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RAPID DETERMINATION OF CORE DIMENSIONS OF CROSSFLOW GAS-TO-GAS HEAT EXCHANGERS

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SUMMARY

A generalized procedure is presented which permits a rapid determination of the core lengths of crossflow gas-to-gas heat exchangers. The inlet states of each fluid, the available pressure drops for each fluid, the required temperature change of one fluid, the fluid heat capacities, and the core configuration must be prescribed. Core dimensions for two selected core configurations with identical fluid conditions and for one configuration with different fluid conditions are determined from several trial solutions and application of a generalized chart.

INTRODUCTION

Recently the application of gas-to-gas heat exchangers for reducing the temperature of the air used for cooling the turbines of high-temperature aircraft engines has received consideration. For this type of application, compressor bleed air can be used for the primary air and ram air can be used for the secondary air. The ram air is used to reduce the temperature of the compressor bleed air, and, as a consequence, to reduce the amount of compressor bleed air required. The inlet states of each fluid, the available pressure drops for each fluid, the required temperature change of one fluid, and the heat capacities of each fluid are usually prescribed. Crossflow through the exchanger is assumed; this generally simplifies the required system of ducting.

Since light weight, small volume, and small frontal area are of primary importance in this type of application, compact heat exchangers (like those reported in ref. 1) are desirable. Normally, under the conditions stated, the determination of the dimensions of a heat exchanger of a given core configuration by a procedure such as that presented in reference 1 becomes a lengthy trial-and-error process. Reference 2, however, presents a graphical procedure which involves the construction of a series of charts and three trial calculations to determine the dimensions of a specified heat-exchanger core configuration for a prescribed set of fluid conditions. This method of determining heat-exchanger size is shorter and less time consuming than the procedure of reference 1.

In many instances the particular core configuration which is most suitable for a given heat-exchanger application cannot be specified readily. Consequently, it is often necessary to consider several core configurations and determine the resulting heat-exchanger dimensions. When the method of reference 2 is applied to a number of different core configurations and fluid conditions, a new set of charts must be constructed for each configuration or condition considered. Thus, even the method of reference 2 becomes relatively lengthy and time consuming when the heat-exchanger size is determined for a number of core configurations and fluid conditions.

Recently a generalized procedure applicable to this problem has been developed at the NACA Lewis laboratory. A chart is constructed in such a way that its use, in conjunction with several trial solutions, will permit the final calculation of heat-exchanger core lengths for a range of prescribed conditions and core configurations and thereby eliminate the construction of new charts for different core configurations or fluid conditions as required by the method of reference 2. The purpose of this report is to present this generalized procedure and illustrate its use in determining the sizes of two of the core configurations shown in figure 1 for identical prescribed fluid conditions and of one core configuration for different prescribed fluid conditions.

SYMBOLS

The following symbols with consistent units are used in this report:

A heat-transfer area

A_F frontal area

A' minimum free-flow area

c_p specific heat at constant pressure

d hydraulic diameter

f friction factor

g acceleration due to gravity

h	heat-transfer coefficient
K	$d_2(1 + \sigma_2^2)$
K'	$2gd_2^2\Delta p_2/\mu_2 v_{i,2}(1 + \sigma_2^2)$
k	thermal conductivity
L	heat-exchanger core dimension
l	one-half fin length
Pr	Prandtl number, $c_p\mu/k$
p	pressure
R	gas constant
Re	Reynolds number, $wd/A'\mu$
St	Stanton number, hA'/wc_p
T	temperature, $^{\circ}R$
Tu	heat-transfer parameter (number of transfer units NTU in ref. 1)
t	fin thickness
v	specific volume
w	weight-flow rate
x	L_2f_2/K_2
y	K'/Re_2^2
α	heat-transfer surface area per unit volume
η_f	fin effectiveness
η_T	thermal effectiveness (ϵ in ref. 1)
η_O	surface effectiveness

μ	viscosity based on bulk temperature
σ	ratio of free-flow area to frontal area, A'/A_F

Subscripts:

ex	exit
f	fin
$F_1,F_2,$ F_3,F_4	function
i	heat-exchanger inlet
max	maximum
min	minimum
n	no-flow direction
1	fluid on primary-air side of heat exchanger
2	fluid on secondary-air side of heat exchanger

ANALYSIS

Core Configurations

Six typical heat-exchanger core configurations are shown in figure 1. Friction and heat-transfer data for numerous geometrical variations of these types of exchangers (88 examples in all) are given in reference 1.

Assumptions

For the particular heat-exchanger application considered herein, the following assumptions are made:

(1) For each core configuration under investigation, the core geometry is prescribed.

(2) The friction factors f_1 and f_2 and the Stanton numbers St_1 and St_2 are known as functions of the Reynolds numbers Re_1 and Re_2 of the two fluids.

(3) Crossflow through the heat exchanger is assumed.

(4) The pressures $p_{1,i}$ and $p_{2,i}$ and the temperatures $T_{1,i}$ and $T_{2,i}$ of both fluids are prescribed at the heat-exchanger inlet.

(5) The heat capacities $w_1 c_{p,1}$ and $w_2 c_{p,2}$ of both fluids are prescribed. (When $c_{p,1} \cong c_{p,2}$, w_1 and w_2 are known.)

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(6) The available pressure drops Δp_1 and Δp_2 of both fluids are prescribed.

(7) The temperature drop $-\Delta T_1$ of one fluid or the temperature rise ΔT_2 of the other fluid is prescribed.

(8) A value of $Pr^{2/3} = 0.75$ is assumed. The heat-transfer data for the core configurations of figure 1 were taken from reference 1 with $Pr^{2/3} = 0.75$.

(9) Heat-exchanger end losses in pressure are neglected.

(10) The surface effectiveness η_0 is assumed.

Basic Equations

According to reference 2, under the foregoing assumptions, consideration of the heat balance, weight flow or heat capacity, pressure drop, and effectiveness relations results in a system of five basic equations in five unknowns. The equations may be written as follows:

$$\frac{L_1}{Re_1 Tu} = \frac{\dfrac{\sigma_1}{\alpha_2}\,\dfrac{c_{p,1}}{c_{p,2}}\,\dfrac{\mu_1}{\mu_2}\,\dfrac{d_2}{d_1}}{\eta_{0,2}(ReSt)_2} + \frac{\sigma_1}{\alpha_1 \eta_{0,1}(ReSt)_1} \tag{1}$$

$$\frac{2gd^2\Delta p}{Re^2\mu^2 v_i(1 + \sigma^2)} - \left(\frac{v_{ex}}{v_i} - 1\right) = \frac{2Lf}{d(1 + \sigma^2)}\left(\frac{v_{ex}}{v_i} + 1\right) \tag{2}$$

where

$$v_i = \frac{RT_i}{p_i}$$

and

$$v_{ex} = \frac{R(T_i \pm \Delta T)}{p_i - \Delta p}$$

(Since equation (2) applies for either fluid, two such equations may be written. The correct algebraic sign must accompany the value of ΔT. ΔT is positive for a temperature rise and negative for a temperature drop.)

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$$w_1 = \sigma_1 Re_1 \frac{\mu_1}{d_1} L_2 L_n \tag{3}$$

(An alternate equation can be obtained by interchanging the subscripts 1 and 2.)

$$\Delta T_2 = - \frac{c_{p,1}\ d_2\ \mu_1\ \sigma_1\ Re_1\ L_2}{c_{p,2}\ d_1\ \mu_2\ \sigma_2\ Re_2\ L_1} \Delta T_1 \tag{4}$$

The surface effectiveness η_0 in equation (1) was assumed to be 0.8 for the fins on both fluid sides of heat exchangers such as those shown in figure 1. For the finned-tube exchanger (fig. 1(f)) $\eta_0 = 0.8$ was assumed for the fins and $\eta_0 = 1.0$ for the tubes. A more exact value of η_0 could be determined from the following equations if desired:

$$\eta_0 = 1 - \frac{A_f}{A} (1 - \eta_f) \tag{5}$$

where

$$\eta_f = \sqrt{\frac{kt}{2h}}\ \frac{1}{l} \tanh \sqrt{\frac{2h}{kt}}\ l \tag{6}$$

The thermal effectiveness η_T necessary for evaluation of Tu from figure 2 (reproduced from ref. 1) for use of equation (1) may be expressed by the equation

$$\eta_T = \left| \frac{\Delta T_{max}}{T_{1,i} - T_{2,i}} \right| \tag{7}$$

The five basic equations (eqs. (1) to (4)) contain the five unknowns L_1, L_2, L_n, Re_1, and Re_2. A purely analytical solution of these equations cannot be obtained, however, because the functions f, η_0, and St are contained within the equations and are generally given in graphical form. Consequently, a combined analytical-graphical procedure will be employed. Friction and heat-transfer relations for flow through rectangular passages are given in reference 3. These relations are

$$f_1 = 0.050\ Re_1^{-0.2} \tag{8}$$

$$St_1 = 0.019\ Re_1^{-0.2}\ Pr^{-2/3} \tag{9}$$

For other flow configurations, friction and heat-transfer data are taken from reference 1.

CALCULATION PROCEDURE

A core configuration is selected; this permits construction of the appropriate f and $ReSt$ curves, samples of which are shown in figure 3. The inlet state of each fluid, the heat capacities and pressure drops of each fluid, and the temperature change of one fluid are also prescribed. The temperature change of the other fluid is determined from the heat-balance equation

$$w_1 c_{p,1}\ \Delta T_1 + w_2 c_{p,2}\ \Delta T_2 = 0 \tag{10}$$

and the number of transfer units Tu is determined from equation (7) and figure 2. If a value of Re_1 is assumed, both terms in the left member of equation (2) can be evaluated; for these calculations, ΔT_2 is determined from equation (10). After the value of f corresponding to the assumed Re_1 is determined from figure 3, L_1 can be evaluated by use of equation (2), which may be written $L_1 = F_1(Re_1)$. Equation (2) can be represented graphically as in figure 4 by a series of straight lines for a selected range of variables. For the assumed Re_1, the value of $(ReSt)_1$ is read from figure 3. Equation (1) may then be solved for $(ReSt)_2$ because $(ReSt)_2 = F_2(Re_1, L_1)$ and Re_2 read from figure 3. A solution of equation (2), $L_2 = F_3(Re_2)$, for this value of Re_2 and the value of ΔT_2 determined from equation (10) then yields a value of L_2. Equation (4), $\Delta T_2 = F_4(Re_1/L_1, L_2/Re_2)$, can then be solved for ΔT_2. If this value does not agree with the ΔT_2 obtained from equation (10), the procedure must be repeated for a new assumed value of Re_1.

After a few such calculations have been completed for different assumed values of Re_1, the final correct values of Re_1, Re_2, L_1, L_2, and L_n can be determined as follows. Plot, on figure 4, the values of the parameter $(v_{ex}/v_i)_2$ corresponding to the values of ΔT_2 determined from equation (4) against the values of the abscissa $x = L_2f_2/K$ where $K = d_2(1 + \sigma_2^2)$. The intersection of a curve through these points with the line representing the value of $(v_{ex}/v_i)_2$ corresponding to the ΔT_2 determined from equation (10) yields a point on figure 4 from which Re_2 can be determined (from the ordinate $y = K'/Re_2^2$ where $K' = 2gd_2^2\Delta p_2/\mu_2^2 v_{i,2}(1 + \sigma_2^2)$); L_2 (from the abscissa $x = L_2f_2/K$ where $K = d_2(1 + \sigma_2^2)$) and the value of f corresponding to this Re_2 from figure 3 can also be determined from this point. Re_1/L_1 can be found from equation (4), and $(ReSt)_1$ can then be found from equation (1). From figure 3, Re_1 is obtained, and L_1 corresponding to this Re_1 is found from equation (2). The values of L_n can then be found from equation (3).

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The procedure described herein permits the rapid determination of the core sizes of gas-to-gas crossflow heat exchangers when a number of core configurations are to be studied. It is equally applicable for studying heat-exchanger core sizes for a selected configuration when different fluid conditions are prescribed. In either case, three trial solutions for each configuration and/or each set of prescribed fluid conditions are sufficient to determine the curve which when inserted on figure 4, or an extension thereof, permits the calculation of the desired final core dimensions. In contrast to this, application of the method of reference 2 requires the construction of a series of new charts for each set of fluid conditions and for each core configuration.

NUMERICAL CALCULATIONS

The heat-exchanger core lengths for two of the heat-exchanger core configurations shown in figure 1 were calculated. The selected core configurations were the plain fin and the finned tube. The values of d, α, and σ for these configurations and the prescribed inlet and exit states of both fluids, prescribed pressure drops of both fluids, and prescribed temperature change of one fluid are listed in table I. For each example, the value of $w_2c_{p,2}$ is taken as twice that of $w_1c_{p,1}$; that is, from equation (10), ΔT_2 is equal to $1/2\left|\Delta T_1\right|$.

The friction and heat-transfer data for the flow of the primary fluid through the finned-tube exchanger are presented in figure 5 and are the graphical representations of equations (8) and (9) with $Pr^{2/3} = 0.75$. Friction and heat-transfer data for the plain-fin exchanger and for the secondary fluid in the finned-tube exchanger are taken from reference 1 and presented in figure 6. These curves are similar to the sample curves of figure 3 discussed in the section PROCEDURE.

Plain-Fin Exchanger

By the procedure outlined previously, three trial solutions were obtained for assumed values of Re_1 equal to 4800, 4600, and 4300. From the first set of conditions, given in table I, values of v_{ex}/v_i for the fluids 1 and 2 were found to be 0.46 and 3.57, respectively. These values were used in the trial solutions. Results of the trial solutions are listed in table II. Corresponding to the assumed values of Re_1, the following points resulting from the trial solutions are plotted on figure 7:

$\left(\frac{v_{ex}}{v_i}\right)_2$	$\frac{f_2 L_2}{K}$
2.83	0.88
3.69	1.44
5.87	2.86

The point of intersection of the curve joining these three points with the curve representing the correct value 3.57 of the parameter $(v_{ex}/v_i)_2$ yielded an ordinate of about 14.90 and an abscissa of about 1.36 (see fig. 7). The final solution obtained by use of this intersection point was found to be

Re_1	L_1, ft	Re_2	L_2, ft	L_n, ft
4650	3.60	3650	0.95	0.46

In order to determine the effect of different fluid conditions on heat-exchanger core dimensions, the plain-fin exchanger was investigated for a second set of fluid conditions (given in table I). The values of v_{ex}/v_i for fluids 1 and 2 for this example were found to be 0.86 and 1.98, respectively. Trial solutions obtained for three assumed values of Re_1 equal to 7500, 7300, and 7000 are listed in table II. The following points, resulting from the trial solutions are plotted in figure 7:

$\left(\frac{v_{ex}}{v_i}\right)_2$	$\frac{f_2 L_2}{K}$
1.95	0.84
2.08	1.11
2.36	1.67

The curve joining these points intersects the line representing the appropriate value of $(v_{ex}/v_i)_2 = 1.98$ at an ordinate of about 6.4 and an abscissa of 0.91. The final solution, corresponding to this point, is

Re_1	L_1, ft	Re_2	L_2, ft	L_n, ft
7370	2.67	4409	0.66	0.49

Finned-Tube Exchanger

In order to demonstrate the difference in core dimensions for different core configurations under identical fluid conditions, a finned-tube exchanger was investigated for fluid conditions identical to those of conditions I for the plain-fin exchanger. Consequently, the values of v_{ex}/v_i for fluids 1 and 2 are identical to those of the plain-fin exchanger, namely, 0.46 and 3.57, respectively. Trial solutions for the finned-tube exchanger for assumed values of Re_1 equal to 21,000, 20,550, and 20,000 were found and are listed in table II. Corresponding to these assumed values of Re_1, the points plotted in figure 7 for this example are

$\left(\frac{v_{ex}}{v_i}\right)_2$	$\frac{f_2 L_2}{K}$
2.92	1.32
3.54	1.81
4.69	2.68

The curve joining these three points intersects the line representing the correct value 3.57 of the parameter $(v_{ex}/v_i)_2$ (this is the same line used for determining the intersection point for the plain-fin exchanger for conditions I, since $(v_{ex}/v_i)_2 = 3.57$ for both exchangers) at an ordinate of about 19.30 and an abscissa of about 1.83 (fig. 7). The final solution was found to be

Re_1	L_1, ft	Re_2	L_2, ft	L_n, ft
20,400	7.14	5570	3.11	0.17

ACCURACY OF SOLUTIONS

The dimensions of the heat-exchanger cores obtained for the core configurations considered herein were used in a conventional calculation procedure presented in reference 1 for determining the pressure drop and temperature changes when core dimensions are known. By this procedure, a check on the accuracy of the present method is possible. For the plain-fin and finned-tube core configurations, deviations in the temperature changes between the two solutions were limited to less than 1 percent and in the pressure drop to less than 3 percent.

GENERAL COMMENTS

From the examples chosen to demonstrate the procedure, one can observe how changes in gas conditions and core configuration appreciably affect the core dimensions. This indicates that an investigation including a number of different configurations may be necessary for every anticipated application and that the configuration best satisfying the space, weight, or frontal-area requirements for the application can then be selected. It is for such investigations that the method presented herein is useful.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, September 25, 1956

REFERENCES

1. Kays, W. M., and London, A. L.: Compact Heat Exchangers - A Summary of Basic Heat Transfer and Flow Function Design Data. Tech. Rep. 23, Dept. Mech. Eng., Stanford Univ., Nov. 15, 1954. (Contract N6-ONR-251, Task Order 6 (NR-065-104) for Office Naval Res.)

2. Eckert, E. R. G., and Diaguila, Anthony J.: Method of Calculating Core Dimensions of Crossflow Heat Exchanger with Prescribed Gas Flows and Inlet and Exit States. NAÇA TN 3655, 1956.

3. Kays, W. M.: Basic Heat Transfer and Flow Friction Design Data for Gas Flow in Circular and Rectangular Cylindrical Tube Heat Exchangers. Tech. Rep. 14, Dept. Mech. Eng., Stanford Univ., June 15, 1951. (Contract N6-ONR-251, Task Order 6 (NR-065-104) for Office Naval Res.)

TABLE I. - PRESCRIBED CONDITIONS AND CORE GEOMETRY

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	Plain-fin exchanger		Finned-tube exchanger
	Conditions		
	I	II	
T_1, ^oR	1600	1000	1600
ΔT_1, ^oR	-1000	-300	-1000
T_2, ^oR	400	600	400
p_1, lb/sq ft abs	5300	5300	5300
Δp_1, lb/sq ft	1000	1000	1000
p_2, lb/sq ft abs	1082	1082	1082
Δp_2, lb/sq ft	400	400	400
$w_1 c_{p,1}/w_2 c_{p,2}$	0.5	0.5	0.5
w_1, lb/sec	2.7	2.7	2.7
d_1, ft	0.00598	0.00598	0.018
d_2, ft	0.00598	0.00598	0.0118
α_1, ft^{-1}	259.88	259.88	48.78
α_2, ft^{-1}	259.88	259.88	229
σ_1	0.388	0.388	0.219
σ_2	0.388	0.388	0.697
$\eta_{0,1}$	0.80	0.80	1.00
$\eta_{0,2}$	0.80	0.80	0.80

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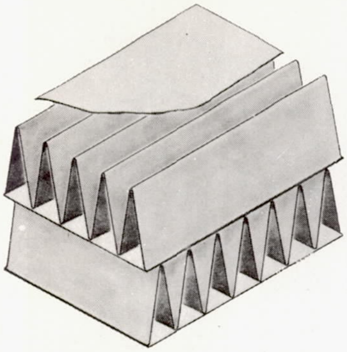
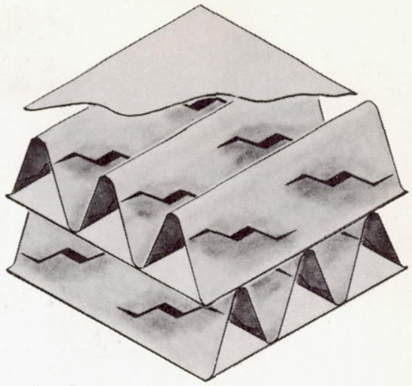
TABLE II. - RESULTS OF TRIAL SOLUTIONS

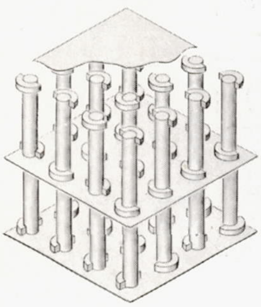
Re_1	L_1, ft	Re_2	L_2, ft	ΔT_2 °R	$\left(\frac{v_{ex}}{v_i}\right)_2$
Plain-fin exchanger (condition I)					
4800	3.38	4320	0.64	314.4	2.83
4600	3.63	3550	.99	529.8	3.69
4300	4.09	2630	1.81	1079.3	5.87
Plain-fin exchanger (condition II)					
7500	2.61	4570	0.61	136.5	1.95
7300	2.72	4060	.79	185.3	2.08
7000	2.92	3380	1.15	290.9	2.36
Finned-tube exchanger					
21,000	6.80	6400	2.26	335.8	2.92
20,550	7.04	5600	3.07	493.0	3.54
20,000	7.37	4700	4.40	782.9	4.69



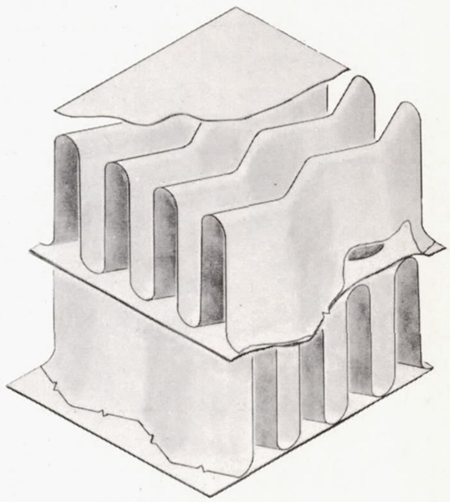
(a) Plain fin.



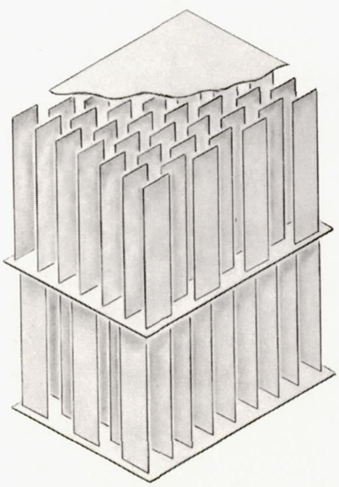
(b) Louvered fin.



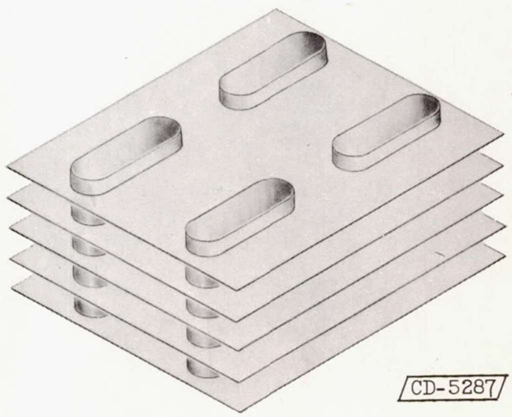
(c) Pin fin.



(d) Wavy fin.



(e) Strip fin.



(f) Finned tube.

Figure 1. - Typical heat-exchanger core configurations.

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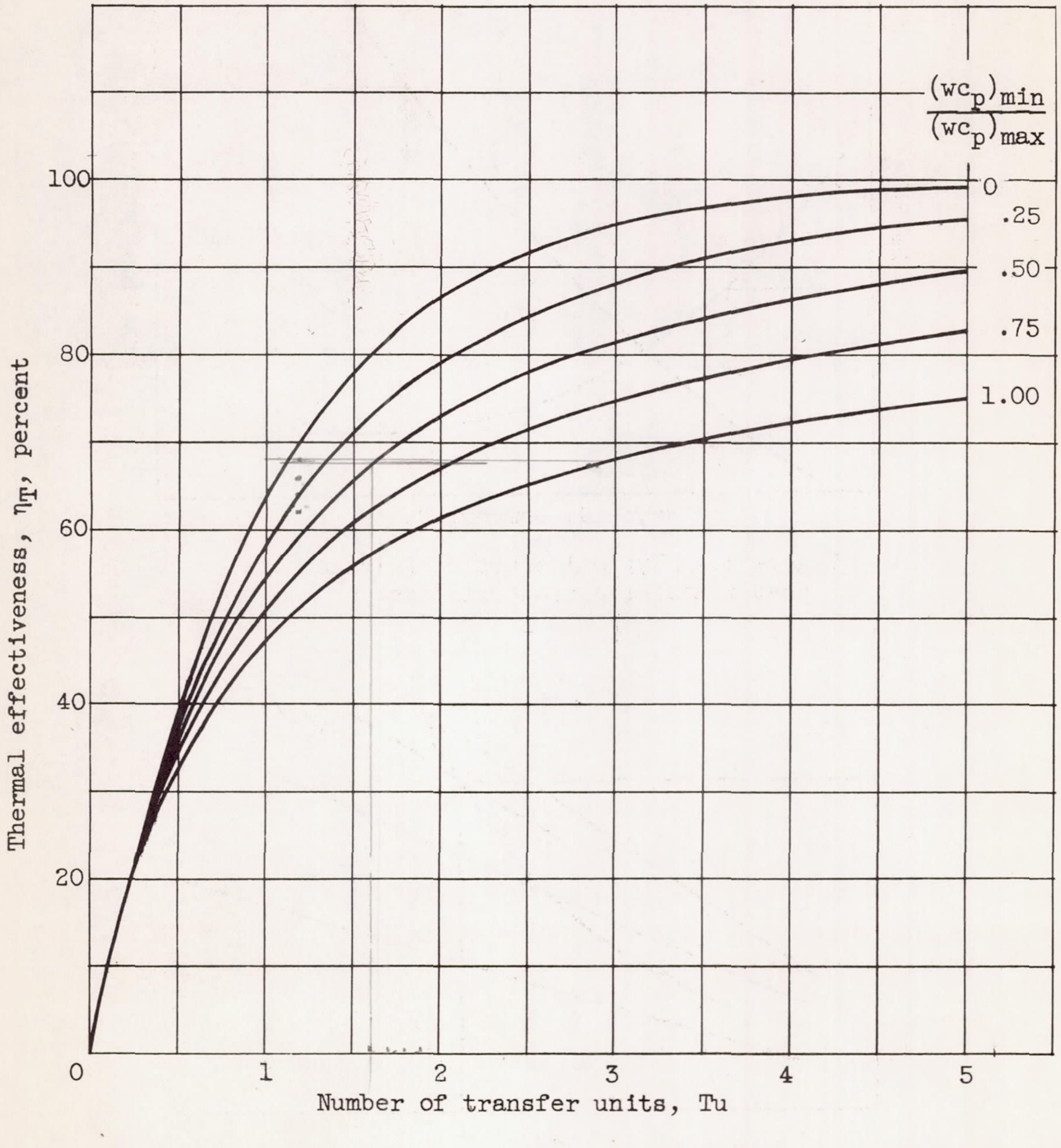


Figure 2. - Performance of crossflow heat exchanger with fluids unmixed (ref. 1).

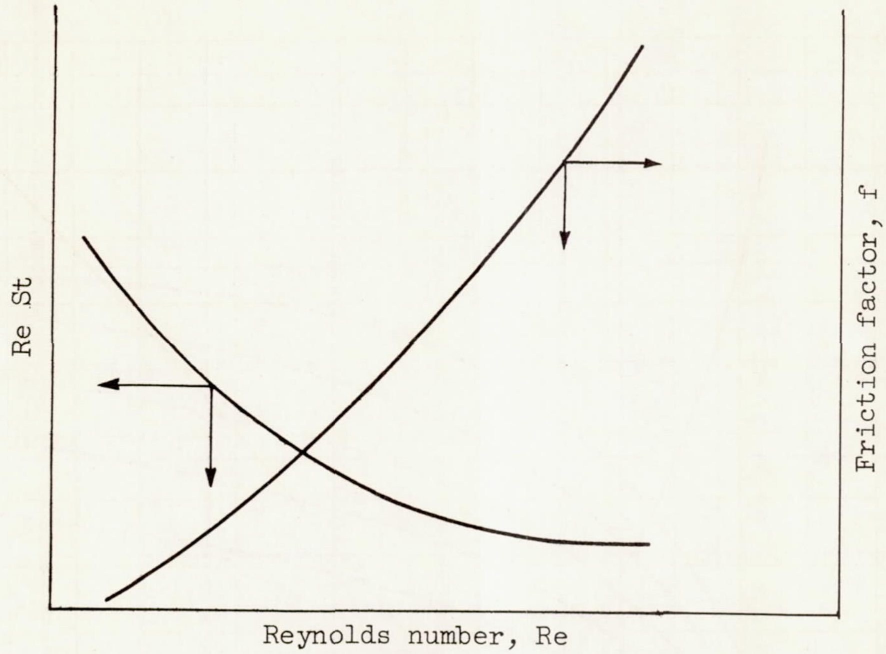


Figure 3. - Typical chart of friction and heat-transfer characteristics.

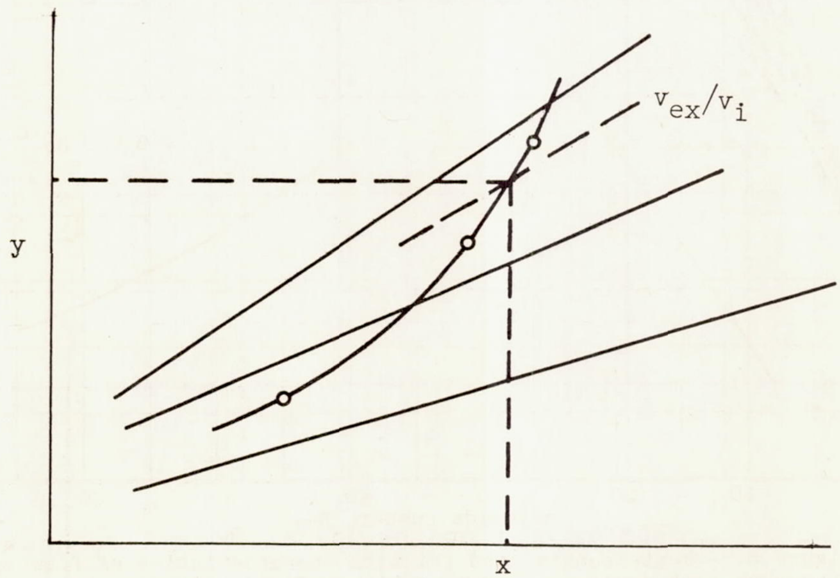


Figure 4. - Typical chart of equation (2) with typical trial solutions.

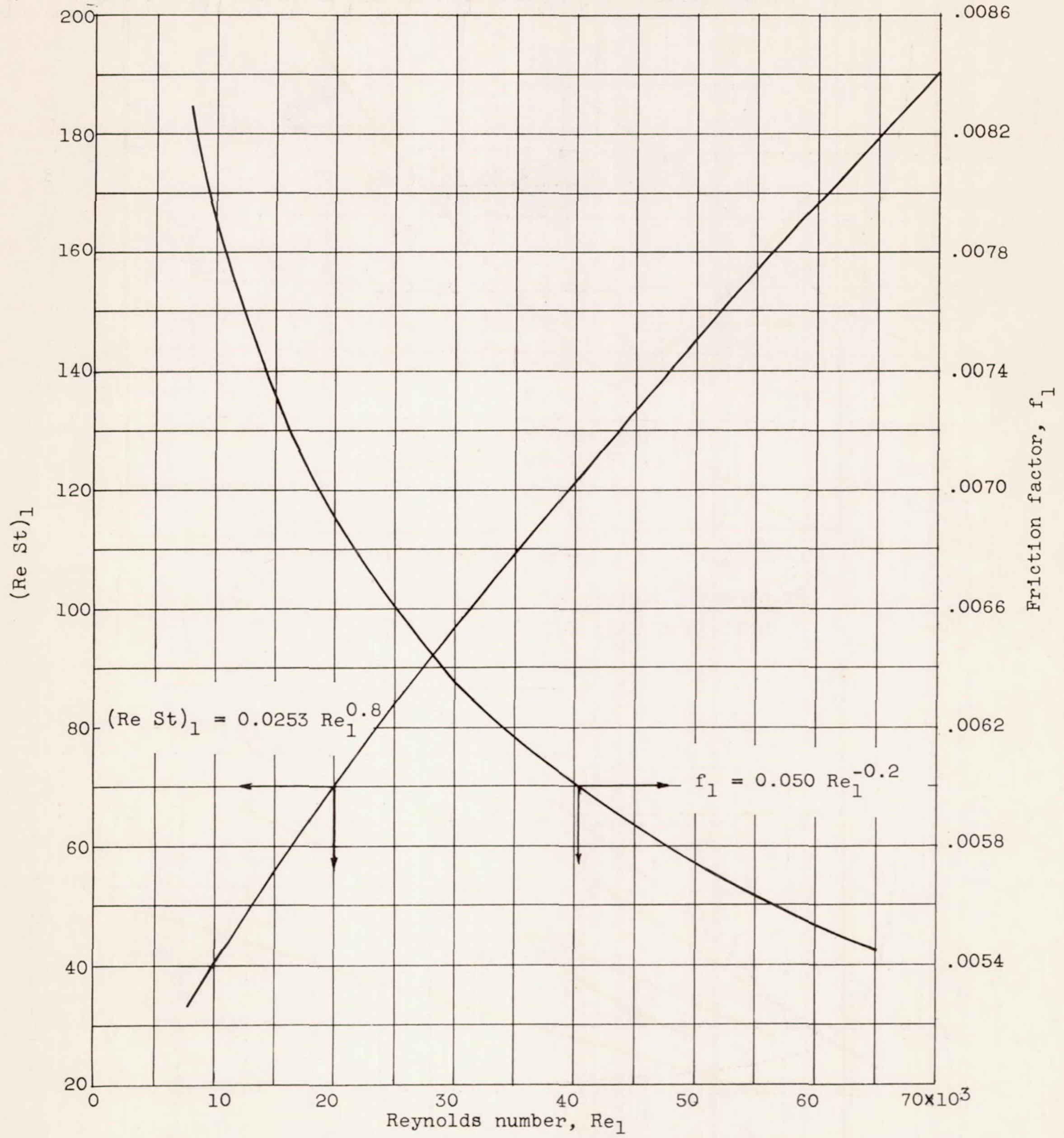
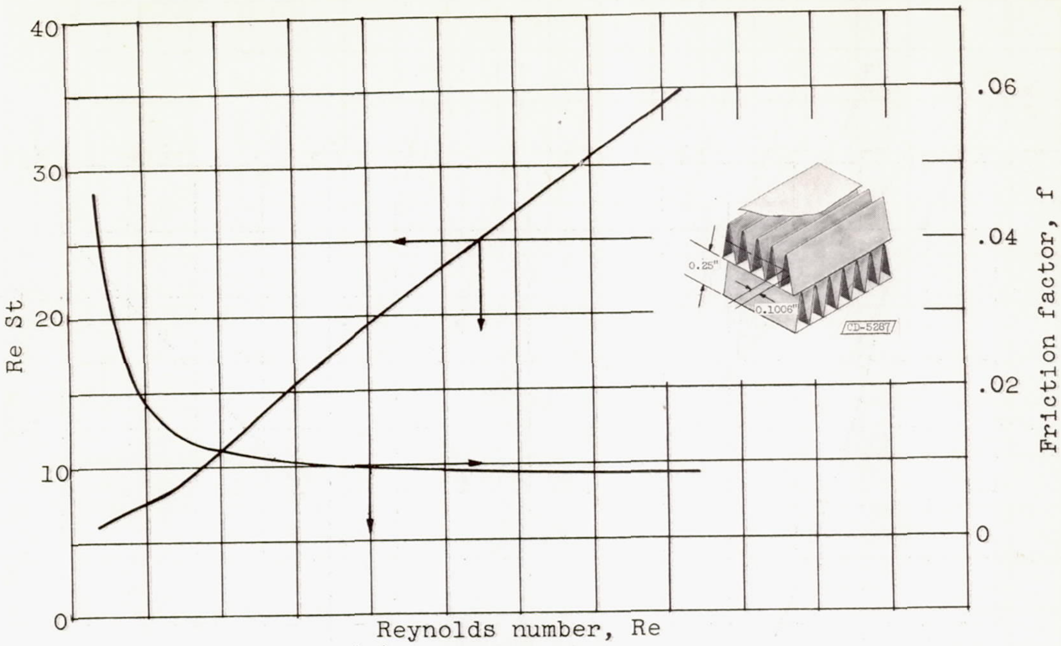
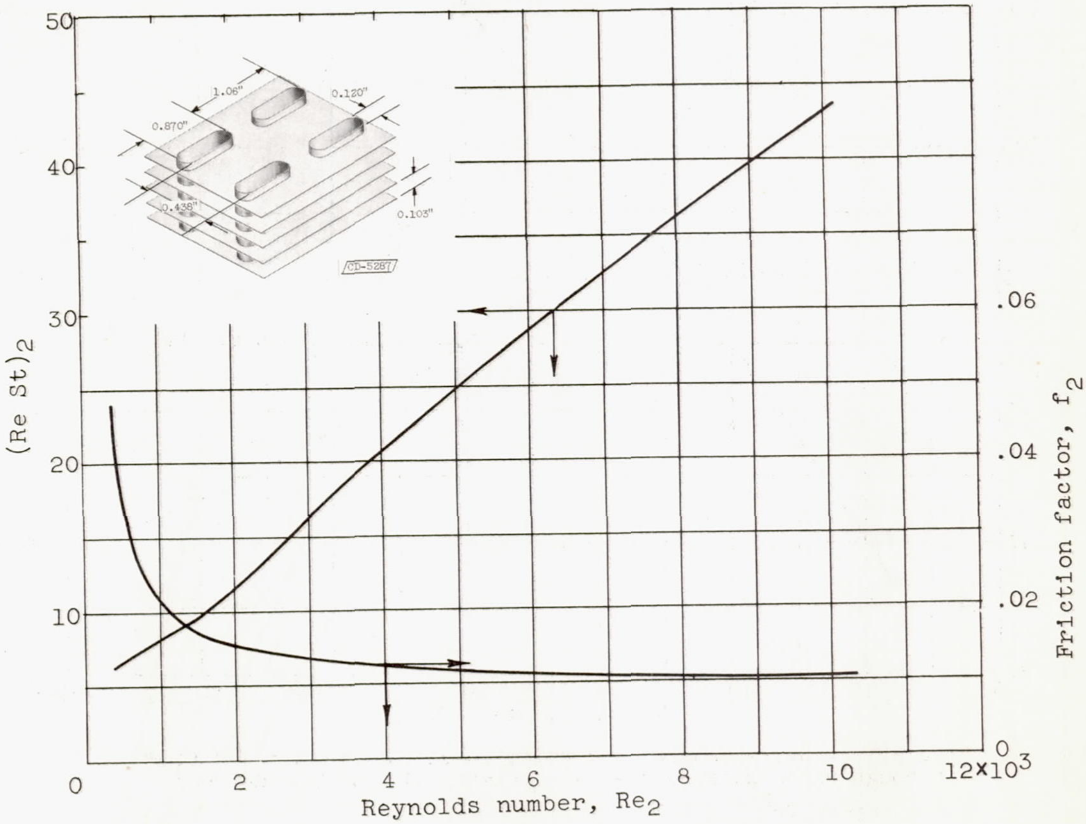


Figure 5. - Heat-transfer and friction characteristics of flow through rectangular passages. Primary fluid; finned-tube exchanger; $Pr^{2/3} = 0.75$.



(a) Plain-fin exchanger.



(b) Finned-tube exchanger; secondary fluid.

Figure 6. - Heat-transfer and friction characteristics of typical heat-exchanger core. $Pr^{2/3} = 0.75$.

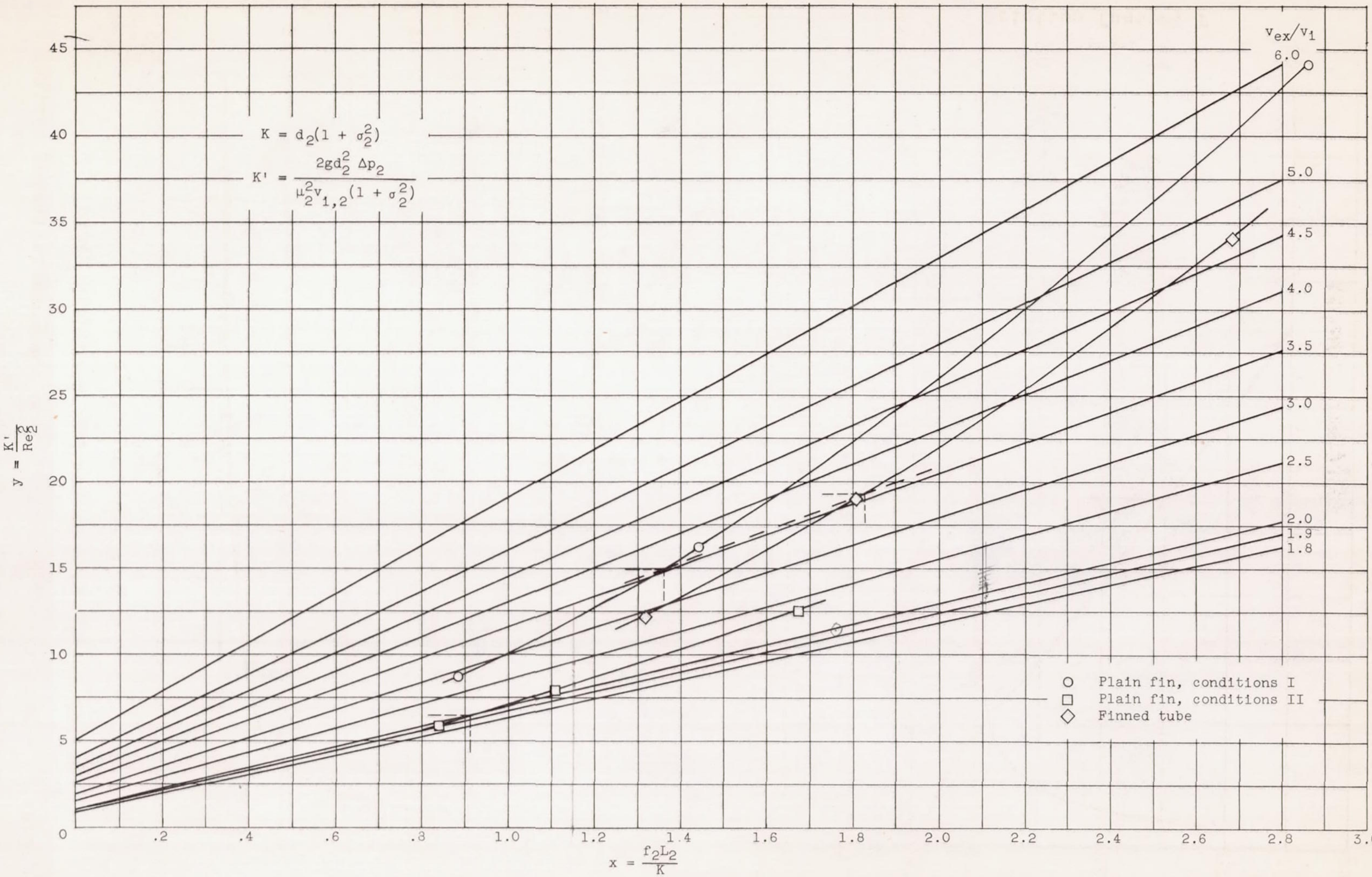


Figure 7. - Graphical representation of equation (2) and trial solutions.