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EFFECT OF MACH AND REYNOLDS NUMBERS

ON MAXIMUM LIFT COEFFICIENT

By John R. Spreiter and Paul J. Steffen

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Figure 16.- The Reynolds number corresponding to the point at
a Mach number of 0.68 should be 5.8 million in-
stead of 0.58 million.

Reynolds number increased nearly linearly with Mach number.

In the supercritical Mach number range, the maximum lift
coefficient of conventional airfoils continued to diminish
with increasing Mach number, while that of the low-drag air-
foils reached a minimum at a Mach number between 0.40 and 0.55

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and began increasing until secondary peak values were reached
at a Mach number between 0.60 and 0.66. At supercritical
Mach numbers, no effects of Reynolds numbers were apparent
for two of the three airplanes on which pertinent data were
obtained. On the third airplane the maximum lift coefficient
was affected by Reynolds number but the phenomenon appeared
to be basically different from that experienced at subcriti-
cal Mach numbers.

INTRODUCTION

A knowledge of the effects of Mach and Reynolds numbers
on the maximum lift coefficient is becoming of greater impor-
tance as the speeds and altitudes attainable by modern air-
planes continually increase. Since most maximum-lift tests
conducted to date have been either at full-scale Reynolds
numbers and low Mach numbers or at high Mach numbers and
small Reynolds numbers, very little is known about the inter-
related effects of these two parameters on the maximum lift
coefficient. In order to obtain quantitative information on
these effects, flight tests were conducted at Ames Aeronauti-
cal Laboratory on six airplanes, three having NACA conven-
tional airfoil sections, two having NACA low-drag sections,
and the sixth having a North American Aviation - NACA compro-
mise low-drag section. The results of these tests which
cover a range of Mach numbers from 0.15 to 0.72 and of Reynolds
numbers from 4,400,000 to 19,500,000 are assembled, together
with a considerable quantity of pertinent wind-tunnel data,
and analyzed in the present report. From this analysis, gen-
eral conclusions have been reached indicating the manner in
which the maximum lift coefficient is influenced by variations
of Mach and Reynolds numbers.

DESCRIPTION OF AIRPLANES AND INSTRUMENTATION

To afford a variety of commonly used airfoil sections,
and to permit   correlation with existing wind-tunnel results,
six particular pursuit-type airplanes were selected for the
flight-test portion of this research. For convenience in
the presentation of the test results and the following dis-
cussion, the type airplane and airplane model will be utilized
rather than the airfoil designation when referring to specific
results. The aircraft used in the conduct of the flight tests

were the Bell P-39N and P-63A airplanes, the Grumman F6F-3
airplane, the Lockheed P-38F and YP-80A airplanes, and the
North American P-51B airplane. The P-38F, P-39N, and the
F6F-3 are equipped with NACA conventional airfoils; the
P-63A and YP-80A are provided with NACA low-drag airfoils,
and the P-51B has a North American Aviation — NACA
compromise low-drag airfoil. Two-view drawings of the test
aircraft together with pertinent specifications are shown
in figure 1. Photographs of the test airplanes are presented
in figures 2(a) through 2(f), and wing-root and -tip airfoil
sections for these airplanes are presented in figure 3.

The P-51B and the YP-80A airplanes had very carefully
filled, waxed, and polished surfaces. The other airplanes
were painted with standard camouflage paint. Of the six air-
planes tested, the P-38F and P-39N airplanes had the roughest
finish and the most openings in the wings.

Standard NACA photographically recording flight instru-
ments were used to measure as a function of time the follow-
ing variables: indicated airspeed, pressure altitude, normal
acceleration, pitching and rolling velocities. The airspeed
heads used for all the airplanes tested were of the freely
swiveling type, to minimize errors due to the large angles of
attack encountered in the tests, and with the exception of
the P-38F airplane were mounted on booms extending approxi-
mately a chord length or more ahead of the leading edge near
the wing tip. The airspeed head of the P-38F airplane was
mounted on a boom extending ahead of the fuselage nose. The
installations were calibrated for position error.

The free-air temperatures were obtained either from
radiosonde observations or from readings of calibrated free-
air-temperature indicators or recorders in the airplane.

Photographs of the 0.167-, 0.350-, and 0.333-scale models
of the P-38, P-39N, and XP-80 airplanes, respectively, as
tested in the Ames 16-foot high-speed wind tunnel are shown in
figures 2(g), 2(h), and 2(i). As may be seen in these photo-
graphs, the XP-80 model was a complete model; the P-38 model
was complete except for the propellers; and the P-39N model
was complete except for the propeller and tail. The forces
and moments were recorded by self-balancing, recording beam
scales.

The airfoil tests conducted in the Ames 1- by $3\frac{1}{2}$-foot
high-speed wind tunnel were of 6-inch-chord metal models
mounted so as to span completely the 1-foot width of the

tunnel test section. The lift was obtained either from inte-
gration of pressure distributions over the airfoils or from
measurements of the reactions on the tunnel walls of the forces
experienced by the airfoils.

TEST PROCEDURE

All flight data in this report were obtained in gradual
stalls made during turns or pull-ups with the airplanes in
the clean condition (flaps and landing gear up). With the
five propeller-driven airplanes, all stalls were made with the
engine throttled and the propeller in the high-pitch setting.
Although the power was on during the stalls of the jet-
propelled YP-80A airplane, the data were considered similar to
the power-off condition of the propeller-driven airplanes as
no slipstream existed over the wing.

The flight maximum lift coefficients were calculated by
the following formula:

$$C_{L_{max}} = \frac{WA_Z}{qS}$$

where

$C_{L_{max}}$ maximum lift coefficient

W weight of the airplane at the time of stall, pounds

A_Z normal acceleration factor, the ratio of the net
          aerodynamic force along the airplane Z-axis at
          the stall (positive when directed upward), to
          the weight of the airplane

S wing area, including area extending through the
          fuselage and nacelles, square feet

q dynamic pressure, pounds per square foot

An error of less than two percent was caused by the assumption
in the preceding formula that the lift was equal to the normal
force WA_Z.

Throughout this report, the Reynolds number was computed

using the mean aerodynamic chord as the characteristic length.

The maximum lift coefficient obtainable in flight may be limited by the effects resulting from a localized stall over a limited portion of the wing or from distortion of the lift curve by compressibility effects; these may be manifested by unstable motions of the airplane, by loss of control effectiveness, or by buffeting of the airplane or controls. Since the extent of such a stall may not be sufficient to prevent attaining higher lift coefficients at greater angles of attack in the wind tunnel, a difference between the maximum lift coefficient obtainable in flight tests and in wind-tunnel tests will usually be anticipated. The characteristics assumed to indicate the stall for each of the six airplanes tested are summarized in the following table:

Airplane	Low Mach numbers	High Mach numbers
F6F-3	Roll-off and slight buffeting	Moderate buffeting with pitch-down, followed by porpoising motions
P-38F	Slight buffeting	Buffeting. Pilot reported apparent ineffectiveness of elevators to increase lift coefficient
P-39N	Buffeting and roll-off	Buffeting and pitch-down
P-51B	Buffeting and roll-off	Severe buffeting with mild roll-off
P-63A	Roll-off and pitch-down	Abrupt roll-off
YP-80A	Roll-off with slight buffeting	Slight buffeting

At low Mach numbers, it is shown that the stalls of all the airplanes except the P-38F were characterized by roll-off. Since stalls characterized by roll-off are clearly defined by the motions of the airplane, the maximum lift coefficients obtained in flight are relatively independent of piloting technique and the amount of control available, and would probably be similar to the values measured in a wind tunnel. At high Mach numbers, however, the stalls of only two airplanes were characterized by roll-off, those of the remaining airplanes

being characterized by buffeting accompanied, in some cases, by pitching motions. Maximum lift coefficients defined by buffeting are probably less than the actual maximum lift coefficient by amounts that are a function of what the pilot considers tolerable buffeting limits. It should be noted that the maximum-lift-coefficient data shown in this report were repeatable even when determined by buffeting considerations indicating that the flight values have significance as the maximum usable lift coefficient of the airplane. A further discussion of this phenomenon is presented in reference 1, together with experimental data for a typical tapered wing showing a comparison of the true maximum lift coefficient and that defined by buffeting.

The major portion of the wind-tunnel data was obtained from tests conducted in the Ames 1- by $3\frac{1}{2}$-foot high-speed wind tunnel and the Ames 16-foot high-speed wind tunnel, and from published reports of tests in the Langley variable-density and two-dimensional wind tunnels. Standard wind-tunnel procedures were used in all these tests.

RESULTS

The variation with Mach number of the maximum lift coefficients of the test airplanes at a constant altitude are shown in figure 4. The test altitudes varied from 20,100 feet to 32,300 feet, depending upon the airplane. The corresponding Reynolds numbers for each are presented in figure 5. For the P-39N (reference 2), P-51B, and P-63A airplanes, additional maximum-lift-coefficient data obtained at several altitudes from 5,000 to 33,000 feet are shown in figure 6 and the corresponding Reynolds number in figure 7.

It should be noted that the data for the P-39N airplane is slightly different from that originally presented in reference 2, due to correction of some small errors.

DISCUSSION

Effect of Reynolds Number on the

Maximum Lift Coefficient

The variation of the maximum lift coefficient with Reynolds

number at constant Mach numbers, obtained by cross-plotting
the data of figures 6 and 7, is shown in figure 8 for the
P-39N, P-51B, and P-63A airplanes. The lower Mach number data
of this figure show that, with increasing Reynolds numbers,
the maximum lift coefficients of the P-51B and P-63A airplanes
at first remain nearly constant, but increase rapidly as the
Reynolds number exceeds a critical value, and finally become
nearly constant again at the higher value. A similar trend
is indicated by the data for the P-39N airplane although the
tests were not extended to Reynolds numbers low enough to de-
fine the critical.

These effects of Reynolds number on the maximum lift
coefficients are qualitatively consistent with those described
in reference 3 and are explainable on the same basis. For pur-
poses of simplification, the following generalized explanation
is given. That is, the constant value of maximum lift coeffi-
cient at low Reynolds numbers is caused by laminar separation
of the boundary layer; the increasing values of maximum lift
coefficient beyond a critical Reynolds number occur as the
boundary-layer separation changes from laminar to turbulent;
and the nearly constant value of the maximum lift coefficient
at the higher Reynolds numbers is produced by turbulent sepa-
ration of the boundary layer. A more complete description of
the mechanism of these changes is provided in reference 3.
Quantitatively the Reynolds number effects found in the pres-
ent tests were much smaller than those of reference 3. This
difference is discussed in a subsequent section of this re-
port.

Effects of Mach Number on the Maximum Lift Coefficient

The curves of figure 9 (obtained by cross-plotting the
data of figs. 6 and 7) show that at constant Reynolds number
the maximum lift coefficient of the P-39N, P-51B, and P-63A
airplanes continually diminish as the Mach number increases
from 0.15 to 0.40. (Application of the methods of reference 4
shows that for the measured variations of maximum lift coeffi-
cient with Mach number, the theoretically computed critical
Mach numbers of the airfoils were attained at Mach numbers of
approximately 0.4 for all the airplanes tested.) Figures 4
and 9 show that, as the Mach number is increased further, the
maximum lift coefficients of the airplanes with conventional
airfoils continue to diminish to the highest test Mach numbers;
whereas those for the airplanes with low-drag airfoils reach
a minimum at a Mach number between 0.40 and 0.50 and then

begin increasing until peak values are reached at a Mach number between 0.60 and 0.66.

The decrease of the maximum lift coefficients of all the airplanes with increasing Mach number in the subcritical Mach number range may be attributed to the separation of the boundary layer at smaller angles of attack induced by the compressibility-steepened adverse pressure gradient.

The reasons for the diverse characteristics exhibited by the low-drag and conventional airfoils at supercritical Mach numbers are illustrated in figure 10 and discussed in reference 5. In brief, the pressure distributions of figure 10 indicate that as the Mach number is increased beyond the critical, a tendency for the pressure peak at the nose of the airfoil to decrease and for the low-pressure region to broaden is evidenced for both the conventional and low-drag airfoils. With the NACA 66,2-215 (a = 0.6) low-drag airfoil, the upper-surface low-pressure region broadens considerably with increasing Mach number, more than offsetting the reduction in the pressure peak so that higher maximum lift coefficients are produced. This effect persists until at Mach numbers between 0.60 and 0.66 the loss in lift due to the decrease of the pressure peak finally offsets the addition of lift due to the broadening of the low-pressure region. With the NACA 23015 conventional airfoil, however, the negative pressure peak broadens so slightly that it is insufficient to counteract the loss in lift produced by the lowering of the pressure peak, and the maximum lift coefficient continues to decrease up to the highest Mach number of the flight tests. Meanwhile the lower-surface pressure distributions on both airfoils remain virtually unchanged throughout the Mach number range shown, indicating that the upper-surface effects account for nearly all the observed changes in maximum lift coefficient.

Interrelated Effects of Mach and Reynolds Numbers

on the Maximum Lift Coefficient

While the foregoing sections have discussed the effects of Mach and Reynolds numbers on the maximum lift coefficient as though they were entirely separate and independent phenomena, the actual effects of each variable are modified to a secondary extent by the value of the other. In general, the interaction of the Mach and Reynolds number effects is such that the variation of the maximum lift coefficient with either variable remains qualitatively as described previously although modified quantitatively.

In this section, the interrelated effects of Mach and
Reynolds numbers on the maximum lift coefficients of the
P-39N, P-51B, and P-63A airplanes as evidenced by the curves
of figures 8 and 9 are discussed. This discussion is divided
into two sections corresponding, respectively, to Mach numbers
less than or greater than the critical Mach number of 0.4,

Subcritical Mach number range.- The curves of figure 8
for the P-51B and P-63A airplanes show that, as the Mach
number increases, the critical Reynolds number becomes greater
while the characteristic effects of Reynolds number become
smaller, finally disappearing at moderately supercritical
Mach numbers. The same general trends may be seen in the
data for the P-39N airplane although the test range does not
extend to Reynolds numbers as small as the critical. Most
of the interrelated effects of Mach and Reynolds numbers are
best illustrated by the graphs of figure 11 and the further
discussion of these phenomena will be concerned mainly with
the data as plotted in that manner.

Figure 11 shows the variation with Mach number of four
of the pertinent parameters describing the variation of the
maximum lift coefficient with Reynolds number. These param-
eters are R_{cr}, the critical Reynolds number; $\Delta C_{L_{max}}$, the
increment in the maximum lift coefficient as the stall changes
from laminar separation to turbulent separation; $C_{L_{max_{lam}}}$,
the maximum lift coefficient corresponding to laminar sepa-
ration; and $C_{L_{max_{turb}}}$, the maximum lift coefficient corre-
sponding to turbulent separation. Each of these factors is
illustrated by a sketch shown in figure 11. The abrupt in-
crease in the maximum lift coefficient of the P-51B airplane
shown in figure 8 at the highest Reynolds numbers at Mach
numbers of 0.40 and 0.50 is essentially a supercritical Mach
number phenomenon and will be discussed later.

The R_{cr} of the P-51B and P-63A airplanes are shown in
figure 11(a) to vary nearly linearly with Mach number through-
out the range tested. There are two effects which could cause
such a variation of R_{cr} with Mach number. One is the in-
crease of kinematic viscosity in the boundary layer of a
compressible fluid due to aerodynamic heating which causes
the ratio of the local Reynolds number (based on boundary-
layer conditions) to the free-stream Reynolds number to
diminish as shown in reference 6. Accordingly, as shown by
references 7 and 8, a larger free-stream Reynolds number
would be necessary to reach the local critical Reynolds

numbers required for transition from a laminar to a turbulent
boundary layer. A second possibility is due to the fact that
increasing the Mach number in the subcritical range has effects
on the upper-surface pressure distributions similar to that
of decreasing the airfoil thickness. The pressure peaks
become sharper and the adverse pressure gradients become
steeper. Decreasing the airfoil thickness is shown in refer-
ence 3 to increase R_{cr}, hence similar effects due to in-
creasing Mach number would be anticipated.

Figure 11(b), which shows the variation with Mach
number of $\Delta C_{L_{max}}$ for the P-51B and P-63A airplanes, in-
dicates that $\Delta C_{L_{max}}$ decreases with increasing Mach number,
finally becoming zero at Mach numbers between 0.50 and 0.60.
To afford a better understanding of this effect curves of
$C_{L_{max_{lam}}}$ and $C_{L_{max_{turb}}}$ are plotted as a function of Mach
numbers in figure 11(c). These curves show that, while
both parameters decrease as the Mach number is increased,
$C_{L_{max_{turb}}}$ decreases at a much greater rate than $C_{L_{max_{lam}}}$.

This characteristic probably may be attributed partially to
the higher local Mach numbers involved at the larger lift
coefficient of $C_{L_{max_{turb}}}$ causing the adverse pressure
gradients to be steepened more than those corresponding to
$C_{L_{max_{lam}}}$. Another contributing factor may be that the
turbulent separation, originating at the trailing edge, is
affected by the compressibility-steepened adverse pressure
gradient along the entire chord, while the laminar sepa-
ration, occurring near the leading edge, would be influenced
mainly by the steepened pressure gradients over only a small
portion of the chord.

Supercritical Mach number range.- At supercritical Mach
numbers, figures 8 and 9 show that the maximum lift coeffi-
cient continues to vary rapidly with changes of Mach number
and that the usual Reynolds number effects, as described
previously, become negligibly small at Mach numbers of 0.55
for all the airplanes tested.

With the P-51B airplane, however, an unusual effect of
Reynolds number was indicated which may have significance as
a general type of phenomenon possible at supercritical Mach
numbers. Figure 8 shows that at moderately supercritical
Mach numbers and at Reynolds numbers greater than those at
which the previously discussed variation of maximum lift

coefficient with Reynolds number occurs, the maximum lift
coefficient increased abruptly as the Reynolds number was
increased. The entire effect disappeared at Mach numbers
greater than about 0.60. The curves of figure 6 for the
P-51B airplane show that the Mach number at which the
maximum lift coefficient starts to increase when in the
supercritical Mach number range is nearly constant at a
value of 0.49 for all altitudes tested above 17,300 feet,
corresponding to Reynolds numbers less than 13,700,000.
At lower test altitudes, or larger Reynolds numbers, however,
the maximum lift coefficient reaches its minimum value at
a lower Mach number. Since increasing values of the maximum
lift coefficient with increasing Mach numbers in the super-
critical Mach number region have been shown to be caused
by the rearward movement of the shock wave, it appears
that these Reynolds number effects at supercritical Mach num-
bers are more the result of the boundary layer influencing
the shock-wave position than the result of the normal
boundary-layer separation, whether laminar or turbulent,
being modified by compressibility effects as at subcritical
Mach numbers. Further research is necessary before a com-
plete understanding of this phenomenon is had.

Comparison of Flight and Wind-Tunnel Data

In order to determine whether the general effects of
Mach and Reynolds numbers indicated by the flight data are
adequate to permit correlation of flight and wind-tunnel
results, wind-tunnel data, although obtained at combinations
of Mach and Reynolds numbers not covered in flight, are com-
pared with the flight data. In successive divisions of this
section, the flight data will be compared with wind-tunnel
data obtained for models tested in the Ames and Langley
16-foot high-speed wind tunnels, those obtained for several
airfoils in the Ames 1- by $3\frac{1}{2}$-foot high-speed wind tunnel,
and those obtained for several airfoils in the Langley
variable-density and two-dimensional wind tunnels.

Comparison of flight data with model data from Ames and
Langley 16-foot high-speed wind tunnels.- Maximum lift-
coefficient data obtained on models of the P-39, P-39N, and
XP-80 airplanes in the Ames 16-foot high-speed wind tunnel
and corrected for trim are presented in figure 13, together
with the high-altitude flight data of figure 4 for the corre-
sponding airplanes. The Reynolds numbers for the tunnel tests
are shown in figure 5 to be considerably lower than those of
the flight tests. A comparison of the maximum lift coefficients

obtained from flight and wind-tunnel data indicates that, al-
though the same qualitative effects of Mach number are shown
in all the data, absolute agreement between the actual values
of the coefficients measured in flight and in the tunnel does
not always exist. Most of the discrepancies, however, may be
accounted for by a consideration of all the pertinent vari-
ables. In general, considering only the difference between
the Reynolds numbers of the flight and wind-tunnel tests, the
comparison should show close agreement existing at the higher
Mach numbers but lower maximum lift coefficients for the tun-
nel data than for the flight data at the lower Mach numbers.
Actually the data for the various airplanes show deviations
from this trend which are of magnitudes that can be related
directly to the relative aerodynamic cleanliness of the de-
signs as discussed in reference 9. Thus the maximum lift co-
efficients of the YP-80A airplane, which had smooth wing
surfaces and no propeller, showed excellent agreement with the
expected trends; those of the P-39N airplane, which had a
rougher surface finish, considerable air leakage, and an idling
propeller, were somewhat lower than the wind-tunnel values.
Although the maximum lift coefficients of the P-38F airplane
were affected by the items mentioned for the P-39N, perhaps an
even more important factor for this airplane was the distor-
tion of the lift curves at high Mach numbers by compressibility
effects.

The importance of this latter consideration is indicated
by the curves of figure 13 which show the variation of the
lift coefficient with angle of attack for models of the P-39N,
XP-80, and P-38 airplanes as measured at several Mach numbers
in the Ames 16-foot high-speed wind tunnel. In contrast to
the lift curves of the P-39N and XP-80 models, which are near-
ly straight lines until near the maximum lift coefficient,
those of the P-38 model have a definite decrease in slope at
moderate angles of attack known, in this case, to result from
air-flow separation over the wing center section. The actual
maximum lift coefficient of the P-38 model then occurs at ex-
tremely high angles of attack. Similar effects were measured
at supercritical Mach numbers in tests of a tapered wing of
NACA 230-series airfoil sections conducted in the Langley
16-foot high-speed wind tunnel and reported in reference 1. At
Mach numbers above 0.55 the angle of attack at which the maxi-
mum lift coefficient was reached was 10° to 12° higher than
that at which pronounced separation of the flow occurred. Con-
sequently, the concept of a range of maximum obtainable lift
coefficients at high Mach numbers was introduced. This range
extends from the lift coefficient corresponding to the initial
stall (arbitrarily defined as being at 2° to 3° above the

angle of attack at which separation of the flow from the wing
initially occurs) to that corresponding to the actual maximum
lift coefficient of the wing. As the initial stall lift coef-
ficient is exceeded, increases in stability and tail buffeting
are likely to occur. These may become sufficiently great that
they would appear to the pilot to define the maximum lift co-
efficient obtainable in flight.

Accordingly, initial-stall and maximum-lift-coefficient
points for the P-38 model, as obtained from figure 13, are
plotted in figure 12. It may be seen that good agreement ex-
ists between the flight maximum lift coefficient and the
initial-stall lift coefficient of the model. Similarly, flight
data for the F6F-3 airplane, the wing of which is similar to
the wing model, agree with the wing model data corresponding
to the initial stall rather than to the maximum lift coeffi-
cient. (The Reynolds number of the tapered wing tests is
shown in fig. 5.) Such considerations have little effect on
the P-39N and XP-80 model data, however, since the initial-
stall lift coefficient is virtually equal to the maximum lift
coefficients for both models. It appears, therefore, that the
maximum lift coefficient obtainable in flight may be better
estimated by considering the value of the initial-stall lift
coefficient of a model rather than its actual maximum lift
coefficient.

Comparison of flight data with Ames 1- by $3\frac{1}{2}$-foot high-
speed airfoil data.- Maximum-lift-coefficient data obtained
in the Ames 1- by $3\frac{1}{2}$-foot high-speed wind tunnel for two-
dimensional models of the NACA 23015, 0015, 66,2-215 and
65_1-212 airfoils which are approximately similar, respectively,
to the root sections of the P-38F and F6F-3, P-39N, P-51B and
P-63A, and YP-80A airplanes, are presented in figure 14, to-
gether with the high-altitude flight data from figure 4 for
these airplanes. For the airfoils and Mach number ranges of
figure 14, no appreciable difference existed between the
initial-stall lift coefficient and the maximum lift coeffi-
cient. The Reynolds numbers of the tunnel tests are shown in
figure 5.

Because of the many differences between a two-dimensional
airfoil model in a wind tunnel and an airplane in flight, only
qualitative verification of the trends indicated by the flight
data should be anticipated and any close quantitative agree-
ment is probably merely coincidental. At the higher Mach num-
bers, very similar effects of Mach number are experienced by
the airplanes and the airfoils; in fact, close quantitative

agreement exists in every case except between that of the
P-38F airplane and the NACA 23015 airfoil. At lower Mach
numbers, however, the maximum lift coefficients of the air-
foils have lower values, and are less affected by changes of
Mach number than those of the airplanes with which they are
compared. These trends are in accord with the previously
discussed effects of Reynolds number which are large at low
Mach numbers and decrease in magnitude as the Mach number
becomes greater.

Comparison of flight data with Langley variable-density-
tunnel and two-dimensional-tunnel airfoil data.- Figures 15(a)
and 15(b) show the variation with effective Reynolds number of
the maximum lift coefficient of several NACA conventional air-
foils as measured in the Langley variable-density wind tunnel
(reference 3). These airfoils were three-dimensional models
having an aspect ratio of 6. Since the tunnel was operated at
constant speed and the Reynolds number was changed by varying
the pressure, the Mach number was almost exactly constant at
a value of 0.06 throughout the entire Reynolds number range.
The data are presented in two groups, figure 15(a) showing the
effects of changes of thickness ratio and figure 15(b) showing
the effects of changes of camber. These curves show that
$C_{L_{max_{lam}}}$ is a very sensitive function of camber but relative-
ly independent of thickness, while R_{cr} varies greatly with
thickness but only slightly with camber. Both camber and
thickness affect $C_{L_{max_{turb}}}$, however, but to a much lesser
extent than they do $C_{L_{max_{lam}}}$ or R_{cr}. Values of the maxi-
mum lift coefficients of several two-dimensional NACA low-drag
airfoil models tested in the Langley two-dimensional wind tun-
nel at three Reynolds numbers (approximately 3,000,000,
6,000,000, and 9,000,000) are presented in figure 15(c). The
various Reynolds numbers were obtained by simultaneous changes
of pressure and airspeed, the Mach numbers corresponding to
the listed test Reynolds numbers being approximately 0.10,
0.14, and 0.15, respectively. Due to the limited Reynolds
number range of these latter tests, the value of R_{cr},
$C_{L_{max_{lam}}}$, and $\Delta C_{L_{max}}$ are unknown, and of the four parame-
only $C_{L_{max_{turb}}}$ is determinable.

A comparison of these airfoil data with the flight data
of figure 8 shows that the variation of the maximum lift coef-
ficient with Reynolds number was much less for the flight
tests than for the tunnel tests and that the R_{cr} was much
higher for the flight tests. These apparent discrepancies,

however, may be accounted for by a consideration of the effects resulting from the difference between the Mach number of the flight and tunnel tests. Since wind-tunnel data of the nature of figure 15 were not available for the exact airfoils used on the test airplanes, airfoils used for comparison with the flight results were selected in accordance with the previously discussed relations shown in figures 15(a) and 15(b). Thus, the airplane $C_{L_{max_{lam}}}$ and $\Delta C_{L_{max}}$ data are compared in figure 11 only with those of airfoils having about the same amount of camber as the airfoils used on the airplanes; whereas the airplane $C_{L_{max_{turb}}}$ data, being relatively insensitive to changes of thickness and camber, are compared with similar data for all the airfoils of figure 15.

Since the Reynolds numbers of the tunnel tests did not quite extend to the point of $C_{L_{max_{turb}}}$ for some of the NACA conventional airfoils, the data were extrapolated slightly and the resulting values plotted in figure 11. In each of the comparisons in figures 11(b) and 11(c) it may be seen that, although the variation of the maximum lift coefficient with Reynolds number was much less for the flight tests than for the tunnel tests, the apparent discrepancies between the Reynolds number effects observed in flight and in the Langley variable-density and two-dimensional wind tunnels may be accounted for by extrapolating the trends indicated by the flight data to very low Mach numbers.

Values of R_{cr} of the airfoils are not shown for comparison with the flight data for the following reason. The flight data were obtained only for the P-51B and P-63A airplanes, which have NACA low-drag airfoils; whereas the only wind-tunnel R_{cr} values available were for NACA conventional airfoils. Values of R_{cr} for NACA conventional airfoils were not believed to be suitable for comparison with those for NACA low-drag airfoils since R_{cr} has been shown to be a very sensitive function of thickness ratio and would, therefore, probably vary considerably with changes of thickness distribution, particularly near the leading edge. It is significant to note, however, that for each of the NACA low-drag airfoils for which data are shown in figure 15(c), R_{cr} has been exceeded at the lowest test Reynolds number of 3,000,000 and Mach number of 0.10. Such low values of R_{cr} are consistent with the trends indicated by the flight data.

Correlation of Maximum-Lift-Coefficient Data

from Several Wind Tunnels

In early efforts to correlate low-speed airfoil data ob-
tained in various wind tunnels having different degrees of
turbulence, the concept of a "turbulence factor" was intro-
duced in references 10 and 11 as a multiplier of the test
Reynolds number to obtain an effective Reynolds number equiv-
alent to nonturbulent free-air conditions. This factor alone
did not prove sufficient for correlating maximum-lift-
coefficient data. The data of this report, however, suggests
the possibility that similar data obtained on airfoil models
tested in various wind tunnels may be correlated by a consid-
eration of the effects of Mach number, as well as Reynolds
number.

Accordingly, from maximum-lift-coefficient data measured
in six wind tunnels on models of the NACA 0012 and 0012-63
airfoils, which are very similar, values corresponding to
several effective Reynolds numbers were selected from faired
curves in the reference reports and replotted in figure 16 as
a function of Mach number. The test conditions are summarized
in table I.

The data from these wind tunnels indicate that it may be
possible to correlate the maximum lift coefficients obtainable
in various wind tunnels by considering the Mach number as well
as the turbulence factor, even though the Mach number may be
very small. These curves for the NACA 0012 airfoil are very
similar to those previously observed in the flight data. It
is shown that the maximum lift coefficient is affected by Mach
numbers as low as 0.15 and that the Reynolds number effects
decrease with increasing Mach number, as indicated by the
flight test data. Similar plots of maximum lift data have been
made for several other airfoils. Although the quantity of data
for any of the other airfoils is not so great as that for the
NACA 0012 airfoil, trends similar to those shown in figure 16
are readily apparent for all the airfoils.

CONCLUSIONS

An analysis of the effects of Mach and Reynolds numbers
on the maximum lift coefficients of several airplanes and
wind-tunnel models has resulted in the following conclusions:

1. When the effects of Mach number were considered, as well as those of Reynolds number, good correlation was found between flight data and available wind-tunnel data, provided buffeting or other factors did not prevent attainment of the actual maximum lift coefficient in flight, in which case the maximum lift coefficient obtained in flight appeared to be related to a pronounced decrease in lift-curve slope. The same considerations provided good agreement among limited airfoil data from various wind tunnels. Data indicated that the maximum lift coefficient was affected by Mach number down to Mach numbers of approximately 0.15.

2. Distinct differences exist between the effects of Mach and Reynolds numbers on the maximum lift coefficient in the subcritical and supercritical Mach number regions.

3. In the subcritical Mach number region, the maximum lift coefficient obtainable in flight by the airplanes tested decreased steadily with increasing Mach number. As the Mach number was increased in the supercritical Mach number region, the maximum lift coefficient of NACA conventional airfoils continued to diminish as at subcritical Mach numbers, while that of NACA low-drag airfoils reached a minimum at a Mach number between 0.40 and 0.55 and then began increasing until secondary peak values were reached at a Mach number between 0.60 and 0.66.

4. In the subcritical Mach number region, effects of Reynolds number on the maximum lift coefficient were qualitatively as described in NACA Report No. 586; quantitatively, the effects of Reynolds number on the maximum lift coefficient decreased progressively with increasing Mach number, becoming nil at a Mach number of approximately 0.55. The critical Reynolds number increased nearly linearly with Mach number.

5. At supercritical Mach numbers, no effects of Reynolds number were apparent for two of the three airplanes on which pertinent data were obtained; on the third airplane the maximum lift coefficient was affected by Reynolds number but in a manner basically different from that experienced at subcritical Mach numbers.

Ames Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
        Moffett Field, California, March 28, 1946.

REFERENCES

1. Pearson, E. O., Jr., Evans, A. J., and West, F. E., Jr.:
   Effects of Compressibility on the Maximum Lift Char-
   acteristics and Spanwise Load Distribution of a
   12-Foot-Span Fighter-Type Wing of NACA 230-Series
   Airfoil Sections. NACA ACR No. L5G10, 1945.

2. Nissen, James M., and Gadeberg, Burnett L.: Effect of
   Mach and Reynolds Numbers on the Power-Off Maximum
   Lift Coefficient Obtainable on a P-39N-1 Airplane as
   Determined in Flight. NACA ACR No. 4F28, 1944.

3. Jacobs, Eastman N., and Sherman, Albert: Airfoil Sec-
   tion Characteristics as Affected by Variations of the
   Reynolds Number. NACA Rep. No. 586, 1937.

4. Heaslet, Max, A.: Critical Mach Numbers of Various Air-
   foil Sections. NACA ACR No, 4G13, 1944.

5. Stack, John, Fedziuk, Henry A., and Cleary, Harold E.:
   Preliminary Investigation of the Effect of Compress-
   ibility on the Maximum Lift Coefficient. NACA ACR,
   Feb. 1943.

6. Allen, H. Julian, and Nitzberg, Gerald E.: The Effect
   of Compressibility on the Growth of the Laminar
   Boundary Layer on Low-Drag Wings and Bodies. NACA
   ACR, Jan. 1943.

7. von Kármán, Th., and Millikan, C. B.: A Theoretical
   Investigation of the Maximum-Lift Coefficient.
   Jour. Appl. Mechanics, vol II, no. 1, March 1935,
   pp, A-21, A-27.

8. Goldstein, S.: Modern Developments in Fluid Dynamics,
   Vol. II, Oxford Univ. Press, 1938, pp. 478, 489.

9. Sweberg, Harold H., and Dingeldein, Richard C.: Summary
   of Measurements in Langley Full-Scale Tunnel of Maxi-
   mum Lift Coefficients and Stalling Characteristics of
   Airplanes. NACA ACR No, L5C24, 1945.

10. Jacobs, Eastman N., and Clay, William C.: Character-
    istics of the N.A.C.A. 23012 Airfoil from Tests in
    the Full-Scale and Variable-Density Tunnels. NACA
    Rep. No, 530, 1935.

11. Platt, Robert C.: Turbulence Factors of N.A.C.A. Wind
        Tunnels as Determined by Sphere Tests. NACA Rep.
        No. 558, 1936.

12. Goett, Harry J., and Bullivant, W. Kenneth: Tests of
        N.A.C.A. 0009, 0012, and 0018 Airfoils in the Full-
        Scale Tunnel. NACA Rep. No. 647, 1938.

13. Muse, Thomas C.: Some Effects of Reynolds and Mach
        Numbers on the Lift of an NACA 0012 Rectangular
        Wing in the NACA 19-Foot Pressure Tunnel. NACA
        CB No. 3E29, 1943.

14. Hilton, W. F., Cowdrey, C. F., and Hyde, G. A. M.:
        Tests of NACA 0012-63 Aerofoil in the High Speed
        Tunnel. A.R.C. 4715, Sept. 1940.

15. Doetsch, H., and Kramer, M.: Systematic Airfoil Tests
        in the Large Wind Tunnel of the DVL. NACA TM No.
        852, 1938.

16. Richter, G.: Messungen am Profil NACA 0012-63 im
        DVL - Hochgeschwindigkeits-Windkanal. Report
        No. 127 Lilienthal-Gesellschaft für Luftfahrt-
        forschung. Sept. 1940.

TABLE I

SUMMARY OF TEST CONDITIONS FOR NACA 0012 AND 0012-63 AIRFOILS AS CONDUCTED IN SEVERAL WIND TUNNELS

Item	Langley full-scale tunnel	Langley variable-density tunnel	Langley 19-foot pressure tunnel	NPL 1-foot high-speed tunnel	DVL 5- by 7-meter tunnel	DVL 2.7-meter high-speed tunnel
Turbulence factor	1	2.64	1	1	1.1	1
Airfoil	NACA 0012	NACA 0012	NACA 0012	NACA 0012-63	NACA 0012	NACA 0012-63
Aspect ratio	6	6	6	Infinite	5	2.74 with end plates
Chord (in.)	72	5	24	2	31.50	19.69
Minimum effective Reynolds number	1,800,000	450,000	1,100,000	440,000	1,200,000	5,900,000
Maximum effective Reynolds number	4,500,000	8,400,000	8,200,000	570,000	3,500,000	(1)
Minimum Mach number	0.04	0.06	0.06	0.40	0.06	0.68
Maximum Mach number	0.11	0.06	0.38	0.55	0.17	(1)
Reference	12	3	13	14	15	16

[1]Only one test point obtained.

P-38F		P-51B	
WING: ROOT NACA 23016	MAC. 7 FT. SPAN 52 FT.	WING: NACA-NAA	MAC. 6.85 FT. SPAN 37 FT.
TIP NACA 4412	TWIST 0 DEG.	COMPROMISE	TWIST 2.0 DEG.
AREA----327 SQ. FT.	WEIGHT. (GROSS) 15,900 LB.	AREA----233.2 SQFT.	WEIGHT (GROSS) 9,300 LB.

P-39N		P-63A	
WING: ROOT NACA 0015	MAC 6.7 FT. SPAN 34 FT.	WING: ROOT NACA 66,2X-116	MAC 6.88FT, SPAN.38.3FT.
TIP NACA 23009	TWIST 0.4 DEG.	TIP NACA 66,2X-216	TWIST 1.7 DEG.
AREA----213.2 SQ. FT.	WEIGHT (GROSS) 7660 LB.	AREA----248 SQ. FT.	WEIGHT (T.O.) 8300 LB.

F6F-3		YP-80A	
WING: ROOT NACA 23016 (MODIFIED)	MAC 8.12FT, SPAN 42.5FT.	WING: ROOT NACA 65,-213	MAC. 6.7 FT, SPAN 38. 9FT.
TIP NACA 23009	TWIST 0 DEG.	TIP NACA 65-213	TWIST 1.3 DEG.
AREA----334 SQ. FT.	WEIGHT (T.O.) 11,500 LB.	AREA----237 SQ. FT.	WEIGHT (GROSS)11,950 LB.

FIGURE 1.- TWO-VIEW DRAWINGS AND PERTINENT SPECIFICATIONS
OF THE AIRPLANES TESTED IN FLIGHT.
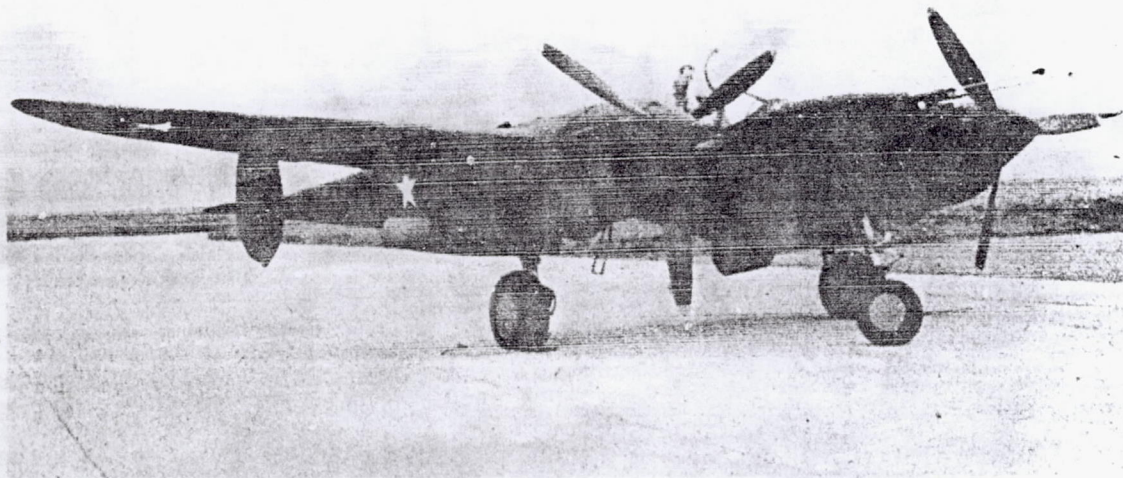
(a) P-38F airplane.
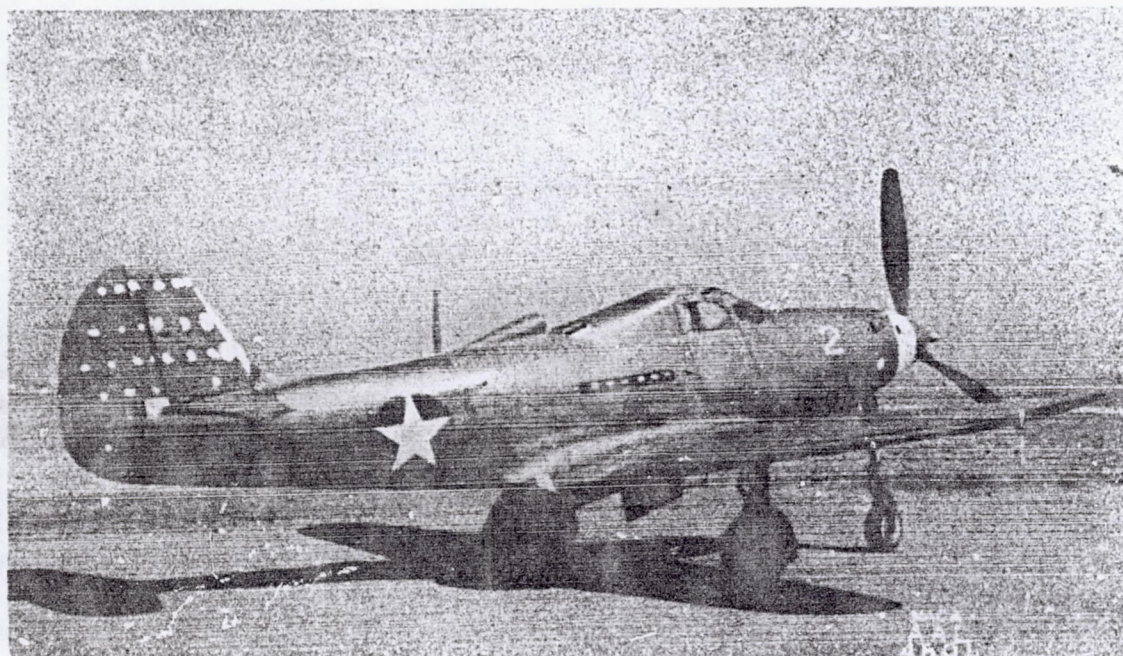
(b) P-39N airplane.

Figure 2(a to i).- Photographs of test airplanes and wind-tunnel models.
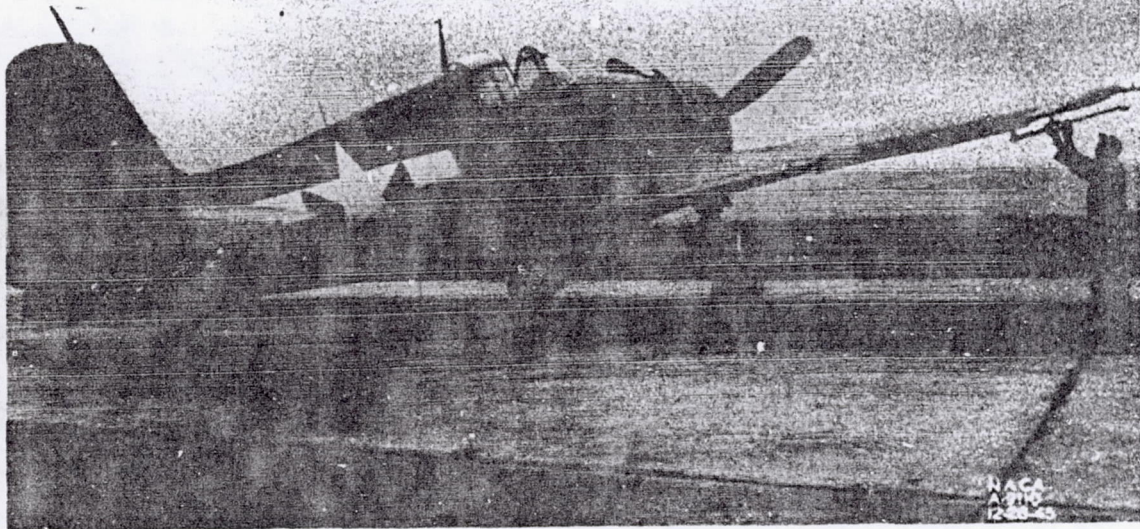
(c) F6F-3 airplane.
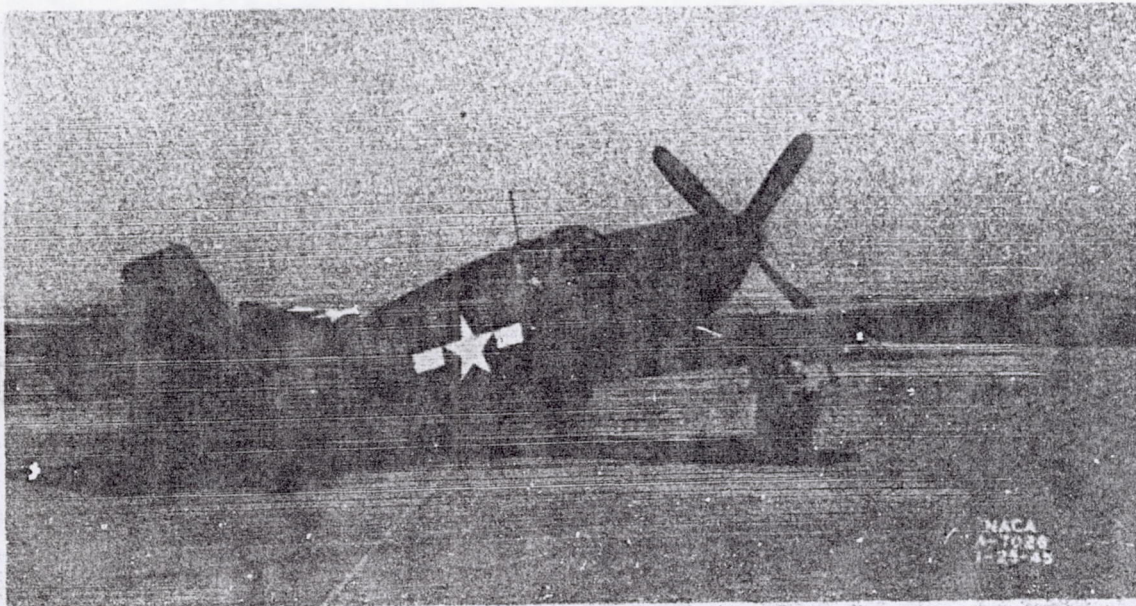
(d) P-51B airplane.

Figure 2.- Continued.
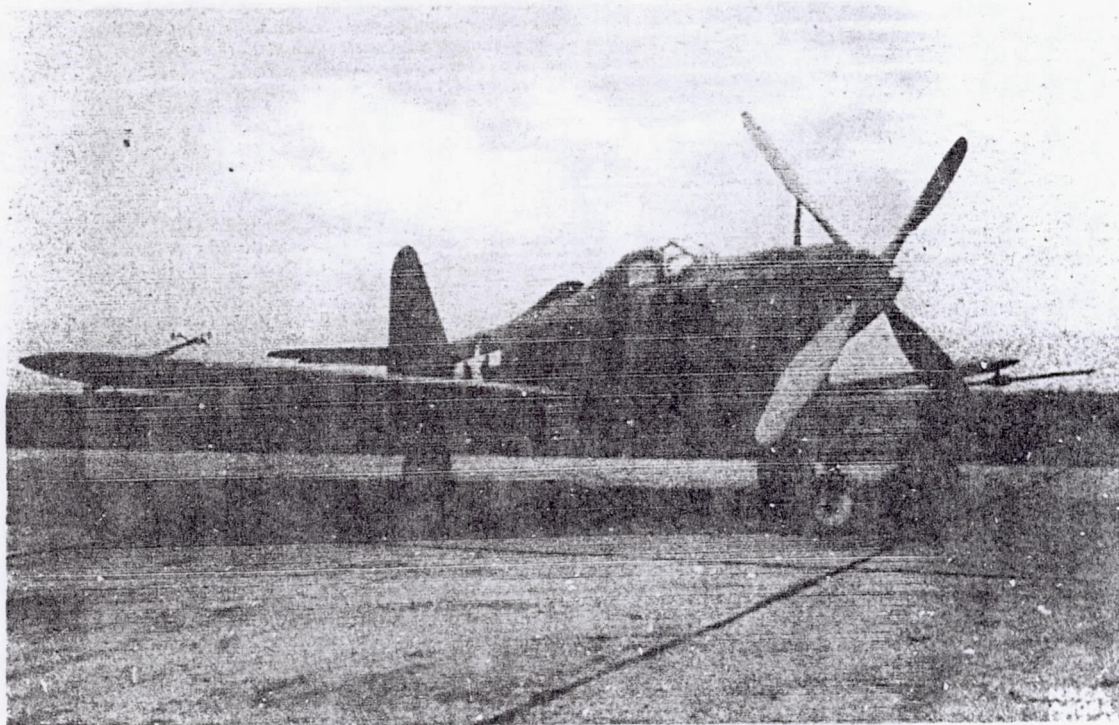
(e) P-63A airplane.
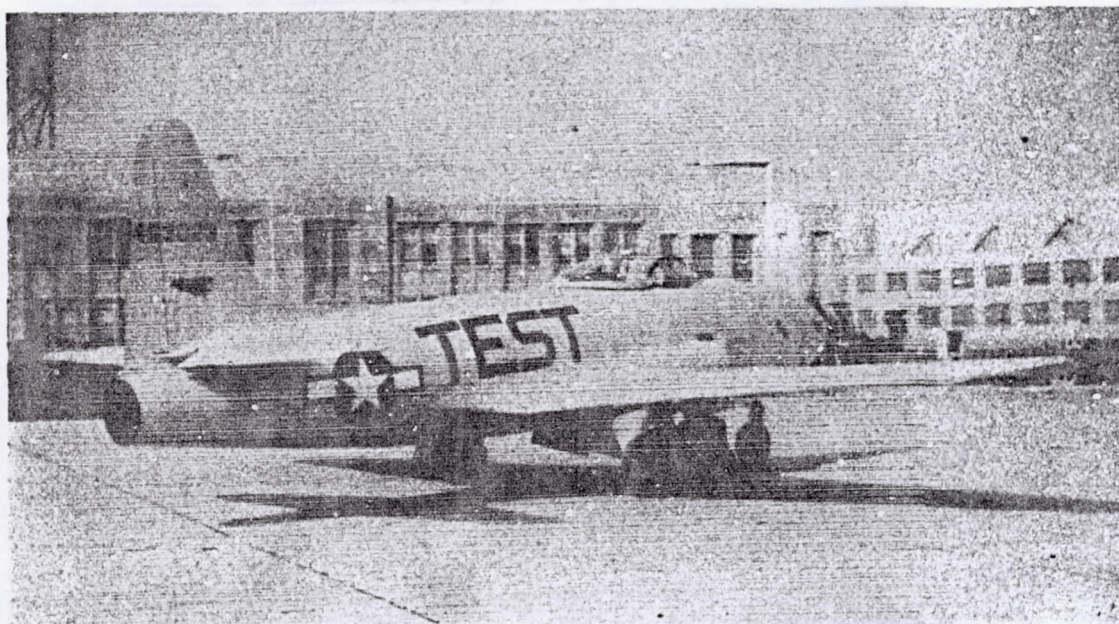
(f) YP-80A airplane.

Figure 2.- Continued.
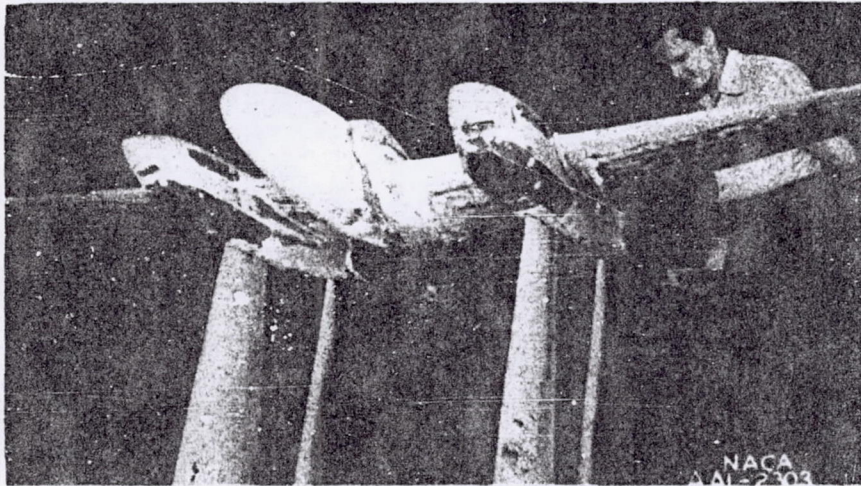
(g) Model of P-38 in Ames 16-foot high-speed wind tunnel.
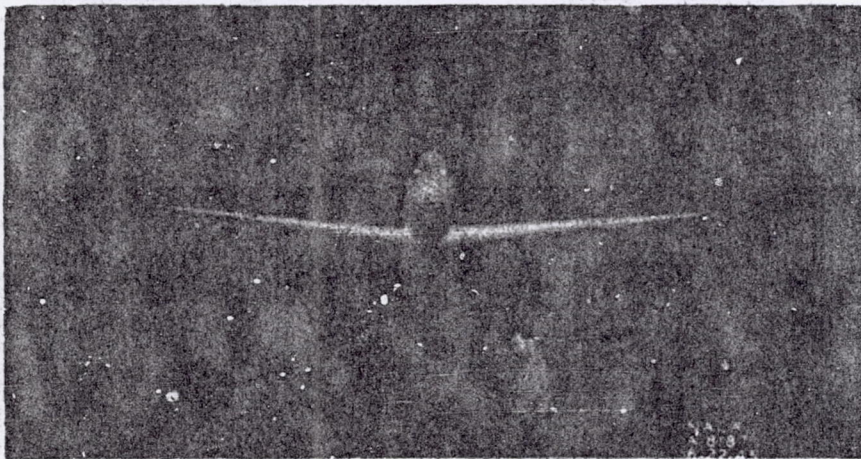
(h) Model of P-39N in Ames 16-foot high-speed wind tunnel.
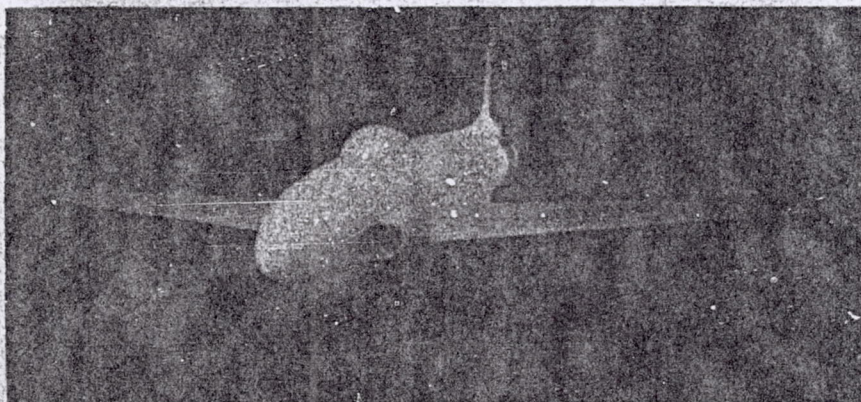
(i) Model of XP-80 airplane in 16-foot high-speed wind tunnel.

Figure 2.- Concluded.

Root section   Tip section

Airplane
P-38F

23016   4412

F6F-3

23015.6 modified   23009

P-39N

0015   23009
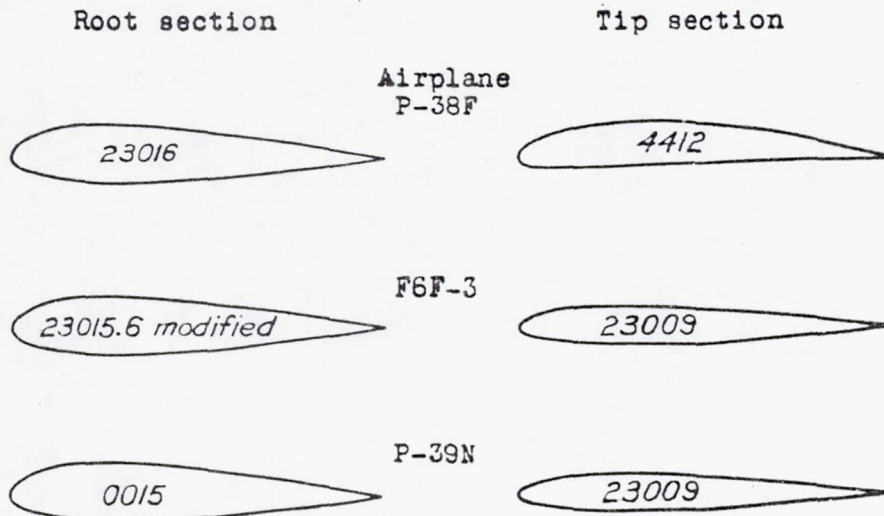
NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

NACA low-drag airfoils

P-51B

NACA-North American   NACA-North American
compromise   compromise

16% thick   11% thick

66,2X-116   P-63A   66,2X-216
a = .6   a = .6

YP-80A
Root and tip

65,-213
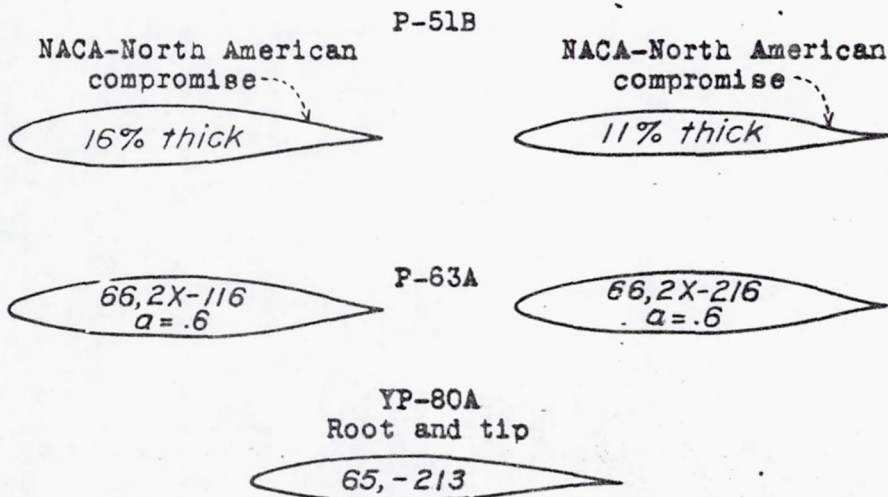
Figure 3.- Profiles of the wing root and tip sections
of the test airplanes.
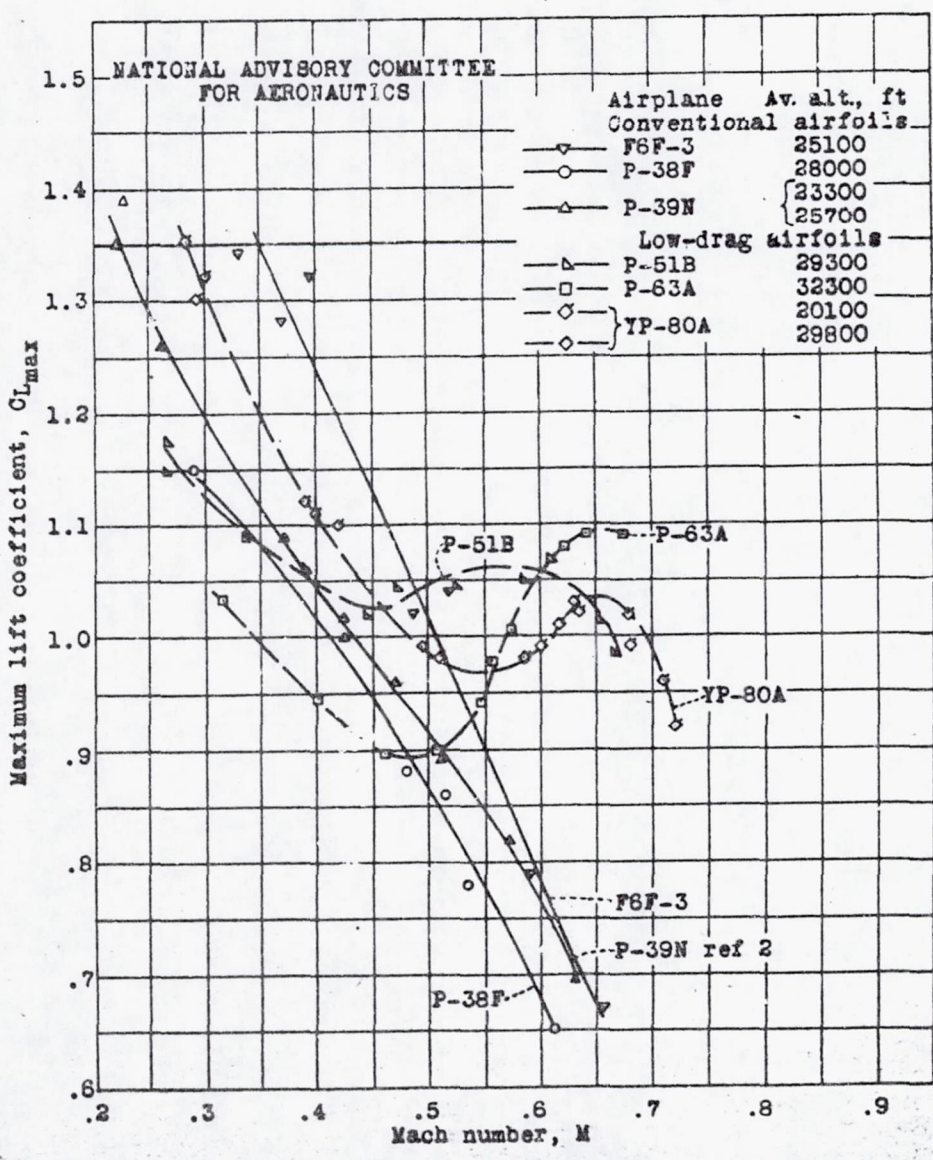
Figure 4.- Variation of maximum lift coefficient with Mach number at high altitude.
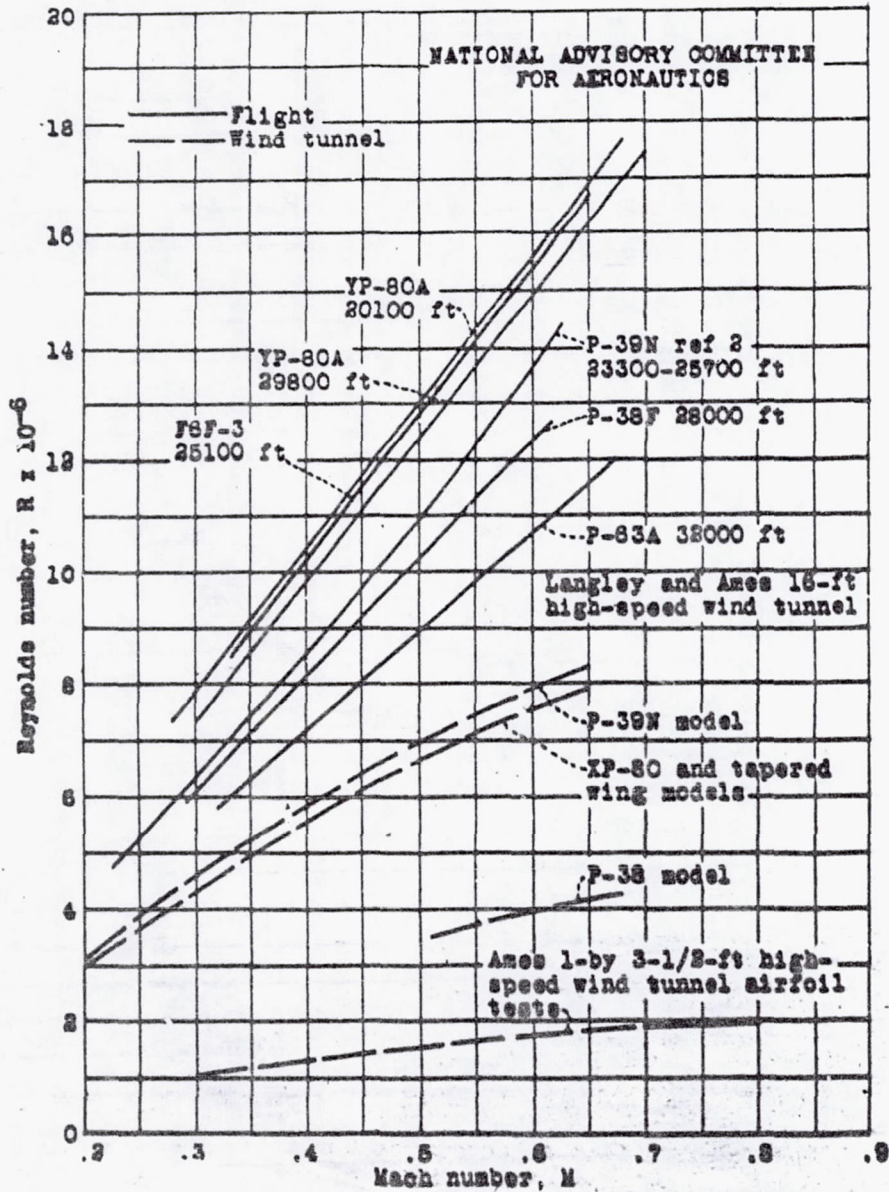
Figure 5.- Variation of Reynolds number with Mach number for the flight test data of figure 4 and the tests in the Langley and Ames 16-foot high-speed wind tunnels and the Ames 1-by 3-1/2-foot high-speed wind tunnel.
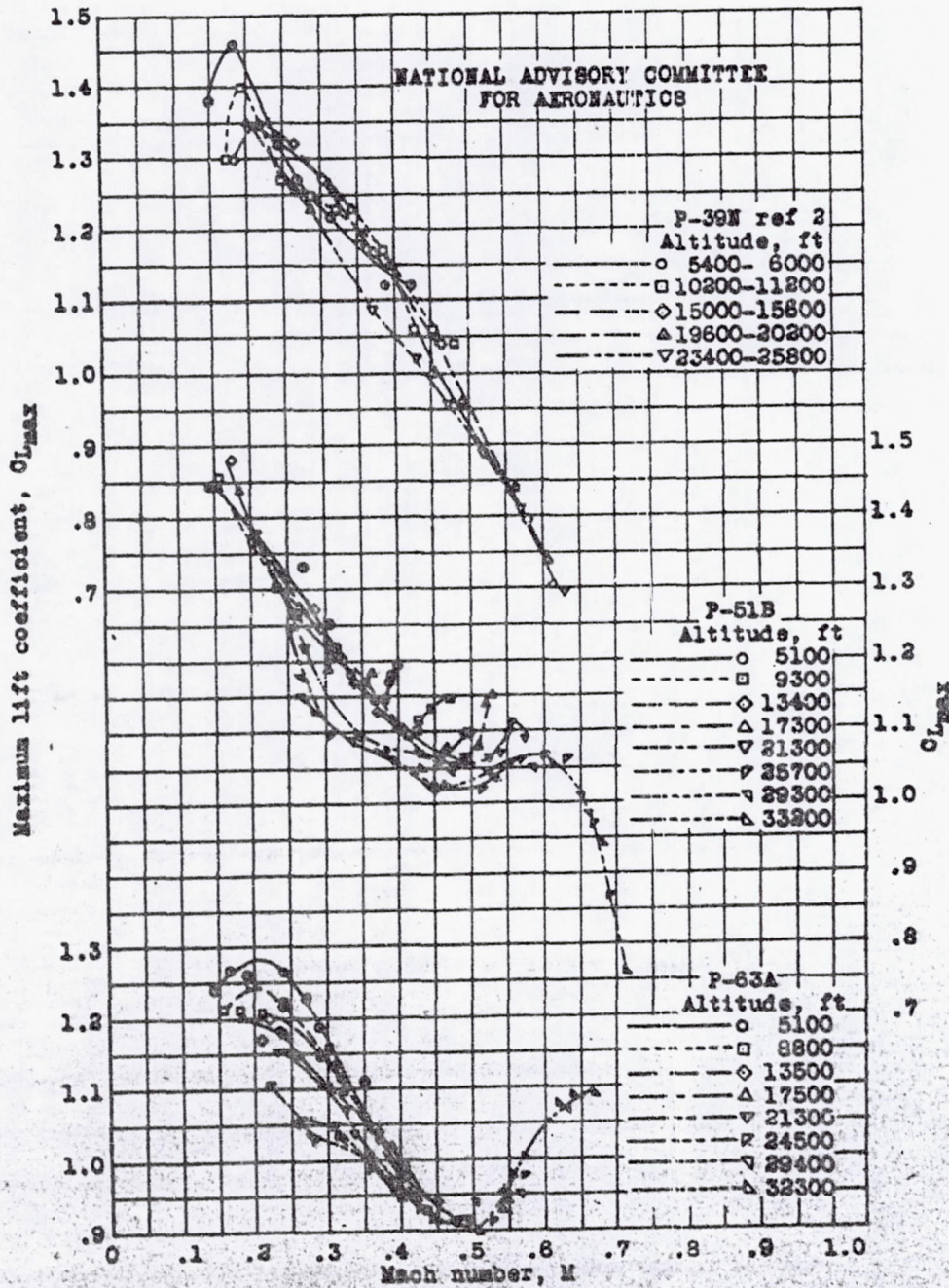
Figure 6.- Variation of maximum lift coefficient with Mach number for various altitudes.

Fig. 7
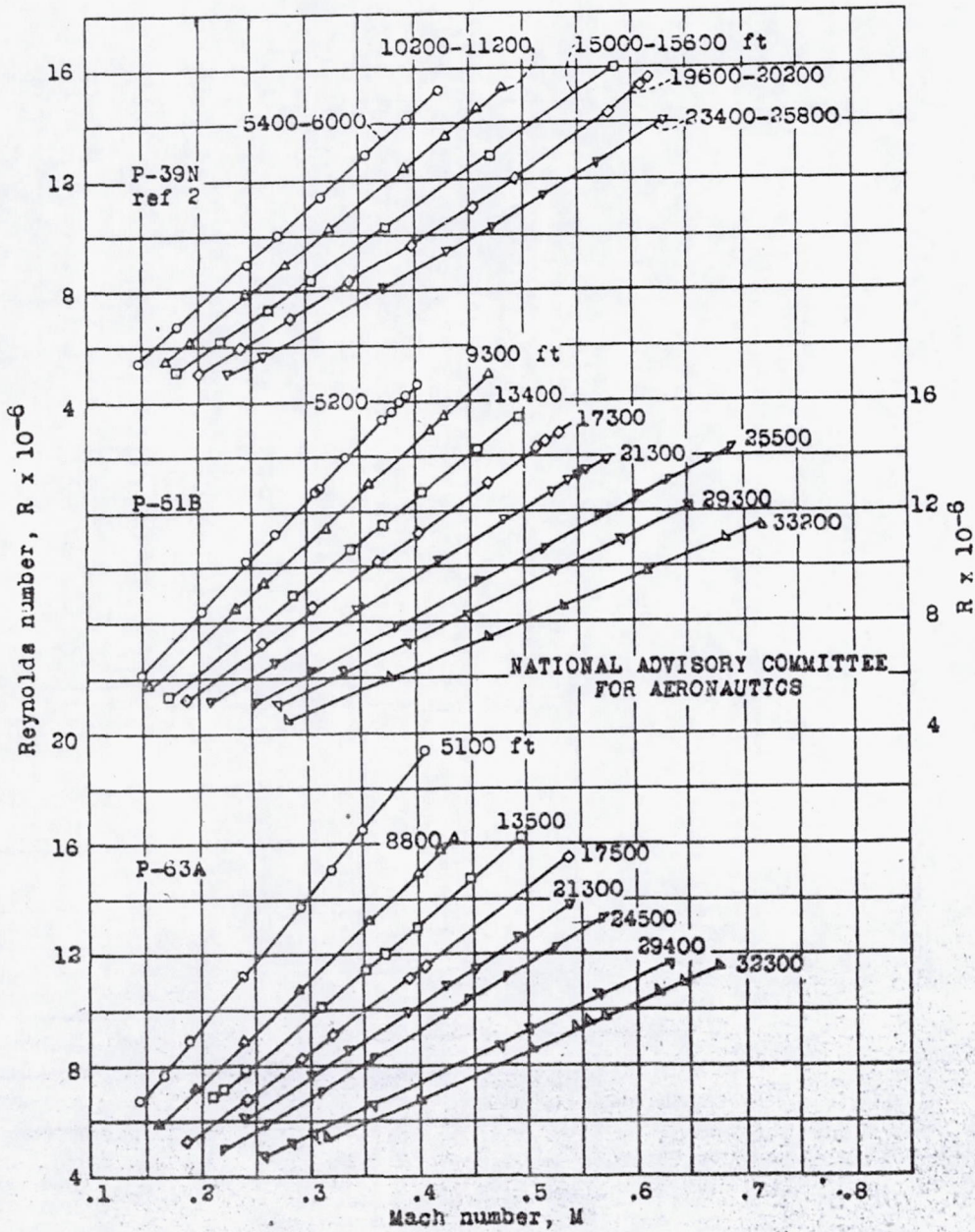
Figure 7.- Variation of Reynolds number with Mach number for the flight test data of figure 6.

Fig. 8
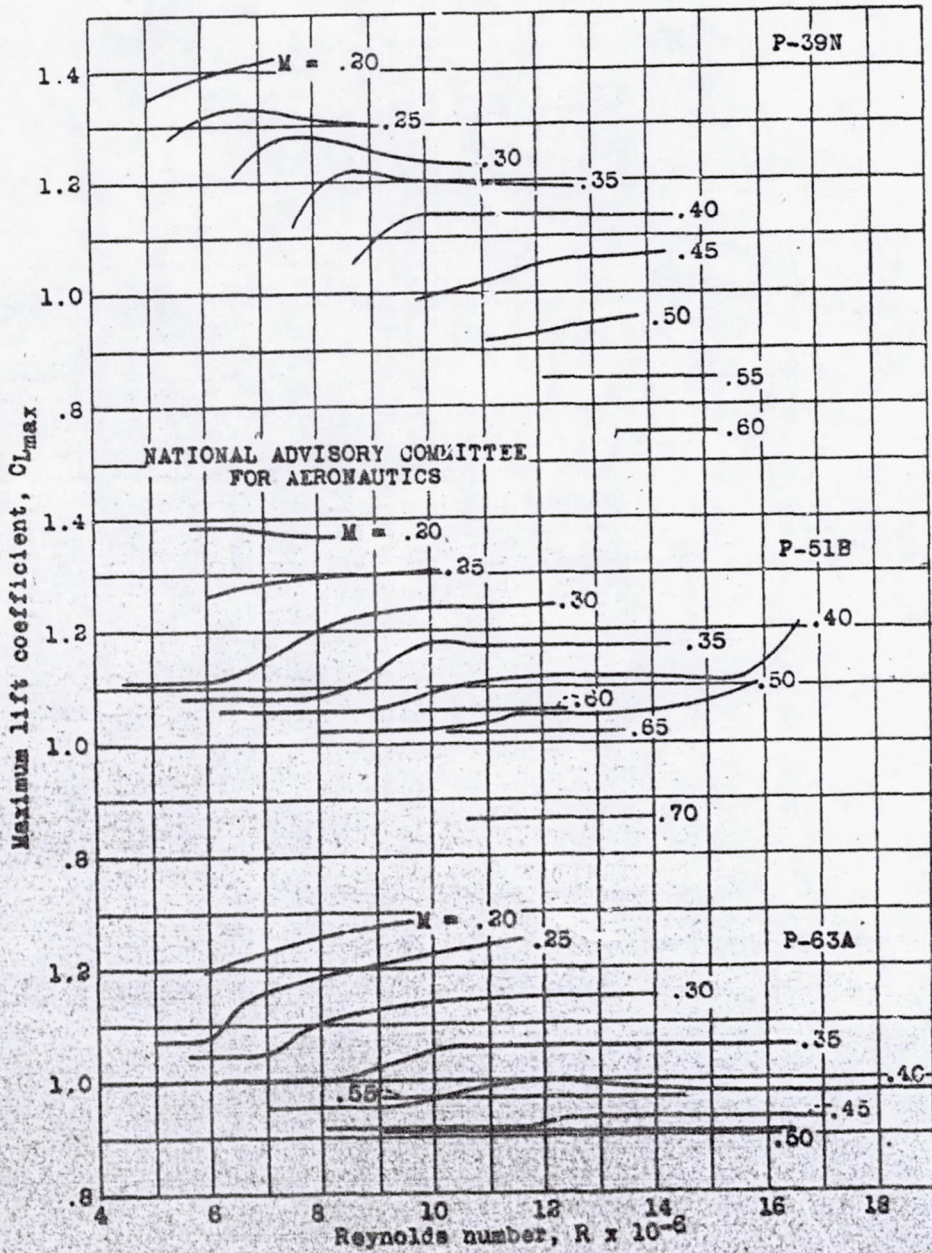
Figure 8.- Variation of maximum lift coefficient with Reynolds number for constant Mach numbers.
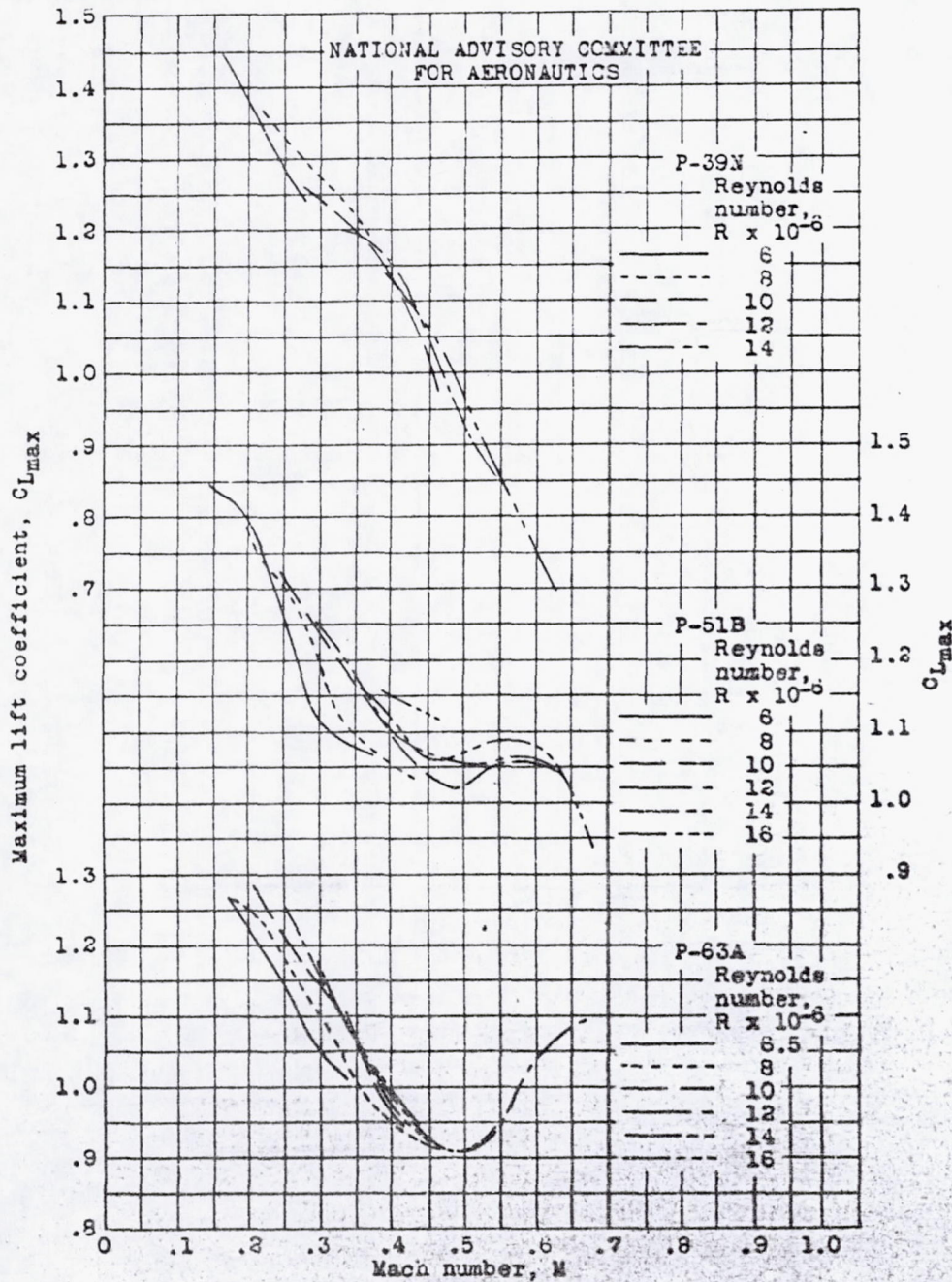
Figure 9.- Variation of maximum lift coefficient with Mach number for various Reynolds numbers.
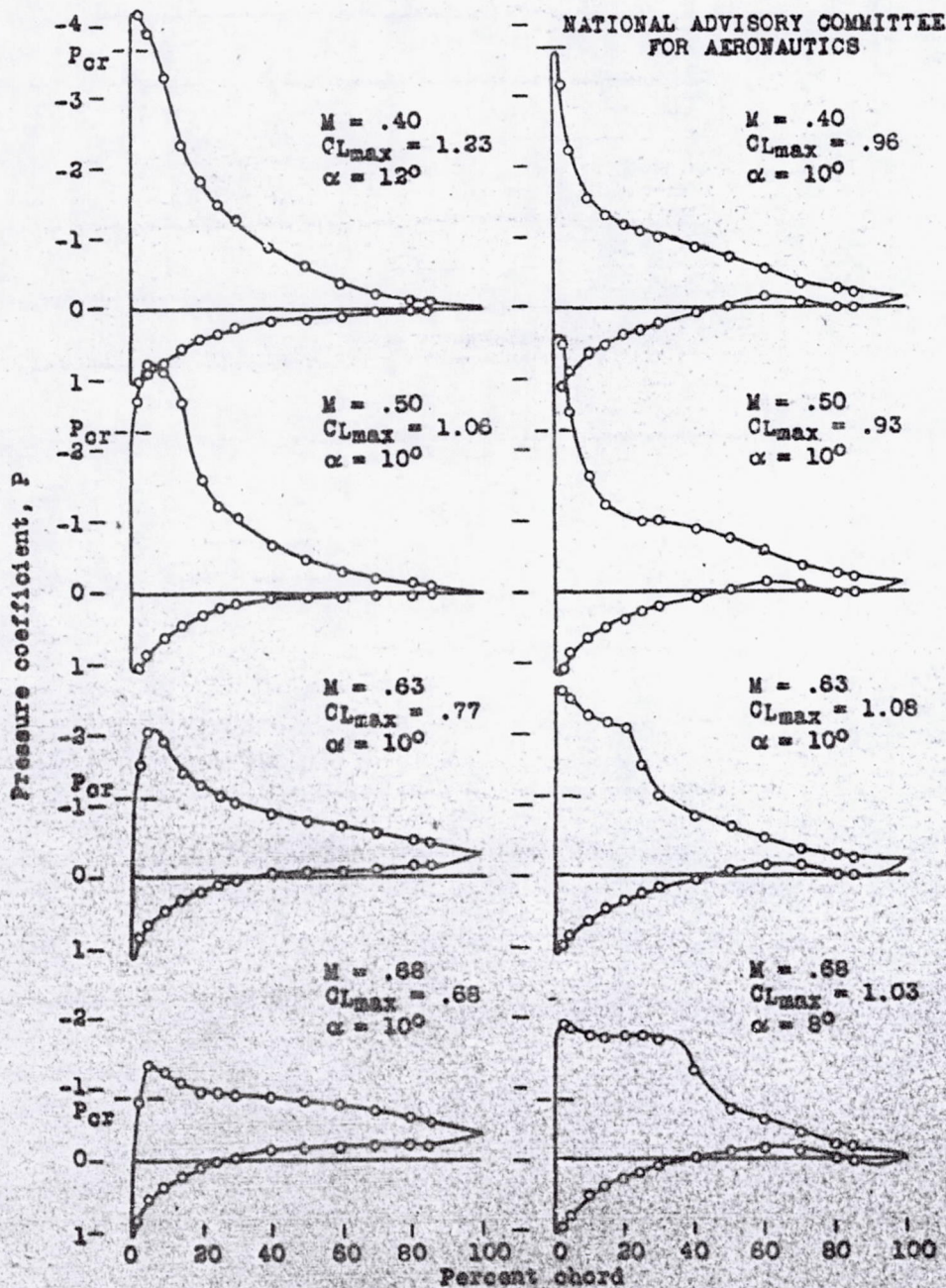
Figure 10.— Typical pressure distributions on NACA conventional and low-drag airfoils at maximum lift coefficient. Ames 1-by 3-1/2-foot high-speed wind tunnel.
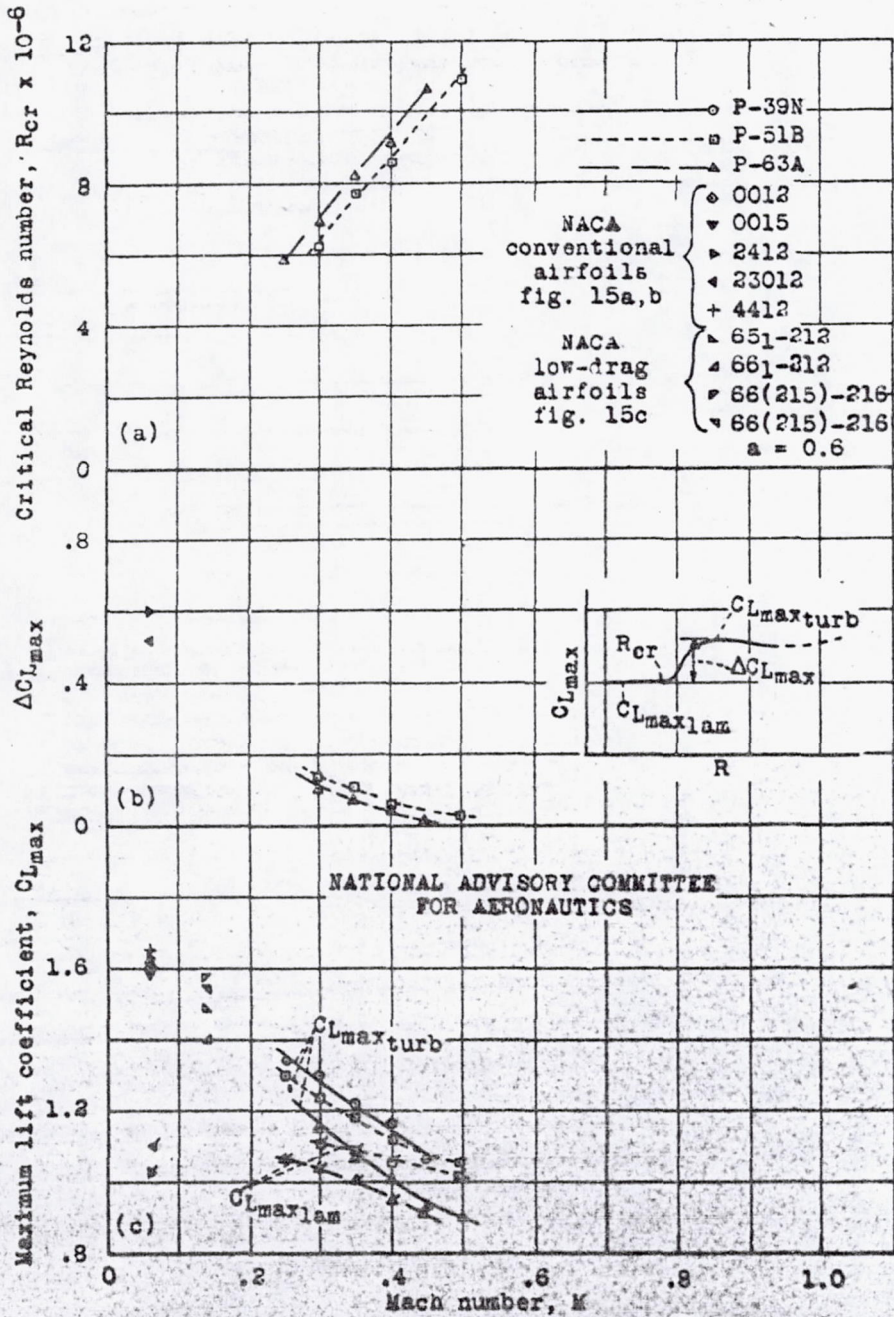
Figure 11.- Effect of Mach number on four parameters indicating the variation of maximum lift coefficient with Reynolds number.
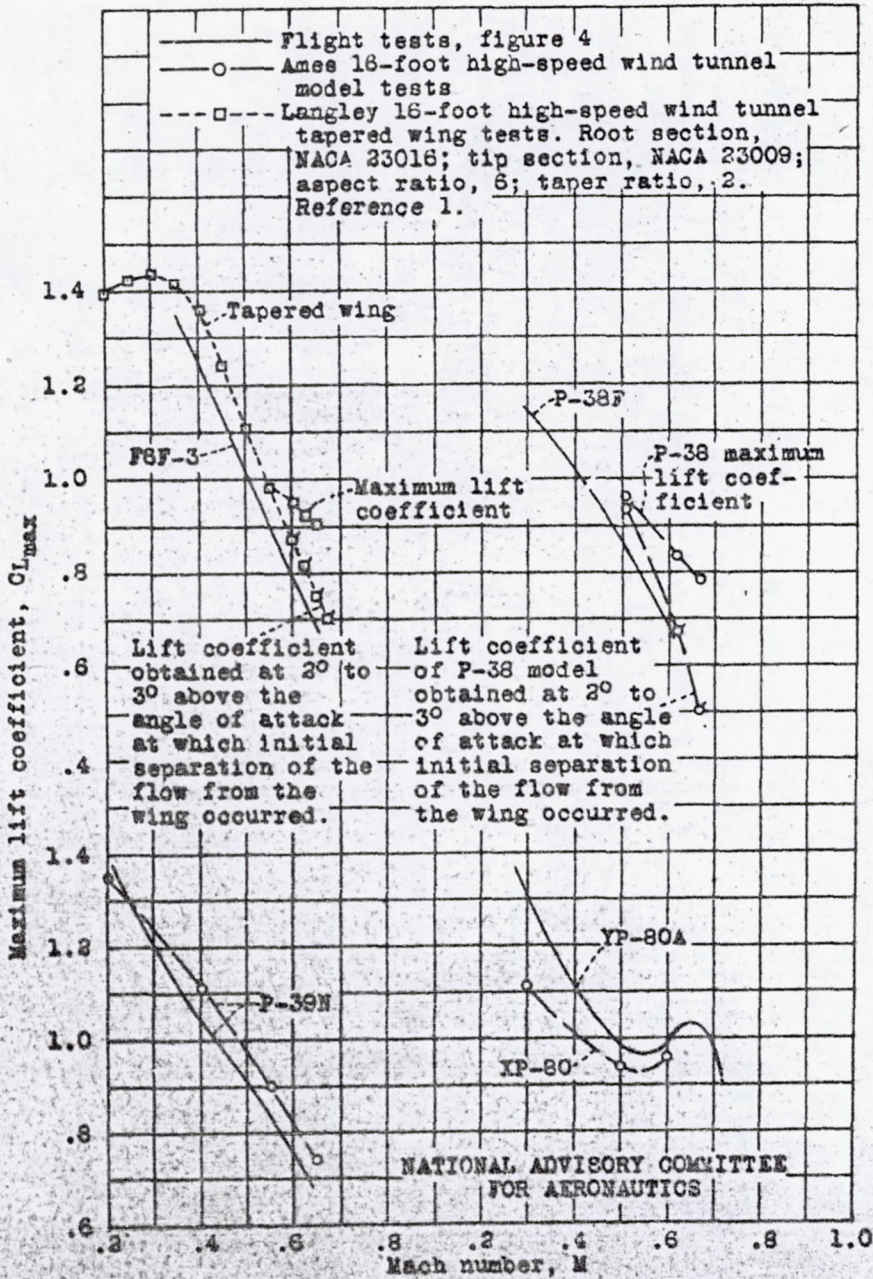
Figure 12.- Comparison of high-altitude flight data with wind-tunnel data showing the variation of maximum lift coefficient with Mach number.
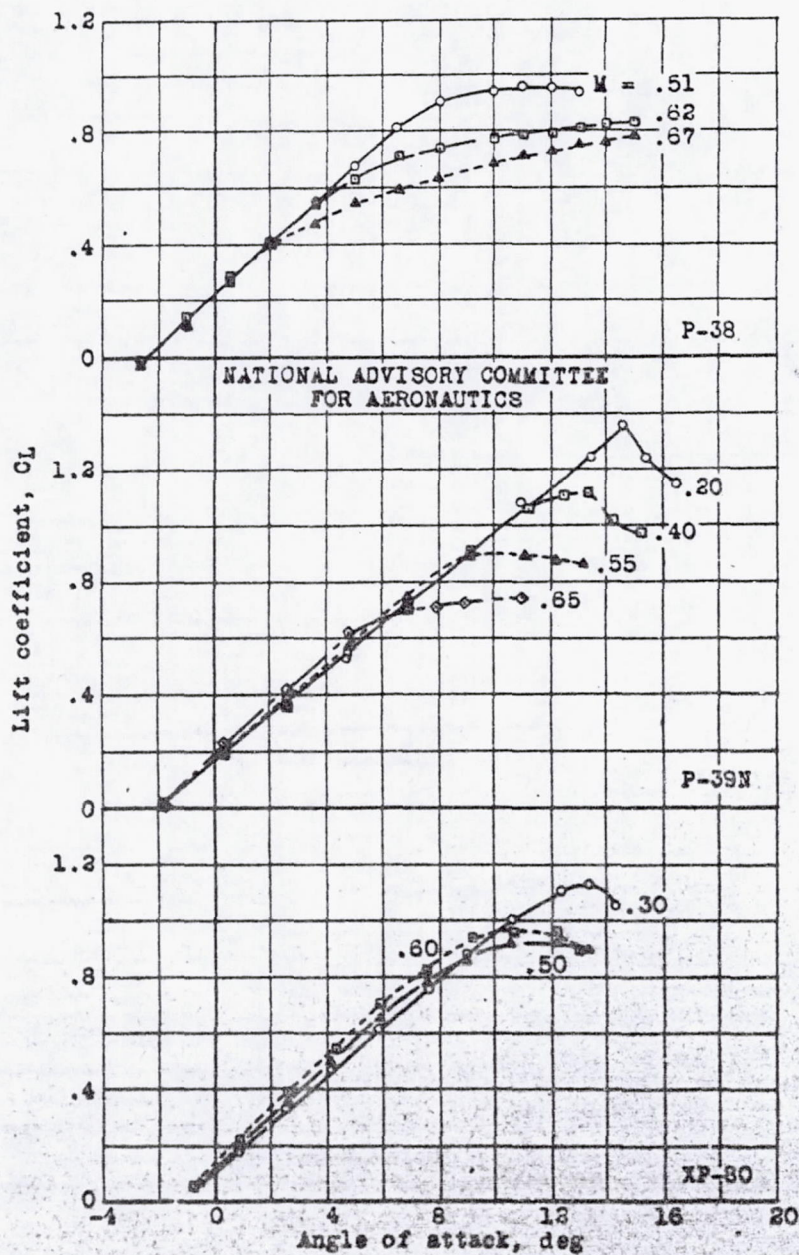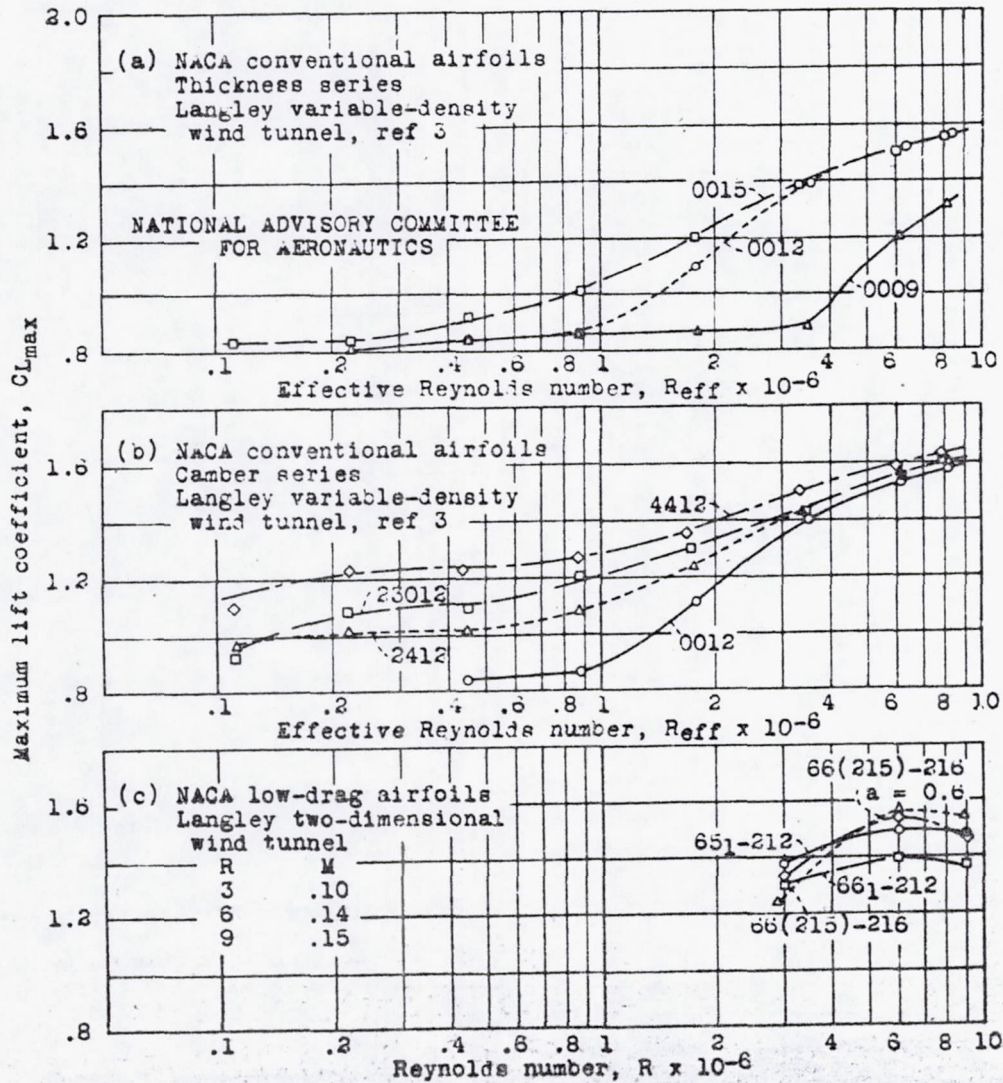
Figure 13.- Variation of lift coefficient with angle of attack at several Mach numbers for three models. Ames 16-foot high-speed wind tunnel.

Fig. 14
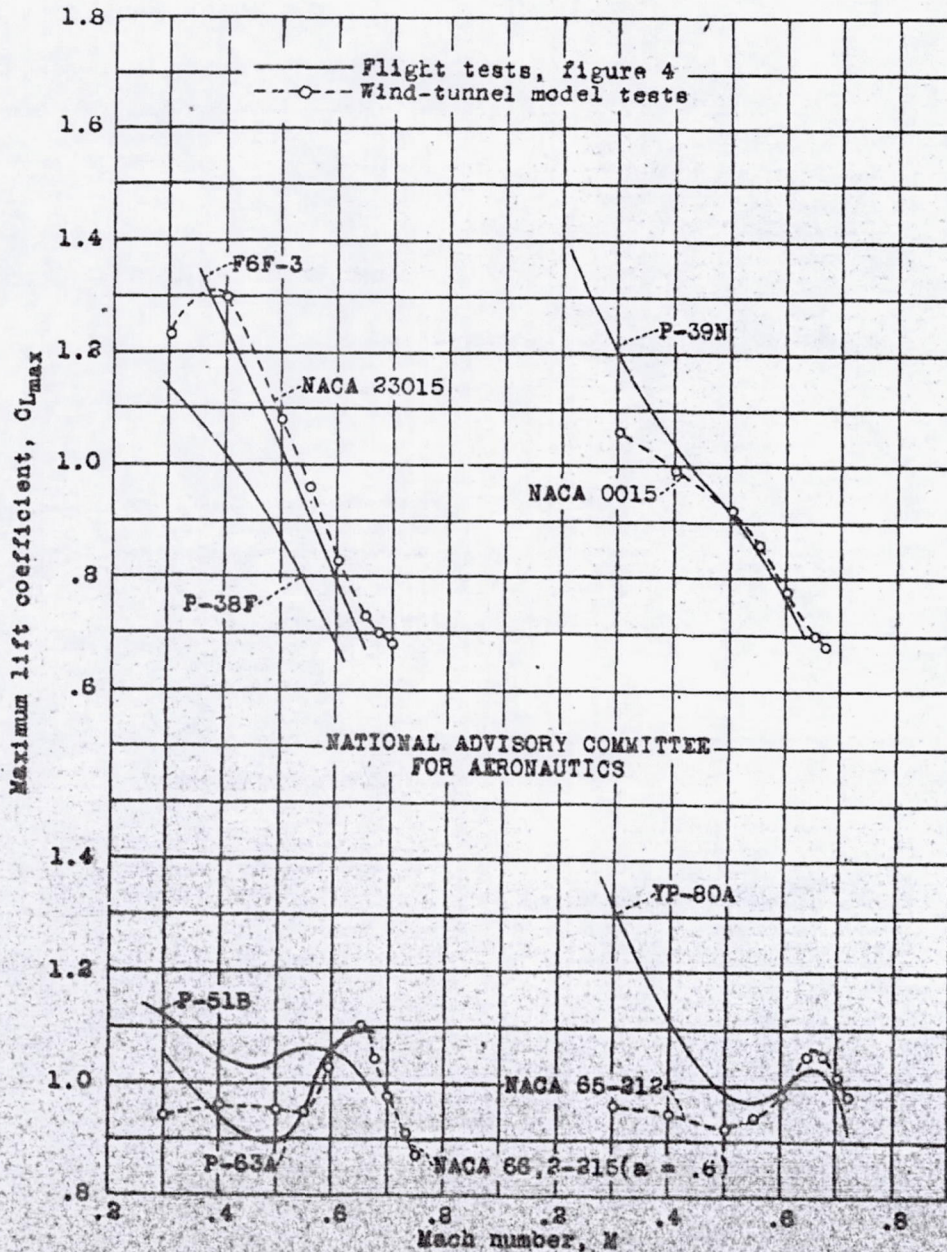
Figure 14.- Comparison of high-altitude flight data with Ames 1-by 3-1/2-foot high-speed wind-tunnel data showing the variation of maximum lift coefficient with Mach number.

Figure 15.- Variation with Reynolds number of the maximum lift
coefficients of several NACA conventional and low-drag
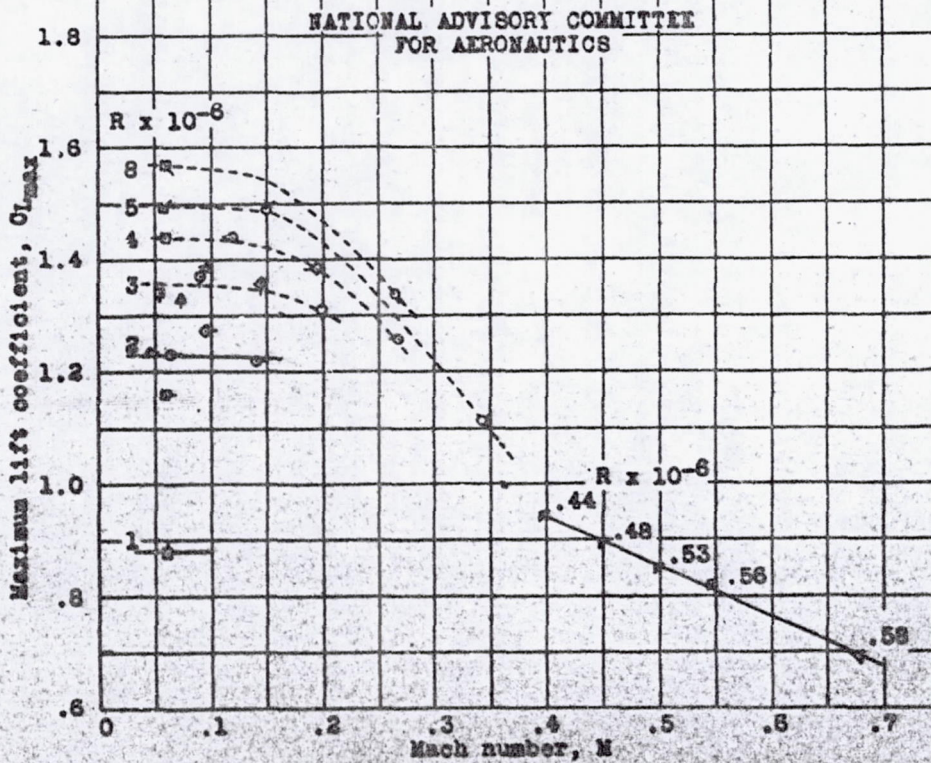airfoils.

Fig. 16

Figure 16.- Maximum lift coefficients of NACA 0012 and 0012-63 airfoils as measured in several wind tunnels.