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EXPERIMENTAL INVESTIGATION OF THE VIBRATIONS OF A BUILT-UP RECTANGULAR BOX BEAM

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SUMMARY

Experimental modes and frequencies of a uniform built-up box beam are presented and comparisons are made between experimental and theoretical frequencies. For bending vibrations, frequencies obtained from an analysis of a substitute-stringer structure which includes the influence of transverse shear deformation and shear lag were found to agree very well with those obtained experimentally. In the case of torsional vibrations, the frequencies obtained from either an elementary or a four-flange beam analysis which includes the effects of restraint of warping were found to be in satisfactory agreement with the experimental frequencies.

INTRODUCTION

The accurate determination of natural modes and frequencies is of basic importance in the dynamic analysis of aircraft structures. The influence of secondary effects such as transverse shear and shear lag in bending vibrations and restraint of warping in torsional vibrations on the frequencies of structures has been the subject of many theoretical papers. (See, for example, refs. 1 to 7.) However, little experimental information pertaining to the importance of these effects in structures typical of aircraft-wing construction seems to exist.

In order to provide such experimental information, vibration tests were conducted on the large-scale uniform built-up box beam of rectangular cross section shown in figure 1. The purpose of this paper is to present the experimental results on the modes and frequencies of this box beam. In addition, comparisons are presented between these experimental frequency results and the results obtained from calculations based on elementary theories and on theories which include the effects of transverse shear, shear lag, and restraint of warping.

EXPERIMENTAL INVESTIGATION

Specimen

The built-up box beam used in the present investigation is shown in figure 1. The spars, cover sheets, and stringers were made from 2024 (formerly 24S-T) aluminum alloy. In order that the specimen would exhibit vibration characteristics similar to those of actual aircraft wings, realistic proportions were used in the construction of the box beam with the exception of the bulkhead weight and spacing. The bulkheads were purposely overdesigned and closely spaced in order to prevent distortions in the plane of the cross section and to lower the natural frequencies. The bulkheads consisted of formed steel sheet with steel backing bars attached to the flanges. In addition to providing extra weight, the backing bars formed a sturdy base for attachment of the shakers and measuring equipment. Even though the bulkheads were very heavy, their resistance to distortions out of the plane of the cross section can be considered to be negligible.

The box beam had a width-depth ratio of 3.59 and a plan-form aspect ratio (length over width) of 13.3. The aluminum material (2024) of the sheets and stringers had a modulus of elasticity of 10.6×10^6 pounds per square inch, a shear modulus of 4.0×10^6 pounds per square inch, and a density of 0.100 pound per cubic inch. The steel material of the bulkheads had a modulus of elasticity of 29.0×10^6 pounds per square inch, a shear modulus of 11.0×10^6 pounds per square inch, and a density of 0.283 pound per cubic inch. The total weight of the beam was 302 pounds.

Test Equipment

Shaker system.- The shaker system consists of four electromagnetic shakers, a control console, and a rotating-machine power supply. Each of the shakers has a maximum force amplitude of 50 pounds over the frequency range of 5 to 500 cycles per second and a maximum travel (double amplitude) of 1 inch for the moving coil. A signal generator, which has a voltage output proportional to the velocity of motion, is attached to the moving coil of each shaker. The total weight of the moving element including the signal generator is 2.0 pounds.

The power supply for the shaker system consists of 7 units: a 12-horsepower a-c motor, two d-c generators, a d-c motor, and three alternators. The a-c motor drives the two d-c generators. One generator provides the field supply for the alternators and the shakers, and the other supplies the power to the variable-speed d-c motor. The d-c motor drives the three alternators. Two of the alternators, connected in parallel, supply the power to drive the shakers in the frequency range from 2.5 to 62.5 cycles per second, and the third supplies the power in the frequency range from 62.5 to 500 cycles per second. The frequency of the system is varied by changing the speed of the variable-speed d-c motor.

All controls necessary to operate the shaker system are housed in the control console shown in figure 2. This console contains the frequency controls, a field current control, and an individual force control and phase switch for each shaker. Meters for indicating the current through the drive coil of each shaker, the total field current, and the total alternator current are provided. A frequency meter for each range of the alternators is also mounted in the control console.

These units provide a versatile system with which the force output and the relative phase (0° or 180°) of force of each shaker can be controlled separately over the entire frequency range.

Measuring equipment.- The amplitude of motion of the test beam was obtained from 10 vibration pickups connected through a switch panel to the galvanometers of a 36-channel recording oscillograph. The pickups are self-generating velocity units with a sensitivity of 94.5 millivolts per inch per second, an impedance of 650 ohms, and an undamped natural frequency of 4.75 cycles per second. The frequency response of the pickups is essentially flat from 8 to 100 cycles per second; however, the usable range is from 5 to 500 cycles per second. In addition to the 10 vibration pickups, the signal generators on the shakers were also used as pickup units. These units have a sensitivity of 158 millivolts per inch per second and an impedance of 700 ohms. The frequency response of the signal generator is flat up to 200 cycles per second, with the usable range up to 500 cycles per second. The recording oscillograph was used to record simultaneously the output of all the pickups. The high output voltage and low internal impedance of the pickups and signal generators made it possible to dispense with amplifier circuits in the measuring system; however, in order to have some control over the amplitude of the signal trace on the records, two channels of the oscillograph were assigned to each pickup, one channel containing a high-sensitivity galvanometer, and the other, a low-sensitivity galvanometer. The high-sensitivity galvanometers have a sensitivity of 2 milliamperes per inch of trace, a frequency response which is flat to 300 cycles per second, and a total resistance of 50 ohms. The low-sensitivity galvanometers have a sensitivity of 13 milliamperes per inch of trace, a frequency response which is flat to 600 cycles per second, and a total resistance of 1,050 ohms. The recording oscillograph is equipped with an automatic record numbering system, a trace identifier, and a 0.01-second timing system.

The current through the drive coil, and hence the force output of each shaker, was determined by measuring the voltage drop across a

0.1-ohm resistor in series with the coil. This voltage signal was recorded with the output of the pickups on the recording oscillograph. A cathode-ray oscilloscope was used for visual observation of the output of any pickup or the force output of any shaker.

The recording cabinet, shown in figure 2, contains the oscilloscope and the 36-channel recording oscillograph. In addition, it contains a selector switch for each pickup and shaker. Each selector switch has an off position, positions for the X- and Y-axes of the oscilloscope, and positions for the high- and low-sensitivity galvanometers assigned to each pickup.

In order to obtain more accurate readings of the frequency of vibration than was possible from the frequency meters on the control console, a Stroboconn frequency indicator was used. In this frequency meter the output from one of the pickups flashes a stroboscopic light onto a series of graduated disks revolving at controlled speeds. The disk speed, and hence the frequency of vibration, is known to an accuracy of 0.01 percent.

Test Setup and Instrumentation

An overall view of the test setup is shown in figure 3. The beam was suspended from the wooden support frames by flexible steel aircraft cables, 1/8 inch in diameter, attached to each end of the beam. This type of support offered negligible resistance to small displacements of the beam in the horizontal direction; therefore, in this direction the beam was considered to be essentially free-free. The four shakers were mounted in pairs on steel pedestals and were attached to the corners of the beam by means of necked-down axial-force connectors. Two of the shakers with necked-down force connectors, with one support cable and mounting bracket, are shown in figure 4.

A small mass was attached to each of the support cables to provide a method for detuning. (See fig. 4.) The positions of these masses could be adjusted to change the natural frequency of the cables during the tests and hence eliminate troublesome resonances of the cables.

The 10 pickups were mounted on one cover of the beam directly over the spars. Eight of the pickups were placed on one-half of the beam and the other two were placed on the other half. (See fig. 5.) Signal leads from the pickups were connected to terminal boards mounted above and below the beam. Leads for each pickup were then connected from the terminal boards to the recording cabinet. In order to have symmetrical mass distribution about the beam center line, each pickup was counterbalanced where necessary with a symmetrically located steel block.

Test Procedure

The study of the vibration characteristics of the test beam was conducted in four parts: symmetrical bending, antisymmetrical bending, symmetrical torsion, and antisymmetrical torsion.

The shakers were attached to each corner of the box beam (as shown in fig. 5) and the phase of each shaker was set to produce the desired motion of the beam. (For example, if symmetrical bending modes were being investigated, all the shakers were vibrated in phase; if antisymmetrical torsional modes were being investigated, shakers A and C were set 180° out of phase with shakers B and D. See fig. 5.) In order to observe visually the motions of the beam, the output of one pickup was switched onto the Y-axis of the oscilloscope and the shaker force signal (current through the coil) was switched onto the X-axis. The power supply was then turned on and the force output of the shakers equalized. With the force controls set at a constant value, the frequency was slowly increased until the amplitude of vibration reached a maximum as determined from the pickup output viewed on the oscilloscope. As an aid in obtaining the resonant frequency, the phase angle between the pickup signal and the force signal was observed from the Lissajous ellipse shown on the oscilloscope. Each pickup signal was viewed in turn on the oscilloscope to get an approximate idea of the type of mode associated with this frequency. The selector switches were set to the position that would give the best record trace on the oscillograph for each signal and a simultaneous record made of the output from the pickups, signal generators, and shaker force signals. The frequency of vibration was then read on the Stroboconn frequency indicator and recorded. After the resonance was established and the data recorded, the frequency was increased again until the next resonance was detected. In this manner all the natural beam modes in the frequency range from 5 to 300 cycles per second were identified and recorded.

After all four parts of the vibration study were completed, the pickups and steel weights were removed and the tests were repeated to obtain the natural frequencies without the effect of these concentrated masses. For these tests, the frequency range was extended to 400 cycles per second. It was found that the natural frequencies of the beam without pickups differed from those with the pickups by approximately 4 percent for bending and 2 percent for torsion.

Experimental Results

In the frequency range covered by the tests on the beam without pickups, the first 14 natural beam frequencies (8 bending and 6 torsion) were obtained and are presented in tables I and II. For the tests with pickups attached, the pickup data were used to plot the deflection shapes for the first 4 bending and 4 torsion modes. Only these 8 modes were plotted because the pickups provided an insufficient number of data points for plotting deflection curves for the higher modes. The deflection curves, shown in figures 6 and 7, were plotted from the data obtained from the 8 pickups and 2 signal generators on one-half of the beam; the other 2 pickups and signal generators were used to check symmetry.

THEORETICAL CALCULATIONS AND COMPARISON WITH EXPERIMENTAL RESULTS

The natural frequencies of the box beam were calculated by several different approaches. The bending frequencies were calculated by using elementary theory, by the equations and curves of reference 6, which include the secondary effect of transverse shear deformation, and by the approach of reference 2, which employs a substitute-stringer idealization (see fig. 8(a)) to include the effects of transverse shear and shear lag. The torsional frequencies were obtained by applying elementary theory and by using a four-flange idealization, as shown in figure 8(b), to include the effects of restraint of warping (ref. 7). In order to utilize the approaches of references 2 and 7 for the experimental box beam, additional derivations were necessary. The derivations presented in the appendix include

(1) A derivation of the frequency equation for antisymmetrical bending vibrations of the substitute-stringer box beam

(2) A derivation of the frequency equations for both symmetrical and antisymmetrical torsional vibrations of a more general four-flange box than was presented in reference 7

Also presented in the appendix are the frequency equation for the symmetrical vibration of a substitute-stringer box derived in reference 2 and the numerical values of the necessary parameters for the box beam.

All these calculations were based on the assumption that the box beam is uniform along the length. Examination of the beam indicates that it can be considered uniform except for the concentrated masses of the moving elements of the shakers at the tips. Before comparisons with experimental results are made, therefore, these effects should be taken into account. For the elementary theory the corrections can be made exactly by consideration of the differential equations and proper boundary conditions. These corrections are shown in figures 9 (bending) and 10 (torsion) as a function of $M/\mu L$, the ratio between the total tip mass to the total beam mass, for bending, or Mr^2/I_pL, the ratio of the total tip mass moment of inertia to the total beam mass moment of inertia, for torsion. From the values of these parameters (0.0265 for bending and 0.0486 for torsion), it can be seen that the correction for tip mass is fairly small. Therefore, instead of making exact calculations for the values of the corrections when the various secondary effects are included, the "elementary" corrections were assumed to apply percentagewise to the other results.

The corrected calculated frequencies for the first 8 bending modes and the first 6 torsion modes, with the corresponding experimental frequencies, are given in tables I and II, respectively. These results are also shown by means of bar graphs in figure 11. In order to show on the same chart the frequencies for all the modes considered, the heights of the bars are plotted on a log scale.

DISCUSSION OF RESULTS

For the bending frequencies there is excellent agreement between the experimental and the calculated results that include the effects of transverse shear and shear lag (substitute-stringer solution). The errors for the first two frequencies are less than 1 percent, with only a 7-percent error in the eighth natural frequency. On the other hand, an error of less than 7 percent is obtained only for the first mode with elementary theory, whereas the eighth mode is in error by over 80 percent. A comparison of the frequencies that include only transverse shear with both the elementary frequencies and those that contain transverse shear and shear lag shows that, for this particular test beam, transverse shear deformation contributes more to the reduction in natural frequency than does shear lag, especially in the higher modes of vibration. Both effects, however, are important.

Although the comparisons between the substitute-stringer theory and experiment are good, the theoretical frequencies are consistently higher than the experimental ones. Some of these discrepancies can be attributed to other secondary effects, such as longitudinal or rotary inertia (refs. 1, 4, 5, and 6) and panel vibrations (ref. 8), which have been neglected in the substitute-stringer calculations.

For torsional frequencies the agreement between the experimental frequencies and the calculated results that include the effects of restraint of warping (four-flange solution) is again very good; errors in the calculated frequencies vary from 1 percent in the first mode to 10 percent in the sixth mode. In this case, however, the agreement between experiment and elementary theory is equally good. In the first three modes the four-flange solution is slightly better; on the other hand, in the higher modes the elementary solution is better. In fact, for the third antisymmetrical and symmetrical modes, the elementary frequencies are higher than those obtained experimentally. No explanation for these discrepancies has been found.

From an examination of the results for both bending and torsion cases, an interesting possibility presents itself. As is shown in the bending case, the substitute-stringer idealization accurately incorporates the effects of transverse shear and shear lag. The same idealization would also incorporate most of the effects of restraint of warping in torsion. Since the effects of restraint of warping in torsion seem to be relatively unimportant, it is entirely probable that the same substitute-stringer structure could be used to calculate the coupled bending-torsion modes and frequencies of a box beam with unsymmetrical cross sections.

CONCLUSIONS

The first 14 natural beam frequencies obtained from vibration tests of a built-up box beam of rectangular cross section are presented. From a comparison between these experimental frequencies and calculated frequencies the following conclusions can be made:

1. The frequency equations which are based on the analysis of a substitute-stringer box and include the influence of transverse shear and shear lag predict the frequencies of bending vibrations of uniform box beams with good accuracy. The errors vary from 1 percent in the first two modes to 7 percent in the eighth mode. On the other hand, elementary theory gives very poor results for the higher modes.

2. The frequency equations which are based on the analysis of a four-flange box and include the influence of restraint of warping predict the frequencies of torsional vibrations of uniform box beams with good accuracy. The errors vary from 1 percent in the first mode to 10 percent in the sixth mode. The equations based on elementary theory, however, give results which are equally good.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., November 8, 1955.

APPENDIX

CALCULATION OF FREQUENCIES OF TEST BEAM

Calculations for Beam Vibration Frequencies

In this appendix the equations used in the calculations of the natural frequencies of the test beam are presented. These equations are obtained from the method of reference 2 for bending vibrations and from the method of reference 7 for torsional vibrations. Since there is some possibility of confusion in the use of symbols, most of the symbols are defined separately in each of the sections; however, the common symbols are as follows:

a	half-depth of beam, 2.53 in.
b	half-width of beam, 9.0 in.
t_C	cover-sheet thickness, 0.051 in.
t_W	web thickness, 0.064 in.
L	half-length of free-free beam, 120 in.
E	modulus of elasticity, 10.6×10^6 lb/sq in.
G	shear modulus of elasticity, 4.0×10^6 lb/sq in.
u_F	longitudinal displacement of point of flange
x	longitudinal coordinate

Beam Bending Frequencies

The frequency equation for the symmetrical modes of a free-free substitute-stringer beam (eq. (B24) from ref. 2) is

$$k_B{}^2\left[\tan k_B k_S + 2\left(k_B k_S\right)^3 \sum_{n=1,3,5}^{\infty} \frac{1}{P_n{}^2 N_n - P_n\left(\frac{n\pi}{2}\right)^2}\right] = 0 \qquad \text{(A1)}$$

where

$$\left.\begin{aligned} P_n &= \left(\frac{n\pi}{2}\right)^2 - k_B{}^2 k_S{}^2 \\ N_n &= 1 + \frac{k_S{}^2\left[\left(\frac{n\pi}{2}\right)^2 + (KL)^2\right]\left(\frac{n\pi}{2}\right)^2}{\frac{A_T}{A_F}\left(\frac{n\pi}{2}\right)^2 + (KL)^2} \end{aligned}\right\} \qquad \text{(A2)}$$

In these equations, k_B is the frequency parameter defined as $k_B = \omega_B\sqrt{\frac{\mu L^4}{EI}}$ where ω_B is the natural circular frequency in radians per second.

The various parameters given in reference 2 are defined and their numerical values based on measured quantities taken from the test beam are given by

A_F cross-sectional area of flange of substitute-stringer structure, 0.342 sq in.

A_L cross-sectional area of substitute stringer, 0.853 sq in.

A_S effective shear-carrying area, $4at_W = 0.642$ sq in.

$A_T = A_F + A_L$

b_C distance between web and centroid of area of half cover, 4.80 in.

b_S distance between web and adjacent substitute stringer, 2.40 in.

C constant

I bending moment of inertia, $4a^2A_T = 30.02 \text{ in.}^4$

K shear-lag parameter, $\sqrt{\frac{Gt_C}{Eb_S}\frac{A_T}{A_FA_L}} = 0.1830$

k_B bending-frequency coefficient, $\left(4.748 \times 10^{-2}\right)\omega_B$

k_S coefficient of shear rigidity, $\frac{1}{L}\sqrt{\frac{EI}{GA_S}} = 0.0919$

T maximum kinetic energy

U maximum strain energy

u_F longitudinal displacement of point of flange in substitute-stringer structure

u_L longitudinal displacement of point of substitute stringer

w vertical displacement of cross section of beam

a_n,b_n,c_n Fourier series coefficients

n integer

μ mass of beam per unit length, 0.00340 slug/in.

Inasmuch as the frequency equation for the antisymmetrical modes of a free-free substitute-stringer beam is not given in reference 2, the basic steps in its derivation are presented in this appendix.

A natural mode of vibration must satisfy the variational equation

$$\delta(U - T) = 0 \tag{A3}$$

where

$$\left.\begin{aligned} U &= 2\int_{-L}^{L}\left[E\left(\frac{du_F}{dx}\right)^2 A_F + E\left(\frac{du_L}{dx}\right)^2 A_L + G\left(\frac{u_F - u_L}{b_S}\right)^2 t_C b_S + \right. \\ &\qquad \left. G\left(\frac{dw}{dx} - \frac{u_F}{a}\right)^2 t_W a\right]dx \\ T &= \frac{1}{2}\int_{-L}^{L}\mu\omega_B{}^2 w^2 dx \end{aligned}\right\} \tag{A4}$$

and the variation is taken independently with respect to u_F, u_L, and w. (See ref. 2.)

Appropriate assumptions for the displacements for antisymmetrical modes of a free-free beam are

$$\left.\begin{aligned} w &= Cx + \sum_{n=2,4,\ldots}^{\infty} a_n \sin\frac{n\pi x}{2L} \\ u_F &= \sum_{n=0,2,4}^{\infty} b_n \cos\frac{n\pi x}{2L} \\ u_L &= \sum_{n=0,2,4}^{\infty} c_n \cos\frac{n\pi x}{2L} \end{aligned}\right\} \qquad \text{(A5)}$$

Substituting expressions (A5) into expression (A4) and using equation (A3), where the variation is with respect to the a's, b's, c's, and C independently, results, after simplification, in the following frequency equation:

$$k_B^2\left[1 - k_Bk_S \cot k_Bk_S + 2\left(k_Bk_S\right)^4 \sum_{n=2,4,6}^{\infty} \frac{1}{P_n^2N_n - P_n\left(\frac{n\pi}{2}\right)^2}\right] = 0 \qquad \text{(A6)}$$

where P_n and N_n are given by equation (A2).

Before these equations could be applied, the box beam used in this investigation had to be idealized into the substitute-stringer structure shown in figure 8(a). Since the flanges and stringers of the substitute structure are assumed to carry all the normal stresses, the effective bending stiffness of the spars (including the portion of the covers directly over the flanges) was incorporated into the flanges of the idealized structure, whereas the bending stiffness of the remaining portion of the covers was incorporated into the substitute stringers. The shear webs and cover-sheet thicknesses of the idealized structure were made equal to the thicknesses of the actual beam. The location of the substitute stringer was chosen as recommended in reference 2, that is, $\frac{b_S}{b_C} = \frac{1}{2}$. The bulkhead spacing was assumed to be close enough for the mass of the beam to be considered uniformly distributed.

Beam Torsion Frequencies

The frequency equation presented in reference 7 is for the special case of symmetrical free-free vibrations of a four-flange box beam with web and cover sheet of equal thicknesses. For a beam with web and cover sheets of unequal thicknesses, the analysis procedure of reference 7 still applies and equations (C5) and (C9) of the reference become, respectively,

$$-4Gab\left(\frac{t_C}{b} - \frac{t_W}{a}\right)u_F' + 4Gab\left(t_C a + t_W b\right)\theta'' + \omega_T^{\,2} I_p \theta = 0 \tag{A7}$$

and

$$4EA_F u_F'' - 4G\left(\frac{t_C}{b} - \frac{t_W}{a}\right)u_F - G\left(t_C a - t_W b\right)\theta' = 0 \tag{A8}$$

where primes denote differentiation with respect to x. The characteristic equation (eq. (C12) of ref. 7) becomes

$$\alpha^4 + \left(k_T^{\,2} K_2^{\,2} - K_1^{\,2}\right)\alpha^2 - k_T^{\,2} K_2^{\,2} = 0 \tag{A9}$$

where

$$\left.\begin{aligned} K_1 &= 2L\sqrt{\frac{Gt_C t_W}{EA_F\left(t_C a + t_W b\right)}} \\ K_2 &= \frac{2\sqrt{abt_W t_C}}{t_C a + t_W b} \end{aligned}\right\} \tag{A10}$$

The parameter k_T is the frequency parameter defined by $k_T = \omega_T\sqrt{\frac{I_p L^2}{GJ}}$ where ω_T is the natural circular frequency in radians per second.

The solutions of equations (A7) and (A8) are obtained as in reference 7, with appropriate boundary conditions, and the resulting frequency equations for the free-free vibrations of a four-flange box beam are

$$\alpha_2\left(K_2^{\,2} k_T - \alpha_2^{\,2}\right)\tanh \alpha_1 - \alpha_1\left(K_2^{\,2} k_T^{\,2} + \alpha_1^{\,2}\right)\tan \alpha_2 = 0 \tag{A11}$$

for the symmetrical modes and

$$\alpha_1\left(K_2^{\,2} k_T^{\,2} + \alpha_1^{\,2}\right)\tanh \alpha_1 + \alpha_2\left(K_2^{\,2} k_T - \alpha_2^{\,2}\right)\tan \alpha_2 = 0 \tag{A12}$$

for the antisymmetrical modes. In equations (A11) and (A12), α_1 and α_2 are the real and imaginary roots, respectively, of equation (A9).

The various parameters used in this section are defined and their numerical values for the test beam are given by

A_o cross-sectional area enclosed by medium line of wall thickness, 89.89 sq in.

A_F cross-sectional area of flange, 0.587 sq in.

I_p mass polar moment of inertia per unit length, 0.1439 lb-sec^2/in.

J torsional stiffness constant, $\dfrac{4A_o^2}{\oint \frac{ds}{t}} = 36.18 \text{ in.}^4$

k_T torsional frequency coefficient, $(14.44 \times 10^{-2})\omega_T$

θ rotation of cross section

The numerical values calculated for the parameters were obtained from measured quantities taken from the test beam. In the quantity I_p, the mass polar moment of inertia of the bulkheads was uniformly distributed along the beam. The polar moment of inertia I_p also included the contribution of the mass polar moments of inertia of the rivet heads and the spar-web stiffeners.

REFERENCES

1. Budiansky, Bernard, and Kruszewski, Edwin T.: Transverse Vibrations of Hollow Thin-Walled Cylindrical Beams. NACA Rep. 1129, 1953. (Supersedes NACA TN 2682.)

2. Davenport, William W., and Kruszewski, Edwin T.: A Substitute-Stringer Approach for Including Shear-Lag Effects in Box-Beam Vibrations. NACA TN 3158, 1954.

3. Anderson, Roger A., and Houbolt, John C.: Effect of Shear Lag on Bending Vibration of Box Beams. NACA TN 1583, 1948.

4. Traill-Nash, R. W., and Collar, A. R.: The Effects of Shear Flexibility and Rotatory Inertia on the Bending Vibrations of Beams. Quarterly Jour. Mech. and Appl. Math., vol. VI, pt. 2, June 1953, pp. 186-222.

5. Shaw, F. S., and Traill-Nash, R. W.: The Exact Natural Frequency Equations for Beams of Uniform Cross-Section, Allowing for Shear and Rotatory Inertia Effects. Rep. SM 131, Department of Supply and Development, Division of Aeronautics (Melbourne), July 1949.

6. Kruszewski, Edwin T.: Effect of Transverse Shear and Rotary Inertia on the Natural Frequency of a Uniform Beam. NACA TN 1909, 1949.

7. Kruszewski, Edwin T., and Kordes, Eldon E.: Torsional Vibrations of Hollow Thin-Walled Cylindrical Beams. NACA TN 3206, 1954.

8. Budiansky, Bernard, and Fralich, Robert W.: Effects of Panel Flexibility on Natural Vibration Frequencies of Box Beams. NACA TN 3070, 1954.

TABLE I

COMPARISONS BETWEEN EXPERIMENTAL AND CALCULATED BENDING

FREQUENCIES OF A BUILT-UP BOX BEAM

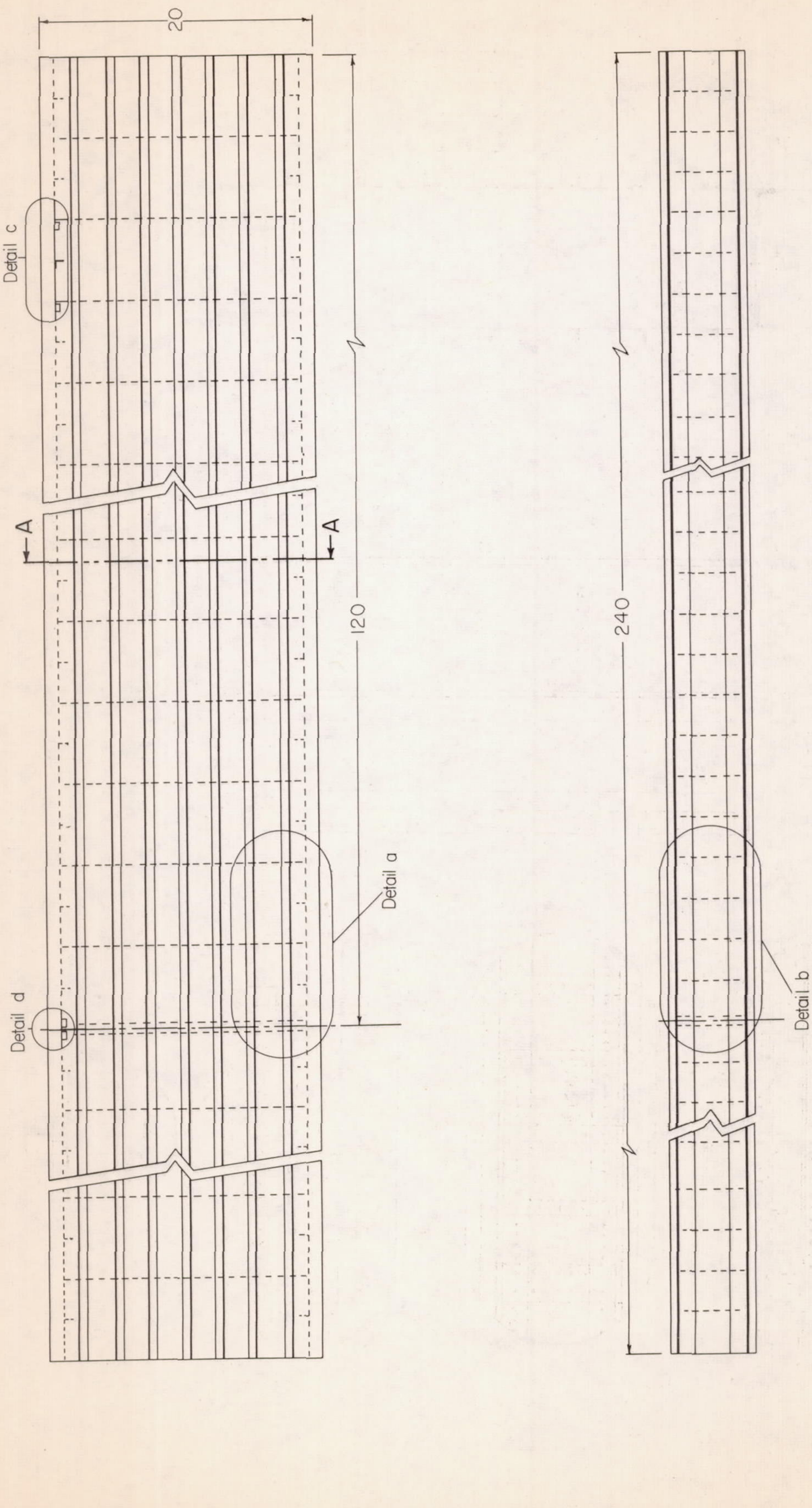
Mode	Experimental frequency, cps	Calculated frequency, cps		
		Elementary theory	Secondary effects included	
			Transverse shear (ref. 6)	Transverse shear and shear lag (ref. 2)
1st symmetrical	18.0	18.4	18.2	18.0
1st antisymmetrical	46.7	50.7	48.3	47.1
2d symmetrical	84.7	101	91.8	86.4
2d antisymmetrical	129.7	166	142	132
3d symmetrical	176.1	247	198	181
3d antisymmetrical	224.6	345	259	233
4th symmetrical	271.0	458	317	285
4th antisymmetrical	318.6	588	380	340

TABLE II

COMPARISONS BETWEEN EXPERIMENTAL AND CALCULATED TORSIONAL

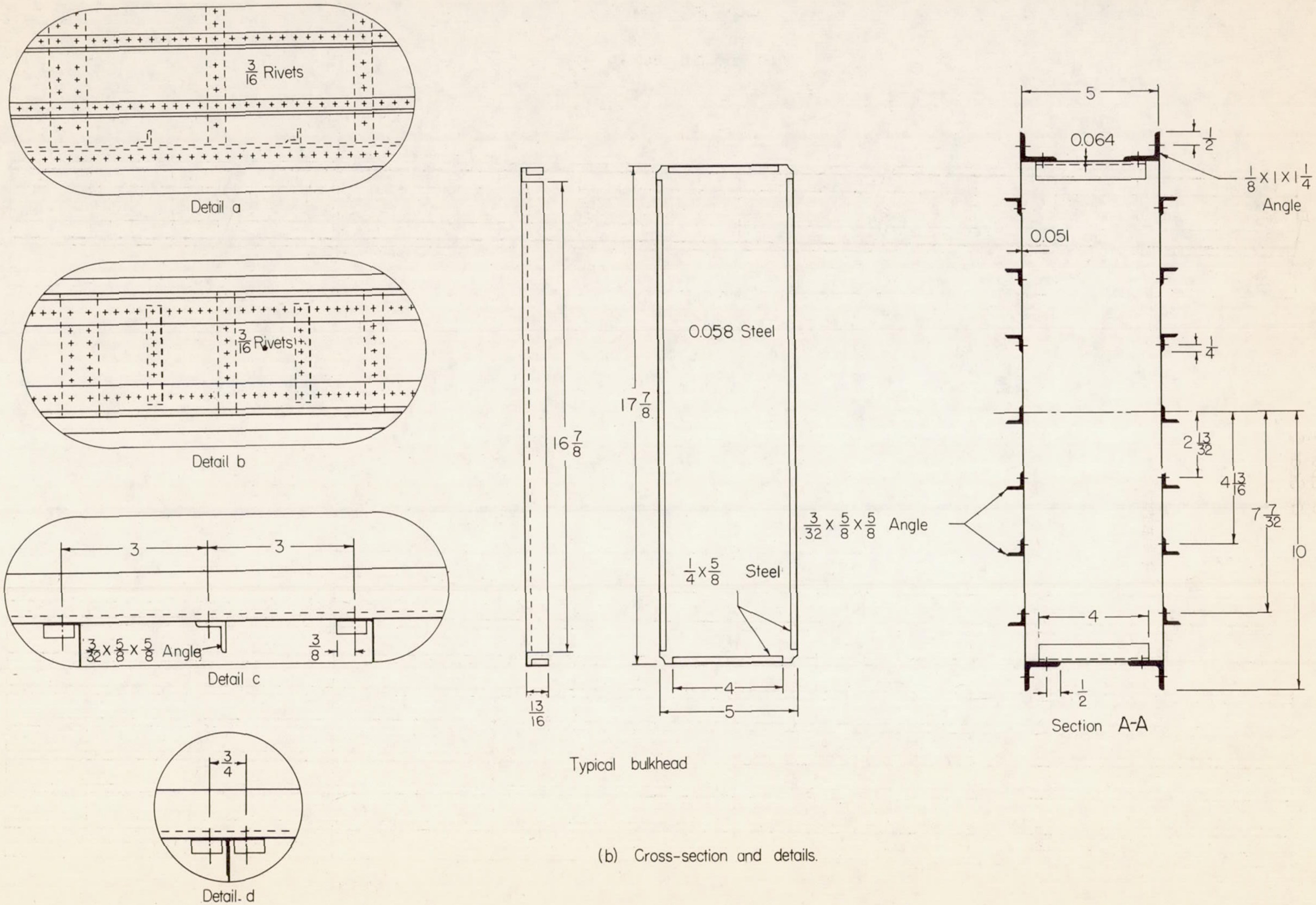
FREQUENCIES OF A BUILT-UP BOX BEAM

Mode	Experimental frequency, cps	Calculated frequency, cps	
		Elementary theory	Four-flange idealization (ref. 7)
1st antisymmetrical	64.7	63.0	63.3
1st symmetrical	129.2	126	127
2d antisymmetrical	194.5	189	193
2d symmetrical	254.8	252	261
3d antisymmetrical	313.3	314	331
3d symmetrical	370.0	378	404



(a) Elevation and plan-form.

Figure 1.—Test beam.



(b) Cross-section and details.

Figure 1.— Concluded

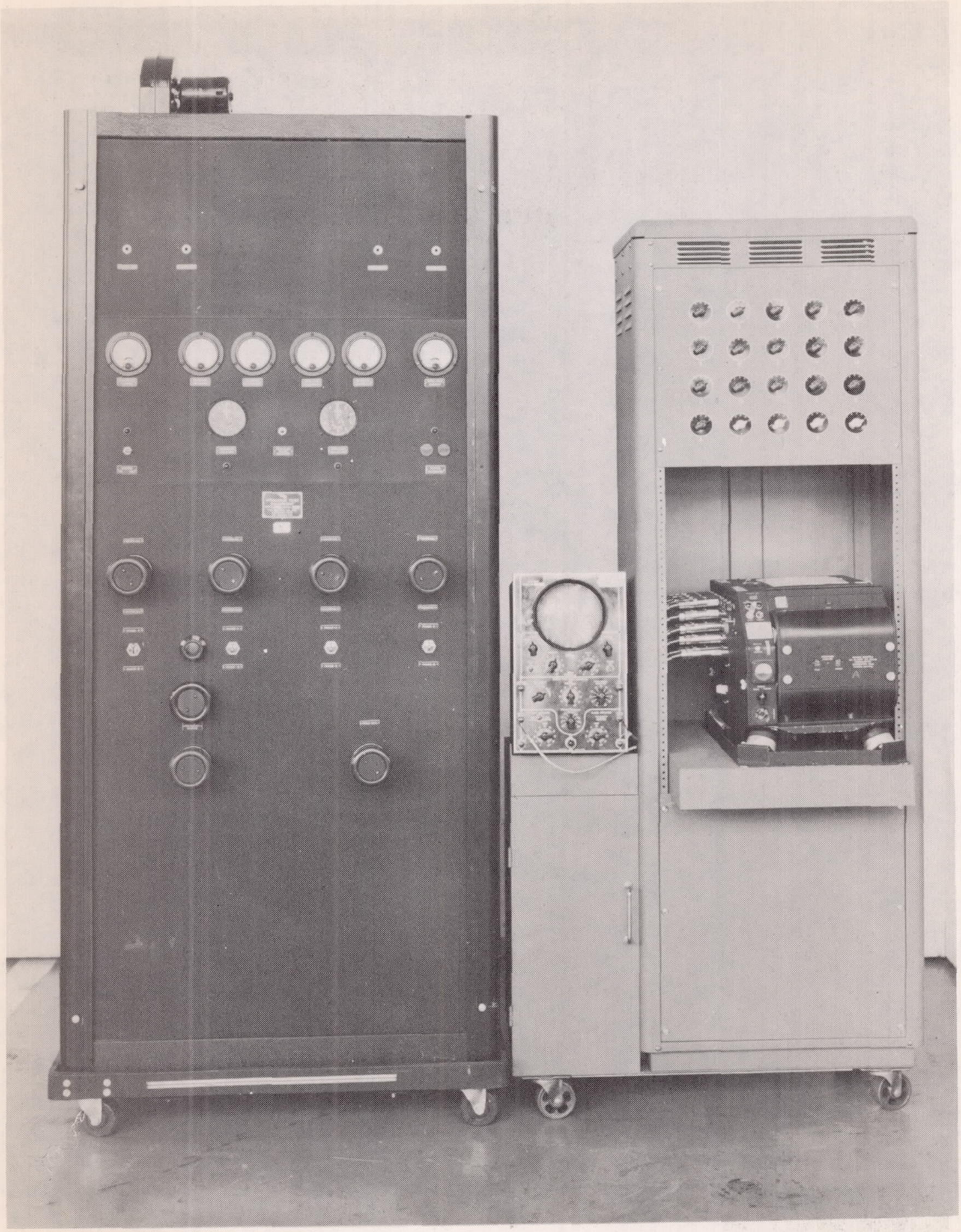
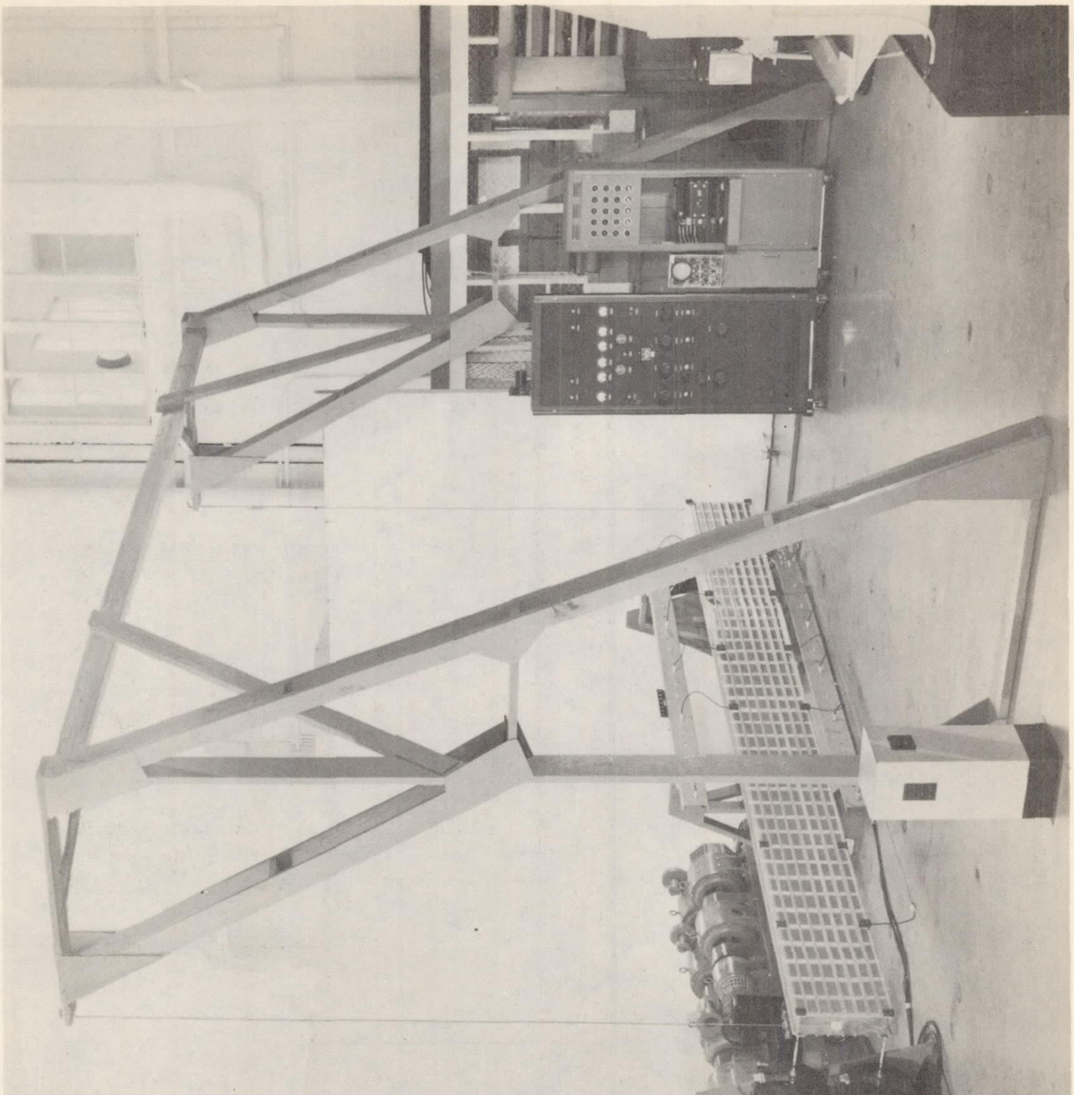


Figure 2.- Control console and recording cabinet. L-86284



L-86281.1

Figure 3.- General test setup.

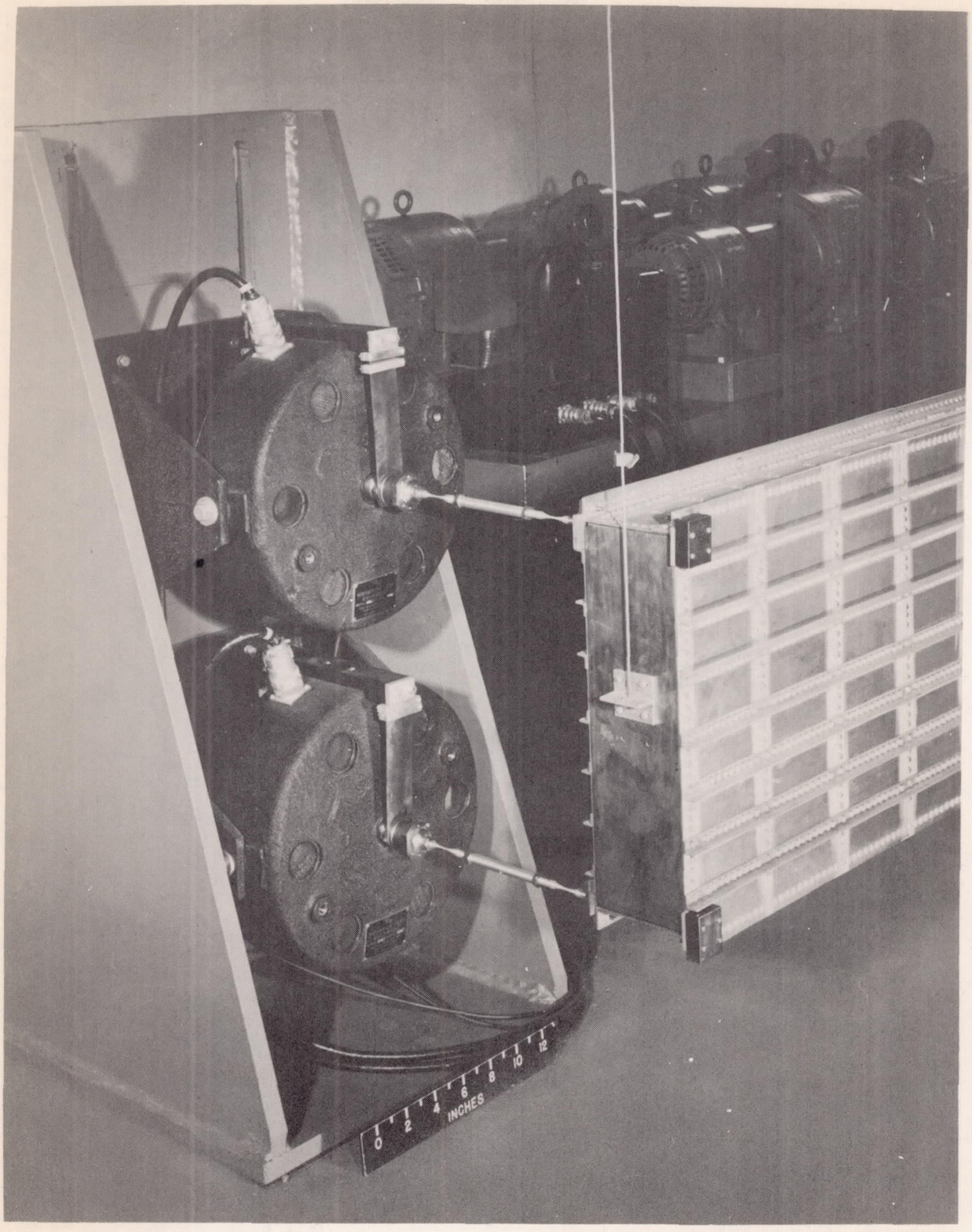


Figure 4.- Shakers and connectors. L-86282.1

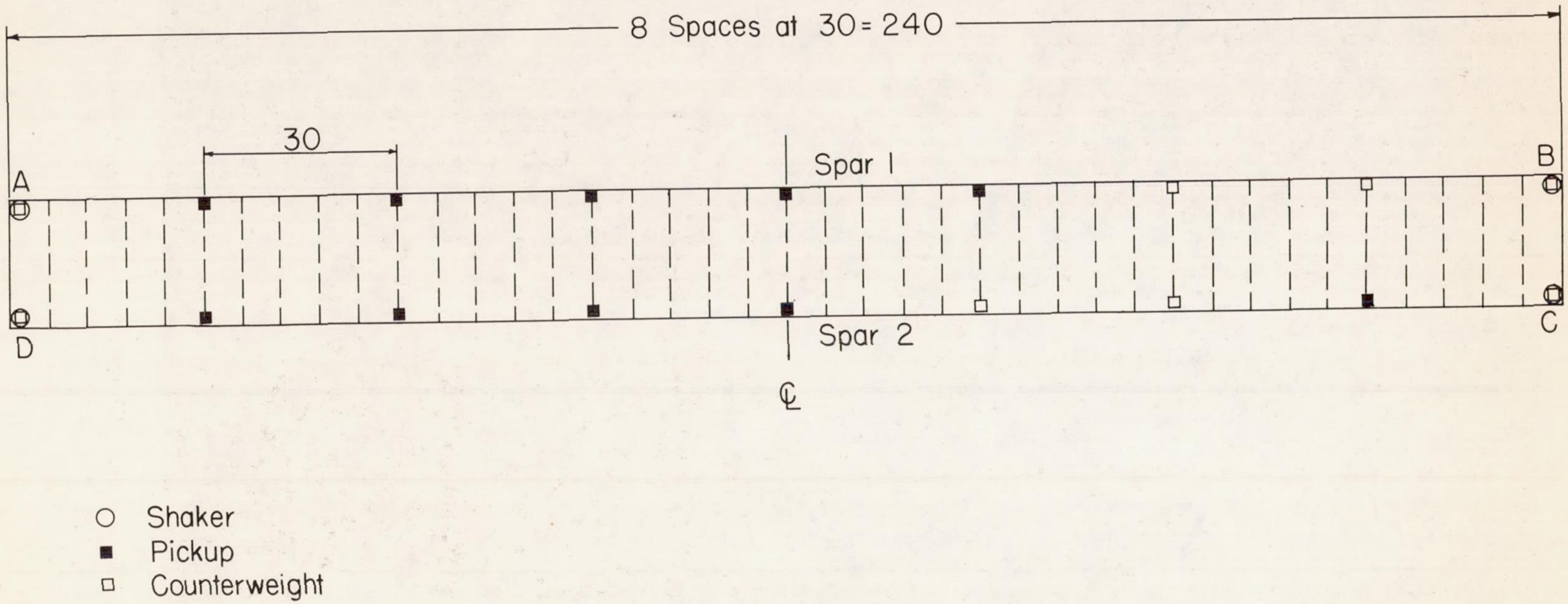


Figure 5.— Location of shakers, pickups, and counterweights.

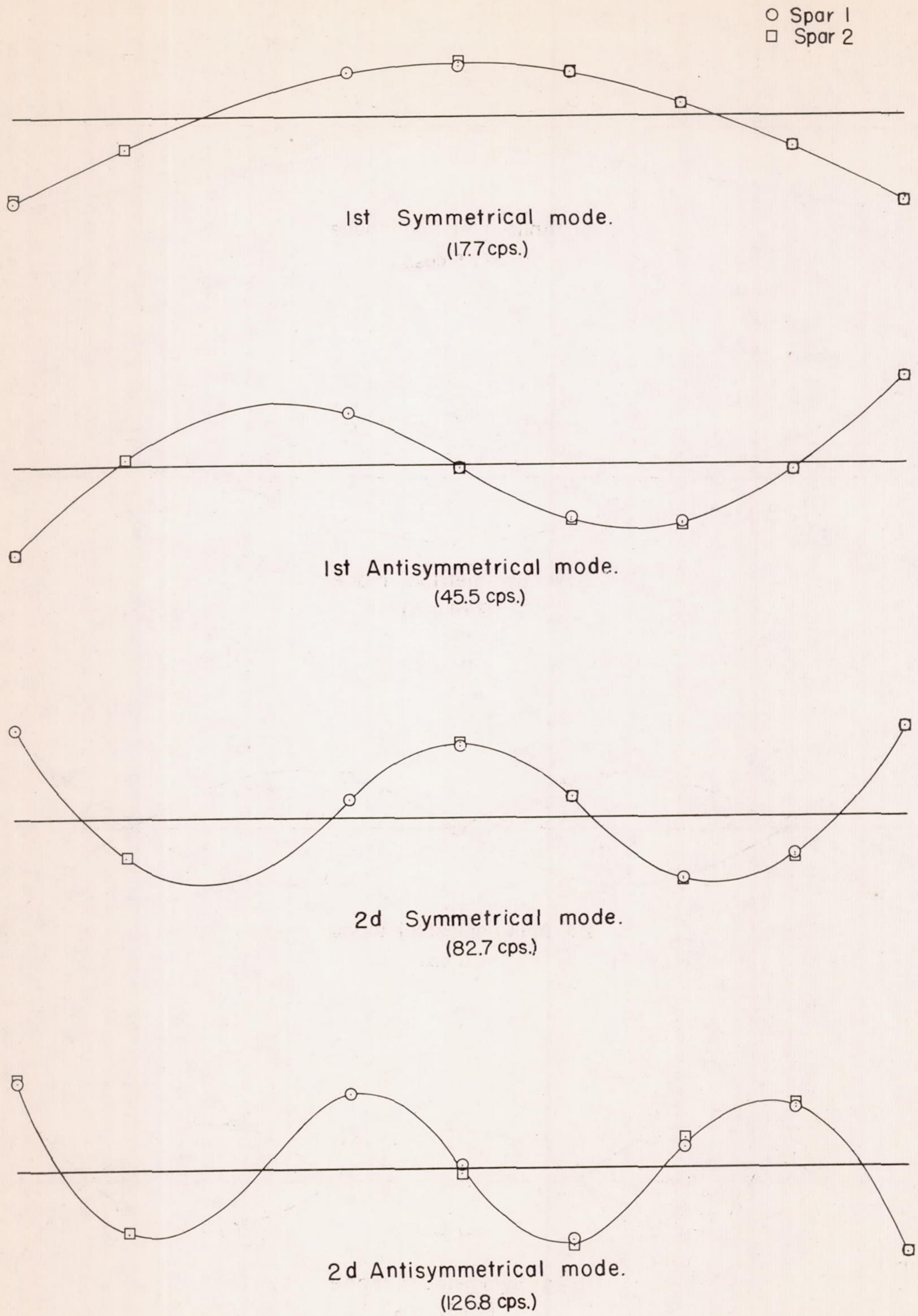


Figure 6.- Experimental bending modes and frequencies of test beam with pickups attached.

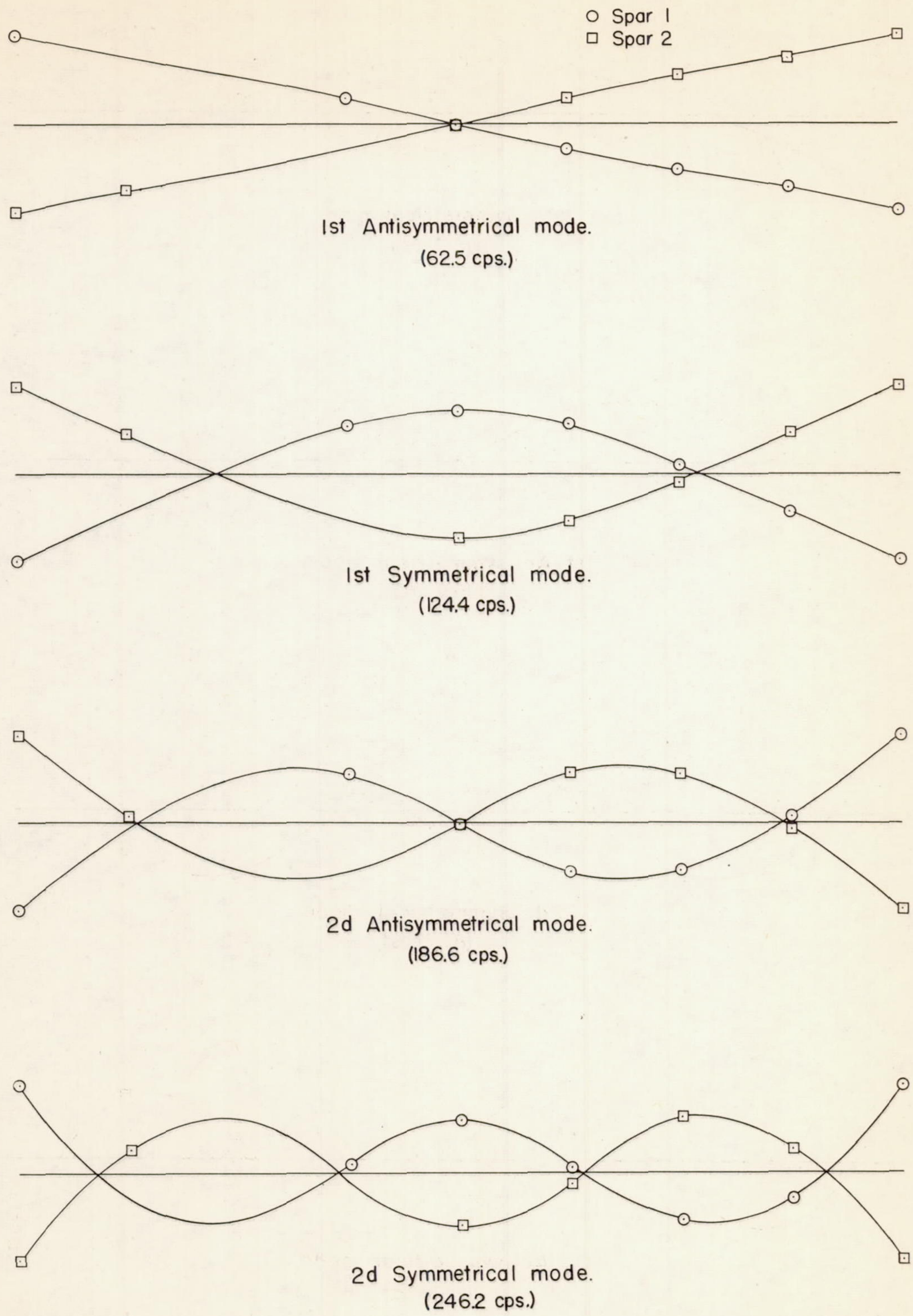
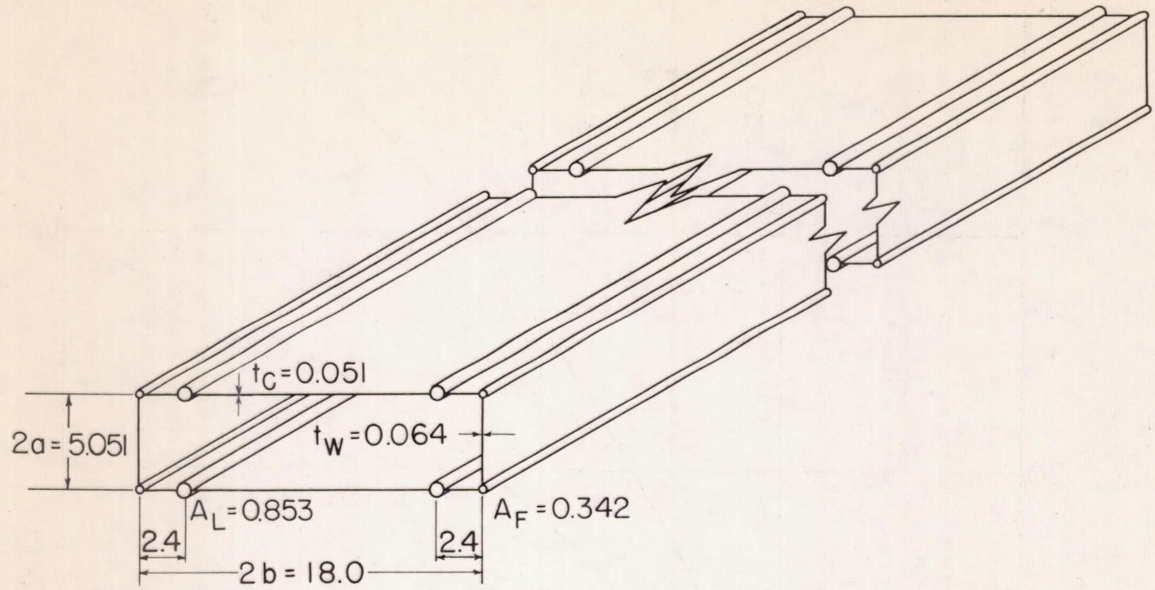
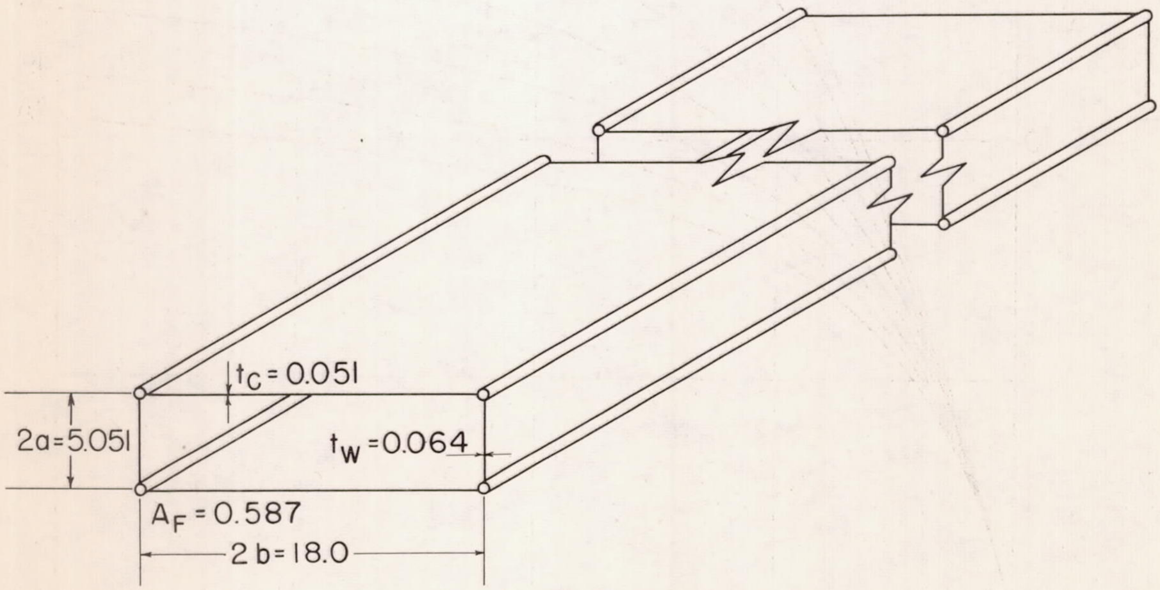


Figure 7.—Experimental torsional modes and frequencies of test beam with pickups attached.



(a) Substitute - stringer idealization.



(b) Four - flange idealization.

Figure 8. - Substitute structure idealizations of box beam test specimen.

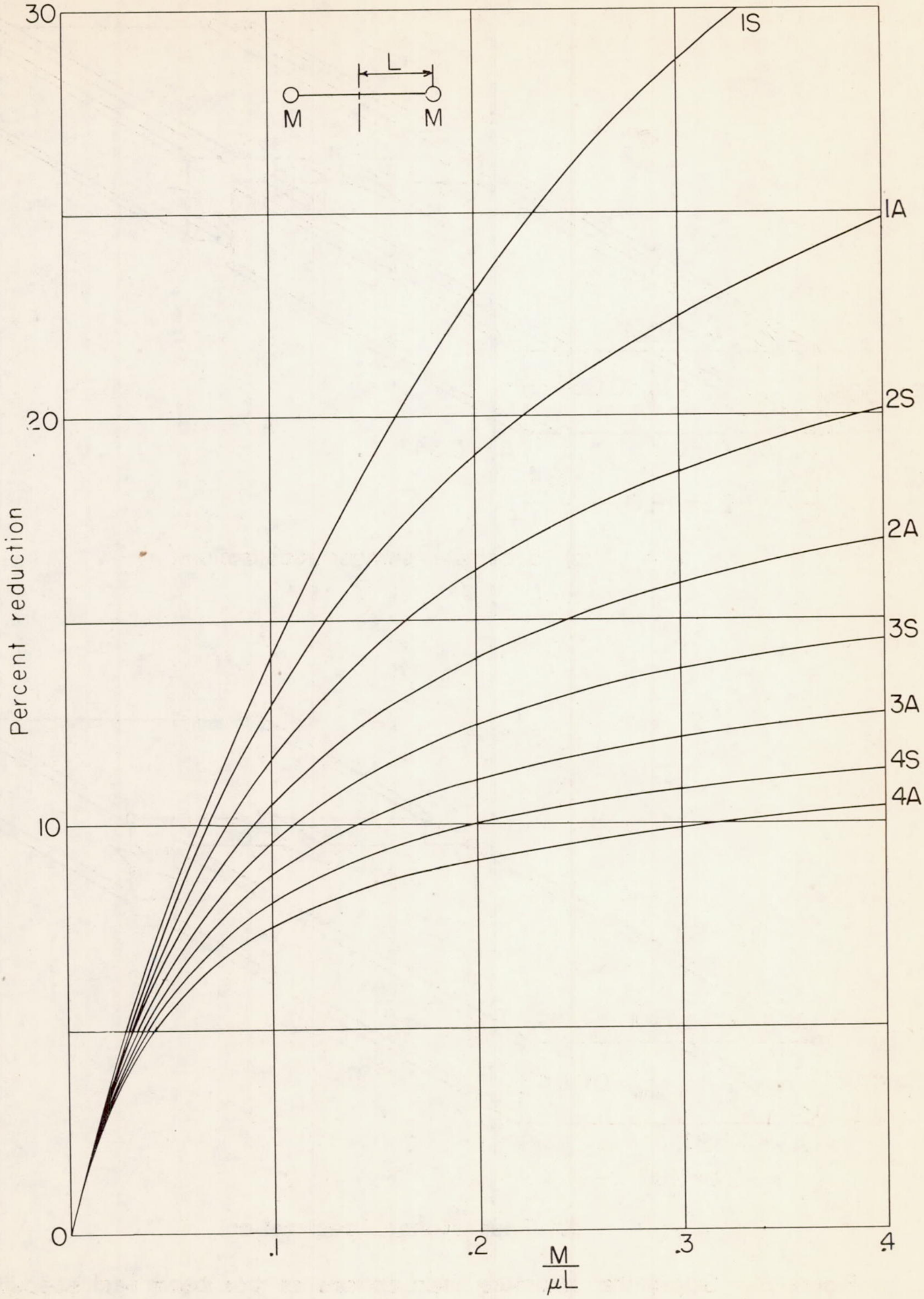


Figure 9.—Per cent reduction of elementary bending frequencies due to concentrated masses, where μ = mass per unit length, and S and A refer to symmetrical and antisymmetrical modes, respectively.

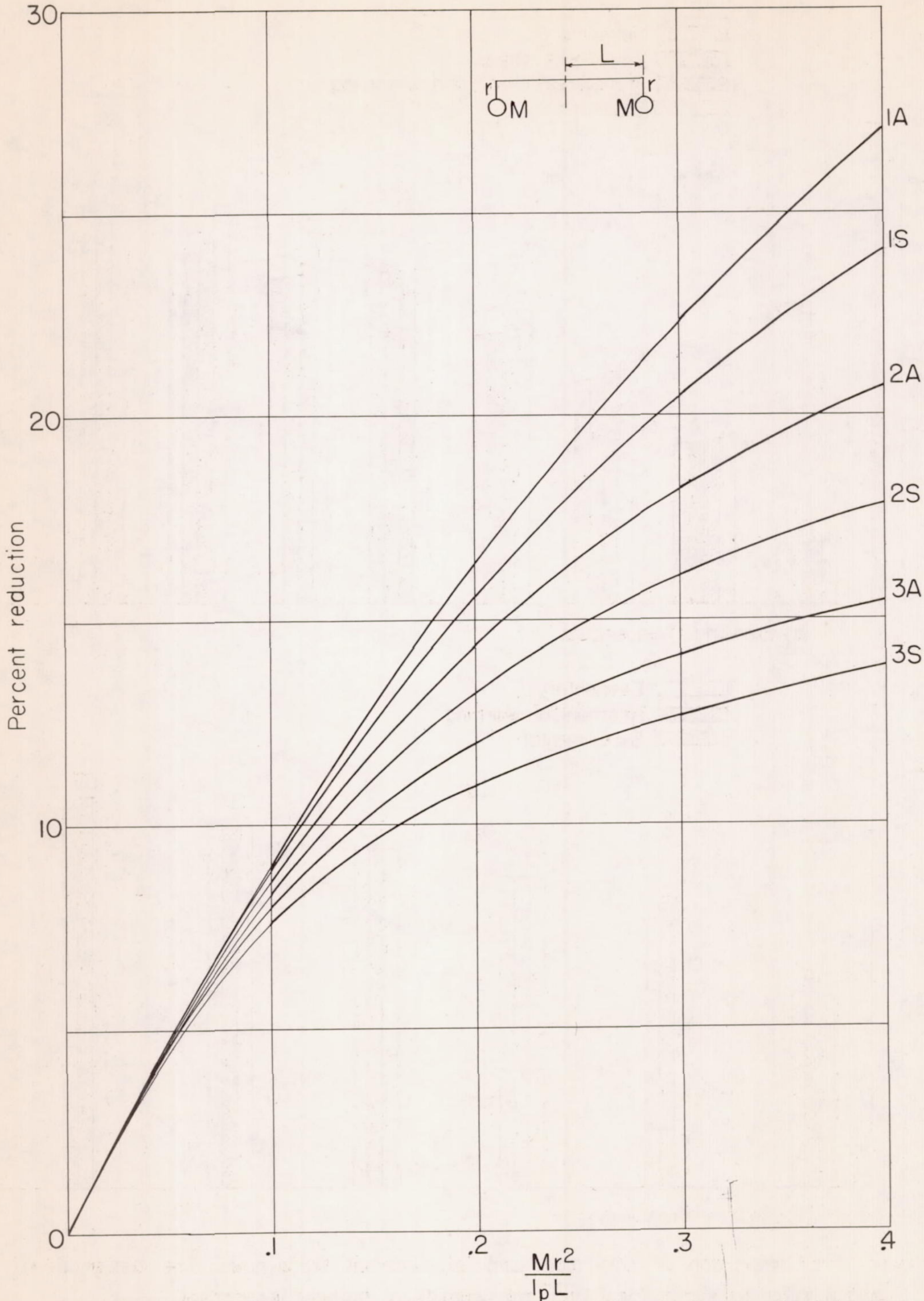


Figure 10.—Per cent reduction of elementary torsional frequencies due to eccentric concentrated masses, where I_p = mass polar moment of inertia per unit length, and S and A refer to symmetrical and antisymmetrical modes, respectively.

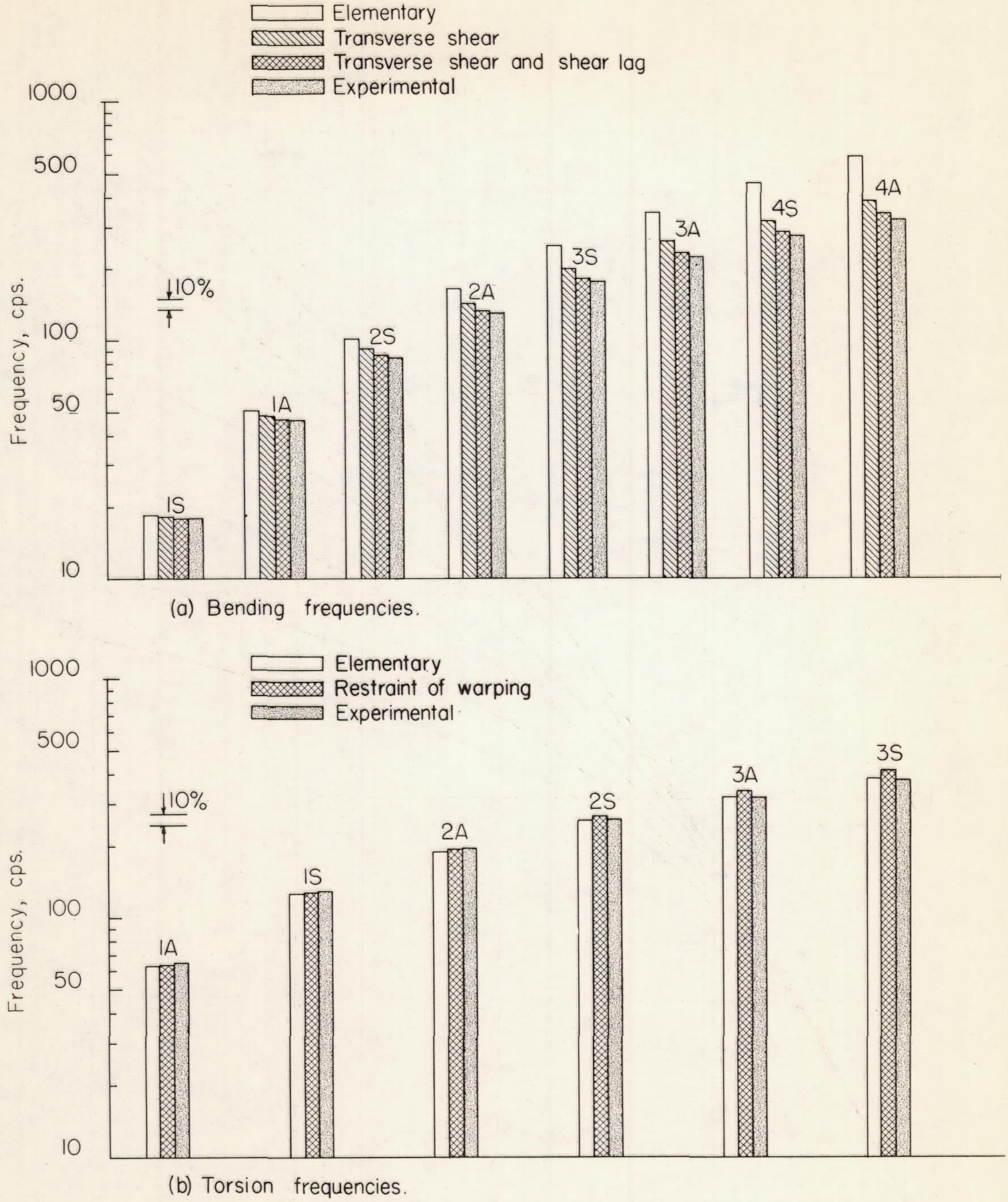


Figure 11.— Comparison of calculated and experimental frequencies. The designations S and A refer to symmetrical and antisymmetrical modes, respectively.