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TECHNICAL NOTE 3713

SELECTION OF OPTIMUM CONFIGURATIONS FOR HEAT EXCHANGER WITH ONE DOMINATING FILM RESISTANCE

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SELECTION OF OPTIMUM CONFIGURATIONS FOR HEAT EXCHANGER WITH ONE DOMINATING FILM RESISTANCE

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SUMMARY

An investigation of the problem of selecting heat-exchanger configurations for optimum performance was made. The fluid on one side of the exchanger was assumed to have negligible heat-transfer resistance, and the amount of heat exchanged per unit time and the mass flow and inlet state of each fluid were prescribed. Any one of the parameters, power expended, weight, volume, or frontal area, can be optimized with respect to any one of the three remaining parameters when the heat exchanger is arranged normal to the approaching primary fluid. When the heat exchanger is inclined at an angle to the upstream direction, any one of the parameters, power, weight, or volume, can be optimized with respect to any one of the two remaining parameters. With this arrangement, the projected frontal area of the inclined heat exchanger will be equal to that of the heat exchanger requiring the minimum duct cross-sectional area when arranged normal to the primary fluid flow.

This method of optimization is illustrated for several compact heat-exchanger configurations where a prescribed amount of heat is transferred. The calculations were made to determine which configuration requires the least energy to drive the primary fluid for a prescribed weight, volume, or frontal area. The calculations also include inclining the heat exchanger at an angle to the upstream direction. For this arrangement, the least energy required to drive the fluid for a prescribed weight or volume was considered for the various configurations.

The results for heat-exchanger configurations for which heat-transfer data are reported in the literature are presented in the form of charts which illustrate the method of analysis and the results that can be drawn.

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INTRODUCTION

The task of optimizing a heat-exchanger design occurs continuously in the development of such equipment. Usually the goal of the optimization is to make the over-all cost for building and operating the heat exchanger a minimum. It is difficult to obtain generally applicable results from such investigations, since the ratio of initial to operating costs and the production costs of different heat-exchanger geometries are subject to continuous fluctuations. In some cases, however, especially for heat exchangers installed in vehicles such as cars, ships, or aircraft, other factors besides the cost become more important. These parameters are the power required to drive the coolants through the heat exchanger and the weight and over-all volume of the heat-exchanger equipment. In addition, quite often the frontal area that the heat exchanger exposes to the approaching stream of coolant is of special importance. For such applications, an optimization is required with respect to these parameters. An investigation with this as a goal can be put on quite a general basis and can result in generally applicable rules.

For general conditions, like finite heat resistances on both sides of the heat-transferring area and different mass-flow rates and heat capacities of the two fluids, the task of optimizing an exchanger is a complex one which can be performed only in several steps. The goal of such an investigation is to distribute the heat-transferring area properly to both sides, to determine the optimum flow velocities for both fluids, and to select surface configurations for both passages. Numerous investigations of this type have been published (refs. 1 to 8), especially on the subject of optimizing regenerators for gas turbines. The complexity of the problem made it necessary to employ simplifying assumptions. Heat transfer and friction were assumed to follow relations in the form of power functions; entrance and end losses were neglected. Heat capacities and mass-flow rates of both coolants were often assumed equal. Results of such calculations are very useful and time saving in approximating optimum conditions; however, they have to be followed by more exact trial-and-error calculations.

On the other hand, conditions are much simpler for heat exchangers in which the heat resistance on one side is so small that it can be neglected (refs. 9 and 10). For such a heat exchanger, an optimization can be carried out on a very general basis and with inclusion of all the effects that have just been mentioned. Such an investigation is desirable for newer applications such as occur in nuclear-reactor design. The present report is concerned with such a study and especially with the task of comparing different heat-exchanger configurations with respect to pressure drop, frontal area, volume, and weight required. The study is based on friction and heat-transfer characteristics of heat-exchanger configurations that have been published recently (refs. 11 and 12). The study also includes the possibility of improving the performance of a heat exchanger by arranging it at an angle to the oncoming coolant stream.

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DEVELOPMENT OF HEAT-EXCHANGER EQUATIONS

Definition of the Optimization Study

For the present investigation, it is assumed that the heat exchanger is arranged in a duct of constant cross section. Generally, an optimization must consider the conditions for both media between which the heat is exchanged. In this report it is assumed, in accordance with the INTRODUCTION, that the heat transfer to the medium on one side of the exchanger is such that any heat resistance on that side can be neglected. The conductive heat resistance in the primary exchanger wall is also considered negligible. Under this stipulation, the heat exchanged depends only on the conditions of the other cooling medium, which is called the primary coolant. The comparison is made between heat exchangers that must transfer identical amounts of heat per unit time in the heat exchanger; the mass flow of both fluids through the exchanger and the states (pressure and temperature) of the fluids at the entry to the exchanger are prescribed. Any one of the parameters, power expended, weight, volume, or frontal area, can be optimized with respect to any one of the three remaining parameters. By inclining the heat exchanger into the primary flow direction, one of the parameters, power expended, weight, or volume, can be optimized with respect to any one of the remaining two. The projected frontal area of the heat exchanger in the inclined position is equal to that of the exchanger requiring the minimum frontal area when arranged normal to the flow. Special consideration will be given to the problem of finding which specific heat-exchanger configuration transfers the prescribed amount of heat from the primary to the secondary medium with the least energy required to drive the primary coolant, for prescribed heat-exchanger weight, volume, or frontal area. Before starting with this study, the parameters that are used to describe a certain heat-exchanger configuration are summarized briefly.

Friction and Heat-Transfer Characteristics

The friction and heat-transfer characteristics of different heat-exchanger configurations are usually published in the form of friction factor f and Stanton number St. For instance, the results of experimental investigations (refs. 11 and 12) on compact heat-exchanger configurations have been published in this form. The present report is based on the data of references 11 and 12; hence, the definitions and nomenclature for the different parameters follow quite closely those in the mentioned references.

Friction factor. - The friction factor is defined by the following equation:

$$\Delta P_{friction} = f \frac{A}{A_c} \rho_m \frac{V_m^2}{2} \tag{1}$$

(Symbols are defined in the appendix and in fig. 1.)

Stanton number. - The Stanton number describing the heat transfer from the primary coolant to the heat-exchanger surface is defined by the equation

$$St = \frac{h}{c_p \rho_m V_m} = \frac{h}{c_p \rho_c V_c} \tag{2}$$

The specific heat c_p will be considered constant. The heat-transfer coefficient h is defined by

$$Q = hA\,(t_{w,m} - t_m) \tag{3}$$

The exchanger surface area A may consist of a finned and an unfinned (in direct contact with the other coolant) surface. In equation (3), $t_{w,m}$ indicates the mean wall surface temperature on the primary-coolant side including the fins. This temperature is lower than the temperature of the unfinned heat-exchanger wall, since the heat-conduction process from the unfinned wall surface into the fins reduces the average temperature of the wall. Equation (3) can also be written in such a way that it contains the temperature of the unfinned wall surface t_w, which is equal to the temperature of the second coolant.

$$Q = \eta_0 hA(t_w - t_m) \tag{4a}$$

In this expression η_0 indicates the surface effectiveness of the total primary-coolant-side heat-exchanger surface. This effectiveness is composed of the effectiveness of the fin proper η_f and the effectiveness of the unfinned surface (which is assumed to be unity) according to the equation

$$\eta_0 = \frac{A_f}{A} \eta_f + \frac{(A - A_f)}{A} \tag{4b}$$

In this equation, A_f is the primary-coolant-side fin surface and A the total (unfinned and finned) surface. The fin effectiveness is given by

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$$\eta_f = \frac{\tanh \sqrt{\frac{2h}{k\tau}}\, l}{\sqrt{\frac{2h}{k\tau}}\, l} \tag{4c}$$

The friction factor and the Stanton number are functions of the Reynolds number describing the flow through the heat exchanger.

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Reynolds number. - The Reynolds number is defined as

$$Re = \frac{\rho_m V_m d_h}{\mu_F} = \frac{\rho_c V_c d_h}{\mu_F} \tag{5}$$

The hydraulic diameter d_h in this equation is defined as

$$d_h = \frac{4A_c}{C} \tag{6}$$

There is some arbitrariness in the definitions of A_c and C for passages that change their shape or cross section in flow direction. In agreement with references 11 and 12, the minimum flow area A_c will be used and the circumference C is calculated from the relation $A = LC$ (for total number of passages). In some heat-exchanger reports the hydraulic radius $r_h = A_c/C$ is used instead of the hydraulic diameter; therefore, care must be taken in using parameters from the literature to determine on which of these characteristic lengths the parameters are based.

In recent investigations at the NACA (ref. 13), it has been established that better correlations are obtained, especially for large temperature differences, when the density in equations (1), (2), and (5) is evaluated at the film temperature. Here, the value in the core of the fluid (at bulk temperature) will be used, since this simplifies the calculations considerably. Only for heat exchangers operating with very large temperature changes are large deviations caused by this simplification.

Thermal effectiveness. - The thermal effectiveness is defined as the ratio of the temperature change of that one of the two media for which the heat-capacity rate is smaller to the initial temperature difference between the two media before they enter the heat exchanger. When the medium with the smaller heat-capacity rate mc_p is the primary coolant, the expression for the thermal effectiveness is

$$\eta_T = \frac{t_e - t_i}{t_i' - t_i} \tag{7a}$$

When $m'c_p'$ is smaller than mc_p, the equation is

$$\eta_T = \frac{t_i' - t_e'}{t_i' - t_i} \tag{7b}$$

The thermal effectiveness is always considered positive. In equations (7a) and (7b) the temperatures are entered in such a way that a positive η_T results for heat flow from the secondary to the primary fluid.

The thermal effectiveness η_T depends on the flow arrangement in the heat exchanger (counterflow, parallel flow, or cross flow) and on the parameter Tu, the number of transfer units. The relation between Tu and η_T can be read from figures 1 to 7 in reference 11 for different flow arrangements. Figure 2 shows this relation for some flow arrangements. It has an especially simple analytical form for $mc_p << m'c_p'$;

$$Tu = \ln \frac{1}{1 - \eta_T} \tag{8a}$$

For counterflow with $mc_p = m'c_p'$,

$$Tu = \frac{\eta_T}{1 - \eta_T} \tag{8b}$$

In the nomenclature used herein, Tu has the form UA/mc_p when $mc_p < m'c_p'$ and $UA/m'c_p'$ when $mc_p > m'c_p'$. For negligible heat resistance on the secondary-coolant side and in the primary heat-transfer wall,

$$U = \eta_0 h \tag{9}$$

The parameter Tu converts to $\eta_0 hA/mc_p$ when $mc_p < m'c_p'$ and $\eta_0 hA/m'c_p'$ when $mc_p > m'c_p'$. With the equation

$$m = \rho_{fr} V_{fr} A_{fr} = \rho_c V_c A_c \tag{10}$$

the following relation holds for $mc_p < m'c_p'$:

$$Tu = \frac{\eta_0 hA}{mc_p} = \eta_0 St \frac{A}{A_c} = \eta_0 St \frac{4L}{d_h} \tag{11a}$$

and for $mc_p > m'c_p'$

$$Tu = \frac{\eta_0 hA}{m'c_p'} = \eta_0 St \frac{A}{A_c} \frac{mc_p}{m'c_p'} = \eta_0 St \frac{4L}{d_h} \frac{mc_p}{m'c_p'} \tag{11b}$$

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The thermal effectiveness determines the length-to-diameter ratio of the passages according to equations (8a) or (8b) and (11a) or (11b) as soon as the parameter $\eta_0 St$ is prescribed. For the heat exchangers investigated later in this report, $\eta_0 St$ varies between 0.002 and 0.02; the larger value belongs to configurations having flow separation and low Reynolds numbers. For $\eta_0 St = 0.02$ and $\eta_T = 0.4$, equations (8a) and (11a) give a value of $L/d_h = 6.39$. This value may well be too small to establish developed flow. However, it will be assumed herein that friction factors and Stanton numbers are independent of the passage length in the direction of flow or, in other words, that the flow is fully developed over the major portion of the passage length. Accordingly, some caution must be exercised in applying the results of this report when the thermal effectiveness of a heat exchanger and the Reynolds number are simultaneously small.

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There is a unique relation between the mean temperature difference $\Delta t_m = t_w - t_m$ in the heat exchanger, the thermal effectiveness, and the initial temperature difference, which in this study is a prescribed parameter. It can be derived from the following equation for $mc_p < m'c_p'$:

$$Q = UA\,\Delta t_m = mc_p \eta_T (t_i' - t_i)$$

or

$$\Delta t_m = \frac{mc_p}{UA} \eta_T (t_i' - t_i) \tag{12}$$

Use of equations (11a) and (9) transforms equation (12) into

$$\Delta t_m = \frac{\eta_T}{Tu} (t_i' - t_i) \tag{13}$$

This equation holds also for $mc_p > m'c_p'$.

Heat-Exchanger Size Parameters

In addition to the friction and heat-transfer parameters, some parameters are required which describe the particular configuration of the exchanger. The ratio of free-flow area A_c for the primary coolant stream through the heat exchanger to the frontal area A_{fr} is denoted by

$$\sigma = \frac{A_c}{A_{fr}} \tag{14}$$

Another parameter is needed to indicate the amount of heat-transfer area A available per unit of total heat-exchanger core volume v. The ratio A/v can be calculated for various heat-exchanger configurations from the information contained in references 11 and 12. In using this value as a parameter, however, the following fact must be considered: When a comparison is to be made between different heat-exchanger configurations, a parameter should be available that is independent of the scale of the particular configurations (where the scale, for instance, is characterized by the hydraulic diameter d_h of the passages). In the parameter A/v, however, the area A increases proportionally to the square of the characteristic length, whereas the volume v varies proportionally to the cube of the characteristic length. Therefore, the value Ad_h/v is a better parameter describing the available surface area per unit volume, since this parameter is independent of the scale in which the particular configuration is produced.

The ratio of surface area A to weight W of the heat-exchanger core is obtainable from the information in references 11 and 12. It is assumed that all different heat exchangers compared in this report are made of the same material; therefore, a density of 1 will be arbitrarily postulated for the solid material of the exchanger. The area weight parameter is then actually the ratio of surface area to volume of the solid material. Again, this parameter has the disadvantage of changing its value when the scale of the particular heat-exchanger configuration is varied. When the heat-exchanger configuration is enlarged to one geometrically similar, this ratio will vary as the ratio A/v; in other words, a parameter Ad_h/W will have a constant value for a specific configuration regardless of its scale. However, the wall thickness of the material from which the heat exchanger is manufactured may be prescribed by the manufacturing process rather than by other considerations. In this case, the wall thickness must be kept constant when the scale (hydraulic diameter d_h) of the heat-exchanger core is changed. The weight then increases proportionally to the thickness s of the material and to the square of the hydraulic diameter, whereas the heat-transfer area increases proportionally to the square of the hydraulic diameter. Correspondingly, a parameter As/W, with s constant, depends only on the geometry and not on the scale.

Basic Heat-Exchanger Equations

Heat transferred per unit volume. - The heat flow transferred in the heat exchanger can be written

$$Q = UA\,\Delta t_m \tag{15}$$

Introducing the over-all heat-transfer coefficient U from equations (2) and (9) and the mean temperature difference from equation (13) gives

$$Q = c_p \rho_c V_c \eta_0 StA \frac{\eta_T}{Tu} (t_i' - t_i) \tag{16}$$

Finally, the mass velocity $\rho_c V_c$ can be expressed by the Reynolds number Re, and the following expression is obtained for the heat flow per unit core volume

$$\frac{Q}{v} = c_p \mu_F (t_i' - t_i) \frac{\eta_0 \mathrm{StRe}}{d_h^2} \frac{Ad_h}{v} \frac{\eta_T}{\mathrm{Tu}} \tag{17}$$

The values on the right side of the equation are grouped together into different parameters. The first group consists of the values that are characteristic for a specific task of the heat exchanger (specific heat, viscosity, and initial temperature difference). The second group comprises the parameters describing heat-transfer and flow characteristics, scale, and configuration of the exchanger. The third group indicates the influence of the thermal effectiveness. The expression Q/v is one of the relations needed for the intended optimization, since it expresses the heat exchanged per unit volume in the parameters that are available for the various exchanger configurations.

Heat transferred per unit weight. - In the same way, the heat exchange per unit weight of heat-exchanger core may be written

$$\frac{Q}{W} = c_p \mu_F (t_i' - t_i) \frac{\eta_0 \mathrm{StRe}}{d_h^2} \frac{Ad_h}{W} \frac{\eta_T}{\mathrm{Tu}} \tag{18}$$

Again the right side of the equation is composed of parameters pertinent to the specific task of the heat exchanger, to the heat-exchanger configuration, and to the thermal effectiveness.

Power required for normal flow. - The drop in total pressure in the air flow through the heat exchanger consists of three parts: the pressure drop due to friction, the pressure drop due to contraction or deceleration at the entrance and exit of the passages, and a pressure drop due to acceleration of the flow in this passage caused by the temperature increase in the primary coolant. The pressure drop due to friction in the passages is characterized by the friction factor and is defined in equation (1). The pressure drop at entrance and exit can be written as

$$\Delta P_{end} = K_c \rho_c \frac{V_c^2}{2} + K_e \rho_e \frac{V_e^2}{2} = \left(K_c + \frac{\rho_c}{\rho_e} K_e\right) \rho_c \left(\frac{V_c^2}{2}\right) \tag{19}$$

The pressure-drop coefficient K_e, for a sudden increase in cross section, is usually calculated by use of the equation $K_e = (\sigma - 1)^2$ from the momentum law. The coefficient K_c depends on the inlet geometry and has to be taken from reference 14 or from various handbooks. The pressure drop due to acceleration is given by

$$\Delta P_{acc} = \frac{\rho_c V_c^2}{2}\left(\frac{\rho_c}{\rho_e} - 1\right) \tag{20}$$

From the total-pressure drop $\Delta P = \Delta P_{friction} + \Delta P_{end} + \Delta P_{acc}$, the power required to drive the air through the heat exchanger can be calculated as

$$N = A_c V_m \Delta P \tag{21}$$

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By introduction of equations (1), (19), and (20) for the pressure drop, dividing equation (21) by the core volume v and replacing A_c/A from equation (11a) gives the following expression for $mc_p < m'c_p'$

$$\frac{N}{v} = \frac{\mu_F^3}{2\rho_m^2 d_h^4} fRe^3 \frac{Ad_h}{v} + \frac{\mu_F^3}{2\rho_c\rho_m d_h^4}\left[K_c + \frac{\rho_c}{\rho_e}K_e + \left(\frac{\rho_c}{\rho_e} - 1\right)\right]\eta_0 StRe^3 \frac{Ad_h}{v}\frac{1}{Tu} \tag{22}$$

This equation expresses the power required per unit heat-exchanger volume as a function of the prescribed parameters. The first term on the right side contains only the surface A of the heat exchanger, whereas the second term depends also on the ratio of surface area to cross-sectional area, which in this equation is contained in Tu. The corresponding equation for the power required per unit weight for $mc_p < m'c_p'$ is

$$\frac{N}{W} = \frac{\mu_F^3}{2\rho_m^2 d_h^4} fRe^3 \frac{Ad_h}{W} + \frac{\mu_F^3}{2\rho_c\rho_m d_h^4}\left[K_c + \frac{\rho_c}{\rho_e}K_e + \left(\frac{\rho_c}{\rho_e} - 1\right)\right]\eta_0 StRe^3 \frac{Ad_h}{W}\frac{1}{Tu} \tag{23}$$

In many cases it is of special interest to know the amount of power expended per amount of heat exchanged. This can easily be obtained by dividing equation (22) by equation (17) or equation (23) by equation (18). The corresponding expression for $mc_p < m'c_p'$ is

$$\frac{N}{Q} = \frac{\mu_F^2}{2\rho_m^2 c_p(t_i' - t_i)} \frac{fRe^2}{d_h^2\eta_0 St}\frac{Tu}{\eta_T} + \frac{\mu_F^2}{2\rho_c\rho_m c_p(t_i' - t_i)}\frac{Re^2}{d_h^2}\left[K_c + \frac{\rho_c}{\rho_e}K_e + \left(\frac{\rho_c}{\rho_e} - 1\right)\right]\frac{1}{\eta_T} \tag{24}$$

OPTIMIZATION FOR NORMAL FLOW

Among the four parameters, power expended, weight, volume, and frontal area, any one can be optimized with respect to any one of the remaining. For instance, it can be determined which of various heat-exchanger cores, all transferring a prescribed amount of heat Q and having the same weight W, requires the smallest power N. The task might also be to determine which core of a heat exchanger designed for a certain heat flow Q and having a prescribed volume v has the smallest frontal area A_{fr}. Such an optimization can be easily made with the help of figures of the type that will be presented for a number of heat-exchanger configurations that are optimized for minimum power expended with respect to frontal area, volume, or weight. The configurations are indicated in figure 3. A representative configuration was chosen from each of the different families of exchangers discussed in references 11 and 12.

In the comparisons made in this section, the end and acceleration losses have been omitted from equations (22) to (24); as a result of these deletions, the following equations are obtained (for $mc_p < m'c_p'$):

$$\frac{N}{v} = \frac{\mu_F^3}{2\rho_m^2 d_h^4} fRe^3 \frac{Ad_h}{v} \tag{25}$$

$$\frac{N}{W} = \frac{\mu_F^3}{2\rho_m^2 d_h^4} fRe^3 \frac{Ad_h}{W} \tag{26}$$

and

$$\frac{N}{Q} = \frac{\mu_F^2}{2\rho_m^2 c_p (t_i' - t_i)} \frac{fRe^2}{d_h^2 \eta_0 St} \frac{Tu}{\eta_T} \tag{27}$$

End losses can be neglected for heat exchangers that have a certain minimum depth so that the friction losses are large compared to the end losses. Figure 4 illustrates the magnitude of the entrance and exit losses in relation to the friction losses. All the curves are not shown, since the ranges of friction and Stanton parameters were not known in the required Reynolds number range. The upper and lower curves for each configuration were obtained by using the corresponding maximum or minimum values of the friction factor and the Stanton number from the data presented in references 11 and 12. The figure shows that, at a thermal effectiveness of about 0.7, the end losses for configuration 1 range from 20 to 30 percent of the friction losses. At an effectiveness of 0.3, the end losses range from 60 to 100 percent of the friction losses. The acceleration losses are small as long as the temperature increase in the heat exchanger remains small compared to the absolute temperature of the approaching air flow.

Optimization of Parameter N/Q with Respect to Frontal Area

For a fixed mass flow m through the exchanger and a prescribed density, the frontal area A_{fr} is inversely proportional to the frontal velocity V_{fr}. Accordingly, the frontal velocity may be used as a basis for comparison instead of the frontal area. Expressed in terms of the Reynolds number, the frontal velocity is

$$V_{fr} = \frac{\mu_F}{\rho_c d_h} \sigma Re \tag{28}$$

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For prescribed inlet conditions and thermal effectiveness, N/Q in equation (27) is proportional to $fRe^2/d_h^2\eta_0 St$. Instead of plotting this parameter against $\sigma Re/d_h$, which is proportional to V_{fr}, the value $f/\sigma^2\eta_0 St$, which is the ratio of $fRe^2/d_h^2\eta_0 St$ to $(\sigma Re/d_h)^2$, can be used. In figure 5 the parameter $f/\sigma^2\eta_0 St$ is plotted against the parameter $\sigma Re/d_h$ for configurations 1 to 7 (fig. 3). The values of f, St, σ, and d_h were determined from references 11 and 12. Values of η_0 were calculated with equation (4b). The value of k in the fin-effectiveness expression (η_f, eq. (4c)) of 100 Btu per hour per foot per $^{\circ}F$ was used. This is indicative of a high-conductivity metal.

For each of the configurations, except 7 (finned tube), the width of the passage for the primary fluid was assumed equal to the width of the passage for the secondary fluid, and the parameter σ was calculated accordingly. This may somewhat favor configuration 7 in its weight, volume, and frontal area in the following comparison, since configuration 7 has a smaller passage area for the secondary fluid. For a comparison of the performance of various heat exchangers with prescribed dimensions, the hydraulic diameter has different values and the parameters as developed before must be used. If the heat-exchanger configurations are compared for the same hydraulic diameter, the value d_h can be dropped from the various parameters.

A comparison of this nature, where d_h is included in one case and eliminated in the other, emphasizes the effect of the scale in which the particular configuration is produced and is presented in figures 5 and 6, respectively.

Stanton numbers for air were used; therefore, figures 5 and 6 hold for air as primary coolant. They can, however, be used for any coolant as long as only ratios of the ordinate values are used for a comparison of various configurations. The absolute values of the ordinate are valid for a fluid with a Prandtl number Pr when the Stanton number St in the parameter on the ordinate is replaced by $St(Pr/0.671)^{2/3}$. The other figures in the report can be generalized in the same way when the change in the Stanton number is made wherever it appears.

Figures 5 and 6 show that the power N required varies considerably for the different heat-exchanger configurations, especially at the higher flow velocity. When a certain frontal area (or frontal velocity) and heat flow Q are prescribed, a substantial saving can be obtained when, for instance, heat exchanger 3 is replaced by heat exchanger 7. Figure 6 indicates that, at a value of $\sigma Re = 3000$, the power N required for heat exchanger 3 is 13.6 times as large as that for heat exchanger 7. An inspection of the different curves shows that the power required generally decreases as the flow through the passage becomes smoother. In 3983 this type of optimization, flow separation and even turbulence should be avoided as much as possible. The relative exchanger volume or weight required for the different configurations can be determined from figures 7 to 10.

Optimization of Parameter N/Q with Respect to Heat Transferred per Unit Volume

The power required per unit heat flow for various heat-exchanger configurations with respect to heat transferred per unit core volume is presented in figures 7 and 8. The characteristic ordinate parameters for N/Q are $fRe^2/\eta_0 St d_h^2$ or $fRe^2/\eta_0 St$ from equation (27). The corresponding abscissa parameters for Q/v are $(\eta_0 StRe/d_h^2)\ Ad_h/v$ or $(\eta_0 StRe)\ Ad_h/v$ from equation (17). Figure 7 compares the heat-exchanger configurations using the dimensions shown in figure 3, whereas figure 8 compares different heat-exchanger configurations assuming that all have the same hydraulic diameter. Comparison of figures 7 and 8 shows that the scale hydraulic diameter is a parameter to be considered in the optimization. For instance, the relative positions of exchangers 1 and 3 are reversed. The order of the configurations in figures 7 and 8 is significantly different from that in figures 5 and 6. Heat exchangers that are poor in the comparison of N/Q with V_{fr} (or A_{fr}) are good in the present comparison, and vice versa. This is especially apparent in figure 8, where the hydraulic diameters of all passages are equal. This means that the selection of the most advantageous configuration will depend upon which of the parameters is especially important in a specific application. This behavior has already become apparent in a study on bare-tube heat exchangers reported in reference 10. The relative frontal area for heat exchangers with the same volume can be determined from the information given in figures 5 and 7 or figures 6 and 8. As would be expected from the discussion in the previous paragraph, that heat exchanger in a group, all having the same core volume, which has an especially low power consumption per unit of heat transferred usually has a low frontal velocity and correspondingly a large frontal area.

Optimization of Parameter N/Q with Respect to Heat Transferred per Unit Weight

Figures 9 and 10 are the bases for a comparison of power expended for a heat exchanger with a certain weight. A study of the figures indicates that this comparison generally reveals the same trends as figures 7 and 8; this means that a heat-exchanger configuration that is good with respect to volume is usually also good with respect to weight. However, there are some exceptions in which a heat exchanger that is better from a weight standpoint will have a poorer performance from a volume standpoint. The corresponding frontal velocities can be obtained from the information presented in figures 5 and 9 or figures 6 and 10.

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In addition to the exchangers studied in figures 5 to 10, similar calculations have been performed for the remainder of the configurations reported in references 11 and 12. Because of space limitations, it was necessary to restrict the study herein to a few representative exchangers. Similar curves are available from the Heat Transfer Laboratory, University of Minnesota for the remainder of the configurations.

MODIFICATION OF EQUATIONS FOR INCLINED HEAT EXCHANGER

In the preceding optimization it became evident that a gain in one of the parameters is usually accompanied by a loss in another; for instance, in the comparison of heat exchangers with the same frontal area, the ones that had an especially low power requirement had a large weight, and vice versa; or in the comparison on the basis of the same weight, the heat exchanger with a low power consumption generally had a large frontal area. In some cases it may be possible to circumvent this difficulty by inclining the heat-exchanger face at an angle to the direction of the oncoming air flow (fig. 11). The air flow is directed into the heat exchanger by a group of turning vanes. Another group of vanes redirects the air as it leaves the heat exchanger. In this arrangement, even when the cross section of the oncoming air stream is fixed, the frontal area of the heat exchanger can be varied by changing the angle through which the heat exchanger is turned from its normal position. This angle is identified as α (fig. 12). The advantage of such an arrangement, however, is restricted to heat exchangers with a length that is small relative to the width of the projected area of the heat-exchanger face; otherwise, the total width of the arrangement becomes considerably larger than the width of the approaching air stream (see fig. 11). In this report, the configurations are compared for equal projected face area; no attempt is made to evaluate the total width of the arrangement.

Turning the flow twice in its direction causes pressure losses, and the question arises as to how much these losses increase the power expenditure and through what angles various heat-exchanger configurations must be turned to create optimum conditions.

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Terms that describe the losses connected with the turning of the flow have to be added to the relations for the energy expended to direct the flow through the heat exchanger (eqs. (22) and (24)). Very little information is contained in the literature on losses in turning vanes that turn the flow through a considerable angle and at the same time decelerate it. For this reason, the turning losses will be introduced into the following optimization for a specific vane configuration for which the losses can be obtained by calculations when the flow is assumed to be incompressible. This configuration consists of a row of straight blades arranged parallel to the direction of the flow passages through the heat exchanger (fig. 12). If the conservation of momentum is considered for a control area, as indicated by the dashed block in the figure, the following result is obtained: The flow enters the control area through the plane 1-1' in the direction of the duct with a velocity V_0. The momentum flow in the direction of the heat-exchanger passages through plane 1-1' is given accordingly by the following expression:

$$mV_0 \cos \alpha$$

The flow leaves the control area through plane 2-2' with a velocity parallel to the heat-exchanger passages of magnitude V_{fr}. The corresponding momentum flow is

$$mV_{fr}$$

From the consideration of continuity, it follows that the two velocities V_0 and V_{fr} are connected by the following relation:

$$V_{fr} = V_0 \cos \alpha$$

This shows that the momentum flows in the heat-exchanger passage through planes 1-1' and 2-2' are equal. If the number of vanes in the duct is sufficient, it has to be expected that the flow conditions through plane 1-2 are exactly the same as through plane 1'-2' so that no net momentum transport into the control area through these planes occurs. Correspondingly, no change of momentum of the flow in passing through the control area occurs; and, as a consequence, the static pressures in the area 1-1' and the area 2-2' have to be equal. In other words, the decrease in velocity occurs without an increase in static pressure. After passing the heat exchanger, the flow has to be turned back in the direction of the duct, and at the same time it is accelerated to the velocity in the smaller duct area. This turning of the flow with the accompanying acceleration can be accomplished with very good efficiency by turning vanes so that in the present calculation it will be assumed that this turning is effected without losses. Correspondingly, Bernoulli's equation describes the pressure drop from the exit plane 3-3' to the plane 4-4' in the duct downstream of the heat-exchanger passage;

$$p_3 + \rho_e \frac{V_3^2}{2} = p_4 + \rho_e \frac{V_4^2}{2}$$

From the continuity it follows that

$$V_4 = \frac{V_3}{\cos \alpha}$$

Therefore,

$$p_3 - p_4 = \frac{\rho_e V_3^2}{2}\left(\frac{1}{\cos^2 \alpha} - 1\right) = \frac{\rho_c^2 V_c^2 \sigma^2}{2\rho_e} \tan^2 \alpha \tag{29}$$

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Because of the turning aspects alone, there is a drop in static pressure from p_1 to p_4 that is equal to $p_3 - p_4$. The difference in total pressure caused by the inlet vanes is found from $p_3 - p_4$ by adding the kinetic energy in plane 1-1', where the velocity is V_{fr}, and subtracting the kinetic energy in plane 4-4'. The total-pressure drop connected with the turning of the flow can be expressed by an equation of the form

$$\Delta P = K_t \rho_c \frac{V_c^2}{2} \tag{30}$$

analogous to the pressure drops for friction and acceleration. This results in the following values of the loss parameter:

$$K_t = \frac{\rho_c}{\rho_e} \sigma^2 \tan^2 \alpha = \frac{\sigma^2}{\cos^2 \alpha}\left(\frac{\rho_c}{\rho_e} - 1\right) \tag{31}$$

The losses calculated in this way should lead to an estimate that is high for the turning losses. By a good design of the decelerating vanes ahead of the heat exchanger, it should be possible to reduce these losses considerably. Addition of the term K_t into the square bracket on the right side of equations (22) to (24) makes them applicable to heat exchangers arranged at an angle α to the oncoming stream and equipped with turning vanes. With this addition, equation (24) becomes

$$\frac{N}{Q} = \frac{\mu_F^2}{2\rho_m^2 c_p(t_i' - t_i)} \frac{fRe^2}{d_h^2 \eta_0 St} \frac{Tu}{\eta_T} + \frac{\mu_F^2}{2\rho_c \rho_m c_p(t_i' - t_i)} \times$$

$$\frac{Re^2}{d_h^2}\left[K_c + \frac{\rho_c}{\rho_e} K_e + \left(\frac{\rho_c}{\rho_e} - 1\right) + \frac{\rho_c}{\rho_e} \sigma^2 \tan^2 \alpha\right] \frac{1}{\eta_T} \tag{32}$$

Equations (17) and (18) for Q/v and Q/W remain as previously presented. For the numerical evaluations in the next section, it is assumed that the heat capacity of the second fluid may be very large $(m'c_p' \gg mc_p)$ so that Tu as given by equation (8a) is now inserted into equations (17) and (18). It may also be specified that the temperature increase in the primary fluid in the heat exchanger is moderate, so that the acceleration-loss term $\frac{\rho_c}{\rho_e} - 1$ in equation (32) can be omitted.

OPTIMIZATION OF INCLINED HEAT EXCHANGERS

The optimization of heat exchangers is studied in the following way: It is assumed that different heat exchangers are to be compared which all have to transfer a certain amount of heat Q with prescribed initial temperature difference, prescribed state of the oncoming air flow, and prescribed thermal effectiveness. Also, the weight or the core volume of the heat exchanger will be assumed as fixed in the comparison. Additionally, the heat-capacity rate of the second fluid may be very large $(m'c_p' \gg mc_p)$ so that Tu is given by equation (8a). Figures 5 and 7 or figures 6 and 8 make it possible to determine the relative frontal areas of the heat-exchanger configurations that are considered for the optimization. The configuration that requires the smallest frontal area corresponding to the largest value of the parameter σRe or $\sigma Re/d_h$ is designated as the standard exchanger. It is assumed that this heat-exchanger core is arranged normal to the oncoming air stream $(\alpha = 0)$. All other heat exchangers then require a larger frontal area and have to be arranged at a certain angle to the duct when the duct area is required to be the same as for the standard exchanger. The angle at which the heat exchanger has to be arranged can be obtained from the condition that for a constant duct area and a constant cooling air mass flow, the two duct velocities $V_{0,a}$ and $V_{0,b}$ must be equal. Here, the subscript a refers to the standard exchanger and the subscript b to some other exchanger. The following equation results for the angle at which heat-exchanger b has to be arranged:

$$\cos \alpha_b = \frac{\sigma_b}{\sigma_a} \frac{V_{c,b}}{V_{c,a}} \tag{33}$$

Replacing the core velocities by the corresponding Reynolds numbers results in the following final equation:

$$\cos \alpha_b = \frac{\sigma_b Re_b}{\sigma_a Re_a} \frac{d_{h,a}}{d_{h,b}} \tag{34}$$

The properties μ and ρ are equal in both cases and, hence, disappear from this relation.

For a prescribed value Q/v or Q/W, the abscissa parameter in figures 7 to 10 can be determined from the appropriate equation (17) or (18). From the corresponding ordinate parameter and figures 5 and 6, the Reynolds number can be obtained. Equation (34) fixes the angle α through which each configuration must be turned to maintain the constant projected face area. When this angle is known, the power required per unit of heat transferred can be calculated from equation (32) with acceleration loss omitted. The values N/Q may now be compared for the different configurations, and that heat-exchanger configuration which requires the least power has to be considered as the optimum configuration. 3983
Such a comparison has been made and is presented in figures 13 to 16. Values for K_c were taken from reference 14 (fig. 52) and various handbooks. The dashed line in each figure represents the standard exchanger (exchanger 2, fig. 3) arranged normal to the main flow ($\alpha = 0$). All other exchangers must be placed at an angle to the duct to obtain the same duct cross-sectional area. Angles are indicated for several values of the abscissa. At each abscissa location, the angles reading from top to bottom refer to the curves reading from top to bottom (e.g., in fig. 13(a), at a value of the abscissa of 10, heat exchanger 1 must be inclined at an angle of 53.2°).

In some cases it may be noted that one or more of the exchangers is missing from the comparison. This situation occurs when the friction and Stanton parameters are not known in the required Reynolds number range. Three plots, which correspond to three values of the thermal effectiveness of the heat exchanger (0.3, 0.5, and 0.7), are presented in each figure.

Figure 13 shows the value that is proportional to N/Q for heat exchangers plotted against a parameter proportional to Q/v with the assumption that the different heat-exchanger configurations are compared for the hydraulic diameters for which each configuration has been investigated (ref. 11 or fig. 3). Figure 14 presents the same comparison when the various configurations have the same hydraulic diameter. Figures 15 and 16 show the results of an analogous comparison based on equal weight for the various heat exchangers.

From figures 13 and 14 it can be observed that, for fixed values of the abscissa, increases in the thermal-effectivenss parameter η_T result in increased values of required power. For instance, figures 13(a), (b), and (c) show that for an abscissa value of 8×10^5 (proportional to Q/v) the values proportional to N/Q corresponding to $\eta_T = 0.3$, 0.5, and 0.7 for the standard heat exchanger are about 1500, 2000, and 4000×10^{10}, respectively. Moreover, for those heat-exchanger configurations in figures 13 and 14 that show crossover, the points of crossover occur at smaller values of the abscissa as the thermal effectiveness parameter η_T increases. Similar results are also noticeable in figures 15 and 16.

Figures 13 to 16 show that the hydraulic diameter should be considered in the optimization of inclined heat exchangers as well as in normal heat exchangers. It can also be recognized from the figures that considerable savings in the power expended can be obtained by a proper choice of the exchanger configuration and by inclined orientation in the duct for the primary fluid, especially when the heat-exchanger weight is of primary importance. In figure 15(a), for instance, the best configuration consumes a power that is approximately one-fifth to one-eighth the power required for the configuration with largest ordinate (proportional to N/Q). In this connection it should be remembered that turning losses were estimated on a conservative basis. Even larger savings in power will be experienced when the turning losses can be reduced by proper turning vanes.

SUMMARY OF RESULTS

The selection of an optimum configuration for a heat exchanger with one dominating film resistance was discussed. The amount of heat transferred per unit time and the mass flow and inlet state of each fluid were prescribed. Power required to drive the primary fluid was optimized with respect to weight, volume, or frontal area for a group of heat exchangers presented in references 11 and 12 as illustrative examples. Results obtained from these optimizations are summarized as follows:

1. No heat-exchanger configuration was found which can be considered the best under all conditions. It is probable that such an optimum configuration does not exist.

2. For a given volume, the heat exchanger that required the least power to drive the primary fluid had the smallest frontal velocity (or the largest frontal area).

3. A heat exchanger that is good with respect to volume is usually also good with respect to weight.

4. In the comparison of heat exchangers with the same frontal area, geometries that produce smooth flow without separation require the least power.

5. In the comparison of heat exchangers with equal volume or weight, geometries requiring the least power are those which encounter a fair amount of flow disturbance and separation.

6. For given weight or volume, the power required to drive the primary fluid through an exchanger inclined at an angle to the approaching fluid is less than that required for a minimum duct-area exchanger arranged normal to the flow when the projected frontal area of the

inclined exchanger equals the minimum duct area of the normal exchanger. This advantage is sufficiently large that it will still persist when the projected frontal area is only moderately larger than the duct area.

7. The effect of change in scale of the various configurations was determined by optimizing for assumed equal hydraulic diameters. The results showed that a change in scale altered the order of preference of the exchanger when minimum power is required for a prescribed weight, volume, or frontal area. 3983

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, February 29, 1956

APPENDIX - SYMBOLS

The following symbols are used in this report:

A	heat-transferring area
A_c	free-flow area
A_f	fin area
A_{fr}	frontal area
C	circumference of passage in heat exchanger
c_p	specific heat at constant pressure of primary fluid
c_p'	specific heat at constant pressure of secondary fluid
d_h	hydraulic diameter
f	friction factor
h	heat-transfer coefficient
K	pressure-drop
k	thermal conductivity
L	duct length
l	one-half fin length
m	mass flow of primary fluid
m'	mass flow of secondary fluid
N	power required
P	total pressure
Pr	Prandtl number, $c_p\mu/k$
p	static pressure
Q	heat flow transferred in exchanger
r_h	hydraulic radius, A_c/C

Re	Reynolds number, $\rho V d_h/\mu$
St	Stanton number, $h/c_p\rho V$
s	thickness of exchanger walls
Tu	number of transfer units
t	primary fluid temperature
t_w	wall temperature
t'	secondary fluid temperature
U	over-all heat-transfer coefficient
V	velocity
v	heat-exchanger core volume
W	heat-exchanger weight
α	angle of heat exchanger towards flow direction
η_o	surface effectiveness
η_f	fin effectiveness, $\dfrac{\tanh\sqrt{\frac{2h}{k\tau}}\,l}{\sqrt{\frac{2h}{k\tau}}\,l}$
η_T	thermal effectiveness
μ	viscosity
ρ	density of air
σ	contraction of heat-exchanger passages, A_c/A_{fr}
τ	fin thickness

Subscripts:

a	standard heat exchangers ($\alpha = 0$)
acc	acceleration

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b	heat exchanger other than standard exchanger ($\alpha \neq 0$)
c	see fig. 1
e	exit
end	end
F	at film temperature
f	fin
fr	frontal
friction	friction
i	inlet
m	mean
t	turning
0,1,1', 2,2', 3,3', 4,4'	see fig. 12

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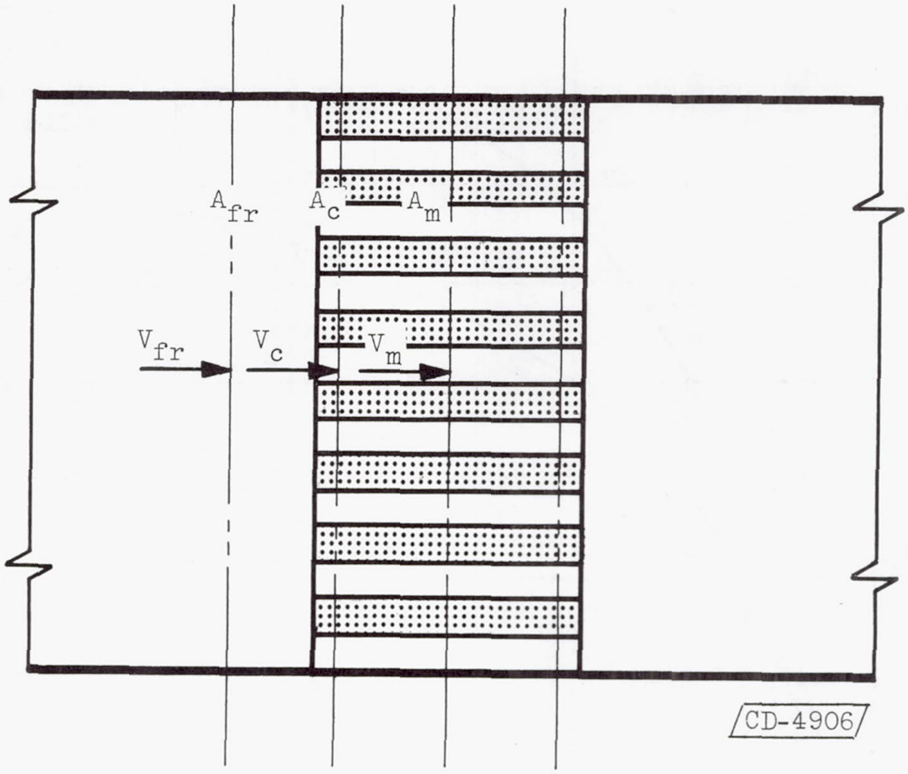

Figure 1. - Schematic representation of heat exchanger and flow passages.

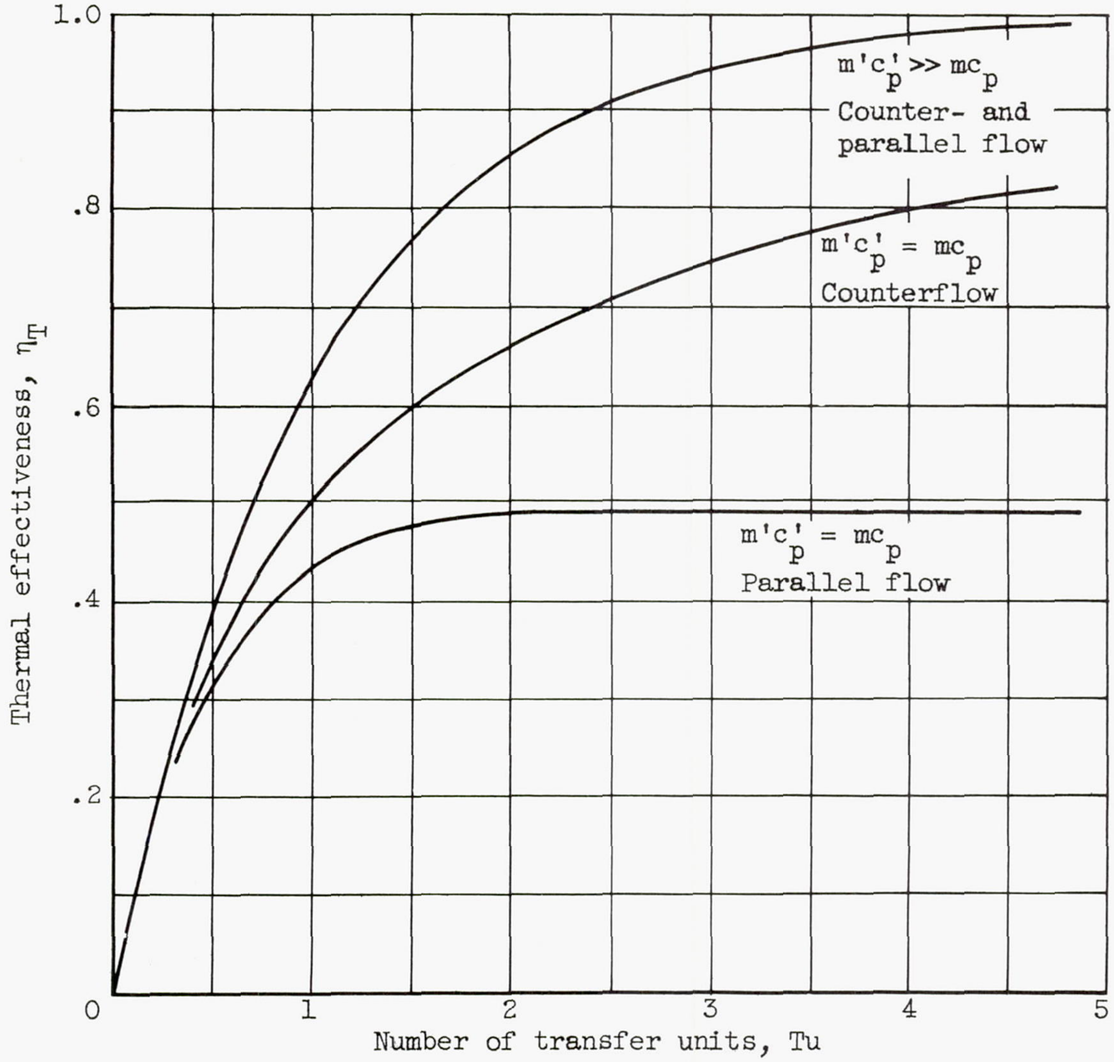

Figure 2. - Thermal effectiveness against number of transfer units for various flow arrangements (ref. 11).

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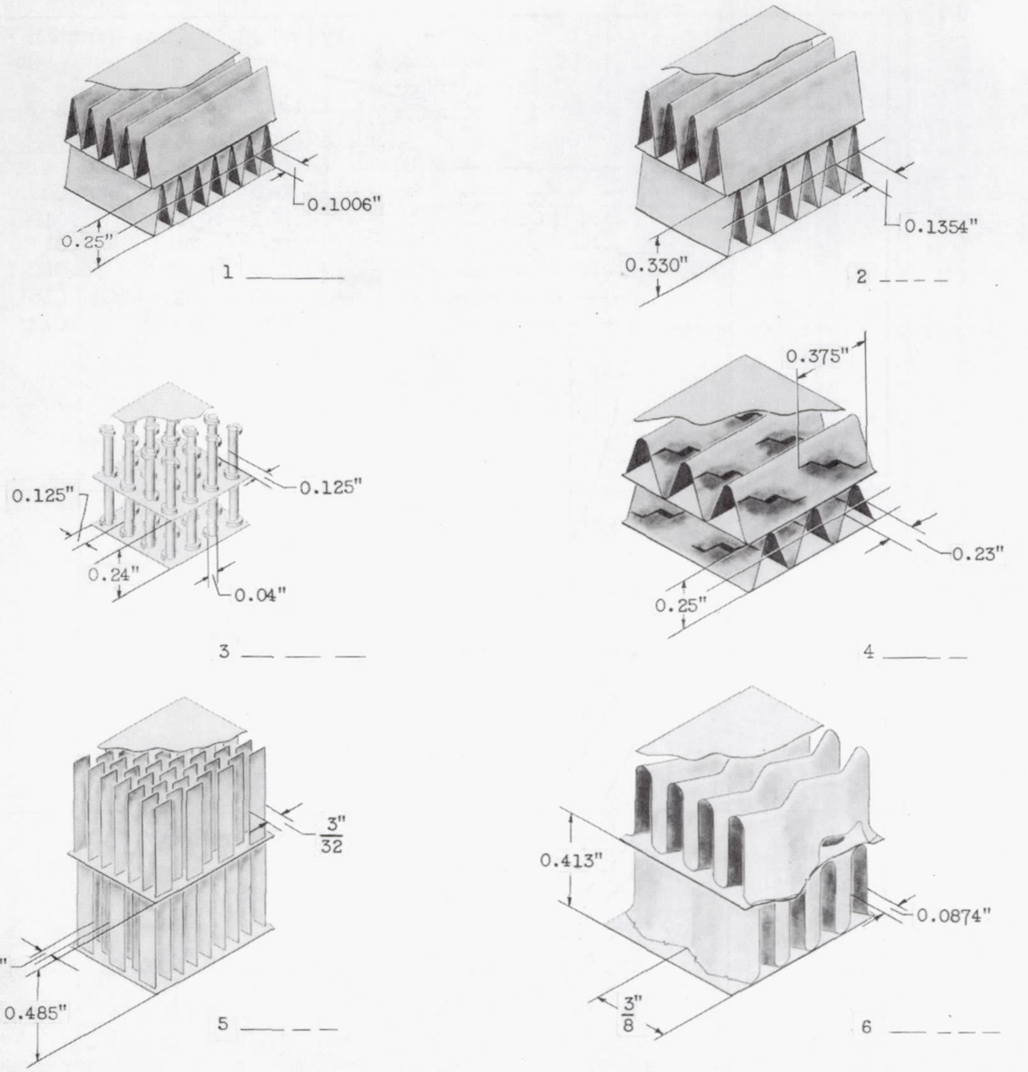

Designation, ref. 11 or 12	Herein	Free-flow to frontal area, σ	Hydraulic diameter, d_h
19.86	1	0.409	0.00598
14.77	2	.437	.00848
AP-1	3	.334	.01494
3/8 in. - 8.7	4	.448	.01196
3/32 - 12.22	5	.472	.0112
11.44 - 3/8 W	6	.458	.00696
9.68-0.87	7	.697	.0118

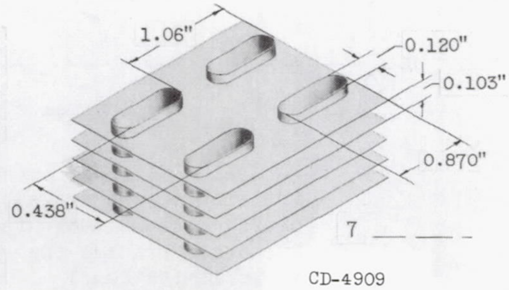

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Figure 3. - Configurations and dimensions of heat exchangers used in this investigation.

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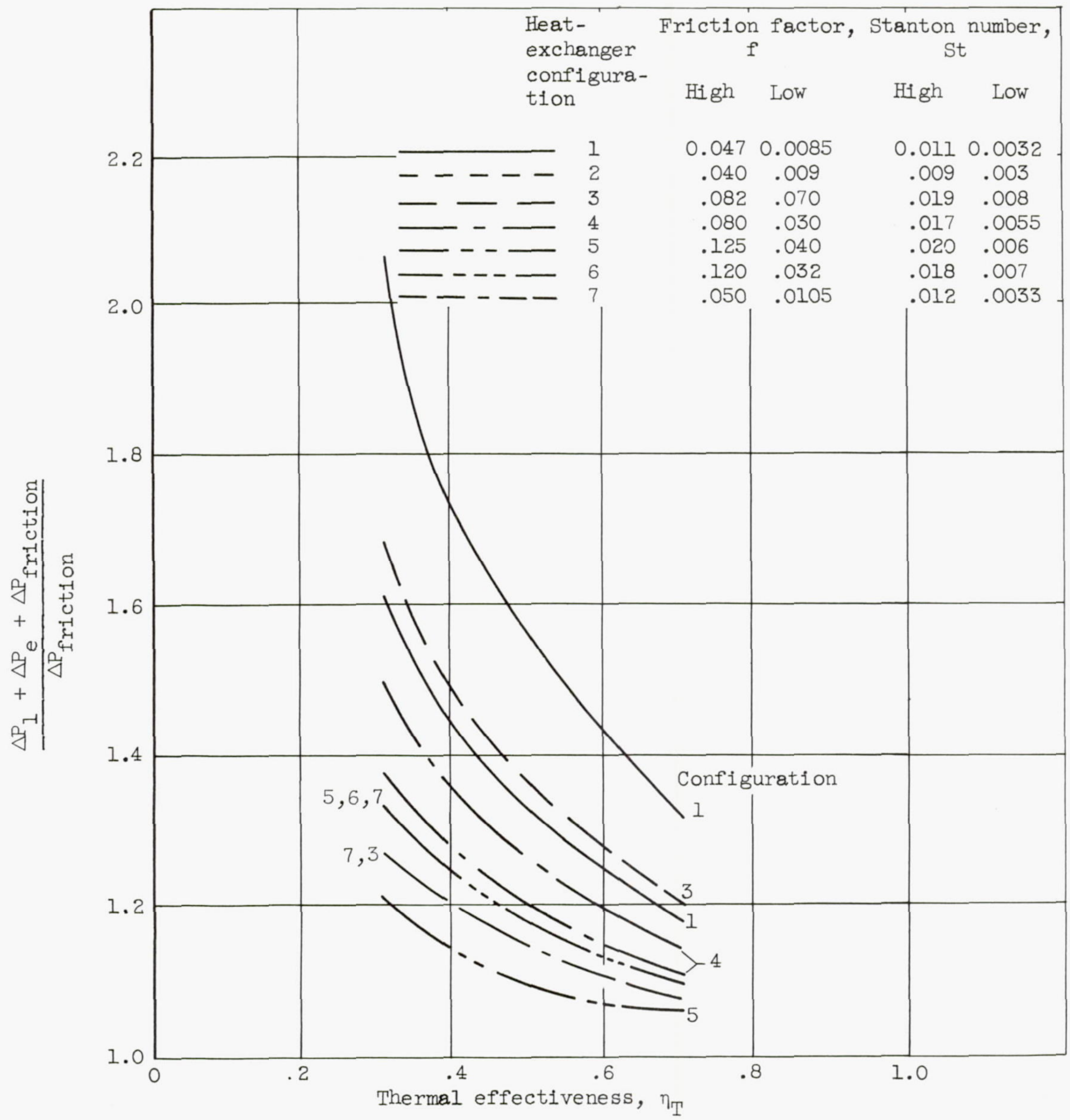

Figure 4. - Ratio of entrance, friction, and exit pressure losses to friction losses against thermal effectiveness for heat-exchanger configurations shown in figure 3. (The two curves for each heat exchanger bracket the range; some curves are omitted because of range limitation.)

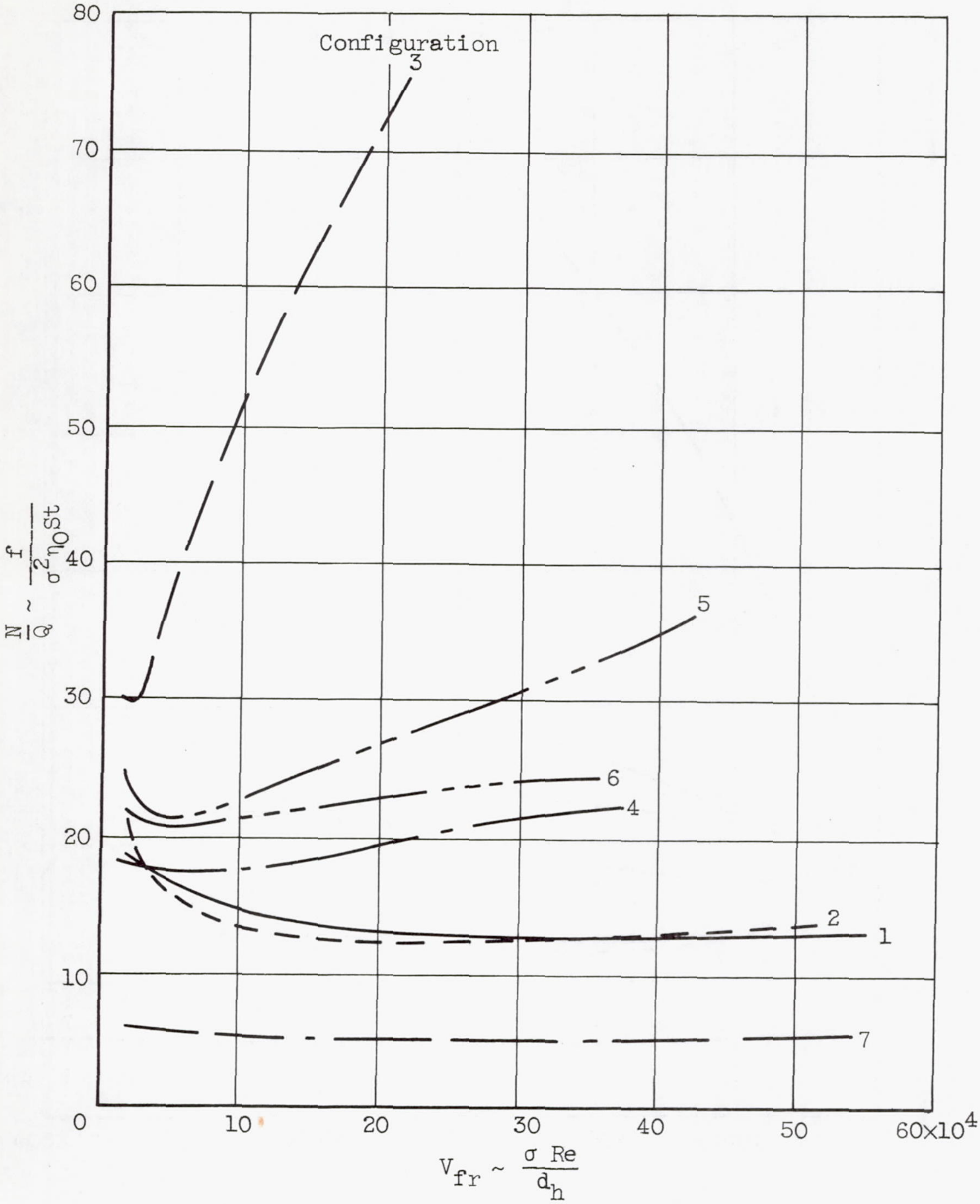

Figure 5. - Performance parameter of heat exchangers against frontal area for hydraulic diameters of configurations shown in figure 3. Primary fluid, air; frontal area inversely proportional to frontal velocity.

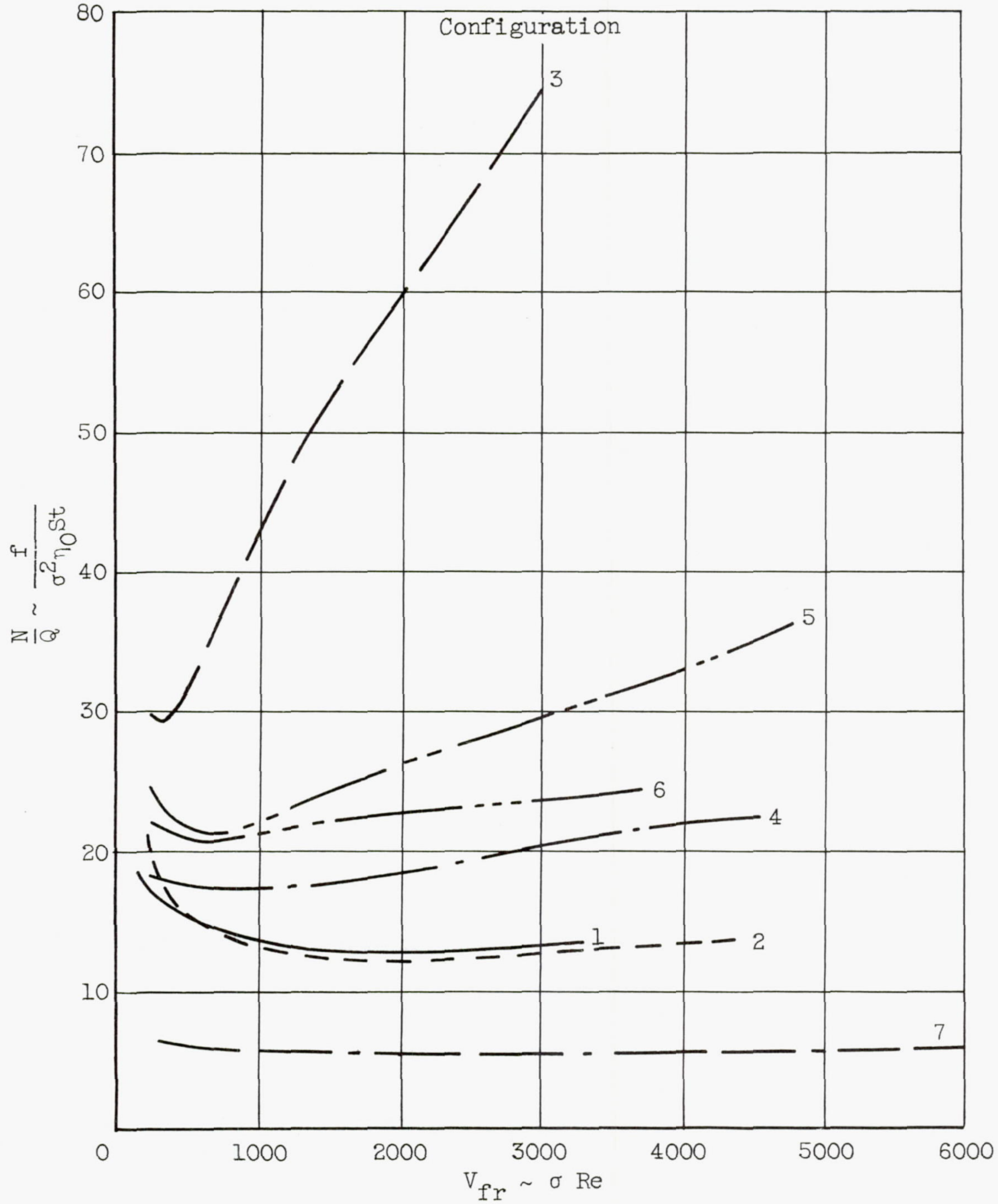

Figure 6. - Performance parameter of heat exchangers against frontal area on basis of equal hydraulic diameters for all exchangers. Primary fluid, air; frontal area inversely proportional to frontal velocity.

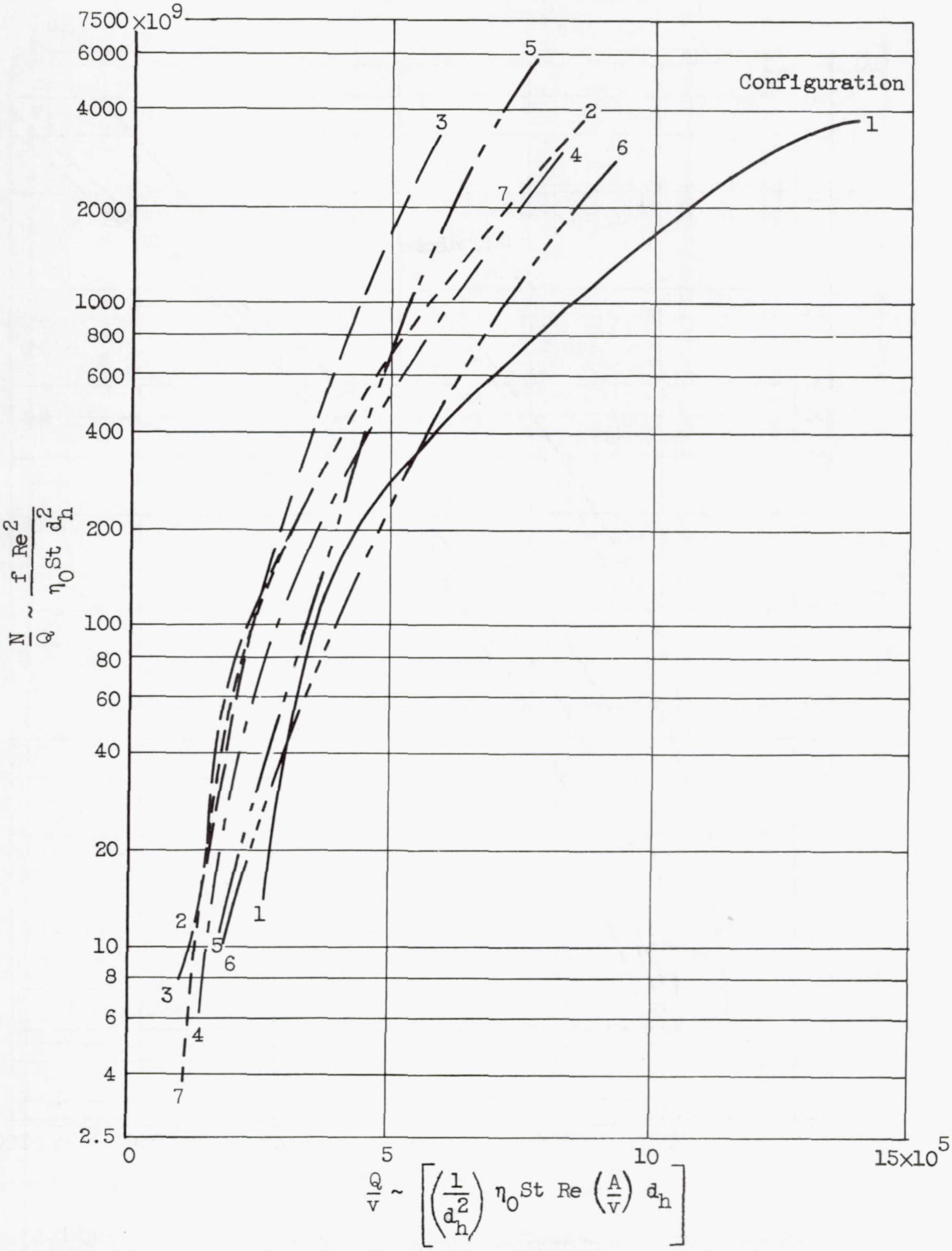

Figure 7. - Performance parameter of heat exchangers against core-volume parameter for hydraulic diameters of configurations shown in figure 3. Primary fluid, air.

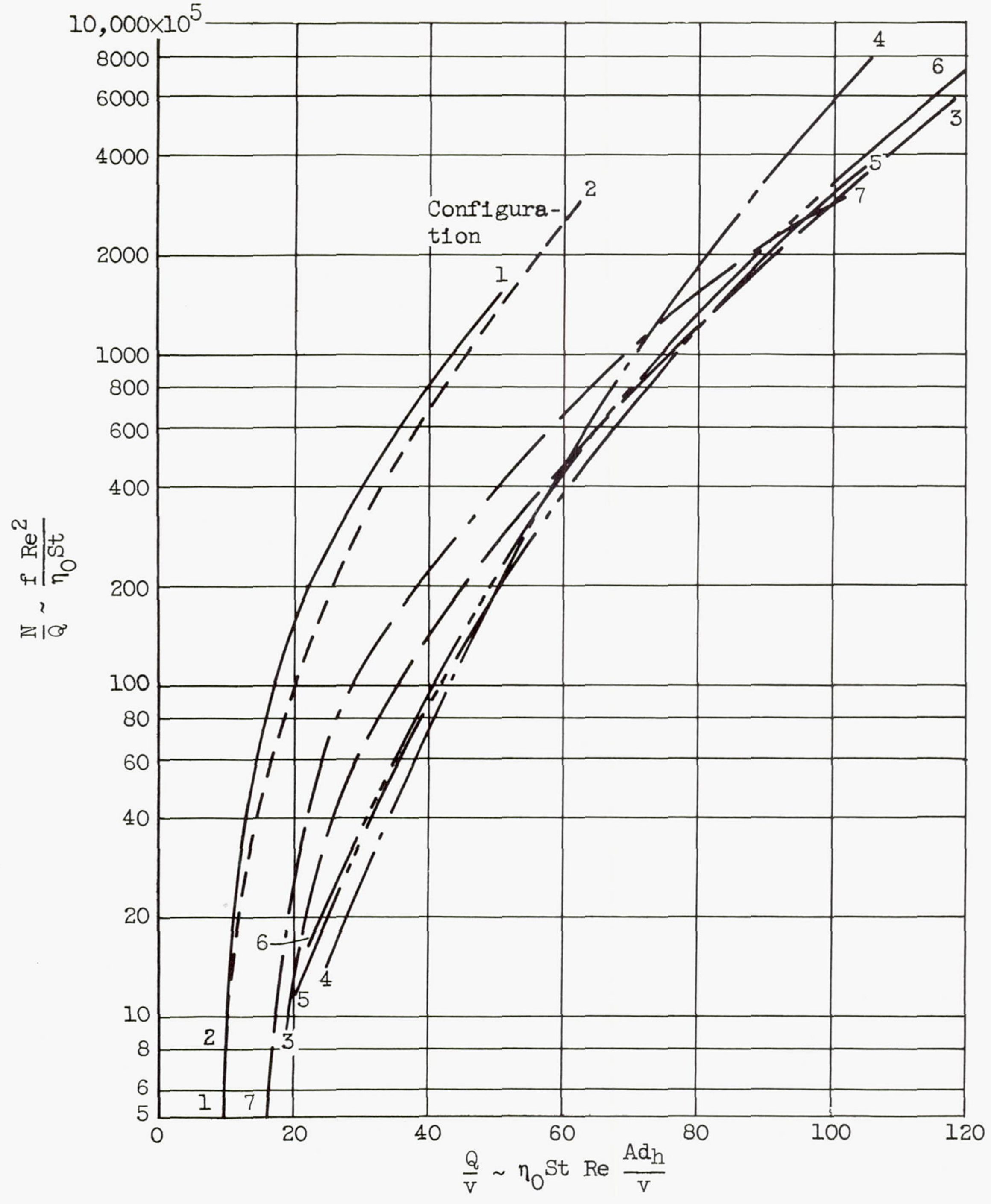

Figure 8. - Performance parameter of heat exchangers against core-volume parameter using equal hydraulic diameters for all exchangers. Primary fluid, air.

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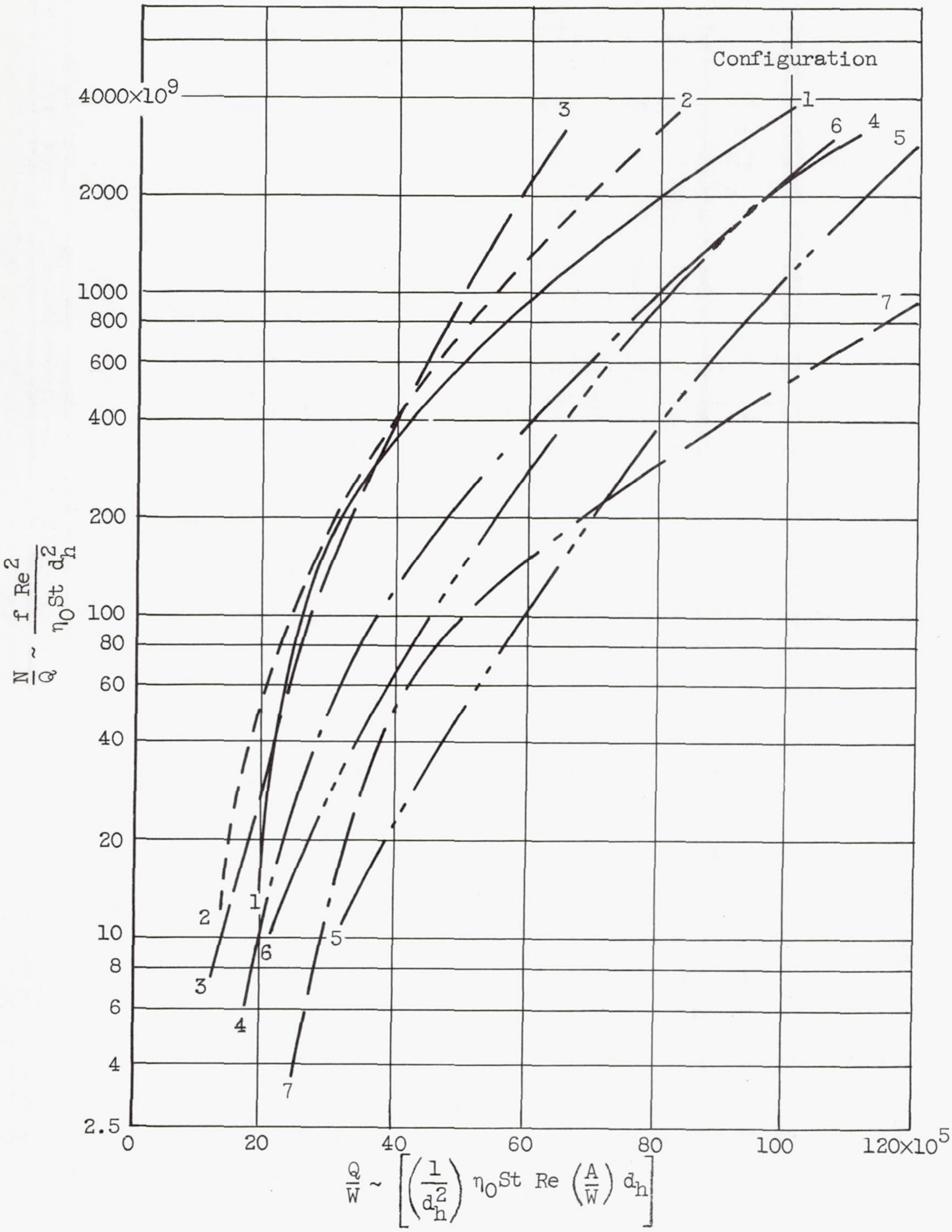

Figure 9. - Performance parameter of heat exchangers against heat-exchanger weight parameter using hydraulic diameters of configurations shown in figure 3. Primary fluid, air.

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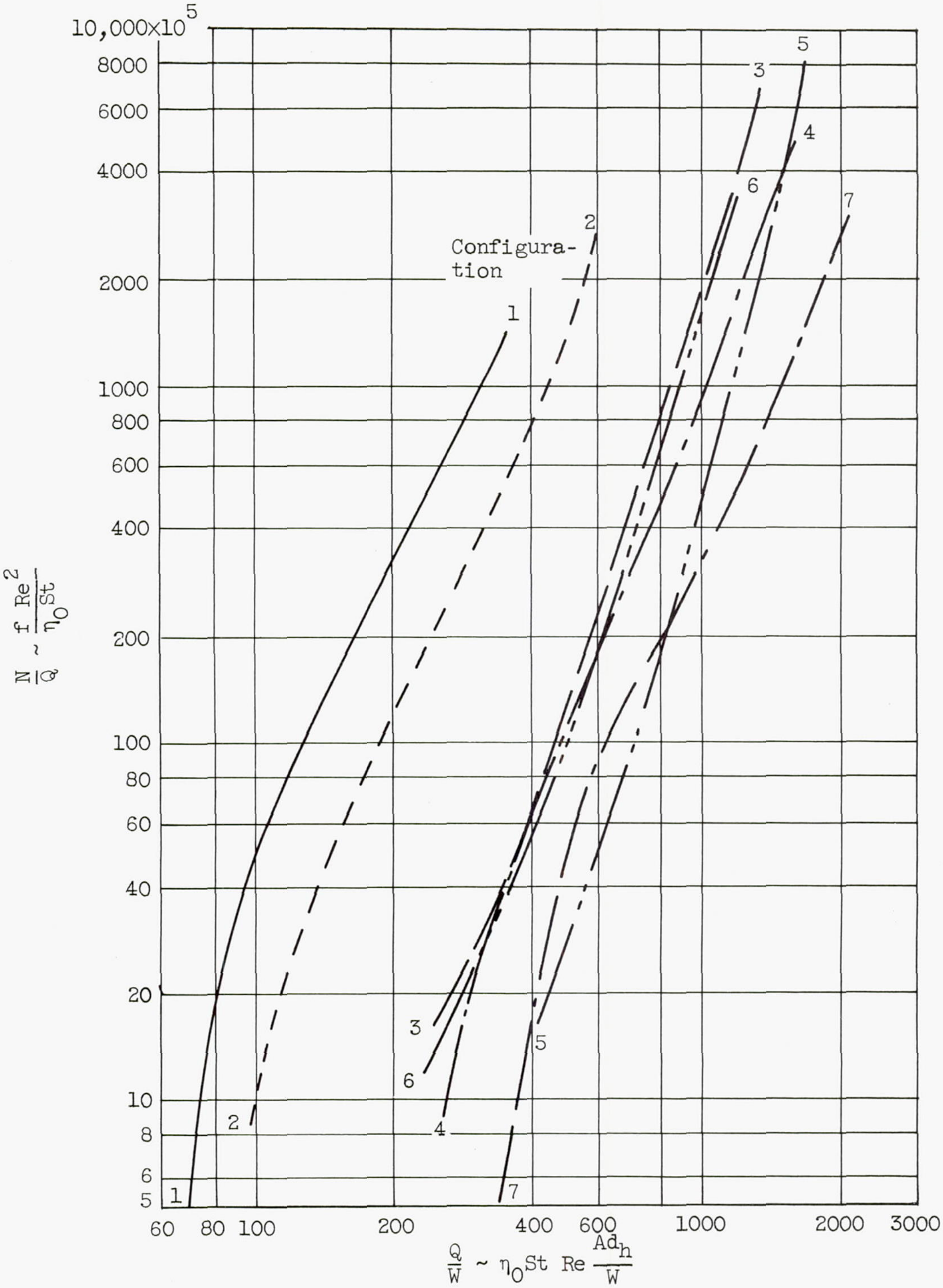


Figure 10. - Performance parameter of heat exchangers against heat-exchanger weight parameter using equal hydraulic diameters for all exchangers. Primary fluid, air.

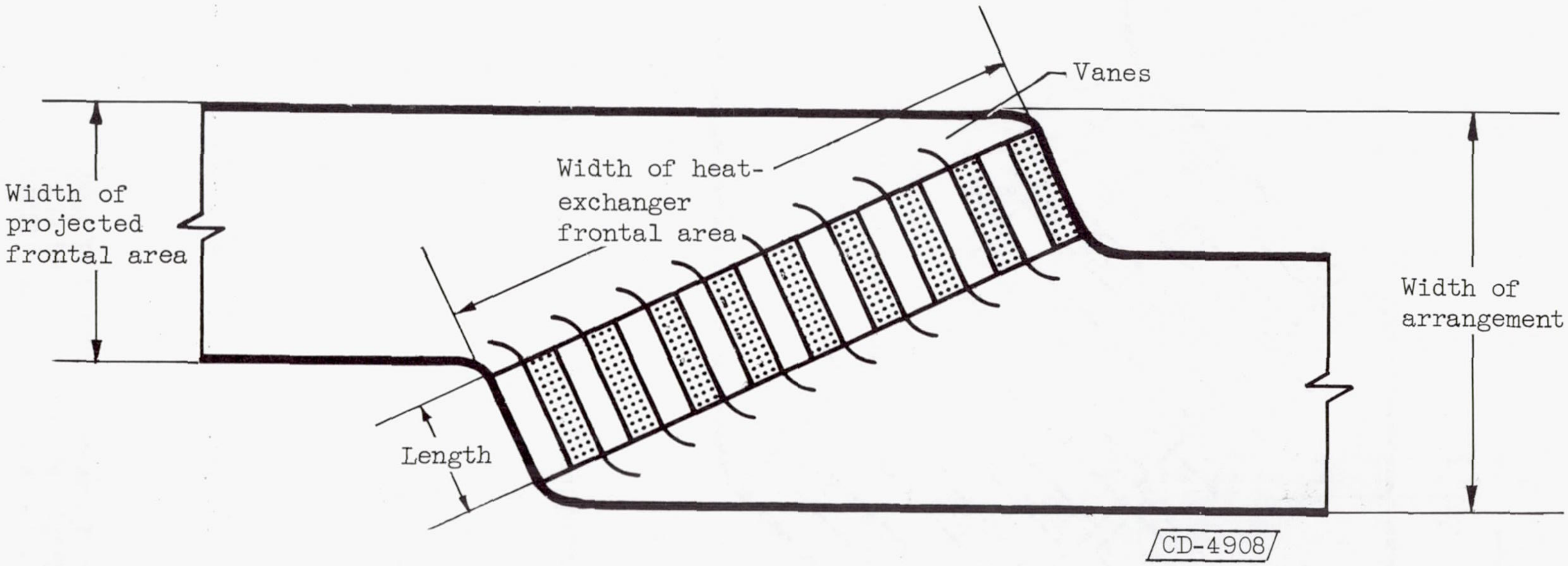


Figure 11. - Schematic representation of heat exchanger arranged at angle in flow passage.

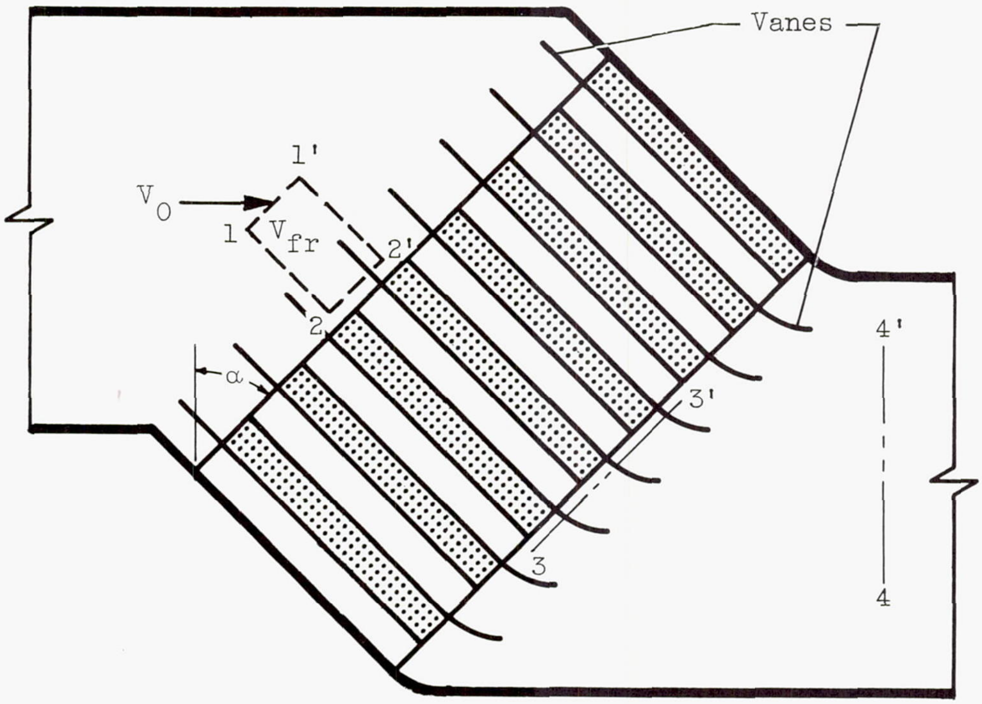
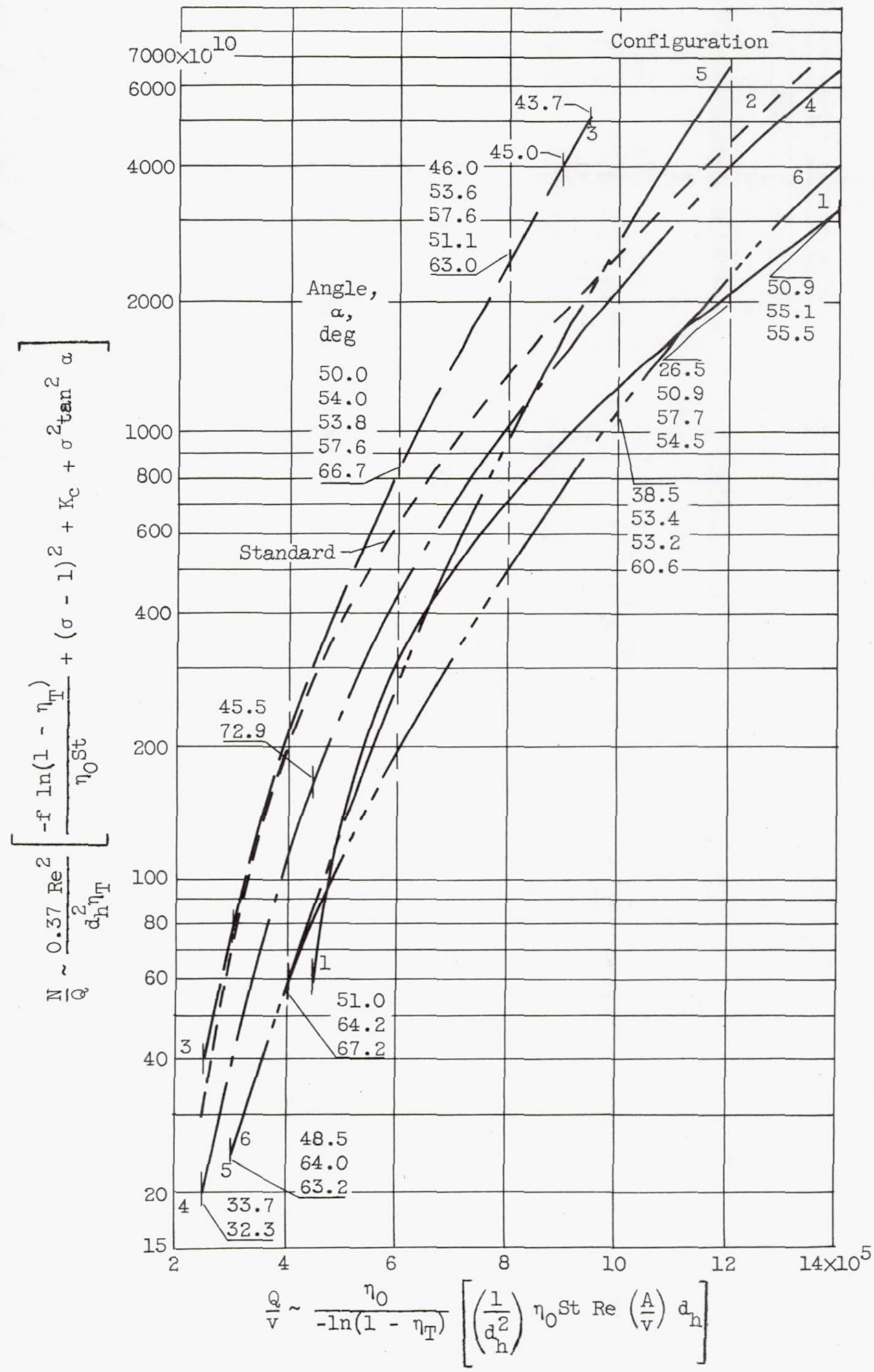


Figure 12. - Assumed configuration for calculation of turning losses.



(a) Thermal effectiveness, 0.3.

Figure 13. - Performance parameter of inclined heat exchangers against core-volume parameter for hydraulic diameters of configurations shown in figure 3. Primary fluid, air; $mc_p << m'c_p'$.

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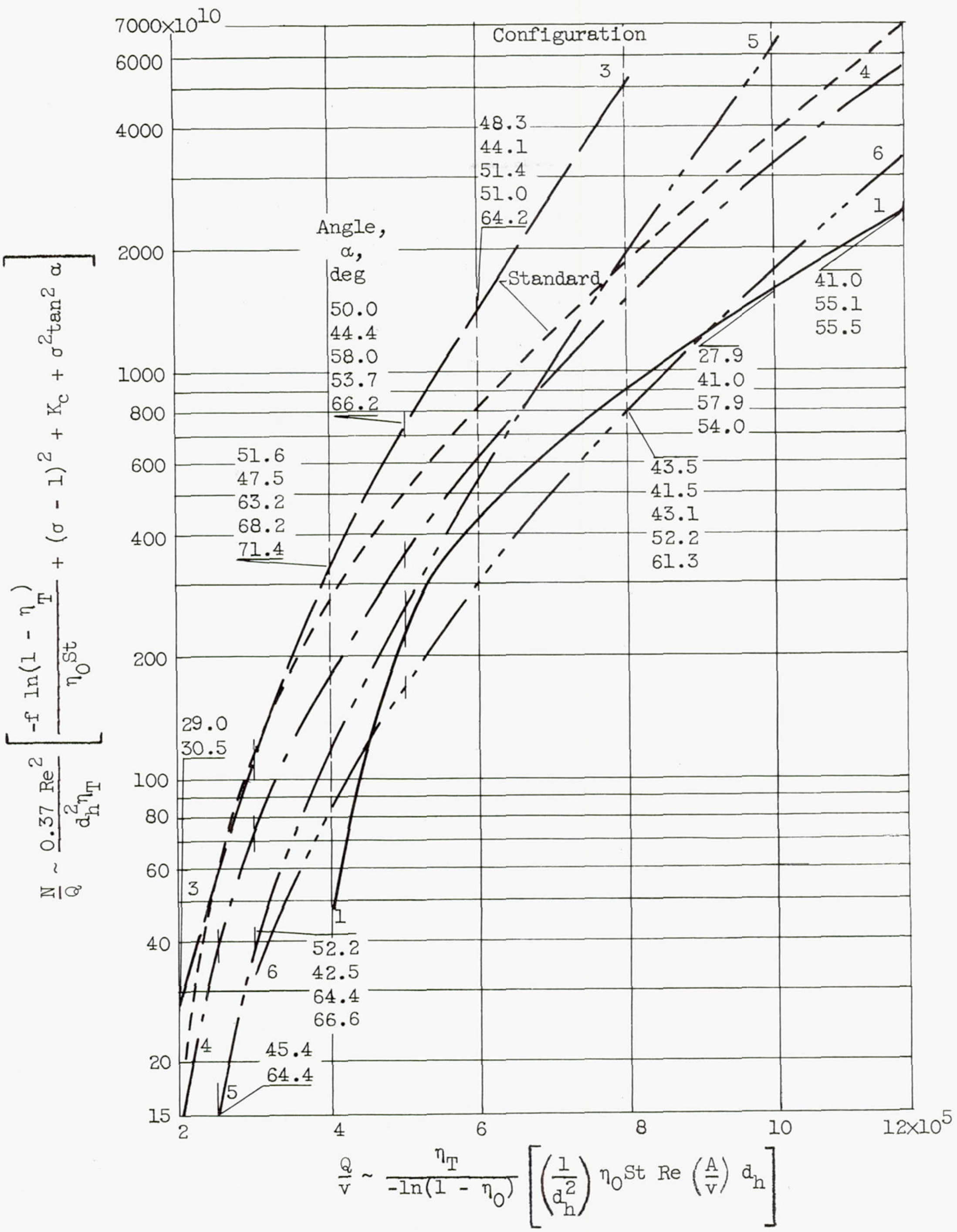
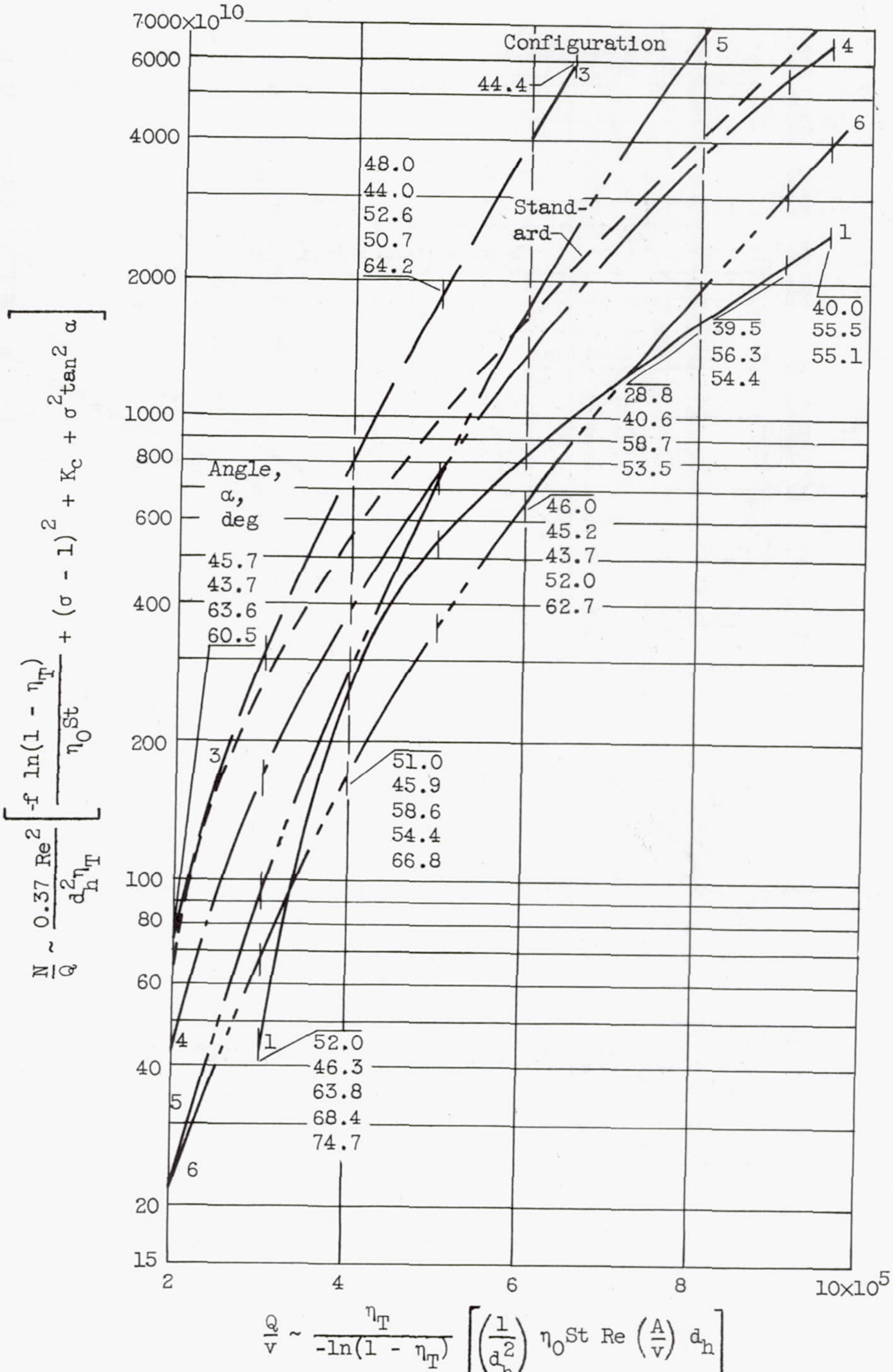


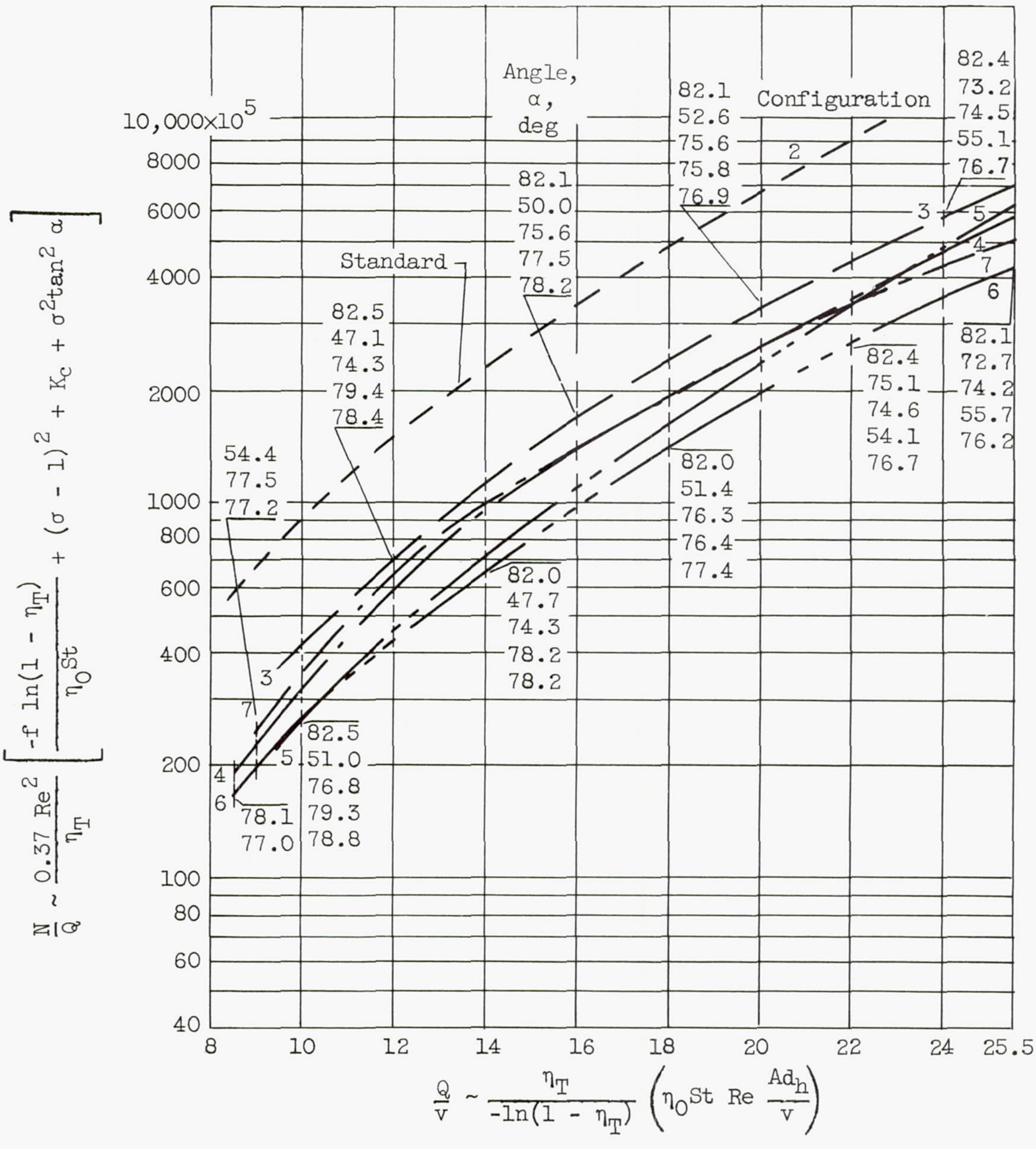
(b) Thermal effectiveness, 0.5.

Figure 13. - Continued. Performance parameter of inclined heat exchangers against core-volume parameter for hydraulic diameters of configurations shown in figure 3. Primary fluid, air; $mc_p << m'c_p'$.



(c) Thermal effectiveness, 0.7.

Figure 13. - Concluded. Performance parameter of inclined heat exchangers against core-volume parameter for hydraulic diameters of configurations shown in figure 3. Primary fluid, air; $mc_p << m'c_p'$.

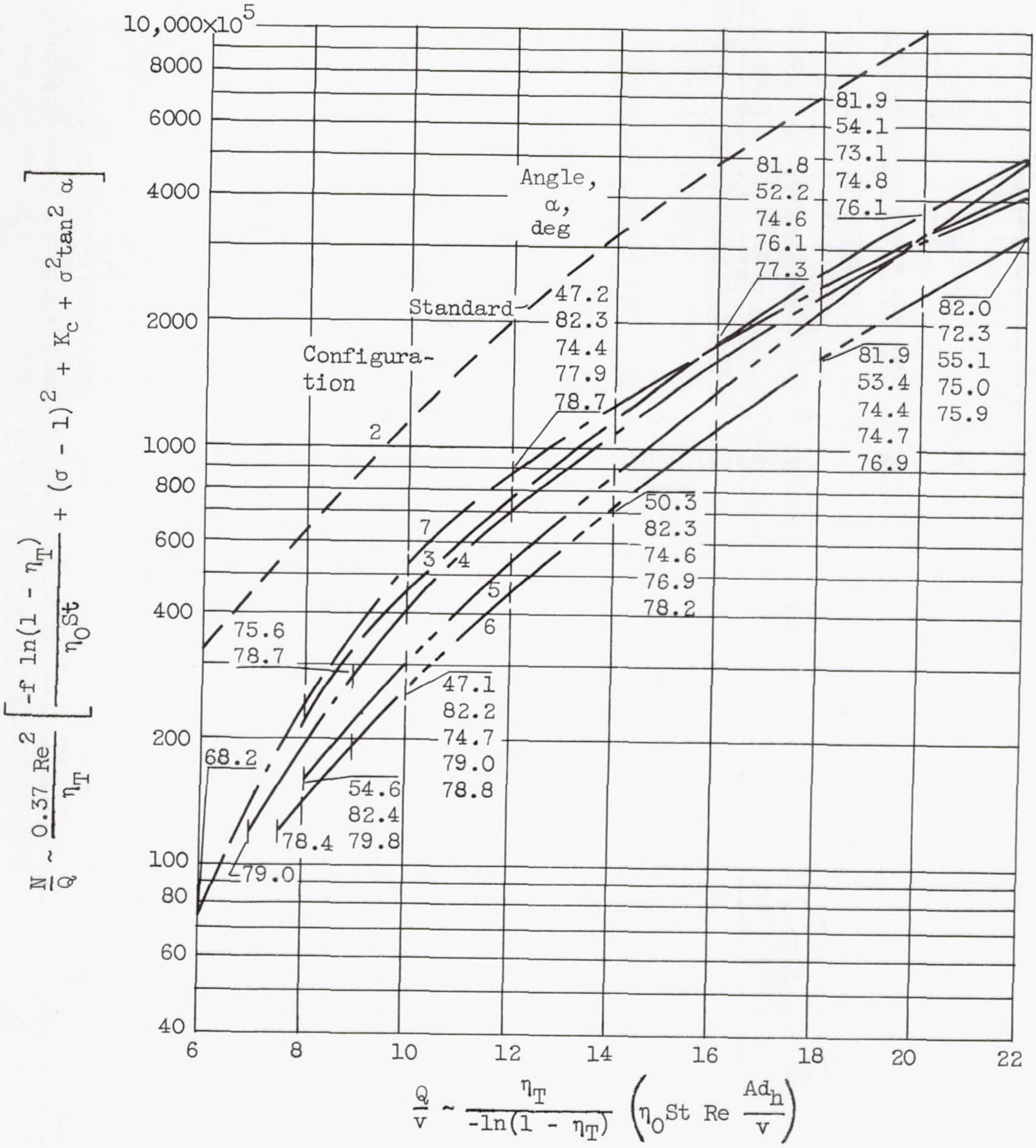


$$\frac{N}{Q} \sim \frac{0.37\ Re^2}{\eta_T}\left[\frac{-f\ \ln(1-\eta_T)}{\eta_O St} + (\sigma - 1)^2 + K_c + \sigma^2 \tan^2 \alpha\right]$$

$$\frac{Q}{v} \sim \frac{\eta_T}{-\ln(1-\eta_T)}\left(\eta_O St\ Re\ \frac{Ad_h}{v}\right)$$

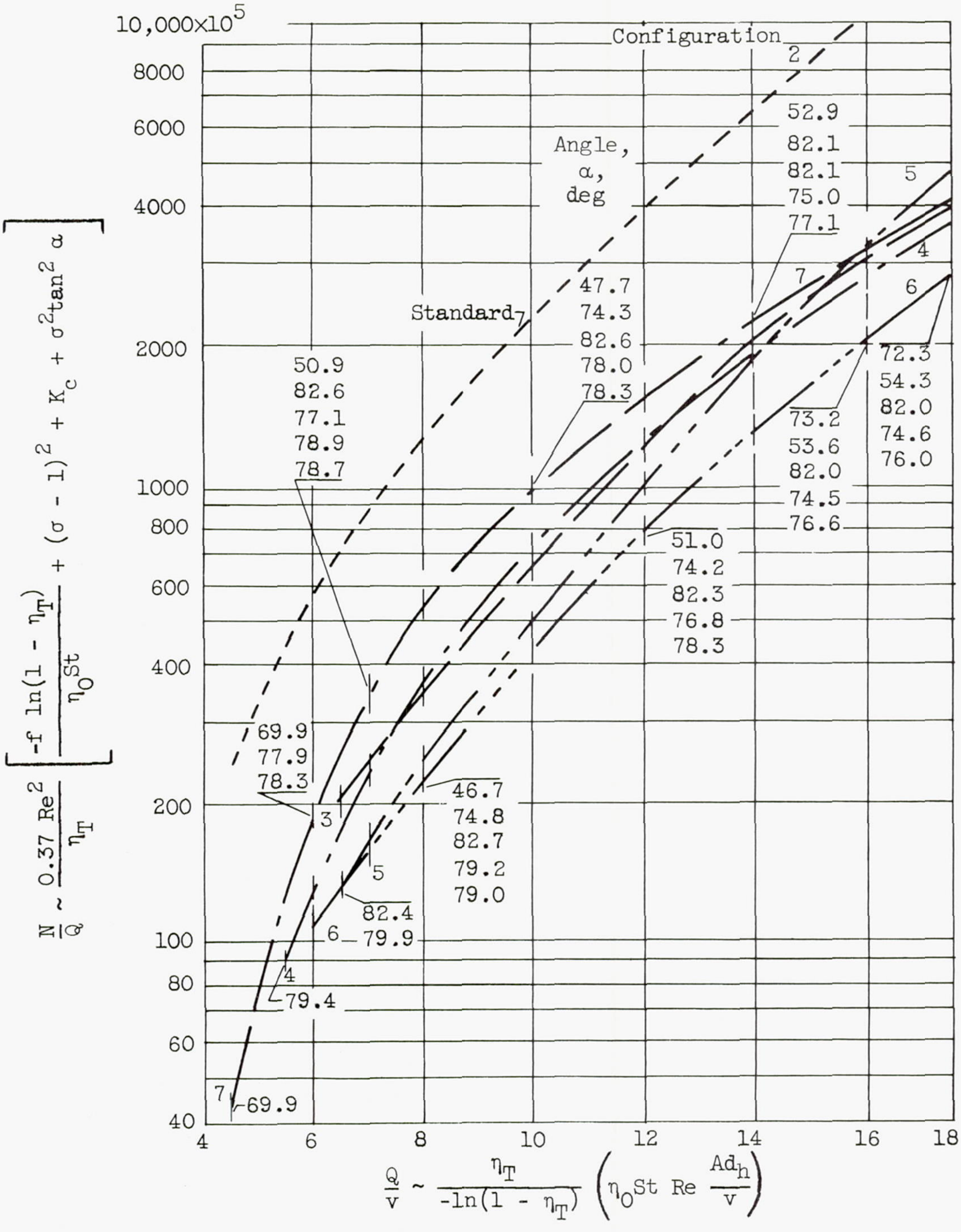
(a) Thermal effectiveness, 0.3.

Figure 14. - Performance parameter of inclined heat exchangers against core-volume parameter using equal hydraulic diameters for all exchangers. Primary fluid, air; $mc_p << m'c_p'$.



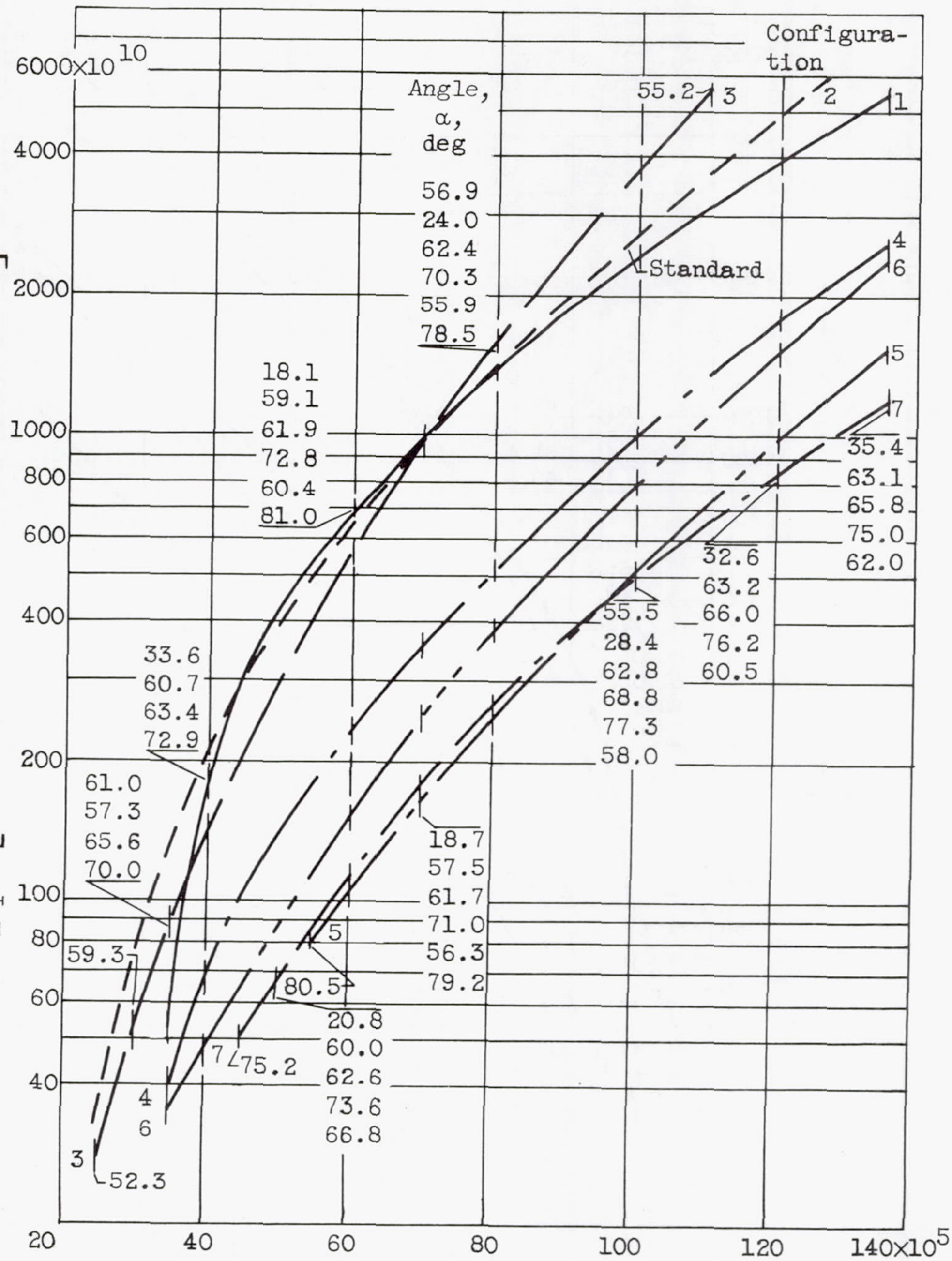
(b) Thermal effectiveness, 0.5.

Figure 14. - Continued. Performance parameter of inclined heat exchangers against core-volume parameter using equal hydraulic diameters for all exchangers. Primary fluid, air; $mc_p << m'c_p'$.



(c) Thermal effectiveness, 0.7.

Figure 14. - Concluded. Performance parameter of inclined heat exchangers against core-volume parameter using equal hydraulic diameters for all exchangers. Primary fluid, air; $mc_p \ll m'c'_p$.

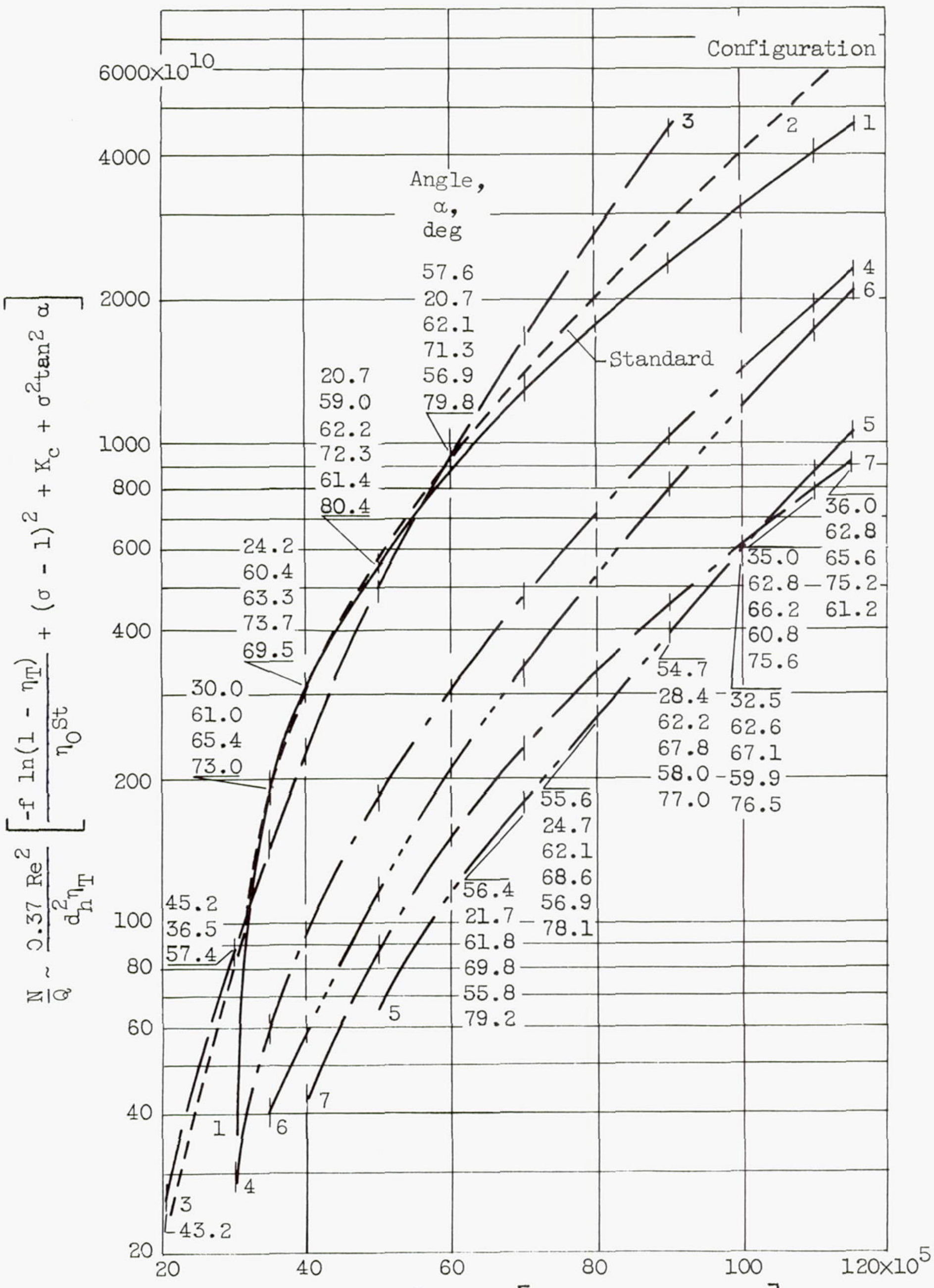


$$\frac{N}{Q} \sim \frac{0.37\ Re^2}{d_h^2 \eta_T}\left[\frac{-f\ \ln(1-\eta_T)}{\eta_O St} + (\sigma - 1)^2 + K_c + \sigma^2 \tan^2 \alpha\right]$$

$$\frac{Q}{W} \sim \frac{\eta_T}{-\ln(1-\eta_T)}\left[\left(\frac{1}{d_h^2}\right)\eta_O St\ Re\left(\frac{A}{W}\right)d_h\right]$$

(a) Thermal effectiveness, 0.3.

Figure 15. - Performance parameter of inclined heat exchangers against core-weight parameter for hydraulic diameters of configurations shown in figure 3. Primary fluid, air; $mc_p << m'c_p'$.

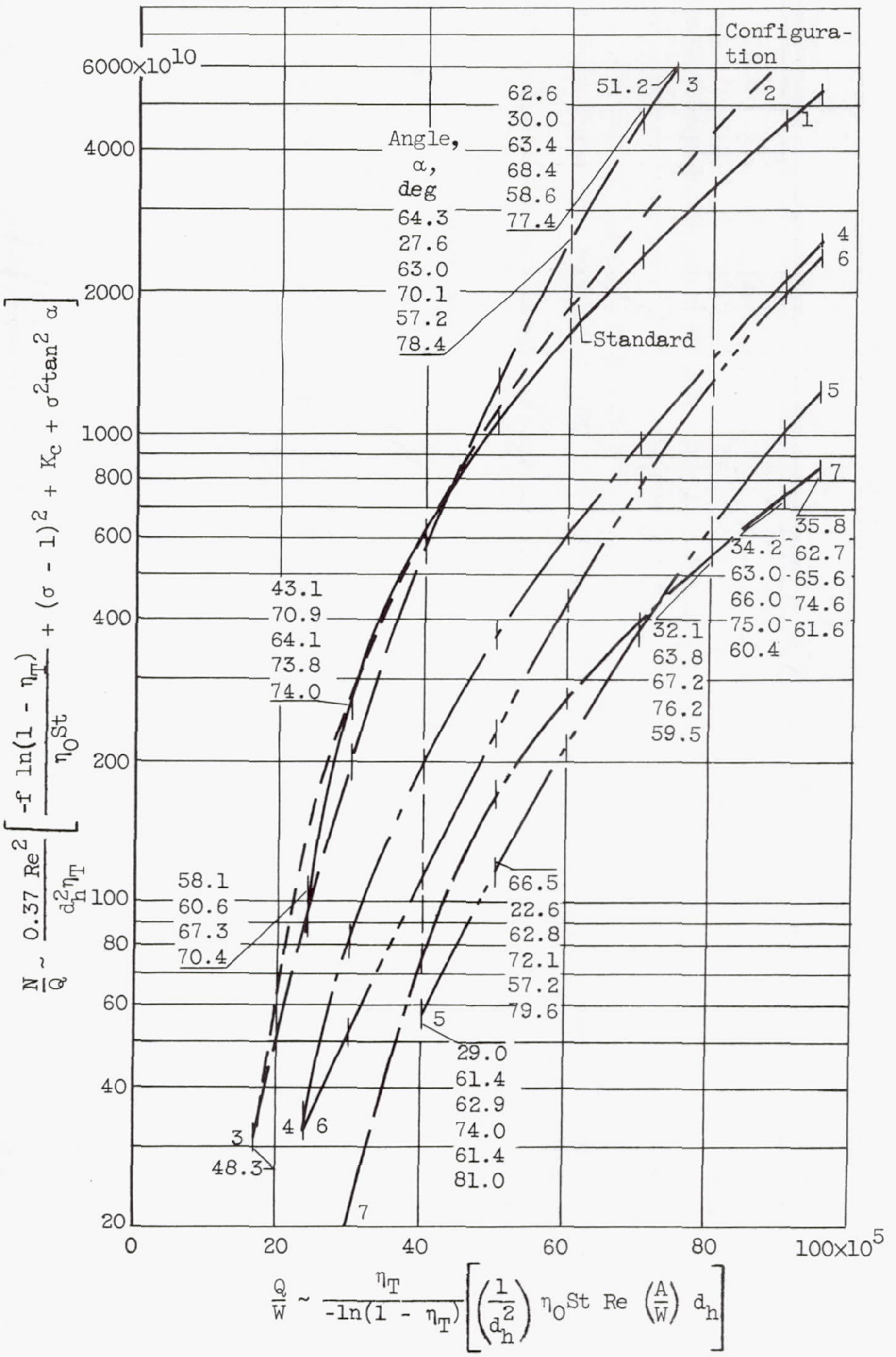


$$\frac{N}{Q} \sim \frac{0.37\ Re^2}{d_h^2 \eta_T} \left[\frac{-f \ln(1 - \eta_T)}{\eta_O St} + (\sigma - 1)^2 + K_c + \sigma^2 \tan^2 \alpha \right]$$

$$\frac{Q}{W} \sim \frac{\eta_T}{-\ln(1 - \eta_T)} \left[\left(\frac{1}{d_h^2}\right) \eta_O St\ Re \left(\frac{A}{W}\right) d_h \right]$$

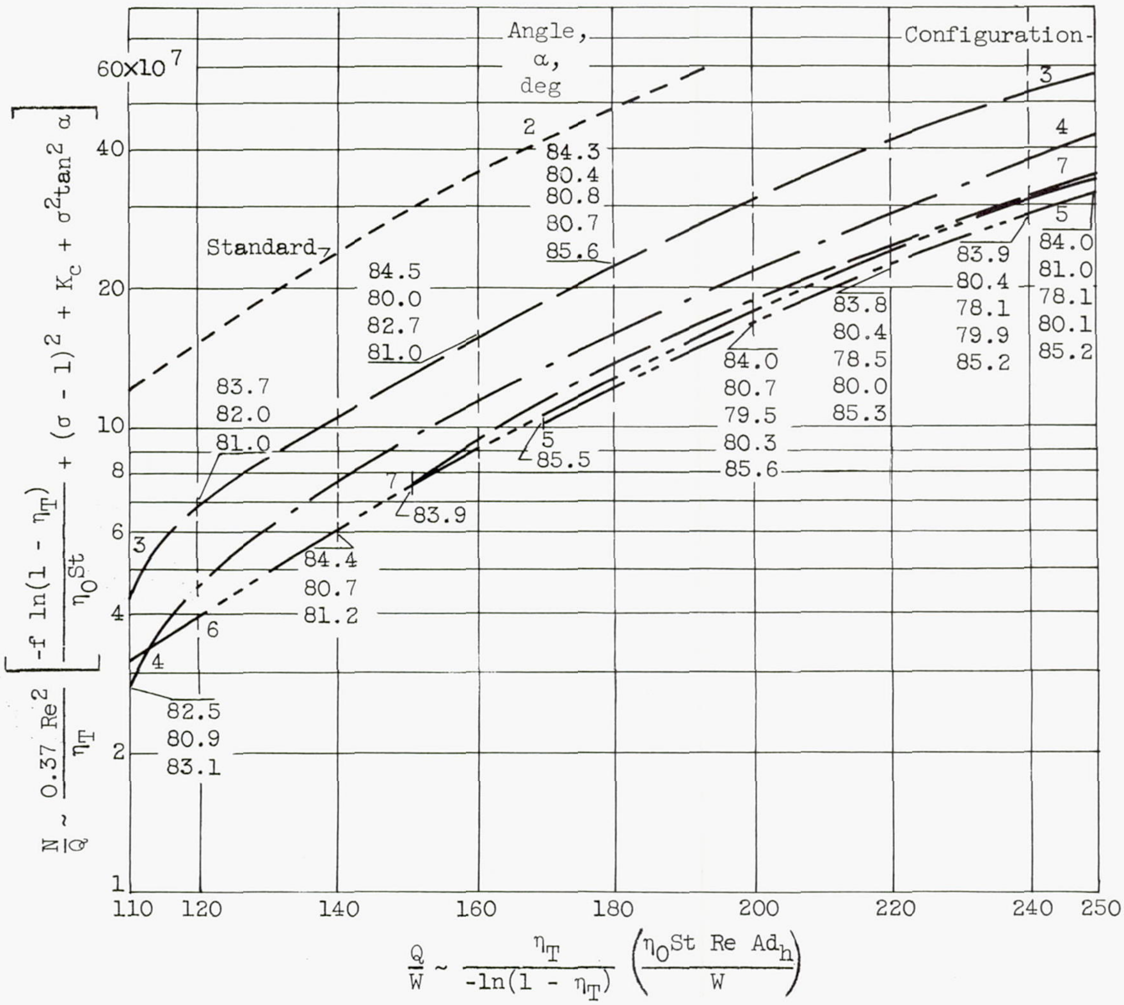
(b) Thermal effectiveness, 0.5.

Figure 15. - Continued. Performance parameter of inclined heat exchangers against core-weight parameter for hydraulic diameters of configurations shown in figure 3. Primary fluid, air; $mc_p \ll m'c_p'$.



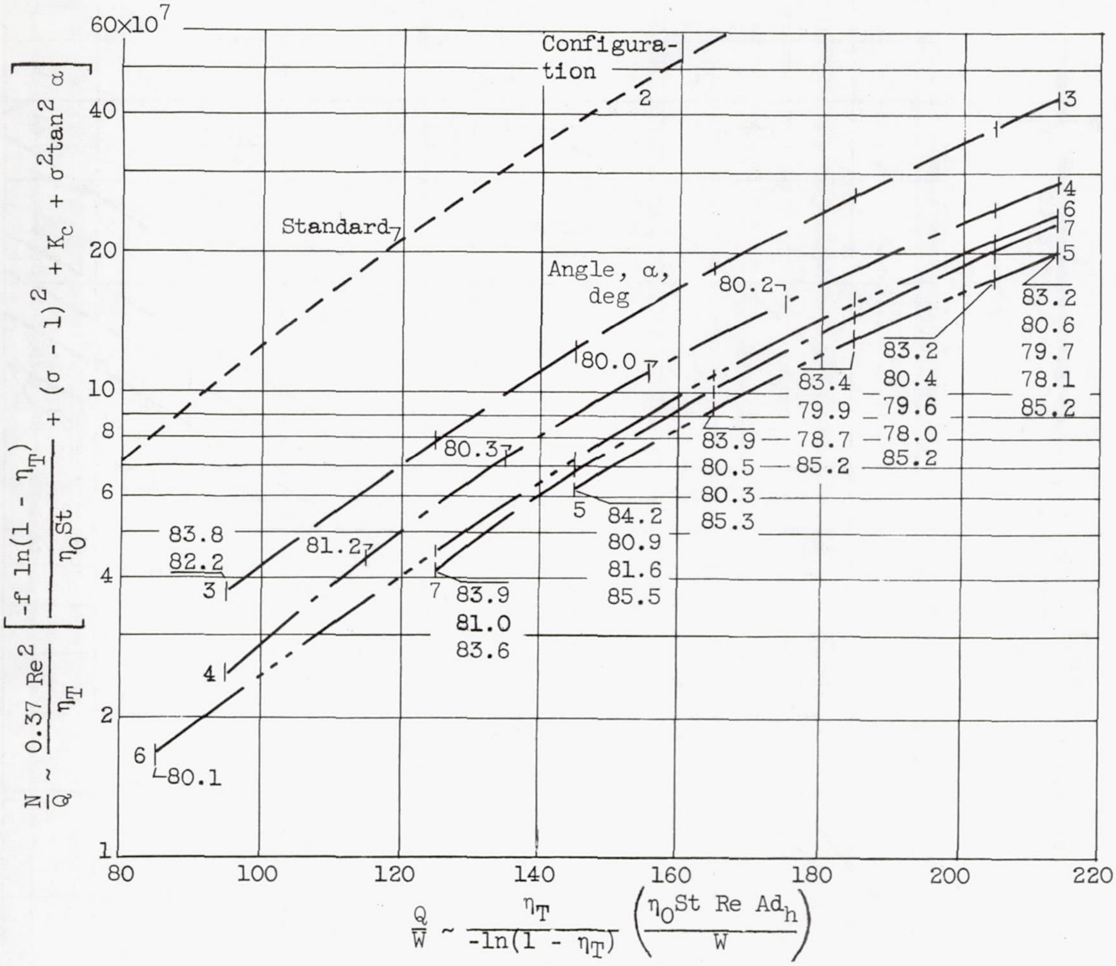
(c) Thermal effectiveness, 0.7.

Figure 15. - Concluded. Performance parameter of inclined heat exchangers against core-weight parameter for hydraulic diameters of configurations shown in figure 3. Primary fluid, air; $mc_p << m'c_p'$.



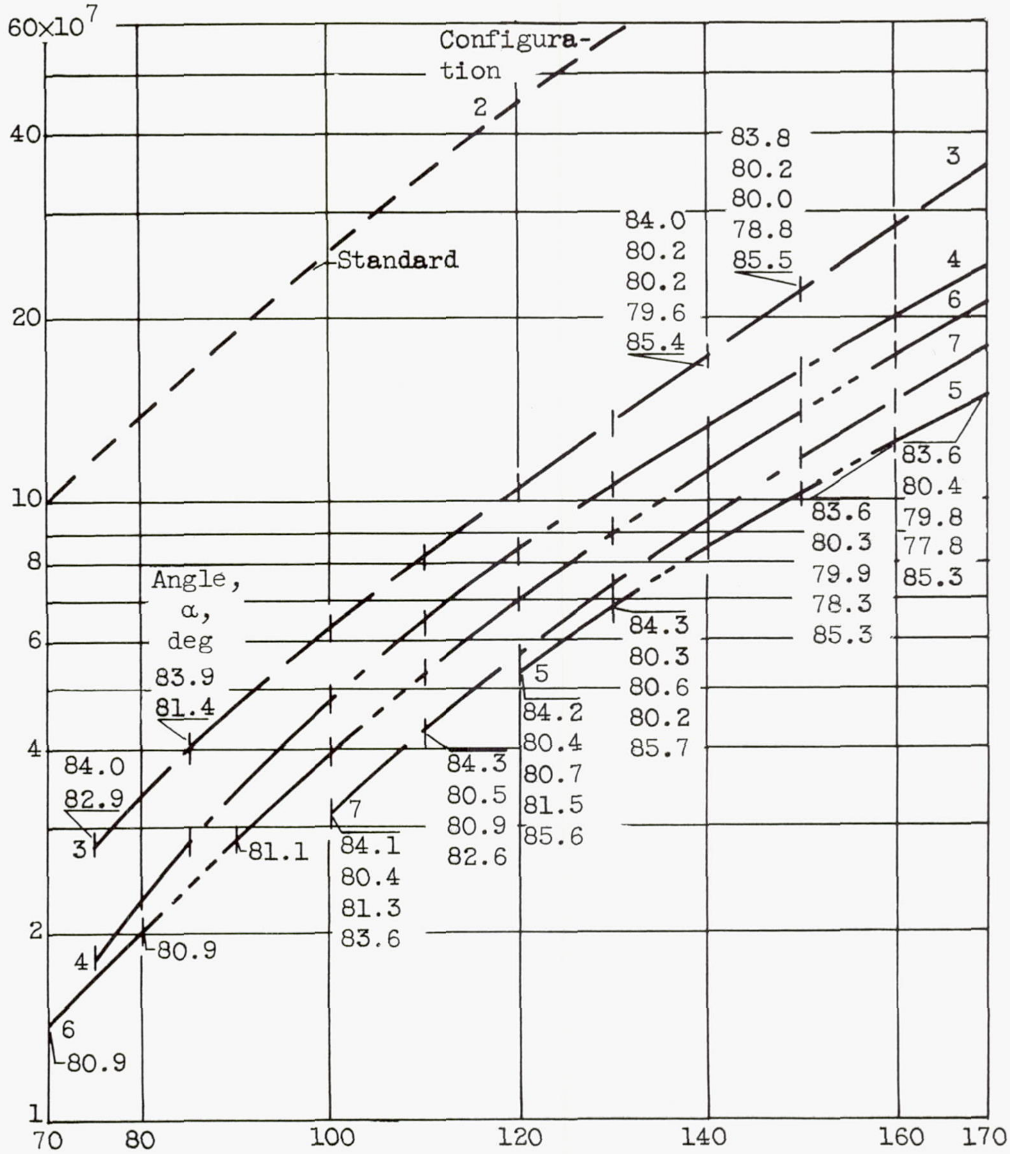
(a) Thermal effectiveness, 0.3.

Figure 16. - Performance parameter of inclined heat exchangers against core-weight parameter using equal hydraulic diameters for all exchangers. Primary fluid, air; $mc_p \ll m'c_p'$.



(b) Thermal effectiveness, 0.5.

Figure 16. - Continued. Performance parameter of inclined heat exchangers against core-weight parameter using equal hydraulic diameters for all exchangers. Primary fluid, air; $mc_p << m'c_p'$.



$\frac{N}{Q} \sim \frac{0.37\ Re^2}{\eta_T}\left[\frac{-f\ \ln(1-\eta_T)}{\eta_0 St} + (\sigma - 1)^2 + K_c + \sigma^2 \tan^2 \alpha\right]$

$\frac{Q}{W} \sim \frac{\eta_T}{-\ln(1-\eta_T)}\left(\frac{\eta_0 St\ Re\ Ad_h}{W}\right)$

(c) Thermal effectiveness, 0.7.

Figure 16. - Concluded. Performance parameter of inclined heat exchangers against core-weight parameter using equal hydraulic diameters for all exchangers. Primary fluid, air; $mc_p \ll m'c_p'$.