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ACOUSTIC ANALYSIS OF RAM-JET BUZZ

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SUMMARY

A one-dimensional analysis of ram-jet buzz is presented. The particular mechanism leading to buzz, which is treated herein, is the amplification of acoustic waves in the ram-jet combustion chamber due to successive reflections from the inlet and exit sections.

It is assumed that the combustion chamber is of constant area and is long compared with the length of the inlet. The inlet of such a configuration operates quasi steadily, or nearly so, during unsteady operation. It is assumed that the exit operates quasi steadily at all times. The configuration is shown to be unstable when the real part of the acoustic impedance of the inlet is greater than a term of the order of the combustion-chamber Mach number. For quasi-steady operation, the impedance of an inlet is proportional to the slope of its characteristic curve. Increasing the combustion-chamber Mach number or decreasing the slope of the inlet characteristic curve during subcritical operation will tend to increase the range of stable operation.

Computations indicate that burning increases the stable operating range of a given configuration. The computations assume a fixed planar flame front and a constant heat release per unit mass.

The requirement of a relatively long combustion chamber is relaxed in the appendix that treats the case of isentropic flow in the combustion chamber.

The applicability of the results for configurations wherein the inlet is replaced by a compressor is mentioned briefly in CONCLUDING REMARKS.

INTRODUCTION

Self-sustained oscillations frequently occur in systems through which a fluid is flowing. The oscillations are induced by that component (or components) of the system that tends to amplify incident

3677

I-00

pressure waves. The surging of a system containing a centrifugal- or axial-flow compressor (e.g., refs. 1 to 4) is an example of a self-sustained oscillation. Inlets designed for supersonic jet engines also have been observed to induce oscillations (e.g., refs. 5 to 11) which are usually referred to as "buzz." The origin of buzz in ram-jet engines is the subject of the present report.

Consider a nose inlet and an exit section connected by a combustion chamber of constant area, as indicated schematically in figure 1. Such a configuration has the essential features of a ram-jet engine. If the exit section is wide open, the normal shock is downstream of the inlet cowl lip, and the engine is said to be operating supercritically (fig. 1(a)). Decreasing the exit area tends to move the shock forward, increasing the pressure in the combustion chamber but leaving the mass flow unaffected. With further decreases in exit area, the shock eventually moves forward of the inlet cowl lip and the inlet is said to be operating subcritically (fig. 1(b)). During subcritical operation, the mass flow decreases with decreases in exit area. Eventually a mass flow is reached below which operation is unstable, this resulting in buzz. A typical plot of combustion-chamber pressure against mass flow is shown in figure 1(c). The slope of this curve is generally positive at the mass flow corresponding to the transition between stable and unstable operation.

The origin of buzz may be attributed to either of two types of phenomenon. These are distinguished, herein, as follows:

(1) Linear instability origin: Consider a weak pressure wave to be propagating upstream in the combustion chamber. The wave will be reflected by the inlet section and will travel downstream where it will be reflected by the exit section. If the successive reflections tend to increase the amplitude of the wave, the system is unstable and oscillations will result. If the magnitudes of the corresponding incident and reflected waves at the inlet are linearly related, then the oscillation is termed herein as having a linear instability origin. The essential feature of this type of instability origin is that the ratio of the magnitudes of the incident and reflected waves at the inlet be constant, the constant depending on the nominal steady flow through the ram jet. This instability can be analyzed by the method of small perturbations. The entire ram-jet configuration must be considered.

(2) Nonlinear instability origin: In some cases, a weak incident wave may create large-scale aerodynamic changes within the inlet (e.g., large-scale separation, shock motion, etc.) such that the magnitude of the reflected wave is not directly related to the magnitude of the incident wave. Oscillations induced by such inlet behavior are termed herein as having a nonlinear instability origin. The weak incident

wave may be thought of as "triggering" the buzz cycle. The occurrence of this type of instability is expected to depend primarily on the aerodynamic behavior of the inlet without particular regard for the remainder of the configuration.

It is desirable to determine criteria defining the stable operating range of ram-jet configurations. In reference 7 it was observed that a series of nose inlets buzzed at that mass flow where a vortex sheet (originating at the upstream shock configuration) just enters the cowl lip. A qualitative analysis of the flow for this condition suggested that the vortex sheet causes separation on the inner surface of the cowling. This separation apparently triggered the buzz cycle. However, reference 11 presents data which indicate that the entrance of the vortex sheet into an inlet cowling need not necessarily result in buzz. In reference 11, it is suggested that the occurrence of buzz in the experimental ram-jet configuration test reported therein might be associated with separation on the inlet centerbody. The role of separation and boundary-layer build-up as mechanisms leading to buzz is also discussed in reference 8. Both references 8 and 11 present extensive experimental data and discussions of the nature of the buzz cycle.

Several papers, such as references 6 and 9, have investigated analytically the stability of ram-jet configurations having a linear instability origin. Reference 6 likens the ram-jet configuration to a Helmholtz resonator having a mean through flow. The system becomes unstable when the slope of the characteristic curve (pressure against mass flow) of the inlet becomes sufficiently positive. Both burning and nonburning cases were considered. For the nonburning case, conditions in the combustion chamber are assumed uniform at any instant, the wave structure in the combustion chamber thus being neglected. This assumption is valid provided that the combustion chamber is very short compared with the wavelength of the oscillations. A similar restriction applies to the analysis of the burning case. When the combustion chamber is not short compared with the wavelength of the oscillations, it is necessary to take into account the existence of waves in the combustion chamber. The stability of a relatively simple configuration of this type is analyzed in reference 9, which considers a configuration consisting of an open-nose inlet, a relatively long constant-area combustion chamber, and a choked exit. The stability of the system is analyzed using transfer functions (ref. 12). (The transfer-function concept provides an extension of classical acoustic-impedance techniques to systems having a mean through flow.) Reference 9 reports that the configuration became unstable at flight Mach numbers above a certain critical value, the critical value depending on the Mach number of the mean flow through the combustion chamber. This theoretical trend was confirmed by experiment.

3677

CO-1 back

A stability analysis, similar to that of reference 9 is presented herein. It is assumed that the ram jet has a constant-area combustion chamber and a linear instability origin. However, the inlet is not specialized to be an open-nose inlet. The flow is considered to be one-dimensional. (Some justification for this assumption can be deduced from ref. 10, which correlates the wave structure in an experimental ram-jet combustion chamber with calculations based on one-dimensional flow theory.) Both burning and nonburning cases are treated.

ANALYSIS

Flow through a tube of constant area is first considered. The fluid is assumed to be slightly perturbed from the condition of uniform through flow, and the corresponding equations of motion are noted. Possible standing waves are indicated and the stability of flow through a simple ram-jet configuration is then investigated.

Basic Equations

Consider a tube of constant area with a through flow of velocity u , pressure p , etc. Assume the flow to be slightly perturbed and represent the perturbations by Δp , Δu , etc. The equations of motion, neglecting viscosity, heat conduction, and second-order terms, are

Momentum:

$$\rho \left[\frac{\partial(\Delta u)}{\partial t} + u \frac{\partial(\Delta u)}{\partial x} \right] + \frac{\partial(\Delta p)}{\partial x} = 0 \quad (1a)$$

Continuity:

$$\frac{\partial \Delta \rho}{\partial t} + u \frac{\partial \Delta \rho}{\partial x} + \rho \frac{\partial \Delta u}{\partial x} = 0 \quad (1b)$$

Energy:

$$\frac{\partial \Delta \eta}{\partial t} + u \frac{\partial \Delta \eta}{\partial x} = 0 \quad (1c)$$

State:

$$\frac{\Delta p}{p} - \frac{\Delta \rho}{\rho} - \frac{\Delta T}{T} = 0 \quad (1d)$$

(Symbols are defined in appendix A.) The entropy perturbation is related to the pressure, temperature, and density perturbations by the relations

$$\frac{\Delta\eta}{c_v} = \frac{\Delta p}{p} - \gamma \frac{\Delta\rho}{\rho} = -(\gamma - 1) \frac{\Delta p}{p} + \gamma \frac{\Delta T}{T} \quad (2)$$

The solution of equations (1) can be expressed in the form

$$\frac{\Delta p}{p} = f[x - (a + u)t] + g[x + (a - u)t] \quad (3a)$$

$$\gamma \frac{\Delta u}{a} = f[x - (a + u)t] - g[x + (a - u)t] \quad (3b)$$

$$\gamma \frac{\Delta\rho}{\rho} = f[x - (a + u)t] + g[x + (a - u)t] - h(x - ut) \quad (3c)$$

$$\frac{\Delta\eta}{c_v} = h(x - ut) \quad (3d)$$

The functions f and g represent waves moving upstream and downstream, respectively, with the speed of sound relative to the steady velocity u . The function h represents an entropy wave that is convected by the steady velocity u .

It is now assumed that the perturbations are sinusoidal. (This assumption is not unduly restrictive since a nonsinusoidal oscillation may be considered as a superposition of sinusoidal waves.) With the use of complex notation, equations (3) may be expressed as

$$\frac{\Delta p}{p} = Fe^{i \frac{k}{a} \left(\frac{x}{1+M} - at \right)} + Ge^{-i \frac{k}{a} \left(\frac{x}{1-M} + at \right)} \quad (4a)$$

$$\gamma \frac{\Delta u}{a} = Fe^{i \frac{k}{a} \left(\frac{x}{1+M} - at \right)} - Ge^{-i \frac{k}{a} \left(\frac{x}{1-M} + at \right)} \quad (4b)$$

$$\gamma \frac{\Delta\rho}{\rho} = Fe^{i \frac{k}{a} \left(\frac{x}{1+M} - at \right)} + Ge^{-i \frac{k}{a} \left(\frac{x}{1-M} + at \right)} - He^{i \frac{k}{a} \left(\frac{x}{M} - at \right)} \quad (4c)$$

$$\frac{\Delta\eta}{c_v} = He^{i \frac{k}{a} \left(\frac{x}{M} - at \right)} \quad (4d)$$

(the real part of the right-hand side being understood) where F , G , H , and $k = \omega + iA$ are complex constants. Equations (4) represent sinusoidal waves with frequency ω and amplification A . The numerical values of ω and A depend on the boundary conditions at the inlet x_1 and the exit x_e of the tube. If $A < 0$, the waves are damped. If $A > 0$, the waves amplify and the system is unstable. If $A = 0$, the waves are neither amplified nor damped and the system is in a state of neutral equilibrium. The case $A = 0$ is of particular interest since it generally defines a limit of stable operation.

Examination of equations (4) indicates that the dimensionless complex ratios $\frac{a}{\gamma p} \frac{\Delta p}{\Delta u}$, $\frac{a}{c_p} \frac{\Delta \eta}{\Delta u}$, etc., are independent of time. This is a consequence of having chosen k to be the same for the various waves in equations (4). The ratios $\zeta \equiv \theta - i\psi \equiv \frac{a}{\gamma p} \frac{\Delta p}{\Delta u}$ and $\xi \equiv \frac{a}{c_p} \frac{\Delta \eta}{\Delta u}$ will be used in later developments. These ratios may be termed acoustic impedance and entropy impedance, respectively, by analogy with classical acoustic theory (ref. 13).¹ The quantity θ is termed the acoustic resistance while ψ is termed the acoustic reactance. Similar complex ratios were used in reference 9.

From equations (4) and the definitions of ζ and ξ ,

$$\zeta \equiv \theta - i\psi \equiv \frac{a}{\gamma p} \frac{\Delta p}{\Delta u} = \frac{1 + \frac{G}{F} e^{-2i \frac{k}{a} \left(\frac{x}{1-M^2} \right)}}{1 - \frac{G}{F} e^{-2i \frac{k}{a} \left(\frac{x}{1-M^2} \right)}} \quad (5a)$$

$$\xi \equiv \frac{a}{c_p} \frac{\Delta \eta}{\Delta u} = \frac{\frac{H}{F} e^{i \frac{k}{a} \left[\frac{x}{M(1+M)} \right]}}{1 - \frac{G}{F} e^{-2i \frac{k}{a} \left(\frac{x}{1-M^2} \right)}} \quad (5b)$$

If ζ and ξ are known at some point x_r then the complex constants G/F and H/F can be expressed in terms of the known quantities ζ_r

¹In classical acoustic theory there is no mean through flow so that ξ does not appear as a parameter. The parameter ζ is referred to as the specific acoustic impedance in ref. 13.

and ξ_r . Equations (5) can then be written in the forms

$$\zeta = \frac{1 + \frac{\zeta_r - 1}{\zeta_r + 1} e^{-2i \frac{k}{a} \left(\frac{x - x_r}{1 - M^2} \right)}}{1 - \frac{\zeta_r - 1}{\zeta_r + 1} e^{-2i \frac{k}{a} \left(\frac{x - x_r}{1 - M^2} \right)}} = \frac{1 - i \zeta_r \cot \left[\frac{k}{a} \left(\frac{x - x_r}{1 - M^2} \right) \right]}{\zeta_r - i \cot \left[\frac{k}{a} \left(\frac{x - x_r}{1 - M^2} \right) \right]} \quad (6a)$$

$$\xi = \frac{2 \xi_r}{1 + \zeta_r} \frac{e^{\frac{ik}{aM} \left(\frac{x - x_r}{1 + M} \right)}}{1 - \frac{\zeta_r - 1}{\zeta_r + 1} e^{-2i \frac{k}{a} \left(\frac{x - x_r}{1 - M^2} \right)}} = \frac{1 - i \cot \left[\frac{k}{a} \left(\frac{x - x_r}{1 - M^2} \right) \right]}{\zeta_r - i \cot \left[\frac{k}{a} \left(\frac{x - x_r}{1 - M^2} \right) \right]} \xi_r e^{\frac{ik}{aM} \left(\frac{x - x_r}{1 + M} \right)} \quad (6b)$$

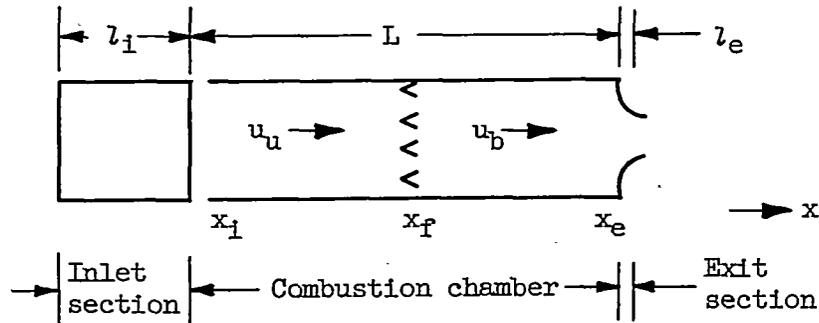
Thus, if ζ and ξ are known at one point in a tube, the values of ζ and ξ everywhere in the tube are defined by equations (6). If the boundary conditions at the tube inlet and exit are specified, then k is so chosen that equations (6) satisfy the given boundary conditions.

Equations (4) to (6) can also be applied to a configuration wherein the undisturbed flow consists of a series of uniform flows separated by interfaces (such as planar flame fronts or shock waves). For each type of interface, there is a corresponding set of boundary conditions that must be satisfied. The boundary conditions that must be satisfied across a flame front are derived in appendix B. Note that each section of uniform flow will, in general, have different values for the constants G/F , H/F , M , and a . The constant k , however, has the same value throughout the system.

3677

Ram-Jet Configuration

The configuration investigated is of the form indicated in sketch (a). A combustion chamber of length $L \equiv x_e - x_i$ is assumed to be of



constant area and to have a planar flame front at x_f . At its upstream end, the combustion chamber is connected to an arbitrary supersonic inlet. At its downstream end, the combustion chamber is connected to a short exit section which is choked. The length l_e (distance from x_e to the throat of the exit section) is assumed very small compared with L so that $l_e/L \lll 1$.

The characteristic length of the inlet l_i will later be assumed small compared with L (i.e., $l_i/L \ll 1$) so that the inlet may be considered as operating quasi steadily, or nearly so, during unsteady operation.² The restriction $l_i/L \ll 1$ may not be applicable for practical ram-jet configurations. This restriction is made because it simplifies the problem of determining the appropriate boundary conditions at x_i . The reason for choosing $l_e/L \lll 1$ is that the exit will at all times be operating quasi steadily.

²Quasi-steady operation implies that at each instant, the flow through a component is as though the operation was steady. Nearly quasi-steady operation implies a small departure from the quasi-steady condition. A component of an acoustical system operates quasi steadily if $\omega l/a \ll 1$, where l is a characteristic length. It will be shown that $\omega L/a = O(1)$ for the fundamental mode of the ram-jet configurations considered. Therefore, $l_i/L \ll 1$ implies that the inlet will operate quasi steadily, or nearly so, during unsteady operation. If $l_e/L \lll 1$, the exit will operate quasi steadily at all times.

Conditions in the burned-gas region ($x_f < x < x_e$) are indicated by the subscript b, while conditions in the unburned gas region ($x_i < x < x_f$) are indicated by the subscript u. When the nonburning case is considered, no special subscripts are used for those regions.

Boundary Conditions

Exit section. - The boundary condition at x_e is derived in appendix C and may be expressed as

$$\left[\zeta + \frac{1}{\gamma - 1} \xi - \frac{2}{M(\gamma - 1)} \right]_e = 0 \quad (7)$$

Equation (7), which is valid provided that the exit section is operating quasi steadily, is assumed to apply throughout this investigation.

Combustion chamber. - The boundary conditions across the flame front are given by equations (B6a) and (B6b) of appendix B. With the heat release per unit mass assumed constant ($\Delta Q = 0$), these equations become

$$\frac{1}{\zeta_{b,f}} - \frac{C_1}{\zeta_{u,f}} + M_b \left[\frac{\gamma_b}{\gamma_u} \frac{\xi_{u,f}}{\zeta_{u,f}} - \frac{\xi_{b,f}}{\zeta_{b,f}} - \left(\frac{\gamma_b}{\gamma_u} - 1 \right) \right] + O(M_u^2) = 0 \quad (8a)$$

$$C_2 \frac{\xi_{b,f}}{\zeta_{b,f}} - \frac{\xi_{u,f}}{\zeta_{u,f}} + C_3 + O(M_u^2) = 0 \quad (8b)$$

where

$$C_1 = \frac{a_b}{a_u} \left[1 - \left(1 - \frac{R_u T_u}{R_b T_b} \right) \frac{\Delta v}{\Delta u_{u,f}} \right]$$

$$C_2 = \frac{c_{v,b} T_b}{c_{v,u} T_u}$$

$$C_3 = (\gamma_u - 1) \left(\frac{R_b T_b}{R_u T_u} - 1 \right)$$

The subscript f indicates that the quantities are evaluated at x_f , the flame position.

Inlet section. - If $\omega l_i/a$ is sufficiently small, the flow through the inlet during unsteady operation may be considered as quasi steady.

3677

2-03

Then ζ_i and ξ_i can be expressed in terms of the slope of the characteristic curve of the inlet under steady operating conditions. If $p = p(u)$ designates the combustion-chamber static pressure as a function of combustion-chamber velocity during steady operation, then

$$\zeta_i = \left(\frac{a}{\gamma p} \right)_i \frac{dp(u)}{du} \quad (9a)$$

$$\xi_i = - (\gamma_i - 1)(\zeta_i + M_i) \quad (9b)$$

Equation (9b) is obtained from equation (2) by noting that, for flows with constant stagnation temperature, $\frac{\Delta T}{T} = - \frac{(\gamma - 1)M \Delta M}{1 + \frac{\gamma - 1}{2} M^2}$ and

$$\frac{\Delta u}{a} = \frac{\Delta M}{1 + \frac{\gamma - 1}{2} M^2}, \text{ so that } \frac{a}{\Delta u} \frac{\Delta T}{T} = - (\gamma - 1)M. \text{ Both } \zeta_i \text{ and } \xi_i, \text{ as}$$

given by equations (9), are purely real.

For slightly larger values of $\omega l_i/a$ (i.e., nearly quasi-steady operation), the inlet impedance can be expressed as

$$\zeta_i = \left[\left(\frac{a}{\gamma p} \right)_i \frac{dp(u)}{du} + \delta\theta_i \right] - i\delta\psi_i \quad (10)$$

where $\delta\theta_i$ and $\delta\psi_i$ are small and represent the departure of θ_i and ψ_i from the quasi-steady values of $\left(\frac{a}{\gamma p} \right)_i \frac{dp(u)}{du}$ and zero, respectively.

A similar expression, representing a small departure from equation (9b), can be written for ξ_i during nearly quasi-steady operation.

A theoretical derivation for the values of ζ_i and ξ_i corresponding to a given inlet appears to be very difficult (even for the cases of quasi-steady and nearly quasi-steady flows). So far, only the open-nose inlet has been treated analytically (ref. 9). No such analytical development will be attempted herein.

Nonburning Case

The configuration indicated in sketch (a) is assumed to be operating without burning. Equation (7) is assumed to apply at all times.

Expressing ζ_e and ξ_e in terms of ζ_i and ξ_i (through eqs. (6)) and substituting into equation (7) yields

$$e^{\frac{-2ikL}{a(1-M^2)}} = - \frac{1 + \zeta_i}{1 - \zeta_i} \frac{1 - \frac{\gamma - 1}{2} M}{1 + \frac{\gamma - 1}{2} M} \left[1 - \frac{M}{1 - \frac{\gamma - 1}{2} M} \frac{\xi_i}{1 + \zeta_i} e^{\frac{ikL}{aM(1+M)}} \right] \quad (11)$$

If ζ_i and ξ_i are specified, the corresponding value of k can be determined from equation (11). This solution for k defines the frequency and amplification of possible standing waves within the tube. If $A > 0$, the waves amplify and the system is inherently unstable.

Another viewpoint is to write equation (11) in the form

$$\zeta_i = \frac{\frac{\gamma - 1}{2} M + i \cot \left[\frac{kL}{a(1 - M^2)} \right]}{1 + i \frac{M}{2 \sin \left[\frac{kL}{a(1 - M^2)} \right]} \left\{ (\gamma - 1) \cos \left[\frac{kL}{a(1 - M^2)} \right] + \frac{\xi_i}{\zeta_i} e^{\frac{ikL}{aM(1-M^2)}} \right\}} \quad (12)$$

which defines the acoustic impedance at x_1 in terms of k and $\frac{\xi_i}{\zeta_i}$, for a tube of length L having a through flow of Mach number M and a short choked exit. If an inlet with a known acoustic impedance is added at x_1 , then k must be determined so that the right-hand side of equation (12) matches the specified ζ_i . Equations (11) and (12) do not require $\lambda_i/L \ll 1$.

It is not generally possible to obtain an explicit expression for k , in terms of ζ_i and ξ_i . A particular solution of equations (11) and (12), applicable for a class of nonburning ram-jet configurations operating near the neutral stability point, will now be presented. (A solution of eqs. (11) and (12) for the case of isentropic flow, is presented in appendix D.)

Consider a ram-jet configuration with the properties

$$\lambda_i/L \ll 1 \quad (13a)$$

3677

CO-2 back

$$A \approx 0 \quad (13b)$$

$$M \ll 1 \quad (13c)$$

By virtue of equation (13a), the inlet will operate quasi steadily, or nearly so, during unsteady operation. Equation (13b) signifies that the ram jet is operating near the point of inherent instability. Under these conditions, it will be shown that $\theta_1 = O(M)$; it then follows that $\zeta_1 = O(M)$ and $\xi_1 = O(M)$.³ Thus, assume, first, that

$$\zeta_1 = O(M); \xi_1 = O(M) \quad (14)$$

This anticipates the result $\theta_1 = O(M)$. By using equations (13) and (14), equations (11) and (12) become

$$e^{-2i \frac{kL}{a}} = - \left[1 + 2\zeta_1 - (\gamma - 1)M + O(M^2) \right] \quad (15a)$$

$$\zeta_1 = \left(\frac{\gamma - 1}{2} M + \frac{AL}{a} \right) + i \left[\frac{\pi}{2} (1 + 2N) - \frac{\omega L}{a} \right] + O(M^2) \quad (15b)$$

Expressing ζ_1 as $\left[\left(\frac{a}{\gamma p} \right)_1 \frac{dp(u)}{du} + \delta\theta_1 \right] - i\delta\psi_1$, where $\delta\theta_1$ and $\delta\psi_1$ are assumed to be $O(M)$, at most (i.e., nearly quasi-steady operation), and solving for k , yield

$$(\omega + iA) \frac{L}{a} = \left[\frac{\pi}{2} (1 + 2N) + \delta\psi_1 \right] + i \left[\left(\frac{a}{\gamma p} \right)_1 \frac{dp(u)}{du} + \delta\theta_1 - \frac{\gamma - 1}{2} M \right] + O(M^2) \quad (16)$$

Thus, the frequencies of the possible standing waves are given by

$$\frac{\omega L}{a} = \frac{\pi}{2} (1 + 2N) + \delta\psi_1 + O(M^2) \quad (17)$$

The fundamental mode is $\frac{\omega L}{a} = \frac{\pi}{2} + \delta\psi_1 + O(M^2)$, which for quasi-steady operation becomes $\frac{\omega L}{a} = \frac{\pi}{2}$. (The latter corresponds to the fundamental

³For nearly quasi-steady operation, ψ_1 is small and may arbitrarily be considered $O(M)$ even though it is not functionally dependent on M . Therefore, if $\theta_1 = O(M)$, then $\zeta_1 = O(M)$. If $\zeta_1 = O(M)$, it then follows that $\xi_1 = O(M)$ (eq. (9b)).

mode of an organ pipe, open at one end and closed at the other.) The system is unstable when

$$\theta_i > \frac{\gamma - 1}{2} M + O(M^2) \quad (18a)$$

or

$$\left(\frac{a}{\gamma p}\right)_i \frac{dp(u)}{du} > \frac{\gamma - 1}{2} M - \delta\theta_i + O(M^2) \quad (18b)$$

which is consistent with the original assumption that $\theta_i = O(M)$. For quasi-steady operation, the system becomes unstable when

$$\left(\frac{a}{\gamma p}\right)_i \frac{dp(u)}{du} > \frac{\gamma - 1}{2} M + O(M^2) \quad (19)$$

That is, the system is unstable when the slope of the characteristic curve exceeds the value indicated by equation (19). With increasing $\omega l_1/a$, the effect of $\delta\theta_i$ must be considered.

It may be concluded that the configuration defined by equations (13a) and (13c) will become unstable when $\left(\frac{a}{\gamma p}\right)_i \frac{dp(u)}{du} = O(M)$, and the fundamental frequency of the standing wave is $\frac{\omega l_1}{a} = \frac{\pi}{2} + O(M)$. Increasing M or decreasing the slope of the inlet characteristic curve,⁴ during subcritical operation, will tend to make a configuration more stable.

⁴The characteristic curve of an inlet is frequently expressed as p_t against M or p_t against W where p_t is the stagnation pressure in the combustion chamber and $W \equiv \rho u S \frac{\sqrt{T_\infty}}{P_\infty}$ is the corrected mass flow referred to free-stream (or other) reference conditions. The relation between $\left(\frac{a}{\gamma p}\right)_i \frac{dp(u)}{du}$ and dp_t/dM or $\frac{dp_t}{dW}$ is given by

$$\begin{aligned} \left(\frac{a}{\gamma p}\right)_i \frac{dp(u)}{du} &= \left(\frac{1}{\gamma p_t} \frac{dp_t}{dM} - M\right) [1 + O(M^2)] \\ &= \frac{1}{\gamma M} \left(\frac{W}{p_t} \frac{dp_t}{dW} - \gamma M^2\right) [1 + O(M^2)] \end{aligned}$$

where it has been assumed that the stagnation temperature in the combustion chamber does not vary with mass flow and that the right-hand side of these equations are of order M (i.e., operation near point of instability). Thus, $\frac{1}{\gamma p_t} \frac{dp_t}{dM} = O(M)$ and $\frac{W}{p_t} \frac{dp_t}{dW} = O(M^2)$ at the point of incipient instability.

In appendix E, it is shown that the exit tends to damp incident waves while the inlet amplifies incident waves (for $\theta_1 > 0$). Thus the inlet is the component of the ram-jet configuration that is responsible for instability. The role of the exit, in damping incident waves, was previously pointed out in references 9 and 10.

Burning Case

The possibility of burning at station x_f (sketch (a)) is now admitted. It is assumed that the configuration satisfies equations (7) and (13) (with eq. (13c) replaced by $M_u \ll 1$).

The boundary condition at x_e is, from equation (7),

$$\frac{M_b}{2} [(\gamma_b - 1)\zeta_e + \xi_e] = 1 \quad (20)$$

while the boundary conditions across the flame front are given by equations (8) (with the heat release per unit mass assumed constant). Take x_f to be the origin of the x -coordinate system ($x_f = 0$), and define the following constants:

$$\left. \begin{aligned} U &= -\frac{kx_i}{a_u} & B &= \frac{kx_e}{a_b} \\ U' &= \frac{U}{M_u(1 + M_u)} & B' &= \frac{B}{M_b(1 + M_b)} \end{aligned} \right\} \quad (21)$$

With use of equations (6); $\xi_{u,f}/\zeta_{u,f}$ can be expressed as

$$\frac{\xi_{u,f}}{\zeta_{u,f}} = \xi_i \frac{1 - i \cot U}{1 - i \zeta_i \cot U} e^{iU'} \quad (22)$$

It was previously found that $\zeta_i = O(M)$ and $\xi_i = O(M)$ for the ram jet operating without burning at $A = 0$. It will be shown that $\zeta_i = O(M_u)$ (and thus $\xi_i = O(M_u)$) for the corresponding burning ram jet. Therefore, to simplify the present analysis, it is assumed, a priori, that

$$\frac{\xi_{u,f}}{\zeta_{u,f}} = O(M_u) \quad (23)$$

which follows from equation (22), when $\zeta_i = O(M_u)$, $\xi_i = O(M_u)$, and $A \approx 0$ (provided $\cot U \ll \infty$). Using equation (23) changes the boundary conditions across the flame front (eqs. (8)) to

$$\frac{1}{\zeta_{b,f}} - \frac{C_1}{\zeta_{u,f}} - M_b \left(\frac{\gamma_b}{\gamma_u} - 1 - \frac{C_3}{C_2} \right) + O(M_u^2) = 0 \quad (24a)$$

$$\frac{\xi_{b,f}}{\zeta_{b,f}} = - \frac{C_3}{C_2} + O(M_u) \quad (24b)$$

Expressing ζ_e and ξ_e in terms of $\zeta_{b,f}$ and $\xi_{b,f}$ and utilizing equations (20) and (24b) give

$$\zeta_{b,f} = \frac{\frac{M_b}{2} (\gamma_b - 1) + i \cot B}{1 + i \frac{M_b}{2} (\gamma_b - 1) \cot B \left[1 - \frac{1}{\gamma_b - 1} \frac{C_3}{C_2} \frac{e^{iB/M_b}}{\cos B} + O(M_u) \right]} \quad (25)$$

Substituting equation (25) into equation (24a) and expressing $\zeta_{u,f}$ in terms of ζ_i give

$$\zeta_i = \left(\frac{\gamma_b - 1}{2} \right) M_b \left\{ \frac{C_1 \cot U + \cot B}{C_1 \cot B + \cot U} - \frac{1}{\gamma_b - 1} \frac{\cot B}{C_1 \cot B + \cot U} \left[2C_5 + \frac{C_3}{C_2} \frac{\cos \left(\frac{B}{M_b} \right)}{\cos B} \right] \right\} + i \left\{ \frac{C_1 \cot B \cot U - 1}{C_1 \cot B + \cot U} - \frac{M_b}{2} \frac{C_3}{C_2} \frac{1}{C_1 \cot B + \cot U} \left[\frac{\sin \left(\frac{B}{M_b} \right)}{\sin B} \right] \right\} + O(M_u^2) \quad (26)$$

where

$$C_5 = \frac{\gamma_b}{\gamma_u} - 1 - \frac{C_3}{C_2}$$

In the derivation of equation (26) it was assumed that

$$\frac{C_1 - \cot U \cot B \left[1 - \frac{1}{\gamma_b - 1} \frac{C_3}{C_2} \frac{e^{i(B+B')}}{\cos B} - \frac{2}{\gamma_b - 1} C_5 \right]}{C_1 \cot B + \cot U} \ll O(1) \quad (27)$$

Since the equation (27) is of order M for the nonburning case, this assumption appears reasonable. Equation (26) indicates that $\theta_1 = O(M_u)$, which leads to $\zeta_1 = O(M_u)$ and $\xi_1 = O(M_u)$ and thus verifies equation (23) (provided that $\cot U \ll \infty$). Equation (26) is the counterpart of equation (12). It defines the acoustic impedance of a constant-area tube having a through flow, a choked exit, and a planar flame front at $x = 0$. If an inlet is added at x_1 , then k must have a value such that the acoustic impedance of the tube matches the acoustic impedance of the inlet. This condition defines k and therefore the stability of the system can be determined.

If it is assumed that the flame is fixed ($\Delta v = 0$), and the gas constants are the same for both the burned- and unburned-gas regions (i.e., $\gamma_b = \gamma_u$, $R_b = R_u$, etc.), equation (26) becomes

$$\zeta_1 = \frac{1}{\sqrt{\lambda} \cot B + \cot U} \left(\frac{\Gamma-1}{2} \sqrt{\lambda} M_u \left\{ \sqrt{\lambda} \cot U + \left[3\lambda - 2 - (\lambda - 1) \frac{\cos\left(\frac{B}{\sqrt{\lambda} M_u}\right)}{\cos B} \right] \frac{\cot B}{\lambda} \right\} + 1 \left[\sqrt{\lambda} \cot B \cot U - 1 - \frac{\Gamma-1}{2} M_u \frac{\lambda-1}{\sqrt{\lambda}} \frac{\sin\left(\frac{B}{\sqrt{\lambda} M_u}\right)}{\sin B} \right] \right) + O(M_u^2) \quad (28)$$

where $\lambda \equiv T_b/T_u$ and equations such as $\frac{a_b}{a_u} = \sqrt{\lambda}$, $\frac{u_b}{u_u} = \lambda \left[1 + O(M_u^2) \right]$, and $\frac{M_b}{M_u} = \sqrt{\lambda} \left[1 + O(M_u^2) \right]$ (based on eqs. (B2) of appendix B) have been used. Equation (28) has been solved numerically for the following special case:

$A = 0$, $\psi_1 = 0$: Assume the configuration is operating at the neutral stability point ($A = 0$), so that U and B are real. Further, assume that $\psi_1 = 0$ (quasi-steady operation). Equation (28) then becomes

$$0 = \sqrt{\lambda} \cot B \cot U - 1 + O(M_u) \quad (29a)$$

$$\frac{\theta_1}{\frac{\Gamma-1}{2} M_u} = \frac{\sqrt{\lambda}}{\sqrt{\lambda} \cot B + \cot U} \left\{ \sqrt{\lambda} \cot U + \left[3\lambda - 2 - (\lambda - 1) \frac{\cos\left(\frac{B}{\sqrt{\lambda} M_u}\right)}{\cos B} \right] \frac{\cot B}{\lambda} \right\} + O(M_u) \quad (29b)$$

Equation (29a) can be solved for ω . Using this value of ω in equation (29b) gives the value of θ_1 at the neutral stability point. A ram-jet configuration is unstable when its value of θ_1 is larger than that indicated by equation (29b).

Since $U = \frac{\omega L}{a_u} \left(1 - \frac{x_e}{L}\right)$ and $B = \frac{1}{\sqrt{\lambda}} \left(\frac{\omega L}{a_u}\right) \frac{x_e}{L}$, equation (29a) can be written as

$$\frac{\omega L}{a_u} = \left(\frac{\omega L}{a_u}\right) \frac{x_e}{L} + \cot^{-1} \left\{ \frac{1}{\sqrt{\lambda}} \tan \left[\frac{1}{\sqrt{\lambda}} \left(\frac{\omega L}{a_u}\right) \frac{x_e}{L} \right] \right\} + O(M_u) \quad (30)$$

which, for a given λ , can be solved directly for $\frac{\omega L}{a_u}$ as a function of $\left(\frac{\omega L}{a_u}\right) \frac{x_e}{L}$. This gives $\frac{\omega L}{a_u}$ as a function of $\frac{x_e}{L}$ for the given λ . Define a frequency parameter Ω according to the relation

$$\Omega \equiv \frac{2}{\pi} \left(\frac{\omega x_e}{a_b} - \frac{\omega x_1}{a_u} \right) \equiv \frac{2}{\pi} \frac{\omega L}{a_u} \left[1 - \frac{x_e}{L} \left(\frac{\sqrt{\lambda} - 1}{\sqrt{\lambda}} \right) \right] \quad (31)$$

This parameter has the value of 1 for $x_e/L = 0$ and $x_e/L = 1$ (considering the fundamental mode) and is somewhat larger than 1 for intermediate values of x_e/L . For a given λ , Ω has a single maximum in the range $0 < \frac{x_e}{L} < 1$. This maximum can be shown to equal

$$(\Omega)_{\max} = \frac{4}{\pi} \tan^{-1}(\lambda^{1/4}) + O(M_u) \quad (32)$$

and occurs at a value of x_e/L given by

$$\frac{x_e}{L} = \frac{\sqrt{\lambda}}{\sqrt{\lambda} + 1} \quad (33)$$

(for $\lambda \neq 1$). Thus $1 \leq [(\Omega)_{\max} + O(M_u)] \leq 2$ for $1 \leq \lambda \leq \infty$. A plot of Ω against x_e/L is presented in figure 2 for $\lambda = 4$ and 9.

Equation (29b) is solved using the value of ω obtained from equation (29a). Substitution of equation (29a) into equation (29b) permits the latter to be written in the more convenient form

$$\frac{\theta_1}{\frac{\gamma-1}{2} M_u} = \frac{1}{1 + \lambda \cot^2 B} \left\{ \lambda + \left[3\lambda - 2 - (\lambda - 1) \frac{\cos \left(\frac{B}{\sqrt{\lambda} M_u} \right)}{\cos B} \right] \cot^2 B \right\} + O(M_u) \quad (34)$$

3677

CO-3

For the nonburning case, $\frac{\theta_i}{\frac{\gamma-1}{2} M_{u1}} = 1$. When there is burning, equation

(34) indicates that $\frac{\theta_i}{\frac{\gamma-1}{2} M_{u1}} > 1$. Thus, a given configuration is made

more stable because of the burning. Equation (34) gives

$$\frac{\theta_i}{\frac{\gamma-1}{2} M_{u1}} = 2 - 1/\lambda \text{ for } x_e/L = 0, \text{ and } \frac{\theta_i}{\frac{\gamma-1}{2} M_{u1}} = \lambda \text{ for } x_e/L = 1.$$

The function $\theta_i / \frac{\gamma-1}{2} M_{u1}$ may have many maximum and minimum points in the range $0 < x_e/L < 1$ because of the relatively rapid oscillations of $\cos(B/\sqrt{\lambda} M_{u1})$ with x_e/L for small M_{u1} . A plot of $\theta_i / \frac{\gamma-1}{2} M_{u1}$ against x_e/L for $\lambda = 9$ and $M_{u1} = 0.01$ and 0.05 is presented in figure 3. The value of $\theta_i / \frac{\gamma-1}{2} M_{u1}$ increases from 1.89 at $x_e/L = 0$ to 9 at $x_e/L = 1$. Oscillations about the mean are more numerous for the $M_{u1} = 0.01$ case than for the $M_{u1} = 0.05$ case.

These results assume a fixed planar flame front and a constant heat release per unit mass. The possibility of a combustion-induced instability ($\Delta Q \neq 0$) has not been considered. If any of these assumptions are relaxed, considerably different stability criteria would be expected.

CONCLUDING REMARKS

A one-dimensional analysis of ram-jet buzz is presented. The particular mechanism leading to buzz, which is treated herein, is the amplification of acoustic waves in the ram-jet combustion chamber due to successive reflections from the inlet and exit sections (i.e., linear instability origin). The system becomes unstable when the real part of the acoustic impedance of the inlet is greater than a term of the order of the combustion-chamber Mach number.

An analytical method for determining the acoustic impedance of an arbitrary inlet would be of interest. As mentioned previously, only the open-nose inlet has been treated analytically (ref. 9) and the analysis therein is restricted to low frequencies.

It should be noted that the previous derivations are generally applicable to configurations wherein the inlet of the present report is

replaced by a compressor. It would appear that only equation (9b) requires modification (to account for the variation of compressor discharge stagnation temperature with mass flow). If the quasi-steady performance of the compressor is characterized by a polytropic exponent n (i.e., $\left[\frac{\Delta p(u)}{p(u)}\right]_i = \frac{n}{n-1} \left[\frac{\Delta T(u)}{T(u)}\right]_i = n \left[\frac{\Delta \rho(u)}{\rho(u)}\right]_i$, etc.) equation (9b) is replaced by

$$\xi_i = \frac{n - \gamma}{n} \zeta_i \quad (35)$$

for n of order 1, ξ_i is at most of the order of ζ_i and the solutions presented in the main body of the report (i.e., eqs. (15) to (34)) are applicable. If the flow in the combustion chamber can be considered isentropic, then appendix D applies.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, August 30, 1955

3677

CO-3 back

APPENDIX A

SYMBOLS

The following symbols are used in this report:

A	constant defining amplification
a	speed of sound
B, B'	constants defined in eqs. (21)
C_1, C_2, C_3, C_4	constants defined in eqs. (B6)
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
F, G, H	complex constants defined by eqs. (4)
f, g, h	functions defined by eqs. (3)
k	complex constant ($\omega + iA$)
L	length of combustion chamber
l	characteristic length
l_i, l_e	characteristic length of inlet and exit section, respectively
M	Mach number in combustion chamber
N	integer (0, 1, 2, . . .)
n	polytropic exponent
p	static pressure of gas in combustion chamber
$p(u)$	combustion-chamber static pressure as function of u for steady operation
Q	heat release per unit mass at flame front
R	gas constant
S	cross-sectional area of combustion chamber

T	temperature of gas in combustion chamber
t	time
U, U'	constants defined in eqs. (21)
u	axial velocity in combustion chamber
v	velocity of planar flame front
W	corrected mass flow
x	coordinate distance in stream direction
γ	ratio of specific heats
Δ	net perturbation in a quantity
δ	perturbation from quasi-steady value
ζ	acoustic impedance, $\theta - i\psi$ (eq. (5a))
η	entropy
θ	acoustic resistance
λ	ratio of temperatures of burned to unburned gases (T_b/T_u)
ξ	entropy impedance, $\frac{a}{c_p} \frac{\Delta\eta}{\Delta u}$ (eq. (5b))
ρ	density of gas in combustion chamber
ψ	acoustic reactance
Ω	frequency parameter defined by eq. (31)
ω	frequency, radians/sec
Subscripts:	
b	burned-gas region
e	combustion-chamber exit section
f	flame front
i	combustion-chamber inlet section

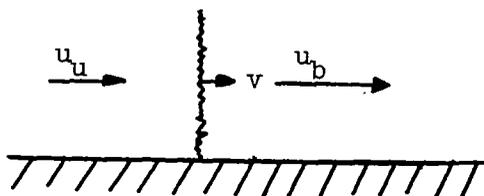
r reference section
u unburned-gas region

APPENDIX B

BOUNDARY CONDITIONS ACROSS PLANAR FLAME FRONT

Consider a flame to be propagating with a small constant velocity v relative to a wall, as indicated in sketch (b). Conditions in the

3677



(b) Flame moving relative to wall.

burned- and unburned-gas regions are related by

Momentum:

$$p_u + \rho_u(u_u - v)^2 = p_b + \rho_b(u_b - v)^2$$

State:

$$\frac{p_u}{\rho_u R_u T_u} = \frac{p_b}{\rho_b R_b T_b} = 1$$

Continuity:

$$\rho_u(u_u - v) = \rho_b(u_b - v)$$

Energy:

$$\frac{\gamma_u}{\gamma_u - 1} \frac{p_u}{\rho_u} + \frac{1}{2} (u_u - v)^2 + Q = \frac{\gamma_b}{\gamma_b - 1} \frac{p_b}{\rho_b} + \frac{1}{2} (u_b - v)^2$$

(B1)

where Q is the energy added, per unit mass. If the flow velocities are small compared with the speed of sound, equations (B1) become, respectively,

$$\left. \begin{aligned} \frac{P_b}{P_u} &= 1 + O(M_u^2) \\ \frac{\rho_b T_b R_b}{\rho_u T_u R_u} &= 1 + O(M_u^2) \\ \frac{T_u R_u (u_b - v)}{T_b R_b (u_u - v)} &= 1 + O(M_u^2) \\ c_{p,b} T_b - c_{p,u} T_u &= Q \left[1 + O(M_u^2) \right] \end{aligned} \right\} \quad (B2)$$

where $M_u = u_u/a_u$. Now, assume that the flow is perturbed. The resulting relations across the flame front are

$$\left. \begin{aligned} \frac{\Delta p_b}{P_b} &= \frac{\Delta p_u}{P_u} \left[1 + O(M_u^2) \right] \\ \frac{\Delta \rho_b}{\rho_b} - \frac{\Delta \rho_u}{\rho_u} &= \left(\frac{\Delta T_u}{T_u} - \frac{\Delta T_b}{T_b} \right) \left[1 + O(M_u^2) \right] \\ \frac{\Delta(u_b - v)}{u_b - v} - \frac{\Delta(u_u - v)}{u_u - v} &= \left(\frac{\Delta T_b}{T_b} - \frac{\Delta T_u}{T_u} \right) \left[1 + O(M_u^2) \right] \\ \frac{c_{p,b} T_b}{c_{p,u} T_u} \frac{\Delta T_b}{T_b} - \frac{\Delta T_u}{T_u} &= \left(\frac{c_{p,b} T_b}{c_{p,u} T_u} - 1 \right) \frac{\Delta Q}{Q} \left[1 + O(M_u^2) \right] \end{aligned} \right\} \quad (B3)$$

From equations (1d) and (2)

$$\left. \begin{aligned} \frac{\Delta \rho}{\rho} &= \frac{1}{\gamma} \left(\frac{\Delta p}{P} - \frac{\Delta \eta}{c_v} \right) \\ \frac{\Delta T}{T} &= \frac{1}{\gamma} \left[\frac{\Delta \eta}{c_v} + (\gamma - 1) \frac{\Delta p}{P} \right] \end{aligned} \right\} \quad (B4)$$

Substitution of equations (B4) into (B3) yields

$$\frac{\Delta p_b}{P_b} = \frac{\Delta p_u}{P_u} \left[1 + O(M_u^2) \right] \quad (B5a)$$

$$\frac{\Delta u_b}{u_b - v} - \frac{\Delta u_u}{u_u - v} \left[1 - \left(1 - \frac{u_u - v}{u_b - v} \right) \frac{\Delta v}{\Delta u_u} \right] = \left(\frac{\Delta \eta_b}{c_{p,b}} - \frac{\Delta \eta_u}{c_{p,u}} + \frac{\gamma_b - \gamma_u}{\gamma_b \gamma_u} \frac{\Delta p_u}{p_u} \right) \left[1 + O(M_u^2) \right] \quad (B5b)$$

$$\frac{c_{p,b} T_b}{c_{p,u} T_u} \frac{1}{\gamma_b} \left[\frac{\Delta \eta_b}{c_{v,b}} + (\gamma_b - 1) \frac{\Delta p_b}{p_b} \right] - \frac{1}{\gamma_u} \left[\frac{\Delta \eta_u}{c_{v,u}} + (\gamma_u - 1) \frac{\Delta p_u}{p_u} \right] = \left(\frac{c_{p,b} T_b}{c_{p,u} T_u} - 1 \right) \frac{\Delta Q}{Q} \left[1 + O(M_u^2) \right] \quad (B5c)$$

3677 Consider the equations (B5) as applied to a ram-jet combustion chamber. Under steady operating conditions, the flame front is stationary (i.e., $v = 0$) and is located at the flameholder ($x = x_f$), which is assumed to be planar. If sinusoidal perturbations are assumed, and these are expressed in complex form, division of equations (B5b) and (B5c) by (B5a) gives

$$\frac{1}{\xi_{b,f}} - \frac{C_1}{\xi_{u,f}} + M_b \left[\frac{\gamma_b}{\gamma_u} \frac{\xi_{u,f}}{\xi_{u,f}} - \frac{\xi_{b,f}}{\xi_{b,f}} - \left(\frac{\gamma_b}{\gamma_u} - 1 \right) \right] + O(M_u^2) = 0 \quad (B6a)$$

$$C_2 \frac{\xi_{b,f}}{\xi_{b,f}} - \frac{\xi_{u,f}}{\xi_{u,f}} + C_3 - C_4 \frac{p_u}{Q} \frac{\Delta Q}{\Delta p_{u,f}} + O(M_u^2) = 0 \quad (B6b)$$

CO-4 where

$$C_1 = \frac{a_b}{a_u} \left[1 - \left(1 - \frac{u_u}{u_b} \right) \frac{\Delta v}{\Delta u_{u,f}} \right]$$

$$C_2 = \frac{c_{v,b} T_b}{c_{v,u} T_u}$$

$$C_3 = (\gamma_u - 1) \left(\frac{R_b T_b}{R_u T_u} - 1 \right)$$

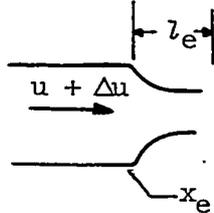
$$C_4 = \gamma_u \left(\frac{c_{p,b} T_b}{c_{p,u} T_u} - 1 \right)$$

The subscript f indicates that the quantity is to be evaluated at x_f . If the heat release per unit mass is assumed constant, $\Delta Q = 0$ in equation (B6b). The choice for the proper values of $\Delta v / \Delta u_{u,f}$ in equation (B6a) depends on the effectiveness of the flameholder in constraining the motion of the flame. For low-frequency oscillations, $\Delta v / \Delta u_{u,f}$ can probably be taken equal to zero (i.e., the flame is considered fixed at x_f). For higher frequencies, $\Delta v / \Delta u_{u,f}$ probably cannot be taken equal to zero (despite the presence of the flameholder) and, in fact, may be complex.

APPENDIX C

BOUNDARY CONDITIONS AT CHOKED EXIT

Assume that the exit section is operating quasi steadily ($\omega l_e/a \ll 1$) and is choked. Under these conditions, the Mach number at x_e (sketch (c)) is constant at all times, its value depending on the ratio of the



(c) Exit section.

tube cross-sectional area to the exit throat area. Thus $(\Delta M)_e = 0$ or $\left(\frac{\Delta u}{u}\right)_e = \left(\frac{\Delta a}{a}\right)_e$. Using $a^2 = \gamma p/\rho$ and equation (2) then yields

$$\left(\frac{\Delta u}{u}\right)_e = \left\{ \frac{1}{2\gamma} \left[(\gamma - 1) \frac{\Delta p}{p} + \frac{\Delta \eta}{c_v} \right] \right\}_e \quad (C1)$$

From the definitions of ζ and ξ , equation (C1) can be written as

$$\left[\zeta + \frac{1}{\gamma - 1} \xi - \frac{2}{M(\gamma - 1)} \right]_e = 0 \quad (C2)$$

which relates ζ and ξ at the exit. An expression similar to equation (C2) is derived in reference 9.

If $\omega l_e/a$ is not small, the appropriate boundary conditions at the exit must be determined by methods analogous to that used in reference 12.

APPENDIX D

SOLUTIONS OF EQUATIONS (11) AND (12) FOR ISENTROPIC FLOW

In the main body of the report, solutions of equations (11) and (12) are presented, which are applicable for ram-jet configurations satisfying equations (13). If it is assumed that the entropy waves in the combustion chamber are negligible (i.e., isentropic flow) then $\xi = 0$, and equations (11) and (12) can be solved without recourse to the restrictions of equations (13). The solution is presented herein.

If $\xi = 0$, equations (11) and (12) become, respectively,

$$e^{-2i \frac{kL}{a(1-M^2)}} = - \frac{1 + \zeta_i}{1 - \zeta_i} \frac{1 - \frac{\gamma-1}{2} M}{1 + \frac{\gamma-1}{2} M} \quad (D1a)$$

and

$$\zeta_i = \frac{\frac{\gamma-1}{2} M + i \cot\left(\frac{kL}{a(1-M^2)}\right)}{1 + i \frac{\gamma-1}{2} M \cot\left(\frac{kL}{a(1-M^2)}\right)} \quad (D1b)$$

Solving equations (D1) for k yields

$$\frac{(\omega + iA)L}{a(1-M^2)} = \frac{\pi}{2} \left[(1 + 2N) + \frac{1}{\pi} \tan^{-1} \frac{2\psi_i}{1 - \theta_i^2 - \psi_i^2} \right] + \frac{i}{4} \ln \left[\frac{(1 + \theta_i)^2 + \psi_i^2}{(1 - \theta_i)^2 + \psi_i^2} \left(\frac{1 - \frac{\gamma-1}{2} M}{1 + \frac{\gamma-1}{2} M} \right)^2 \right] \quad (D2)$$

The frequency of the standing waves is therefore

$$\frac{\omega L}{a} = \frac{\pi}{2} \left[(1 + 2N) + \frac{1}{\pi} \tan^{-1} \frac{2\psi_i}{1 - \theta_i^2 - \psi_i^2} \right] (1 - M^2) \quad (D3)$$

If ψ_i and θ_i are functions of ω , then equation (D3) must be considered as providing an implicit, rather than an explicit, solution for ω .

When $\psi_i = 0$, the frequencies are given by $\frac{\omega L}{a} = \frac{\pi}{2} (1 + 2N)(1 - M^2)$.

3677

CO-4 back

Increasing the Mach number tends to decrease the frequencies. The system will become unstable when

$$1 - \sqrt{1 - \left[\frac{(\gamma - 1)M}{1 + \left(\frac{\gamma - 1}{2} M\right)^2} \right]^2 (1 + \psi_i^2)} < \frac{(\gamma - 1)M}{1 + \left(\frac{\gamma - 1}{2} M\right)^2} \theta_i < 1 + \sqrt{1 - \left[\frac{(\gamma - 1)M}{1 + \left(\frac{\gamma - 1}{2} M\right)^2} \right]^2 (1 + \psi_i^2)} \quad (D4)$$

If $\psi_i = 0$ (but M is not necessarily small), equation (D4) becomes

$$\frac{\gamma - 1}{2} M < \theta_i < \frac{2}{\gamma - 1} \frac{1}{M} \quad (D5)$$

which agrees with previous results for quasi-steady operation. If $M \ll 1$ (but ψ_i is not necessarily small), equation (D4) becomes

$$\frac{\gamma - 1}{2} M(1 + \psi_i^2) \left[1 + O(M^2) \right] < \theta_i < \frac{2}{(\gamma - 1)M} \left[1 + O(M^2) \right] \quad (D6)$$

Equation (D6) indicates that a nonzero value of ψ_i will tend to increase the stability of a configuration, but that the effect is negligible when $\psi_i^2 \ll 1$. Note that equations (D5) and (D6) give an upper bound, as well as a lower bound, to the value of θ_i which lead to instability. In all cases, positive values of θ_i are required.

Equations (D1) to (D6) are valid provided that the exit section is short compared with the wavelength of the oscillations and provided that entropy waves are negligible. There is no restriction on the relative length of the combustion chamber as compared with the inlet.

APPENDIX E

SUPPLEMENTARY DISCUSSION OF RAM-JET STABILITY

Additional insight into the source of ram-jet instability can be gained by considering the ratio of the magnitudes of the reflected to incident waves at the inlet and exit of the combustion chamber. For convenience, the nonburning case is considered.

The pressure and velocity perturbations in the combustion chamber are related by

$$\left. \begin{aligned} \Delta p^+ &= \rho a \Delta u^+ \\ \Delta p^- &= -\rho a \Delta u^- \end{aligned} \right\} \quad (\text{E1})$$

where the superscripts (+) and (-) designate quantities associated with downstream and upstream traveling waves, respectively. The net perturbation at a point is given by

$$\left. \begin{aligned} \Delta p &= \Delta p^+ + \Delta p^- \\ \Delta u &= \Delta u^+ + \Delta u^- \end{aligned} \right\} \quad (\text{E2})$$

If sinusoidal waves are assumed, and the perturbations are expressed as the real part of complex quantities, then $\frac{\Delta p}{\rho a \Delta u} = \zeta$ and equations (E1) and (E2) give

$$\frac{\Delta p^+}{\Delta p^-} = \frac{\zeta + 1}{\zeta - 1} \quad (\text{E3})$$

The ratio of the magnitude of the downstream to the upstream wave at any point is

$$\left| \frac{\Delta p^+}{\Delta p^-} \right| = \left| \frac{\zeta + 1}{\zeta - 1} \right| \quad (\text{E4})$$

The ratio of the magnitude of the reflected to incident wave at the inlet is then

$$\left| \left(\frac{\Delta p^+}{\Delta p^-} \right)_i \right| = \left| \frac{\zeta_i + 1}{\zeta_i - 1} \right| \quad (\text{E5})$$

The ratio of the magnitude of the reflected to the incident wave at the exit is

$$\left| \left(\frac{\Delta p^-}{\Delta p^+} \right)_e \right| = \left| \frac{\zeta_e - 1}{\zeta_e + 1} \right| \quad (\text{E6})$$

The condition that the system be unstable is that successive reflections of a wave (at the inlet and exit) should result in an increase in the magnitude of the wave. That is, the condition for instability is given by

$$\left| \left(\frac{\Delta p^+}{\Delta p^-} \right)_i \left(\frac{\Delta p^-}{\Delta p^+} \right)_e \right| > 1 \quad (\text{E7})$$

or, from equations (E5) and (E6)

$$\left| \frac{\zeta_i + 1}{\zeta_i - 1} \frac{\zeta_e - 1}{\zeta_e + 1} \right| > 1 \quad (\text{E8})$$

Since $\zeta = \theta - i\psi$, equation (E8) can also be written as

$$\frac{(\theta_i + 1)^2 + \psi_i^2}{(\theta_i - 1)^2 + \psi_i^2} \frac{(\theta_e - 1)^2 + \psi_e^2}{(\theta_e + 1)^2 + \psi_e^2} > 1 \quad (\text{E9})$$

which agrees with the stability criteria indicated by equations (16) and (D2).

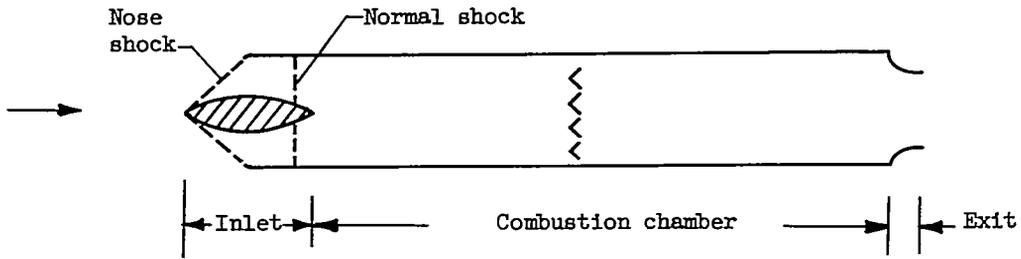
For a short choked exit, $\zeta_e = \frac{2}{(\gamma - 1)M}$ (neglecting entropy waves), so that

$$\left| \left(\frac{\Delta p^-}{\Delta p^+} \right)_e \right| = 1 - (\gamma - 1)M + o(M^2) \quad (\text{E10})$$

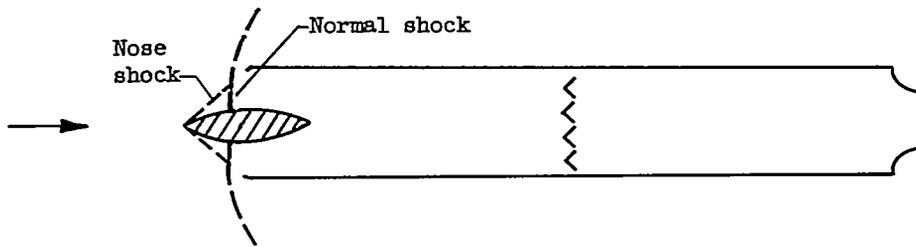
Thus, the reflected wave is weaker than the incident wave, and the exit tends to damp the wave system. The larger the value of M , the greater the damping effect. The system becomes unstable when the inlet amplifies incident waves by a greater amount than they are damped at the exit. The values of θ_i , for which this occurs, is given by equation (D4). As indicated in appendix D, θ_i must be positive for the system to be unstable. Thus, it is the inlet section that is responsible for the instability of ram-jet engines.

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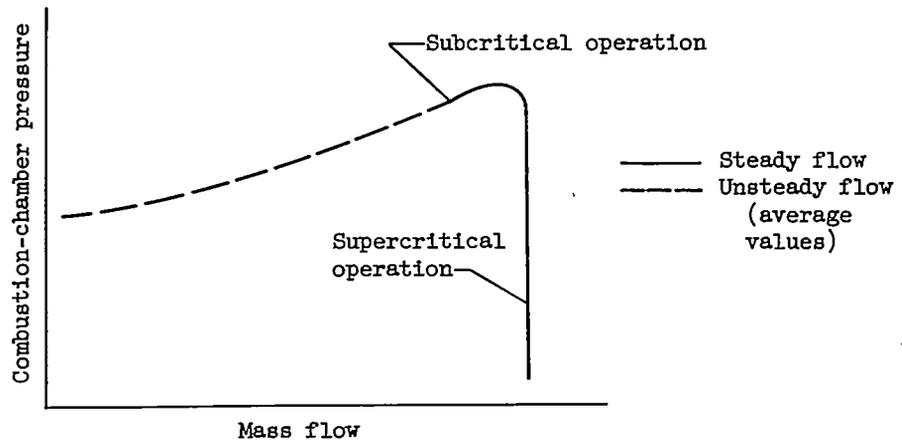
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(a) Supercritical operation.



(b) Subcritical operation.



(c) Typical performance curve.

Figure 1. - Ram-jet configuration and typical performance curve.

3677

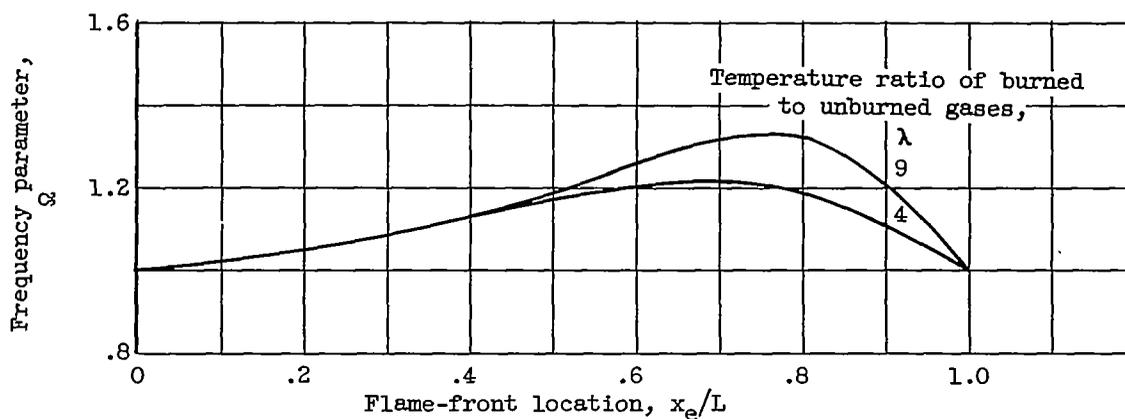


Figure 2. - Frequency parameter as function of flame-front location. (Eqs. (29a) and (31).)

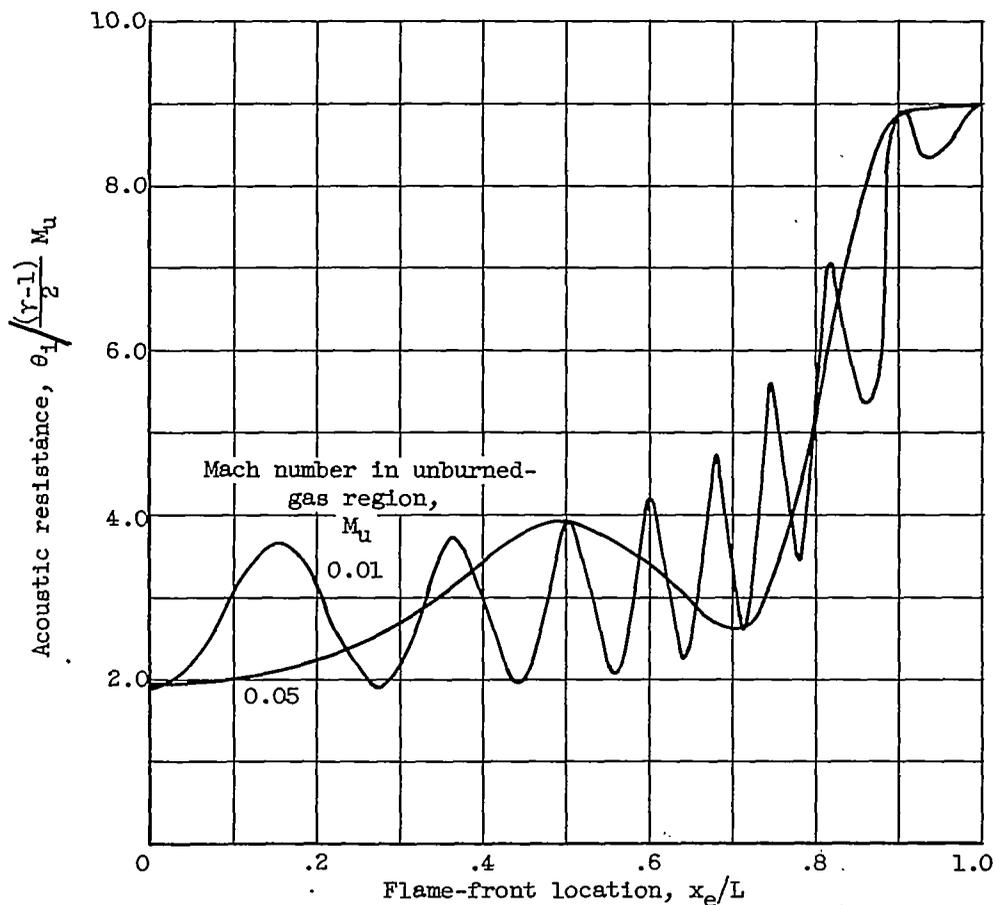


Figure 3. - Effect of planar flame front on acoustic resistance of inlet at point of neutral stability. Temperature ratio of burned to unburned gases, 9. (Eq. (29b).)