was determined for each case. All could then be represented to a common scale in units of  $\sigma$  and, when superimposed at the point  $\gamma = 0.5$ , they appear as shown in figure 2. Here x is the distance from the leading edge of the plate,  $\bar{x}$  being the distance to the point where  $\gamma = 0.5$ . The curve is a Gaussian integral curve. Values of  $\sigma$  ranged from 0.3 to 0.8 foot.

It is probably of some significance that transition regions are statistically similar whether long or short and whether disturbances are strong or weak and irrespective of whether they are introduced from the free stream or from a roughness element on the surface. It does not follow that transition regions would show the same distribution of  $\gamma$  in the presence of a pressure gradient. The close resemblance to a Gaussian integral curve would seem to indicate a near randommess, which may indicate that transition depends on random perturbations superimposed on the more or less regular amplified oscillations in the boundary layer. The skewness evidenced by the departure from the curve at the downstream end of the region may be related in some way to the structure of a transition region as discussed in the section "Structure of Transition Region and Nature of Transition.'f

# CHAMGES IN MEAN CHARACTERISTICS THROUGH TRANSITION REGION

Some of the more fsmilisr progressive changes occurring through a transition region are examined next. The first and best known is the

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gradual change in mean-velocity profile which is shown in figure 3. Here the transition region begins at 5.25 feet fram the leading edge of the plate and ends at 8 feet. The profiles up to the  $5.25$ -foot position are of the Blasius type characteristic of the laminar boundary layer with zero pressure gradient. From this point on the profiles change progressively and reach that characteristic of a fully developed boundary layer at  $8$  feet. These measurements were made in the conventional manner with a small total-head tube. The free-stream velocity was 80 feet per second, and the stream turbulence level was about 0.03 percent. It should be remarked that the position of transition would shift somewhat from day to day, and check runs had to be made to be sure that this did not occur during a set.

Observe next the change in the fluctuation profiles shown in figure  $4$ . Here  $u'$  is the root-mean-square value of the x-component of the velocity fluctuations, measured by conventional hot-wire techniques employing a wire 0.0001 inch in diameter and 0.03 inch long. In this case the turbulence level of the free stream was again about 0.03 percent, and under this condition fluctuations in the laminar layer near the beginning of the transition region consist mainly of the strongly amplified oscillations. These account for the fluctuation level at the  $5-$  and  $5.25$ -foot positions. As the transition region is entered the fluctuation level increases markedly near the surface and develops a strong maximum around  $y \approx 0.03$  inch. Referring to figure 1, it is noticed that there is an abrupt step-up in velocity from a lamhar region to a turbulent region, indicating that the transition region contains a mixture of the two distinct mean-velocity profiles, one pertaining to the lsminsx flow and the other pertaining to the turbulent flow. The steps from the one to the other register on the hot-wire as fluctuations and account largely for the high fluctuation level in the vicinity of the maximum. Referring to figure  $\mathfrak{Z}_2$ , it is seen that the step between lsminar and turbulent profiles disappears around 0.1 inch and is smalJ percentagewise at greater distances. This is about the location of the foot of the peak in figure  $4$ . It is seen also that the peak subsides as the downstream end of the transition region is approached. The fluctuation profile at  $8$  feet is similar to that observed for fully developed turbulent flow.

It is clesr that turbulent characteristics are fully developed at the end of the transition region. This is perhaps not surprising since this is alresdy far frm many points of initial transition. An interesting question is, how close to the point of actual breakdown sre the steady-state characteristics of turbulent profiles to be found? At this stage of the investigation it appesmd that the real obstacle to obtaining the answer to this questicm, as well as to studying the mechanism of transition itself, was the fluctuating character of transition which made measurements at a transition point practically impossible. Since the well-known wedge of turbulence behind a single

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roughness element appeared to offer the possibility of a sharp line of transition well away trom the disturbing element, attention was turned to this case.

## TURBULENCE WEDGE

If a particle of sufficient size is placed on the surface in a region of laminar flow, transition occurs at the particle, and a wedgeshaped region of turbulent flow extends downstream. Such wedges have frequently been observed on airfoils starting frcm particles of dirt or similar surface irregularities. It has also been observed that when the particle size or the velocity is reduced, the wedge may begin some distance downstream from the particle. In any case the particle introduces disturbances which cause breakdown of the laminar flow.

It appears that various observers agree on nearly the same value for the half-angle of the wedge and report values in the neighborhood of 10°. Thus the width of the turbulent region increases more rapidly and is always much wider than the wake of the object initiating the wedge. Charters (ref. 5) appears to have been the first to call attention to this more rapid spreading and termed the effect "transverse contamination."

The wedge was introduced into the present investigation mainly in the hope that it would provide a reasonably sharp, stationary line of demarcation between laminar and turbulent flow at which phenomena at the position of changeover could be studied. However, it was soon discovered that the previously reported sharp outllne was only an average condition and that the wedge was in reality bounded by an intermittent region as shown in figure 5. It was further found that only when the particle was sufficiently large or the velocity was sufficiently high did the sides become straight and the angle attain a constant value. When these conditions were not met, even though transition occurred at the particle, the sides initially curved outward. approaching the proper angle asymptotically.

Figure 5 shows a fully established wedge produced at a free-stream velocity of 80 feet per second by a  $1/8$ -inch sphere cemented on the surface of the plate 2 feet from the leading edge. This case was investigated in some detail with a hot-wire probe and a total-head tube. Beyond a fully turbulent core, subtending a half-angle of  $6.4^{\circ}$ , was a region in which the turbulence was intermittent, as illustrated by the oscillograms in the figure. The outer limit of the intermittent region subtended a half-angle of  $10.6^{\circ}$ . It is interesting to note that this value agrees reasonably well with the value of  $9\frac{1}{6}$  found by Charters to be the angle of transverse contamination.

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Since one purpose in using the wedge was to learn how soon after abrupt transition the fully developed, steady-state condition would be found, mean-velocity profiles were determined with a total-head tube. Within the turbulent core the profiles were those characteristic of a fully developed turbulent boundary layer. While the so-called transition in this case was not sharp, the intermittent region was short compsred with that found in free transition on a flat plate. Thus it appears that the only requirement for the existence of the fully developed character is that the flow be turbulent all of the time.

The oscillograms in figure 5 show typical conditions at several points. Here, as in figure 1, the time scale rums from right to left, and the interval between timing dots is 1/60 second. The top record shows the sudden velocity step-up at the beginning of a turbulent region and the ending followed by a slow fall in velocity which was always characteristic of a record obtained with a hot-wire close to the surface. A possible explanation of the regular repetition of turbulent bursts is given in the section "Artificially Tnitiated Turbulent Spot." The second record shows a very short pulse near the outer limit having a suggestion of the characteristic shape. The third record obtained near the apex of the wedge, although giving the appearance of being composed largely of spikes jutting upwsrd, has traces of the characteristic shape. The fourth record shows disturbances in the laminar layer caused by the presence of the wedges. Without the wedge no visible disturbance would have appeared here for the same signal amplification.

At this stage two facts had become clear. One was that the socalled transition point could not be made stationary. The other was that alJ records in a transition **region,** except those pertaining to the intermittent region very near the apex of the wedge, showed two very distinct features, namely, the beginning of a turbulent section with an abrupt velocity increase and the ending followed by a slow, exponentiallike velocity decrease. The reason for these facts was unknown. It was, of course, not unreasonable that a transition point should be bobbing about, but the characteristic features of the records were very puzzling. It was conjectured that the abrupt rise might mean that the flow somehow tumbled and that the turbulence was the result of the tumbling. However, there was no definite evidence of a momentarily large disturbance in the laminar flow to cause the tumbling. In particular, it was noted that deep in a transition region, where the flow was turbulent for a large percentage of the time, oscillations in the remaining laminar regions were conspicuously absent.

Much time was spent in attempts to find evidence of mmentary flow separation in transition regions. The most trustworthy methd consisted of emitting iodine vapor from a small hole in the plate and noting by the discoloration of a starch film on the surface whether vapors ever traveled upstream. No evidence of separation could be found in any part of a transition region.

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Bits of information like this, while so far not proving anything, aroused the suspicion that all was not welJ with the concept of breakdown along a wobbly line which fluctuated randomly upstream and downstream. Perhaps patches of turbulence like those observed on a water table by Emmons (ref. 1) and later by Mitchner (ref. 6) were actually being encountered. This would at least explain why a transition point could never be held stationary.

Early in the investigation a number of film records had been obtained of the simultaneous response of two hot-wire probes placed one behind the other with a spacing of from 3 to 5 inches. some offsetting was necessary to avoid interference. Inspection of a large number of records showed that only very rarely was turbulence absent at the downstream station when it was present at the upstream station. Ih these few cases it appeared as though turbulent patches might have moved across the probes, striking the upstream one first and the downstream one second; but, because of the rarity of this occurrence, turbulent patches seemed to be the exception rather than the rule. As previously mentioned, a similar condition was found by Evvard, Tucker, and Burgess (ref.  $3$ ) in their studies of transition on a 10<sup>°</sup> cone at supersonic speeds. In a few cases their short-exposure schlieren photographs showed regions of turbulent boundary layer followed downstream by laminar flow. As a rule transition appeared to occur abruptly at various points, followed downstream by flow that was predominantly turbulent.

While these experiments showed that isolated turbulent spots could exist, they were interpreted as evidence against a mechanics of txansition involving the growth of spots. This interpretation, in addition to being wrong, had the unfortunate effect of delaying the crucial experiment described in the next section.

### ARTIFICIALLY INITIATED TURBULENT SPOT

If a particle which produces a turbulence wedge is suddenly removed, the turbulence will recede downstream followed by laminar flow. If a particle or equivalent disturbance appears only for a brief instant, a turbulent spot is produced which moves downstream. Both Emmons and Mitchner had shown that spots could readily be produced in their watertable experiments by allowing, say, a drop of water to strike the surface. Mitchner (ref. 7) had also shown that a spot could be initiated in a boundary layer by au electric spark, and he was able to study its form and rate of propagation. He commented, however, that the ability to prcduce a spot artificially did not offer a complete verification of Emmons's theory by any means and that considerable work remained to be done. The investigation of the spot was undertaken here with the feeling

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that, whether or not it had any significant connection with natural transition, it was unquestionably a phenomenon of intrinsic interest. Fortunately it turned out to be most fruitful.

A spsrk was made to jump to the flat plate from a fine needle placed  $1/4$  inch or less from the surface. Short duration was obtained by discharging a condenser previously charged from a high-voltage source. Spsrks were prduced one at a time while a hot-wire probe was placed in various locations to detect the passage of the spot. From film records containing l/60-second timing marks, the times for the spot to reach and pass over the probe were readily determined. Figure **6** shows the velocities and shape of the spot as derived from such records. Ihcluded in the figure is a sample record of the passage of a spot over a hot-wire 0.015 inch from the surface. The electrical disturbance frm the spark picked up by the circuit showed on the trace in all cases and served as a convenient and accurate means of indicating the time of discharge.

The records were at once seen to contain much more information than the time of yassage of a spot. The first significant fact recognized was that these records had the same features as all records previously obtained in transition regions, namely, the start of the turbulent section with an abrupt step-up in velocity and the ending of the section with a slow exponentiallike fall. For comparison a typical record of natural transition is included in figure  $6$ . This feature was characteristic of the passage of a spot. The abrupt rise occurred when the spot met the wire, and the slow fall occurred when it left. This was not proof that all turbulence actually originated at points and grew into spots, as did the spark-initiated spot of figure 6, but it was almost indisputable evidence that what had been observed in transition regions was the passage of turbulent patches as they moved downstream. This subject will again be considered in the section "Structure of Transition Region and Nature of Transition" where experiments till be described that completely confirmed this evidence.

The characteristics of the spot shown in figure 6 were derived from detailed observations at free-stream velocities of 30 feet per second and at positions 2.7 and  $4.7$  feet downstream from the spark with the spark 2.3 feet from the leading edge. Measurements at a free-stream velocity of 50 feet per second showed the same behavior. Additional information was obtained on the velocity of propagation at various distances along the center line and on the angle  $\alpha$  when the spark was 3 inches from the leading edge.

The center-line observations showed that the streamwise propagation velocity of the kading end of the spot (downstream end) was independent of distance from the spark but that the velocity of the trailing end (upstresm end) apparently decreased smewhat for distances less than 1.7 feet. However, the accuracy was low.within 1 foot because of the

shortness of the time interval and the uncertainty in defining the beginning and ending of a spot. Furthermore, a duration of the effect of the spark of only  $0.005$  second, such as might have arisen from the size of the initial disturbance, would have produced an apparent reduction in velocity of the trailing end at short distances of the amount observed. Accordingly, it was concluded that the reduction was apparent rather than real. The velocities were further found to be directly proportional to the free-stream velocity  $U_1$ . The values at the surface were: At the trailing end,  $(0.50 \pm 0.01)U_1$  and, at the leading end,  $(0.88 \pm 0.02)$ U<sub>1</sub>.

As indicated by the shape of the **trailing** end shown in the elevation view of figure 6, the velocity of this end was independent of y out to about the normal thickness of the laminar layer  $\delta_{\lambda}$ . The velocity of the leading end was at first thought to be about the same as  $U_1$ , but more detailed observations revealed that it varied from a value of 0.88U<sub>1</sub> at the surface to  $U_1$  at the end of the overhanging tip shown in figure 6. The turbulence is, in fact, transported downstream with the free-stream velocity, and the lag at the surface is due to the time required for propagation inward through the slower moving air of the laminar layer. Probably the chief significance of the slower progress nesr the surface is that it gives rise to an overhanging leading edge. Since the velocity at the surface evidently depends on the time required to reach the surface, it is not clesr that it should bear a constant ratio to U<sub>1</sub> nor, correspondingly, that the overhang should remain proportional to the length of the spot. Dependence on the thickness of the laminar layer would also be expected. Nevertheless, the ratio was constant and proportions were preserved as far as could be ascertained for the limited range covered in the investigation.

Only on the center line were sufficient data taken to define the thickness distribution of the spot. The position of the outer edge was determined from the known positions of the hot-wire and spark and the time required for the spot to meet and pass over the wire for different values of y. In this way rates of propagation of the edges were determined, and from the rates the relative positions could be found. As far as could be determined fran the measurements, the general shape shown in the elevation view of figure 6 was preserved as the spot grew. So far there is no ready explanation for the existence of the hump; but, using the top of the hump as a measure of the thiclmess of the spot, it was found that the thickness increased with distance like a fully developed turbulent boundary layer with an initial thickness equal to that of the laminar layer at the position of the spark.

The plan form of the spot was determined in detail by placing the hot-wire at different distances z from the center line. A fixed distance,  $y = 0.015$  inch, was chosen as being sufficiently close to

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the surface to give results indicative of the condition at the surface. As far as could be ascertained, the shape remained similar as the spot grew. Except for some degee of bluntness near the center line, the leading sides (or edges) were on the average straight lines extending to about the maximum width, and the angle  $\theta$  had a constant value of 15.3°. The downstream propagation velocity was everywhere the same along these sides and equal to the velocity at the center. The trailing edge was found to preserve the somewhat indented appearance shown in figure 6; and, except for a slight variation in velocity to account for this shape, the velocity was essentially constant along the trailing edge and equsl to  $0.5U_1$ , the value on the center line. The shape of the spot shown here is qualitatively like that found by Mitchner (ref. 7). With respect to the velocity of propagation, it is interesting to note in reference 8 that turbulent bursts, which were attributed to surface roughness, were observed on a body of revolution at a Mach nunber of 3.5. The velocity was estimated from the shock wave emanating from the upstream edge of the burst and reported to be 60 percent of the free-stresm velocity.

b discussing this problemwith Dr. Hugh L. Dryden, he suggested that it would be interesting to find out whether a spot would grow laterally when the Reynolds number based on the displacement thickness of the laminar layer was below the critical value of about  $450$ , for which complete stability is predicted on the basis of small-perturbation theory. Accordingly, the growth envelope, corresponding to the wedge of angle  $\alpha$  in figure 6, was investigated when the spark was fired 3 inches from the leading edge of the plate at free-stream velocities of 10 and 30 feet per second. At 10 feet per second the Reynolds number was below critical for a distance of about 1 foot frm the spsrk. There was, however, a small favorable pressure gradient in this region which probably extended the stable range.

The envelopes corresponding to.10 and 30 feet per second are shown in figure 7. Beginning from the initial size of the disturbance, the growth is at first slow but approaches a linear increase in both cases with inclinations  $10^{\circ}$  and  $8.6^{\circ}$ , respectively, for 30 and 10 feet per second. If the 8.6 $^{\circ}$  line is extrapolated to the axis, the app origin is at a position corresponding to a Reynolds number of  $480$ . This is better agreemat with stability theory than could have been expected under the circumstances and may be partly fortuitous. However, there is definitely a lag in growth for the extent of the stable range.

The difference between  $\alpha = 10^{\circ}$  in figure 7 and  $\alpha = 11.5^{\circ}$  in figure **6 may** be accounted for by the fact that in the latter the initial size of the spot was neglected. However, the difference between  $\alpha$  at 30 and 10 feet per second probably reflects the effect of Reynolds nuuiber. Mitchner's value of  $\alpha$ , which was found at a Reynolds number in this low range, was  $8.6^\circ$ .

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The remarkable similarity between the growth envelope of a spot and the turbulence wedge and the agreement between the angles in the two cases is evidently far more than mere coincidence. It will be noted two cases is evidently far more than mere coincidence. in figure 5 that the occurrence of alternately turbulent and laminar flow has a striking regularity. It is lmown, furthermore, that the manner of beginning and ending of turbulent sections signifies passage downstream of turbulent regions. These facts suggest that the turbulence wedge may be something on the order of a succession of turbulent spots telescoped one into the other just far enough to form the fully turbulent core.

A more plausible connection with spots may be found in an important phenmenon associated with receding turbulence. This is the stable state left in the laminar flow following the passage of turbulence. The existence of this phenomenon was suspected when it was noted from the ending of turbulent records that the velocity near the surface was left high after the passage of turbulence and then decreased to Its normal value. It was concluded that this meant that the entire velocity profile of the laminar layer was left turbulentlike and then gradually changed back to a normal type of profib. While the layer was in this state, it was highly stable and no breakdown was likely to occur. This interval wi12. be termed "the recovery trail." The remarkable calm following this trail and some of the consequences of it will be discussed in the section "Calming EEYect." For the present it is merely noted that receding turbulence is automatically followed by laminar flow at least in this trail unless overtaken by another spot. Applying this to the turbulence wedge and realizing that the sides will move downstream unless held by transverse spreading, it appears that any irregularity will grow into a  $$ sequence of turbulent protuberances moving downstream and separated from one another by the length of the recovery trail. The record of figure 5 would.certainly support this conclusion.

Returning to a consideration of the spot, the streamwise propagation may be rationalized as follows: The leading end is projected downstream with the free-stream velocity and the turbulent state penetrates the slower moving air of the laminar layer causing the lag of increasing amounts toward the surface. The movement of the trailing end may be visualized as just the reverse, namely, a blowing away of the turbulence by the outer stream and a lagging behind of the turbulence in the slower moving air near the surface. The mean-velocity profile here is such that air traveling slower than the trailing edge constitutes a very thin layer near the surface in which turbulence is not self-sustained. In other words, it dissipates rapidly because of both viscosity and diffusion outward. Because of the stability of the profile, the turbulence diffused outward cannot sustain itself in the outer strata. If this were not the case, the velocity of the trailing end would be fixed by the velocity of air which was never turbulent, namely, the velocity at the height of the laminar sublayer, which is about  $0.301$ .

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The triangular shape of the spot with vertex pointing downstream may he accounted for by the fact that the downstream end does not have the time that the upstream end has in which to grow laterally. It was noted by Dr. Dryden that a rate of spot expansion could be estimated in the following way: Assuming a mean velocity of the spot to be the average of  $0.50U_1$  and  $0.94U_1$ , where  $0.94U_1$  is the mean of  $0.88U_1$  and  $U_1$ , a value of 0.72 $U_1$  is obtained. Denoting a velocity of spot growth by  $cU_1$ , the value of c vupstream and downstream becomes the difference between the value 0.72 and that of the two ends, or  $c = 0.22$ . Since the vertex angle of the leading edge is  $15.3^\circ$ , one should find  $c = 0.72 \sin 15.3^\circ = 0.19$ . While these values are in reasonable agreement, the meaning of the first value of c is vague because of the different conditions at the leading and trailing edges. It is felt that more information must be obtained on the factors controlling spot growth before this problem is understood.

By this time there seemed to be little doubt that a transition region consisted of turbulent patches going downstream ard finally merging to form the completely turbulent region. It was pertinent therefore to inquire whether two spots side by side would merge at their normal rate of growth or whether they would close in more rapidly because of a mutual effect on the flow between them. Accordingly, two sparks separated in the z-direction were set off simultaneously and spot growth was studied. As far as could be detected, the two behaved independently, and the mutual effect, if any, was small. Evidence of this fact had been obtained earlier by observing that two intersecting turbulence wedges showed little tendency to affect one another.

## STRUCTURE OF TRANSITION REGION AND NATURE OF TRANSITION

Before concluding that a transition region consists of patches of turbulence going downstream, one item of contradictory evidence must be disposed of. Why had it been inferred from earlier experiments that laminar flow was rarely to be found downstream from turbulent flow? If present ideas are right, just the opposite is the rule in a transition region. Tn order, therefore, to confirm these ideas, it must he possible to detect turbulence at an upstream point before it arrives at a downstream point.

In the earlier experiments the hot-wire probes were separated longitudinally from 3 to 5 inches and displaced in the z-direction about  $5/4$  inch to avoid interference. Consequently, much could happen in the space.between them. Accordingly, new experiments were arranged in which the separation was 1 inch with one wire directly behind the other. In this case interference was avoided by making the leading wire longer than the rear wire and using very fine prongs. Both wires were 0.016 inch from the surface. Runs in which the signals from the two wires were recorded simultaneously by photographing the screen of a dual-beam oscilloscope were made in a natural transition region. Since the time to travel 1 inch was short and the film speed was limited to about 8 feet per second, the spacing between events was small. A representative sample of the record obtained at a free-stream velocity of 80 feet per second is shown in figure 8. Turbulence is seen to arrive first at the upstieam wire and a short time later at the downstream wire and to leave first at the upstream wire and later at the downstream wire. Many such records were inspected. Except for a few cases where terminations were poorly defined, all showed the same phenomenon. Furthermore, the velocities for the leading and trailing edges calculated from the time delay checked those for the spot to about 10 percent. No better accuracy would be expected with such small time resolutions.

While this completely confirmed the passage of turbulent patches, a further experiment was performed to see whether spot characteristics in the y-direction could be detected. Two wires were placed at the same x-position but at distances of  $0.013$  and  $0.143$  inch from the surface. The simultaneous records are shown by the second sample in figure 8. It is clearly seen that the turbulence arrives first and leaves first at the outermost wire. This was all sround confirmation not only that a transition region consists of patches of turbulence moving downstream but that the patches behave as do artificially produced spots. This statement does not infer that the patches have the shapes shown in figure 6; for, granted that they may have had this shape early in their history, they would sooner or later merge and give a resultant of almost any shape.

It now becomes completely clear that what were always seen in a transition region were patches of turbulence moving downstream. Points of initial breakdown were rarely if ever seen. Yhe term "transition," meaning the change from the laminar to turbulent state, may be retained, but it must now be recognized that it has two parts: (1) The initial breakdown of laminar flow due to a perturbation and (2) the growth of turbulence into the surrounding laminar region.

Since the first part was universally missed in surveying a transition region, one can so far only infer something about it from circumstantial evidence. First, the mere fact that it was missed means that the probability of having a probe at the point where the event occurred was small. This is evidence that the breakdown is pointlike as opposed to a simultaneous breakdown along a Une or over a considerable area. If the breakdown is pointlike, then there exists the opportunity for lateral growth. Of this there is ample evidence. For example, observed times of arrival of large patches at two points separated laterally by as little as  $3/4$  inch were markedly different, and the same was true of times of leaving. This indicates that patches have edges at least as

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sloping as those of the single spot of figure  $6$ . Sloping sides automatically mean the occurrence of lateral spreading. The existence of lateral spreading also explains why, when two probes were separated longitudinally by several inches, the downstream probe so rarely showed laminsr flow when the upstieam probe showed turbulent flow. The opportunity for conditions to be otherwise was simply made small by the cross feeding of turbulence into the space between the two probes. The same explanation can be applied to the appearance of schlieren photographs of tmmsition on a bcdy of revolution at supersonic speeds. Here the silhouette view of phenomena along only one generator of the bdy would rarely show the result of laminar breakdown along this generator but, in general, the turbulence that reached this generator from the sides. If the breakdown had occurred in the form of a line, say a ring or partial ring around the body, the chances for obseming gaps of laminar flow would have been increased.

The concept of transition along a continuous line stems from the old idea of a front diviting the laminar and turbulent regimes. The argument was that, if the flow was sufficiently unstable to break down at a given value of x, it would be even more unstable at a greater value of x and therefae would be completely turbulent up to the first point. It has now been shown that this argument is false. It is nevertheless true that the amplitude of amplified waves is on the average greater at the greater values of  $x$ ; and, therefore, if these cause the breakdown, it should generally have occurred at the downstream point. The explanation of this appsrent inconsistency is to be found in the observed interference patterns which produce trains of high and low amplitude advancing in the x-direction. In regions of high amplitude isolated peaks are found, usually jutting toward the higher velocities. From observations made with two wires spaced  $3/4$  inch in the z-direction, it appears that these irregularities occur in a spotty fashion over the xz-plane. tical breakdown, probably at the peaks, would be expected to be equally spotty.

Peaks exe observed when the hot-wire is close to the surface, say  $0.01$  or  $0.02$  inch away, and indicate a momentary increase in velocity near the surface. Closely spaced peaks found near the apex of a turbulence wedge give the records a one-sided, spiked appearance. Some interesting observations were made in an adverse pressure gradient strong enough to shorten the transition region to a few inches. Here amplitudes increased rapidly with x and developed short trains in which the frequency was approximately doubled and the mean level of the group was shifted toward the higher velocity. This shift, whether accompanying a pulse or a train, evidently means that shearing stresses are brought into play which increase the velocity near the surface much as do the stresses in turbulent flow. Not only must such phenomena be out of the linear range, but one would suppose that they would be three dimensional as well. The nexk step is the appearance of turbulence, but what is involved in this step is still unknown.

It may be remarked that momentary flow separation, which has been postulated as a prelude to transition, is ruled out by the evidence available in these cases. While it is true that the velocity near the surface could reverse during part of a cycle when the amplitude is high, this apparently is prevented by the action of the shearing sixesses which become high at the same time.

Since the present evidence for pointlike breakdown is circumstantial, this must be left an open question. The possibility that conditions for breakdown may be met practically simultaneously over a region of some extent cannot be ruled out. Cases may differ in this respect. It is hoped that further experiments will provide a definite answer to this question. It is known, however, that the breakdown is moving downstream as it is occurring. Ih short, all parts of the process, the wave and the breakdown as well as the resulting spot or patch, are moving downstream.

Having disposed of the mechanism of initial breakdown as best as can be done with the meager information at hand, attention is now turned to the structure of the transition region. At a given value of  $x_1$ , defined as the beginning of the transition region, points of breakdown begin. While this position may vary somewhat, it is fairly definite. From here on spots form, grow, amd merge with one another. At the position  $x_1$  waves and irregular disturbances in the laminar flow are much in evidence. After  $x_1$  waves and general perturbations in the laminar regions are less in evidence, and finally in about the last half of the transition region they are nesrly absent in what is left of laminar flow. The beginning of the disappearances can be attributed to the spotty nature of waves, regions of high amplitude having been taken out by breakdowns. The more ccmplete disappearance in the later stages is the result of a calming effect which will be discussed more fully in the next section. This effect has already been mentioned in connection with the outline of the turbulence wedge. It will be found that trailing each turbulent patch is a region of calm by virtue of which the patch prohibits a fresh breakdown behind it.

Thus finally a transitim region is found divided roughly into two parts: (1) An initial stage where initial breakdowns occur and spots form and begin to grow; (2) a final stage consisting primarily of spot growth with new breakhwns prohibited. The change from one to the other is, of course, progressive. Patches of irregular shape form from the merging of spots. The stable trail of a patch does not prevent the leading edge of a following patch from overtaking it. Finally, all patches close and the transition region is ended.

The striking fact is that turbulence is always moving downstream and in a very real sense tends to blow away. In a manner of speaking, it experiences a struggle for survival and can only maintain itself by

fresh breakdowns in the following laminsr flow. It wilIlbe seen in the next section that it does not maintain itself when ode interferes with the struggle for survival.

# CAIMING EFFECT

The brief reference to the recovery trail and the attention called to trailing effects in the two preceding sections have perhaps given some hint that this subject deserves further attention. Its importance stems from the fact that a boundary layer passed over by a recovery trail is left in a state of calm for a pericd until the disturbed condition again sets in. Transition will not occur within this period.

The possible existence of such an effect was anticipated because of the highly stable condition of the recovery trail. A probably more logical argument runs as follows: At the trailing edge of receding turbulence the flow becomes laminar, but the velocity near the surface is higher than it would be for a boundary layer in its normal state. The normal state returns in a relatively short interval termed the recovery trail. The normal layer that results is a reconstructed layer, built for the most part of old properties contributed by the oncoming flow but of only those old properties that have arrived. Since the reconstructed layer is.laid down at a rate of one-half the free-stream velocity and the disturbances travel at a slower rate, there is a progressively widening interval in which the layer lacks the disturbmces. ' According to stability theory and experiment (ref.  $4$ ) two-dimensional waves travel downstream at a velocity around one-third the free-stream velocity, the exact value depending on the Reynolds number, boundarylayer thickness, and wave length.

Having made the foregoing prediction, it was decided to try an experiment in which a recovery trail would be made to pass through a region of natural transiticm. What was desired was an instantaneous line source of turbulence which would span a considerable distance z and so sweep over a considerable width of the layer. Presumably a row of simultaneously fired sparks would have produced this result; but, since the necessary equipment was not at hand, a single spark was fired sufficiently far upstream to grow into a large spot by the time it reached the transition region. The conditions were free-stream turbulence, 0.03 percent;  $U_1 = 80$  feet per second; beginning of transition, 5.5 feet; and spark, 0.25 feet from the leading edge. The maximum lateral width of the spot was calculated to be 2 feet at the 5.5-foot position.

Oscillograph records were obtained titi a hot-wire about 0.015 inch from the surface on the center line at 5.5, 7, 8, and 8.5 feet from the leading edge. Examples are shown in figure 9. The first record of each

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set shows the natural condition; the second shows the passage of the trail. The record pertaining to 5.5 feet shows the period of calm following the recovery trail. The other records show how laminar flow is brought to  $7, 8,$  and  $8.5$  feet. The laminar condition lasts until the flow again breaks down at some upstream position and the leading edge of the resulting turbulence reaches the point under observation. The position is assumed to be that for the beginning of natural transition, which in this case is 5.5 feet. Since the leading edge is traveling with a velocity 1.76 times that of the trail, it catches up and progressively shortens the lsminar section.

From the length of the laminar sections at 7, 8, and 8.5 feet the perid of calm at 5.5 feet during which transition did not occur was estimated on the basis of the velocities of trailing and leading edges, of a spot as given in figure 6. These came out to be 0.122 second from 7 feet, 0.098 second from 8 feet, and 0.078 second from 8.5 feet. The progressively decreasing time is an indication of systematic error or is evidence of edge effects from the turbulent field about the relatively narrow laminar strip. Judging by the record at 5.5 feet, the period of calm was probably around o.1 second. ..

If the starting point for the wave reaching the  $5.5$ -foot position were lmown, its velocity could be calculated. One needs to know the nearest upstream point outside of the growth envelope of the spot from which a wave can arrive at the center line at the 5.5-foot position. If it is assumed that a wave cannot travel on a diagonal course and therefore had to start at the origin of the spot, it would be required to travel a distance of  $5.25$  feet. If it started to follow the spot immediately its time of travel would be the time for the trailing edge of the spot to reach the 5.5-foot position plus the O.1-second perial of calm. The time is found to be 0.231 second. The wave velocity  $c_r$ is found to be 23 feet per second, or  $c_r/U_1 = 0.29$ . If, on the other hand, a wave does travel on a diagonal, say  $10^{\circ}$  to the mean flow, or about as the half-angle of a turbulence wedge, then it would be required to travel only 3 feet in a time of 0.175 second;  $c_r/U_1$  is then found to be 0.21. While the present conditions differed considerably, the results axe of the same order.

Because of the foregoing uncertainties an attempt was made to produce a breakdown of short duration along a line and to obtain a trail of considerable extent in the z-direction. A piece of twine about 0.03 inch in diameter was attached at the floor and ceiling of the tunnel by rubber bands and was thus held under tension about an inch from the plate. This was drawn out like the string of a bow and allowed to slap the plate. The object was to make the twine enter the boundary layer, disrupt the flow, and then leave. Since the twine would vibrate like any plucked string, damping was required.

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A position about  $\frac{1}{4}$  feet from the leading edge was chosen. At a free-stream velocity of 78 feet per second the twine contacted the surface at 4.17 feet along a line of about **2** feet in extent in the z-direction. At this position and this velocity waves were already well developed. On this particular occasion the beginning of natural transition appeared to be at 6 feet. Two observations were made with the hot-wire at  $5$  feet and three were made at  $9$  feet where the layer was completely turbulent. Some calming was noted at 5 feet, but the duration was uncertain. A period of laminar flow was observed at 9 feet for each of the three trials. From these the times that the flow remained calm at  $6$  feet without transition were 0.067, 0.06, and 0.Q5 second.

**Since the wave was** already present at the **4.17-foot** position, it was assmed that the wave continued to progress frm this point and that it approached the 6-foot position during the time that the trailing end of the receding turbulence traveled from  $4.17$  feet to 6 feet plus the estimated calm period at 6 feet. The values of  $c_r/U_1$  were calculated from the three trials to be 0.21, 0.22, and  $0.24$ .

These values agree fairly well with those obtained with the spot. The mean of all is 0.23 with a maximum deviation of 13 percent. While this value seems rather definite, it should be remembered that it depends on assumptions that may be questionable. If the value  $0.23$  represents a wave velocity, it is apparently lower than that of the two-dimensional wave calculated in stability theory. Furthermore, one should probably be comparing group velocity which for boundary-layer waves should be somewhat higher than that for a single wave. There exists also the possibility that the calming may have other effects which lengthen the period before breakdown is resmed.

Obviously much more study is required in order to clarify this subject. The properties of three-dimensional waves about which little is known should be examined. The waves may le required to be three dimensional, or to have a three-dimensional perturbation superposed on them, to prcduce breakdown. There also remains the question of the extent of calming when the impressed disturbances are already large, as occurs when the free-stieam turbulence is high or when roughness elements are present on the surface. Cases where pressure gradients exist also need to be investigated.

The calming effect interrupts the formation of spots and thus allows established turbulence to pass on downstream. The results can be an extension of laminar flow. It has already been noted in the preceding section that breakdowns are prohibited by this effect in the latter part of the transition region. This results in a lengthening of the transition region. The action here, however, is reduced to a minor role because of the rapidity with which upstream patches overrun the regions of calm. If turbulence, initially in the form of a line, were made to sweep over the transition region from its beginning, then the calming

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effect would be fulJy effective. obviously the extent of laminsr regime following the Une depends on the length of time during which a state of calm exists. This in turn depends on where one chooses to initiate the Mne and on the difference between the velocity of the trailing edge of the resulting turbulent strip  $(0.501)$  and that of the following wave.

When the velocity of the wave is sufficiently small, say of the order of O.25U1, it appears to be possible to derive a net gain in the average extent of laminar flow by artificially starting turbulence along a line somewhere ahead of a region of natural transition, provided the turbulence so introduced begins as a Une (negligible width at the start) and passes downstream with velocities of leading and trailing edges as given for the spot. However, until more is known about this effect, any attempt to assess potential benefits is purely in the realm of speculation. It is interesting to note, however, that turbulence properly injected at proper time intervals can in principle alleviate the severity of the turbulence "disease." The parallel to medical practice is obvious. The difficulty here is that the patient will suffer just as much from the mild cases as from the disease proper if the periods of immunity are too short.

National Bureau of Standards, Washington, D. C., February 28, 1955.

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![](_page_23_Figure_0.jpeg)

Figure 3.- Mean-velocity profiles through transition region.<br> $U_1 = 80$  feet per second; free-stream turbulence, 0.03 percent.

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![](_page_24_Figure_0.jpeg)

Figure 1.- Oscillograms of u-fluctuation in transition regions with hotwire probe 0.013 inch from surface. Time interval between dots,  $1/60$  second.

 $1,0$  $\frac{u'}{U_i}$  (FREE STREAM)  $U_{\parallel}$ , FPS 0.0003 80 ×  $.8$  $104$ .0004 Ó  $\overline{\phantom{a}}$ 33 .0002(I-MM WIRE)  $\blacktriangle$  $\gamma$ 35  $.010$  (GRID)  $\mathbf{o}$  $.6$ GAUSSIAN  $\cdot$  $\cdot$ .2  $\Omega$  $Q - 4$  $-2$  $\bullet$  $\overline{c}$  $-3$  $-1$  $\overline{\mathbf{3}}$  $\mathbf{I}$  $rac{x-\overline{x}}{\sigma}$ 

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Figure 2.- Intermittency factor  $\gamma$  in transition regions. Range of standard deviation  $\sigma$ , 0.3 to 0.8 foot. Curve is Gaussian integral curve matched at  $\gamma = 0.5$ .

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Figure 4.- Profiles of  $u'/u_1$  through transition region.  $U_1 = 80$  feet per second; free-stream turbulence, 0.03 percent.

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Figure 5.- Turbulence wedge produced by three-dimensional roughness<br>element (1/8-inch sphere) on surface. Time interval between dots,<br> $1/60$  second;  $U_1 = 80$  feet per second.

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![](_page_28_Figure_0.jpeg)

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Figure 6.- Turbulent spot initiated by electric spark between needle electrode and surface. Oscillograms with 1/60-second timing dots shown at top of figure; time progression from right to left; upper record shows spark discharge on right and spot passage on left; lower record shows natural transition.

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![](_page_29_Figure_0.jpeg)

Figure 7.- Envelopes of spot growth.  $R_{\beta\ast}$ , Reynolds mumber based on laminar-boundary-layer displacement thickness.

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![](_page_30_Figure_0.jpeg)

Figure 8.- Simultaneous signals from two hot-wires in region of natural transition. Time progression from right to left; time interval between dots, 1/60 second; upper record, 1-inch separation in x; lower record, 0.15-inch separation in y.

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![](_page_31_Figure_1.jpeg)

Figure 9.- Oscillograms comparing natural and calmed conditions. Time progression from right to left; time interval between dots, 1/60 second;  $U_1$  = 80 feet per second.

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