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A METHOD UTILIZING DATA ON THE SPIRAL, ROLL-SUBSIDENCE, AND DUTCH ROLL MODES FOR DETERMINING LATERAL STABILITY DERIVATIVES FROM FLIGHT MEASUREMENTS

By Bernard B. Klawans and Jack A. White

Langley Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

A method for determining lateral stability derivatives from flight measurements is obtained by arranging the lateral equations of motion in such form that information from each of the three modes of lateral motion may be utilized. This method permits determination of all the important derivatives without requiring an estimation of any of the derivatives. The results of an error analysis are given to show the effects of errors in the measured quantities on the accuracy of each stability derivative for three representative airplanes.

INTRODUCTION

Calculated and measured wind-tunnel values of the aerodynamic lateral stability derivatives are necessary in the design of any modern airplane. Because of the uncertainty of determination of many of the derivatives from theory or from wind-tunnel tests, however, flight measurements of the derivatives are desirable to check the values assumed in the design and to provide a basis for further improvement of the aerodynamic characteristics.

Several methods have been proposed for determining lateral stability derivatives from flight tests by analyzing transient or frequency-response data. (For example, see refs. 1 to 3.) These methods allow determination of all the important derivatives. One method, known as the vector method (ref. 3), allows some insight into the effect of errors in the flight measurements on the accuracy of the derivatives. In this method, which utilizes data from the Dutch roll mode alone, two of the derivatives must be estimated or assumed in order to evaluate the others.

The purpose of this paper is to present a method in which the characteristics of the spiral and roll-subsidence modes as well as those of the Dutch roll mode are utilized in an effort to obtain all the important derivatives. In general, separate flight tests are needed to measure as accurately as possible the characteristics of the three modes. An error analysis is made to show the accuracy of flight measurements required to produce a desired accuracy of each stability derivative for three representative airplanes.

SYMBOLS

a_y lateral acceleration, ft/sec^2

b wing span, ft

C_L trim lift coefficient, $\dfrac{W}{\frac{1}{2}\rho V^2 S}$

C_l rolling-moment coefficient, $\dfrac{\text{Rolling moment}}{\frac{1}{2}\rho V^2 Sb}$

$C_{l_p} = \dfrac{\partial C_l}{\partial \frac{pb}{2V}}$

$C_{l_r} = \dfrac{\partial C_l}{\partial \frac{rb}{2V}}$

$C_{l_\beta} = \dfrac{\partial C_l}{\partial \beta}$

C_n yawing-moment coefficient, $\dfrac{\text{Yawing moment}}{\frac{1}{2}\rho V^2 Sb}$

$C_{n_p} = \dfrac{\partial C_n}{\partial \frac{pb}{2V}}$

$C_{n_r} = \dfrac{\partial C_n}{\partial \frac{rb}{2V}}$

$C_{n_\beta} = \frac{\partial C_n}{\partial \beta}$

C_Y lateral-force coefficient, $\frac{\text{Lateral force}}{\frac{1}{2}\rho V^2 S}$

$C_{Y_p} = \frac{\partial C_Y}{\partial \frac{pb}{2V}}$

$C_{Y_r} = \frac{\partial C_Y}{\partial \frac{rb}{2V}}$

$C_{Y_\beta} = \frac{\partial C_Y}{\partial \beta}$

D differential operator, d/ds

D_1 Dutch roll root

$D_{1,r}$ real portion of Dutch roll root

$D_{1,i}$ imaginary portion of Dutch roll root

D_2 root of roll-subsidence motion

D_3 root of spiral motion

$\left(\frac{D\phi}{\beta}\right)_1, \left(\frac{D\phi}{\beta}\right)_2, \left(\frac{D\phi}{\beta}\right)_3$ ratio of nondimensional rolling velocity to angle of sideslip in Dutch roll, roll-subsidence, and spiral modes, respectively

$\left(\frac{D\psi}{\beta}\right)_1, \left(\frac{D\psi}{\beta}\right)_2, \left(\frac{D\psi}{\beta}\right)_3$ ratio of nondimentional yawing velocity to angle of sideslip in Dutch roll, roll-subsidence, and spiral modes, respectively

h altitude, ft

K_n constants, functions of measurable flight quantities (n varies from 1 to 39)

K_X	nondimensional radius of gyration in roll about longitudinal stability axis, $\sqrt{\left(\frac{k_{X,O}}{b}\right)^2 \cos^2\eta + \left(\frac{k_{Z,O}}{b}\right)^2 \sin^2\eta}$
K_Z	nondimensional radius of gyration in yaw about vertical stability axis, $\sqrt{\left(\frac{k_{Z,O}}{b}\right)^2 \cos^2\eta + \left(\frac{k_{X,O}}{b}\right)^2 \sin^2\eta}$
K_{XZ}	nondimensional product-of-inertia parameter, $\left[-\left(\frac{k_{Z,O}}{b}\right)^2 + \left(\frac{k_{X,O}}{b}\right)^2\right] \sin\eta \cos\eta$
$k_{X,O}$	radius of gyration in roll about principal longitudinal axis, ft
$k_{Z,O}$	radius of gyration in yaw about principal vertical axis, ft
m	mass of airplane, slugs
p	rolling angular velocity, $d\phi/dt$, radians/sec
r	yawing angular velocity, $d\psi/dt$, radians/sec
S	wing area, sq ft
s	nondimensional time parameter based on span, Vt/b
t	time, sec
V	airspeed, ft/sec
v	lateral velocity, ft/sec
W	weight of airplane, lb
X,Y,Z	stability coordinate axes (defined in fig. 1)
β	angle of sideslip, $\frac{v}{V}$, radians
η	inclination of principal longitudinal axis of airplane with respect to flight path, positive when principal axis is above flight path at nose, deg
μ	relative density factor, $m/\rho Sb$

ρ mass density of air, slugs/cu ft

ϕ angle of roll, radians

ψ angle of yaw, radians

DEVELOPMENT OF METHOD

Equations

The present method for determining lateral stability derivatives from flight measurements depends on arranging the lateral equations of motion in such a form that information from each of the three lateral modes of motion may be utilized. In this section the original equations of motion are given in standard form and then in modified form.

Assumptions.- The usual assumptions of lateral stability theory are made:

(1) There are only three degrees of freedom: sideslip β, roll ϕ, and yaw ψ

(2) After the airplane is disturbed, all aerodynamic controls are fixed in their trim position

(3) The disturbed motions are assumed to be small.

Modifications of standard equations of lateral motion.- The linear, second-order, simultaneous differential equations of lateral motion referred to stability axes (see fig. 1) for the condition of controls fixed in level flight are as follows:

Side force:

$$\beta\left(2\mu D - C_{Y_\beta}\right) + \phi\left(-C_L - \frac{1}{2}C_{Y_p}D\right) + \psi\left(2\mu D - \frac{1}{2}C_{Y_r}D\right) = 0 \tag{1}$$

Rolling moment:

$$\beta\left(-C_{l_\beta}\right) + \phi\left(2\mu K_X^2 D^2 - \frac{1}{2}C_{l_p}D\right) + \psi\left(-2\mu K_{XZ}D^2 - \frac{1}{2}C_{l_r}D\right) = 0 \tag{2}$$

Yawing moment:

$$\beta\left(-C_{n_\beta}\right) + \phi\left(-2\mu K_{XZ}D^2 - \frac{1}{2}C_{n_p}D\right) + \psi\left(2\mu K_Z{}^2D^2 - \frac{1}{2}C_{n_r}D\right) = 0 \qquad (3)$$

Equations (1) to (3) are expressed in a form best suited for determination of the resultant motions (β, ϕ, and ψ) from known system constants. The present paper is concerned with the converse problem, that of determination of the system constants (stability derivatives) from measured resultant motions. In this procedure, the known quantities are the values of period and damping, or time constants, of the various modes of motion expressed mathematically as certain values of the operator D and ratios of the measured quantities ϕ, ψ, and β in each mode of motion. If equations (1) to (3) are each divided by β and the terms regrouped, the equations are obtained in a form suitable for the present analysis.

Side force:

$$-C_{Y_\beta} - \frac{1}{2}C_{Y_p}\frac{D\phi}{\beta} - \frac{1}{2}C_{Y_r}\frac{D\psi}{\beta} + 2\mu D\left(1 + \frac{\psi}{\beta}\right) - C_L\frac{\phi}{\beta} = 0 \qquad (4)$$

Rolling moment:

$$-C_{l_\beta} - \frac{1}{2}C_{l_p}\frac{D\phi}{\beta} - \frac{1}{2}C_{l_r}\frac{D\psi}{\beta} + 2\mu D\left(\frac{D\phi}{\beta}K_X{}^2 - \frac{D\psi}{\beta}K_{XZ}\right) = 0 \qquad (5)$$

Yawing moment:

$$-C_{n_\beta} - \frac{1}{2}C_{n_p}\frac{D\phi}{\beta} - \frac{1}{2}C_{n_r}\frac{D\psi}{\beta} + 2\mu D\left(\frac{D\psi}{\beta}K_Z{}^2 - \frac{D\phi}{\beta}K_{XZ}\right) = 0 \qquad (6)$$

Basis of the Method

The present method is based on certain relations developed in the theory of linear differential equations with constant coefficients. These relations are proved in most textbooks on differential equations.

The operator D may be handled as an algebraic quantity in the equations of motion. In the usual process of solving these equations, the roots of the characteristic equation are obtained (ref. 4). A complex value of D corresponding to one of these roots represents an oscillatory mode, the frequency of which is related to the imaginary part of D and the damping to the real part. A real value of D represents an exponential convergence or divergence. In the case of the lateral equations of motion, a small real root represents the spiral mode, a large negative real root represents the roll-subsidence mode, and a complex root represents the Dutch roll mode.

In each mode of motion, the variables ϕ, ψ, and β maintain given ratios. In the case of nonoscillatory mode, these ratios are real quantities. In the case of oscillatory modes these ratios are complex quantities, the moduli of which give the amplitude ratios between the variables, and the angles of which give the phase angles. These ratios may also be treated as algebraic quantities in equations (4), (5), and (6). This procedure was pointed out in reference 5.

If the airplane is disturbed in any manner and the controls returned to the trim position, the transient motion may be expressed as the sum of contributions of the three modes of motion. Because the equations of motion are linear, the principle of superposition applies. This principle states that, if any transient responses which satisfy the equations of motion are added, the resulting response also satisfies the equations of motion. Since any response which satisfies the equations of motion consists of contributions of the three modes, each of these modes must separately satisfy the equations of motion. This result is used in the present analysis by substituting into equations (4), (5), and (6) the measured values of certain characteristics of each mode. In this way, a number of simultaneous equations are obtained from which the stability derivatives may be obtained. The details of the procedure are now described.

Method of Solution

Tabulation of known and unknown quantities in the equations.- It is possible to make ground measurements of all pertinent mass characteristics of an airplane. (See ref. 6.) Furthermore, the air density and the trim lift coefficient are easily obtained. Theoretically, all the characteristics of each mode of motion could be measured in flight. This information includes the value of the root D and the ratios ϕ/β and ψ/β or some derivative of these quantities. These values may be substituted into equations (4), (5), and (6) to yield three equations for each nonoscillatory mode. The Dutch roll mode yields six equations inasmuch as the real and imaginary parts of the equations must equal zero separately. Under these conditions, 12 equations could be written to determine the nine unknown stability derivatives, namely, C_{Y_β}, C_{n_β}, C_{l_β}, C_{Y_p}, C_{n_p}, C_{l_p}, C_{Y_r}, C_{n_r}, and C_{l_r}. Because the number of equations available is greater than the number of unknowns, several alternative procedures may be used for determining the unknowns. Some of the equations may be omitted from the analysis or some of the quantities measurable in flight describing the modes of motion may be eliminated. In practice, not all the flight measurements can be made with equal ease or accuracy. Elimination of the measurements which yield the least accurate results would therefore appear to be desirable. For the present, one particular set of measurable quantities is assumed in order to illustrate the method of solution. Further investigation to determine the set of measurable quantities which yields the most accurate results would be desirable. The assumption is made that all the characteristics of the Dutch roll mode will be used but that, for the roll-subsidence and spiral modes, only the roots and not the ratios of the variables will be utilized. The following quantities are considered as knowns and unknowns:

Type of measurement	Known quantities	Unknown quantities
Dutch roll	$D_{1,r}, D_{1,i}, (D\phi/\beta)_1, (D\psi/\beta)_1$	None
Roll subsidence	D_2	$(D\phi/\beta)_2, (D\psi/\beta)_2$
Spiral motion	D_3	$(D\phi/\beta)_3, (D\psi/\beta)_3$
General	$K_X^2, K_Z^2, K_{XZ}, \mu, C_L$	None
Stability derivatives	None	$C_{Y_\beta}, C_{n_\beta}, C_{l_\beta}, C_{Y_p}, C_{n_p}, C_{l_p}, C_{Y_r}, C_{n_r}, C_{l_r}$

Thus, there are 13 unknowns and 12 equations.

Solution of equations.- Since in the case under consideration the number of equations is less than the number of unknowns, some assumption is necessary to reduce the number of unknowns. Experience has shown that the values of C_{Y_p} and C_{Y_r} may often be neglected in calculating the lateral motion. It is therefore assumed in the present analysis that these quantities equal zero. A calculation to show the errors introduced by this assumption is given in a later section.

Substitution of the known quantities into equations (4) to (6) yields 12 relations, but one of these, the side-force equation for the Dutch roll imaginary quantities, is neglected inasmuch as it involves only known quantities if C_{Y_p} and C_{Y_r} are assumed to be equal to zero. The other 11 equations suffice to determine the 11 remaining unknowns. The method of solving the equations is arbitrary, but the procedure outlined in the following section has been found to be convenient in practice.

For the purpose of the present report, actual flight measurements were not used to obtain the values of the "known quantities" listed in the preceding table. Instead, numerical values were chosen for the stability derivatives, mass parameters, and operating conditions of three representative airplanes. These values are listed in table I. These selected values were then used to calculate the characteristics of the resultant modes of lateral motion. These characteristics, given in table II, were then used to supply the "known quantities" listed in the preceding table and were used as the starting point in the determination of the derivatives by the present method. This procedure assures that a consistent set of known quantities are used and, in the subsequent error analysis, allows study of the effects of errors in one measurable quantity at a time. The practical problems of developing procedures for making flight measurements and for working up the desired characteristics from flight data are not discussed in the present report.

The procedure for solving the 11 simultaneous equations is now discussed. Usually the equations will be solved numerically rather than in symbolic form. A numerical example is given in the appendix. In the following equations, algebraic combinations of known quantities have been represented by K_n (n varies from 1 to 39) for the sake of brevity. These K values have been worked out in terms of the known quantities, but because the resulting expressions are long and because they are not needed in a numerical solution, they are not presented here. The following procedure in terms of algebraic symbols should be used simply as an outline of the order to be followed in solving the equations. The numerical example given in the appendix shows the actual procedure and illustrates how the numerical results may be obtained without specific expression for the K values. From the Dutch roll mode, the following relations are obtained:

Equations for imaginary part of Dutch roll mode:

$$-\frac{1}{2}C_{l_p} - \frac{1}{2}C_{l_r}K_1 + K_2 = 0 \qquad (7)$$

$$-\frac{1}{2}C_{n_p} - \frac{1}{2}C_{n_r}K_3 + K_4 = 0 \qquad (8)$$

Equations for real part of Dutch roll mode:

$$-C_{Y_\beta} + K_5 = 0 \qquad (9)$$

$$-C_{l_\beta} - \frac{1}{2}C_{l_p}K_6 - \frac{1}{2}C_{l_r}K_7 + K_8 = 0 \qquad (10)$$

$$-C_{n_\beta} - \frac{1}{2}C_{n_p}K_9 - \frac{1}{2}C_{n_r}K_{10} + K_{11} = 0 \qquad (11)$$

From these equations, the derivative C_{Y_β} is found and the values of C_{l_β}, C_{n_β}, C_{l_p}, and C_{n_r} may be determined in terms of C_{l_r} and C_{n_p} as shown in the appendix.

The solution of the Dutch roll equations is very similar to the vector method of reference 3. In the method of reference 3, values of two of the derivatives such as C_{n_p} and C_{l_r} are assumed. The remaining derivatives may then be determined by plotting vector diagrams representing the equations of motion for the Dutch roll mode. The closure of the vector diagrams is the graphical equivalent of setting the real and imaginary parts of the Dutch roll equations equal to zero. In the present method, by using the roll-subsidence and spiral as well as the Dutch roll characteristics, all the important derivatives are evaluated.

The equations from the roll-subsidence mode may be put in the following form. In these equations, the K-values are functions only of the known quantities, and the previously derived relations have been used to express C_{l_β}, C_{n_β}, C_{l_p}, and C_{n_r} in terms of C_{l_r} and C_{n_p}:

$$\left(\frac{D\phi}{\beta}\right)_2 + K_{12}\left(\frac{D\psi}{\beta}\right)_2 + K_{13} = 0 \tag{12}$$

$$\left(\frac{D\phi}{\beta}\right)_2\left(K_{14}C_{l_r} + K_{15}\right) + \left(\frac{D\psi}{\beta}\right)_2\left(C_{l_r} + K_{16}\right) + K_{17}C_{l_r} + K_{18} = 0 \tag{13}$$

$$\left(\frac{D\phi}{\beta}\right)_2\left(C_{n_p} + K_{19}\right) + \left(\frac{D\psi}{\beta}\right)_2\left(K_{20}C_{n_p} + K_{21}\right) + K_{22}C_{n_p} + K_{23} = 0 \tag{14}$$

The values of $\left(\frac{D\phi}{\beta}\right)_2$ and $\left(\frac{D\psi}{\beta}\right)_2$ may be eliminated from equations (12), (13), and (14) to obtain a relation between C_{l_r} and C_{n_p}

$$C_{l_r} + K_{24}C_{n_p} + K_{25} = 0 \tag{15}$$

Likewise, the equations from the spiral mode may be put in the following form:

$$\left(\frac{D\phi}{\beta}\right)_3 + K_{26}\left(\frac{D\psi}{\beta}\right)_3 + K_{27} = 0 \tag{16}$$

$$\left(\frac{D\phi}{\beta}\right)_3\left(K_{28}C_{l_r} + K_{29}\right) + \left(\frac{D\psi}{\beta}\right)_3\left(C_{l_r} + K_{30}\right) + K_{31}C_{l_r} + K_{32} = 0 \tag{17}$$

$$\left(\frac{D\phi}{\beta}\right)_3\left(C_{n_p} + K_{33}\right) + \left(\frac{D\psi}{\beta}\right)_3\left(K_{34}C_{n_p} + K_{35}\right) + K_{36}C_{n_p} + K_{37} = 0 \tag{18}$$

The values of $\left(\frac{D\phi}{\beta}\right)_3$ and $\left(\frac{D\psi}{\beta}\right)_3$ may be eliminated from equations (16), (17), and (18) to obtain another relation between C_{l_r} and C_{n_p}

$$C_{l_r} + K_{38}C_{n_p} + K_{39} = 0 \tag{19}$$

Equations (15) and (19) may be solved simultaneously to obtain C_{n_p} and C_{l_r}. Finally, the derivatives C_{l_β}, C_{n_β}, C_{l_p}, and C_{n_r} may be determined from the equations derived from equations (7) to (11).

DISCUSSION

Probable Errors in Measurements

The entire discussion to this point has been based on the premise of precisely determinable values. Since flight research at best is an inexact science, some knowledge of the errors likely to be involved in the measurement of the so-called "known" quantities appears to be in order. The errors depend, of course, on the accuracy of instrumentation employed, the reading accuracy, and the technique of making the flight measurements.

The spiral root is usually the most difficult to measure. A method of measuring the characteristics of the spiral mode is given in reference 7. The spiral root for a fighter airplane, for which the time to double amplitude was about 30 seconds, was measured to an estimated ±9-percent error; however, it would be expected that, as the root approaches zero, the percent error would increase. Errors in the measurement of the spiral root are expected to be more of an absolute type of error than a percentage error.

In order to make a preliminary assessment of the results, the following ranges of errors, based on flight experience with the type of instrumentation employed by the NACA, may be used as a rough guide.

Quantity	Error
Dutch roll period, percent	±5
Dutch roll damping, percent	±3
Amplitude ratios, $(D\psi/\beta)_1$ and $(D\phi/\beta)_1$, percent	±5
Phase angle of $(D\psi/\beta)_1$ and $(D\phi/\beta)_1$, deg	±6
Roll-subsidence root, percent	±6
η, deg	±1
Mass and inertia characteristics, percent	±2

The mass and inertia characteristics can be measured accurately on the ground for a known loading condition, but in flight they are subject to uncertainty because of errors in the measurement of fuel consumption.

In order to determine the effect of errors in the flight measurements, calculations were made for three airplanes of low, medium, and high relative densities. The stability derivatives, operating conditions, and mass parameters which were chosen are typical of the particular class of airplane. These values are listed in table I. These selected values were then used to calculate the characteristics of the resultant modes of lateral motion. (See table II.)

Each flight or ground measurable quantity in turn was varied from the correct value and used to recalculate the stability derivatives by the method given. The results of these calculations are shown in figures 2 to 8. Thus, for example, in figure 2, a 5-percent error in μ alone (with all the other measurable flight quantities correct) causes C_{Y_β} to be calculated as -0.73 rather than -0.69 and C_{n_p} to be calculated as -0.03 rather than as -0.025 for the airplane with the low relative density.

Errors in Derivatives

Static stability derivatives.- Examination of figures 2 to 8 shows that the static stability derivatives C_{Y_β}, C_{l_β}, and C_{n_β} are, in general, not unduly sensitive to errors in the measured data. The percent error in these derivatives is usually about the same or less than that in a given measured quantity. The errors usually increase somewhat as the value of μ increases. One exception is the relatively high sensitivity of C_{Y_β} to the phase angle of $D\psi/\beta$ for the Dutch roll mode (fig. 6).

A surprising result is the insensitivity of C_{n_β} to the frequency of the Dutch roll $D_{1,i}$. (See fig. 7.) The frequency is usually considered to be proportional to the square root of C_{n_β} for constant values of the other derivatives. In the present example, the large simultaneous variations of the other derivatives, especially C_{n_p}, probably account for the small variation of C_{n_β}. It is not known whether this result would be found in all cases.

Damping derivatives.- The damping derivatives C_{l_p} and C_{n_r} become progressively more difficult to determine as the value of μ increases. This result is related to the fact that the damping derivatives have less effect on the motion as μ increases. Accurate determination of these derivatives is therefore less important at higher values of μ. For the range of μ considered, reasonably accurate measurements of C_{l_p} were obtained. The value of C_{n_r} can be determined only for the airplanes with low and medium relative density.

Cross derivatives.- The cross derivatives C_{l_r} and C_{n_p} are evidently the most difficult to obtain accurately. The value of C_{l_r} is well-defined for the airplane with the low relative density but rather poorly defined for the airplanes with medium and high relative density. The value of C_{n_p} is not determined with reasonable accuracy for any of the cases. It is greatly affected by errors in Dutch roll period and in the phase angles of $D\phi/\beta$ and $D\psi/\beta$.

Effect of Side-Force Rate Derivatives

A check was made to determine the effects of variations in the side-force rate derivatives C_{Y_r} and C_{Y_p} which were assumed to equal zero in the previous analysis. In practice, the values of C_{Y_r} and C_{Y_p} may lie in the range from about 0.3 to -0.3. The values of all the other derivatives were calculated with these values for C_{Y_r} and C_{Y_p}. The results of these calculations are shown in figure 9. The derivative C_{Y_r} has very little or no effect on any of the other derivatives. The derivative C_{Y_p} has a small effect on C_{Y_β} and C_{n_p} for all three airplanes and a small or no effect for the other derivatives. In view of these results it appears that a slight improvement in accuracy could be gained by using an estimated value for C_{Y_p} in the analysis rather than by assuming it to be zero.

Effect of an Error in β

The separate errors assumed in the quantities $D\phi/\beta$ and $D\psi/\beta$ may be considered as errors in $D\phi$ and $D\psi$. If it is assumed, however, that an error exists in the magnitude or phase angle of β, both $D\phi/\beta$ and $D\psi/\beta$ will be affected in the same way. Additional calculations have been made, therefore, to determine the effects of the combined errors in these quantities which would result from an error in β and the results of these calculations are given in figure 10. The derivative C_{Y_β} is highly sensitive to the phase angle β but shows little sensitivity to the magnitude of β. The derivatives C_{l_r}, C_{n_p}, and C_{n_r} become progressively more sensitive as the value of μ increases whereas C_{l_β}, C_{l_p}, and C_{n_β} for all three airplanes show very small or no sensitivity to β.

Improvements in the Method

Measurement of C_{Y_β}.- A deficiency in the method described is the sensitivity of C_{Y_β} to an error in the phase angle $D\psi/\beta$. (See fig. 6.) The source of this error may be seen by examining the side-force equation for the Dutch roll mode. (See eq. (4).) Since C_{Y_β} is the only unknown involved, this equation alone is required for its determination.

$$-C_{Y_\beta} + 2\mu D_{1,r} + 2\mu\left(\frac{D\psi}{\beta}\right)_{1,r} - C_L\left(\frac{\phi}{\beta}\right)_{1,r} = 0 \tag{20}$$

Since $D\psi/\beta$ is almost a pure imaginary quantity (see table II), changes in the phase angle affect the real part of this quantity directly. This value is then multiplied by the large factor μ which introduces errors in C_{Y_β}.

An alternate method of determining C_{Y_β}, pointed out in reference 3, is to measure lateral acceleration a_y by means of an accelerometer. The value $\frac{ma_y}{\beta qS}$ gives the sum of the terms $2\mu D + 2\mu\left(\frac{D\psi}{\beta}\right) - C_L\left(\frac{\phi}{\beta}\right)$, which according to equation (20) equals C_{Y_β}. In addition, the use of this procedure may allow determination of $D\psi/\beta$ to a higher degree of accuracy than would be possible by measurements of $D\psi$ and β directly.

Alternate sets of flight measurements.- The set of flight measurements discussed in the present report is not necessarily the best for any particular airplane. Since determination of C_{n_p} by the present method requires extreme accuracy in the measurements, it is likely that the use of additional data, such as the value of $D\phi/\beta$ for the roll-subsidence mode, would allow more accurate determination of C_{n_p}. Preliminary calculations indicate that greater accuracy in the measurement of C_{n_p} can be obtained by this method.

Another possibility which may prove desirable in some cases is elimination of measurements of the spiral mode and substitution of more complete data on the roll-subsidence mode. Accurate determination of the spiral characteristics is difficult in some airplanes because of the changes in lateral trim caused by motion of fuel in wing tanks or because of the tendency of wings to take a slight twist due to structural hysteresis following application of lateral control.

CONCLUDING REMARKS

A method is presented for determining lateral stability derivatives from flight measurements. This method utilizes data from each of the three modes of lateral motion and allows determination of all the important stability derivatives. An error analysis is made to show the effects of errors in the measured quantities on the accuracy of each stability derivative for three representative airplanes.

The static stability derivatives C_{Y_β}, C_{l_β}, and C_{n_β} may be determined with good accuracy by the proposed method. The damping derivatives C_{l_p} and C_{n_r} become progressively more difficult to determine as the value of the airplane density factor μ increases. The values of the cross-rate derivatives C_{l_r} and C_{n_p} are the most difficult to obtain accurately. The value of C_{l_r} is well-defined for low values of μ but is poorly defined at medium or high values of μ. The value of C_{n_p} is not determined with sufficient accuracy for any of the cases studied.

The proposed method may be modified to utilize different sets of flight measurements without changing the basic procedure. Improvements in the accuracy of determination of some of the derivatives might result from the use of different sets of flight measurements.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., May 7, 1957.

APPENDIX

NUMERICAL EXAMPLE

The data given for the fighter airplane in tables I and II are used in the example. Values are substituted into equations (4), (5), and (6) of the text.

Imaginary Part of Dutch Roll Mode

The side-force equation disappears because the two unknowns C_{Y_p} and C_{Y_r} are assumed to be equal to zero. The rolling-moment equation is:

$$-\frac{1}{2}C_{l_p}\left(\frac{D\phi}{\beta}\right)_{1,i} - \frac{1}{2}C_{l_r}\left(\frac{D\psi}{\beta}\right)_{1,i} + 2\mu D_1\left[\left(\frac{D\phi}{\beta}\right)_1 K_X^{\;2} - \left(\frac{D\psi}{\beta}\right)_1 K_{XZ}\right] = 0$$

$$-\frac{1}{2}C_{l_p}(0.103i) - \frac{1}{2}C_{l_r}(-0.302i) + 2(13)(-0.0354 + 0.304i)\Big[(-0.211 + 0.103i)(0.0171) - (0.0100 - 0.302i)(0)\Big] = 0$$

In the products $(0.0354 + 0.304i)(-0.211 + 0.103i)$ and $(-0.0354 + 0.304i)(0.0100 - 0.302i)$, only the imaginary parts are used.

$$-0.0515iC_{l_p} + 0.151iC_{l_r} + 0.0302i = 0$$

$$C_{l_p} = 2.90C_{l_r} - 0.585 \tag{A1}$$

The yawing-moment equation is:

$$-\frac{1}{2}C_{n_p}\left(\frac{D\phi}{\beta}\right)_{1,i} - \frac{1}{2}C_{n_r}\left(\frac{D\psi}{\beta}\right)_{1,i} + 2\mu D_1\left[\left(\frac{D\psi}{\beta}\right)_1 K_Z^2 - \left(\frac{D\phi}{\beta}\right)_1 K_{XZ}\right] = 0$$

$$-\frac{1}{2}C_{n_p}(0.103i) - \frac{1}{2}C_{n_r}(-0.302i) + 2(13)(-0.0354 + 0.304i)\Big[(0.0100 -$$

$$0.302i)(0.0492) - (-0.211 + 0.103i)(0)\Big] = 0$$

Again, in the products $(-0.0354 + 0.304i)(0.0100 - 0.302i)$ and $(-0.0354 + 0.304i)(-0.211 + 0.103i)$, only the imaginary parts are used.

$$-0.0515iC_{n_p} + 0.151iC_{n_r} + 0.0175i = 0$$

$$C_{n_r} = 0.340C_{n_p} - 0.116 \tag{A2}$$

Real Part of Dutch Roll Mode

The side-force equation is:

$$-C_{Y_\beta} - \frac{1}{2}C_{Y_p}\left(\frac{D\phi}{\beta}\right)_{1,r} - \frac{1}{2}C_{Y_r}\left(\frac{D\psi}{\beta}\right)_{1,r} + 2\mu D_{1,r}\left[1 + \left(\frac{\psi}{\beta}\right)_{1,r}\right] - C_L\left(\frac{\phi}{\beta}\right)_1 = 0$$

If the following equalities are used, the substitution in the side-force equation may be followed with ease.

$$2\mu D_{1,r}\left[1 + \left(\frac{\psi}{\beta}\right)_{1,r}\right] = 2\mu D_{1,r} + 2\mu\left(\frac{D\psi}{\beta}\right)_{1,r}$$

$$\left(\frac{\phi}{\beta}\right)_1 = \frac{\left(\frac{D\phi}{\beta}\right)_1}{D_1}$$

Substituting the values in the side-force equation gives:

$$-C_{Y_\beta} + 0 + 0 + 2(13)(-0.0354) + 2(13)(0.0100) - 0.071\left(\frac{-0.211 + 0.103i}{-0.0354 + 0.304i}\right) = 0$$

In the quotient of $\left(\frac{-0.211 + 0.103i}{-0.0354 + 0.304i}\right)$, only the real part is used; thus,

$$-C_{Y_\beta} - 0.69 = 0$$

$$C_{Y_\beta} = -0.69 \tag{A3}$$

The rolling-moment equation is:

$$-C_{l_\beta} - \frac{1}{2}C_{l_p}\left(\frac{D\phi}{\beta}\right)_{1,r} - \frac{1}{2}C_{l_r}\left(\frac{D\psi}{\beta}\right)_{1,r} + 2\mu D_1\left[\left(\frac{D\phi}{\beta}\right)_1 K_X^2 - \left(\frac{D\psi}{\beta}\right)_1 K_{XZ}\right] = 0$$

$$-C_{l_\beta} - \frac{1}{2}C_{l_p}(-0.211) - \frac{1}{2}C_{l_r}(0.0100) + 2(13)(-0.0354 + 0.304i)\Big[(-0.211 +$$

$$0.103i)(0.0171) - (0.0100 - 0.302i)(0)\Big] = 0$$

In the products $(-0.0354 + 0.304i)(-0.211 + 0.103i)$ and $(-0.0354 + 0.304i)(0.0100 - 0.302i)$, only the real parts are used; thus,

$$-C_{l_\beta} + 0.106C_{l_p} - 0.005C_{l_r} - 0.0105 = 0$$

Substituting the value of C_{l_p} from equation (A1) gives:

$$-C_{l_\beta} + 0.106\left(2.90C_{l_r} - 0.585\right) - 0.005C_{l_r} - 0.0105 = 0$$

$$C_{l_\beta} = 0.300C_{l_r} - 0.0720 \tag{A4}$$

The yawing-moment equation is:

$$-C_{n_\beta} - \frac{1}{2}C_{n_p}\left(\frac{D\phi}{\beta}\right)_{1,r} - \frac{1}{2}C_{n_r}\left(\frac{D\psi}{\beta}\right)_{1,r} + 2\mu D_1\left[\left(\frac{D\psi}{\beta}\right)_1 K_Z{}^2 - \left(\frac{D\phi}{\beta}\right)_1 K_{XZ}\right] = 0$$

$$-C_{n_\beta} - \frac{1}{2}C_{n_p}(-0.211) - \frac{1}{2}C_{n_r}(0.0100) + 2(13)(-0.0354 + 0.304i)\Big[(0.0100 -$$

$$0.302i)(0.0492) - (-0.211 + 0.103i)(0)\Big] = 0$$

In the products $(-0.0354 + 0.304i)(0.0100 - 0.302i)$ and $(-0.0354 + 0.304i)(-0.211 + 0.103i)$, only the real parts are used; thus,

$$-C_{n_\beta} + 0.106C_{n_p} - 0.005C_{n_r} + 0.116 = 0$$

Substituting the value of C_{n_r} from equation (A2) gives:

$$-C_{n_\beta} + 0.106C_{n_p} - 0.005\left(0.340C_{n_p} - 0.116\right) + 0.116 = 0$$

$$C_{n_\beta} = 0.103C_{n_p} + 0.117 \qquad \text{(A5)}$$

Roll Subsidence

The side-force equation is:

$$-C_{Y_\beta} - \frac{1}{2}C_{Y_p}\left(\frac{D\phi}{\beta}\right)_2 - \frac{1}{2}C_{Y_r}\left(\frac{D\psi}{\beta}\right)_2 + 2\mu D_2\left[1 + \left(\frac{\psi}{\beta}\right)_2\right] - \left(\frac{\phi}{\beta}\right)_2 C_L = 0$$

$$0.69 + 0 + 0 + 2(13)(-0.499) + 2(13)\left(\frac{D\psi}{\beta}\right)_2 - \left(\frac{D\phi}{\beta}\right)_2\left(\frac{0.071}{-0.499}\right) = 0$$

$$\left(\frac{D\phi}{\beta}\right)_2 = 183\left(\frac{D\psi}{\beta}\right)_2 - 86.5 \qquad \text{(A6)}$$

The rolling-moment equation is:

$$-C_{l_\beta} - \frac{1}{2}C_{l_p}\left(\frac{D\phi}{\beta}\right)_2 - \frac{1}{2}C_{l_r}\left(\frac{D\psi}{\beta}\right)_2 + 2\mu D_2\left[\left(\frac{D\phi}{\beta}\right)_2 K_X{}^2 - \left(\frac{D\psi}{\beta}\right)_2 K_{XZ}\right] = 0$$

$$-C_{l_\beta} - \frac{1}{2}C_{l_p}\left(\frac{D\phi}{\beta}\right)_2 - \frac{1}{2}C_{l_r}\left(\frac{D\psi}{\beta}\right)_2 + 2(13)(-0.499)\left[\left(\frac{D\phi}{\beta}\right)_2(0.0171) - \left(\frac{D\psi}{\beta}\right)_2(0)\right] = 0$$

Substitutions from equations (A1), (A4), and (A6) give:

$$-(0.300C_{l_r} - 0.0720) - \frac{1}{2}\left(2.90C_{l_r} - 0.585\right)\left[183\left(\frac{D\psi}{\beta}\right)_2 - 86.5\right] -$$

$$\frac{1}{2}C_{l_r}\left(\frac{D\psi}{\beta}\right)_2 + 2(13)(0.499)\left[183\left(\frac{D\psi}{\beta}\right)_2 - 86.5\right](0.0171) = 0$$

$$\left(12.8 - 265C_{l_r}\right)\left(\frac{D\psi}{\beta}\right)_2 - 5.95 + 125C_{l_r} = 0$$

$$\left(\frac{D\psi}{\beta}\right)_2 = \frac{-5.95 + 125C_{l_r}}{-12.8 + 265C_{l_r}} \tag{A7}$$

The yawing-moment equation is:

$$-C_{n_\beta} - \frac{1}{2}C_{n_p}\left(\frac{D\phi}{\beta}\right)_2 - \frac{1}{2}C_{n_r}\left(\frac{D\psi}{\beta}\right)_2 + 2\mu D_2\left[\left(\frac{D\psi}{\beta}\right)_2 K_Z{}^2 - \left(\frac{D\phi}{\beta}\right)_2 K_{XZ}\right] = 0$$

$$-C_{n_\beta} - \frac{1}{2}C_{n_p}\left(\frac{D\phi}{\beta}\right)_2 - \frac{1}{2}C_{n_r}\left(\frac{D\psi}{\beta}\right)_2 + 2(13)(-0.499)\left[\left(\frac{D\psi}{\beta}\right)_2(0.0492) - \left(\frac{D\phi}{\beta}\right)_2(0)\right] = 0$$

Substituting the values obtained from equations (A2), (A5), and (A6) gives:

$$-\left(0.103C_{n_p} + 0.117\right) - \frac{1}{2}C_{n_p}\left[183\left(\frac{D\psi}{\beta}\right)_2 - 86.5\right] - \frac{1}{2}\left(\frac{D\psi}{\beta}\right)_2\left(0.340C_{n_p} - 0.116\right) +$$

$$(2)(13)(-0.499)\left(\frac{D\psi}{\beta}\right)_2(0.0492) = 0$$

$$\left(-91.7C_{n_p} - 0.58\right)\left(\frac{D\psi}{\beta}\right)_2 + 43.3C_{n_p} - 0.117 = 0$$

$$\left(\frac{D\psi}{\beta}\right)_2 = \frac{-0.117 + 43.3C_{n_p}}{0.58 + 91.7C_{n_p}} \tag{A8}$$

Substitute the value of $(D\psi/\beta)_2$ from equation (A7) into equation (A8).

$$\frac{-5.95 + 125C_{l_r}}{-12.8 + 265C_{l_r}} = \frac{-0.117 + 43.3C_{n_p}}{0.58 + 91.7C_{n_p}}$$

$$103.5C_{l_r} + 8.63C_{n_p} - 4.95 = 0 \tag{A9}$$

Spiral Motion

The side-force equation is:

$$-C_{Y_\beta} - \frac{1}{2}C_{Y_p}\left(\frac{D\phi}{\beta}\right)_3 - \frac{1}{2}C_{Y_r}\left(\frac{D\psi}{\beta}\right)_3 + 2\mu D_3\left[1 + \left(\frac{\psi}{\beta}\right)_3\right] - C_L\left(\frac{\phi}{\beta}\right)_3 = 0$$

$$0.69 + 0 + 0 + 2(13)(-0.0000725) + 2(13)\left(\frac{D\psi}{\beta}\right)_3 - \left(\frac{0.071}{-0.0000725}\right)\left(\frac{D\phi}{\beta}\right)_3 = 0$$

$$26\left(\frac{D\psi}{\beta}\right)_3 + 980\left(\frac{D\phi}{\beta}\right)_3 + 0.688 = 0$$

$$\left(\frac{D\psi}{\beta}\right)_3 = -37.7\left(\frac{D\phi}{\beta}\right)_3 - 0.0265 \tag{A10}$$

The rolling-moment equation is:

$$-C_{l_\beta} - \frac{1}{2}C_{l_p}\left(\frac{D\phi}{\beta}\right)_3 - \frac{1}{2}C_{l_r}\left(\frac{D\psi}{\beta}\right)_3 + 2\mu D_3\left[\left(\frac{D\phi}{\beta}\right)_3 K_X{}^2 - \left(\frac{D\psi}{\beta}\right)_3 K_{XZ}\right] = 0$$

$$-C_{l_\beta} - \frac{1}{2}C_{l_p}\left(\frac{D\phi}{\beta}\right)_3 - \frac{1}{2}C_{l_r}\left(\frac{D\psi}{\beta}\right)_3 +$$

$$2(13)(-0.0000725)\left[\left(\frac{D\phi}{\beta}\right)_3(0.0171) - \left(\frac{D\psi}{\beta}\right)_3(0)\right] = 0$$

Substituting the values obtained from equations (A1), (A4), and (A10) gives:

$$-\left(0.300C_{l_r} - 0.072\right) - \frac{1}{2}\left(2.90C_{l_r} - 0.585\right)\left(\frac{D\phi}{\beta}\right)_3 -$$

$$\frac{1}{2}C_{l_r}\left[-37.7\left(\frac{D\phi}{\beta}\right)_3 - 0.0265\right] - 0.00188\left(\frac{D\phi}{\beta}\right)_3(0.0171) = 0$$

$$\left(17.0C_{l_r} + 0.292\right)\left(\frac{D\phi}{\beta}\right)_3 - 0.300C_{l_r} + 0.0720 = 0$$

$$\left(\frac{D\phi}{\beta}\right)_3 = \frac{0.0720 - 0.300C_{l_r}}{-0.292 - 17.0C_{l_r}} \tag{A11}$$

The yawing-moment equation is:

$$-C_{n_\beta} - \frac{1}{2}C_{n_p}\left(\frac{D\phi}{\beta}\right)_3 - \frac{1}{2}C_{n_r}\left(\frac{D\psi}{\beta}\right)_3 + 2\mu D_3\left[\left(\frac{D\psi}{\beta}\right)_3 K_Z{}^2 - \left(\frac{D\phi}{\beta}\right)_3 K_{XZ}\right] = 0$$

$$-C_{n_\beta} - \frac{1}{2}C_{n_p}\left(\frac{D\phi}{\beta}\right)_3 - \frac{1}{2}C_{n_r}\left(\frac{D\psi}{\beta}\right)_3 + 2(13)(-0.0000725)\left[\left(\frac{D\psi}{\beta}\right)_3(0.492) - \left(\frac{D\phi}{\beta}\right)_3(0)\right] = 0$$

Substituting the values from equations (A2), (A5), and (A10) gives:

$$-\left(0.103C_{n_p} + 0.117\right) - \frac{1}{2}C_{n_p}\left(\frac{D\phi}{\beta}\right)_3 - \frac{1}{2}\left(0.340C_{n_p} - 0.116\right)\left[37.7\left(\frac{D\phi}{\beta}\right)_3 - 0.0265\right] - 0.00188\left[-37.7\left(\frac{D\phi}{\beta}\right)_3 - 0.0265\right](0.0492) = 0$$

$$\left(5.40C_{n_p} - 2.20\right)\left(\frac{D\phi}{\beta}\right)_3 - 0.099C_{n_p} - 0.119 = 0$$

$$\left(\frac{D\phi}{\beta}\right)_3 = \frac{-0.119 - 0.099C_{n_p}}{2.20 - 5.40C_{n_p}} \tag{A12}$$

Substitute the value of $(D\phi/\beta)_3$ from equation (A11) into equation (A12) to obtain

$$\frac{0.0720 - 0.300C_{l_r}}{-0.292 - 17.0C_{l_r}} = \frac{-0.119 - 0.099C_{n_p}}{2.20 - 5.40C_{n_p}}$$

$$-2.68C_{l_r} - 0.420C_{n_p} + 0.124 = 0$$

$$C_{l_r} = -0.157C_{n_p} + 0.046 \tag{A13}$$

Substitute the value of C_{l_r} from equation (A13) into equation (A9)

$$103.5\left(-0.157C_{n_p} + 0.046\right) + 8.63C_{n_p} - 4.95 = 0$$

$$-7.67C_{n_p} - 0.19 = 0$$

$$C_{n_p} = -0.025$$

Substitute the value of C_{n_p} into equation (A13) to obtain

$$C_{l_r} = -0.157(-0.025) + 0.046$$

$$C_{l_r} = 0.050$$

Substitute the value of C_{l_r} into equation (A1) to obtain

$$C_{l_p} = 2.90(0.05) - 0.585$$

$$C_{l_p} = -0.44$$

Substitute the value of C_{n_p} into equation (A2) to obtain

$$C_{n_r} = 0.34(-0.025) - 0.116$$

$$C_{n_r} = -0.125$$

Substitute the value of C_{l_r} into equation (A4) to obtain

$$C_{l_\beta} = 0.30(0.05) - 0.072$$

$$C_{l_\beta} = -0.057$$

Substitute the value of C_{n_p} into equation (A5) to obtain

$$C_{n_\beta} = 0.103(-0.025) + 0.117$$

$$C_{n_\beta} = 0.115$$

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TABLE I

CHARACTERISTICS OF REPRESENTATIVE AIRPLANES USED IN CALCULATIONS

Characteristic	Fighter	Medium Bomber	High-altitude fighter
μ	13.0	31.83	182
K_X^2	0.0171	0.0311	0.01557
K_Z^2	0.0492	0.072	0.156
K_{XZ}	0	0	0.0020
C_L	0.071	0.443	0.49
h, ft	0	35,000	50,000
V, ft/sec	700	700	776
b, ft	41.6	116	25
C_{Y_β}	-0.69	-0.61	-0.58
C_{Y_p}	0	0	0
C_{Y_r}	0	0	0
C_{l_β}	-0.0573	-0.14	-0.18
C_{l_p}	-0.44	-0.44	-0.33
C_{l_r}	0.05	0.149	0.22
C_{n_β}	0.115	0.12	0.25
C_{n_p}	-0.025	0.0276	-0.049
C_{n_r}	-0.125	-0.156	-0.68

TABLE II

CALCULATED CHARACTERISTICS OF THE RESULTANT MODES FOR THE REPRESENTATIVE AIRPLANES

[All roots are given in nondimensional form.]

Airplane	μ	Roots of characteristic equation			$D\phi/\beta$			$D\psi/\beta$		
		Dutch roll	Roll subsidence	Spiral motion	Dutch roll	Roll subsidence	Spiral motion	Dutch roll	Roll subsidence	Spiral motion
Fighter	13.0	$-0.0354 \pm 0.3039i$	-0.4993	-0.0000725	$-0.2113 + 0.1028i$	24.77	-0.04947	$0.01003 - 0.3022i$	0.3375	1.84
Medium bomber	31.83	$-0.00447 \pm 0.1679i$	-0.1284	-0.000419	$-0.215 + 0.2828i$	4.36	-0.095	$0.00684 - 0.159i$	-0.1177	1.56
High-altitude fighter	182	$0.00258 \pm 0.0665i$	-0.0410	-0.000770	$-0.197 + 0.3745i$	2.75	-0.49	$0.00325 - 0.0622i$	-0.0508	0.856

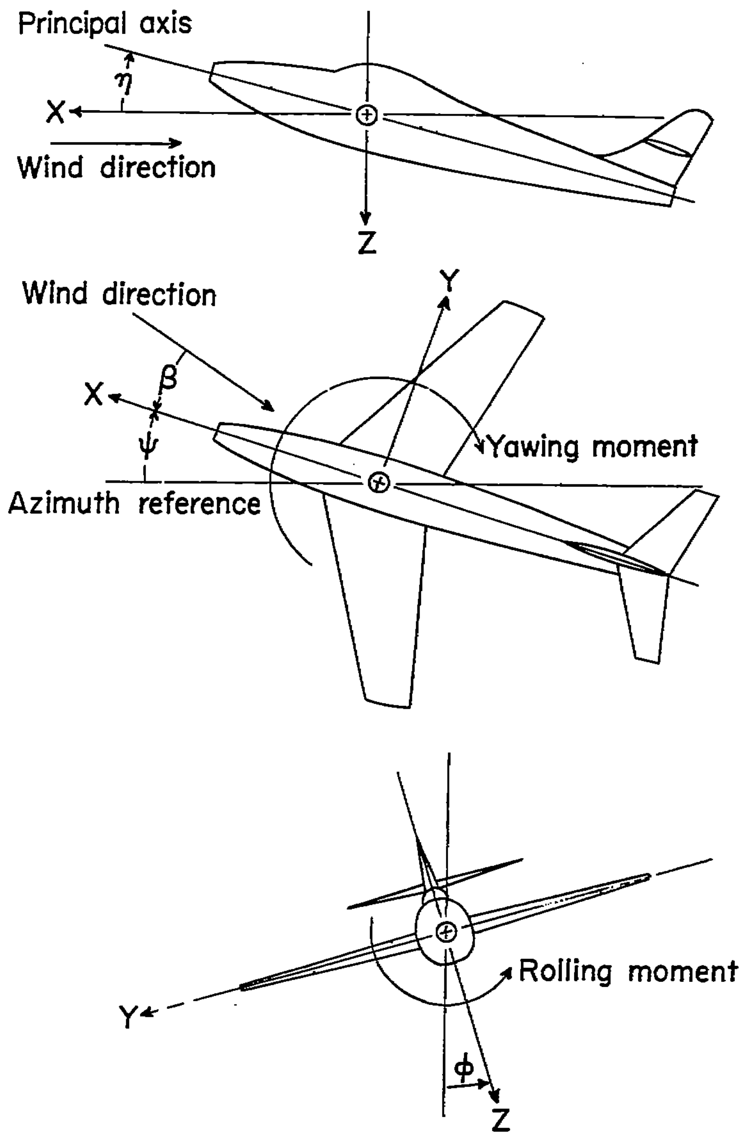

Figure 1.- Stability axes system employed with positive direction of forces, moments, and displacements shown.

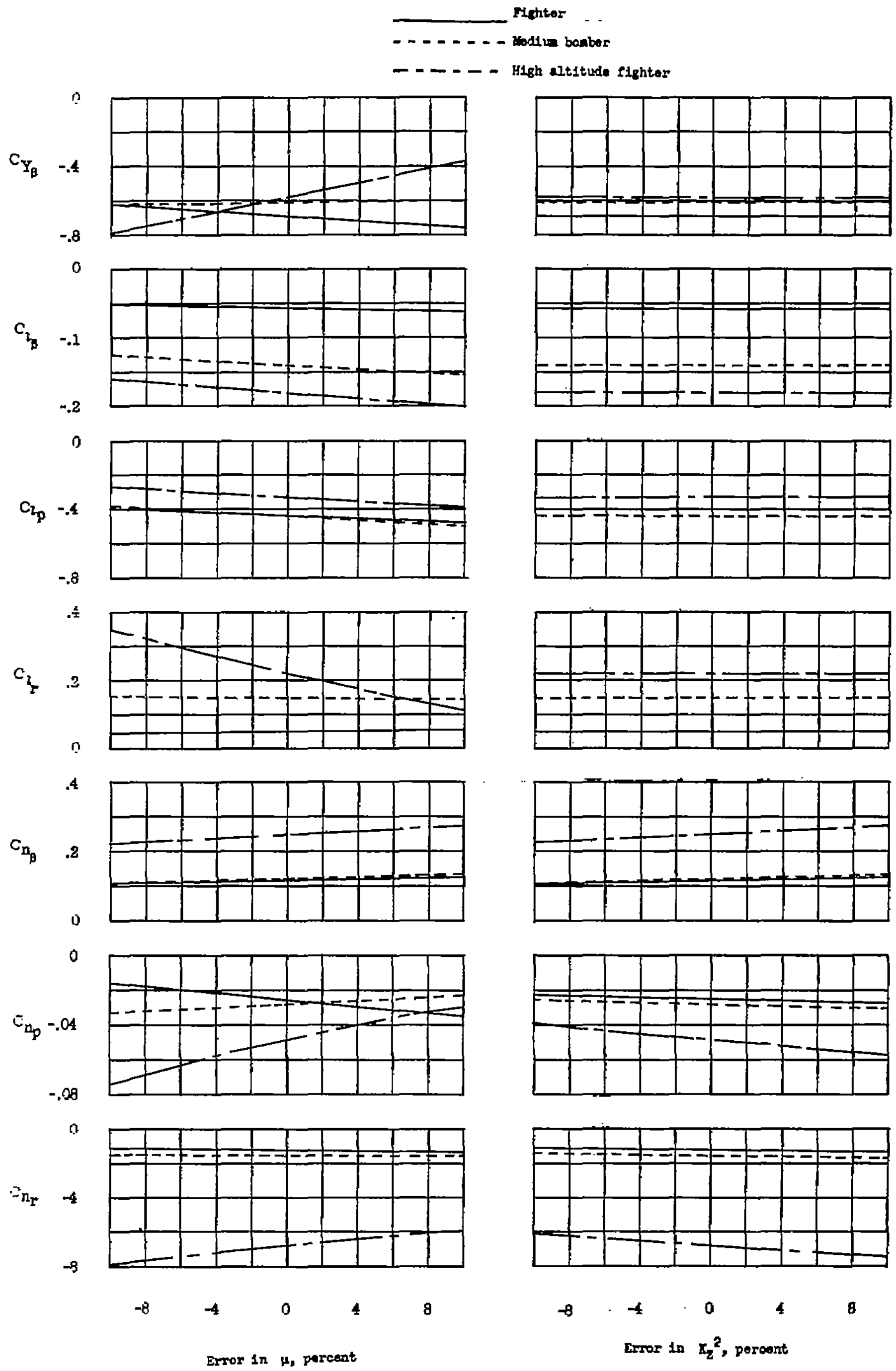

Figure 2.- Variation of the stability derivatives produced by errors in the relative-density factor and the nondimensional radius of gyration in yaw about the vertical stability axis.

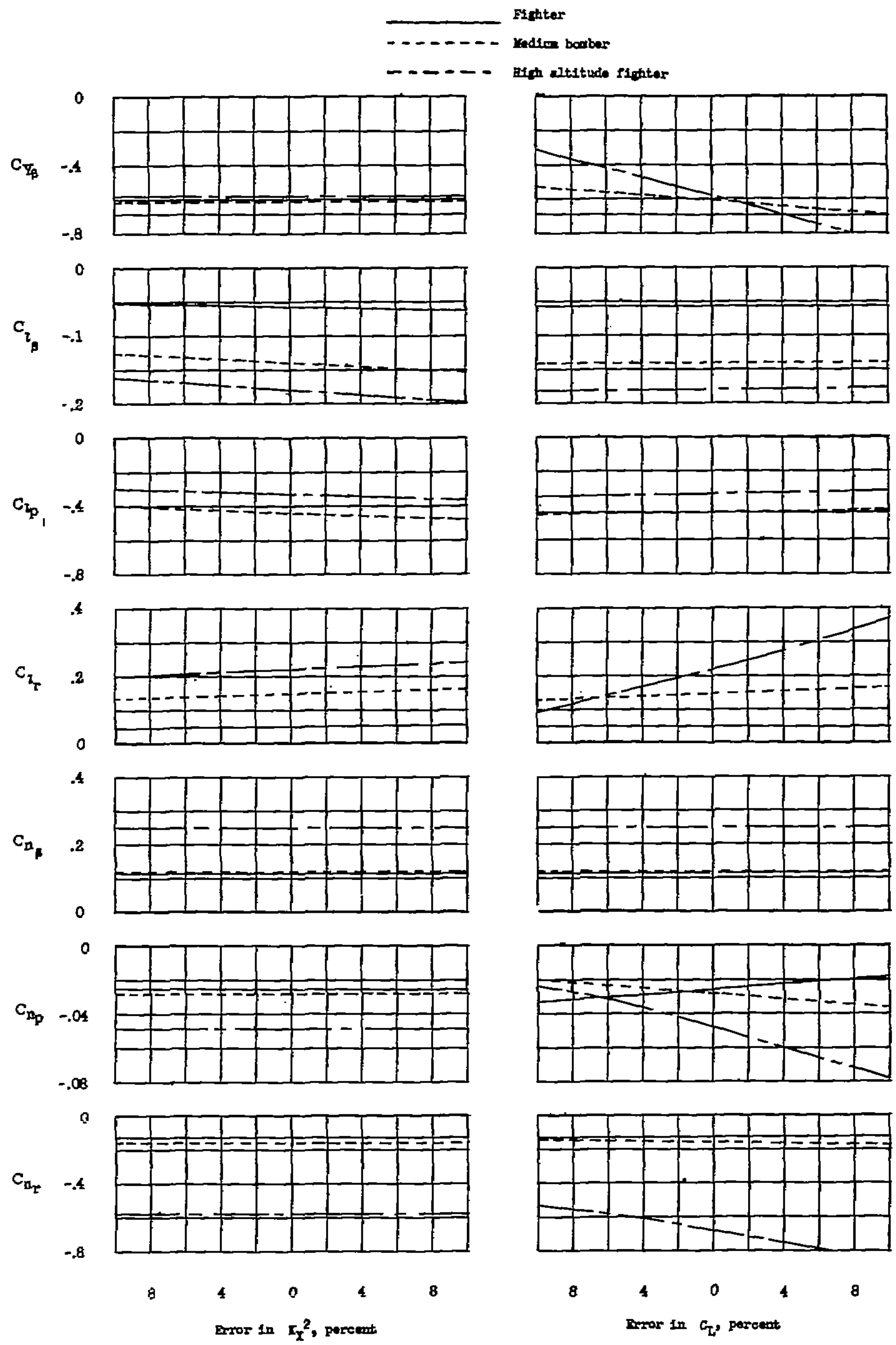

Figure 3.- Variation of the stability derivatives produced by errors in the nondimensional radius of gyration in roll about the longitudinal stability axis and the lift coefficient.

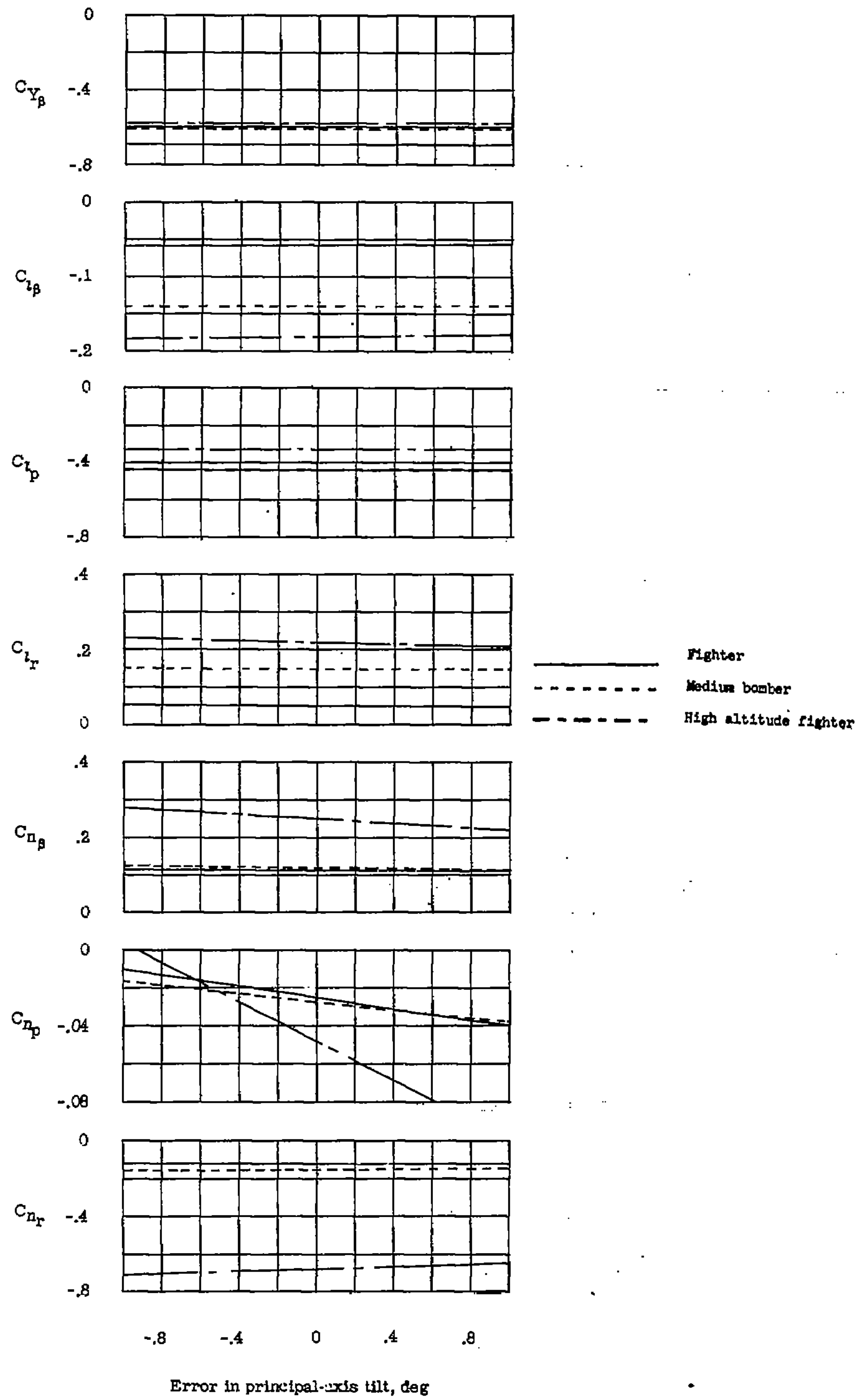

Figure 4.- Variation of the stability derivatives produced by an error in principal-axis tilt.

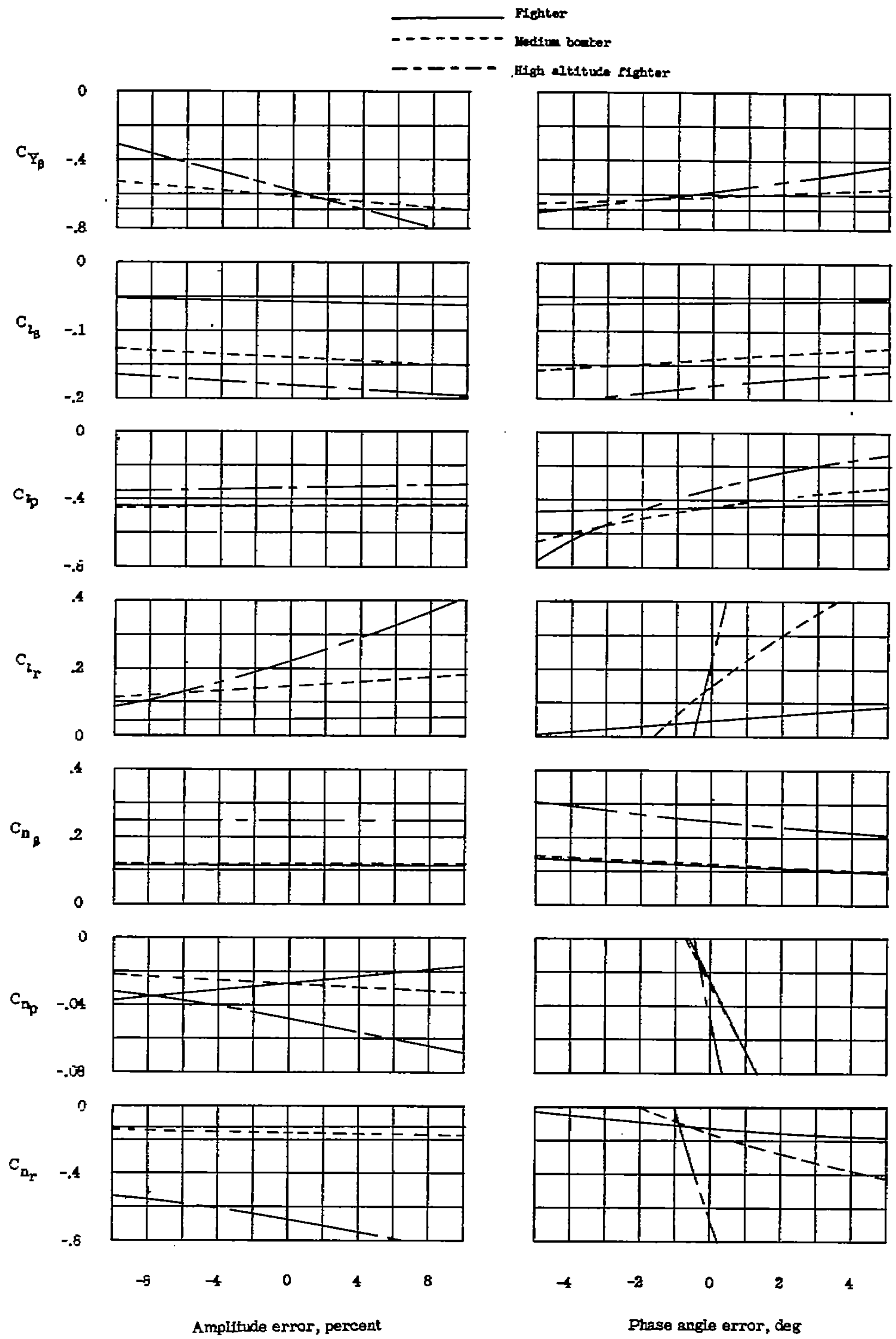

Figure 5.- Variation of the stability derivatives produced by errors in the amplitude and phase angle of the Dutch roll ratio $D\phi/\beta$.

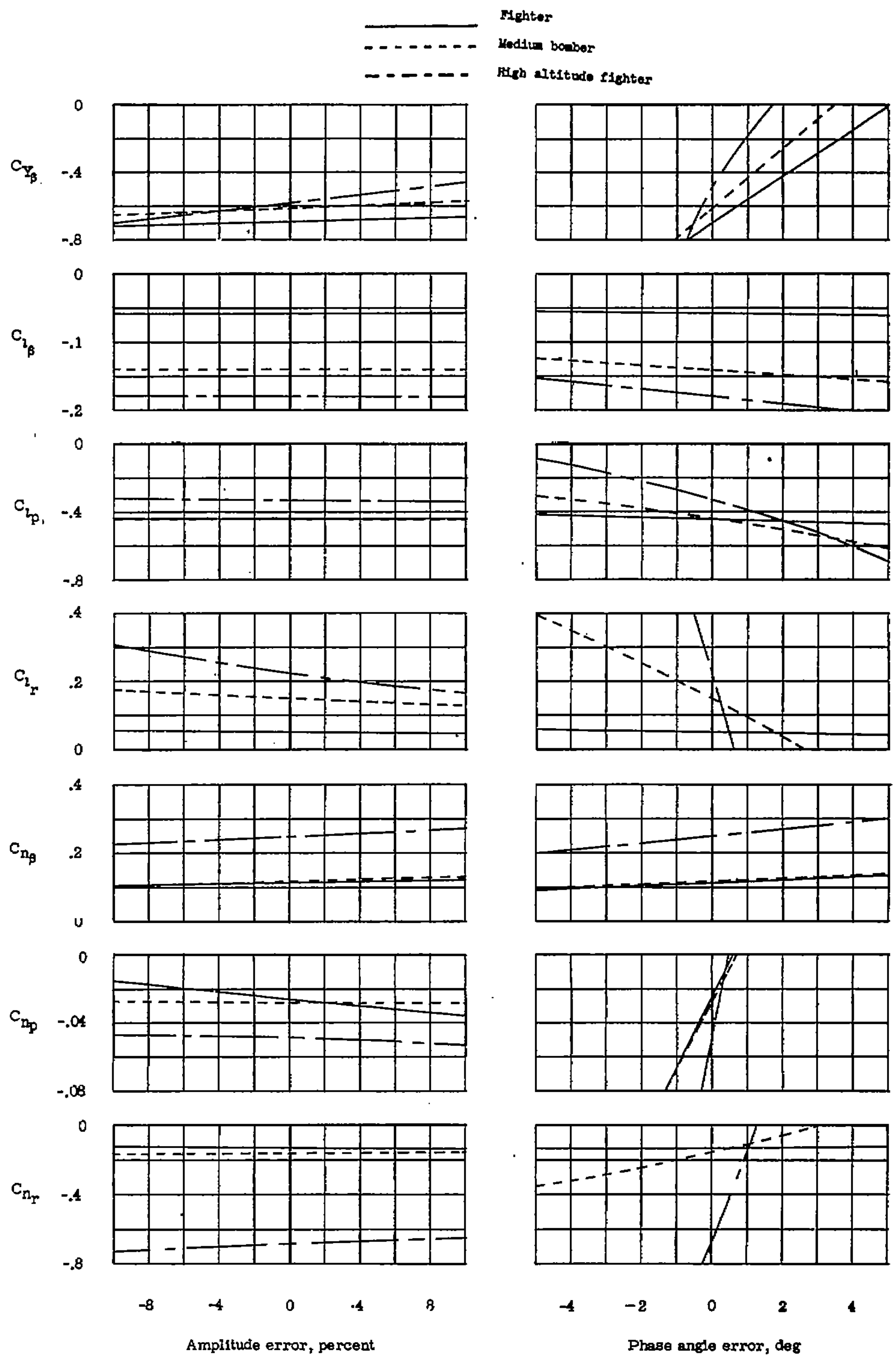

Figure 6.- Variation of the stability derivatives produced by errors in the amplitude and phase angle of the Dutch roll ratio $D\psi/\beta$.

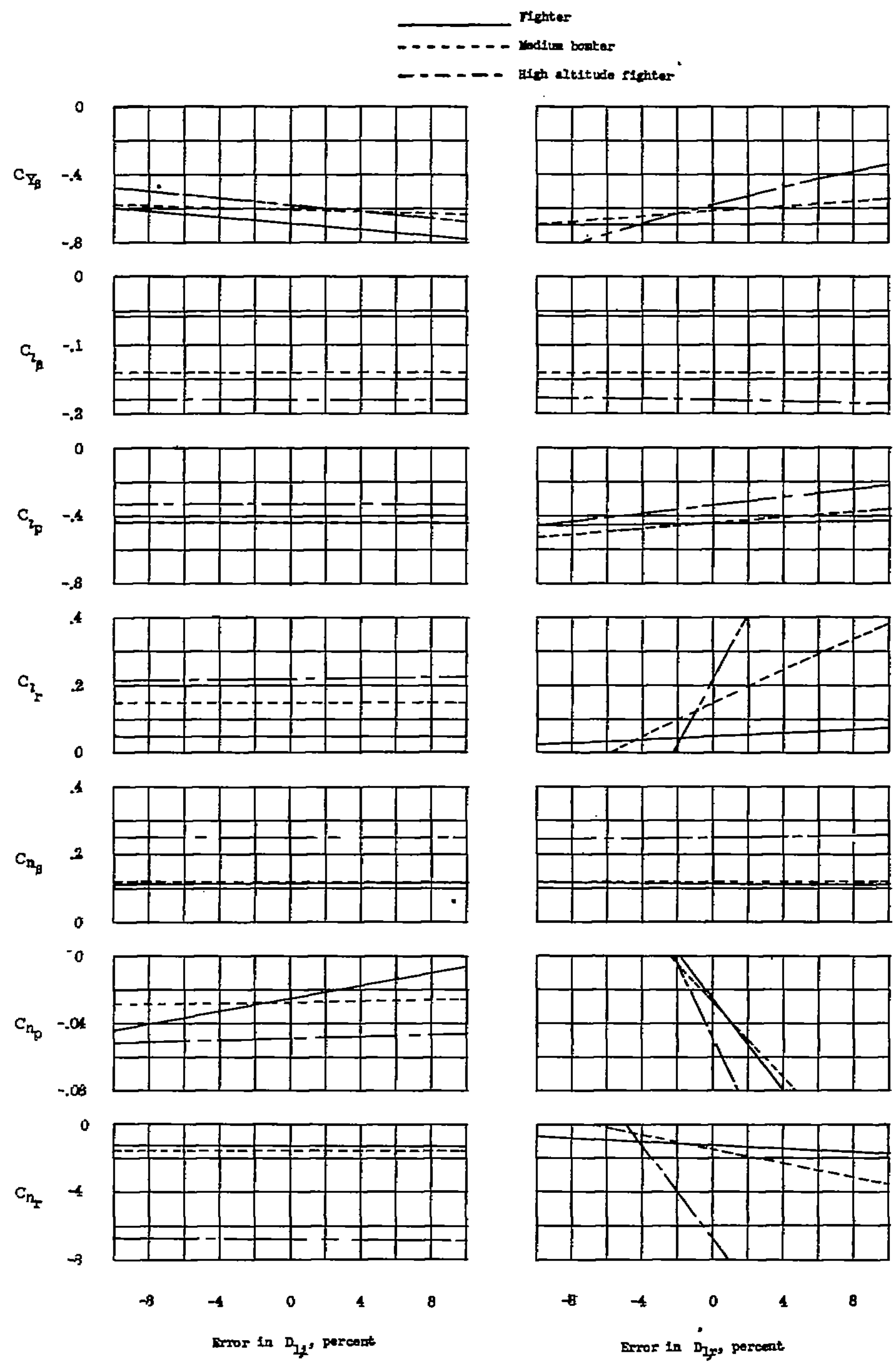

Figure 7.- Variation of the stability derivatives produced by errors in the real and imaginary parts of the Dutch roll root of the characteristic equation.

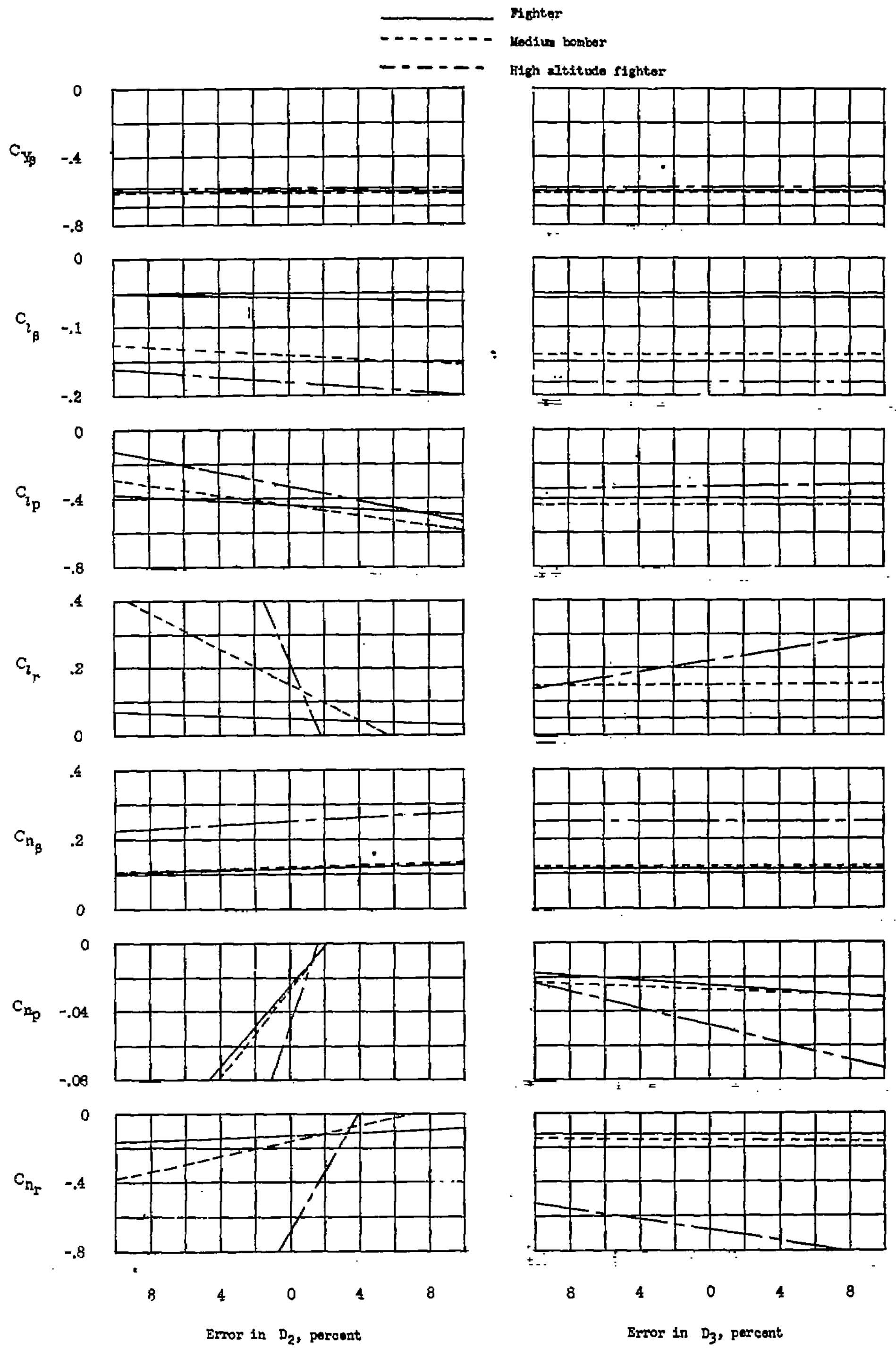

Figure 8.- Variation of the stability derivatives produced by errors in the roll-subsidence and spiral roots of the characteristic equation.

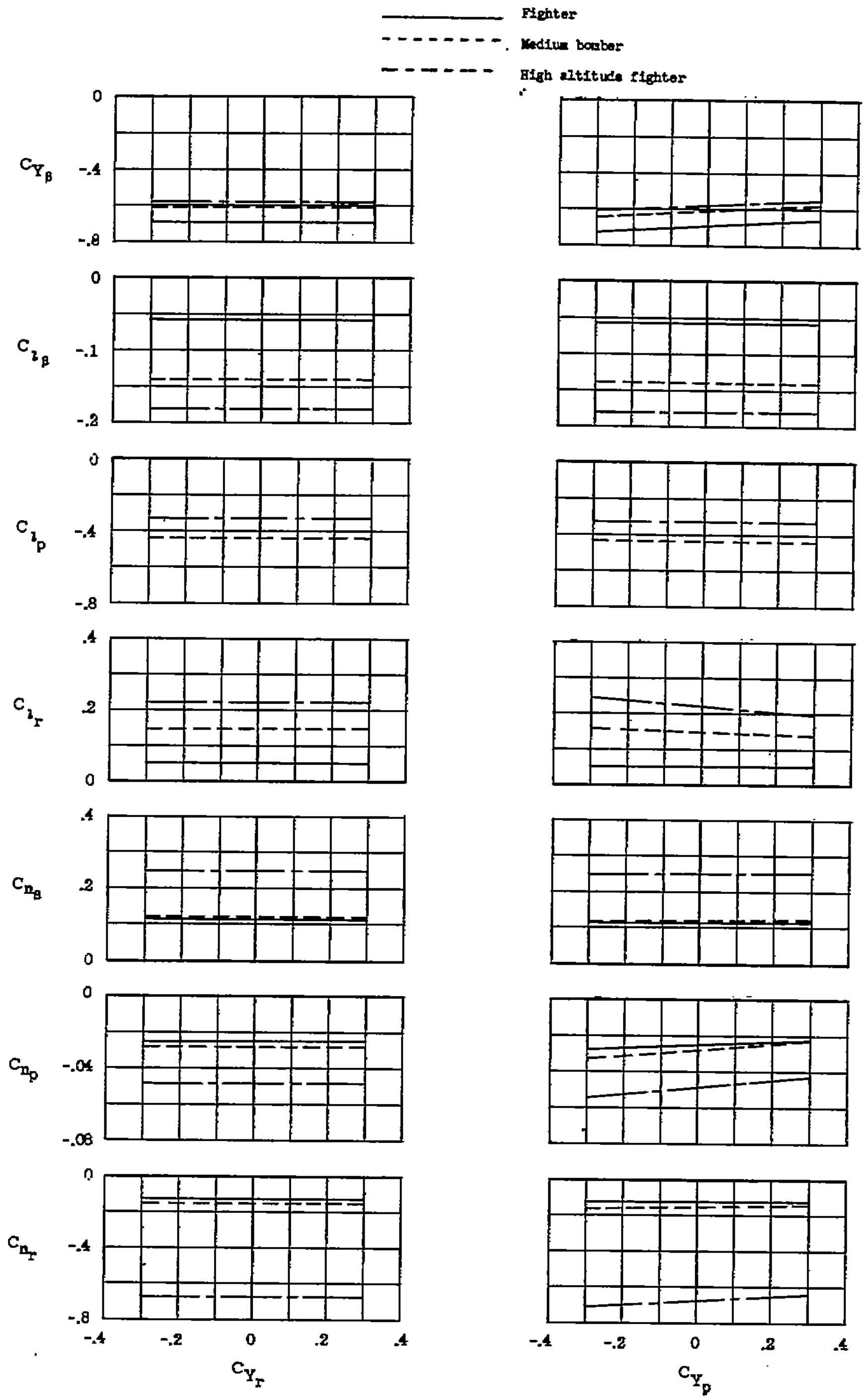

Figure 9.- Variation of the stability derivatives produced by the maximum and minimum values of C_{Y_r} and C_{Y_p}.

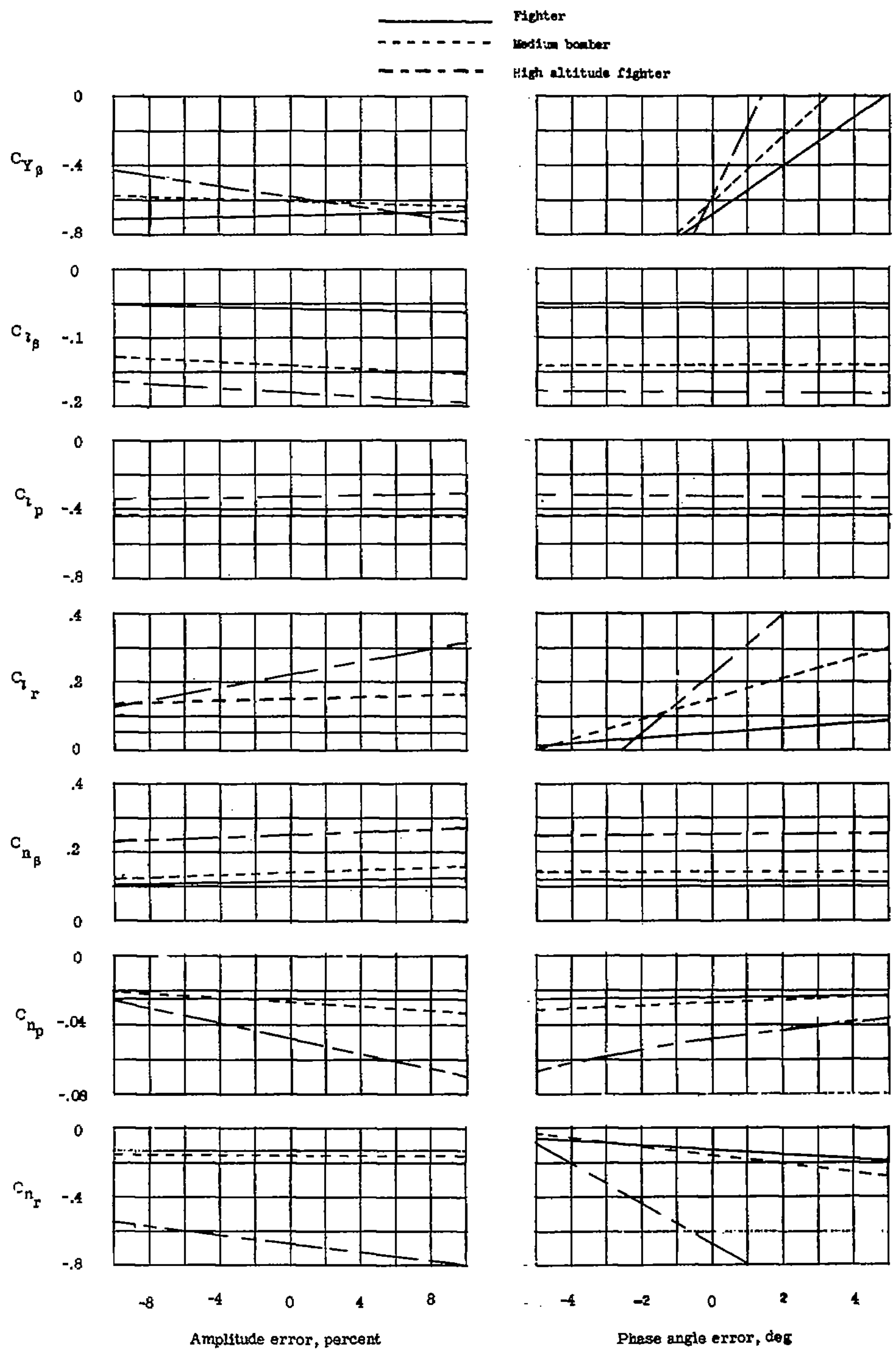

Figure 10.- Variation of the stability derivatives produced by errors in the amplitude and phase angle of β.