# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

## TECHNICAL NOTE 3597

# ANALYSIS OF A VANE-CONTROLLED GUST-ALLEVIATION SYSTEM

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## SUMMARY

A theoretical study using an electronic analog computer has been made of the response to step gusts of an airplane equipped with a system in which modified wing flaps and a modified elevator are operated to reduce accelerations in rough air. These surfaces are actuated by an automatic control system in response to the indications of an angle-of-attack vane. The effect of interconnection of the flap operating mechanism with the pilot's control system is also included in the study.

The analysis indicates that modified trailing-edge wing flaps and a coupled elevator operated by a vane-controlled servo system are effective in reducing accelerations due to step gusts when the flap-elevator system maintains constant lift and zero pitching moment about the wing aerodynamic center. The device would not completely alleviate the accelerations due to flight through rough air, however, because of the lag in the response of the servomechanism. Desirable dynamic stability and control characteristics of this arrangement may be obtained without greatly altering the acceleration-alleviation characteristics of the system by providing a small but reasonable static margin for the airplane. Interconnection of the flap operating mechanism and the pilot's elevator control appears to allow adequate longitudinal control of the airplane. In addition, the pilot's control motion results in a more rapid change in normal acceleration than is obtained by elevator control alone.

## INTRODUCTION

A theoretical analysis (based on frequency-response methods) of various means of increasing the smoothness of flight through rough air is presented in reference 1. One of the most promising of the methods analyzed utilized an automatic control system in which the flaps and elevators were operated in accordance with the indications of an angle-of-attack vane to maintain approximately constant lift and zero pitching moment on the airplane.

A vane-controlled gust-alleviation system that operates in accordance with the findings of reference 1 has been incorporated in a twin-engine light transport airplane. Because this modified airplane and control system are not sufficiently similar to the type previously analyzed (ref. 1), it was desired to make a more refined analysis applicable to this particular case. The refinements included: consideration of the characteristics of the servomechanism being used; simulation of the true time lag between the action of gusts hitting the vane, wing, and tail; and inclusion of a device to return the flaps slowly to the neutral position in case of steady angle-of-attack changes due to changes in airspeed or weight.

The airplane chosen was flight tested to measure some of the stability derivatives, and the flaps were modified in an attempt to obtain the desired pitching-moment and downwash characteristics. In addition, the servo system designed to operate the gust-alleviation controls was tested and it was found that the servomechanism could be described by a first-degree second-order differential equation.

The analysis in the present paper includes a study of the transient response of the airplane for both gust disturbances and longitudinal-control inputs. The aerodynamic characteristics necessary for optimum gust alleviation are derived and the response of this system is compared with that of the basic airplane. Some of the pertinent aerodynamic parameters affecting the airplane response are investigated; these include static margin, flap lift and pitching-moment characteristics, and variations in downwash obtained by different flap configurations. The time constant describing the canceling network used to force the flaps to return to neutral is varied to determine the effects of such a system on the transient response of the airplane. The effects of variations in the natural frequency of the servomechanism used to operate the flap controls are also investigated. The solutions of the derived equations of motion were performed by an electronic analog computer and the disturbances considered were step inputs in gust angle of attack (uniform across the span of the airplane) or pilot's elevator-control motions.

## SYMBOLS

$C_m$ pitching-moment coefficient, $\dfrac{M}{\frac{1}{2}\rho V^2 S\bar{c}}$

$C_Z$ vertical-force coefficient, $\dfrac{F_Z}{\frac{1}{2}\rho V^2 S}$

$\bar{c}$ mean aerodynamic chord of wing

| | |
|---|---|
| D | differential operator, $d/ds$ |
| e | base of natural logarithms |
| E | servo-system input signal |
| $F_Z$ | vertical force (positive downward) |
| f | frequency |
| g | acceleration due to gravity |
| $K_Y$ | nondimensional radius-of-gyration factor, $k_y/\bar{c}$ |
| $K_c$ | ratio between main-elevator deflection and longitudinal-control deflection |
| $K_{cw}$ | gain constant of signal proportional to integral of flap deflection |
| $K_e$ | ratio between vane deflection and main-elevator deflection for basic airplane at a steady-state condition |
| $k_y$ | radius of gyration of airplane about Y-axis |
| $K_1$ | ratio between main-flap deflection and quantity measured by gust detector with elevator fixed |
| $K_2$ | ratio between auxiliary-flap deflection and main-flap deflection |
| $K_3$ | ratio between auxiliary-elevator deflection and main-flap deflection |
| $l$ | ratio of distance between center of gravity and tail to mean aerodynamic chord of wing |
| $l_n$ | ratio of distance between center of gravity and angle-of-attack vane to mean aerodynamic chord of wing |
| M | pitching moment (positive upward) |
| m | mass of airplane |
| $N_{Fr}$ | Froude number, $g\bar{c}/V^2$ |
| n | normal acceleration, g units |

| | |
|---|---|
| $S$ | wing area |
| $s$ | distance measured in chords, $tV/\bar{c}$ |
| $t$ | time |
| $V$ | velocity of center of gravity of airplane with respect to still air |
| $w$ | velocity along Z-axis |
| $w_g$ | vertical velocity of gust (positive upward) |
| X,Y,Z | airplane body axes |
| $\alpha_g$ | angle of attack due to gust, $w_g/V$ |
| $\alpha_o$ | angle of attack between X-axis and velocity vector $V$ |
| $\alpha_t$ | angle of attack of tail |
| $\alpha_w$ | angle of attack of wing |
| $\delta_c$ | longitudinal-control deflection |
| $\delta_e$ | main-elevator deflection |
| $\delta_f$ | flap deflection |
| $\delta_v$ | deflection of angle-of-attack vane |
| $\epsilon$ | downwash at tail or error signal |
| $\zeta$ | fraction of critical damping of servo system |
| $\theta$ | angle of pitch |
| $\rho$ | air density |
| $\mu$ | relative-density factor, $m/\rho S\bar{c}$ |
| $\omega_n$ | nondimensional natural frequency of servo system |

Subscripts:

| | |
|---|---|
| e | elevator |
| $e_a$ | auxiliary elevator |
| f | flap system |
| $f_a$ | auxiliary flap |
| $f_m$ | main flap |
| t | tail |
| $t_0$ | time for gust to reach vane |
| $t_1$ | time for gust to reach wing |
| $t_2$ | time for gust to reach tail |
| tot | total for airplane |
| v | angle-of-attack vane |
| w | wing |

A dot over a quantity indicates differentiation with respect to time.

Stability derivatives are indicated by subscript notation; for example, $C_{Z_\alpha} = \frac{\partial C_Z}{\partial \alpha}$. Subscript following a stability derivative indicates component of airplane which contributes derivative; for example,

| | |
|---|---|
| $\left(C_{m_{\delta_f}}\right)_w$ | variation of pitching-moment coefficient with flap deflection contributed by wing |
| $\left(C_{Z_{\delta_e}}\right)_t$ | variation of vertical-force coefficient with elevator deflection contributed by tail |

All distances are absolute values.

## PHYSICAL DESCRIPTION OF SYSTEM

The twin-engine light transport airplane considered in this analysis (fig. 1) is equipped with an angle-of-attack vane, a servo system, and modified flaps and elevators. The angle-of-attack vane is located on a boom attached to the nose of the airplane so that it can sense and measure a gust in sufficient time to cause the servo system to operate the flaps and elevators and thus reduce the response of the airplane to the gust. The control surfaces used for gust alleviation consist of the main flap which utilizes the aileron for increased lift, an auxiliary flap, and an auxiliary elevator. These controls are capable of moving in both an up and a down direction and have proportional deflections when actuated by the servo system. The purpose of the main flap is to change the lift of the wing so that, despite changes in angle of attack due to gusts, the deflection of the main flaps holds the total lift constant. In order to overcome the pitching moment due to flap deflection, the auxiliary elevator (a portion of the original elevator) is geared directly to the main flaps. Satisfactory gust alleviation requires consideration of pitching resulting from changes of lift at the tail due to gusts. The angle of attack at the tail due to gust input can be compensated for by the change in downwash from the wing. The auxiliary flaps are also geared to the main flaps and are used to control the downwash from the wing in the vicinity of the tail (refs. 2 and 3). In this method, the time lags due to gust input and downwash are the same and there is no necessity to use a precise method incorporating time lag to deflect the control surface. However, use of elevator deflection to compensate for change in lift on the tail due to gusts would require consideration of time lag.

A block diagram of the vane-controlled gust-alleviation system is illustrated in figure 2. In this diagram the gust measurement of the vane is transmitted to an amplifier as an input signal. This signal is fed through the amplifier to the servo which operates the gust-alleviation controls in the airplane. If the airplane with the gust-alleviation system has some positive stability, then, as the airplane enters the gust, it will pitch into the gust until the trim angle of attack is reached and the controls are driven back to the neutral position. If the airplane with the gust-alleviation system has neutral stability, then the airplane will have no tendency to pitch into the gusts. The flaps must then be returned to neutral by some method other than the pitching of the airplane. In addition, a change in airspeed or airplane weight will cause a variation in trim angle of attack, and it would be desirable for the flaps to return to neutral under these conditions. In the system investigated, this return of the flaps is accomplished by means of the integrating network used as a canceling circuit and shown in figure 2. The integral of the flap position is fed back to the servo input in such a manner that it cancels the error signal from the vane and returns the gust-alleviation controls to neutral at a rate described by a predetermined time constant. This control motion causes the airplane to pitch, but the time constant can be chosen so that the rate of pitch is small and easily corrected by the pilot's control motion.

With the gust-alleviation system (as just described) in operation, any change in angle of attack of the airplane would be picked up by the vane. Normally, control of the airplane by the elevator is accomplished by changing the angle of attack. If the lift increment due to changing the angle of attack is eliminated by the vane-controlled flaps, the elevator will be ineffective in changing the direction of the flight path. This problem of longitudinal control can be overcome by adding to the signal from the vane a signal that is proportional to the pilot's control deflection. (See fig. 2.) This signal from the pilot's control may be made equal and opposite to the signal from the vane when the airplane reaches the equilibrium angle of attack in a steady pull-up for the particular elevator position. In effect, the signal from the pilot's control introduces a simulated gust signal which operates the flap system in conjunction with the elevator control motion. The gust-alleviation controls thus give an immediate change in lift which may be made equal to the steady-state lift for the elevator deflection.

## EQUATIONS OF MOTION

The development of the equations of longitudinal motion is similar to that given in reference 1. The equations are set up with respect to body axes. The airplane weight and steady-state lift force required to balance the weight are omitted from the equations because only changes from the trim condition are considered and each of the variables is defined as the change in this quantity from steady flight. No unsteady lift conditions have been considered for the operation of the control surfaces or the changes in airplane lift due to gusts. Variations of forward velocity are also neglected.

The nondimensional equations of motion which separately express the summation of the forces and moments contributed by the wing-fuselage combination, the tail, the flap system, and the elevator as given in reference 1 are

$$2\mu\, D\left(\frac{w}{V} - \theta\right) = \alpha_w\left(C_{Z_\alpha}\right)_w + \alpha_t\left(C_{Z_\alpha}\right)_t + \delta_f\left(C_{Z_{\delta_f}}\right)_w + \delta_e\left(C_{Z_{\delta_e}}\right)_t \tag{1a}$$

$$2\mu K_Y^2 D^2\theta = \alpha_w\left(C_{m_\alpha}\right)_w + \alpha_t\left(C_{m_\alpha}\right)_t + \delta_f\left(C_{m_{\delta_f}}\right)_w + \delta_e\left(C_{m_{\delta_e}}\right)_t \tag{1b}$$

The angles of attack of the wing and the tail are given in reference 1 as

$$\alpha_w = \alpha_o + \alpha_g$$

and

$$\alpha_t = \alpha_o - \left(\alpha_o \frac{\partial\epsilon}{\partial\alpha}\right)_{s=-l} + \left(\alpha_g\right)_{s=-l} - \left(\alpha_g \frac{\partial\epsilon}{\partial\alpha}\right)_{s=-l} - \left(\delta_f \frac{\partial\epsilon}{\partial\delta_f}\right)_{s=-l} + l\ D\theta$$

The lift and pitching-moment coefficients and the downwash derivative of the flap are defined (in this report) as

$$\left.\begin{aligned}
\left(C_{Z_{\delta_f}}\right)_w &= \left(C_{Z_\delta}\right)_{f_m} + K_2\left(C_{Z_\delta}\right)_{f_a} + K_3\left(C_{Z_\delta}\right)_{e_a} \\
\left(C_{m_{\delta_f}}\right)_w &= \left(C_{m_\delta}\right)_{f_m} + K_2\left(C_{m_\delta}\right)_{f_a} + K_3\left(C_{m_\delta}\right)_{e_a} \\
\frac{\partial\epsilon}{\partial\delta_f} &= \left(\frac{\partial\epsilon}{\partial\delta_f}\right)_{f_m} + K_2\left(\frac{\partial\epsilon}{\partial\delta_f}\right)_{f_a}
\end{aligned}\right\} \qquad (2)$$

The equations for the angle of attack at the wing and tail can be rewritten to express the gust at the time it reaches the wing as $\alpha_{g,t_1}$ and at the time it reaches the tail as $\alpha_{g,t_2}$ and to approximate the time lag in the downwash from the wing at the time it reaches the tail by multiplying the downwash terms by the operator $\frac{1}{1 + l\ D}$. The exact time lag $l$ in the occurrence of the downwash is expressed by the operator $e^{-l\ D}$. The operator $\frac{1}{1 + l\ D}$ was used to approximate the lag in downwash on the computer because it was more convenient than the more common expression $1 - l\ D$ and gave a reasonably good approximation to the phase and amplitude variations of the downwash at the tail. Then,

$$\left.\begin{aligned}
\alpha_w &= \alpha_o + \alpha_{g,t_1} \\
\alpha_t &= \alpha_o - \alpha_o \frac{\partial\epsilon}{\partial\alpha}\frac{1}{1 + l\ D} + \alpha_{g,t_2} - \alpha_{g,t_1}\frac{\partial\epsilon}{\partial\alpha}\frac{1}{1 + l\ D} - \\
&\quad \delta_f \frac{\partial\epsilon}{\partial\delta_f}\frac{1}{1 + l\ D} + l\ D\theta
\end{aligned}\right\} \qquad (3)$$

The equations of motion of the airplane configuration can then be found by combining equations (1a) and (1b) with equations (3). The resulting equations are

$$D\theta \left\{ -Dl\left[2\mu + l\left(C_{Z_\alpha}\right)_t\right] - \left[2\mu + l\left(C_{Z_\alpha}\right)_t\right] \right\} + \alpha_o \left\{ 2\mu l\, D^2 + D\left[2\mu - l\left(C_{Z_\alpha}\right)_w - l\left(C_{Z_\alpha}\right)_t\right] - \left[\left(C_{Z_\alpha}\right)_w + \left(C_{Z_\alpha}\right)_t - \frac{\partial\epsilon}{\partial\alpha}\left(C_{Z_\alpha}\right)_t\right] \right\} =$$

$$\delta_f \left\{ Dl\left(C_{Z_{\delta_f}}\right)_w + \left[\left(C_{Z_{\delta_f}}\right)_w - \frac{\partial\epsilon}{\partial\delta_f}\left(C_{Z_\alpha}\right)_t\right] \right\} + \delta_e\left[Dl\left(C_{Z_\delta}\right)_e + \left(C_{Z_\delta}\right)_e\right] +$$

$$\alpha_{g,t_1} \left\{ Dl\left(C_{Z_\alpha}\right)_w + \left[\left(C_{Z_\alpha}\right)_w - \frac{\partial\epsilon}{\partial\alpha}\left(C_{Z_\alpha}\right)_t\right] \right\} + \alpha_{g,t_2}\left[Dl\left(C_{Z_\alpha}\right)_t + \left(C_{Z_\alpha}\right)_t\right] \tag{4a}$$

and

$$D\theta \left\{ 2\mu K_Y^2 l\, D^2 + D\left[2\mu K_Y^2 - l^2\left(C_{m_\alpha}\right)_t\right] - l\left(C_{m_\alpha}\right)_t \right\} + \alpha_o \left\{ -Dl\left[\left(C_{m_\alpha}\right)_w + \left(C_{m_\alpha}\right)_t\right] - \left[\left(C_{m_\alpha}\right)_w + \left(C_{m_\alpha}\right)_t - \frac{\partial\epsilon}{\partial\alpha}\left(C_{m_\alpha}\right)_t\right] \right\} = \delta_f \left\{ Dl\left(C_{m_{\delta_f}}\right)_w +\right.$$

$$\left.\left[\left(C_{m_{\delta_f}}\right)_w - \frac{\partial\epsilon}{\partial\delta_f}\left(C_{m_\alpha}\right)_t\right] \right\} + \delta_e\left[Dl\left(C_{m_\delta}\right)_e + \left(C_{m_\delta}\right)_e\right] +$$

$$\alpha_{g,t_1} \left\{ Dl\left(C_{m_\alpha}\right)_w + \left[\left(C_{m_\alpha}\right)_w - \frac{\partial\epsilon}{\partial\alpha}\left(C_{m_\alpha}\right)_t\right] \right\} + \alpha_{g,t_2}\left[Dl\left(C_{m_\alpha}\right)_t + \left(C_{m_\alpha}\right)_t\right] \tag{4b}$$

A block diagram of the vane-controlled gust-alleviation system is shown in figure 2. The expression for the vane deflection in nondimensional notation is

$$\delta_v = \alpha_o + \alpha_{g,t_0} - D\theta\, l_n \tag{5}$$

where $\alpha_{g,t_0}$ is the expression for the gust at the time it reaches the vane.

The input from the longitudinal controller which allows the pilot to trim the system so that the airplane can fly at various angles of attack is proportional to the elevator deflection and is added to the signal from the vane. The error signal to the amplifier is

$$\epsilon = \delta_v - K_e\delta_e \tag{6}$$

In order to assure that the gust-alleviation controls eventually return to zero position for changes in airspeed or weight and after the gust disturbance, a signal proportional to the negative integral of the flap deflection is added to the signal from the amplifier. The input of the servosystem is then

$$E = K_1\epsilon - K_{cw}\frac{\delta_f}{D} \tag{7}$$

It was found from laboratory bench tests that the servomechanism could be approximated by a second-order single-degree-of-freedom system so that the flap motion could be represented by the linear differential equation

$$\left(D^2 + 2\zeta\omega_n D + \omega_n^2\right)\delta_f = \omega_n^2 E \tag{8}$$

where $\omega_n$ is a circular frequency expressed in radians per chord traveled and is defined by the formula

$$\omega_n = 2\pi f \frac{\bar{c}}{V} \tag{9}$$

where $f$ is the frequency in cycles per second. By combining equations (5), (6), (7), and (8), the equation of flap motion becomes

$$D\theta\left(K_1 l_n\omega_n^2\right) + \alpha_o\left(-K_1\omega_n^2\right) + \delta_f\left(D^2 + 2\zeta\omega_n D + \omega_n^2 + \omega_n^2\frac{K_{cw}}{D}\right) =$$

$$\alpha_{g,t_0}\left(K_1\omega_n^2\right) + \delta_e\left(-K_1K_e\omega_n^2\right) \tag{10}$$

Equations (4a), (4b), and (10) are the equations of motion for the airplane gust-alleviation system with both a gust input and an elevator input. The responses of $\alpha_o$, $D\theta$, and $\delta_f$ may be found for either a specified $\delta_e$ or $\alpha_g$, and the normal acceleration may be obtained from the equation

$$n = \frac{D\left(\alpha_o - \theta\right)}{N_{Fr}} \tag{11}$$

where the Froude number $N_{Fr} = \frac{g\bar{c}}{V^2}$. (See ref. 1.)

## REQUIREMENTS FOR CONTROLLABILITY

In the simplified theory presented in reference 1, certain relationships between the aerodynamic parameters and the gearing constants of the gust-alleviation system were shown to produce optimum gust alleviation. These values were used as a logical starting point for the range of parameters covered in the present study. The values of the parameters for optimum gust alleviation are those required for zero change in vertical force and pitching moment due to a gust. These values can be determined by equating the vertical-force and pitching-moment coefficients to zero as in the following relations:

$$\left.\begin{aligned}\left(C_{Z_\alpha}\right)_{tot} &= \left(C_{Z_\alpha}\right)_w + \left(C_{Z_\alpha}\right)_t\Delta\alpha_t + K_1\left(C_{Z_{\delta_f}}\right)_w = 0\\ \left(C_{m_\alpha}\right)_{tot} &= \left(C_{m_\alpha}\right)_w + \left(C_{m_\alpha}\right)_t\Delta\alpha_t + K_1\left(C_{m_{\delta_f}}\right)_w = 0\end{aligned}\right\} \tag{12}$$

where

$$\Delta\alpha_t = \left(1 - \frac{\partial\epsilon}{\partial\alpha} - K_1\frac{\partial\epsilon}{\partial\delta_f}\right) = 0$$

and $K_1$ is the steady-state ratio of flap deflection to vane deflection. The lift and pitching-moment coefficients and downwash derivatives of the flap system are defined in equations (2). It is obvious that if both $\left(C_{Z_\alpha}\right)_{tot}$ and $\left(C_{m_\alpha}\right)_{tot}$ are zero, perfect gust alleviation can be achieved because changes in angle of attack will have no effect on the airplane. In order to accomplish this condition, it is necessary, as shown in equations (12), that there be no change in angle of attack at the tail. As noted previously, the change in angle of attack at the tail due to the gust is compensated for by the change in downwash produced by the auxiliary flaps. In practice, however, it may not be possible to achieve a sufficient change of downwash with the auxiliary flaps to accomplish this requirement. In such a case, the gearing between the main and auxiliary flaps should be selected to keep the change in angle of attack at the tail to a minimum.

Although the conditions as shown by equations (12) give optimum gust alleviation, it is obvious that the airplane will have neutral static stability. The deficiencies in flying qualities that result from neutral stability are well-known. A compromise could be made, however, by allowing the airplane to have a small static margin when $\left(C_{Z_\alpha}\right)_{tot} = 0$. This condition would be expected to give satisfactory gust alleviation while retaining positive stability of the airplane gust-alleviation system. Both the neutral-stability and small positive-stability cases have been investigated and the results are presented subsequently.

If the pilot is to be able to control the airplane, it is necessary to couple his control to the flap as is defined by equation (6). The error signal $\epsilon$ must equal zero in a steady pull-up if the flaps are to return to neutral. Thus,

$$\delta_v - K_e\delta_e = 0$$

If gust inputs to the vane are neglected,

$$K_e\delta_e = \alpha_o - D\theta\ l_n$$

Dividing by $\delta_e$ gives

$$K_e = \frac{\alpha_o}{\delta_e} - \frac{D\theta}{\delta_e} l_n \qquad (13)$$

where $\frac{\alpha_o}{\delta_e}$ and $\frac{D\theta}{\delta_e}$ are the respective steady-state angle of attack and pitching velocity for a unit elevator deflection in a steady pull-up where the gust-alleviation system is inoperative.

The constant of the integrating circuit $K_{cw}$ is defined as a time constant which regulates the rate of decay of the flap deflection after a step vane or elevator input is applied. The significance of this constant may be shown by assuming that a perfect servo operates the flaps so that $\delta_f = E$. Then, by considering only a vane input, from equations (6) and (7),

$$\delta_f = -K_1\delta_v - K_{cw}\frac{\delta_f}{D}$$

Solving this equation for the response of flap deflection to a step vane deflection gives

$$\frac{\delta_f}{\delta_v} = -K_1 e^{-K_{cw}s}$$

where $s$, the time in chords, equals $t\frac{V}{\bar{c}}$. The physical significance of this equation may be shown when $s = \frac{3}{K_{cw}}$; then, $\delta_f = \frac{1}{e^3}K_1\delta_v$ or the flap deflection has decayed to 5 percent of $K_1\delta_v$ after a time

$$t = \frac{3}{K_{cw}}\frac{\bar{c}}{V} \tag{14}$$

This simplified analysis of $K_{cw}$ does not consider feedback of angle of attack and pitching-velocity change to the vane. The integrating-circuit constant $K_{cw}$ should be chosen so that the servo system returns the flap system to neutral at a rate that will allow the airplane motion to be corrected by the pilot's control motion.

## RESULTS AND DISCUSSION

The physical characteristics of the airplane for which the response calculations have been made are given in table I. These values are representative of a light transport airplane. The calculations were made for an airspeed of 150 miles per hour at standard sea-level conditions.

Airplane Response to Gust

The responses of the airplane to the step-gust disturbance, for the 10 cases defined in table II, are shown in figures 3 to 15.

Response of basic airplane.- The effect of a step gust on the airplane when the gust-alleviation system is inoperative is given in figure 3. The effect of the gust on the wing and the tail of the airplane can be seen at 0 and 0.10 second, respectively. During the period when the gust acts on the wing alone, the positive pitching moment of the wing about the airplane center of gravity causes an adverse pitching velocity. The slight reduction of normal acceleration during this period is due to the vertical motion of the airplane as it reacts to the gust. The resulting decrease in angle of attack due to the upward velocity of the airplane is greater than the increase in angle of attack due to the positive pitching moment of the gust acting on the wing. This fact is illustrated by the curve of $\alpha_o/\alpha_g$ which for this case is always negative despite the positive value of $\dot{\theta}/\alpha_g$. After 0.10 second, the gust and the changing downwash from the wing act on the tail to give a negative pitching moment which causes the airplane to pitch into the gust and return to the equilibrium angle of attack.

Optimum gust alleviation.- When the gust-alleviation system is designed to meet the requirements of $(C_{Z_\alpha})_{tot} = 0$, $(C_{m_\alpha})_{tot} = 0$, and $\Delta\alpha_t = 0$ given in the preceding theory (eq. (12)), the airplane gust-alleviation system has the response shown in figure 4. The sudden deflection of the flaps which occurs between -0.07 and 0 second produces a sharp increase in normal force equal and opposite to the normal force that was produced on the basic airplane. Then at 0 second the gust reaches the wing and the normal force is reduced to approximately 10 percent of the normal force attained in case 1 (basic airplane) in addition to reducing the time the acceleration is applied by approximately two-thirds. Although the gust-alleviation system was not as effective in the reduction of pitching velocity, the results were still satisfactory compared with the basic airplane because the maximum pitching velocity was reduced by approximately one-fourth and the period of application by approximately one-third.

"Perfect" gust alleviation, which would be represented by straight lines on both normal-acceleration and pitching-velocity curves, because $(C_{Z_\alpha})_{tot}$ and $(C_{m_\alpha})_{tot}$ for the airplane would always equal zero, is impossible to achieve with the finite time interval necessary for the servo system to deflect the flap system. The effects of the flaps deflecting in anticipation of the gust may be seen in both the normal-acceleration and pitching-velocity curves. The direct effect of this anticipation is found in the period from -0.07 to 0 second where the flaps change the lift and pitching moment to oppose those of the anticipated gust when it reaches the wing. The secondary effect occurs between 0.03 and 0.10 second as the downwash from the flaps anticipates the angle-of-attack change due to the gust at the tail in the same manner as the lift from the flaps anticipates the lift due to the gust acting on the wing.

It is obvious that, by decreasing the finite time necessary for the flap system to deflect, the necessary anticipation time of the gust is decreased and the flap system can more closely compensate for the gust. At the same time it should be noted that the high peak in normal force in figure 4 is a direct result of the step-type input used. It is highly improbable that such a sharp-edge gust would be encountered in the atmosphere. If a ramp type of input were considered which would more closely approximate gusts in the atmosphere, the "spikes" in the normal-force curves would become less significant. The rather large initial variations in acceleration for the gust-alleviation system indicates a high-frequency response which might excite structural-oscillation modes of an airplane to a greater extent than the jump in normal acceleration experienced by the basic airplane on encountering a step gust. In order to determine the frequency response of the basic and gust-alleviated airplanes, a harmonic analysis of the transient response to a step gust was made with a Coradi harmonic analyzer (Dent-Draper model). The results of this analysis (see fig. 5) indicate that the gust-alleviated airplane has considerably less response than the basic airplane for gust frequencies up to about 3 cps. However, for gust frequencies above 3 cps, the gust-alleviated airplane has a greater response than the basic airplane. In an effort to determine the significance of the frequency-response data obtained, the square of the amplitude ratio of the data obtained from the harmonic analysis was multiplied by the power spectrum of turbulence in the atmosphere. The gust spectrum was approximated by a function that varied inversely as the square of the frequency. Such a variation was indicated by the results presented in reference 4. The power spectra of normal acceleration thus obtained for the basic and gust-alleviated airplanes are presented in figure 6. The reduction in normal acceleration afforded by the gust-alleviation system in the low frequency range is emphasized by these data. In addition, figure 6 indicates that the increased response in normal acceleration of the gust-alleviated airplane to gust frequencies greater than 3 cps has little significance since the spectra of both the basic and gust-alleviated airplanes contain almost negligible power at the higher gust frequencies. From the standpoint of normal acceleration, then, it appears as though the gust-alleviation system would have no more tendency to excite structural oscillations than would the normal-acceleration response of the basic airplane. It should be noted that the analysis made in the present paper considers the airplane to be a rigid structure. If the flexibility of the structure were considered, it might be found desirable to use a servomechanism with a frequency response which would be highly attenuated in the frequency range occurring at the natural modes of the structure. Consideration of variations in gust velocity across the span which were previously noted in reference 1 may also dictate the frequency-response characteristics of the gust-alleviation system.

Effect of canceling network.- The effect of using the signal $K_{cw}\frac{\delta_f}{D}$ to return the flaps to neutral may be seen by comparing figures 7 and 8 with figure 4. During the initial part of the transient response there is no perceptible difference in any of the curves presented, but after approximately three-fourths second in figure 7 $\left(K_{cw} = 0.001\right)$ and after approximately one-fourth second in figure 8 $\left(K_{cw} = 0.01\right)$, the canceling signal becomes strong enough to effect the return toward neutral of the flap system and thus force the airplane to pitch into the gust. A low-frequency oscillation, the period of which decreases with increasing values of $K_{cw}$, occurs as a result of the integrating-circuit signal. The maximum value of $K_{cw}$ should therefore be limited to the point where this oscillation is slow enough to be readily controllable by the pilot.

It should be noted that the equations of motion used in the analysis neglect variations in forward speed so that the results for the longer period oscillations resulting from the canceling network may be in error. Nevertheless, this type of procedure can still be used to obtain a good approximation of the proper value of $K_{cw}$.

Effect of positive stability.- It does not appear practical to maintain neutral stability for the airplane gust-alleviation system as used in the optimum case because any small errors in gearing constants or stability derivatives may cause the airplane gust-alleviation system to become unstable. A small negative value of $\left(C_{m_\alpha}\right)_{tot}$ would provide a margin of safety for uncertain factors in the system. Such a value of $\left(C_{m_\alpha}\right)_{tot}$ would also force the airplane to pitch into the gust so that the angle-of-attack feedback would cause the flap system to return to neutral as the airplane approached its equilibrium angle of attack.

The effects of positive stability may be seen in figures 9 and 10. There is little change in the normal-acceleration curves from that of the optimum gust-alleviation case (fig. 4). The pitching-velocity curves, however, are changed so that smaller positive and greater negative pitching velocities are attained as a result of the negative value of $\left(C_{m_\alpha}\right)_{tot}$. Figure 10, which has the greatest change in the pitching-velocity curve, has the positive peak reduced by approximately 50 percent, whereas the increase in the range of pitching velocity is less than 5 percent.

The use of a negative value of $\left(C_{m_\alpha}\right)_{tot}$ appears to be a more desirable method of bringing the flaps back to neutral than is the integrating circuit, since the long-period oscillation is avoided. The integrating circuit, however, is still required to take care of variations in angle of attack due to speed and weight changes.

Effect of downwash at tail.- The downwash due to the flap deflection is used to counteract the effects of the gust at the tail of the airplane. This means of compensation may be accomplished because the time lag for the downwash from the wings and flaps to reach the tail is the same as the time required for the gust to travel from the wing to the tail of the airplane. During the initial part of the transient response, exact correction of the gust at the tail is not achieved, as was seen for case 2 (fig. 4), because the downwash at the tail due to flap deflection leads the gust at the tail in the same way the lift due to flap deflection leads the gust. Figures 10, 11, and 12 show the effects of varying the downwash while retaining a constant steady-state value of $\left(C_{m_\alpha}\right)_{tot}$ of -0.28 for the airplane gust-alleviation system. The normal-acceleration curves are essentially the same in all three cases, whereas the pitching-velocity curves differ markedly. Case 7 (fig. 11), a condition with excessive reversed downwash due to flap deflection at the tail, has an increase of about 60 percent in its peak pitching velocity over that of case 6 (fig. 10).

The effect of the downwash due to flap deflection leading the gust at the tail by 0.03 to 0.10 second can be easily seen in the pitching-velocity curve for case 7 (fig. 11). In this figure the sum of the pitching moments due to flap deflection and the gust acting on the wing is negative and the airplane has an increasing negative pitching velocity. At 0.03 second the downwash from the flaps reaches the tail and the resultant positive pitching moment causes the airplane pitching velocity to decrease until 0.10 second when the combined change in downwash from the wing and the change in angle of attack due to the gust reaching the tail restore the negative pitching moment to the airplane and the negative pitching velocity again increases. In the cases presented earlier, the downwash due to flap deflection is small and the resultant change in the slope of the pitching-velocity curve is negligible compared with the change in sign of the slope of the pitching-velocity curve found in figure 11.

Figure 12 illustrates an extreme case in which the downwash is not much more favorable than would be obtained by using a continuous full-span flap for gust-alleviation purposes. The higher pitching velocity for this case (approximately 70 percent greater than that of case 6) may be attributed to the large pitching moment due to the auxiliary-elevator deflection necessary to compensate for the combined pitching moment of the downwash due to flap deflection and the gust at the tail since, in this case, they act in unison rather than compensating for each other as in the first six cases.

Effect of servomechanism natural frequency.- The time it takes the flaps to deflect to correct for a gust is controlled by the natural frequency and damping of the servo system. This lag in the system is compensated for, at any one design speed, by the distance the vane is mounted ahead of the airplane wing. The effects of changing the servomechanism natural frequency for a fixed vane location and a fixed value of damping may be seen by comparing figures 10, 13, and 14 where the servomechanism natural frequencies are 11, 16.55, and 3.5 cps, respectively. The difference between the normal-acceleration and pitching-velocity curves of figures 10 and 13 is negligible. The only noticeable difference is that the initial slope of the normal-acceleration curve is greater in figure 13. Figure 14, however, is considerably different from figure 10. In figure 14, the servomechanism natural frequency is sufficiently low so that the flap deflects approximately one-half the required distance before the gust reaches the wing. This deflection allows the initial normal-acceleration pulse to be "split" so that the initial part of the pulse is due to the flap deflection, whereas the second part is due to the gust hitting the wing and the effects of the gust being reduced by the continued deflection of the flap system. The actual reduction of the peak normal acceleration in figure 14 as compared with figure 10 is approximately 50 percent. The maximum pitching velocity of figure 14, however, was increased by about 7 percent over that of figure 10. The importance of splitting the initial response in normal acceleration, however, becomes less significant as gusts with longer gradients are considered. It should also be noted that the normal acceleration following the initial peaks is higher and of a longer duration than for higher servomechanism natural frequencies and may be considered more detrimental from the standpoint of passenger comfort.

The effects of variations of speed on the normal acceleration and pitching velocity of the airplane may be found by using the figures showing the effects of servomechanism natural frequency. A change in speed is reflected as a change in nondimensional natural frequency (see eq. (9)) of the servosystem and, therefore, cases 9 and 10 might be considered for other speeds and servomechanism frequencies when the appropriate changes in the vertical and horizontal scales are made. It should be noted that the same scale corrections are applied to the response curves of the basic airplane for changes in airspeed as are applied to the gust-alleviated airplane. Thus, the percentage of gust alleviation by the system remains the same at all speeds, provided the nondimensional natural frequency of the servosystem does not change.

From equation (11) it can be seen that the normal acceleration is proportional to $V^2$ so that an airplane has approximately an additional 55-percent reduction of normal acceleration due to a gust because of a decrease in airspeed from 150 to 100 mph. At other speeds and with unit angular gust disturbances, the scale of the ordinate of the pitching-velocity curve is directly proportional to the speed of the airplane. The ordinates of the flap-deflection and angle-of-attack curves do not change for gusts of unit angle of attack.

For given gust velocity, the angle of attack varies inversely as the forward speed of the airplane and, therefore, maintaining equal angle of attack of the gust rather than equal vertical velocity for comparison of airplane response to gusts might lead to a distorted interpretation of the effects of changing airspeed. If the condition of a gust of equal vertical velocity over a range of forward speeds is to be considered, the ordinates of the response curves for unit angle-of-attack gust inputs at these speeds may be obtained by multiplying the ordinates for the new airspeed and unit angle of attack by the ratio of the standard airspeed to the new airspeed. Because the distance traveled per unit time is proportional to the speed, the time scale is then inversely proportional to the speed.

## Airplane Response to Control Motion

As pointed out previously, the gust-alleviation system prevents normal control of the airplane by the elevator because the airplane is no longer sensitive to changes in angle of attack. In order to overcome this deficiency, it is necessary to allow the pilot to control the flaps as well as the elevator in order to be able to maneuver the airplane. This type of control motion is a relatively new concept and it is desirable to study the airplane response to pilot's control motion in an effort to determine the characteristics of the system and compare it with the conventional method of control.

Response of basic airplane.- The response of the airplane to a step input in elevator angle with the gust-alleviation system inoperative is given in figure 15. In this figure a step elevator deflection forces the airplane to pitch to a new angle of attack until the desired normal acceleration is achieved.

Response of airplane gust-alleviation system to stick input.- The response of the gust-alleviated airplane is presented in figure 16 and was determined by using the same gearing and aerodynamic coefficients as were used to obtain the response shown in figure 4. The normal acceleration of the airplane in this case builds up almost immediately in response to flap deflection, and the steady-state value is reached in about 8 percent of the time required for the basic airplane. There is a slight overshoot of about 3 percent. The flaps must remain deflected to maintain the normal acceleration because the static margin and variation of lift with angle of attack are zero in this case and the elevator cannot produce the angle-of-attack change necessary to maintain the normal acceleration and return the flaps to the neutral position.

Effect of canceling network.- The effects of the canceling network for the case of zero static margin are presented in figure 17 where the gain constant proportional to the integral of flap deflection is 0.01. The initial change in normal acceleration is similar to the response presented in figure 16, but, as the flaps are forced toward the neutral position by the canceling network, the normal acceleration is reduced and causes the pitching velocity to change the angle of attack in the direction necessary to restore the normal acceleration. This method of returning the flaps causes overshoot and a long-period oscillation similar to that shown in figure 8 for the gust response. Here again, it is necessary to choose the gain constant $K_{cw}$ so that the motion of the airplane will be easily controllable by the pilot.

Effect of stability.- The response of the airplane to a step input of the stick for the case where the gust-alleviation system has a positive static margin is illustrated in figure 18. The initial part of the response is still similar to the case of zero static margin, but, as the angle of attack increases because of pitching as a result of elevator deflection, the flaps return to neutral and the airplane remains at the angle of attack called for by the elevator.

Effect of downwash.- Figure 19 illustrates a condition comparable to an airplane using full-span flaps for stick control of the gust-alleviation system. The only aerodynamic change for the condition represented in figure 19 from that shown in figure 18 is a positive downwash due to flap deflection that results from using full-span flaps. In this case the airplane angle of attack changes at a rate greater than in the previous cases and, hence, the flaps return to neutral at a correspondingly greater rate.

Effect of servomechanism natural frequency.- The effects of changing the servomechanism natural frequency are illustrated by the time histories presented in figures 20 and 21. As expected, the increased natural frequency of the servo system decreased the response time of the gust-alleviation system to a stick input, and the decreased natural frequency increased the response time to a stick input. The decreased natural frequency of the servo system also had the effect of producing an oscillation (approximately 1.5 cps) in normal acceleration which subsided in about 1 second. This oscillation was undesirable in that it resulted in an overshoot of about 25 percent over the desired normal acceleration.

## CONCLUDING REMARKS

The use of an electronic analog computer allowed several refinements of the analysis previously presented in NACA TN 2416. These refinements included: consideration of the characteristics of the servomechanism; simulation of the true time lag between the action of the gust hitting the vane, wing, and tail; and inclusion of a device to return the flaps slowly to a neutral position in case of steady angle-of-attack changes due to changes in airspeed or weight.

The results indicated that the device would not completely alleviate the accelerations due to a step gust because the time lag in the response of the servomechanism did not exactly counteract the effect of finite time lag between the action of the gust hitting the vane and the wing. As a result, some rather high frequency variations in normal acceleration occurred at the start of the gust encounter. These high frequency variations could be minimized by selecting the correct servomechanism natural frequency. These oscillations might be expected to excite structural-oscillation modes of an airplane to a greater extent than the jump in normal acceleration experienced by the basic airplane on encountering a step gust. The analysis showed, however, that the power spectra of the normal-acceleration response to gusts of both the basic airplane and the gust-alleviated airplane were small for frequencies above about 3 cps.

The use of an integrating network, which would be desirable to return the flaps to the neutral position because of speed variations and changes in weight, was found to lead to a poorly damped long-period oscillation when used to return the flaps in the case of zero static margin. The period of the oscillation decreased with increasing gain of the integrating signal and, for this reason, the gain of the integrating signal should be kept small. The provision of a small, negative value of static margin was effective in returning the flaps to neutral following a gust encounter at constant airspeed and did not lead to the poorly damped oscillation obtained with the integrating network.

The findings of the present paper corroborate the results of the frequency-response analysis given in NACA TN 2416. The general conclusion made therein pertaining to a vane-controlled flap system to eliminate accelerations due to gusts is applicable herein also. The methods of the design and operation of this system as proposed in NACA TN 2416 and modified in the present paper still appear to be good for reducing the normal acceleration of an airplane in flight through rough air.

The response to control motion of the system showed the rapid change in normal acceleration caused by flap deflection which had been predicted in NACA TN 2416. Reducing the natural frequency of the servo system reduced the speed of response somewhat, and, for a very low servomechanism natural frequency, an undesirable overshoot in normal acceleration was encountered.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., October 27, 1955.

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2. Silverstein, Abe, Katzoff, S., and Bullivant, W. Kenneth: Downwash and Wake Behind Plain and Flapped Airfoils. NACA Rep. 651, 1939.

3. Silverstein, Abe, and Katzoff, S.; Design Charts for Predicting Downwash Angles and Wake Characteristics Behind Plain and Flapped Wings. NACA Rep. 648, 1939.

4. Chilton, Robert G.: Some Measurements of Atmospheric Turbulence Obtained From Flow-Direction Vanes Mounted on an Airplane. NACA TN 3313, 1954.

# TABLE I

## CHARACTERISTICS OF AIRPLANE USED IN CALCULATIONS

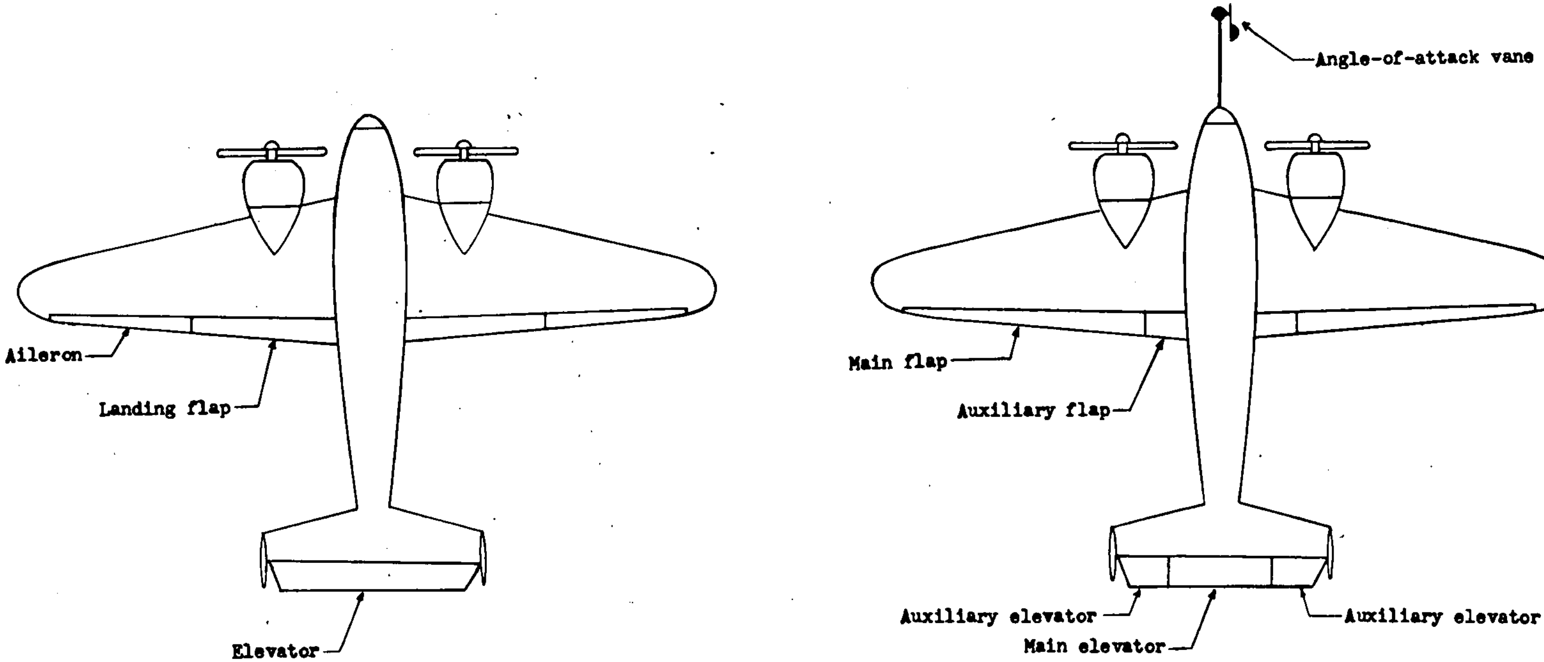
| | |
|---|---|
| **Dimensional data:** | |
| Weight, lb | 8,000.00 |
| Wing area, S, sq ft | 349.00 |
| Wing mean aerodynamic chord, $\bar{c}$, ft | 8.05 |
| Distance from center of gravity to tail, ft | 22.50 |
| Distance from center of gravity to angle-of-attack vane, ft | 15.00 |
| Radius of gyration about Y-axis, $k_Y$, ft | 5.88 |
| **Nondimensional parameters:** | |
| Ratio of distance between center of gravity and tail to mean aerodynamic chord of wing, $l$ | 2.79 |
| Ratio of distance between center of gravity and angle-of-attack vane to mean aerodynamic chord of wing, $l_n$ | 1.86 |
| Relative-density factor, $\mu$ | 37.20 |
| Nondimensional radius-of-gyration factor, $K_Y$ | 0.732 |
| **Aerodynamic parameters of components (based on wing area):** | |
| $\left(C_{Z_\alpha}\right)_t$, per radian | -0.634 |
| $\left(C_{Z_\delta}\right)_{f_m}$, per radian | -0.80 |
| $\left(C_{Z_\delta}\right)_{f_a}$, per radian | -0.30 |
| $\left(C_{Z_\delta}\right)_{ea}$, per radian | -0.158 |
| $\left(C_{Z_\delta}\right)_e$, per radian | -0.163 |
| $\left(C_{m_\alpha}\right)_w$, per radian | 0.432 |
| $\left(C_{m_\alpha}\right)_t$, per radian | -1.78 |
| $\left(C_{m_\delta}\right)_{f_m}$, per radian | -2.20 |
| $\left(C_{m_\delta}\right)_{f_a}$, per radian | -0.085 |
| $\left(C_{m_\delta}\right)_{ea}$, per radian | -0.435 |
| $\left(C_{m_\delta}\right)_e$, per radian | -0.454 |
| $\left(C_{Z_\alpha}\right)_w$, per radian | -5.30 |
| Downwash derivative of wing, $\frac{\partial\epsilon}{\partial\alpha}$ | 0.44 |
| Downwash derivative of main flap, $\frac{\partial\epsilon}{\partial\delta_{f_m}}$ | -0.05 |
| Downwash derivative of auxiliary flap, $\frac{\partial\epsilon}{\partial\delta_{f_a}}$ | 0.15 |

TABLE II

FLAP CHARACTERISTICS AND AUTOMATIC-CONTROL CHARACTERISTICS FOR VANE-CONTROLLED GUST ALLEVIATOR USED IN FIGURES

[Response to step gust and to step control-column input when gearing between control column and main elevator $K_c$ is 4.0 and damping ratio of servo system $\zeta$ is 0.707]

| Case | Figure number for- $\alpha_g$ input | Figure number for- $\delta_c$ input | $K_1$ | $K_2$ | $K_3$ | $K_{cw}$ | f, cps | $\left(C_{Z_{\delta_f}}\right)_w$ | $\left(C_{m_{\delta_f}}\right)_w$ | $\frac{\partial\epsilon}{\partial\delta_f}$ | $\left(C_{m_\alpha}\right)_{tot}$ | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 15 | 0 | 0 | 0 | 0 | 11 | 0 | 0 | 0 | -0.565 | Basic airplane |
| 2 | 4 | 16 | -7.98 | -.135 | -.604 | 0 | 11 | -.664 | .054 | -.070 | 0 | Optimum gust alleviation |
| 3 | 7 | -- | -7.98 | -.135 | -.604 | .001 | 11 | -.664 | .054 | -.070 | 0 | Effects of canceling network |
| 4 | 8 | 17 | -7.98 | -.135 | -.604 | .01 | 11 | -.664 | .054 | -.070 | 0 | |
| 5 | 9 | -- | -8.01 | -.133 | -.620 | 0 | 11 | -.662 | .061 | -.070 | -.057 | Effects of positive stability |
| 6 | 10 | 18 | -8.07 | -.129 | -.664 | 0 | 11 | -.656 | .080 | -.069 | -.283 | |
| 7 | 11 | -- | -8.50 | -.600 | -.897 | 0 | 11 | -.479 | .221 | -.140 | -.283 | Effects of downwash |
| 8 | 12 | 19 | -6.30 | .600 | -.272 | 0 | 11 | -.923 | -.115 | .040 | -.283 | |
| 9 | 13 | 20 | -8.07 | -.129 | -.664 | 0 | 16.55 | -.656 | .080 | -.069 | -.283 | Effects of servo-system natural frequency |
| 10 | 14 | 21 | -8.07 | -.129 | -.664 | 0 | 3.5 | -.656 | .080 | -.069 | -.283 | |




(a) Original airplane.

(b) Modified airplane.

Figure 1.- Sketch of light transport airplane used in analysis and location of angle-of-attack vane and control surfaces.

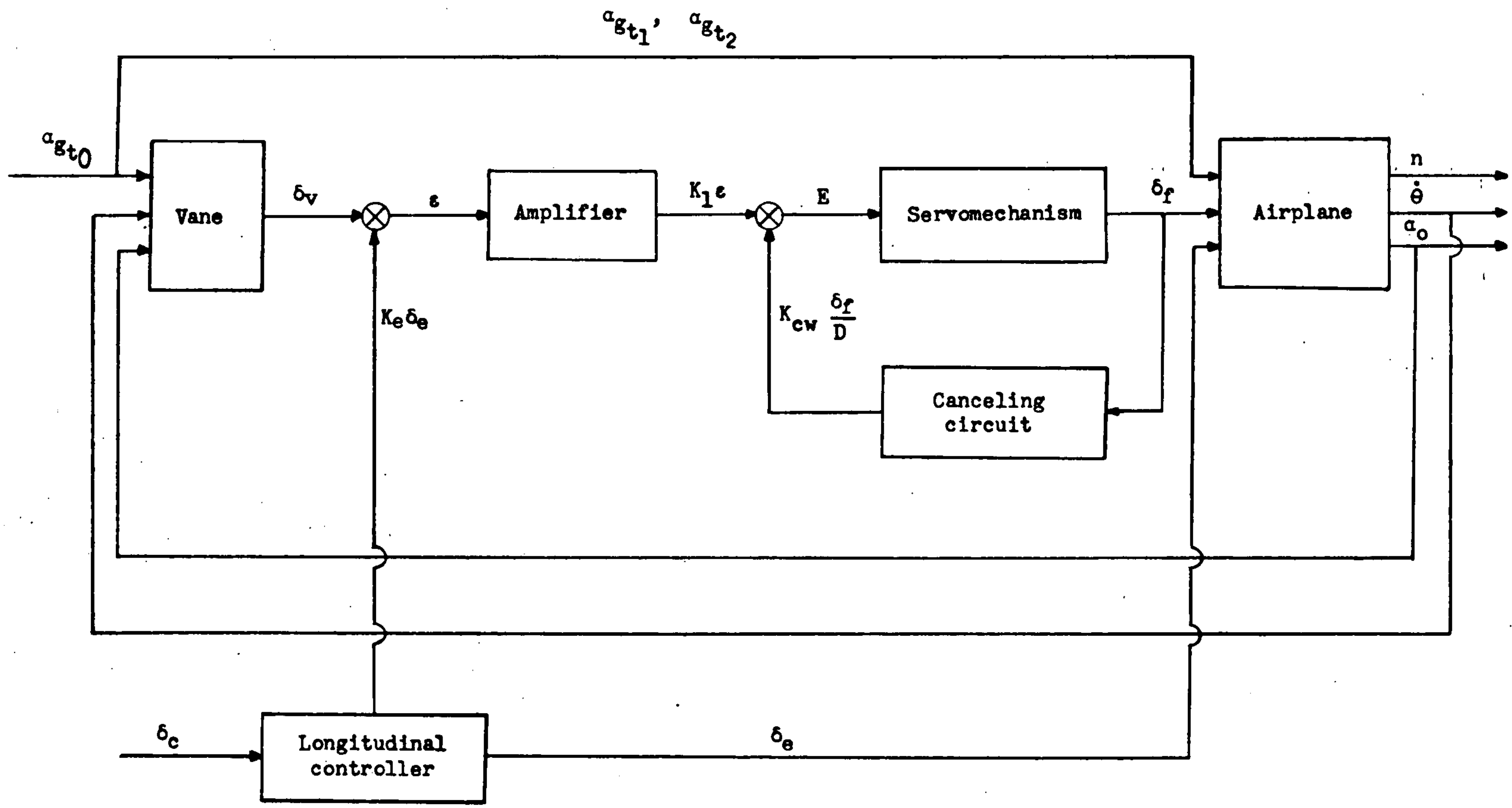



Figure 2.- Block diagram of gust-alleviation system incorporating vane control and pilot's control input to allow longitudinal control of airplane.

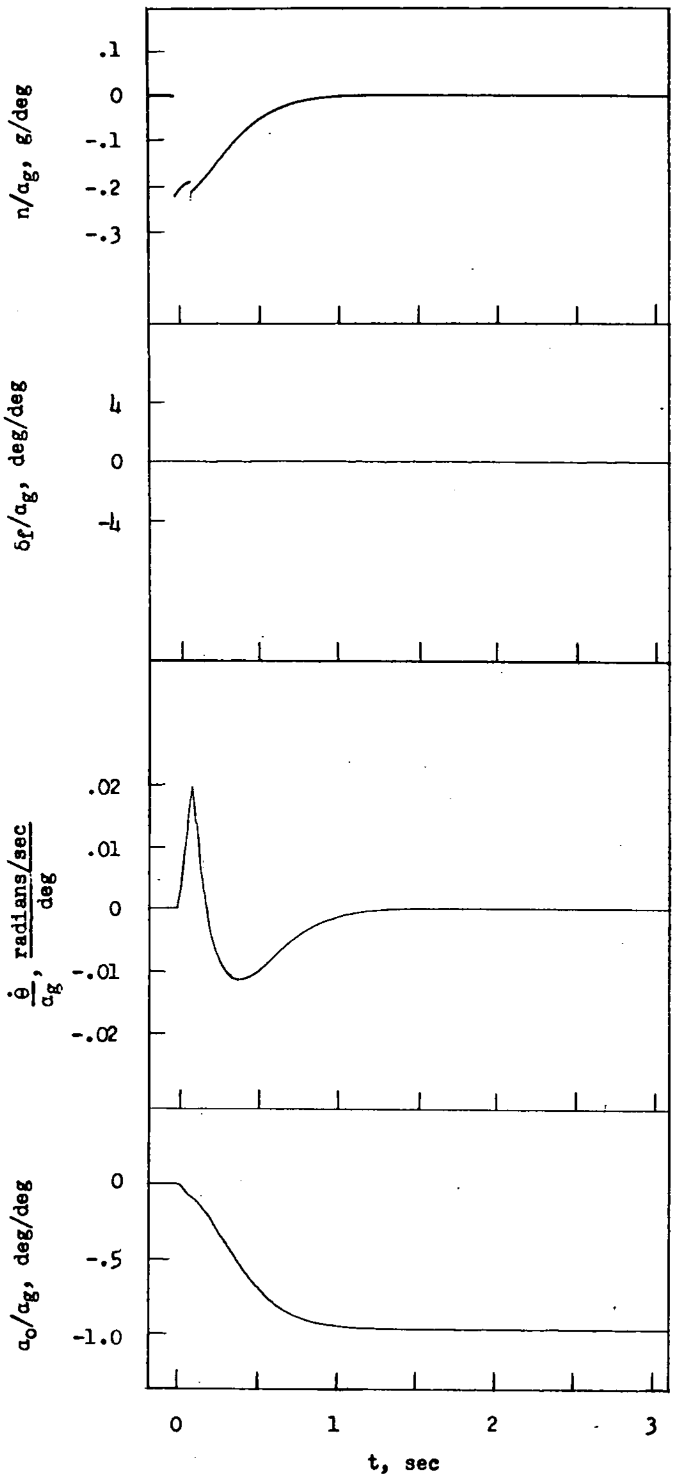



Figure 3.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from a step-gust input for case 1 (basic airplane).

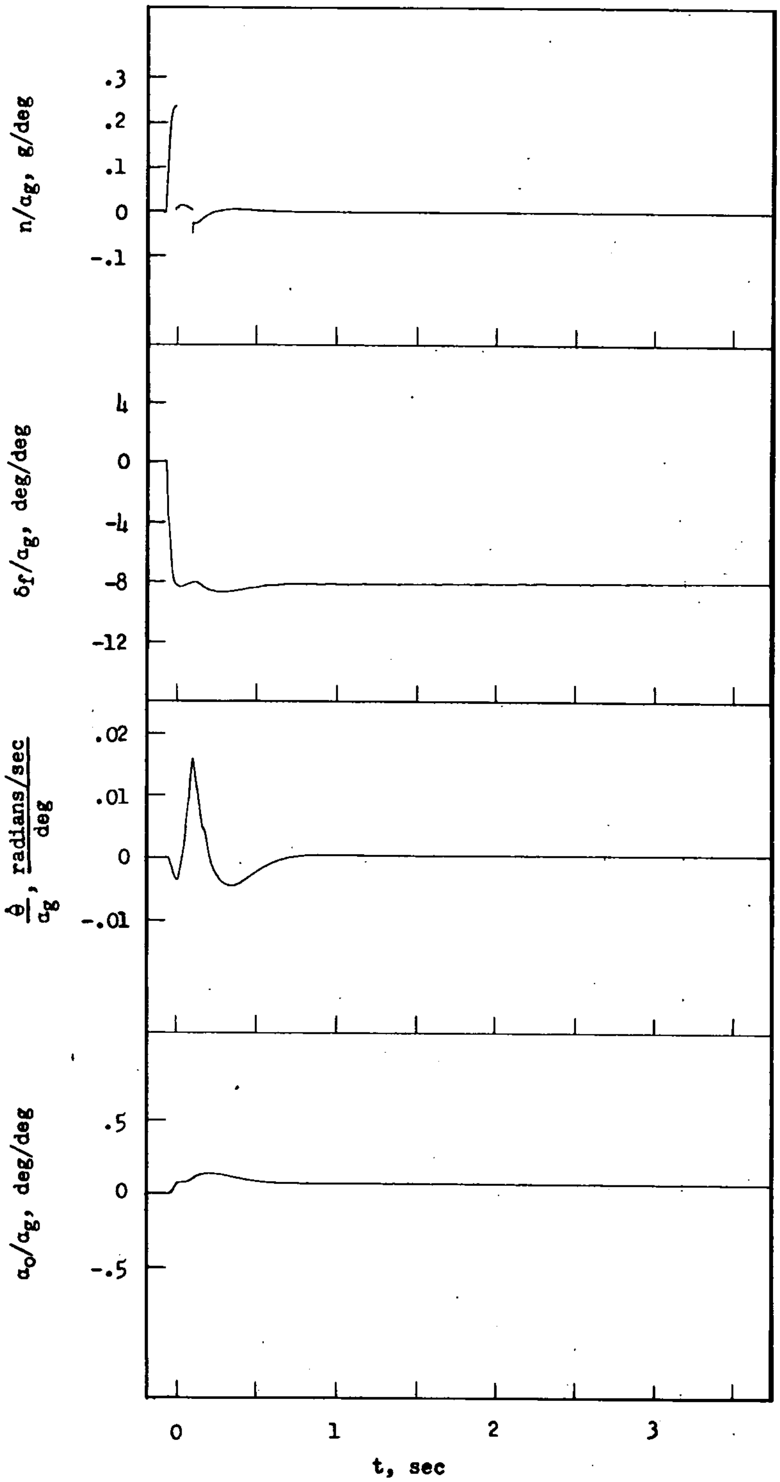



Figure 4.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from a step-gust input for case 2 (airplane with optimum gust-alleviation system).

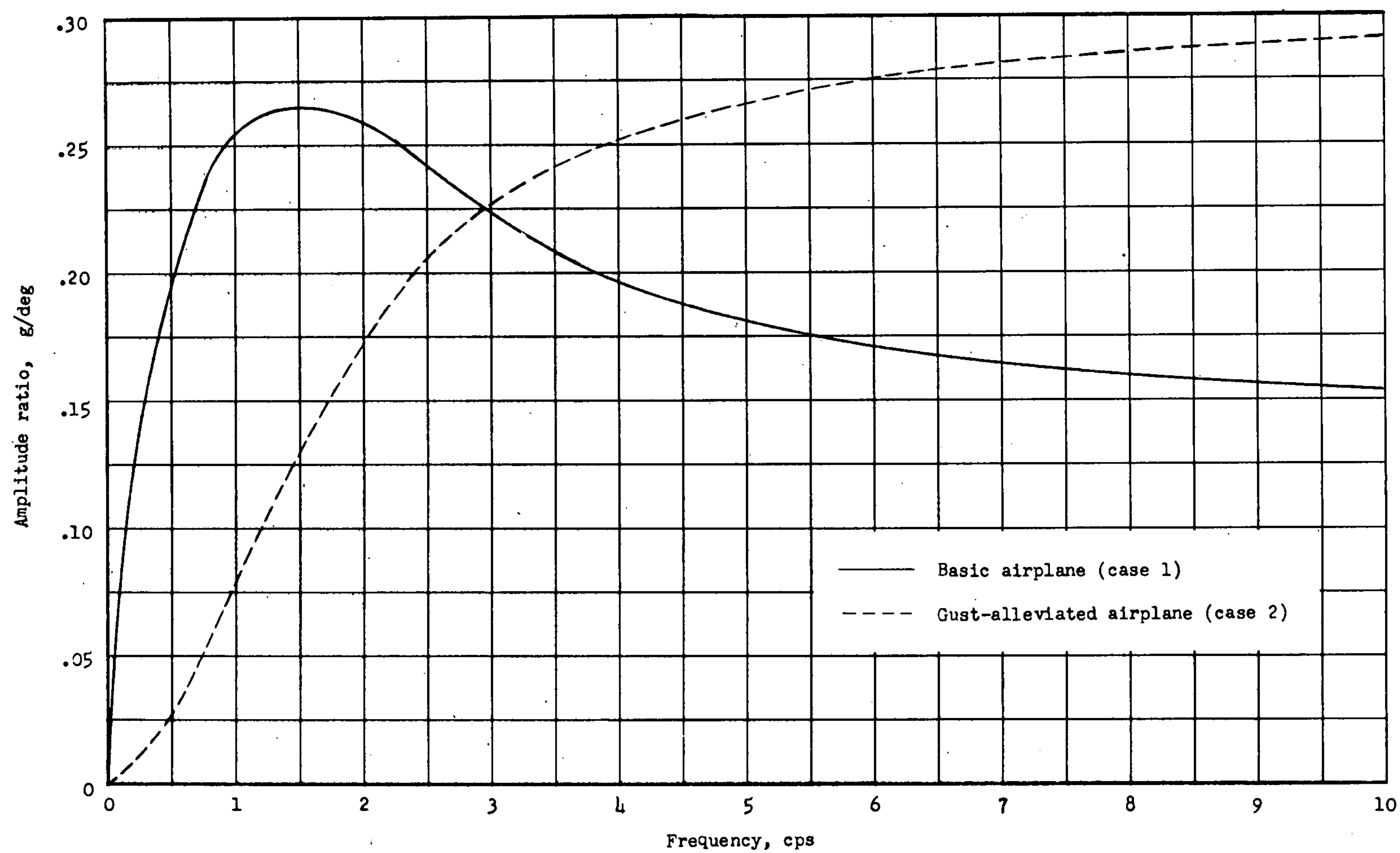



Figure 5.- Comparison of variations with gust frequency of normal acceleration resulting from sinusoidal gusts for case 1 (basic airplane) and case 2 (gust-alleviated airplane).

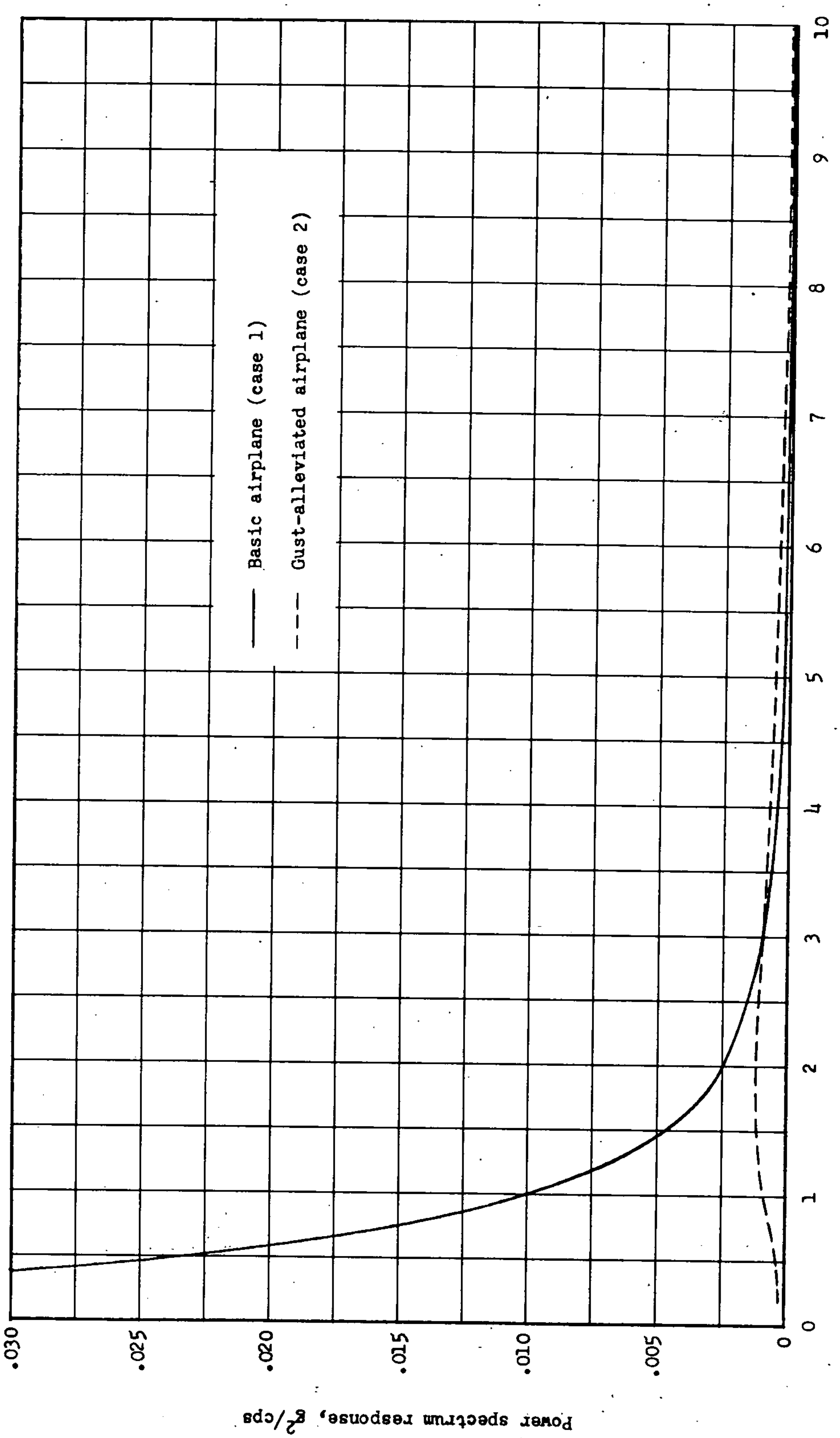



Figure 6.- Comparison of variations with gust frequency of normal acceleration resulting from gust power spectrum for case 1 (basic airplane) and case 2 (gust-alleviated airplane).

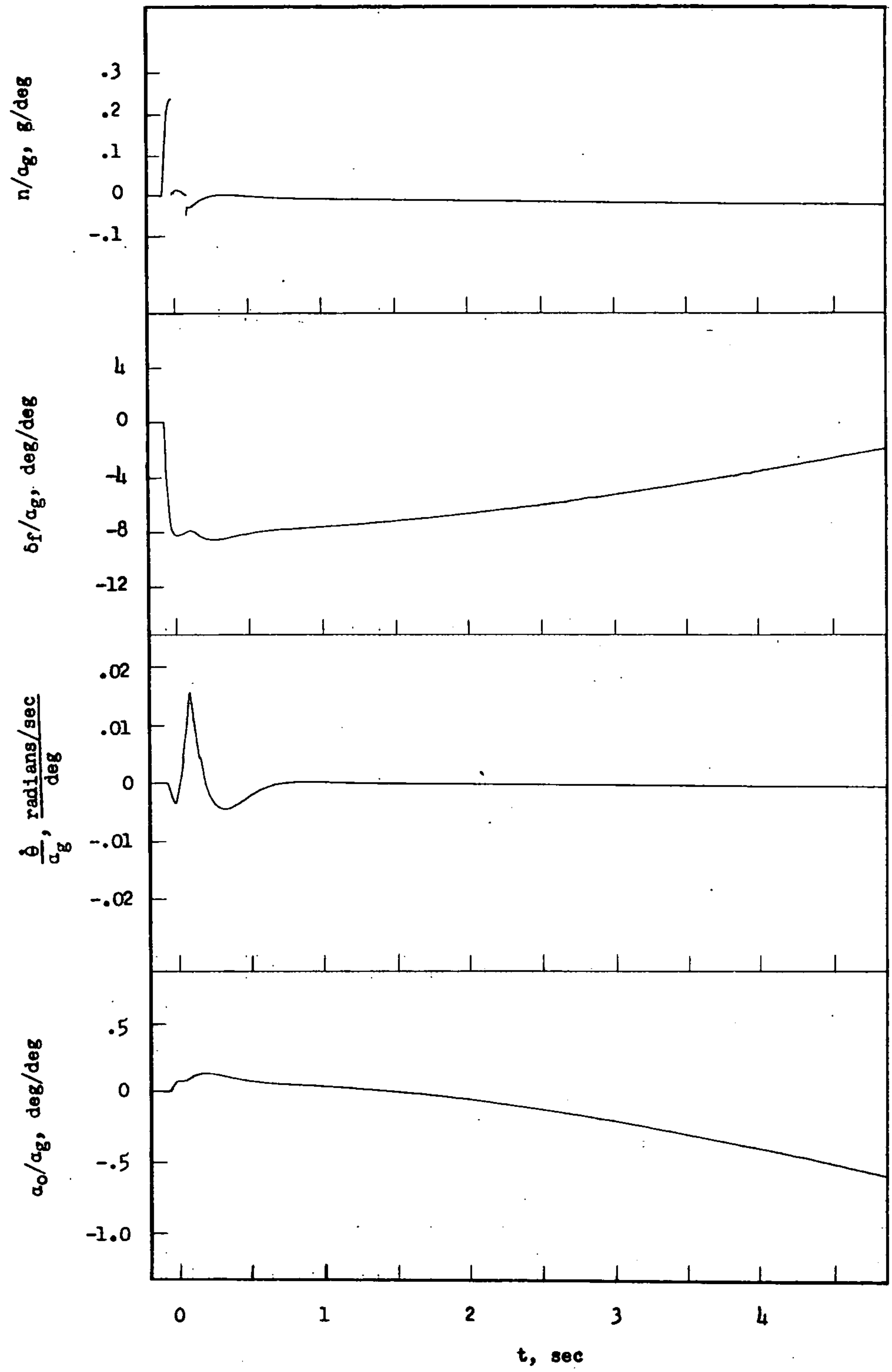



Figure 7.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-gust input for case 3 (airplane with gust-alleviation system).

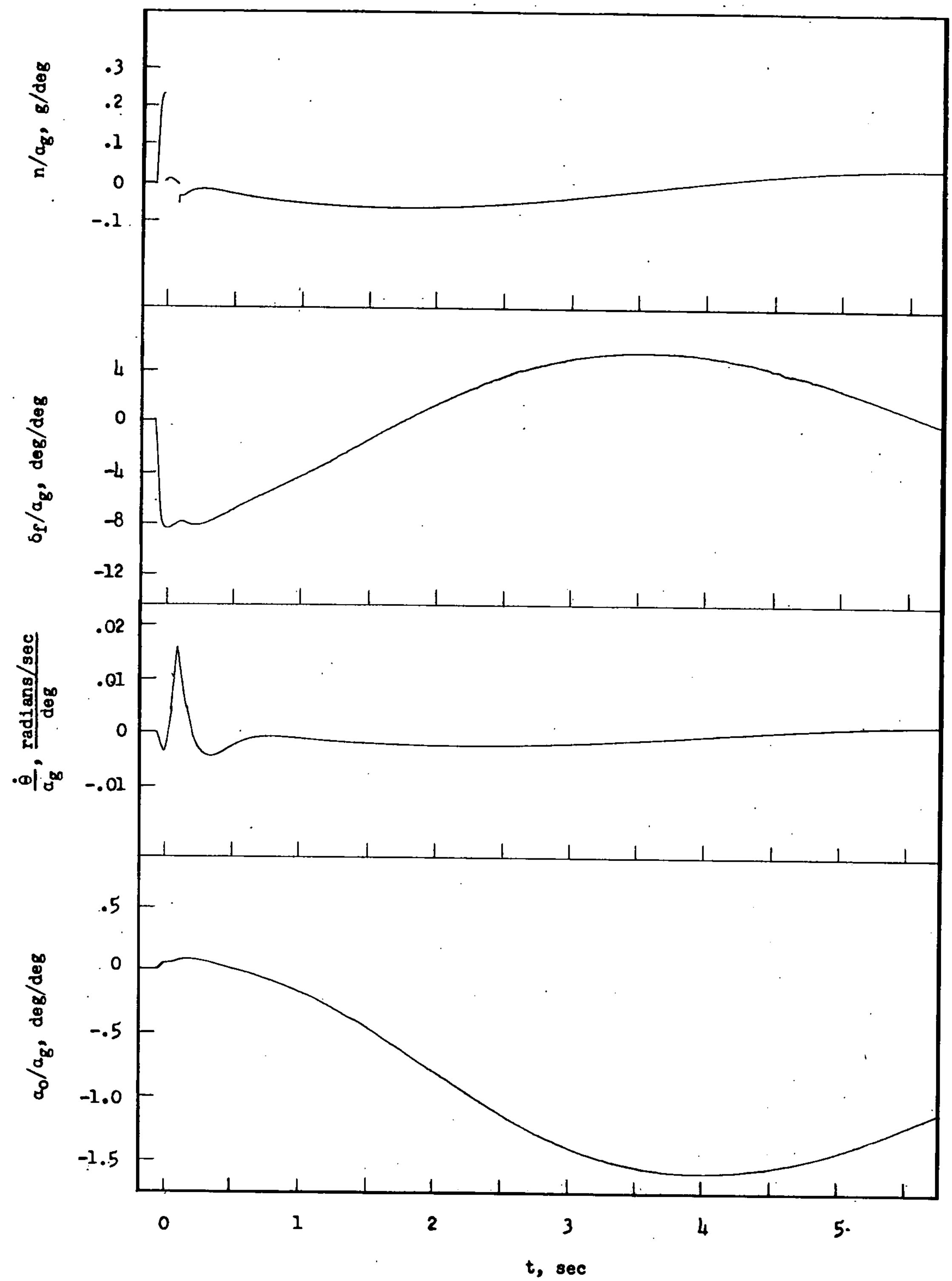



Figure 8.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-gust input for case 4 (airplane with gust-alleviation system).

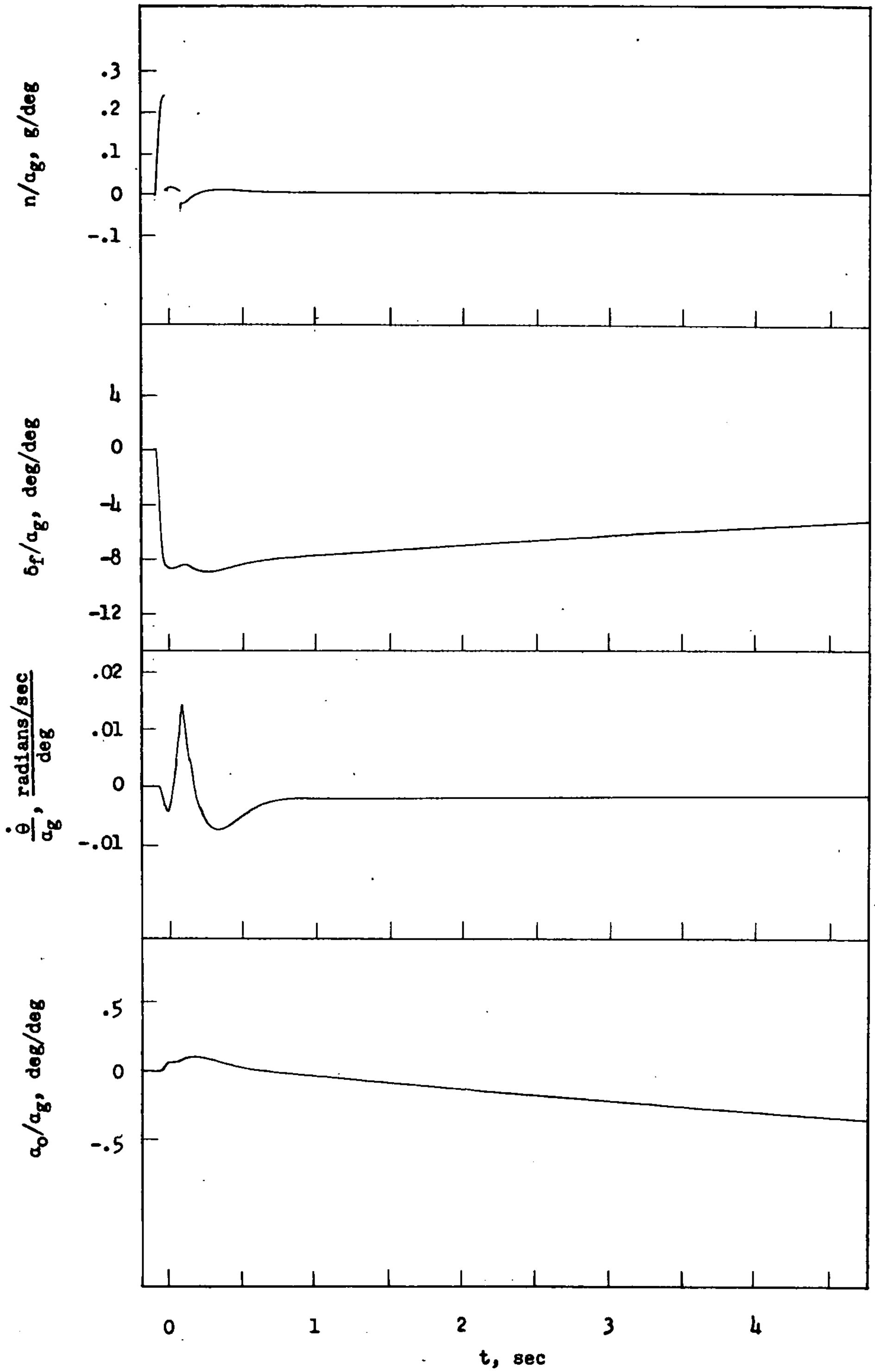



Figure 9.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-gust input for case 5 (airplane with gust-alleviation system).

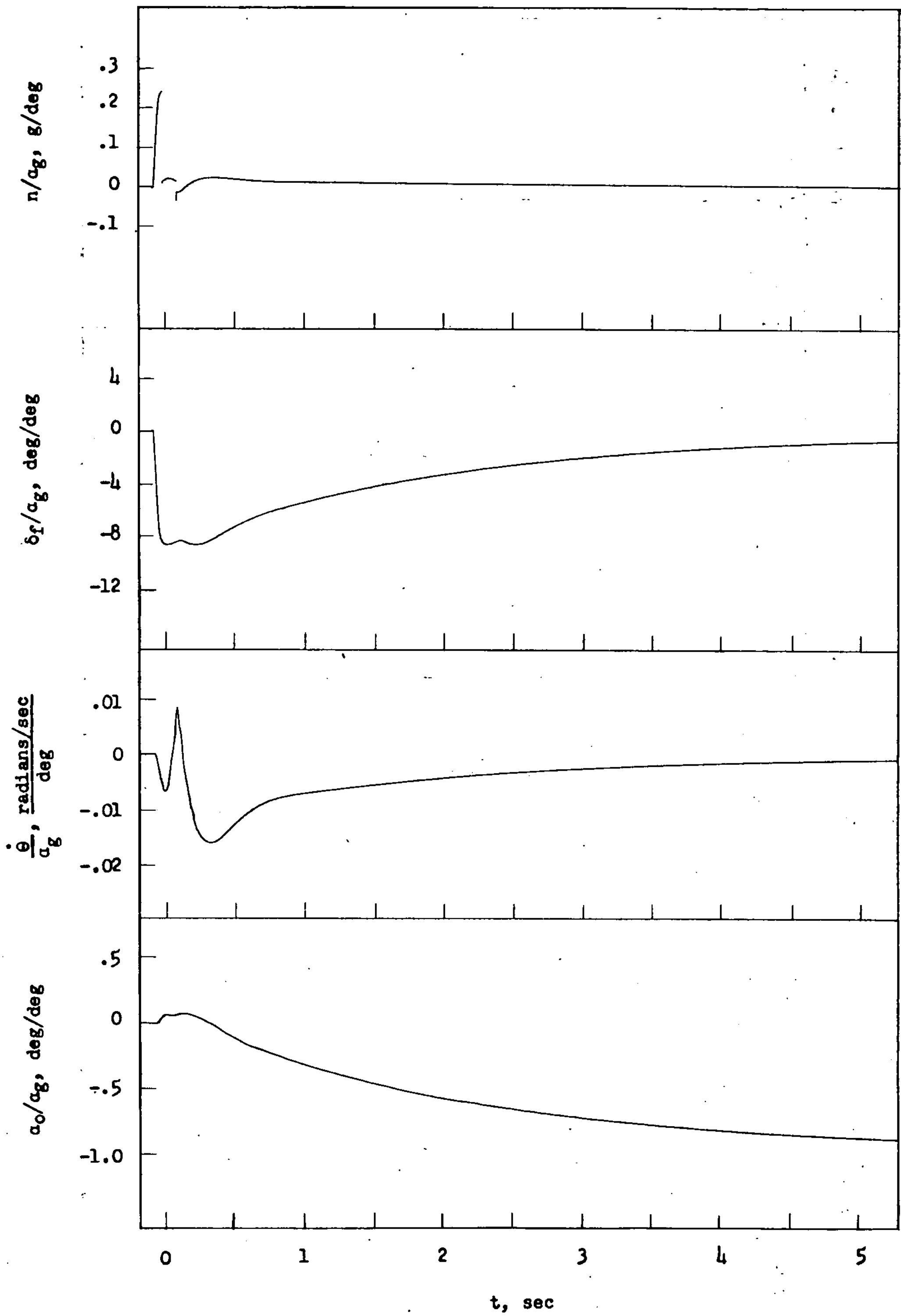



Figure 10.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-gust input for case 6 (airplane with gust-alleviation system).

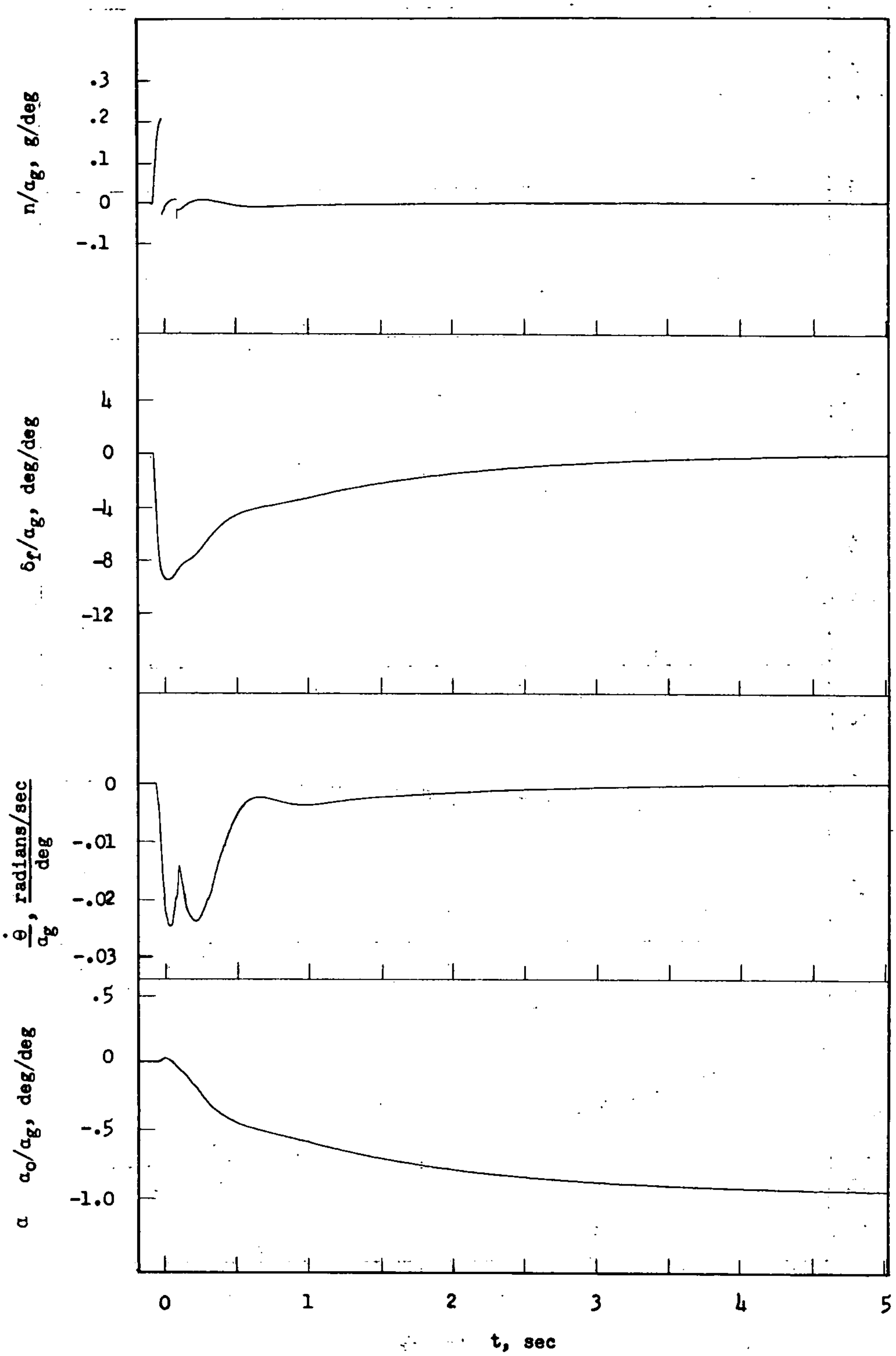



Figure 11.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-gust input for case 7 (airplane with gust-alleviation system).

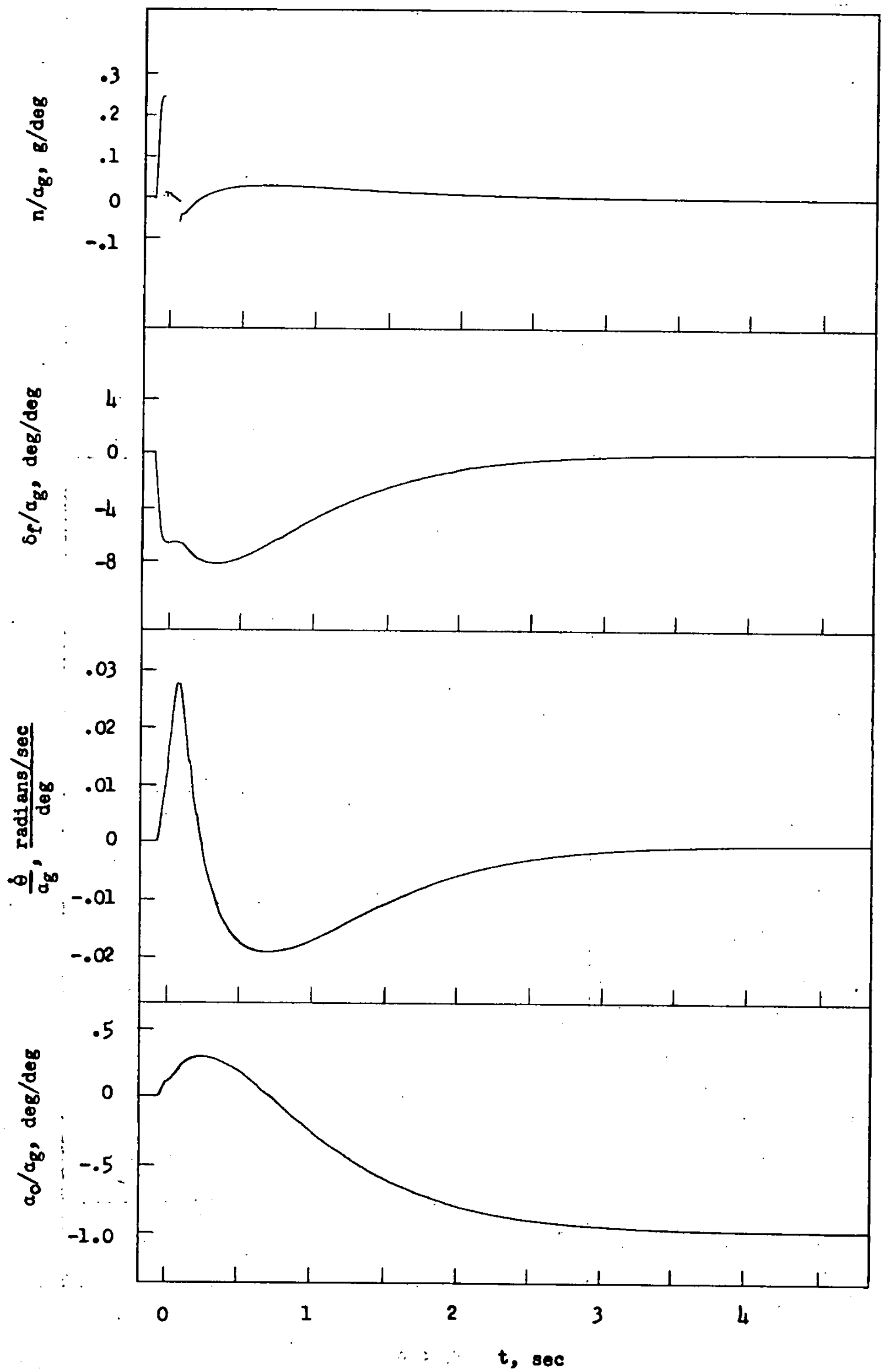



Figure 12.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-gust input for case 8 (airplane with gust-alleviation system).

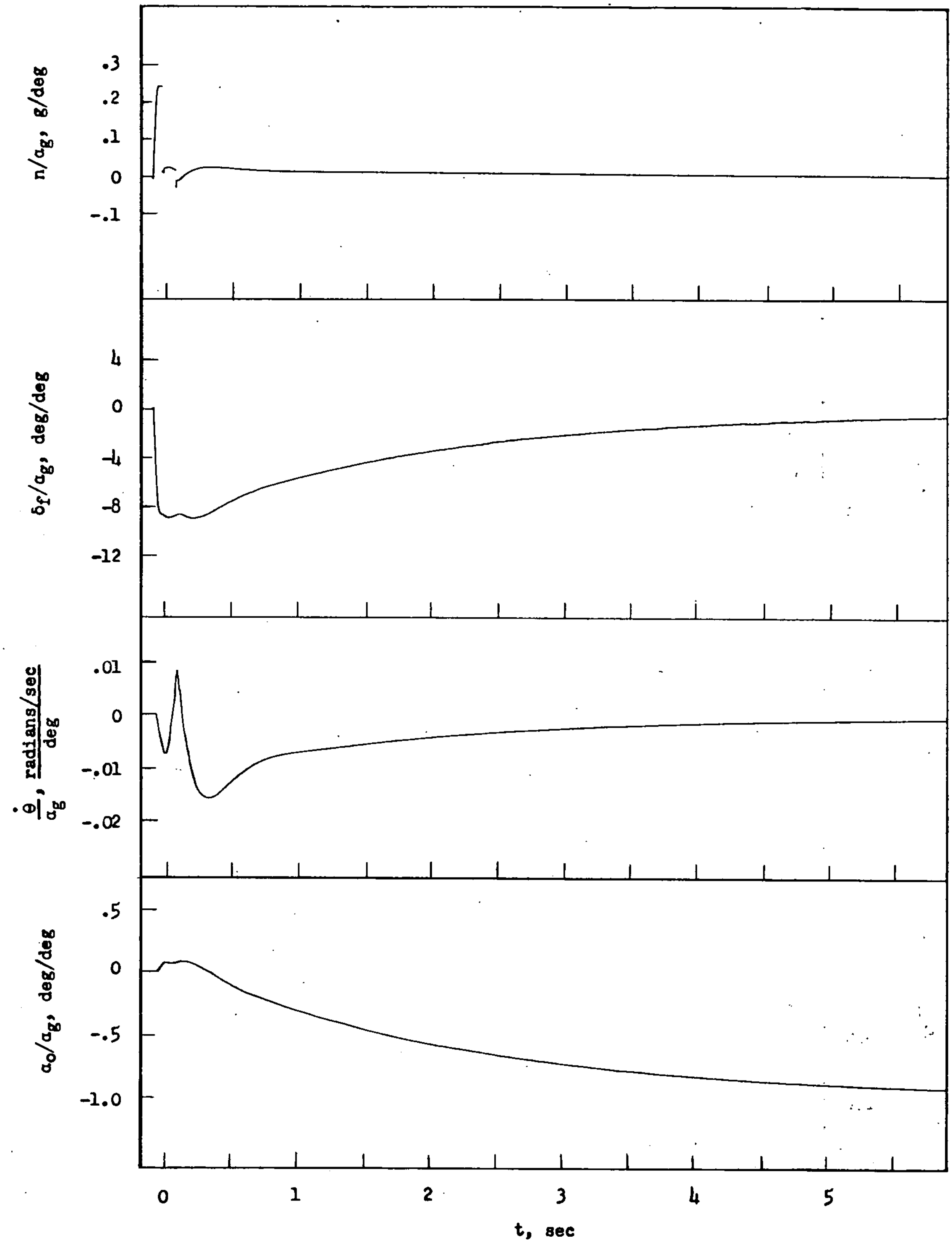



Figure 13.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-gust input for case 9 (airplane with gust-alleviation system).

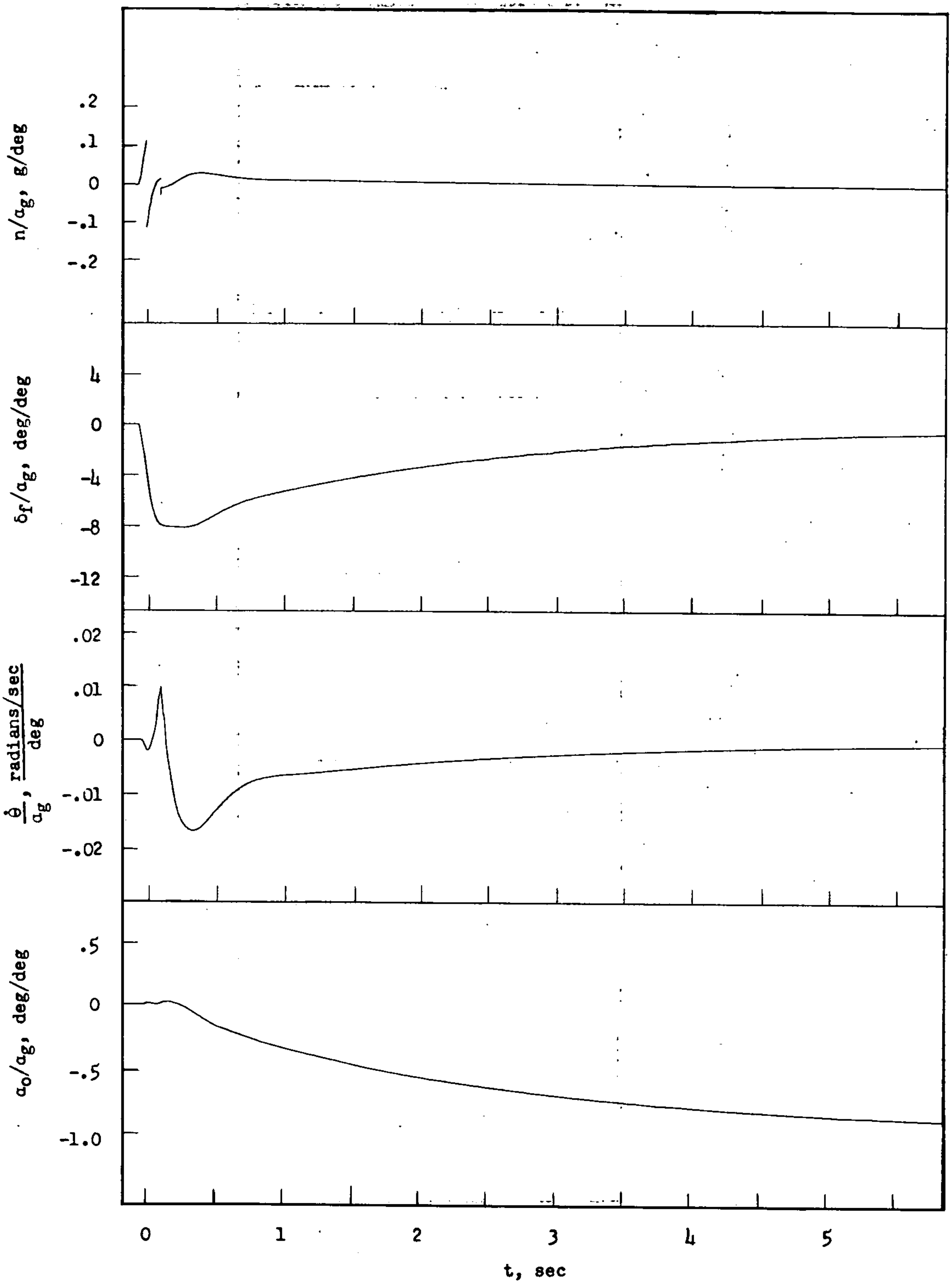



Figure 14.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-gust input for case 10 (airplane with gust-alleviation system).

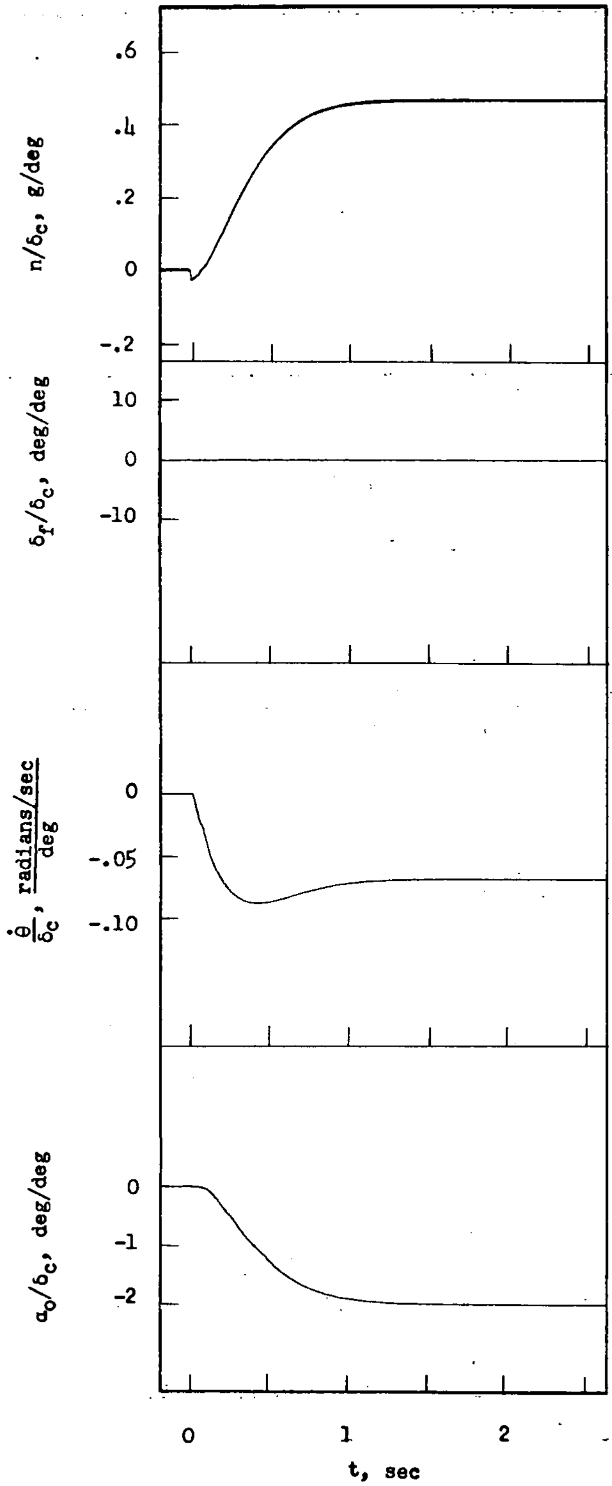



Figure 15.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-elevator input for case 1 (basic airplane).

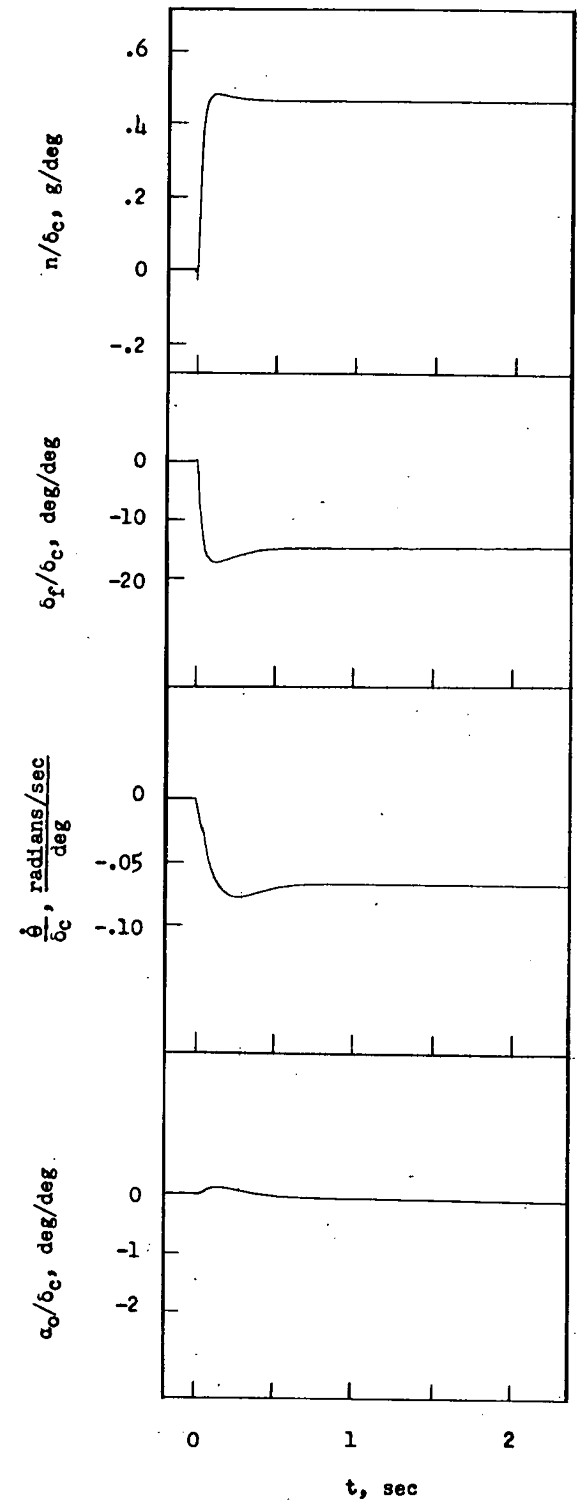



Figure 16.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-elevator input for airplane with pilot's longitudinal-control input of case 2.

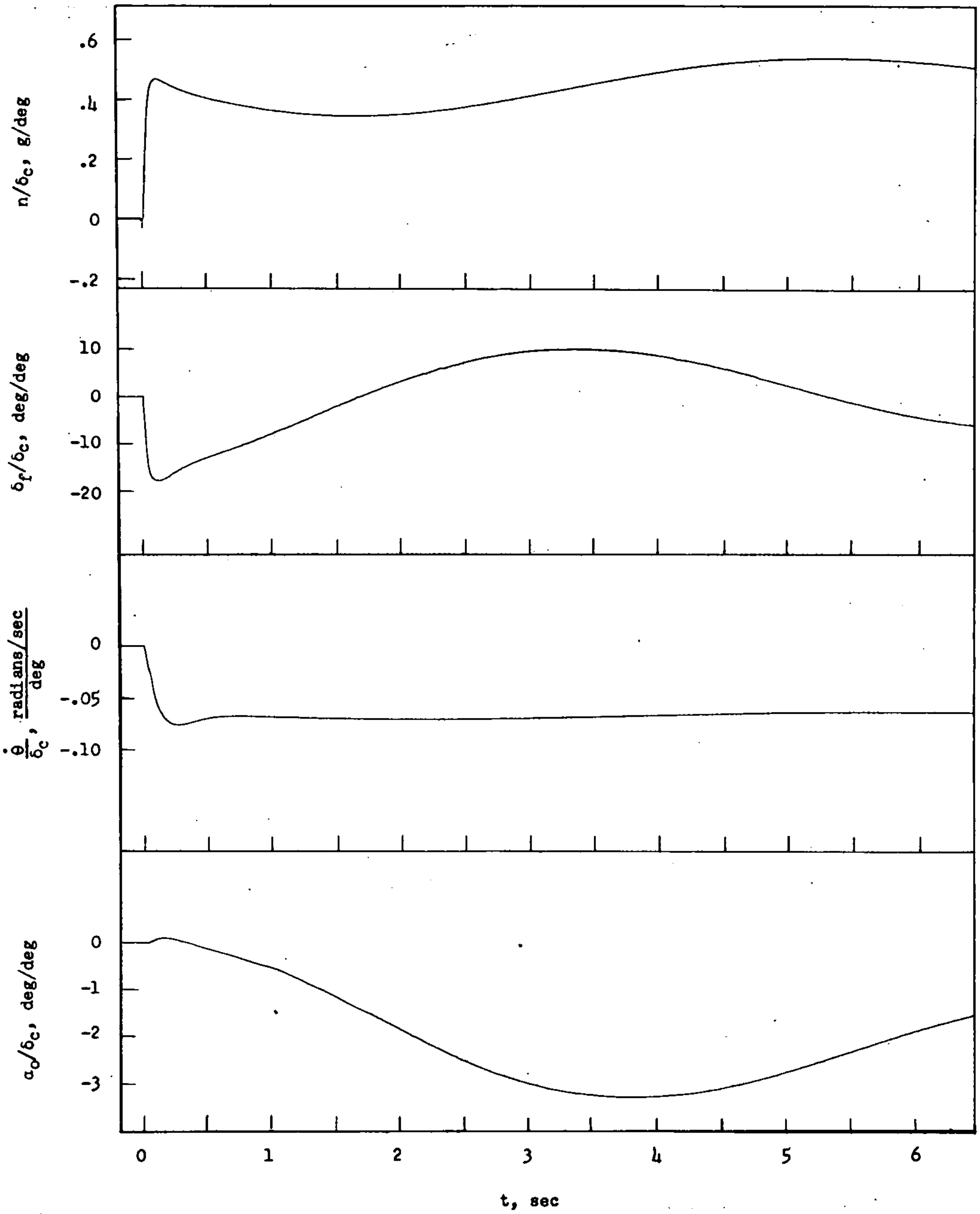



Figure 17.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-elevator input for airplane with pilot's longitudinal-control input of case 4.

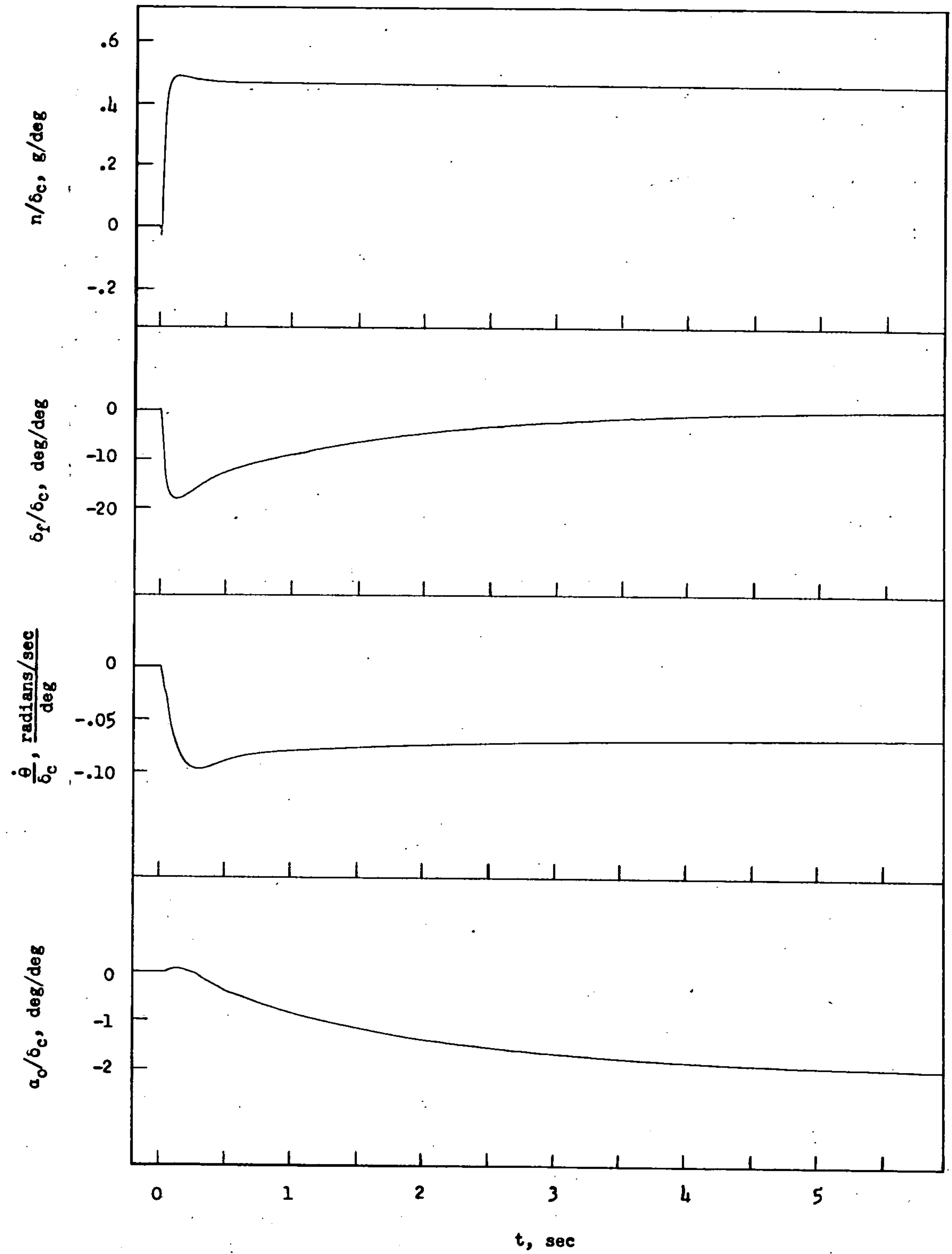



Figure 18.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-elevator input for airplane with pilot's longitudinal-control input of case 6.

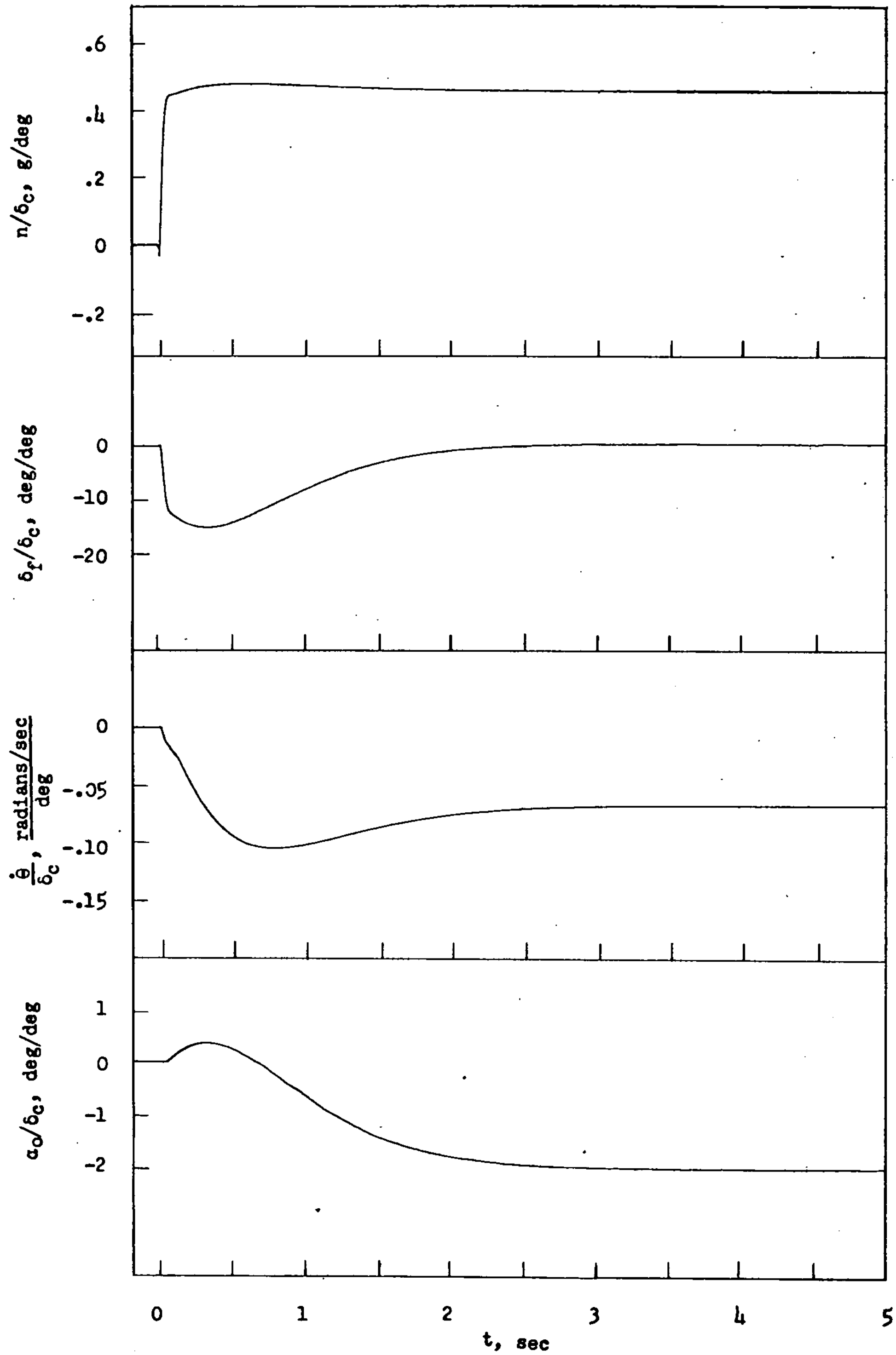



Figure 19.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-elevator input for airplane with pilot's longitudinal-control input of case 8.

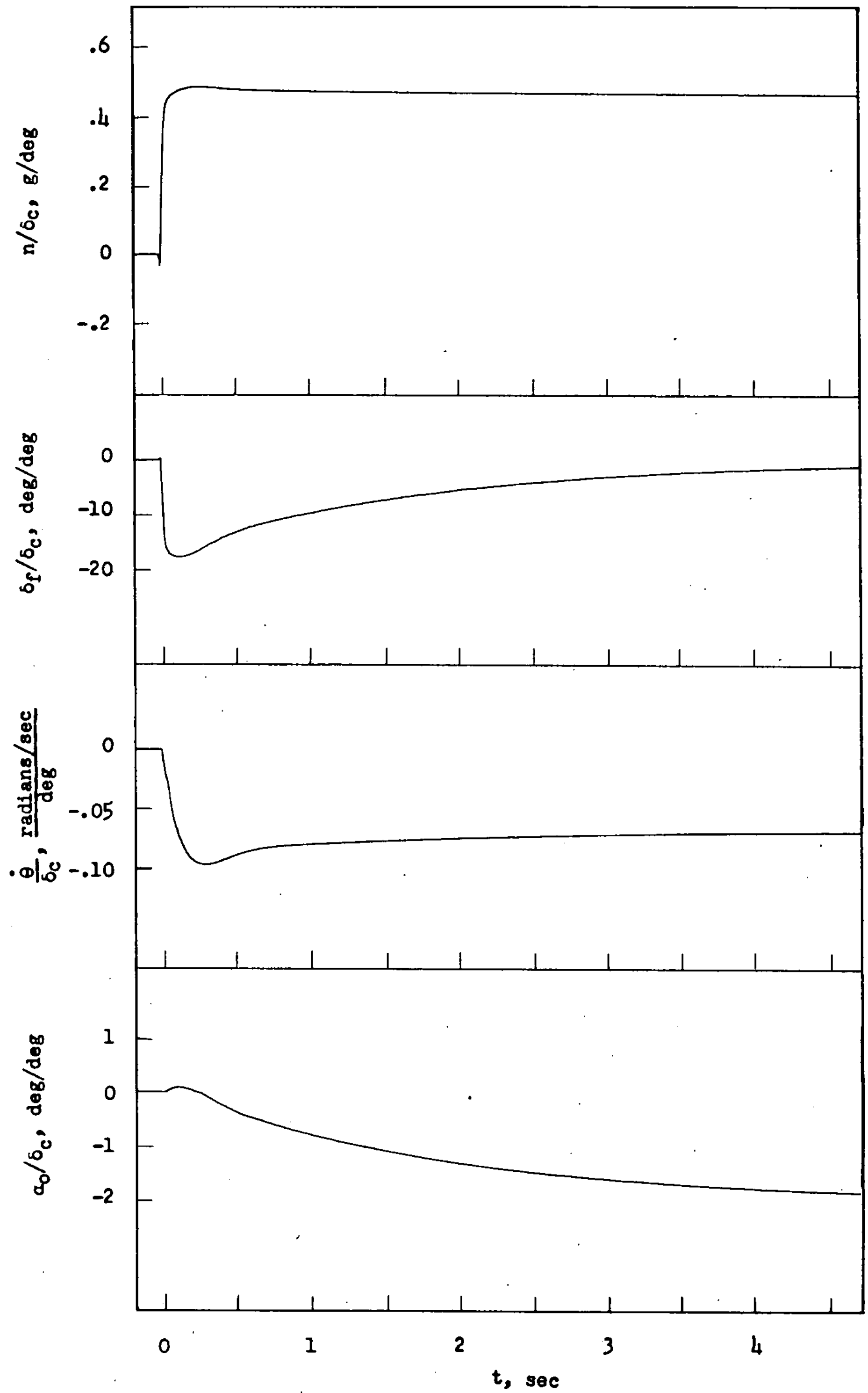



Figure 20.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-elevator input for airplane with pilot's longitudinal-control input of case 9.

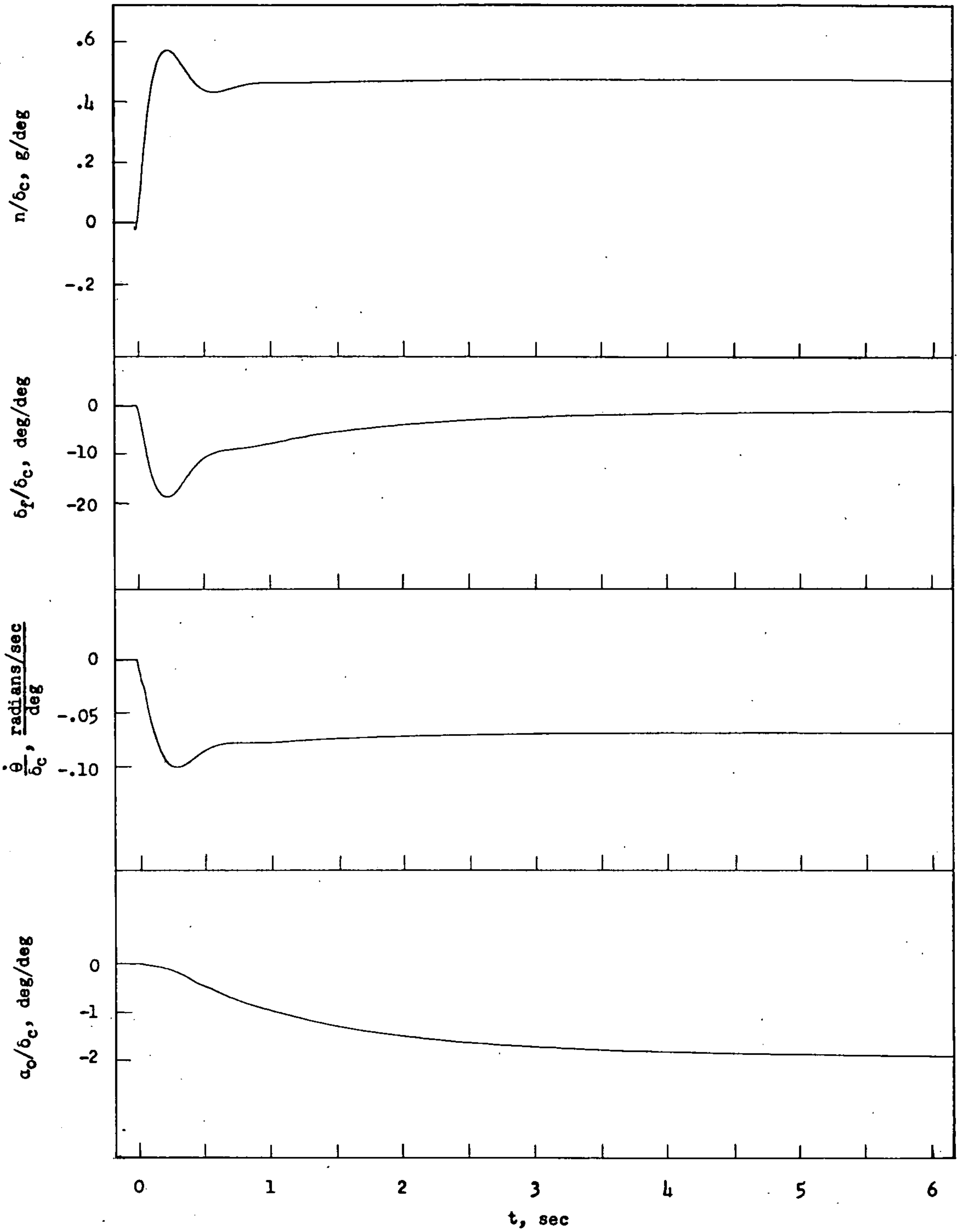



Figure 21.- Variations with time of normal acceleration, flap deflection, pitching velocity, and angle of attack which result from step-elevator input for airplane with pilot's longitudinal-control input of case 10.