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BOUNDARY-LAYER STABILITY DIAGRAMS FOR ELECTRICALLY CONDUCTING FLUIDS IN THE PRESENCE OF A MAGNETIC FIELD

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SUMMARY

The effectiveness of a magnetic field in stabilizing the laminar flow of an incompressible, electrically conducting fluid is studied. The neutral stability curves pertaining to a two-dimensional sinusoidal disturbance are presented for flow over a semi-infinite flat plate in the presence of either a coplanar or transverse magnetic field and for channel flow in the presence of a coplanar magnetic field. As is to be expected, the magnetic field stabilizes the flow unless the velocity profile is distorted by the magnetic field to an inherently unstable shape. This occurs when a transverse magnetic field is fixed relative to a semi-infinite flat plate.

INTRODUCTION

Mere mention of the possibility of controlling the motion of electrically conducting fluids with a magnetic field stimulates one's imagination to conceive flow fields which may furnish certain ideal characteristics. All too often the configurations are too complicated to be amenable to analysis and one must be content with a greatly simplified version of the original idea. A survey of the literature shows that a number of basic solutions are being accumulated. A large portion of the effort is directed at the theoretical evaluation of the effectiveness of a magnetic field in stabilizing a given laminar flow so that transition to turbulent flow is inhibited. Some of the earliest work on problems of this type was carried out by S. Chandrasekhar. He found that a magnetic field would inhibit the onset of convection in a fluid heated from below (ref. 1), and would impede the transition to turbulence of fluid between rotating cylinders of nearly the same diameter (ref. 2). In a later paper, reference 3, it is found that a layer of fluid heated from below and subject to rotation is, under certain conditions, destabilized by application of a small magnetic field. The motion is stabilized by increasing the magnetic field strength beyond a certain amount.

The effect of a magnetic field on the stability of the flow of an incompressible electrically conducting fluid in a two-dimensional channel has been studied for a coplanar magnetic field by Stuart (ref. 4) and for a transverse magnetic field by Lock (ref. 5). The transverse magnetic field is found to be the more effective in stabilizing the flow field. The high degree of stabilization brought about is attributed (to the order of accuracy of the analysis) entirely to the change in the velocity profile caused by the interaction of the fluid and magnetic field. When the magnetic-field lines are parallel to the stream direction, the favorable effect on the stability of a disturbance is brought about by the electromotive resistance encountered when a fluid element leaves its normal path of motion in an effort to form a turbulent eddy, thereby crossing magnetic lines of force.

The effect of a coplanar magnetic field on the stability of a laminar mixing region was studied by Curle (ref. 6). The Reynolds numbers at which a small disturbance becomes unstable are generally quite small for this type flow field (generally less than 100) but increase rapidly with increasing magnetic parameter. Complete stabilization is predicted for a magnetic parameter over 0.301.

An experimental example of flow instability caused by a magnetic field is given by Lehnert in reference 7. It is found that a shallow layer of mercury over a copper disk with two concentric copper rings is destabilized by application of a vertical magnetic field. The rotation of the inner copper ring produces a shear layer in the mercury which is intensified by the magnetic field to the extent that an eddy-type flow results. It is pointed out by Lehnert that a generalization concerning the effect of a magnetic field on the flow field cannot then be made, and each situation must be studied to find out if the beginning of amplification of a disturbance is actually delayed to a higher Reynolds number by the magnetic field.

The flow of an incompressible electrically conducting fluid over a semi-infinite flat plate in the presence of a magnetic field perpendicular to the surface of the plate was studied in reference 8. The effect of the magnetic field on the stability of the flow has not as yet been studied for the case when the magnetic lines of force are perpendicular to or alined with the stream direction. It is the intent of this paper firstly to present an analysis of the stabilizing effect brought about by a coplanar magnetic field acting on an electrically conducting fluid flowing over a semi-infinite flat plate. The analysis is restricted to infinitesimal sinusoidal disturbances of the Tollmien-Schlichting type. In the course of the investigation it is necessary to evaluate a large portion of the numerical work for the corresponding two-dimensional channel problem. Since the method of analysis is slightly different from that of reference 4, these results are presented. Secondly, the effect of a transverse magnetic field is considered. As was found for the channel (Lock, ref. 5), the change in the critical Reynolds number for the flat plate is controlled primarily by the change in the velocity profile brought about by the interaction of the fluid and magnetic field. The velocity profile shapes which are considered are taken from the two simplest cases analyzed in reference 8. The first case assumes that the transverse magnetic field is fixed relative to the plate and the second that it is fixed relative to the fluid far from the plate.

The method of analysis which is used is patterned after the procedure developed and described by C. C. Lin in references 9, 10, and 11. A history of the development and of the various physical problems which have been studied is given in a monograph by Lin in reference 12. A brief outline of the method is given in the introduction to the present analysis. The neutral stability curves are presented for several values of the magnetic parameter.

SYMBOLS

a	$\sqrt{1-c}$
B	imposed magnetic induction
c	wave speed of disturbance
F(z)	Tietjen's function (see eq. (19))
m	magnetic parameter, $\frac{\sigma B^2}{\rho U_\infty}$, per unit length
p	pressure
$q_0, q_1, q_2, \ldots$	inviscid perturbation amplitude functions (see eq. (8))
R	Reynolds number based on boundary-layer thickness, $\frac{\delta U_\infty}{\nu}$
R_{x*}	Reynolds number based on distance from leading edge of flat plate, $\frac{x^* U_\infty}{\nu}$
u	x component of velocity
U	$\frac{\tilde{U}}{U_\infty}$
$\tilde{U}$	velocity in the stream direction of the flow field to be perturbed
v	y component of velocity
x,y	rectangular coordinates

z	$y_o(U_o'\alpha R)^{1/3}$
α	wave number of disturbance
δ	boundary-layer thickness, $6\sqrt{\frac{x^*\nu}{U_\infty}}$
ϵ	$\frac{1}{(\alpha R)^{1/3}}$
ψ	perturbation stream function
φ	amplitude function
ν	kinematic viscosity
η	$\frac{y-y_o}{\epsilon}$
ρ	density of fluid
σ	electrical conductivity
$\chi_{(0)}, \chi_{(1)}, \chi_{(2)}, \ldots$	viscous perturbation stream functions (see eq. (16))

Subscripts

∞	edge of boundary layer, or free stream
o	critical layer where $U = c$
d	disturbance

Superscripts

$\rightarrow$	vector
$'$	derivative with respect to y
$*$	dimensional quantities

ANALYSIS

The present state of stability theory requires that a number of simplifications be made in the analysis so that the method can be applied to physical situations without a prohibitive amount of labor. The method developed by C. C. Lin (refs. 9, 10, and 11) is a compromise between accuracy and effort required to analyze a given flow field. The present analysis is therefore patterned after it.

Resumé of Steps in Analysis

The desired result is a stability diagram of exciting wave number α and Reynolds number R. At the beginning, the undisturbed steady-state solution to the magnetohydrodynamic flow problem being considered is assumed to be known. This information together with the equations of motion, the continuity equation, Maxwell's equations, Ohm's law for a moving fluid, the electromotive force relation, the wave nature of the disturbance, plus various approximations go to make up a complex fourth-order ordinary differential equation for the amplitude function φ. The various steps will now be explained. Sketch (a) was designed to orient the reader in the subsequent analysis which, in view of its well established nature, is discussed only briefly.

The flow field is at some time assumed to be a steady two-dimensional stream of incompressible electrically conducting fluid. A two-dimensional infinitesimal sinusoidal disturbance of a given wave number α is then impressed on the fluid to test for the stability of the stream. A sinusoidal disturbance is chosen because many disturbances which are likely to occur in nature can be Fourier analyzed and thereby reduced to a sum of sinusoidal disturbances. The magnitude of the disturbance is assumed to be vanishingly small or infinitesimal so that the analysis may be simplified by retaining only those terms which are linear in a disturbance or perturbation quantity. The wave nature of the disturbance is introduced by the disturbance stream function

$$\psi = \delta U_\infty \varphi e^{i\alpha^*(x^*-c^*t-ic_i^*t)}$$

where α^* is the wave number, c^* is the velocity of the wave in the stream direction, and c_i^* is the rate of growth of the wave amplitude. The disturbance velocities are then given by $u^* = \partial\psi/\partial y^*$ and $v^* = -(\partial\psi/\partial x^*)$. The starred quantities have physical dimensions, whereas the unstarred counterparts have been made dimensionless by dividing by the free-stream velocity U_∞ or by the boundary-layer thickness δ as the case may be. It is assumed that the disturbance velocity and magnetic-field components are characterized by this exponential and depend on it to a first power. The object of the analysis is to find the conditions when the wave will just begin to grow[1] (i.e., will be neither damped nor amplified but neutral), the factor $c_i{}^*$ of the exponential is set equal to zero and the exponential reduces to $e^{i\alpha^*(x^*-c^*t)}$. This function describes the propagation of the wave in the stream direction for a given station x^* as a function of time. The velocity of propagation for a given disturbance is independent of the distance along and perpendicular to the plate. It remains to find the circumstances under which the wave amplitude neither grows nor diminishes but is neutral. It is found that the neutral point of wave growth occurs when the wave speed c^* is equal to the local velocity $\tilde{U}$ of the fluid. The region in the fluid where this happens is referred to as the "critical layer" and the distance from the wall as $y_o{}^*$.

In the actual flow problem one knows that the disturbance may be of either the two- or three-dimensional type. It has, however, been shown by Squire (ref. 14) that if the flow field is unstable to a three-dimensional disturbance it will be unstable to a two-dimensional disturbance at a lower Reynolds number. The extension of this proof to the type of magnetohydrodynamic problems being considered here is made by Michael in reference 15. Only two-dimensional disturbances will then be considered because they are the most unstable.

When the information just described is introduced into the equations relevant to the problem, a complex ordinary fourth-order differential equation is obtained for the amplitude function φ (sketch (a)). It is complex because imaginary quantities are introduced by the exponential used to describe the perturbations. The terms which contain products or squares of the disturbance quantities are discarded. It is also assumed that the station in question is far enough downstream so that the variables are not changing in the free-stream direction.

Even though a number of simplifying assumptions are made, the form of the differential equation is such that a simple solution has not yet been found. It is necessary then to find four linearly independent solutions by reducing the complete differential equation to two simpler differential equations by a power series expansion in $1/R$ and $\epsilon = (1/\alpha R)^{1/3}$ as indicated in sketch (a). The zero-order terms in $1/R$ are the only ones retained. The resulting differential equation is sometimes referred to as the inviscid form of the differential equation because all terms involving viscosity have been dropped. Proceeding down to the next step

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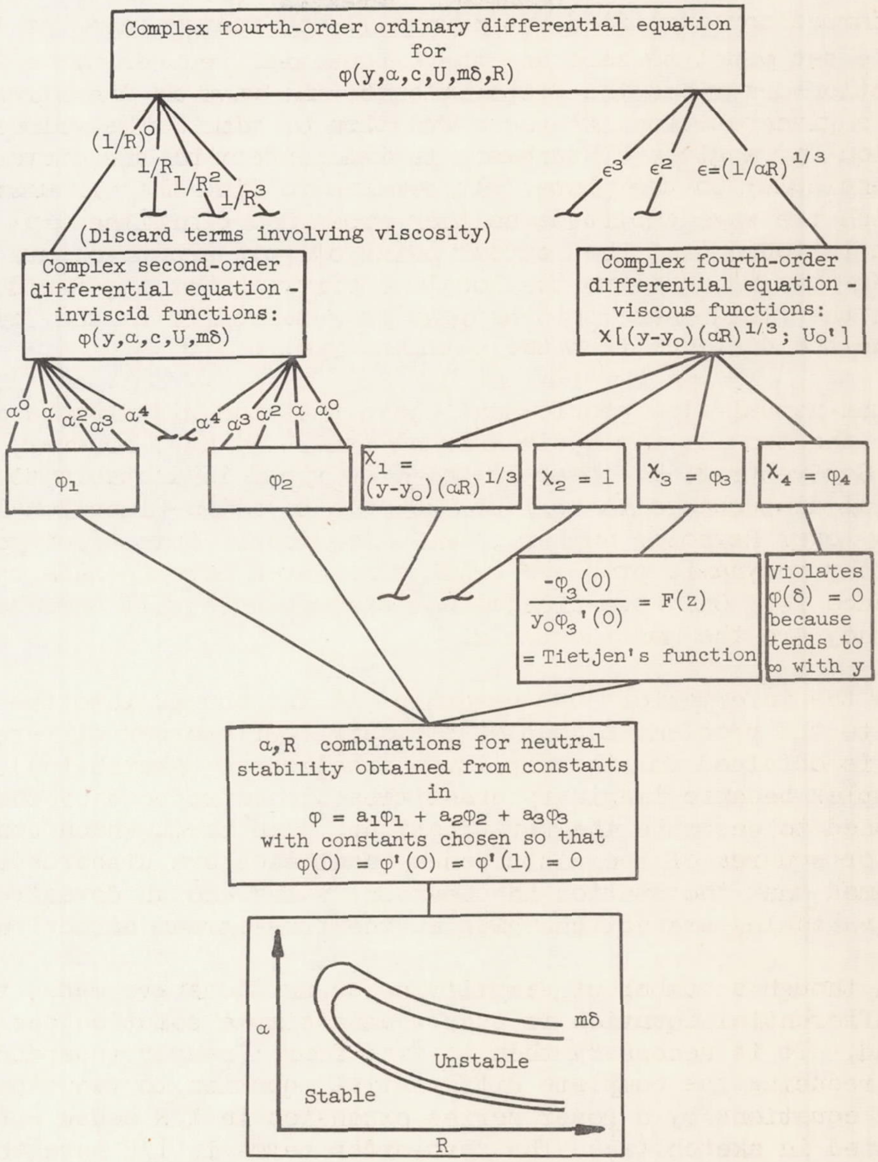
[1]The stability curves corresponding to a number of growth rates, $c_i > 0$, have been computed by S. F. Shen (ref. 13) for flat plate and channel flow using an extension of Lin's method.



Sketch (a)

in sketch (a), the first two linearly independent solutions φ_1 and φ_2 are found by introducing another series which consists of positive powers of the wave number α and whose coefficients depend on the wave speed c and velocity U in the flow field. Once again, only the first few terms in α are retained.

The boxes on the right of sketch (a) indicate that the solution to the first-order term in the set of differential equations which results from the expansion in $\epsilon = (1/\alpha R)^{1/3}$ is the only one which is found. As pointed out by Lin, higher order functions could be found by quadratures but in most cases sufficient accuracy is obtained by considering only the first-order term. This differential equation has four linearly independent solutions which can be used. Two of these solutions, χ_1 and χ_2, are discarded because they are too simple in form for curved velocity distributions. An examination of χ_4 or φ_4 shows that it increases without limit with y and thereby violates the boundary condition that disturbances must die out as y approaches infinity. For this reason it is not used in the problems treated in this paper. The function φ_3 is generally used in the form known as Tietjen's function (sketch (a)).

The three remaining linearly independent solutions are then combined in such a way that the boundary conditions are satisfied. The disturbance velocities will vanish at the wall and edge of the boundary layer when

$$\varphi = a_1\varphi_1 + a_2\varphi_2 + a_3\varphi_3$$

with the constants a_1, a_2, and a_3 chosen so that

$$\varphi(0) = \varphi'(0) = \varphi'(1) = 0$$

This is possible only for a certain combination of α and R when the magnetic parameter $m\delta$, velocity distribution $\tilde{U}$, and wave speed c have been specified. The end result from several such computations is a graph of the wave number α versus the Reynolds number R for various values of the magnetic parameter $m\delta$. Since these curves denote the values of α and R for neutral stability of the wave, a combination of α and R, which lies on the side of the curve denoted as unstable, warns that the amplitude of the disturbance will grow under those conditions. In the stable region the wave is damped.

The number of approximations which are made might cause one to doubt the accuracy of the end results. Estimates made by Lin in reference 11 indicate that the stability curves should not be in error by much more than a few percent and are therefore accurate enough for most engineering purposes.

The analysis of the problems being considered in this paper is presented in the following sections. Since the method is well defined in references 9, 10, and 11, only the essential parts of the analysis are presented.

Coplanar Magnetic Field

Differential equation.- The differential equation for the function φ will now be derived for the magnetic-field lines alined with the stream direction. The result is general enough that it can be applied to the flow in channels and over flat plates. Maxwell's equations for the incompressible-flow problems being considered are

$$\left.\begin{aligned} \text{Div}\ \vec{E} &= 0 \\ \text{Div}\ \vec{H} &= 0 \\ \text{Curl}\ \vec{H} &= 4\pi\vec{j} \\ \text{Curl}\ \vec{E} &= -\mu\,\frac{\partial\vec{H}}{\partial t} \end{aligned}\right\} \tag{1}$$

where $\vec{E}$, $\vec{H}$, $\vec{j}$, and μ are the electric field intensity, magnetic intensity, electric current density, and magnetic permeability, respectively. Ohm's law for a moving fluid is

$$\vec{j} = \sigma(\vec{E} + \vec{U} \times \vec{B}) \tag{2}$$

where $\vec{B} = \mu\vec{H}$ and $\vec{U}$ is the local velocity vector. The equation of continuity is

$$\text{Div}\ \vec{U} = 0 \tag{3}$$

The Navier-Stokes equation modified to include the electromotive force term (so-called Lorentz force) arising from the relative motion between the fluid and magnetic field is

$$\frac{\partial\vec{U}}{\partial t} + (\vec{U} \cdot \text{grad})\vec{U} - \frac{1}{\rho}(\vec{j} \times \vec{B}) + \frac{1}{\rho}\,\text{grad}\ p = \nu\nabla^2\vec{U} \tag{4}$$

where the excess charge density and applied electric field are assumed to be zero.

The relation between the input wave number, α, of the disturbance and the Reynolds number, R, of the flow at which the amplitude of the disturbance neither increases nor decreases (neutral) will be found by introducing the quantities,

$$\left.\begin{aligned} u^* &= \tilde{U} + u_d^*(y^*)e^{i\alpha^*(x^*-c^*t)} \\ v^* &= v_d^*(y^*)e^{i\alpha^*(x^*-c^*t)} \\ B_x &= B + b(y^*)e^{i\alpha^*(x^*-c^*t)} \\ B_y &= \beta(y^*)e^{i\alpha^*(x^*-c^*t)} \\ p^* &= \tilde{p}^* + p_d^*(y^*)e^{i\alpha^*(x^*-c^*t)} \end{aligned}\right\} \qquad (5)$$

where $c^* = c_r^* + ic_i^*$. When the disturbance is classified as neutral (neither amplified nor damped) c_i^* is zero. Since the problem will be to find only the neutral disturbance curves, the quantity c^* will hereafter be used to denote only the real part c_r^*, that is, the wave speed of the disturbance. The quantity α^* is the wave number of the disturbance.

It will be assumed that:

1. The location of the instability is far enough downstream of the entrance to the channel or leading edge of the plate that the velocity normal to the boundary is negligible in comparison with the velocity parallel to the boundary.

2. The fluid is of uniform density and conductivity, and the applied magnetic field, B, is uniform throughout the flow field.

3. The boundaries are perfect conductors in order to complete the circuit for electric currents in the fluid.

4. Terms which contain products or squares of the disturbance quantities are negligible.

5. The disturbances are neutrally stable at values of the Reynolds number high enough so that a series in $(1/\alpha R)^{1/3}$ converges rapidly.

Following the method used by Stuart in reference 4, equations (1) through (5) may be combined and simplified using the foregoing assumptions to yield a complex ordinary differential equation for the dimensionless amplitude function φ.

$$(U-c)(\varphi''-\alpha^2\varphi) - \varphi U'' + im\delta\alpha\varphi = \frac{1}{i\alpha R}(\varphi'''' - 2\alpha^2\varphi'' + \alpha^4\varphi) \qquad (6)$$

where $m = \sigma B^2/\rho U_\infty$, $R = \delta U_\infty/\nu$, and U denotes the local velocity divided by the velocity at the edge of the boundary layer, U_∞. The symbols α and c in equation (6) denote the dimensionless form of the wave number α^* and wave speed c^*, respectively. The amplitude function φ is a function of $y = y^*/\delta$. The primes denote differentiation with respect to the distance y normal to the nearest bounding surface. Hereafter, only the dimensionless unstarred quantities will be used in the analysis unless it is noted otherwise.

The boundary conditions are,

$$\varphi = \varphi' = 0 \qquad \text{at } y = 0$$

$$\varphi = \varphi' = 0 \qquad \begin{cases} \text{at center of channel, } y = 1 \\ \text{or} \\ \text{at } y = \infty \text{ for the flat plate} \end{cases}$$

Four linearly independent solutions to equation (6) will now be found by the technique explained in references 9, 10, and 11. The first two solutions, φ_1 and φ_2, will be derived from a series expansion in $1/R$ and are designated as the inviscid solutions. The two remaining solutions φ_3 and φ_4 result from a series expansion in $\epsilon = (1/\alpha R)^{1/3}$ and are called the viscous solutions.

Inviscid solutions.- If the terms involving $1/\alpha R$ in equation (6) are assumed small, the remaining terms constitute the differential equation which φ_1 and φ_2 must satisfy.

$$(U-c)(\varphi''-\alpha^2\varphi) - \varphi U'' + im\delta\alpha\varphi = 0 \tag{7}$$

A solution to equation (7) is found by the method of Heisenberg (see, e.g., ref. 12). It is assumed that the solution is of the form

$$\varphi = q_o + \alpha q_1 + \alpha^2 q_2 + \alpha^3 q_3 + \ldots \tag{8}$$

When equation (8) is inserted into equation (7) and the terms containing the same power of α are equated, the following set of linear ordinary differential equations is found.

$$q_o'' - \frac{U''}{U-c} q_o = 0 \tag{9a}$$

$$q_1'' - \frac{U''}{U-c} q_1 = - \frac{im\delta}{U-c} q_o \tag{9b}$$

$$q_n'' - \frac{U''}{U-c} q_n = q_{n-2} - \frac{im\delta}{U-c} q_{n-1}, \qquad n = 2, 3, \ldots \tag{9n}$$

The two linearly independent solutions of equation (9a) are

$$q_{o_1} = U - c \tag{10a}$$

$$q_{o_2} = (U-c)\int_0^y \frac{dy}{(U-c)^2} \tag{10b}$$

The two inviscid solutions for the function φ may then be written as

$$\begin{aligned}\varphi_1(y) = (U-c)\Bigg[1 + \alpha^2\int_0^y \frac{1}{(U-c)^2}\int_0^{y_1}(U-c)^2 dy_2 dy_1 + \\ \alpha^4\int_0^y \frac{1}{(U-c)^2}\int_0^{y_1}(U-c)^2\int_0^{y_2}\frac{1}{(U-c)^2}\int_0^{y_3}(U-c)^2 dy_4 dy_3 dy_2 dy_1 + \ldots\Bigg] - \\ (U-c)\Bigg\{im\delta\alpha\Bigg[\int_0^y \frac{1}{(U-c)^2}\int_0^{y_1}(U-c) dy_2 dy_1 + \\ \alpha^2\int_0^y \frac{1}{(U-c)^2}\int_0^{y_1}(U-c)^2\int_0^{y_2}\frac{1}{(U-c)^2}\int_0^{y_3}(U-c) dy_4 dy_3 dy_2 dy_1 + \ldots\Bigg] + \ldots\Bigg\} + \ldots\end{aligned} \tag{11}$$

and

$$\begin{aligned}\varphi_2(y) = (U-c)\Bigg\{\int_0^y \frac{dy}{(U-c)^2} + \alpha^2\int_0^y \frac{1}{(U-c)^2}\int_0^{y_1}(U-c)^2\int_0^{y_2}\frac{1}{(U-c)^2} dy_3 dy_2 dy_1 + \ldots - \\ im\delta\alpha\Bigg[\int_0^y \frac{1}{(U-c)^2}\int_0^{y_1}(U-c)\int_0^{y_2}\frac{1}{(U-c)^2} dy_3 dy_2 dy_1 + \ldots\Bigg] + \ldots\Bigg\} + \ldots\end{aligned} \tag{12}$$

where only linear terms in $m\delta$ have been retained.

The integrals in equations (11) and (12) may be changed to a more convenient form by the transformation employed by Lin in reference 11. At the wall, $y = 0$, and at the edge of the boundary layer, $y = 1$, the inviscid functions and their derivatives then become

$$\left.\begin{aligned}\varphi_1(0) &= -c \\ \varphi_1'(0) &= U'(0) \\ \varphi_2(0) &= 0 \\ \varphi_2'(0) &= -\frac{1}{c}\end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned}
\varphi_1(1) &= \frac{1-c}{1-\alpha^2 H_2} - im\delta\alpha(1-c)p_2 + \ldots \\
\varphi_1'(1) &= \frac{1}{1-c}\left[\frac{\alpha^2 H_1 - \alpha^4 M_3 + \ldots}{1-\alpha^2 H_2} - im\delta\alpha(p_1 + \alpha^2 p_3 + \ldots)\right] \\
\varphi_2(1) &= (1-c)\left[\frac{K_1}{1-\alpha^2 H_2} - \alpha^2 N_3 + im\delta\alpha(q_3 - K_1 p_2 - \ldots)\right] \\
\varphi_2'(1) &= \frac{1}{1-c}\left\{\frac{\alpha^2 K_1(H_1 - \alpha^2 M_3 + \ldots)}{1-\alpha^2 H_2} + 1 - \alpha^2 H_2 - \ldots - \right. \\
&\qquad \left. im\delta\alpha\left[K_1(p_1 + \alpha^2 p_3 + \ldots) - p_2 - \ldots\right]\right\}
\end{aligned}\right\} \tag{14}$$

where

$$p_1 = \int_0^1 (U-c)\,dy \tag{15a}$$

$$p_2 = \int_0^1 \frac{1}{(U-c)^2}\int_0^{y_1}(U-c)\,dy_2\,dy_1 \tag{15b}$$

$$p_3 = \int_0^1 (U-c)^2\int_0^{y_1}\frac{1}{(U-c)^2}\int_0^{y_2}(U-c)\,dy_3\,dy_2\,dy_1 \tag{15c}$$

$$q_3 = \int_0^1 \frac{1}{(U-c)^2}\int_0^{y_1}(U-c)\int_{y_2}^1\frac{1}{(U-c)^2}\,dy_3\,dy_2\,dy_1 \tag{15d}$$

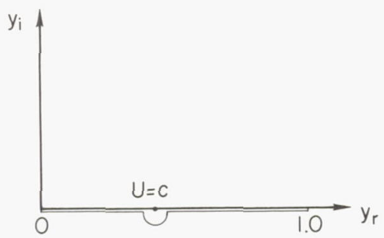
$$H_1 = \int_0^1 (U-c)^2\,dy \tag{15e}$$

$$H_2 = \int_0^1 \frac{1}{(U-c)^2}\int_0^{y_1}(U-c)^2\,dy_2\,dy_1 \tag{15f}$$

$$M_3 = \int_0^1 (U-c)^2\int_{y_1}^1\frac{1}{(U-c)^2}\int_0^{y_2}(U-c)^2\,dy_3\,dy_2\,dy_1 \tag{15g}$$

$$N_3 = \int_0^1 \frac{1}{(U-c)^2} \int_0^{y_1} (U-c)^2 \int_{y_2}^1 \frac{1}{(U-c)^2} \, dy_3 dy_2 dy_1 \tag{15h}$$

$$K_1 = \int_0^1 \frac{1}{(U-c)^2} \, dy \tag{15i}$$



Sketch (b)

The path of integration, according to reference 10, lies along the real y axis with an indentation along a semicircular path under the singular point, $y = y_o$ (i.e., where $U = c$) as shown in sketch (b).

Viscous solutions.- The two remaining independent solutions, the so-called viscous solutions φ_3 and φ_4, are found by introducing the small parameter $\epsilon = (1/\alpha R)^{1/3}$ and the function χ as

$$\left.\begin{aligned} \epsilon\eta &= y - y_o \\ \varphi(y) &= \chi_{(o)}(\eta) + \epsilon\chi_{(1)}(\eta) + \epsilon^2\chi_{(2)}(\eta) + \ldots \\ &\text{and} \\ U - c &= U_o'(\epsilon\eta) + \frac{U_o''}{2!}(\epsilon\eta)^2 + \ldots \end{aligned}\right\} \tag{16}$$

The subscript o indicates that the quantity is to be evaluated at the point where $U = c$. If the equations (16) are introduced into equation (6) and the terms containing the same power of ϵ are equated, the following set of ordinary differential equations results.

$$\eta U_o' \chi_{(o)}'' + i\chi_{(o)}'''' = 0 \tag{17a}$$

$$\eta U_o' \chi_{(1)}'' + i\chi_{(1)}'''' = U_o'' \chi_{(o)} - \frac{\eta^2}{2} U_o'' \chi_{(o)}'' + im\delta\chi_{(o)}'' \tag{17b}$$

etc.

The solutions to equation (17a) are the only ones in this series which are found. As pointed out in the introduction to the analysis two of these four linearly independent solutions are discarded on the grounds that they are trivial. It is also found that the function χ_4

or φ_4 increases with y indefinitely and thereby violates the boundary condition that the disturbance velocities must die out at the edge of the boundary layer. The form of the solution required is then

$$\frac{\varphi_3(0)}{\varphi_3'(0)} \equiv -y_o F(z) \tag{18}$$

where

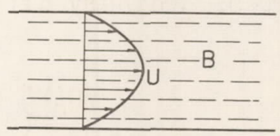
$$z = y_o(U_o'\alpha R)^{1/3}$$

$$F(z) = \frac{\int_{\infty}^{-z} d\zeta \int_{\infty}^{\zeta} \zeta^{1/2} H_{1/3}\left[\frac{2}{3}(i\zeta)^{3/2}\right] d\zeta}{-z\int_{\infty}^{-z} \zeta^{1/2} H_{1/3}\left[\frac{2}{3}(i\zeta)^{3/2}\right] d\zeta} \tag{19}$$

and $H_{1/3}(\ )$ is a Hankel function of the first kind and of order 1/3. The function $F(z)$ is sometimes referred to as the Tietjen's function. The tabulated values of references 9 and 16 are plotted in figure 1. The viscous solution is not modified by the presence of the magnetic field to the order of accuracy of the analysis.

The inviscid solutions, equations (13) and (14), together with equation (18) make it possible to find the change in the neutral disturbance curve caused by a coplanar magnetic field.

Channel flow (parabolic velocity profile).- The flow of a viscous fluid between parallel planes (Poiseuille flow - sketch (c)) gives rise to a parabolic velocity profile if the station in question is not near the entrance to the channel. The effect of a coplanar magnetic field on the growth of a two-dimensional disturbance has already been studied by Stuart in reference 4. The difference between the analysis carried out here and in reference 4 lies in the larger number of terms retained here for the inviscid solution φ_1 and in the form of equation (22) which is used to find the proper α-R combination. The end results of the two analyses should, however, be about the same. Since the integrals (15) must be evaluated for a parabolic velocity profile in order to make application to the flat-plate flow field, only a small amount of additional effort is required to find the neutral disturbance curves for the channel.



Sketch (c)

The velocity distribution is written as

$$U = 2y - y^2 \tag{20}$$

The integrals (15) can be evaluated in closed form for arbitrary values of the wave speed c.

$$p_1 = a^2 - \frac{1}{3} \tag{21a}$$

$$p_2 = \frac{1}{6}\left(\frac{1}{a^2} + \ln\frac{1-a^2}{a^2} + \frac{3a^2-1}{2a^3}\ln\frac{1+a}{1-a}\right) + \frac{\pi i}{12a^3}(3a^2-2a^3-1) \tag{21b}$$

$$\begin{aligned} p_3 = {} & \frac{2\pi i a^2}{45}(3a^2-2a^3-1) + \frac{3a^2-1}{18}\left(\frac{8a^2}{5}\ln a + \frac{8a^4-6a^2+3}{4a^2} - \frac{211a^2}{300}\right) + \\ & \frac{2a^3+3a^2-1}{60a^3}(1+a)^3\left[\frac{211a^2}{180} + \frac{1-17a/4}{5} - \left(1-3a+\frac{8a^2}{3}\right)\ln(1+a)\right] + \\ & \frac{1+2a^3-3a^2}{60a^3}(1-a)^3\left[\frac{211a^2}{180} + \frac{1+17a/4}{5} - \left(1+3a+\frac{8a^2}{3}\right)\ln(1-a)\right] \end{aligned} \tag{21c}$$

$$\begin{aligned} q_3 = {} & \frac{1}{72a^5}\ln\left(\frac{1+a}{1-a}\right)\left[\frac{3(1+a)^2}{2a}(2a-1)\ln(1+a) + \frac{3(1-a)^2}{2a}(2a+1)\ln(1-a) + 3(a^2+2)\right] + \\ & \frac{1}{12a^3}\left\{2L_2(1) - L_2\left(\frac{a-1}{2a}\right) - L_2\left(\frac{2a}{1+a}\right) - \frac{1}{a} - \frac{(\ln 2a)^2}{2} - \frac{[\ln(1+a)]^2}{2} + (\ln 2a)[\ln(1-a)]\right\} + \\ & \frac{i\pi}{16a^6}\left[\frac{4a^3}{3}\ln(1-a^2) - \frac{8a^3}{3}\ln 2a + \frac{2a(2-5a^2)}{3} + 2\left(a^2-\frac{1}{3}\right)\ln\left(\frac{1+a}{1-a}\right) + \frac{i\pi}{3}(3a^2-2a^3-1)\right] \end{aligned} \tag{21d}$$

where $a^2 = 1-c$, and $L_2(\ )$ is the dilogarithmic integral. Numerical values for the relations (21) for several values of the parameter c are presented in table I. The functions L_2 are tabulated in references 17 and 18. The remaining integrals in the group (15) are written and tabulated in reference 11.

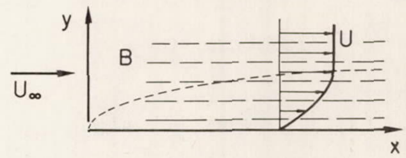
It remains now to combine the inviscid and viscous solutions so that the boundary conditions at the wall and at the edge of the boundary layer are satisfied. It is found that the wave number of an antisymmetric disturbance and the Reynolds number of the flow field must be chosen so that

$$F(z) = -\frac{\varphi_3(0)}{y_o\varphi_3'(0)} = \frac{c\varphi_2'(1)}{y_o\left[U'(0)\varphi_2'(1) + \frac{1}{c}\varphi_1'(1)\right]} \tag{22}$$

With all of the individual functions known, an iteration scheme is employed to find the correct wave number and Reynolds number combination. A graphical method was used to find the intersection of the curves of the functions on the left and right sides of equation (22) for several values of the parameter c, whereas a numerical iteration scheme was used in reference 11.

The neutral disturbance curves for several values of the magnetic parameter $m\delta$ are shown in figure 2(a). Since the parameter $m\delta\alpha$ was held constant in the analysis of reference 4, a direct comparison with the neutral stability curves of that paper cannot be made. The critical Reynolds numbers found by the two analyses will be compared in the discussion.

Flat-plate velocity profile.- When an incompressible viscous fluid flows past a semi-infinite flat plate of zero thickness, the velocity profile can be predicted theoretically and is generally referred to as the Blasius profile (sketch (d)). The neutral stability curve in the nonmagnetic case has been computed in references 9, 10, and 11. The effect of the magnetic field on these results will now be found.



Sketch (d)

The integrals (15a) through (15d), evaluated by the approximate method suggested in reference 11, are tabulated in table II for specific values of c. The real and imaginary parts of K_1 are computed by the relations given in reference 11 as

$$K_{1_r} = -\frac{1}{cU'(0)} + 0.1465 + 1.2467c + 1.045c^2 + 2.039c^3 + 4.078c^4 + 2.423c^5 + \ldots + \frac{9}{8}\left(c^2 + \frac{21}{8}c^5 + \ldots\right)\left(\ln\frac{0.8-c}{c} + i\pi\right) \tag{23}$$

$$K_{1_i} = -\pi\frac{U_o''}{(U_o')^3} \tag{24}$$

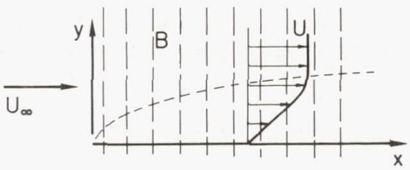
The expression which determines the proper values of wave number α and the Reynolds number R for the flat-plate problem is

$$F(z) = \frac{c[\varphi_2'(1) + \alpha\varphi_2(1)]}{y_o\left\{U'(0)[\varphi_2'(1) + \alpha\varphi_2(1)] + \frac{1}{c}[\varphi_1'(1) + \alpha\varphi_1(1)]\right\}} \tag{25}$$

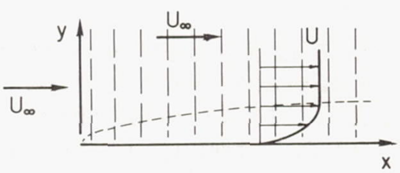
The neutral stability curves for several values of the magnetic parameter $m\delta$ are shown in figure 2(b).

Transverse Magnetic Field

The change in the boundary-layer velocity profile for flow over a flat plate in the presence of a transverse magnetic field was found in reference 8. It was found that the skin friction and heat transfer are reduced if the magnetic field is fixed relative to the plate (sketch (e)) and increased if it is fixed relative to the fluid outside of the boundary layer (sketch (f)). The possibility exists, however, that the magnetically induced velocity profile may be more or less stable to transition to turbulent flow. An estimate of the change in the stability of an infinitesimal sinusoidal disturbance induced by the transverse magnetic field will now be found.



Sketch (e)



Sketch (f)

The differential equation for the disturbance stream function is found by the technique used by Lock in reference 5 which is to combine equations (1) through (5) and then simplify the result by applying the five assumptions outlined in the analysis of the coplanar field. The differential equation for the perturbation stream function is then

$$(U-c)(\varphi''-\alpha^2\varphi) - U''\varphi = \frac{im\delta}{\alpha}\varphi'' + \frac{1}{i\alpha R}(\varphi''''-2\alpha^2\varphi''+\alpha^4\varphi) \tag{26}$$

It is shown by Lock in reference 5 that the forms of the inviscid and viscous solutions are not affected to the order of the analysis by the additional magnetic term in equation (26). In other words, the change in the velocity profile caused by the transverse magnetic field dominates the stabilizing action of the magnetic field. The neutral stability curves for several values of the magnetic parameter mx are found by the method outlined in the appendix of reference 11. The inviscid solutions are found by using the numerical data in tables I and II of reference 8 to determine the velocity profiles at $mx = 0.05$ and 0.10. The numerical results for the integrals (15e) through (15h) are tabulated in tables III and IV.

The real part of the integral (15i) is evaluated by expanding in a series about the critical point $y = y_o$ where $U = c$. The result which was used in the computations for the transverse magnetic field is

$$K_{1_r} = -\frac{0.4}{y_o(0.4-y_o)(U_o')^2} - \frac{U_o''}{(U_o')^3}\ln\left|\frac{0.4-y_o}{y_o}\right| + 0.3\frac{(U_o'')^2}{(U_o')^4} - \frac{(U_o'')^3}{(U_o')^5}(0.04-0.2y_o) +$$

$$\frac{5}{16}\frac{(U_o'')^4}{(U_o')^6}\left(\frac{0.064}{3} - 0.16y_o + 0.4y_o^2\right) + \dots +$$

$$\frac{1}{4(1-c)}\left(\frac{1}{0.75-c} - \frac{1}{\sqrt{1-c}}\ln\left|\frac{1-2\sqrt{1-c}}{1+2\sqrt{1-c}}\right|\right) + \frac{0.1}{(1-c)^2} \tag{27}$$

The imaginary part K_{1_i} is evaluated by use of equation (24). The velocity U in the integrals (15e) through (15h), (24), and (27) is referred to the velocity at the edge of the boundary layer at the particular station being considered. When the magnetic field is fixed relative to the plate the undisturbed stream velocity and the velocity at the edge of the boundary layer are not the same.

The neutral disturbance curves are shown in figures 3(a) and 3(b).

DISCUSSION

The neutral stability curves shown in figures 2 and 3 indicate that the presence of a magnetic field may stabilize or destabilize the flow of an incompressible, electrically conducting fluid. It is seen from these results that the flow over a flat plate is stabilized by either a coplanar magnetic field or by a transverse magnetic field fixed relative to the fluid, but a transverse magnetic field fixed relative to the plate is generally destabilizing. The portion near the top of the $mx = 0.1$ curve in figure 3(a) indicates an opposite trend for a small range in wave number. As pointed out in the introduction, another example of flow instability caused by a magnetic field is presented by Lehnert in reference 7.

A given flow field will probably contain disturbances covering a wide range of wave number due to imperfections in the walls and entrance to the flow field. A conservative value for the critical Reynolds number is then the lowest value at which it is first possible for any of the waves to be amplified. The critical Reynolds numbers for the flow problems considered in references 4 and 5 and for the coplanar magnetic-field cases studied in this paper are shown in figure 4 as a function of the magnetic parameter $m\delta$. The results for the transverse magnetic field as a function of mx are also shown in figure 4. It is seen that the results of Stuart in reference 4 are in essential agreement with the present analysis. The difference between the results is attributable to the smaller number of terms retained in the analysis of reference 4 for the inviscid solution. The results for a laminar mixing region obtained by Curle in reference 6 are not shown in figure 4 because the Reynolds numbers are too small for the scale of the graph.

It is quite evident from figure 4 that a magnetic field is more effective when applied to channel flow than to flat-plate flow. In particular, the transverse magnetic field is so effective in stabilizing the flow in a channel that the curve is a vertical line to the scale of the graph.

When the magnetic field is coplanar, the large difference in the shape of the critical Reynolds number curves for the channel and flat-plate flow fields is attributable to the infinite extent of the flow field above the flat plate. As is shown by Lin in reference 9, the asymptotic form of the disturbance stream function as the distance y^* approaches and exceeds the boundary-layer thickness, δ, introduces additional terms in the equation determining the neutral stability curves. This is obvious when equations (22) and (25) are compared. These additional terms de-emphasize the terms involving the magnetic parameter and result in a much smaller stabilizing effect for the flat plate than for the channel flow.

The magnetic parameter and the Reynolds number for the flow over a flat plate at which an infinitesimal disturbance will grow (figs. 2(b) and 3) are based on the boundary-layer thickness δ taken[2] as $6/\sqrt{U_\infty/\nu x^*}$, where $\tilde{U}/U_\infty \approx 0.999$. The distance along the plate from the leading edge is then related to the boundary-layer thickness by the relationship

$$\delta = \frac{6x^*}{\sqrt{R_{x^*}}}$$

where, $R_{x^*} = U_\infty x^*/\nu$. Therefore,

$$m\delta = \frac{6mx^*}{\sqrt{R_{x^*}}}$$

and

$$R = 6\sqrt{R_{x^*}}$$

[2]Standard texts on boundary-layer theory usually define the thickness as $\delta = 5/\sqrt{U_\infty/\nu x^*}$, where $\tilde{U}/U_\infty \approx 0.99$. As explained in reference 11, more accuracy is achieved by defining a thicker boundary layer to a evaluate the inviscid integrals.

It was found in reference 8 that a magnetic field perpendicular to a flat plate changes the velocity profile in the boundary layer. Even a small magnetic field fixed relative to the plate will cause an inflection point[3] in the velocity profile near the surface. As is shown in figures 3(a) and 4, this causes the flow to be less stable with a magnetic field than in the nonmagnetic field case. The results in figures 3(b) and 4 indicate that a magnetic field fixed relative to the fluid far from the plate changes the velocity profile to a shape which is more stable. The results of reference 8 indicate that the skin friction and heat transfer are reduced in the former and increased in the latter case. Care must then be exercised if one attempts to reduce either the skin friction or heat transfer by imposing a magnetic field across (perpendicular to) the flow field and not in relative motion with the plate, because the laminar flow is destabilized by this technique. Likewise, the increase in the skin friction and heat transfer brought about by a transverse magnetic field sweeping past the plate at the velocity of the free stream would eventually experience a moderate compensating effect in the form of increased stability of the laminar stream.

The results of this paper, in conjunction with that of reference 8, point out the fact that it is not certain whether the skin friction and heat transfer are lowered or raised by using a transverse magnetic field to alter the flow over a flat plate. The magnetic field alters the velocity profile and changes the rate of growth of small disturbances so that the two effects tend to compensate each other. Individual situations must then be considered separately to determine whether an advantage can be achieved.

CONCLUSIONS

The analysis carried out in this report for the flow over a flat plate indicates the effect of a magnetic field on the stability of a disturbance of the Tollmien-Schlichting type. In particular it is found that:

1. The flow is stabilized by a coplanar magnetic field. The increase in the critical Reynolds number is small compared with the increase achieved in a channel with a coplanar or transverse magnetic field.

2. A transverse magnetic field fixed relative to the flat plate changes the velocity profile to an inherently unstable shape which lowers the critical Reynolds number.

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[3]It is noted in figure 3(a) that the maximum value of the wave number first increases and then decreases with increasing mx. This is caused by the rapid change in the curvature of the velocity profile with mx.

3. A transverse magnetic field fixed relative to the fluid far from the plate changes the velocity profile in the boundary layer to a shape which is more stable and thereby raises the critical Reynolds number.

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National Advisory Committee for Aeronautics
Moffett Field, Calif., May 1, 1958

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TABLE I.- COEFFICIENTS FOR INVISCID SOLUTIONS; PARABOLIC VELOCITY PROFILE (COPLANAR MAGNETIC FIELD)

c	p_1	p_2	p_3	q_3
0	0.6667	0.3977	0.09281	∞
.05	.6167	.4101	.0858	0.6765
.1	.5667	.4224	.07900	.6529
.15	.5167	.4351	.07247	.6675
.2	.4667	.4480	.06617	.7035
.25	.4167	.4615	.06010	.7587
.3	.3667	.4756	.05424	.8355
.35	.3167	.4905	.04855	.9401
.4	.2667	.5062	.04302	1.0828
.45	.2167	.5230	.03762	1.2818

TABLE II.- COEFFICIENTS FOR INVISCID SOLUTIONS; BLASIUS PROFILE (COPLANAR MAGNETIC FIELD)

c	p_1	p_2	p_3	q_3
0	0.7133	0.3817	0.1272	∞
.05	.6633	.3941	.1202	.5603
.1	.6133	.4064	.1134	.5367
.15	.5633	.4191	.1068	.5512
.2	.5133	.4321	.10053	.5872
.25	.4633	.4455	.09546	.6424
.3	.4133	.4597	.08860	.7192
.35	.3633	.4745	.08261	.8238
.4	.3133	.4902	.07739	.9665
.45	.2633	.5070	.07198	1.1656

TABLE III.- COEFFICIENTS FOR INVISCID SOLUTIONS; TRANSVERSE MAGNETIC FIELD FIXED RELATIVE TO PLATE

mx	c	H_1	H_2	M_3	N_3	K_{1_r}	K_{1_i}	U_o'	y_o
0.05	0	0.6022	0.2337	0.07907	0.1700	$-\infty$	0	1.8273	0
	.05	.5372	.2225	.06916	.1806	-10.87	-.0310	1.8284	.0274
	.1	.4772	.2107	.06052	.1929	-5.327	-.0338	1.8302	.0547
	.15	.4221	.1983	.05310	.2044	-3.414	-.0102	1.8312	.0820
	.2	.3721	.1854	.04682	.2158	-2.394	.0397	1.8303	.1093
	.25	.3271	.1718	.04162	.2266	-1.721	.1159	1.8263	.1366
	.3	.2870	.1576	.03741	.2385	-1.209	.2199	1.8172	.1641
	.35	.2520	.1428	.03409	.2466	-.7729	.3553	1.8009	.1917
	.4	.2220	.1274	.03151	.2472	-.2995	.5209	1.7844	.2197
	.45	.1969	.1112	.02956	.2389	.2253	.7250	1.7509	.2477
.10	0	.5733	.2223	.06975	.1732	$-\infty$	0	1.6473	0
	.05	.5089	.2111	.05984	.1838	-12.28	-.0914	1.6498	.0303
	.1	.4495	.1993	.05120	.1962	-6.156	-.1394	1.6556	.0606
	.15	.3951	.1869	.04378	.2076	-4.051	-.1456	1.6625	.0907
	.2	.3456	.1740	.03750	.2190	-2.939	-.1132	1.6686	.1207
	.25	.3012	.1604	.03230	.2298	-2.228	-.0447	1.6724	.1507
	.3	.2618	.1462	.02809	.2417	-1.717	.0580	1.6719	.1806
	.35	.2274	.1314	.02477	.2498	-1.323	.1950	1.6657	.2105
	.4	.1980	.1160	.02219	.2504	-1.0126	.3726	1.6520	.2408
	.45	.1736	.0998	.02024	.2421	-.7831	.5959	1.6310	.2718

TABLE IV.- COEFFICIENTS FOR INVISCID SOLUTIONS; TRANSVERSE MAGNETIC FIELD FIXED RELATIVE TO FLUID

mx	c	H_1	H_2	M_3	N_3	K_{1_r}	K_{1_i}	U_o'	y_o
0.05	0	0.6458	0.2536	0.09507	0.1704	$-\infty$	0.4537	2.3245	0
	.05	.5753	.2424	.08516	.1810	-7.937	.4621	2.2862	.0218
	.1	.5099	.2306	.07652	.1934	-3.699	.4748	2.2481	.0438
	.15	.4494	.2182	.06910	.2048	-2.248	.4936	2.2101	.0662
	.2	.3940	.2052	.06282	.2162	-1.4584	.5319	2.1707	.0891
	.25	.3435	.1917	.05762	.2270	-.9174	.5874	2.1293	.1124
	.3	.2981	.1775	.05341	.2389	-.4697	.6642	2.0856	.1360
	.35	.2576	.1627	.05009	.2470	-.0392	.7692	2.0370	.1603
	.4	.2222	.1472	.04751	.2476	.4347	.9107	1.9826	.1852
	.45	.1917	.1310	.04556	.2393	1.0287	1.0983	1.9219	.2108
.10	0	.6585	.2604	.10067	.1725	$-\infty$	.6430	2.6073	0
	.05	.5768	.2492	.09076	.1831	-6.648	.6606	2.5390	.0196
	.1	.5202	.2374	.08212	.1954	-2.966	.6869	2.4712	.0395
	.15	.4586	.2250	.07470	.2068	-1.6996	.7194	2.4045	.0600
	.2	.4019	.2121	.06842	.2183	-1.0201	.7611	2.3380	.0812
	.25	.3503	.1985	.06322	.2291	-.5522	.8189	2.2712	.1028
	.3	.3036	.1843	.05901	.2410	-.1525	.8923	2.2029	.1252
	.35	.2620	.1695	.05569	.2491	.2445	.9911	2.1320	.1483
	.4	.2253	.1541	.05311	.2497	.7030	1.1244	2.0584	.1721
	.45	.1937	.1379	.05116	.2414	1.3022	1.3015	1.9803	.1968

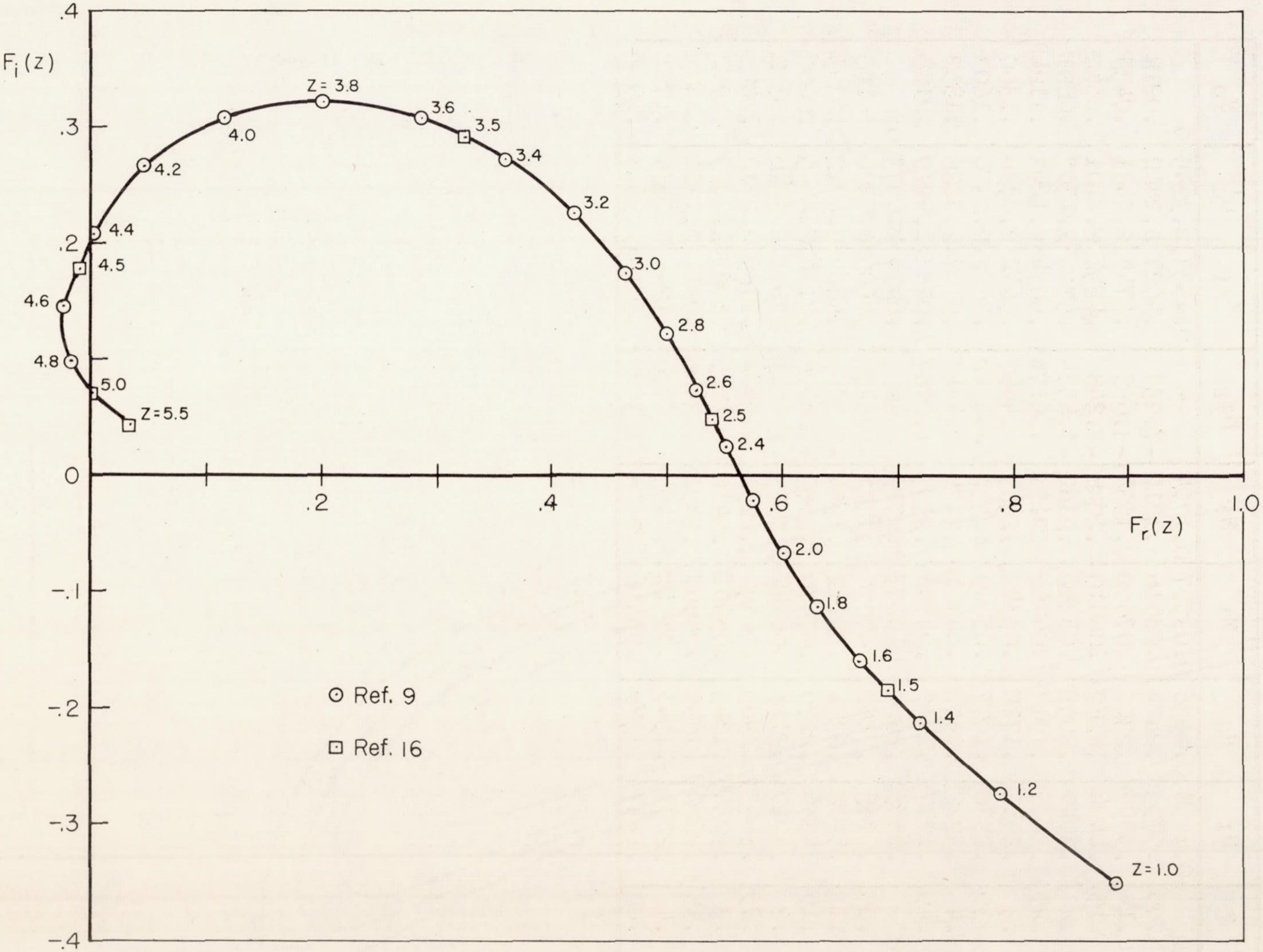
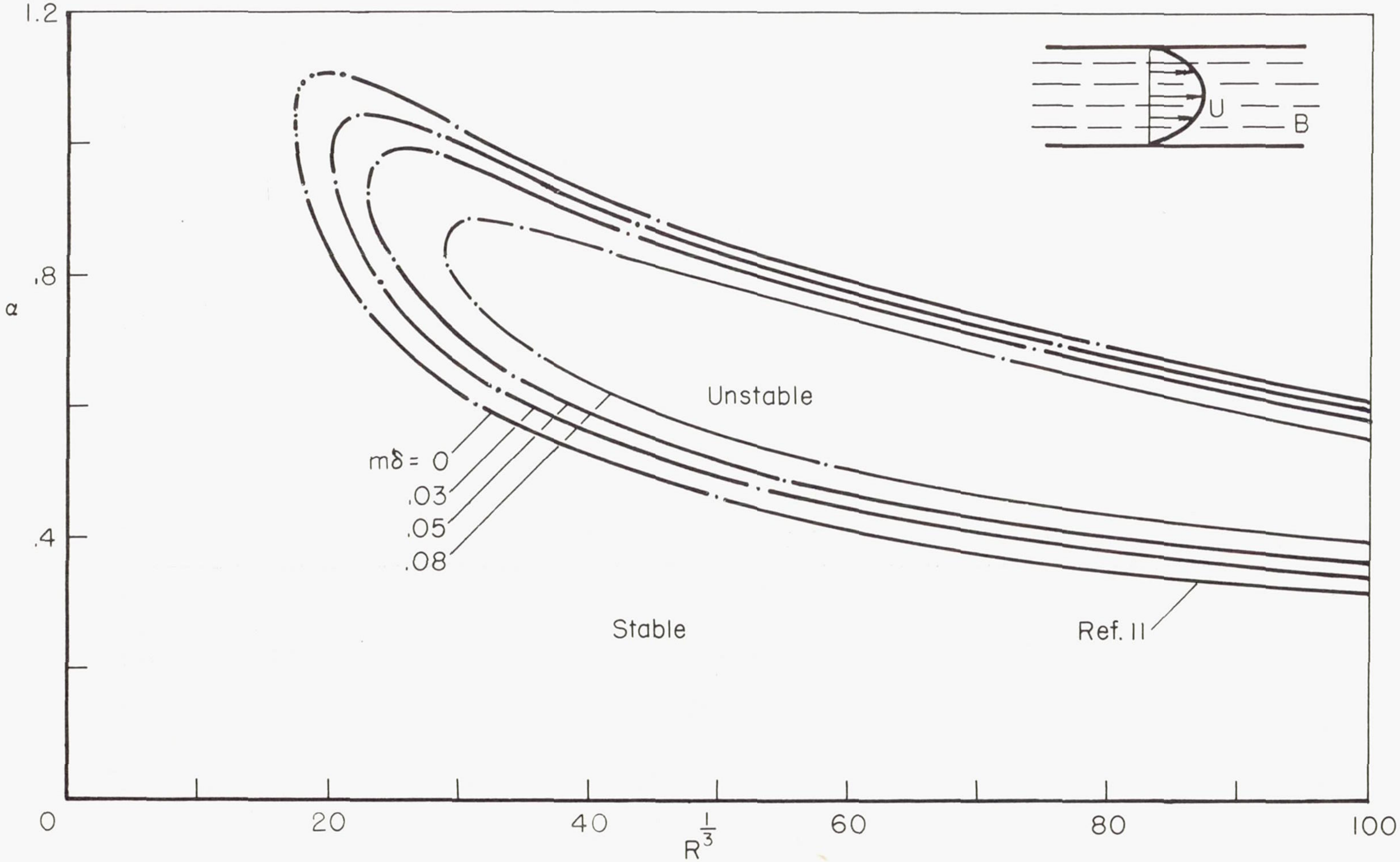
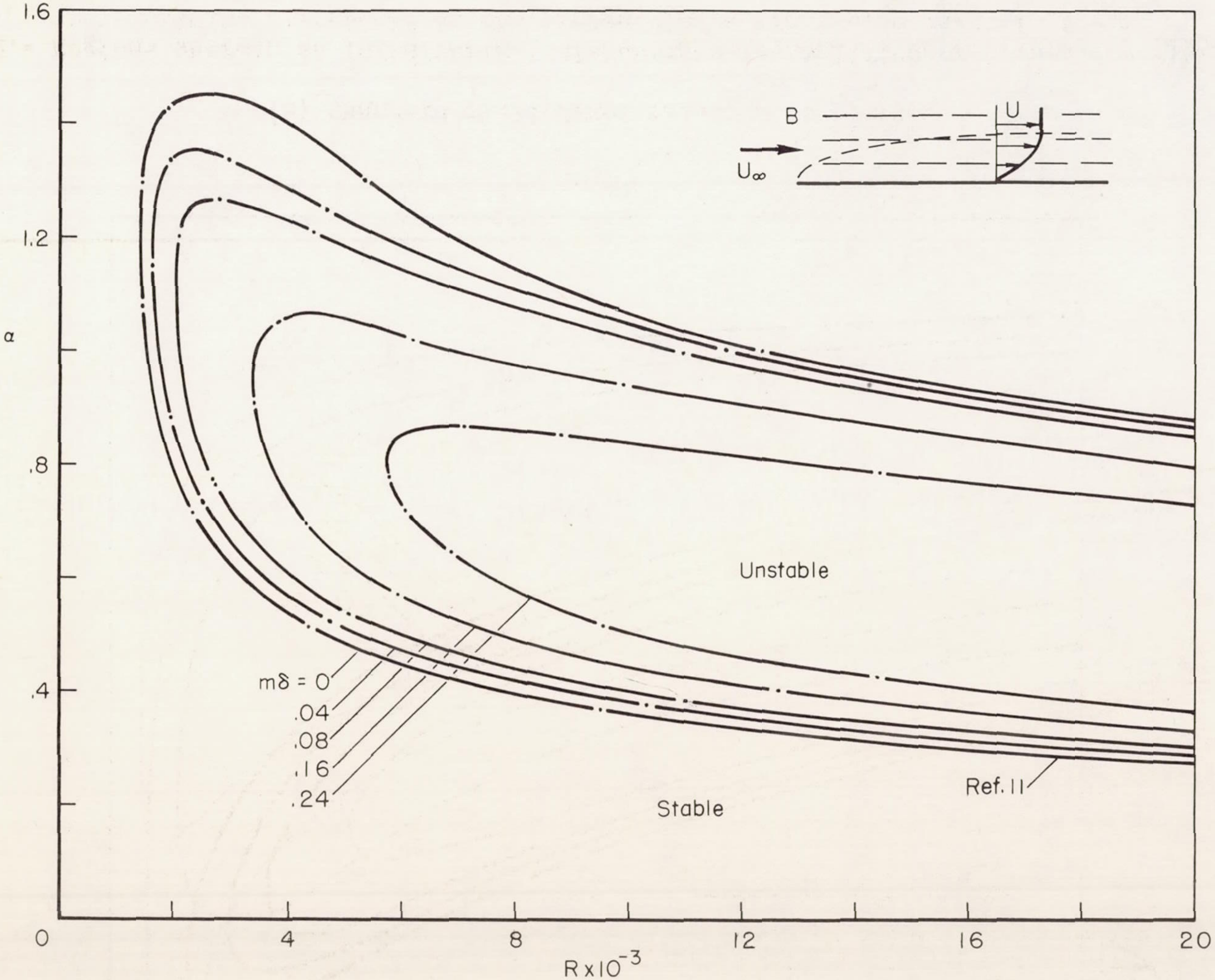


Figure 1.- Tietjen's function, $F(z) = F_r(z) + iF_i(z)$ where $z = y_o(U_o'\alpha R)^{1/3}$.



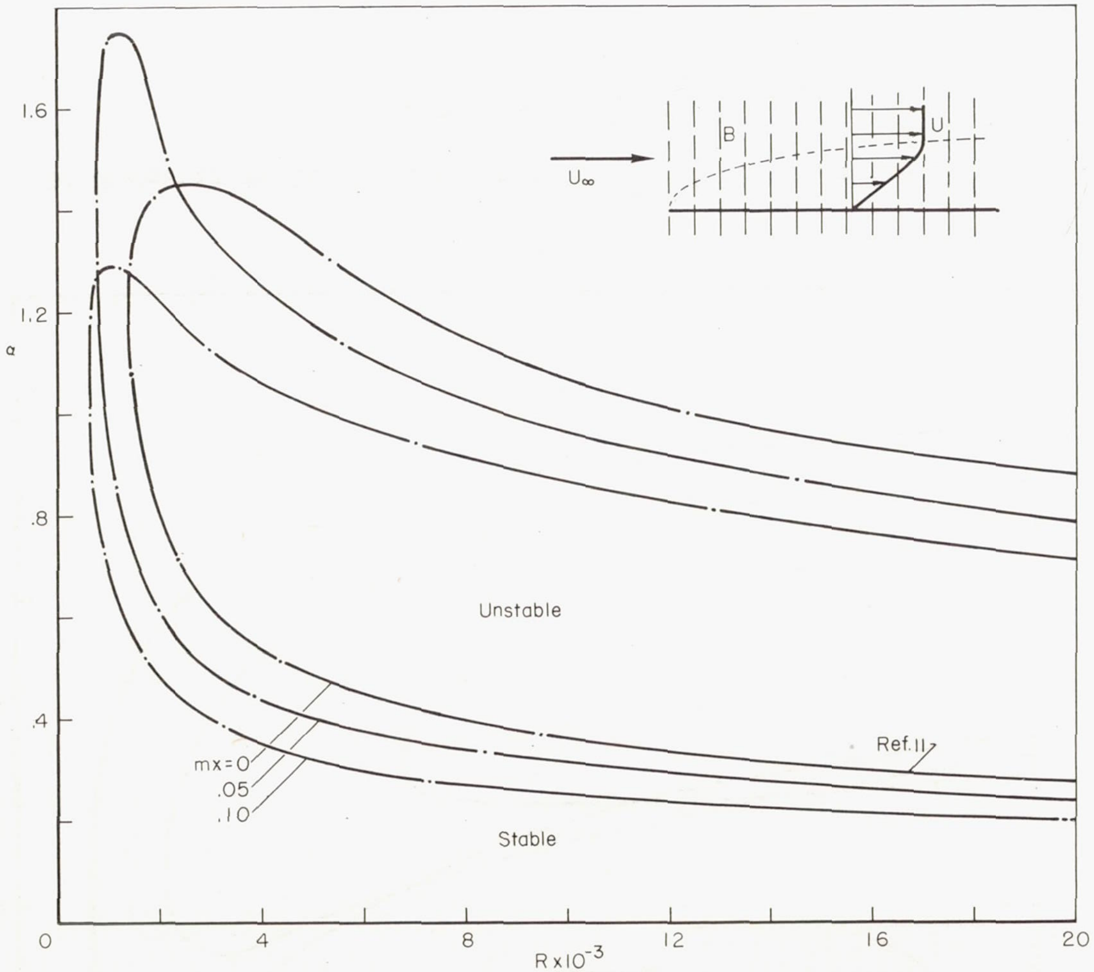
(a) Channel flow; parabolic velocity profile.

Figure 2.- Regions wherein an infinitesimal sinusoidal disturbance is amplified or damped in the presence of a coplanar magnetic field.



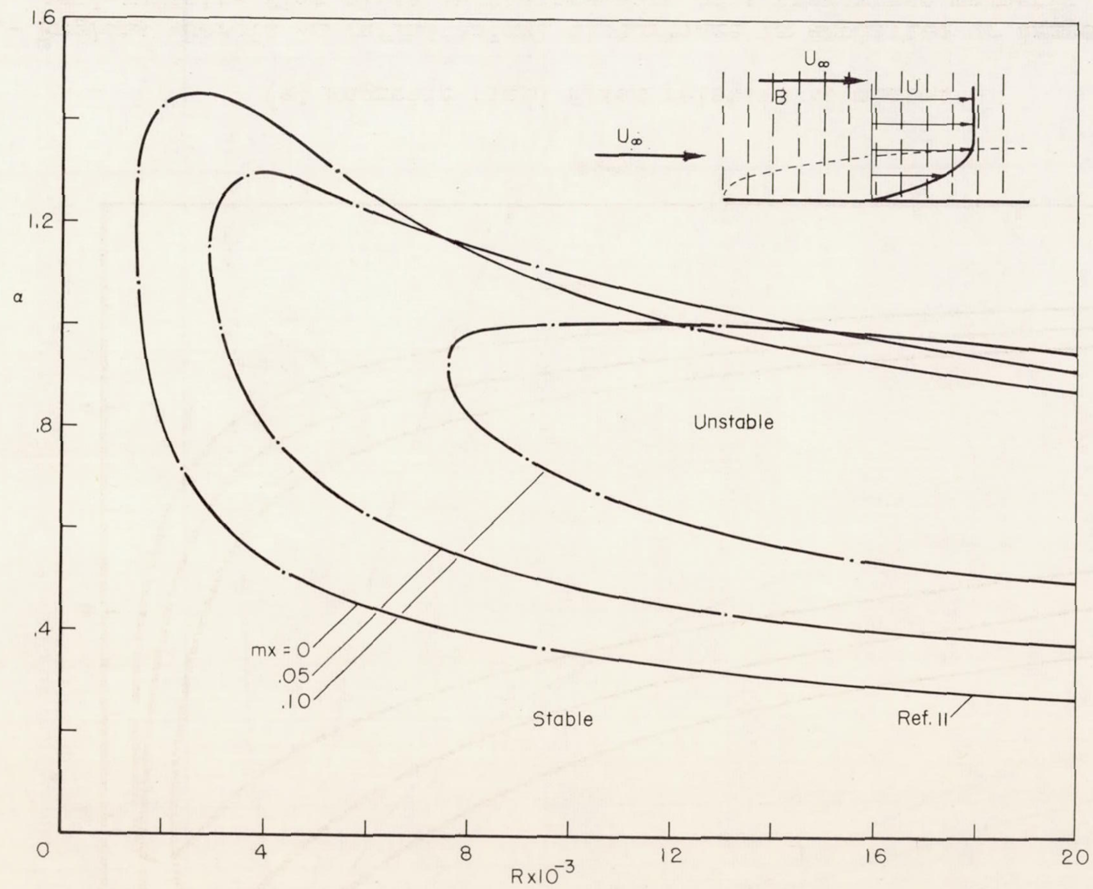
(b) Flat plate; Blasius boundary-layer profile.

Figure 2.- Concluded.



(a) Magnetic field fixed relative to plate.

Figure 3.- Regions wherein an infinitesimal disturbance is amplified or damped for flow over a semi-infinite flat plate in the presence of a transverse magnetic field.



(b) Magnetic field fixed relative to fluid far from the plate.

Figure 3.- Concluded.

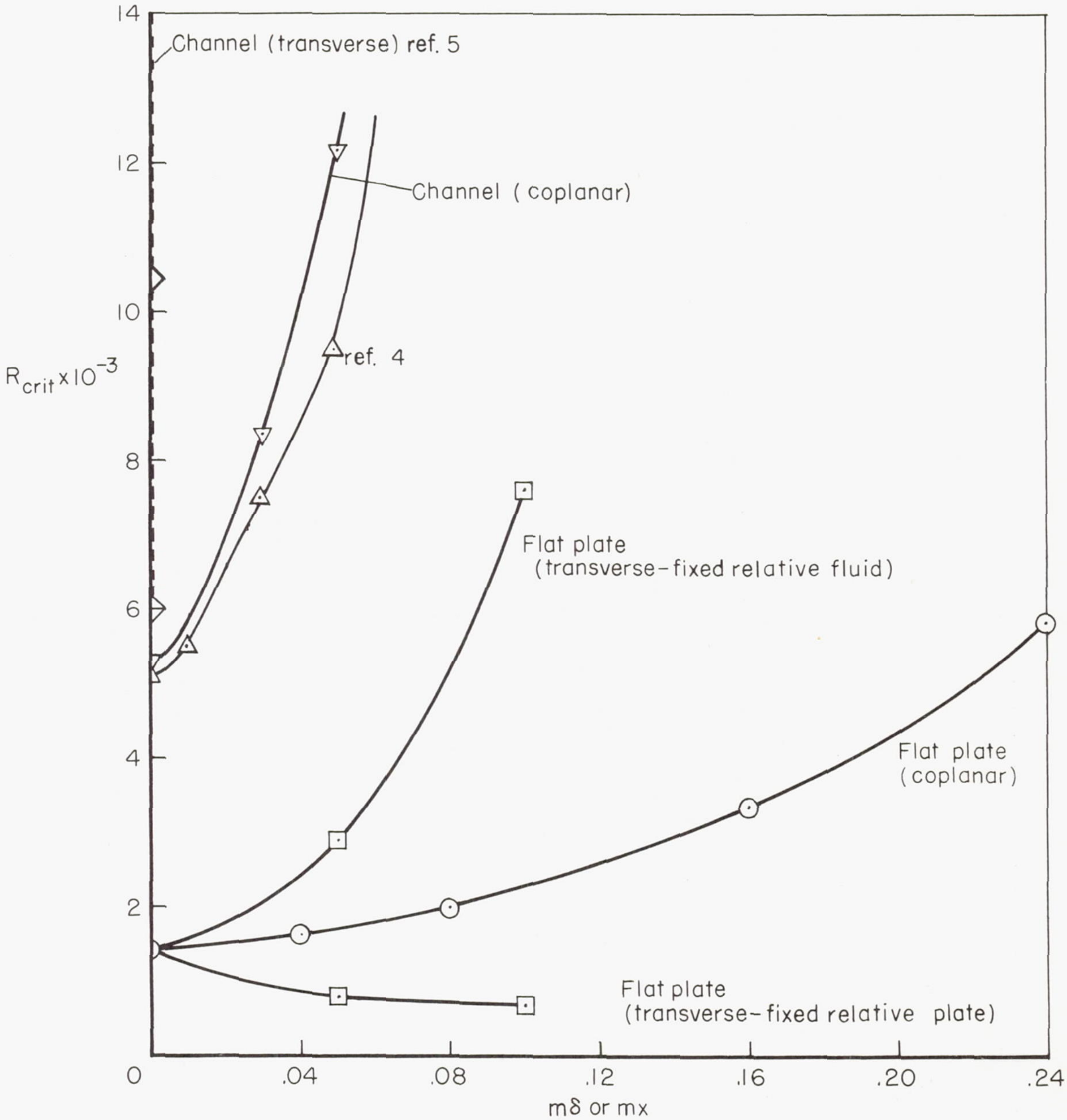


Figure 4.- Critical Reynolds number as a function of the magnetic parameter.