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## METHOD OF CALCULATING CORE DIMENSIONS OF CROSSFLOW HEAT EXCHANGER WITH PRESCRIBED GAS FLOWS AND INLET AND EXIT STATES

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# METHOD OF CALCULATING CORE DIMENSIONS OF CROSSFLOW HEAT EXCHANGER WITH PRESCRIBED GAS FLOWS AND INLET AND EXIT STATES

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## SUMMARY

A calculation procedure is described by which the dimensions of the core of a gas-to-gas crossflow heat exchanger with prescribed heat-transfer surface can be determined rapidly. The procedure is based on a number of charts that may be prepared for each surface with prescribed entrance conditions, pressure drops, and temperature changes. The Reynolds numbers that determine the flow velocities on both sides of the heat exchanger as well as the three basic dimensions of the heat-exchanger core are found by reading values from the charts and performing a few simple calculations.

## INTRODUCTION

Gas-to-gas heat exchangers find many uses in modern aircraft and missiles. They are used, for instance, in air-conditioning units, which are necessary for cabin cooling, and for the cooling of electronic and other equipment. Such heat exchangers may also be used in turbojet or turboprop engines to reduce the temperature of the air used for cooling the turbine or other engine parts subjected to high gas temperatures. This air is usually bled from the compressor of the engine at various stages. The amount of air required may be substantially reduced if it is cooled in a heat exchanger to temperatures below the bleed-off temperature. Ram air can be used in this application as secondary air, which cools compressor bleed air before it is ducted to the turbine or to other engine parts.

Usually a large number of calculations have to be carried out for the design of such a heat exchanger in order to determine a size that will fulfill the specified requirements, especially if the heat-exchanger characteristics are studied over a range of conditions. It is therefore of advantage to have a simple method available by which the dimensions of the heat-exchanger core can be rapidly calculated. In applications of this nature, the heat-exchanger inlet-air conditions, the pressure drops available, and the air-flow quantities along both sides of the heat exchanger are usually prescribed as well as the required temperature change of the primary air. Small weight and volume are of prime importance. This latter requirement can be met by using compact heat-exchanger surfaces such as the ones investigated at Stanford University (ref. 1). Crossflow of the primary and secondary air in the heat exchanger is usually applied because it simplifies the ducting of both gas streams.

Many calculations of this nature were recently made at the NACA Lewis laboratory. From the experience collected in this way a procedure was developed that appears to give the desired results. It is the purpose of this report to describe this calculation procedure and to present an example of its use. Heat transfer from air to air is studied in the example; however, the method can be used generally for gas-to-gas heat exchangers. 3948

## SYMBOLS

The following symbols with consistent units are used in this report:

| | |
|---|---|
| $A$ | heat-transfer area |
| $A'$ | free-flow area |
| $C_c$ | constant (see eq. (17)) |
| $C_e$ | constant (see eq. (17)) |
| $c_p$ | specific heat at constant pressure |
| $d$ | hydraulic diameter |
| $f$ | friction factor |
| $G$ | mass velocity, $m/A'$ |
| $h$ | heat-transfer coefficient |
| $K_c$ | pressure-loss coefficient for abrupt contraction |
| $K_e$ | pressure-loss coefficient for abrupt expansion |
| $K_1$ | constant (see eq. (19)) |
| $k$ | thermal conductivity |

| | |
|---|---|
| L | heat-exchanger core dimension |
| $l$ | one-half fin length (see fig. 3) |
| m | mass-flow rate |
| Pr | Prandtl number, $c_p\mu/k$ |
| p | pressure |
| R | gas constant |
| Re | Reynolds number, $Gd/\mu$ |
| s | fin thickness |
| St | Stanton number, $h/Gc_p$ |
| T | temperature, $^{\circ}R$ |
| Tu | heat-transfer parameter (number of transfer units) |
| U | over-all heat-transfer coefficient |
| v | specific volume |
| $\alpha$ | heat-transfer surface area per unit volume |
| $\eta_f$ | fin effectiveness |
| $\eta_T$ | thermal effectiveness |
| $\eta_O$ | surface effectiveness |
| $\mu$ | viscosity |
| $\sigma$ | ratio of free-flow area to frontal area |

Subscripts:

| | |
|---|---|
| ex | exit |
| f | fin |
| i | inlet |

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m mean

max maximum

min minimum

n no-flow direction

1 fluid on one side of heat exchanger

2 fluid on other side of heat exchanger

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## ANALYSIS

The purpose of a heat exchanger is to transfer heat from one fluid to another. In this analysis one of the fluids will arbitrarily be denoted by the subscript 1, the other by 2, without specifying which one is cooled and which is heated in the exchanger.

### Prescribed Conditions and Geometry

The following values are assumed as prescribed in this analysis of a heat exchanger: the entrance conditions for both fluids $p_1$, $T_1$, $p_2$, and $T_2$; the mass-flow rates of the two fluids $m_1$ and $m_2$; the difference between the exit and entrance temperatures of one of the fluids in the heat exchanger $\Delta T_1$ or $\Delta T_2$; and the available pressure drops for both fluids $\Delta p_1$ and $\Delta p_2$.

The temperature change for the second medium flowing through the exchanger is immediately fixed by the heat-balance equation, which (neglecting heat losses from the heat exchanger to the surroundings) is

$$m_1 c_{p,1} \Delta T_1 + m_2 c_{p,2} \Delta T_2 = 0 \tag{1}$$

In this report a positive value of $\Delta T$ always denotes a temperature increase. Cooling of a fluid in the exchanger is indicated by a negative value of $\Delta T$.

It is also assumed that a specific geometry has been chosen for the heat exchanger (for instance a finned-tube arrangement as shown in fig. 1) and that, correspondingly, the parameters describing this geometry are known. These parameters are the hydraulic diameters for the passages of both fluids $d_1$ and $d_2$, the thickness of fins $s_1$ and $s_2$, which may

be used on both sides of the heat-exchanger surface, the ratio of free-flow to frontal area for both flow passages $\sigma_1$ and $\sigma_2$, and the heat-transfer area per unit volume of the heat-exchanger core $\alpha_1$ and $\alpha_2$. In addition, it is assumed that the friction factors $f_1$ and $f_2$ and the Stanton numbers $St_1$ and $St_2$ of the particular geometry are known as functions of the respective Reynolds numbers $Re_1$ and $Re_2$. Such information is included in reference 1 for a large number of geometries.
3948 In addition, the flow configuration for the heat exchanger is prescribed as crossflow.

## Required Effectiveness and Number of Transfer Units

The heat-transfer parameter Tu, which usually is referred to as "number of transfer units," can be determined as a function of the thermal effectiveness $\eta_T$ of the heat exchanger. Diagrams for this relation are presented in reference 1, and the example for a crossflow heat exchanger is reproduced in figure 2.

The thermal effectiveness is defined as

$$\eta_T = \left| \frac{\Delta T_{max}}{T_{i,1} - T_{i,2}} \right| \tag{2}$$

The temperature difference $\Delta T_{max}$ is the larger of the temperature differences $\Delta T_1$ and $\Delta T_2$, and the value of $\eta_T$ will be considered positive regardless of the sign of the right side of equation (2). The effectiveness $\eta_T$ can be calculated from the information prescribed for the heat exchanger. The heat-transfer parameter Tu can then be read from figure 2 for crossflow.

The aim of the calculation is to determine the dimensions of the heat-exchanger core ($L_1$, $L_2$, and $L_n$ in fig. 1). In addition, the velocities with which the two fluids enter the heat exchanger are not known. Instead of the velocity, the Reynolds number will be used, since the information on friction factors and Stanton numbers is usually given as a function of Reynolds number. This means that five unknowns ($L_1$, $L_2$, $L_n$, $Re_1$, and $Re_2$) are connected with the problem; and, therefore, five equations must be found to determine these values.

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## Flow-Continuity Equation

Two equations are available that describe the flow continuity for the two fluids. The mass flow of fluid 1 entering the heat exchanger is given by the equation

$$m_1 = A_1'G_1 \tag{3}$$

With the expressions for Reynolds number and free-flow area, $Re_1 = G_1 d_1/\mu_1$ and $A_1' = \sigma_1 L_2 L_n$, equation (3) can be transformed into

$$m_1 = \sigma_1 Re_1 \frac{\mu_1}{d_1} L_2 L_n \tag{4}$$

A corresponding equation can be obtained for fluid 2 by interchanging subscripts 1 and 2.

## Pressure-Drop Equation

Two equations are also available that describe the pressure drop of the two fluids in the heat exchanger. Generally, the pressure drop of a fluid flowing through a passage is described by the following equation (see ref. 1):

$$\frac{\Delta p}{p_i} = \frac{G^2 v_i}{2p_i} \left[ \left(K_c + 1 - \sigma^2\right) + 2\left(\frac{v_{ex}}{v_i} - 1\right) + f \frac{A}{A'} \frac{v_m}{v_i} - \left(1 - \sigma^2 - K_e\right) \frac{v_{ex}}{v_i} \right] \tag{5}$$

The four terms within the brackets of this equation describe the entrance loss, the pressure drop connected with the acceleration due to heating of the fluid, the pressure drop caused by friction, and the exit pressure loss, respectively. The factors $K_c$ and $K_e$ describing the entrance and exit losses may be found in reference 1 and are generally functions of Reynolds number $Re$ and the ratio of free-flow area to frontal area $\sigma$. If the two fluids are assumed to be ideal gases, the specific volumes can be expressed as

$$v_i = \frac{RT_i}{p_i} \quad \text{and} \quad v_{ex} = \frac{R(T_i + \Delta T)}{p_i - \Delta p} \tag{6}$$

The average specific volume appearing in the third term in the brackets of equation (5) is approximated by

$$v_m = \frac{v_i + v_{ex}}{2} \tag{7}$$

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By use of the parameters

$$Re = \frac{Gd}{\mu} \quad \text{and} \quad \frac{A}{A'} = \frac{4L}{d} \tag{8}$$

and equations (6) and (7), equation (5) is transformed into

$$\frac{\Delta p}{p_i} = \frac{(Re)^2 \mu^2 RT_i}{2d^2 p_i^2} \left[ (K_c - 1 - \sigma^2) + (1 + \sigma^2 + K_e)\left( \frac{1 + \frac{\Delta T}{T_i}}{1 - \frac{\Delta p}{p_i}} \right) + 2\frac{L}{d} f \left( \frac{2 - \frac{\Delta p}{p_i} + \frac{\Delta T}{T_i}}{1 - \frac{\Delta p}{p_i}} \right) \right] \tag{9}$$

Algebraic signs must accompany the value of $\Delta T$. ($\Delta T$ is positive for a temperature rise and negative for a temperature drop.) Two such equations may be written, one for each fluid flowing through the heat exchanger.

## Heat-Transfer and Surface-Effectiveness Equations

A fifth relationship may be obtained by consideration of the heat transfer in the exchanger. The equation defining the number of transfer units is

$$Tu_{max} = \frac{A_1 U_1}{(mc_p)_{min}} \tag{10}$$

The value $(mc_p)_{min}$ is the smaller of the two values $m_1 c_{p,1}$ and $m_2 c_{p,2}$. The over-all heat-transfer coefficient $U_1$ is given by

$$\frac{1}{U_1} = \frac{1}{\eta_{0,1} h_1} + \frac{1}{\eta_{0,2} \frac{A_2}{A_1} h_2} \tag{11}$$

when the thermal resistance in the primary surface is considered negligible, which usually is the case for gas-to-gas heat exchangers. The numerator on the right side of equation (10) may also be written as $A_2 U_2$ where the over-all heat-transfer coefficient $U_2$ is properly defined (interchanging indexes 1 and 2). It is assumed in equation (11) that the resistance to heat flow through the wall of the primary heat-exchanger surface can be neglected. The resistance of heat flow through extended heat-exchanger surface areas (fins that may exist on both heat-exchanger sides) are taken into account by the corresponding surface effectiveness parameters $\eta_{0,1}$ and $\eta_{0,2}$.

The surface effectiveness $\eta_0$ is connected with the effectiveness of the fins themselves $\eta_f$ by the equation

$$\eta_0 = \frac{A_f}{A} \eta_f + \frac{A - A_f}{A} = 1 - \frac{A_f}{A} (1 - \eta_f) \tag{12}$$

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Relations for the fin effectiveness are available only for comparatively simple geometries like fins with rectangular or triangular cross sections on plane surfaces or on circular tubes, or for cylindrical or conical fins. For actual heat-exchanger geometries the configurations are often considerably more complicated, and the fin effectiveness has in this case to be estimated from the simple geometry that comes closest to the actual one. In very many cases the relation for a rectangular fin on a plane wall is used for such an approximation. This fin effectiveness is described by the equation

$$\eta_f = \sqrt{\frac{ks}{2h}} \frac{1}{l} \tanh \sqrt{\frac{2h}{ks}} l \tag{13}$$

The heat-transfer coefficient in this equation is given by

$$h = \frac{\mu c_p}{d} \text{ReSt} \tag{14}$$

Figure 3 shows the values of the fin effectiveness of straight fins. Also included are curves for fins of rectangular cross section on cylindrical surfaces. According to the preceding relations, the fin effectiveness has to be regarded as a function of Reynolds number.

For the present calculation the fluid with the larger temperature change in the heat exchanger will be denoted as fluid 1 $(\Delta T_{max} = \Delta T_1)$. Then the fluid with the smaller value of $mc_p$ is fluid 1 $(mc_p)_{min} = m_1 c_{p,1}$.

Considering this, combining equations (10) and (11), and introducing

$$h = Gc_p \text{ St}, \quad \text{Re} = \frac{Gd}{\mu}, \quad \text{and} \quad A_1 = \alpha_1 \frac{A_1'}{\sigma_1} L_1 \tag{15}$$

give the equation

$$\eta_{O,2}\,(ReSt)_2 = \frac{\frac{\alpha_1}{\alpha_2}\left(\frac{c_p\mu}{d}\right)_1\left(\frac{d}{c_p\mu}\right)_2}{\left(\frac{\alpha}{\sigma Re}\right)_1\frac{L_1}{Tu} - \frac{1}{\eta_{O,1}\,(ReSt)_1}} \tag{16}$$

Equations (4) and (9) for fluids 1 and 2 and equation (16) are five relations that can be used to determine the five unknown parameters. The equations cannot be solved analytically, since the functions $f$, $\eta_O$, and $St$ are contained within the equations and are generally given as functions of Reynolds number in graphical form. The length parameters $L_1$, $L_2$, and $L_n$ are not connected with any empirical functions; therefore, they can be eliminated from the equations. The result of this procedure is a system of two equations for the two parameters $Re_1$ and $Re_2$, which have to be solved by some graphical or iteration procedure. However, these equations become quite unwieldy, and it is more advantageous to use the original system of five equations to obtain an immediate solution. The procedure described in the following section was evolved after a large number of trials as the one which most rapidly leads to a solution.

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## CALCULATION PROCEDURE

In the proposed procedure the aim is to determine the dimensions of a crossflow heat-exchanger core; therefore, equation (9) is solved for L:

$$L = \frac{d}{2f}\left(\frac{1 - \frac{\Delta p}{p_i}}{2 - \frac{\Delta p}{p_i} + \frac{\Delta T}{T_i}}\right)\left[\frac{p_i^2\Delta p 2d^2}{\mu^2 T_i p_i R(Re)^2} - C_e\frac{1 + \frac{\Delta T}{T_i}}{1 - \frac{\Delta p}{p_i}} - C_c\right] \tag{17}$$

where

$$C_c = (K_c - 1 - \sigma^2)$$

and

$$C_e = (1 + \sigma^2 + K_e)$$

Equation (17) is used to determine the lengths of the passages for both fluids 1 and 2. It has to be kept in mind that the initial state, the pressure drop of the fluid, and the heat-exchanger geometry will be different for each equation. Again, as noted in equation (9), the proper sign must accompany $\Delta T$. Equation (1) can be written in the form

$$\Delta T_2 = - \frac{m_1 c_{p,1}}{m_2 c_{p,2}} \Delta T_1 \tag{18}$$

By using equation (4) for fluids 1 and 2, equation (18) becomes

$$\Delta T_2 = - K_1 \frac{Re_1}{Re_2} \frac{L_2}{L_1} \Delta T_1 \tag{19}$$

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where the constant $K_1$ is

$$K_1 = \frac{c_{p,1}}{c_{p,2}} \frac{d_2}{d_1} \frac{\mu_1}{\mu_2} \frac{\sigma_1}{\sigma_2}$$

In equations (16), (17)(for both fluids), and (19) the length $L_n$ does not appear, so that the four unknowns remaining in the equation can be calculated. After these parameters have been obtained, the length $L_n$ can be calculated from equation (4). To obtain the four unknown parameters it is best to start by assuming a value for the Reynolds number $Re_1$. Then the length $L_1$ is calculated from equation (17) for fluid 1, $Re_2$ from equation (16), and $L_2$ from equation (17) for fluid 2. Equation (19) is now used to calculate the value $\Delta T_2$. A comparison of this value with the prescribed value indicates how good the original assumption of $Re_1$ was. By iteration, repeating the same calculation procedure, the correct values of $Re_2$ and all the remaining parameters are determined.

The described procedure is effective when only a few calculations on a specific heat-exchanger geometry have to be made. If, however, a large number of such calculations have to be done, the calculation can be considerably facilitated by the use of charts instead of the equations. Figure 4 indicates schematically the charts that have to be prepared. Chart I is obtained by a solution of equation (17) for fluid 1 and gives the connection between $Re_1$ and $L_1$ for a specific heat-exchanger configuration with prescribed entrance conditions, prescribed temperature drop $\Delta T_1$, and prescribed pressure drops $\Delta p$. Chart II is obtained from equation (16) and chart I, and presents the connection between the two Reynolds numbers $Re_1$ and $Re_2$. The chart is calculated by assuming a value of $Re_1$. From chart I, the corresponding value $L_1$ can be obtained, and inserting the parameters $Re_1$ and $L_1$ into equation (16) the parameter $Re_2$ is obtained. The chart is based on the same prescribed conditions as chart I. Chart III presents equation (17) for fluid 2 and describes the connection between $Re_2$ and $L_2$ for various values of $\Delta T_2$.

In using these charts it is actually advantageous to start with an assumed Reynolds number $Re_2$. Chart II gives the corresponding Reynolds number $Re_1$, chart I the corresponding length $L_1$, and chart III the corresponding length $L_2$. Equation (19) is now used to check the validity of the assumed Reynolds number $Re_2$. The temperature difference $\Delta T_2$ is calculated with this equation and compared with the prescribed temperature difference. The procedure has to be repeated with various assumed parameters $Re_2$ until the temperature difference obtained from equation (19) agrees with the prescribed temperature difference. This iteration procedure can be made in chart III in the following way: For each $L_2$ value the actual value of the temperature difference $\Delta T_2$ calculated with equation (19) is inserted. A curve is drawn through the $\Delta T_2$ values, which are obtained from equation (19); where this curve intersects the horizontal line through the prescribed $\Delta T_2$, the correct values of $L_2$ and Reynolds number $Re_2$ are found.

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Chart II can be prepared for the situation in which the surface effectiveness $\eta_0$ varies with Reynolds number as prescribed before. In actual calculations it was found that the variation of the surface effectiveness with Reynolds number over a considerable range is comparatively small; therefore, a simplification of the prescribed calculation procedure is possible by assuming an average value of the surface effectiveness.

The described calculation procedure will be demonstrated on a numerical example in the following section.

## NUMERICAL EXAMPLE

### Prescribed Condition and Geometry

The specific configuration to be used in this example is a finned-tube arrangement. The heat-transfer, friction, and geometry characteristics are presented in figure 5. This information was obtained from reference 1. The fluid conditions on each side of the heat exchanger are assumed as follows:

| Fluid 1 | Fluid 2 |
| --- | --- |
| $T_{i,1} = 1410^\circ$ R | $T_{i,2} = 880^\circ$ R |
| $p_{i,1} = 5300$ lb/sq ft | $p_{i,2} = 1080$ lb/sq ft |
| $\Delta p_1 = 1000$ lb/sq ft | $\Delta p_2 = 400$ lb/sq ft |
| $m_1 = 2.70$ lb/sec | $m_2 = 5.40$ lb/sec |

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A temperature drop $\Delta T_1$ of $-300^\circ$ is prescribed for fluid 1. Air is the fluid on both sides of the heat exchanger. Fluid 1 flows inside the tube, and fluid 2 flows along the fins. The specific heat of air $c_p$ is assumed constant in the temperature range considered; therefore, the ratio of $m_1/m_2$ is approximately equal to $\Delta T_2/\Delta T_1$ (eq. (18)). For this example, $m_1/m_2 = 1/2$, $\Delta T_1 = -300$; and therefore $\Delta T_2$ must equal $150^\circ$.

## Required Effectiveness and Number of Transfer Units

The effectiveness of the heat exchanger is given by equation (2):

$$\eta_T = \left| \frac{-300}{1410 - 880} \right| = 0.566$$

With the specific heats of the two fluids assumed constant, $mc_{p,1}/mc_{p,2} = 0.5$, and the required number of transfer units Tu obtained from figure 2 is 1.068.

## Method of Determining Lengths and Heat-Transfer Requirements

Chart for heat-exchanger length $L_1$. - The chart of equation (17) for fluid 1 is presented in figure 6 for the required temperature drop of $300^\circ$. For this example $K_c$ and $K_e$ for the terms $C_c$ and $C_e$, respectively, in equation (17) are set equal to zero. However, values of $K_c$ and $K_e$ are readily available in reference 1. The initial state of the air was given previously. The viscosity $\mu$ is based on the arithmetical mean of the inlet and outlet temperatures. When the dimensions have to be determined exactly, the calculation procedure should be repeated with viscosities based on film temperature. For preliminary calculations the introduction of viscosity at bulk temperature appears sufficient. The hydraulic diameter ($d_1 = 0.018$ ft) and the ratio of the free-flow area to the frontal area ($\sigma_1 = 0.219$) are determined from the geometry of the core configuration (see fig. 5). The friction factor $f$ is obtained from the equation presented in reference 2 for flow inside rectangular tubes ($f = 0.050\ Re^{-0.2}$). At this point the range of Re is arbitrarily chosen, since friction data for rectangular tubes are readily available.

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Chart for heat-transfer equation. - The chart of equation (16) is shown in figure 7. Because of the finned-tube arrangement, $\eta_{0,1} = 1.0$. Again the viscosity $\mu$ and specific heat $c_p$ are based on the arithmetical mean temperature of the inlet and outlet conditions. On fluid 2 side, $d_2 = 0.0118$ feet, and the heat-transfer surface area per unit volume $\alpha_2 = 229$ square feet per cubic foot (see fig. 5). The values for $d_1$ and $\sigma_1$ for fluid 1 side used in equation (17) are used here, and $\alpha_1 = 48.76$ square feet per cubic foot. The value of Tu (1.068) was determined previously, and values of $L_1$ for the corresponding assumed values of $Re_1$ can be determined from figure 6. The Stanton number St for fluid 1 side was determined for the assumed $Re_1$ by the equation

$$St = 0.019\ Re_1^{-0.2}/Pr^{2/3}$$

as given in reference 2 for flow in rectangular tubes. (The values of $Pr^{2/3}$ are assumed to be 0.75 for fluids 1 and 2. For the conditions imposed on this heat exchanger, an effectiveness $\eta_{0,2}$ of 0.80 is assumed. If the heat-transfer results of figure 5 are plotted as $(ReSt)_2$ against $Re_2$ (fig. 8), values of $Re_2$ can be readily determined for values of $(ReSt)_2$.

The limits on the range of Reynolds numbers $Re_2$ to be considered in this expression are primarily determined by the availability of the data on the fin side of the heat exchanger. For this example $Re_2$ ranges from about 300 to 10,000. This limitation fixes the range of $Re_1$, which in turn determines the $Re_1$ range in figure 6.

Chart for heat-exchanger length $L_2$. - The chart of equation (17) for fluid 2 is presented in figure 9. Again, $K_c$ and $K_e$ are set equal to zero. The geometry, state, and properties of the fluid are taken for that of fluid 2 side of the heat exchanger. The friction factor $f$ for this equation is obtained from the data presented in figure 5.

In the ANALYSIS section, $Re_1$ was assumed and $Re_2$ was determined from equation (16) for the assumed $Re_1$. This was necessary because $L_1$ appears in the right side of equation (16) and, according to equation (17), is also a function of $Re_1$. For the actual calculation of the heat-exchanger size by use of the charts presented, however, this procedure is altered slightly. The chart presented in figure 9 contains a series of curves for values of $Re_2$, which are spaced at intervals of either 250, 500, or 1000. To avoid unnecessary interpolation (assumed values of $Re_1$ would not normally yield values of $Re_2$ used as the parameter for the curves shown in fig. 9), it appears advisable to assume a value of $Re_2$ and determine the corresponding value of $Re_1$ from figure 7. A step-by-step outline of the calculation procedure which employs the use of the charts is given in the following section.

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Use of charts to determine $L_2$ and $L_1$. - The procedure to use in determining the heat-exchanger lengths on the sides is as follows:

(1) Assume an $Re_2$ (7000) and find the corresponding $Re_1$ (29,400) for the required Tu (1.068) from figure 7.

(2) For the assumed $Re_2$ (7000) and given $\Delta T_2$ (150), find $L_2$ (7.8 in.) from figure 9.

(3) Using figure 6, for $Re_1 = 29{,}400$ and $\Delta T_1 = 300$, $L_1 = 29.5$ inches.

(4) Make the necessary substitution in equation (19), solving for $\Delta T_2$:

$$\Delta T_2 = -\left[\left(\frac{0.0118}{0.0180}\right)\left(\frac{225}{187}\right)\left(\frac{10^{-7}}{10^{-7}}\right)\left(\frac{0.219}{0.697}\right)\right]\left(\frac{29{,}400}{7{,}000}\right)\left(\frac{7.8}{29.5}\right)(-300)$$

$$\Delta T_2 = 82.2^{\circ} \quad (c_p \text{ assumed constant})$$

This temperature drop is not in agreement with the required value of $\Delta T_2 = 150^{\circ}$, and the calculation procedure has to be repeated with a new assumption for $Re_2$.

(5) If a lower value of $Re_2$ ($Re_2 = 6000$) is assumed, $Re_1 = 28{,}800$ (fig. 7). Repeating steps (2), (3), and (4) gives $L_2 = 11.2$ inches, $L_1 = 30.8$ inches, and $\Delta T_2 = 129.3^{\circ}$. The value of $Re_2 = 6000$ is still too high.

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(6) An assumed value of $Re_2 = 5000$ gives an $Re_1 = 28,000$, $L_2 = 16.9$ inches, $L_1 = 31.9$ inches, and $\Delta T_2 = 219.8^\circ$. This value of $Re_2$ is slightly low; however, for estimating purposes, this value may satisfy the calculations.

(7) If, however, a more exact value is required, the results of equation (19), for the three assumed values of $Re_2$, can be plotted on figure 9. These points are $\Delta T_2 = 82.2^\circ$ and $L_2 = 7.8$ inches, $\Delta T_2 = 129.3^\circ$ and $L_2 = 11.2$ inches, and $\Delta T_2 = 219.8^\circ$ and $L_2 = 16.9$ inches. A curve can be drawn through the three points. The exact value of $L_2$ is read from the intersection of this curve with a horizontal line at the given prescribed value of $\Delta T_2$ (at $\Delta T_2 = 150^\circ$, $L_2 = 12.45$ in. and $Re_2 = 5750$).

(8) Returning to figure 7, when $Re_2 = 5750$, $Re_1 = 28,600$ and $L_1 = 31.00$ inches (fig. 6).

## No-Flow-Length Calculation

By use of equation (4), the no-flow length $L_n$ can be readily determined. In this example,

$$L_n = \frac{2.70 \times 0.018 \times 12}{28,600 \times 225 \times 10^{-7} \times 0.219 \times 12.45} = 0.334 \text{ ft} = 4.00 \text{ in.}$$

## Surface-Effectiveness Check

For this example the heat-transfer coefficient was $11.24 \times 10^{-3}$ Btu/(sec)(sq ft)($^\circ$R), the conductivity of the metal was assumed to be $8.89 \times 10^{-3}$ Btu/(sec)(ft)($^\circ$R), the fin thickness was $s = 3.33 \times 10^{-4}$ feet, and the fin length $l = 0.316/24$ feet. From figure 3, for $l\sqrt{2h/ks} = 1.147$, $\eta_f = 0.72$. Substituting $\eta_f$ values in equation (12) gives

$$\eta_0 = 1 - \frac{A_f}{A}(1 - \eta_f)$$

$$= 1 - 0.795\ (1 - 0.72)$$

$$= 0.78$$

a value within reasonable agreement with the assumed 0.80.

## CHECK OF PROCEDURE

The dimensions of the heat-exchanger core evolved from this procedure were used in a conventional calculation procedure presented in reference 1 for determining the pressure drop and temperature changes when the core dimensions are known. For this example the pressure drop agreed within 2 percent and the temperature change within 4 percent with the initially assumed value. The latter deviation is mainly caused by the limited accuracy with which values can be read in the left part of figure 2 and can easily be improved by referring to tabulated values in reference 1.

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Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, December 8, 1955

## REFERENCES

1. Kays, W. M., and London, A. L.: Compact Heat Exchangers - A Summary of Basic Heat Transfer and Flow Friction Design Data. Tech. Rep. 23, Dept. Mech. Eng., Stanford Univ., Nov. 15, 1954. (Contract N6-ONR-251, Task Order 6 (NR-065-104) for Office Naval Res.)

2. Kays, W. M.: Basic Heat Transfer and Flow Friction Design Data for Gas Flow in Circular and Rectangular Cylindrical Tube Heat Exchangers. Tech. Rep. 14, Dept. Mech. Eng., Stanford Univ., June 15, 1951. (Contract N6-ONR-251, Task Order 6 (NR-065-104) for Office Naval Res.)

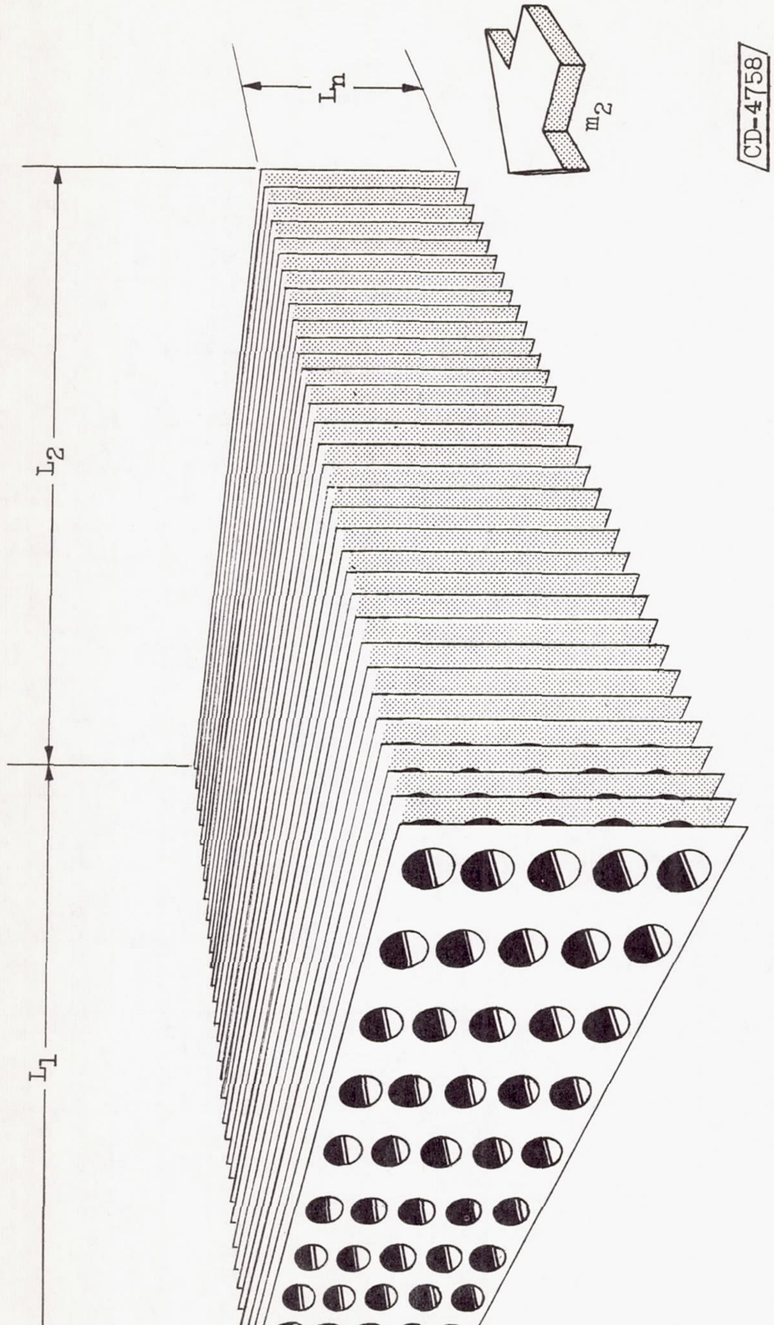


Figure 1. - Crossflow finned-tube heat-exchanger core.

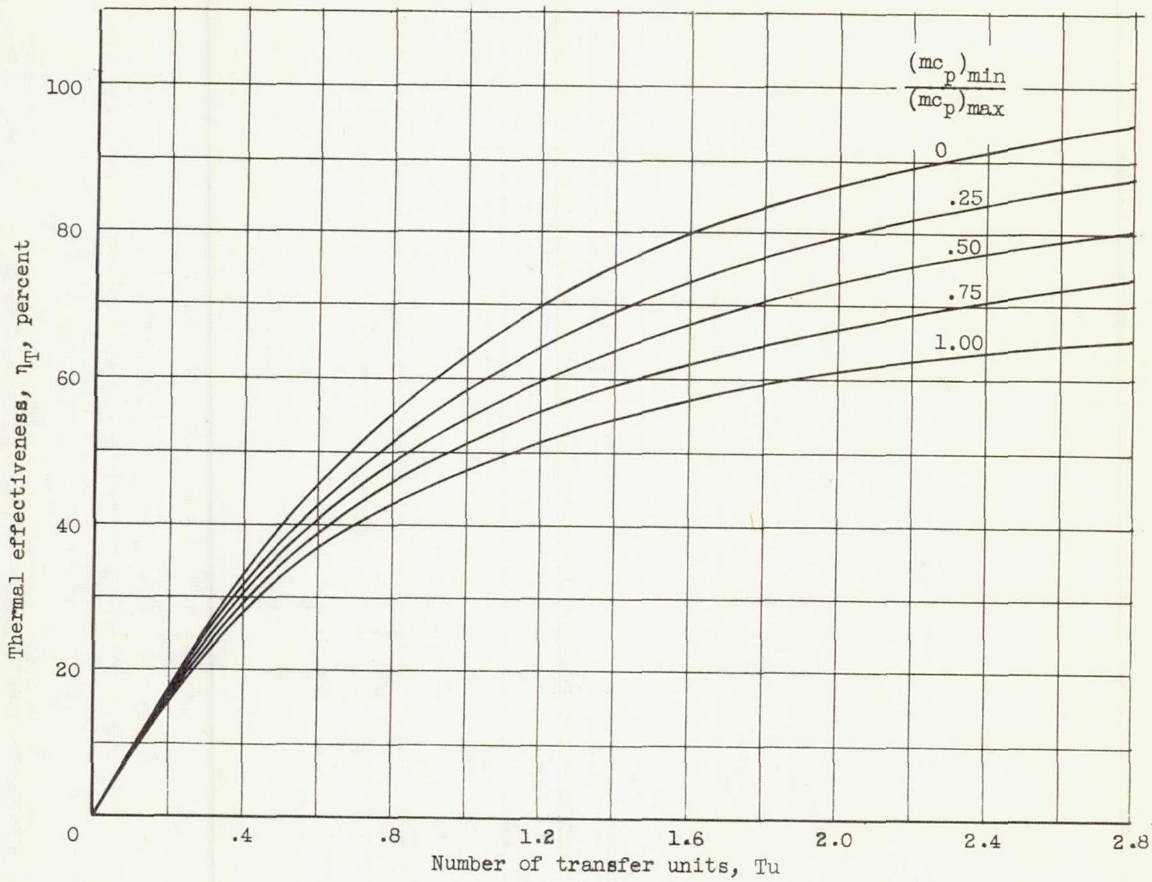


Figure 2. - Performance of crossflow heat exchangers with fluids unmixed.

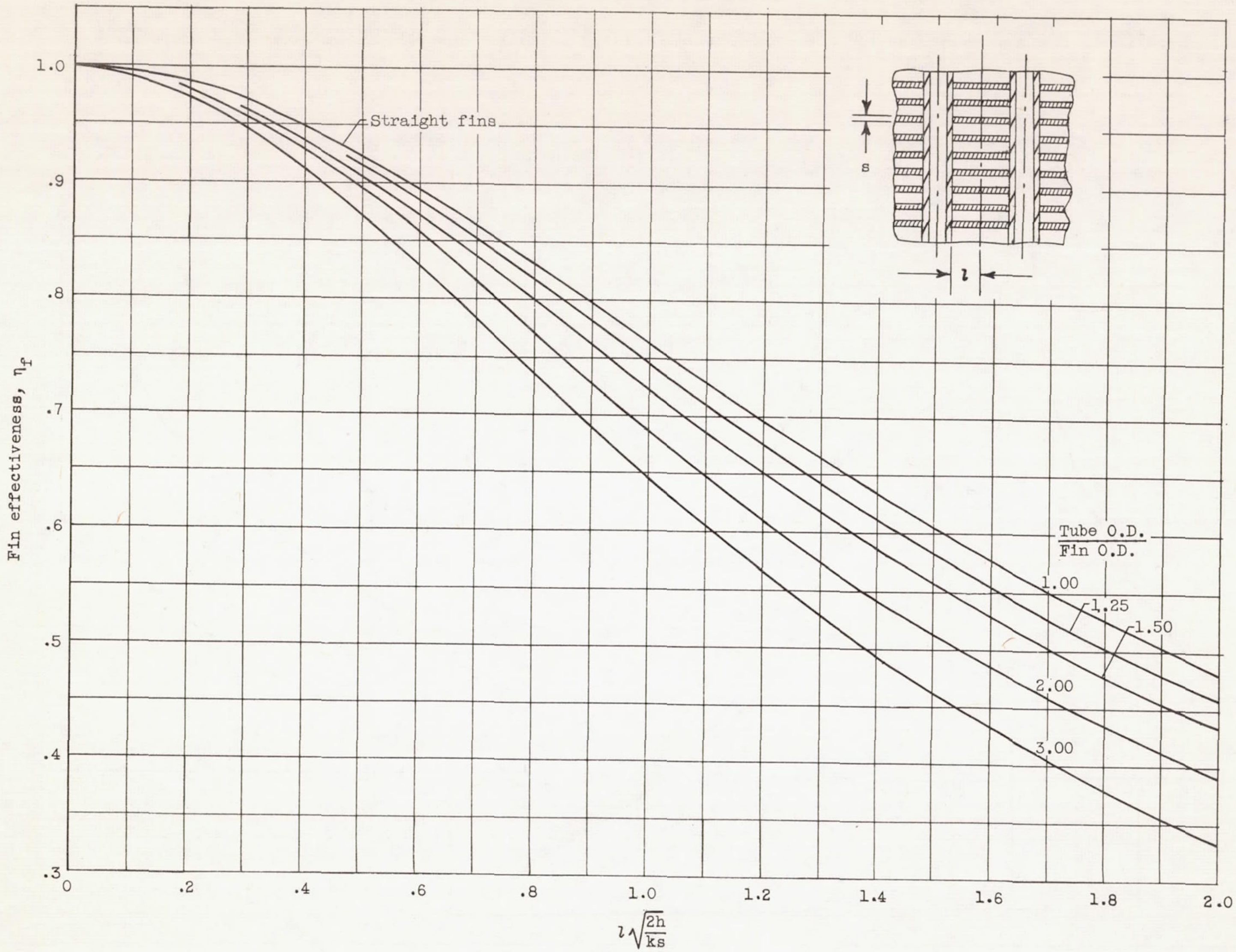


Figure 3. - Effectiveness of circular and straight fins.

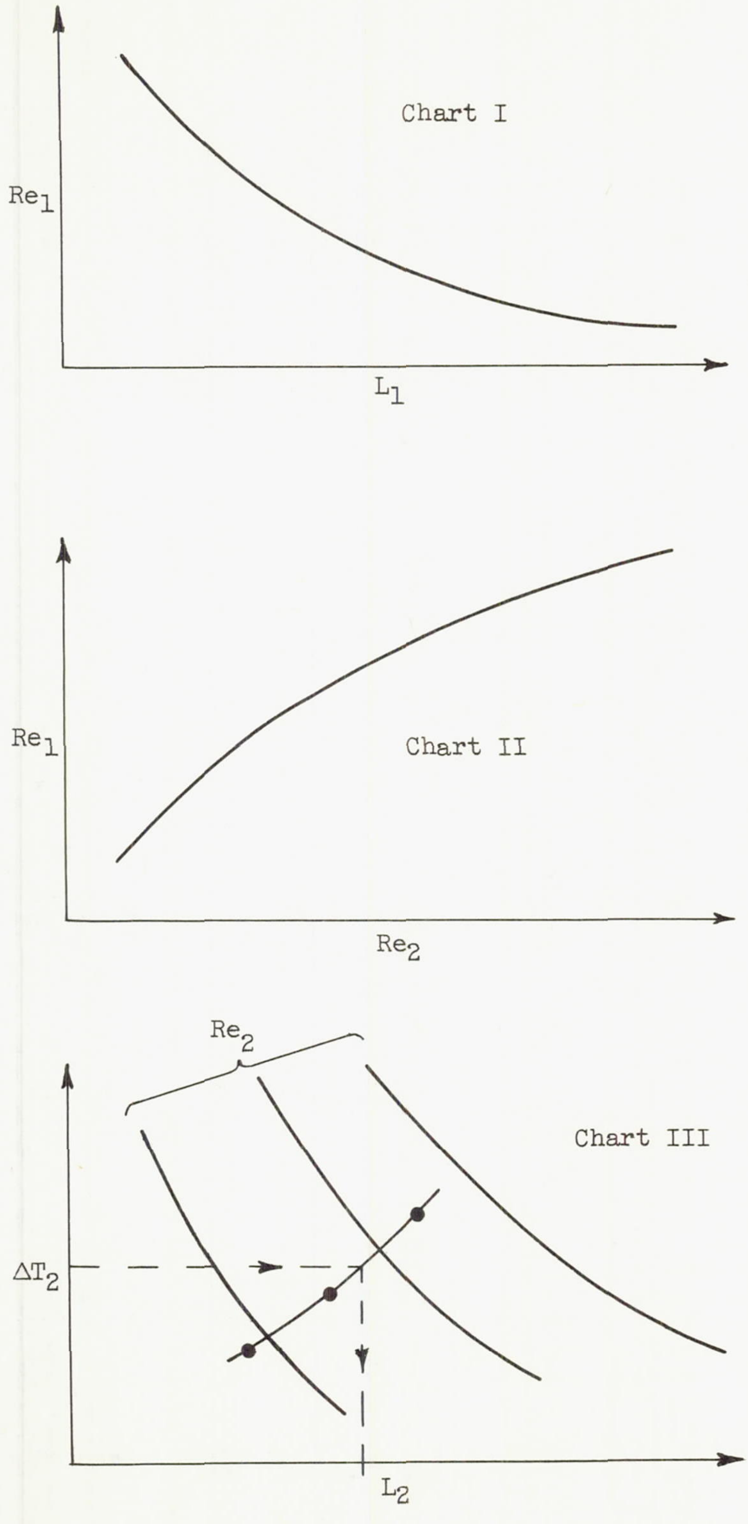


Figure 4. - Charts required for calculation procedure.

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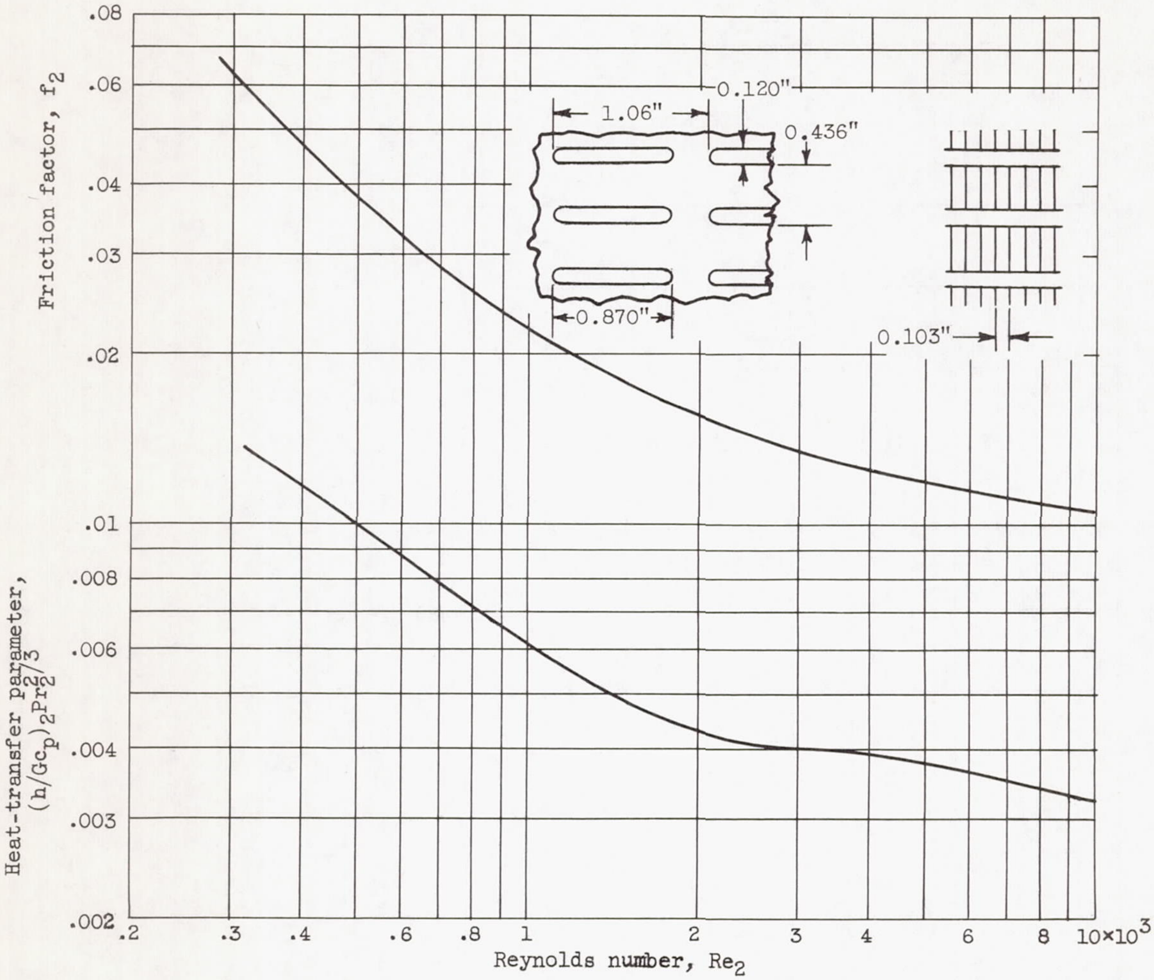


Figure 5. - Heat-transfer and friction data for finned-tube heat-exchanger core (ref. 1, fig. 102).

| | |
|---|---|
| Fin area/total area, $A_f/A$ . . . . . . . . . . . . . . . . . . . . . | 0.795 |
| Fin pitch, 1/in. . . . . . . . . . . . . . . . . . . . . . . . . . . | 9.681 |
| Fin thickness, s, in. . . . . . . . . . . . . . . . . . . . . . . . | 0.004 |
| Hydraulic diameter, fin side, $d_2$, ft . . . . . . . . . . . . . . . | 0.0118 |
| Hydraulic diameter, tube side, $d_1$, ft . . . . . . . . . . . . . . . | 0.018 |
| Free-flow area/frontal area, fin side, $\sigma_2$ . . . . . . . . . . . | 0.697 |
| Free-flow area/frontal area, tube side, $\sigma_1$ . . . . . . . . . . . | 0.219 |
| Heat-transfer area/volume, fin side, $\alpha_2$, sq ft/cu ft . . . . . | 229 |
| Heat-transfer area/volume, tube side, $\alpha_1$, sq ft/cu ft . . . . . | 48.76 |

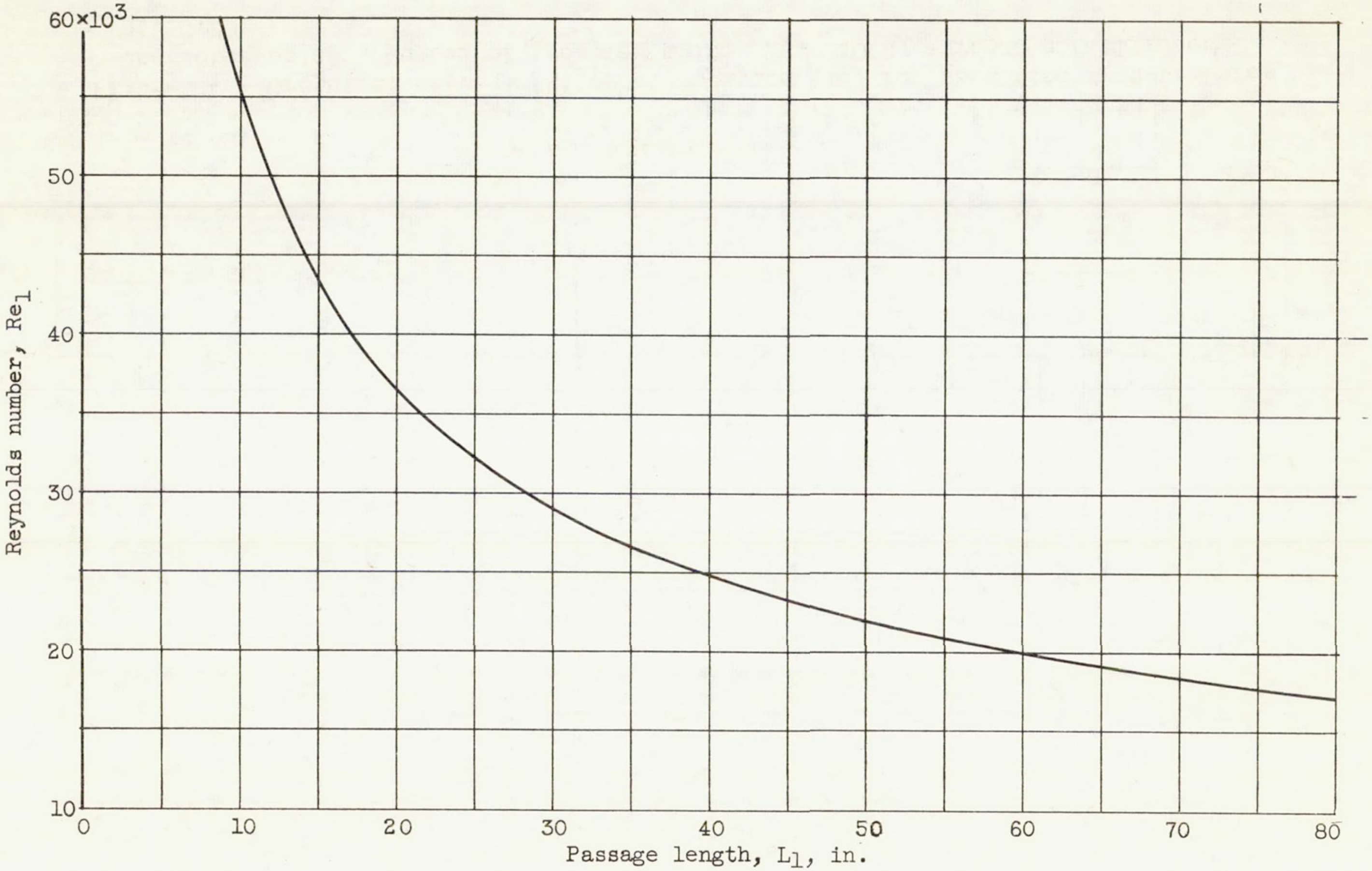


Figure 6. - Graphical representation of equation (17) for specified inlet condition, temperature and pressure drops, and flow geometry for fluid 1.

| | | | |
|---|---|---|---|
| $\Delta p_1$, lb/sq ft | 1000 | $d_1$ | 0.018 |
| $p_{i,1}$, lb/sq ft | 5300 | $\Delta T_1$ | -300 |
| $T_{i,1}$, $^{\circ}R$ | 1410 | $m_1$, lb/sec | 2.70 |
| $\sigma_1$ | 0.219 | | |

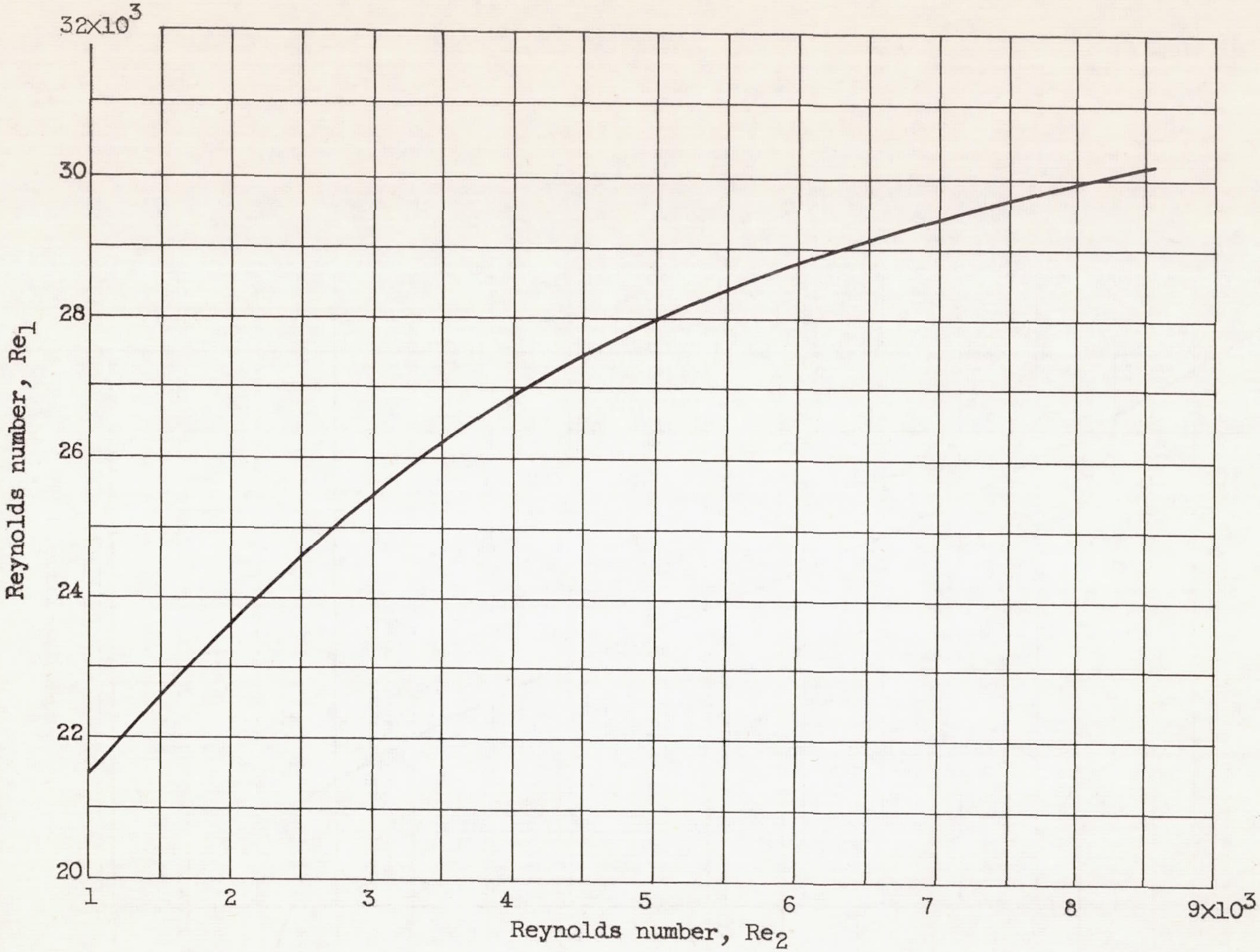


Figure 7. - Graphical representation of equation (16) for specified heat-transfer characteristics. Number of transfer units Tu, 1.068; surface effectiveness $\eta_0$, 0.80.


Figure 8. - Heat-transfer characteristics of finned-tube heat-exchanger core ($Pr^{2/3}$, 0.750).

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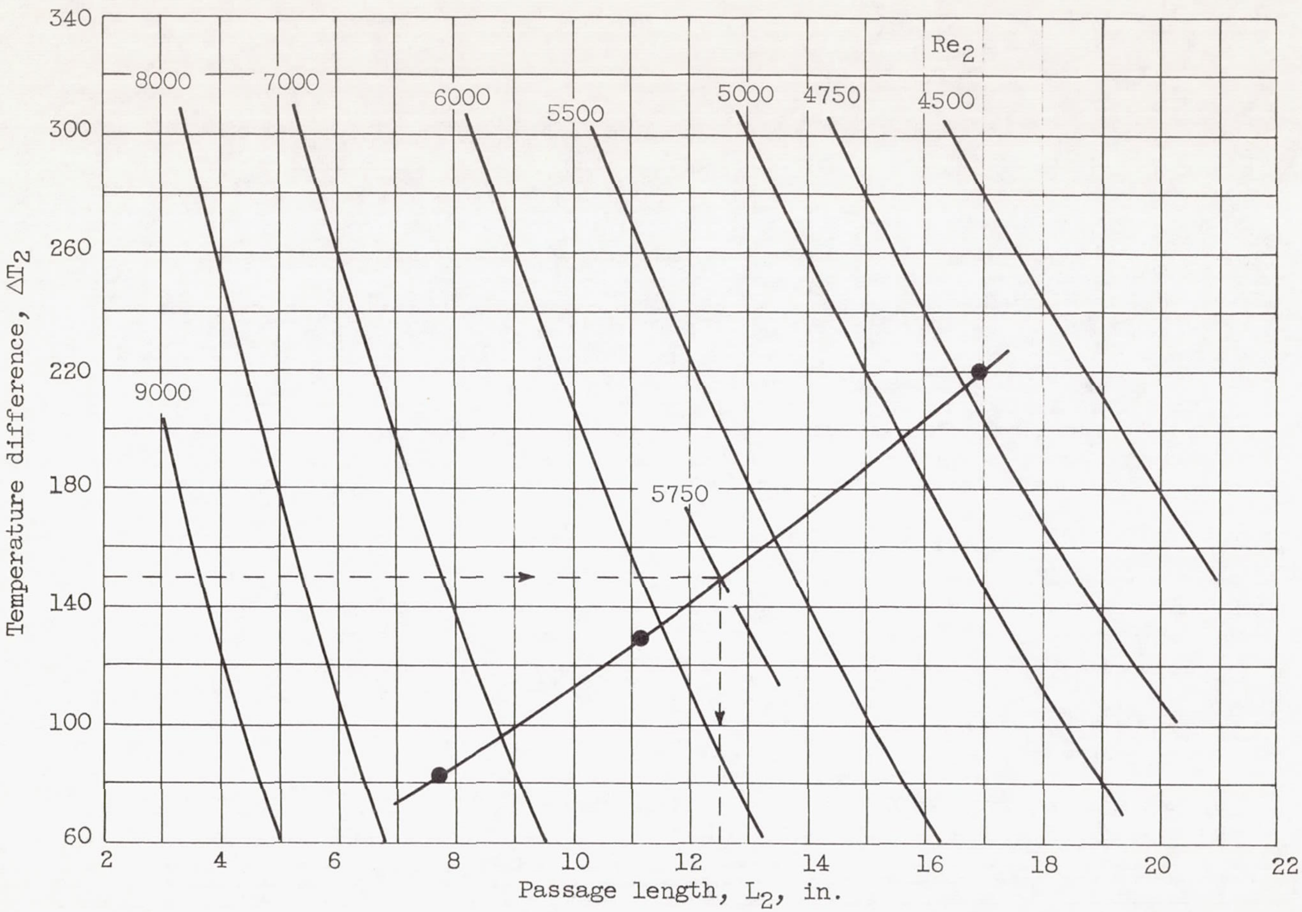


Figure 9. - Graphical representation of equation (17) for specified inlet conditions, pressure drop, and flow geometry for fluid 2 for variations in temperature rise.

| | | | |
|---|---|---|---|
| $\Delta p_2$, lb/sq ft . . . . . | 400 | $\sigma_2$ . . . . . . . . | 0.697 |
| $\Delta p_{i,2}$, lb/sq ft . . . | 1080 | $d_2$, ft . . . . | 0.0118 |
| $T_{i,2}$, $^{\circ}R$ . . . . . . . . | 880 | $m_2$, lb/sec . . . . | 5.4 |

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