NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4346

APPROXIMATE METHOD FOR CALCULATING MOTIONS IN ANGLES OF ATTACK AND SIDESLIP DUE TO STEP PITCHING- AND YAWING-MOMENT INPUTS DURING STEADY ROLL

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Washington
September 1958

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SUMMARY

The simplified method of NACA Report 1344 for calculating motions in angles of attack and sideslip resulting from a trim angle of attack and steady rolling velocity has been extended in the present paper to the condition of pitching- and yawing-moment inputs. The resulting formulas are intended primarily for rolling-velocity conditions in which rolling divergence is not encountered. From the calculated angles of attack and sideslip, preliminary estimates of aircraft loads for design purposes can be made. Motions calculated with the simplified method are compared with exact solutions of the five-degree-of-freedom equations.

INTRODUCTION

Many high-performance aircraft encounter inertia coupling in rolling maneuvers. As a result this condition must be investigated during the design of new aircraft. A thorough analysis of rolling flight requires the solution of the nonlinear equations of motion, and analog or digital computers are usually utilized for this problem. However, for preliminary design purposes, simplified methods of analysis are desirable.

A simplification used by many investigators in analyses of roll-coupled motions is that of linearizing the equations of motion by assuming rolling velocity to be constant. In early investigations of the stability of steady-rolling aircraft (refs. 1 and 2), the concept of critical rolling velocities was defined. Steady-state solutions of these linearized equations also yield useful results as to the magnitudes of the responses to be expected in rolling maneuvers and have been discussed in references 2 and 3. Transient solutions of the equations of motion for steady rolling can be obtained, but such calculations are lengthy. Approximate expressions presented in reference 4 permit simple calculation of transient responses of angles of attack and sideslip due to a trim angle of attack.

In this paper the method of reference 4 is extended to the calculation of motions resulting from step pitching- and yawing-moment inputs. This method is applicable to inputs such as elevator deflection, rudder deflection, aileron yaw, and yawing moment due to rolling velocity. Transient responses of angles of attack and sideslip are presented for a range of rolling velocities for a current swept-wing fighter airplane assumed to be flying at a Mach number of 0.7 and an altitude of 32,000 feet. The method is intended primarily for conditions of rolling velocities up to the critical value. In the region of rolling divergence, nonlinearities in the equations of motion and airplane aerodynamics are known to be significant, and the simplified, linearized results are not expected to be good approximations of the five-degree-of-freedom nonlinear results.

SYMBOLS

Airplane equations of motion are referenced to a system of principal body axes in this paper.

A,B,C,D,E coefficients of equation (2)

b wing span, ft

$\bar{c}$ wing mean aerodynamic chord, ft

C_L lift coefficient, $\frac{\text{Lift}}{qS}$

C_l rolling-moment coefficient, $\frac{\text{Rolling moment}}{qSb}$

C_m pitching-moment coefficient, $\frac{\text{Pitching moment}}{qS\bar{c}}$

C_n yawing-moment coefficient, $\frac{\text{Yawing moment}}{qSb}$

C_Y side-force coefficient, $\frac{\text{Side force}}{qS}$

I_X moment of inertia of airplane about X-axis, slug-ft^2

I_Y	moment of inertia of airplane about Y-axis, slug-ft^2
I_Z	moment of inertia of airplane about Z-axis, slug-ft^2
K_n	amplitude coefficients (n = 0, 1, 2, and 3)
m	airplane mass, slugs
M_Y	pitching moment, ft-lb
M_Z	yawing moment, ft-lb
p	rolling velocity, radians/sec
p_o	steady rolling velocity, radians/sec
q	pitching velocity, radians/sec; dynamic pressure, lb/sq ft
r	yawing velocity, radians/sec
S	wing area, sq ft
t	time, sec
V	airplane velocity, ft/sec
α	angle of attack, radians
α_o	initial angle of attack of airplane principal axis, radians
$\Delta\alpha$	incremental change in angle of attack, radians
β	angle of sideslip, radians
θ	phase angle, radians
λ_1, λ_2	real roots of characteristic equation
$a \pm i\omega$	complex roots of characteristic equation

$$\omega_\theta^2 = \frac{-C_{m_\alpha} qS\bar{c}}{I_Y {p_o}^2}$$

$$\omega_\psi^2 = \frac{C_{n_\beta} qSb}{I_Z p_o^2}$$

$$C_{L_\alpha} = \frac{\partial C_L}{\partial \alpha}$$

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha}$$

$$C_{m_q} = \frac{\partial C_m}{\partial (q\bar{c}/2V)}$$

$$C_{Y_\beta} = \frac{\partial C_Y}{\partial \beta}$$

$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta}$$

$$C_{n_p} = \frac{\partial C_n}{\partial (pb/2V)}$$

$$C_{n_r} = \frac{\partial C_n}{\partial (rb/2V)}$$

$$C_{l_\beta} = \frac{\partial C_l}{\partial \beta}$$

$$C_{l_p} = \frac{\partial C_l}{\partial (pb/2V)}$$

$$L_\alpha = C_{L_\alpha} qS$$

$$L_\beta = C_{l_\beta} qSb$$

$$L_p = C_{l_p} \frac{qSb^2}{2V}$$

$$M_\alpha = C_{m_\alpha} qS\bar{c}$$

$$M_q = C_{m_q} \frac{qS\bar{c}^2}{2V}$$

$$N_\beta = C_{n_\beta} qSb$$

$$N_r = C_{n_r} \frac{qSb^2}{2V}$$

$$N_p = C_{n_p} \frac{qSb^2}{2V}$$

$$Y_\beta = C_{Y_\beta} qS$$

Dot over symbol indicates derivative with respect to time.

ANALYSIS

Equations of Motion

If rolling velocity and forward speed are assumed constant and weight components are neglected, the airplane equations of motion referenced to principal axes are

Pitching:

$$\dot{q} - \frac{M_q}{I_Y} q - \left(\frac{I_Z - I_X}{I_Y}\right) p_o r - \frac{M_\alpha}{I_Y} \Delta\alpha = \frac{M_Y}{I_Y}$$

Yawing:

$$\dot{r} - \frac{N_r}{I_Z} r - \left(\frac{I_X - I_Y}{I_Z}\right) p_o q - \frac{N_\beta}{I_Z} \beta = \frac{M_Z}{I_Z} \qquad (1)$$

Side force:

$$\dot{\beta} - \frac{Y_\beta}{mV} \beta + r - p_o \Delta\alpha = p_o \alpha_o$$

Normal force:

$$\dot{\alpha} + \frac{L_\alpha}{mV} \Delta\alpha - q + p_o \beta = 0$$

The terms on the right-hand side of the equations are disturbances; the pitching-moment disturbance M_Y results from elevator deflection, while the yawing-moment disturbance M_Z may result from rudder deflection, aileron deflection (aileron yaw), or rolling velocity $\frac{N_p}{I_Z} p_o$. Airplane motions due to pitching- and yawing-moment inputs are the subject of this investigation. Motions due to the term $\alpha_o p_o$ were treated in reference 4 and are omitted in this investigation.

The characteristic equation for steady roll is given by the expansion of the determinant formed by the terms on the left-hand side of equations (1) and is

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0 \qquad (2)$$

where

$$A = 1$$

$$B = -\frac{N_r}{I_Z} - \frac{M_q}{I_Y} - \frac{Y_\beta}{mV} + \frac{L_\alpha}{mV}$$

$$C = -\left(\frac{I_X - I_Y}{I_Z}\right)\left(\frac{I_Z - I_X}{I_Y}\right)p_o^2 - \frac{M_\alpha}{I_Y} + \frac{N_\beta}{I_Z} + \frac{M_q}{I_Y}\frac{N_r}{I_Z} + p_o^2 + \frac{Y_\beta}{mV}\frac{N_r}{I_Z} + \frac{Y_\beta}{mV}\frac{M_q}{I_Y} -$$

$$\frac{L_\alpha}{mV}\frac{M_q}{I_Y} - \frac{L_\alpha}{mV}\frac{Y_\beta}{mV} - \frac{L_\alpha}{mV}\frac{N_r}{I_Z}$$

$$D = -\left(\frac{N_r}{I_Z} + \frac{M_q}{I_Y}\right)p_o^2 - \frac{M_q}{I_Y}\frac{N_\beta}{I_Z} + \frac{M_\alpha}{I_Y}\frac{N_r}{I_Z} - \frac{M_q}{I_Y}\frac{N_r}{I_Z}\frac{Y_\beta}{mV} + \frac{M_q}{I_Y}\frac{N_r}{I_Z}\frac{L_\alpha}{mV} + \frac{N_\beta}{I_Z}\frac{L_\alpha}{mV} +$$

$$\frac{M_\alpha}{I_Y}\frac{Y_\beta}{mV} + \frac{L_\alpha}{mV}\frac{Y_\beta}{mV}\frac{N_r}{I_Z} + \frac{L_\alpha}{mV}\frac{Y_\beta}{mV}\frac{M_q}{I_Y} + \frac{Y_\beta}{mV}\left(\frac{I_X - I_Y}{I_Z}\right)\left(\frac{I_Z - I_X}{I_Y}\right)p_o^2 -$$

$$\frac{L_\alpha}{mV}\left(\frac{I_X - I_Y}{I_Z}\right)\left(\frac{I_Z - I_X}{I_Y}\right)p_o^2$$

$$E = -\left(\frac{I_X - I_Y}{I_Z}\right)\left(\frac{I_Z - I_X}{I_Y}\right)p_o^4 - \frac{M_\alpha}{I_Y}\left(\frac{I_X - I_Y}{I_Z}\right)p_o^2 - \frac{N_\beta}{I_Z}\left(\frac{I_Z - I_X}{I_Y}\right)p_o^2 +$$

$$\frac{M_q}{I_Y}\frac{N_r}{I_Z}p_o^2 - \frac{M_\alpha}{I_Y}\frac{N_\beta}{I_Z} - \frac{M_q}{I_Y}\frac{N_\beta}{I_Z}\frac{L_\alpha}{mV} - \frac{M_\alpha}{I_Y}\frac{N_r}{I_Z}\frac{Y_\beta}{mV} - \frac{M_q}{I_Y}\frac{N_r}{I_Z}\frac{L_\alpha}{mV}\frac{Y_\beta}{mV} +$$

$$\frac{L_\alpha}{mV}\frac{Y_\beta}{mV}\left(\frac{I_X - I_Y}{I_Z}\right)\left(\frac{I_Z - I_X}{I_Y}\right)p_o^2$$

Approximate Expressions for Angles of Attack and Sideslip

In reference 4, responses of angles of attack and sideslip due to the disturbance $\alpha_o p_o$ were obtained from equations (1) by the Laplace transform method (ref. 5). It was shown that the amplitude of the higher frequency mode appearing in the solutions was small and could be neglected. Thus, the solutions were simplified to a constant term plus either an oscillatory mode or two aperiodic modes, depending on the rolling velocity. In addition, the modal coefficients were simplified by omitting some terms of negligible value. The same procedure has been adopted in this investigation to derive approximate expressions for responses due to unit step inputs of M_Y/I_Y and M_Z/I_Z.

When the significant mode is a long-period oscillation, as for subsequent conditions in this paper with $p_o = -1.0$, -1.5, -2.5, and -3.0 radians/sec, the response is written:

$$\left.\begin{array}{c} \dfrac{\beta}{M_Z/I_Z} \\ \dfrac{\beta}{M_Y/I_Y} \\ \dfrac{\Delta\alpha}{M_Z/I_Z} \\ \dfrac{\Delta\alpha}{M_Y/I_Y} \end{array}\right\} = K_0 + K_1 e^{at}\cos(\omega t + \theta) \tag{3}$$

When the significant modes are two exponentials, as for $p_o = -1.86$, -2.0, and -2.33 radians/sec in this paper, the response is written:

$$\left.\begin{array}{c} \dfrac{\beta}{M_Z/I_Z} \\ \dfrac{\beta}{M_Y/I_Y} \\ \dfrac{\Delta\alpha}{M_Z/I_Z} \\ \dfrac{\Delta\alpha}{M_Y/I_Y} \end{array}\right\} = K_0 + K_2 e^{\lambda_1 t} + K_3 e^{\lambda_2 t} \tag{4}$$

After simplification, the amplitude coefficients are given by the following expressions. In these expressions, approximate values of the characteristic roots, as obtained by the method of the appendix of reference 4, are used.

In the expressions for $\frac{\beta}{M_Z/I_Z}$,

$$K_0 = \frac{\frac{M_q}{I_Y}\frac{L_\alpha}{mV} + \frac{M_\alpha}{I_Y} + \left(\frac{I_Z - I_X}{I_Y}\right){p_o}^2}{E}$$

$$K_1 = \frac{2\sqrt{G^2 + H^2}}{L\sqrt{E/C}} \qquad \theta = \tan^{-1}\left(\frac{-H\omega - Ga}{Ha - G\omega}\right)$$

where

$$G = \omega^2 - a^2 + a\left(\frac{M_q}{I_Y} - \frac{L_\alpha}{mV}\right) + K_0 E$$

$$H = -2a\omega + \omega\left(\frac{M_q}{I_Y} - \frac{L_\alpha}{mV}\right)$$

$$L = 4\left(3a^2\omega - \omega^3\right) + 2C\omega$$

$$K_2 = \frac{-{\lambda_1}^2 + \lambda_1\left(\frac{M_q}{I_Y} - \frac{L_\alpha}{mV}\right) + K_0 E}{\lambda_1\left(2C\lambda_1 + D\right)}$$

$$K_3 = \frac{-{\lambda_2}^2 + \lambda_2\left(\frac{M_q}{I_Y} - \frac{L_\alpha}{mV}\right) + K_0 E}{\lambda_2\left(2C\lambda_2 + D\right)}$$

In the expressions for $\frac{\beta}{M_Y/I_Y}$,

$$K_0 = \frac{-p_o\left[\left(\frac{I_X - I_Y}{I_Z}\right)\frac{L_\alpha}{mV} + \frac{N_r}{I_Z}\right]}{E}$$

$$K_1 = \frac{-2H}{L\sqrt{E/C}} \qquad \theta = \tan^{-1}\left(\frac{-H\omega - Ga}{Ha - G\omega}\right)$$

where

$$G = p_o a\left[1 - \left(\frac{I_X - I_Y}{I_Z}\right)\right] + K_0 E$$

$$H = p_o \omega\left[1 - \left(\frac{I_X - I_Y}{I_Z}\right)\right]$$

$$L = 4\left(3a^2\omega - \omega^3\right) + 2C\omega$$

$$K_2 = \frac{p_o\lambda_1\left[1 - \left(\frac{I_X - I_Y}{I_Z}\right)\right] + K_0 E}{\lambda_1\left(2C\lambda_1 + D\right)}$$

$$K_3 = \frac{p_o\lambda_2\left[1 - \left(\frac{I_X - I_Y}{I_Z}\right)\right] + K_0 E}{\lambda_2\left(2C\lambda_2 + D\right)}$$

In the expressions for $\frac{\Delta\alpha}{M_Z/I_Z}$,

$$K_0 = \frac{-p_o\left[\frac{M_q}{I_Y} + \left(\frac{I_Z - I_X}{I_Y}\right)\frac{Y_\beta}{mV}\right]}{E}$$

$$K_1 = \frac{-2p_o\omega\left[1 + \left(\frac{I_Z - I_X}{I_Y}\right)\right]}{L\sqrt{E/C}} \qquad \theta = \tan^{-1}\left(\frac{\omega}{-a}\right)$$

where

$$L = 4\left(3a^2\omega - \omega^3\right) + 2C\omega$$

$$K_2 = \frac{p_o\lambda_1\left[1 + \left(\frac{I_Z - I_X}{I_Y}\right)\right] + K_0E}{\lambda_1\left(2C\lambda_1 + D\right)}$$

$$K_3 = \frac{p_o\lambda_2\left[1 + \left(\frac{I_Z - I_X}{I_Y}\right)\right] + K_0E}{\lambda_2\left(2C\lambda_2 + D\right)}$$

In the expressions for $\frac{\Delta\alpha}{M_Y/I_Y}$,

$$K_0 = \frac{\frac{N_r}{I_Z}\frac{Y_\beta}{mV} + \frac{N_\beta}{I_Z} + \left(\frac{I_X - I_Y}{I_Z}\right)p_o^2}{E}$$

$$K_1 = \frac{2|G|}{L\sqrt{E/C}} \qquad \theta = \tan^{-1}\left(\frac{-H\omega - Ga}{Ha - G\omega}\right)$$

where

$$G = a^2 - \omega^2 - a\left(\frac{Y_\beta}{mV} + \frac{N_r}{I_Z}\right) + K_0E$$

$$H = 2a\omega - \omega\left(\frac{Y_\beta}{mV} + \frac{N_r}{I_Z}\right)$$

$$L = 4\left(3a^2\omega - \omega^3\right) + 2C\omega$$

$$K_2 = \frac{\lambda_1^2 - \lambda_1\left(\frac{Y_\beta}{mV} + \frac{N_r}{I_Z}\right) + K_0E}{\lambda_1\left(2C\lambda_1 + D\right)}$$

$$K_3 = \frac{\lambda_2^2 - \lambda_2\left(\frac{Y_\beta}{mV} + \frac{N_r}{I_Z}\right) + K_0E}{\lambda_2\left(2C\lambda_2 + D\right)}$$

DISCUSSION

Airplane Stability in Steady Roll

The stability of an airplane in steady roll is determined from the stability equation, equation (2), and has been treated generally in reference 1 and, for the airplane of this investigation, in reference 4. The airplane physical and aerodynamic characteristics and flight conditions used in this paper are the same as in reference 4 and are given in table I. Exact and approximate roots of the characteristic equation were published in reference 4 and are repeated in table II. The rolling divergence boundaries, as developed in reference 1, are presented in figure 1 for this airplane. For this flight condition there is no divergent mode for any rolling velocity since the divergence boundary is not intersected. At low roll rates (-1.0 and -1.5 radians/sec) the airplane has two stable oscillatory modes. At roll rates for which the airplane is near the divergence boundary (-1.86, -2.0, and -2.33 radians/sec), there is a stable oscillatory mode and two convergent aperiodic modes, one of which is approaching neutral stability. At the higher roll rates, -2.5 and -3.0 radians/sec, the airplane again has two stable oscillatory modes.

Sample Responses From Simplified Method

Angle-of-attack and sideslip responses to unit step inputs of M_Y/I_Y and M_Z/I_Z have been calculated from the simplified expressions for a range of rolling velocities from -1.0 to -3.0 radians/sec and are shown in figures 2 and 3 along with the exact steady-rolling responses. These motions are for the airplane and flight condition given in table I. The good agreement between the simplified and exact motions justifies the use of the simplified expressions for steady-rolling calculations.

The steady-state values of the responses shown in figures 2 and 3 are given in table III. Since rolling maneuvers are generally of short duration, the steady-state condition is seldom attained.

Note that for the larger roll rates of figures 2 and 3 only a portion of the responses shown might be of practical interest. For example, at a roll rate of -3.0 radians/sec a 360° roll would be performed in slightly over 2 seconds, whereas the responses are shown for a duration of 8 seconds.

Comparison With Five-Degree-of-Freedom Responses

The nonlinear five-degree-of-freedom equations of motion are used universally for predicting airplane motions in large-amplitude rolling maneuvers. In order to determine whether the simplified results are good approximations of the airplane motions experienced in practical rolling maneuvers, a comparison was made of the simplified and five-degree-of-freedom solutions. The airplane equations of motion with constant forward speed were programed on an analog computer and responses were obtained for a step aileron deflection in combination with (1) a step rudder deflection, (2) a step elevator deflection, and (3) combined step elevator and rudder deflections. At the time of these calculations, more complete aerodynamic data which were available for this airplane (table IV) were used. The flight conditions and inertias were the same as for the calculations for figures 2 and 3, and the angle of attack of the principal axis was assumed zero. The results of these calculations are presented in figure 4 with corresponding motions from the simplified method.

The motions in angles of attack and sideslip are presented in figure 4(a) for a step yawing-moment input M_Z/I_Z of -0.2092, equivalent to 5° of rudder deflection. The variation of rolling velocity with time from the nonlinear equations is presented, from which trial values are selected for use with the simplified method. The values chosen were -2.5 radians/sec, the average value during a 360° roll, -3.0 radians/sec, and -3.2 radians/sec, the steady-state value. During the latter part of the motions the simplified method with a roll rate of -3.0 radians/sec provided the best agreement with the nonlinear motions. For the first second of the response, better agreement in the β motions is obtained when a roll rate of -1.5 radians/sec is used. This is the average value during the first second of the nonlinear rolling-velocity response. There is poor agreement initially in the angle-of-attack response because the longitudinal mode is neglected in the simplified method.

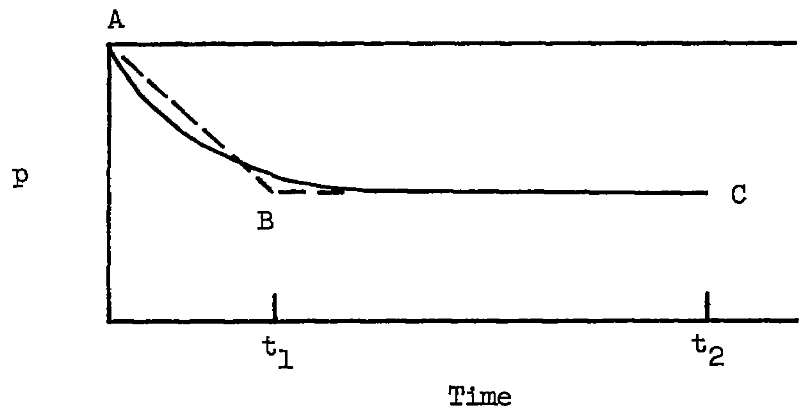
The response to a step pitching-moment input M_Y/I_Y of 0.5879, equivalent to about -3° of elevator deflection, is shown in figure 4(b) for the nonlinear system and for the simplified method for roll rates of -1.5, -2.5, -2.7, and -3.0 radians/sec. The roll-rate response of the nonlinear system is not shown but is similar to the curve in figure 4(a) except that the steady-state value is -2.7 radians/sec. For the first second the simplified results for a roll rate of -1.5 radians/sec, the average value, provide the best agreement with the nonlinear response. For the remainder of the time history, simplified results for the steady-state rolling velocity, -2.7 radians/sec, are good approximations of the nonlinear response.

The results for combined pitching- and yawing-moment inputs are shown in figure 4(c). The steady-state rolling velocity is -2.6 radians/sec, and the nonlinear response is approximated in two steps as were the motions due to the individual inputs.

Suggested Procedure for Use of Approximate Method

In airplane-design calculations, the simplified methods of this paper may be employed as follows: Determine the maximum roll rate of interest and roll-rate response time for a specific flight condition from one-degree-of-freedom roll calculations and experience with similar aircraft. When the maximum roll rate is established, select intermediate values to cover the rolling-velocity range of the airplane.

For a selected roll rate, plot the one-degree-of-freedom response (see sketch):



For times up to t_1, compute $\Delta\alpha$ and β responses by using the average value of rolling velocity during this time. Compute the $\Delta\alpha$ and β responses for the steady-state roll rate as well for times beyond t_1. From these responses, maximum values of $\Delta\alpha$ and β for the maneuver are determined, with which preliminary load calculations may be made. A tabulation or plot of these data for the range of roll rates will disclose any serious conditions. Needless to say, critical conditions should be examined further by five-degree-of-freedom studies.

A word of caution is advanced regarding the use of one-degree-of-freedom roll calculations. In roll-coupled maneuvers, the one-degree-of-freedom roll equation will not generally provide an accurate relationship between aileron deflection and rolling velocity because of the effect of sideslip in damping or augmenting the roll. The use of the one-degree-of-freedom equation is intended only for obtaining estimates of response time and selecting a range of roll rates. Steady-state solutions of the five-degree-of-freedom equations are required to relate accurately aileron deflection and rolling velocity.

Angle of Attack of Principal Axis

Angle of attack coupled with roll rate is a major input to the equations for steady roll but was neglected in this paper in order to isolate the effects of pitching- and yawing-moment inputs. The effect of angle of attack was treated in reference 4 and the results can be algebraically added to the results of this paper for calculations in which both types of inputs are considered.

Recovery Motions

Recovery is the portion of the rolling maneuver in which the control is reversed to stop the roll, and the motions are defined by the nonlinear equations of motion. In many cases the maximum excursions in angles of attack and sideslip (and, consequently, loads) occur during this time. For approximating recovery motions the classical three-degree-of-freedom lateral equations and two-degree-of-freedom longitudinal equations with initial conditions and approximations for the coupling terms were used. These equations are given in the appendix.

Sample recovery motions from the five-degree-of-freedom equations and two sets of approximate motions are presented in figure 5. These recovery motions are the latter part of a 360° roll after the aileron has been neutralized to stop the roll. Motions in angles of attack and sideslip, as calculated from the linear, uncoupled, two-degree-of-freedom longitudinal and three-degree-of-freedom lateral equations, are shown and differ in magnitude and phase with the nonlinear responses. When approximations to the coupling terms are included, the agreement with the nonlinear response is better. The remaining phase difference is attributed primarily to the 50 deg/sec rate at which the aileron was taken off in the analog calculations. When these approximate equations are used for recovery calculations, occasional checks with the five-degree-of-freedom equations should be made.

A dangerous recovery situation, for which the linear equations are invalid, is possible in flight. After the ailerons are neutralized for recovery from a rapid roll, it is possible for rolling velocity to be maintained at a large value instead of being reduced to zero. The airplane enters an autorotational state, a steady-state condition of large angular and linear velocities. Five-degree-of-freedom solutions are required to predict this condition.

Aircraft Loads

In many rolling maneuvers the critical factor is the load encountered. In pure rolls the vertical-tail load is usually the one of concern. In a roll-coupled maneuver in which large excursions in angles of attack and sideslip occur, structural failures of the wing, horizontal tail, or vertical tail are possible. Preliminary estimates of surface loads can be made by using the calculated angles of attack and sideslip.

CONCLUDING REMARKS

The simplified method of NACA Report 1344 for calculating motions in angles of attack and sideslip in roll-coupled maneuvers has been extended in the present paper to the condition of pitching- and yawing-moment inputs. The resulting formulas are intended primarily for rolling-velocity conditions in which rolling divergence is not encountered. From the calculated angles of attack and sideslip, preliminary estimates of aircraft loads for design purposes can be made. Motions calculated with the simplified method are compared with exact solutions of the five-degree-of-freedom equations.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 5, 1958.

APPENDIX

ROLL-RECOVERY EQUATIONS

The five-degree-of-freedom equations were separated into the three-degree-of-freedom lateral and two-degree-of-freedom longitudinal equations for approximating roll-recovery motions. As the coupling terms were known to have significant effects on recovery motions, they were included in simplified form.

In the lateral equations the coupling terms appear as variables. These terms were linearized by assuming pitching velocity and angle of attack to be constant. After the Laplace transform is applied, the lateral equations are written as

$$\left(\lambda - \frac{L_p}{I_X}\right)p - \left(\frac{I_Y - I_Z}{I_X}\right)q_o r - \frac{L_\beta}{I_X}\beta = p_o$$

$$-\left(\frac{I_X - I_Y}{I_Z}\right)q_o p + \left(\lambda - \frac{N_r}{I_Z}\right)r - \frac{N_\beta}{I_Z}\beta = \frac{M_Z/I_Z}{\lambda} + r_o$$

$$-\alpha_o p + r + \left(\lambda - \frac{Y_\beta}{mV}\right)\beta = \beta_o$$

where M_Z/I_Z is the yawing-moment input applied in this investigation and the subscript o denotes values of the variables at the start of recovery.

In the longitudinal equations the coupling terms appear as inputs since they consist of lateral variables only. The inputs represented by these terms are large at zero time and decrease to zero as the rolling velocity decays. Hence, these terms were approximated by assuming β and r to be constant and p to decay as given by the one-degree-of-freedom roll response.

After the Laplace transform is applied, the longitudinal equations are written as

$$\left(\lambda - \frac{M_q}{I_Y}\right)q - \frac{M_\alpha}{I_Y}\alpha = \frac{M_Y/I_Y}{\lambda} + q_o + \left(\frac{I_Z - I_X}{I_Y}\right)\frac{p_o r_o}{(\lambda - \lambda_r)}$$

$$-q + \left(\lambda + \frac{L_\alpha}{mV}\right)\alpha = \alpha_o - \frac{p_o \beta_o}{(\lambda - \lambda_r)}$$

where M_Y/I_Y is the pitching-moment input applied in this investigation and λ_r is the one-degree-of-freedom characteristic rolling root.

REFERENCES

1. Phillips, William H.: Effect of Steady Rolling on Longitudinal and Directional Stability. NACA TN 1627, 1948.

2. White, R. J., Uddenberg, R. C., Murray, D., and Graham, F. D.: The Dynamic Stability and Control Equations of a Pivoted-Wing Supersonic Pilotless Aircraft, With Downwash, Wake, and Interference Effects Included. Doc. No. D-8510, Boeing Airplane Co., Jan. 9, 1948.

3. Rhoads, Donald W., and Schuler, John M.: A Theoretical and Experimental Study of Airplane Dynamics in Large-Disturbance Maneuvers. Jour. Aero. Sci., vol. 24, no. 7, July 1957, pp. 507-526, 532.

4. Sternfield, Leonard: A Simplified Method for Approximating the Transient Motion in Angles of Attack and Sideslip During a Constant Rolling Maneuver. NACA Rep. 1344, 1958. (Supersedes NACA RM L56F04.)

5. Churchill, Ruel V.: Modern Operational Mathematics in Engineering. McGraw-Hill Book Co., Inc., 1944.

TABLE I

MASS AND AERODYNAMIC CHARACTERISTICS OF AIRPLANE

m, slugs	745
I_X, slug-ft^2	10,976
I_Y, slug-ft^2	57,100
I_Z, slug-ft^2	64,975
C_{L_α}, per radian	3.85
C_{m_α}, per radian	-0.36
C_{m_q}, per radian	-3.5
C_{Y_β}, per radian	-0.28
C_{n_β}, per radian	0.057
C_{n_r}, per radian	-0.095
V, ft/sec	691
S, sq ft	377
$\bar{c}$, ft	11.3
b, ft	36.6
q, lb/sq ft	197

TABLE II

ROOTS OF THE CHARACTERISTIC EQUATION

p_o	Exact		Approximate	
0	-0.488 ± 2.30i	-0.0729 ± 1.54i	. .	
-1.0	-0.362 ± 2.89i	-0.199 ± 0.942i	-0.352 ± 3.10i	-0.209 ± 0.876i
-1.5	-0.337 ± 3.33i	-0.224 ± 0.483i	-0.340 ± 3.42i	-0.221 ± 0.469i
-1.86	-0.327 ± 3.66i	-0.322, -0.145	-0.332 ± 3.71i	-0.311, -0.146
-2.0	-0.324 ± 3.79i	-0.453, -0.020	-0.329 ± 3.83i	-0.444, -0.020
-2.33	-0.318 ± 4.08i	-0.374, -0.111	-0.322 ± 4.13i	-0.363, -0.113
-2.5	-0.316 ± 4.24i	-0.245 ± 0.258i	-0.320 ± 4.29i	-0.241 ± 0.255i
-3.0	-0.311 ± 4.70i	-0.250 ± 0.760i	-0.312 ± 4.80i	-0.248 ± 0.743i

TABLE III

STEADY-STATE VALUES OF MOTIONS PRESENTED IN FIGURES 2 AND 3

p_o, $\frac{\text{radians}}{\text{sec}}$	$\frac{\beta}{M_Z/I_Z}$, sec^2	$\frac{\Delta\alpha}{M_Z/I_Z}$, sec^2	$\frac{\beta}{M_Y/I_Y}$, sec^2	$\frac{\Delta\alpha}{M_Y/I_Y}$, sec^2
-1	-0.58	-0.058	-0.063	0.213
-1.5	-1.07	-.216	-.236	.249
-1.86	-3.57	-1.35	-1.47	-.106
-2.0	-13.34	-7.03	-7.65	-3.45
-2.33	-.552	-1.52	-1.65	-2.08
-2.5	.171	-.503	-.547	-.898
-3.0	.21	-.097	-.106	-.282

TABLE IV

AERODYNAMIC CHARACTERISTICS FOR FIVE-DEGREE-OF-FREEDOM PROBLEMS

C_{L_α}, per radian	4.15
C_{m_α}, per radian	-0.406
C_{m_q}, per radian	-4.5
C_{Y_β}, per radian	-0.63
C_{n_β}, per radian	0.0515
C_{n_r}, per radian	-0.175
C_{l_p}, per radian	-0.325
C_{l_β}, per radian	-0.03
$\frac{M_Y}{I_Y}$, radians/sec^2	0.5879
$\frac{M_Z}{I_Z}$, radians/sec^2	-0.2092

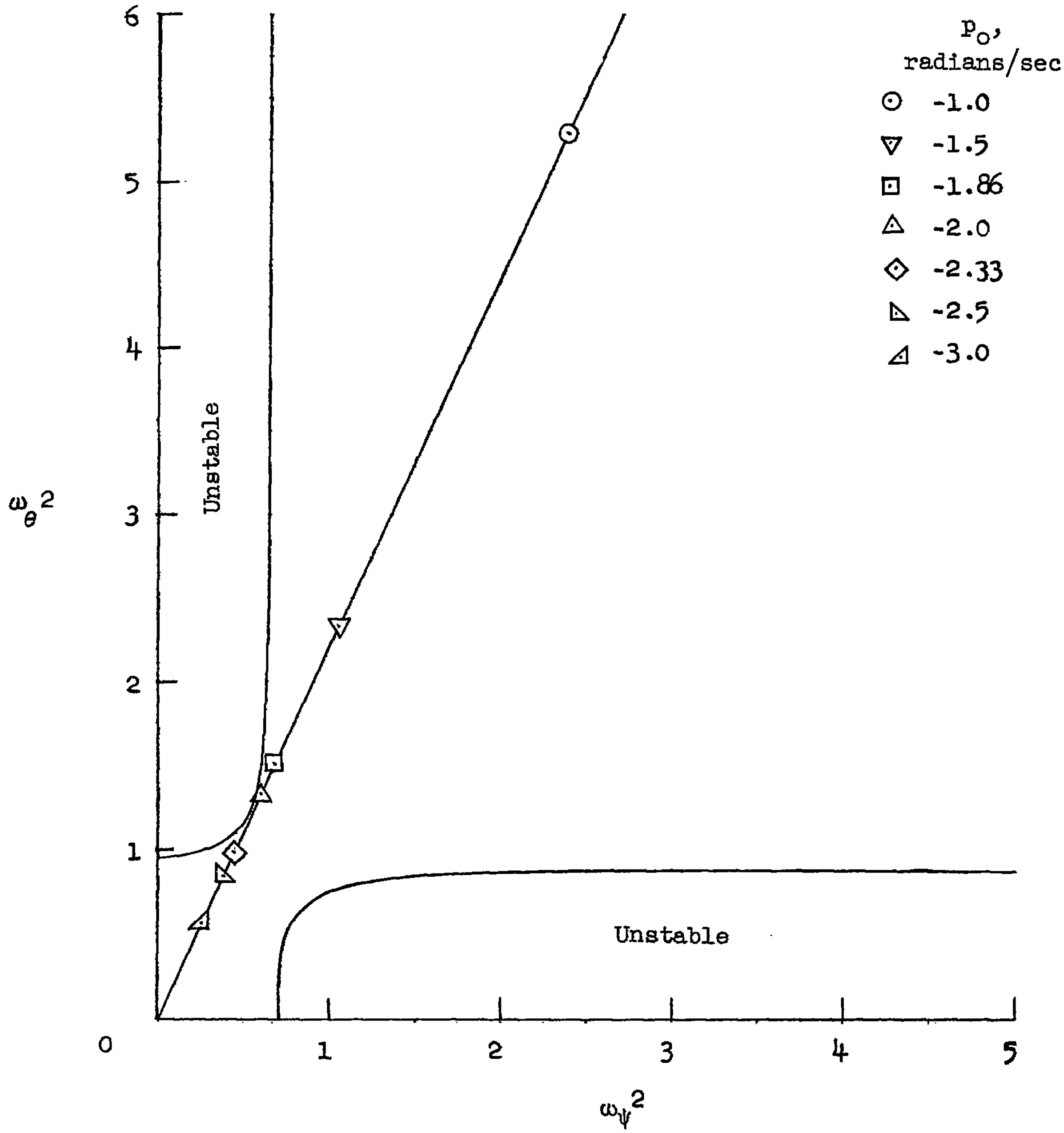
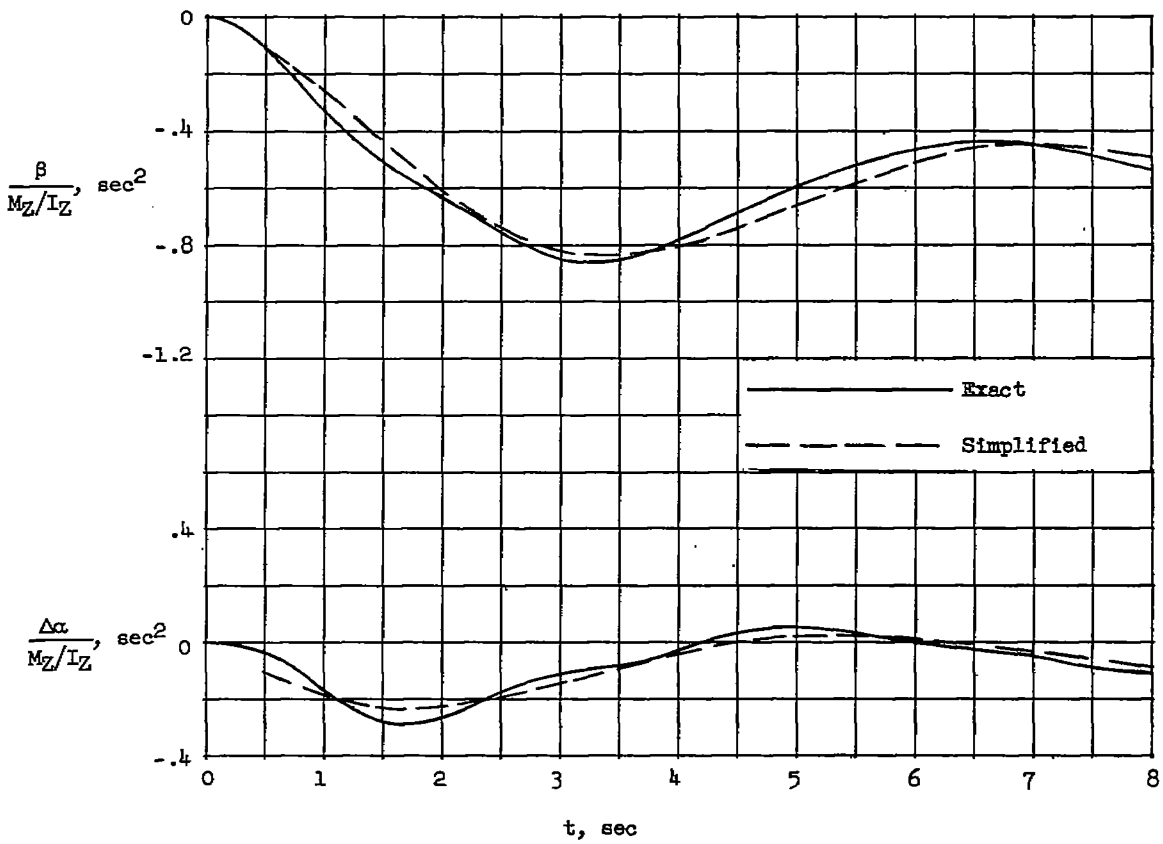
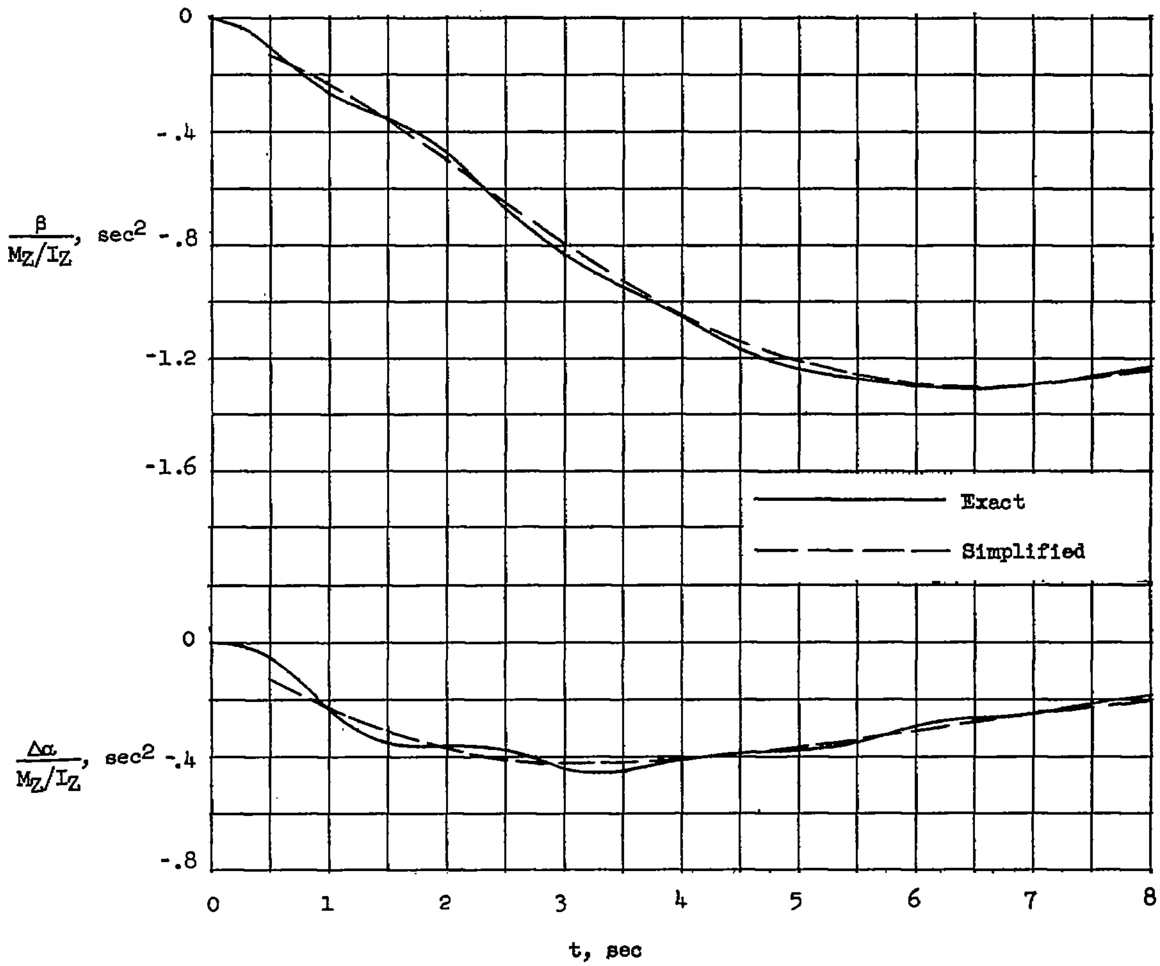


Figure 1.- Boundaries in the ω_θ^2, ω_ψ^2 plane which define regions of aperiodic divergence for example airplane.



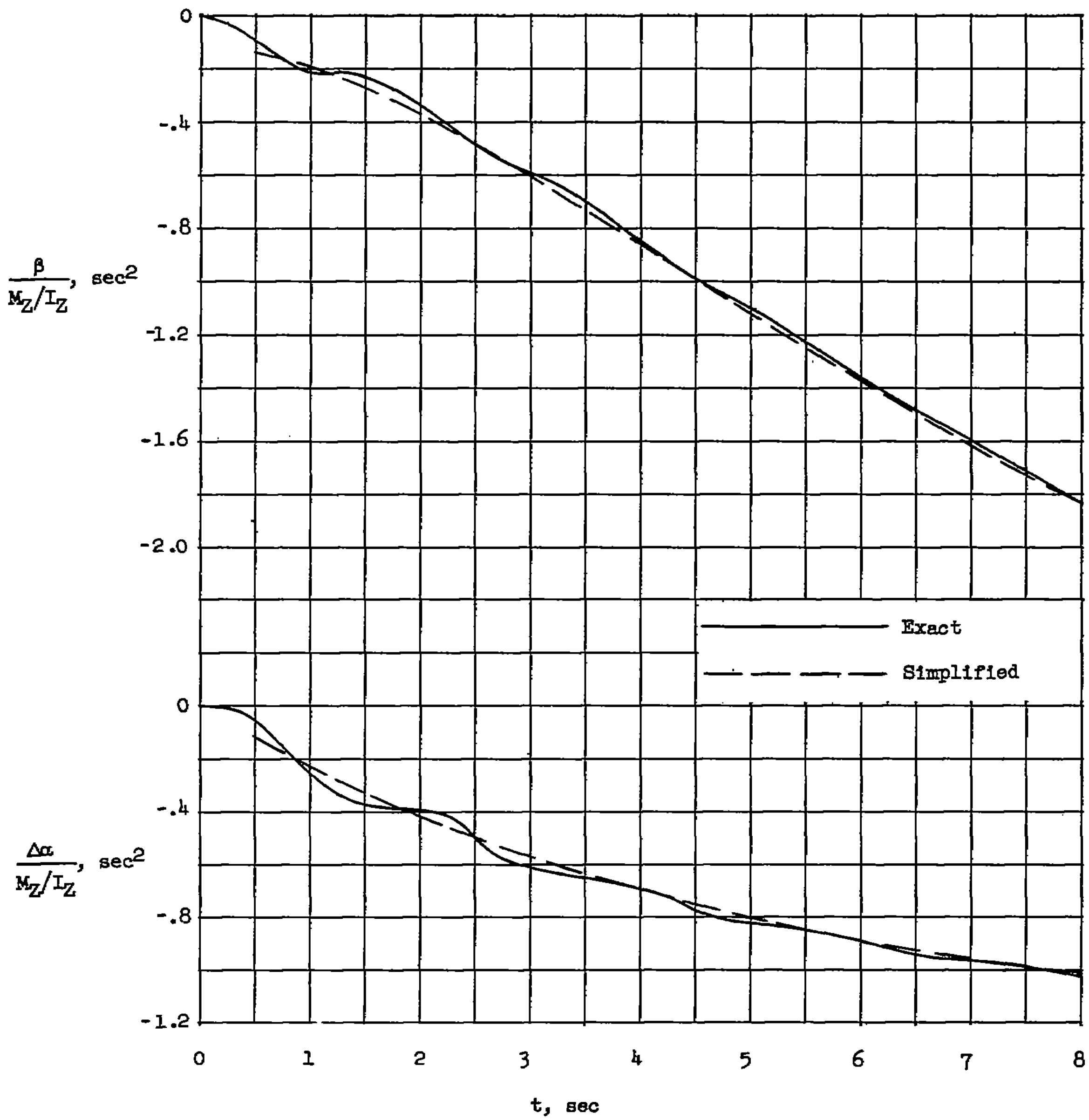
(a) $p_o = -1.0$ radian/sec.

Figure 2.- Time histories of $\frac{\beta}{M_Z/I_Z}$ and $\frac{\Delta\alpha}{M_Z/I_Z}$ for exact and simplified solutions of the four-degree-of-freedom equations.



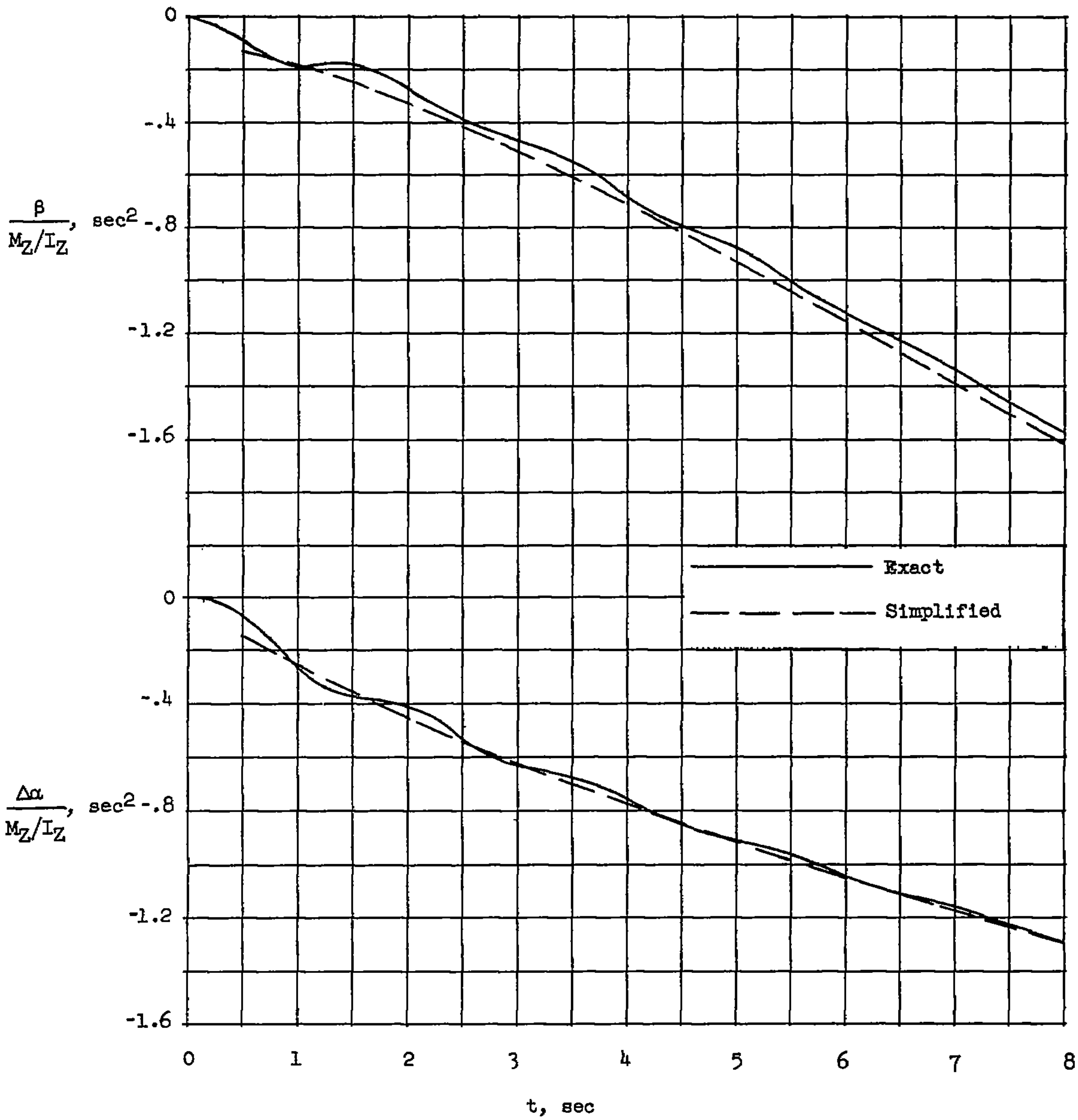
(b) $p_o = -1.5$ radians/sec.

Figure 2.- Continued.



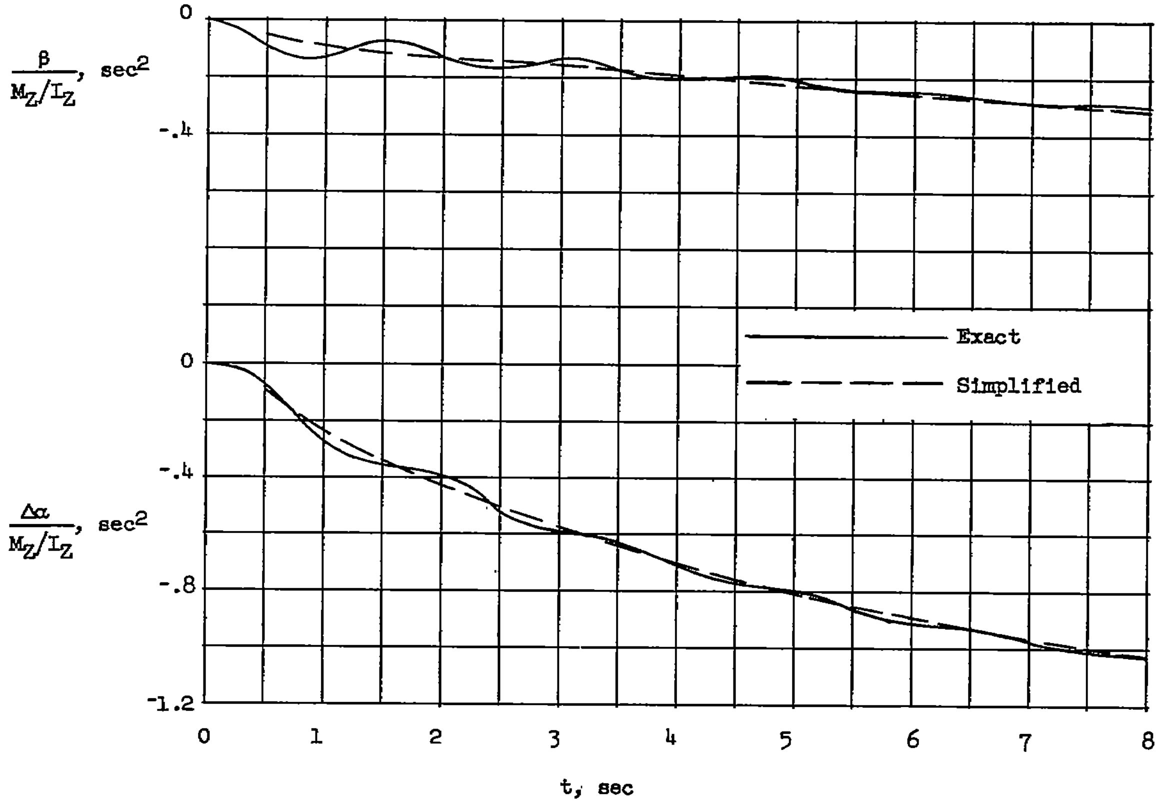
(c) $p_o = -1.86$ radians/sec.

Figure 2.- Continued.



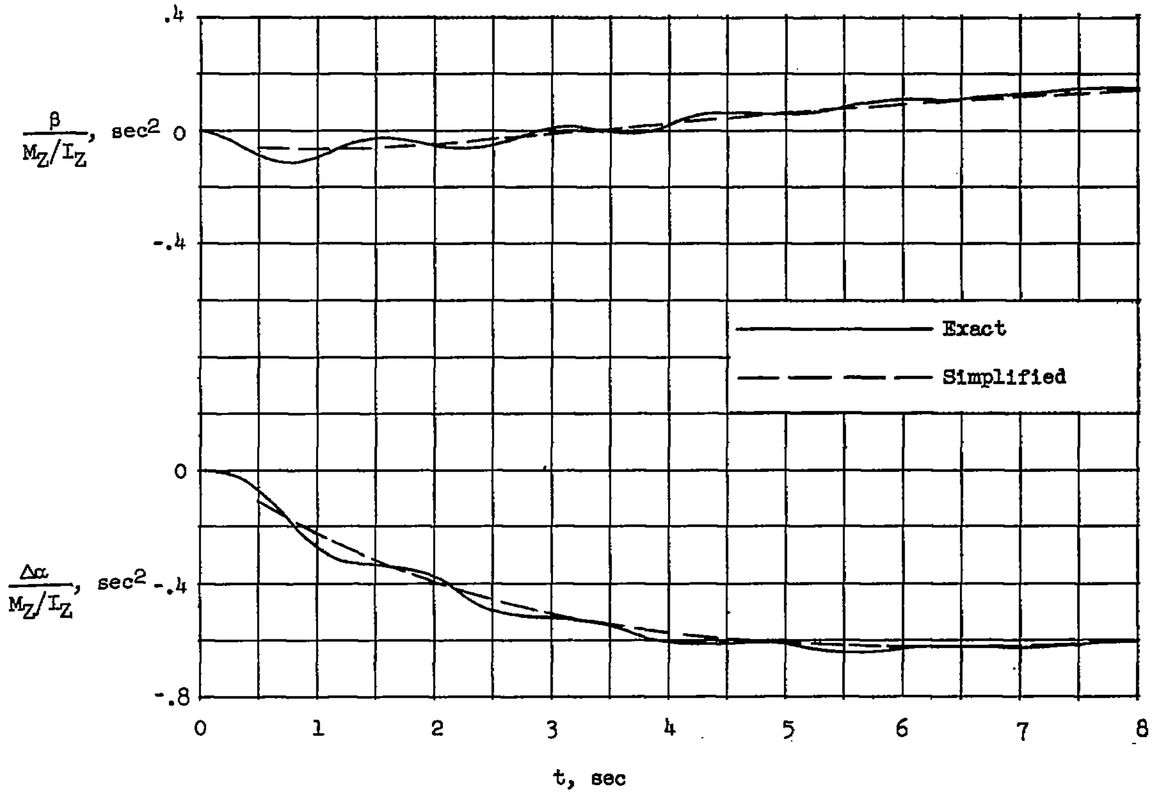
(d) p_o = -2.0 radians/sec.

Figure 2.- Continued.



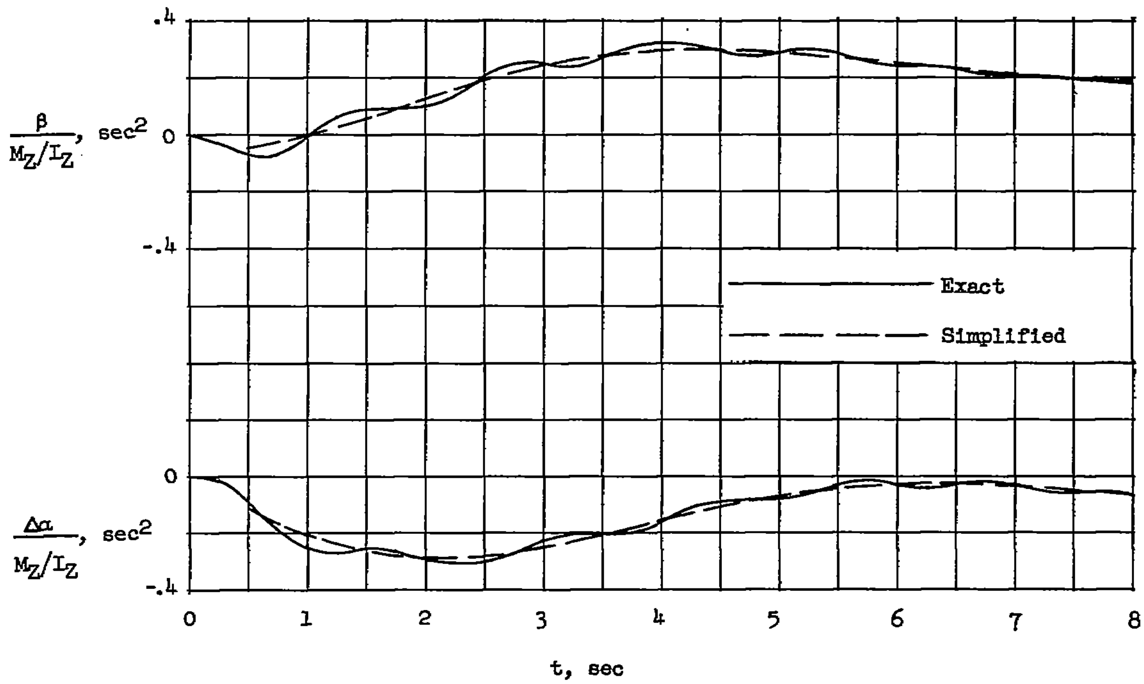
(e) $p_o = -2.33$ radians/sec.

Figure 2.- Continued.



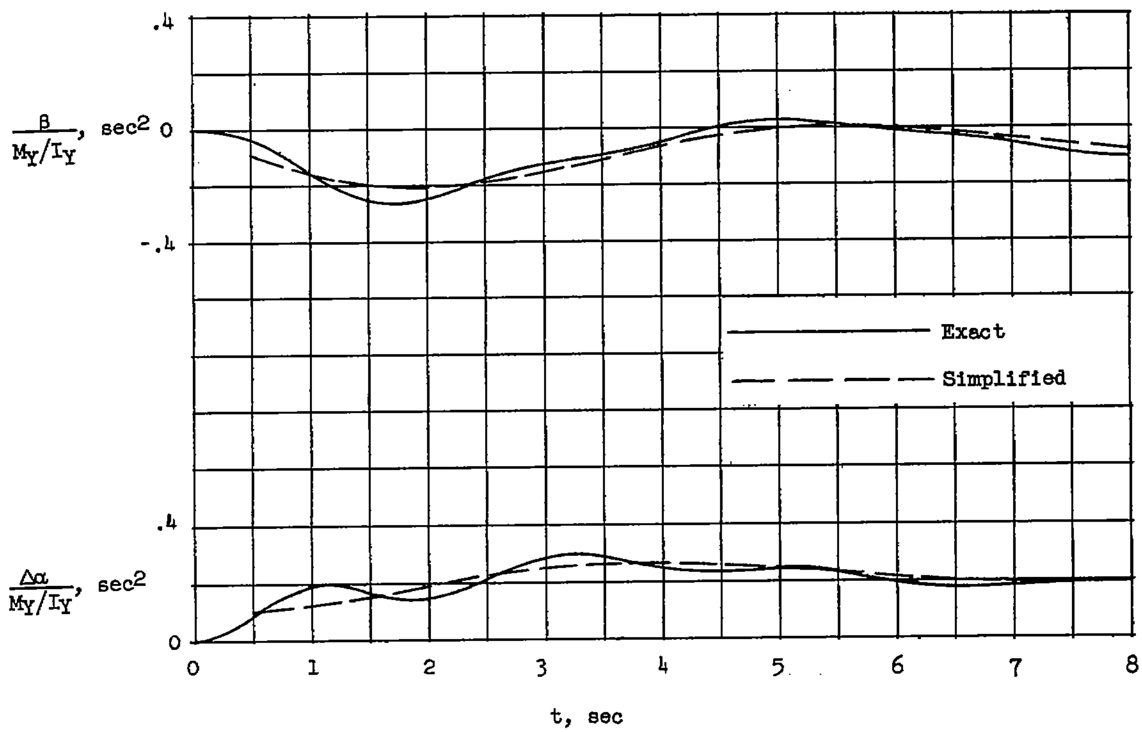
(f) p_o = -2.5 radians/sec.

Figure 2.- Continued.



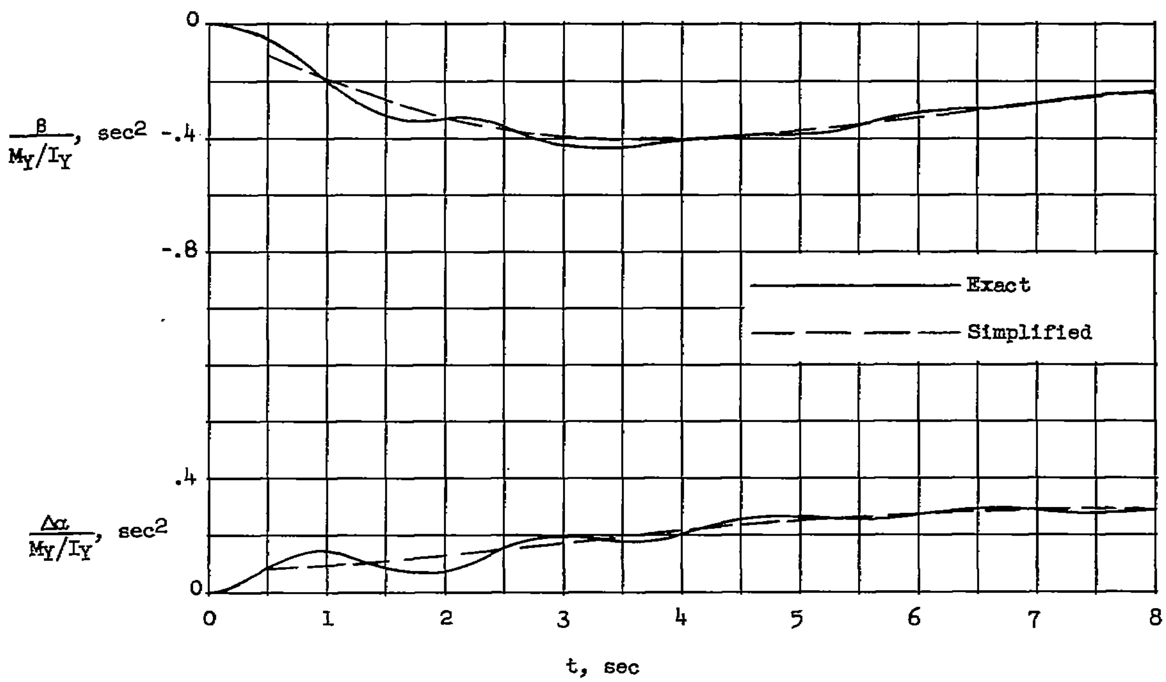
(g) p_o = -3.0 radians/sec.

Figure 2.- Concluded.



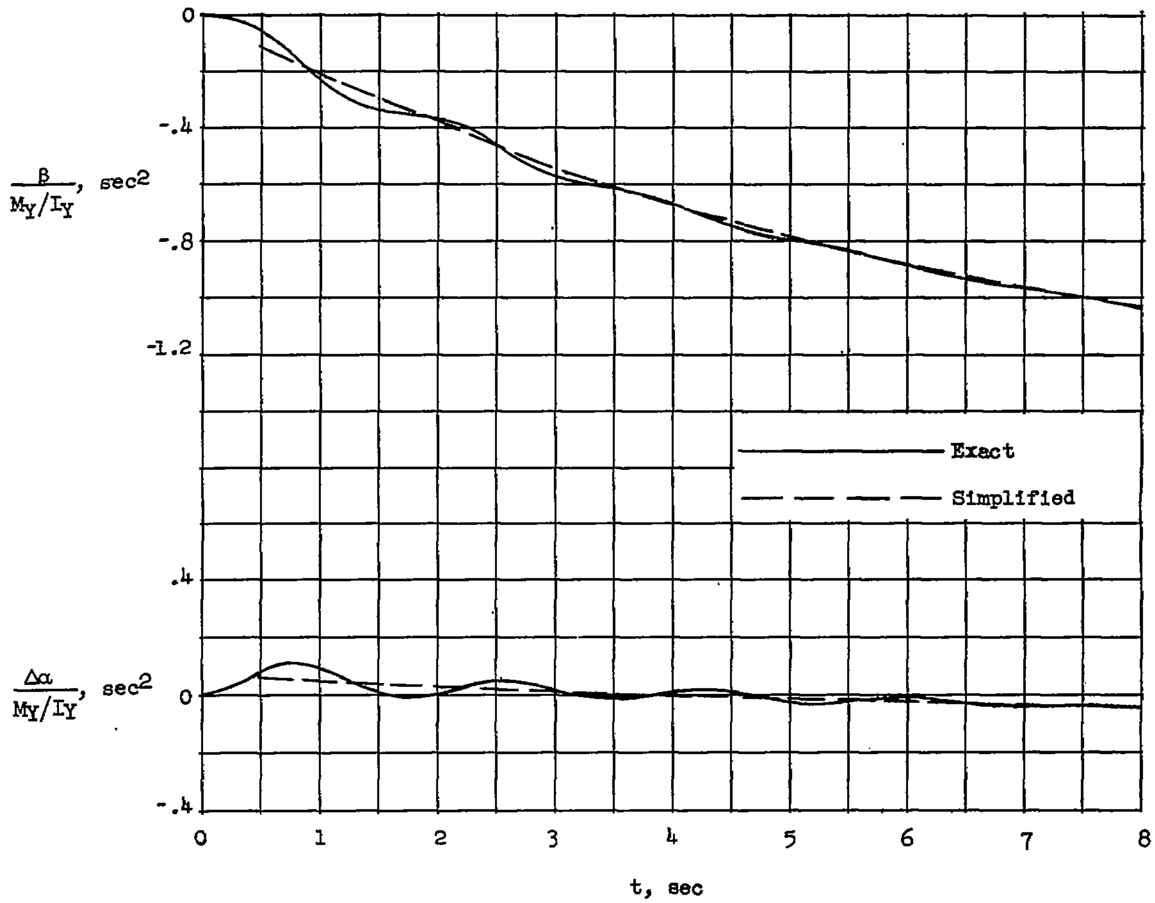
(a) $p_o = -1.0$ radian/sec.

Figure 3.- Time histories of $\frac{\beta}{M_Y/I_Y}$ and $\frac{\Delta\alpha}{M_Y/I_Y}$ for exact and simplified solutions of the four-degree-of-freedom equations.



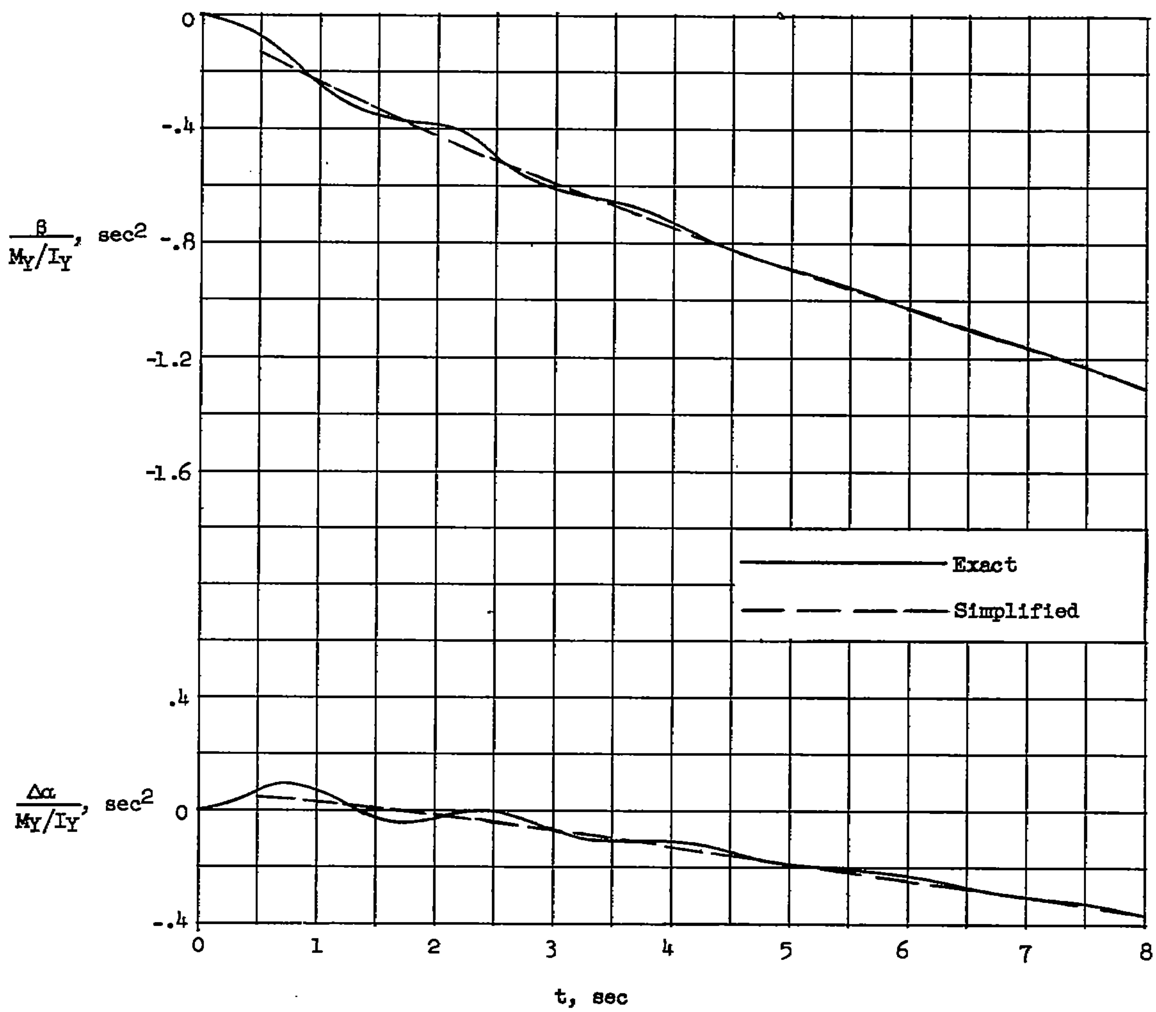
(b) p_o = -1.5 radians/sec.

Figure 3.- Continued.



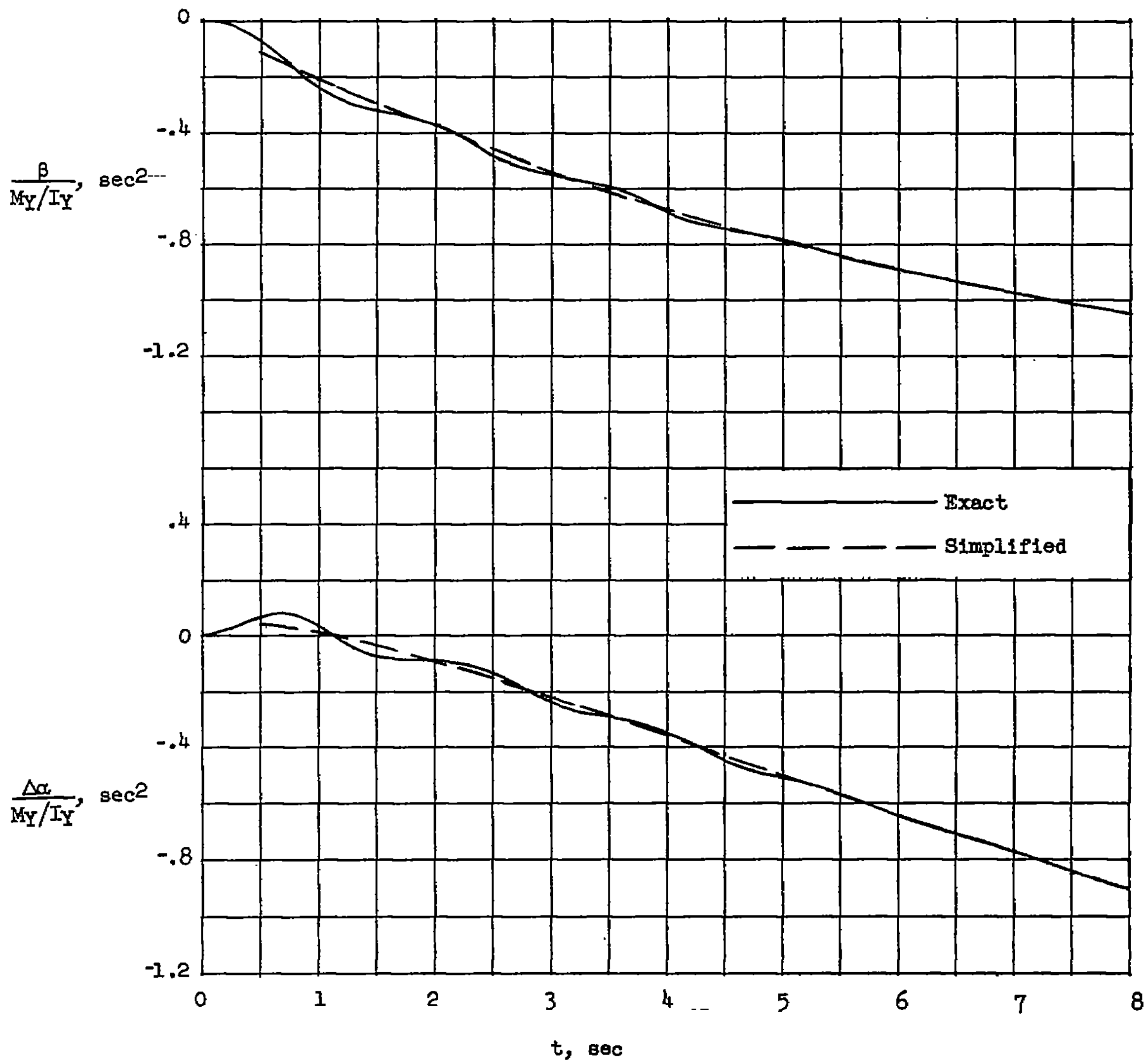
(c) $p_o = -1.86$ radians/sec.

Figure 3.- Continued.



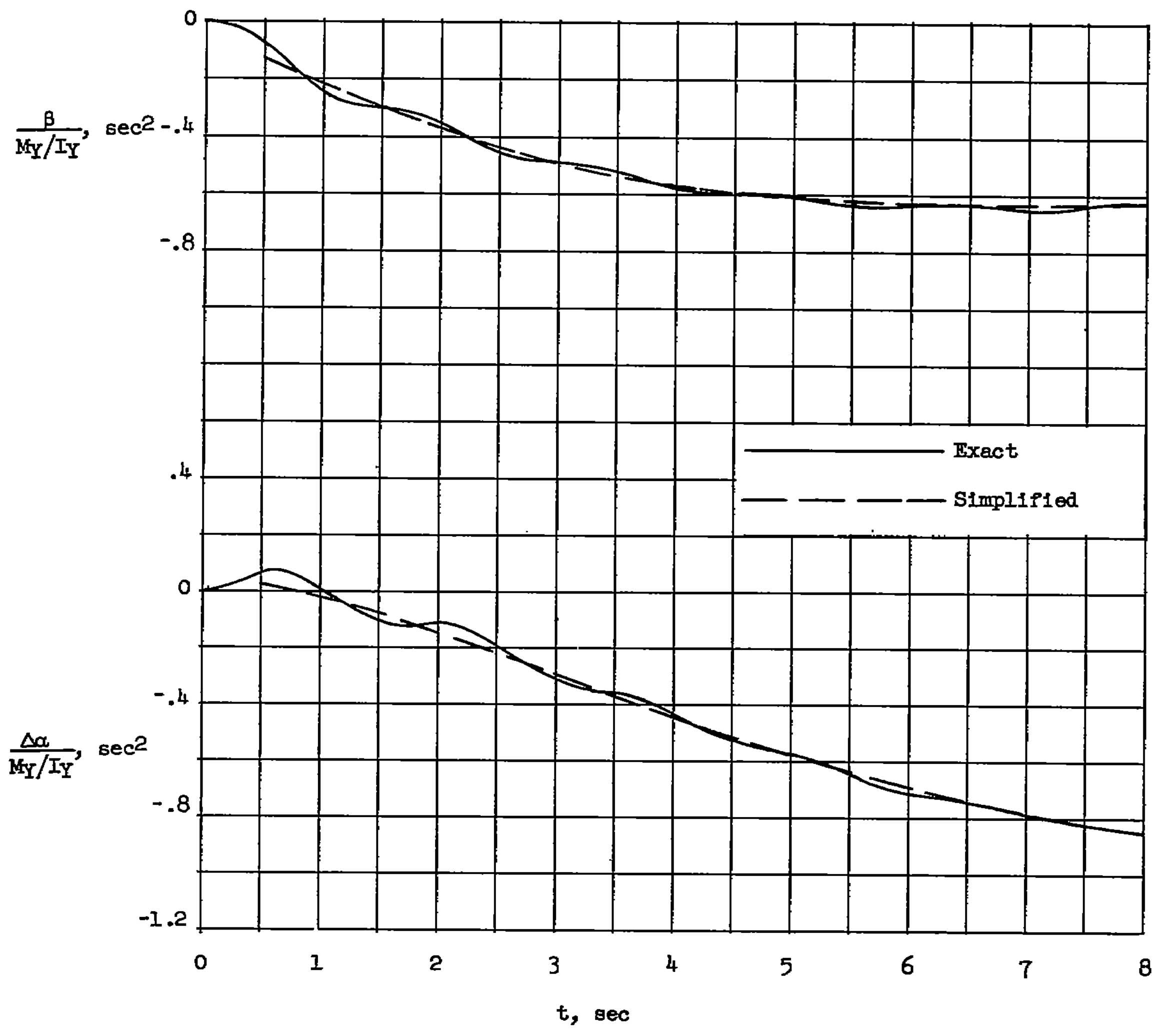
(d) $p_o = -2.0$ radians/sec.

Figure 3.- Continued.



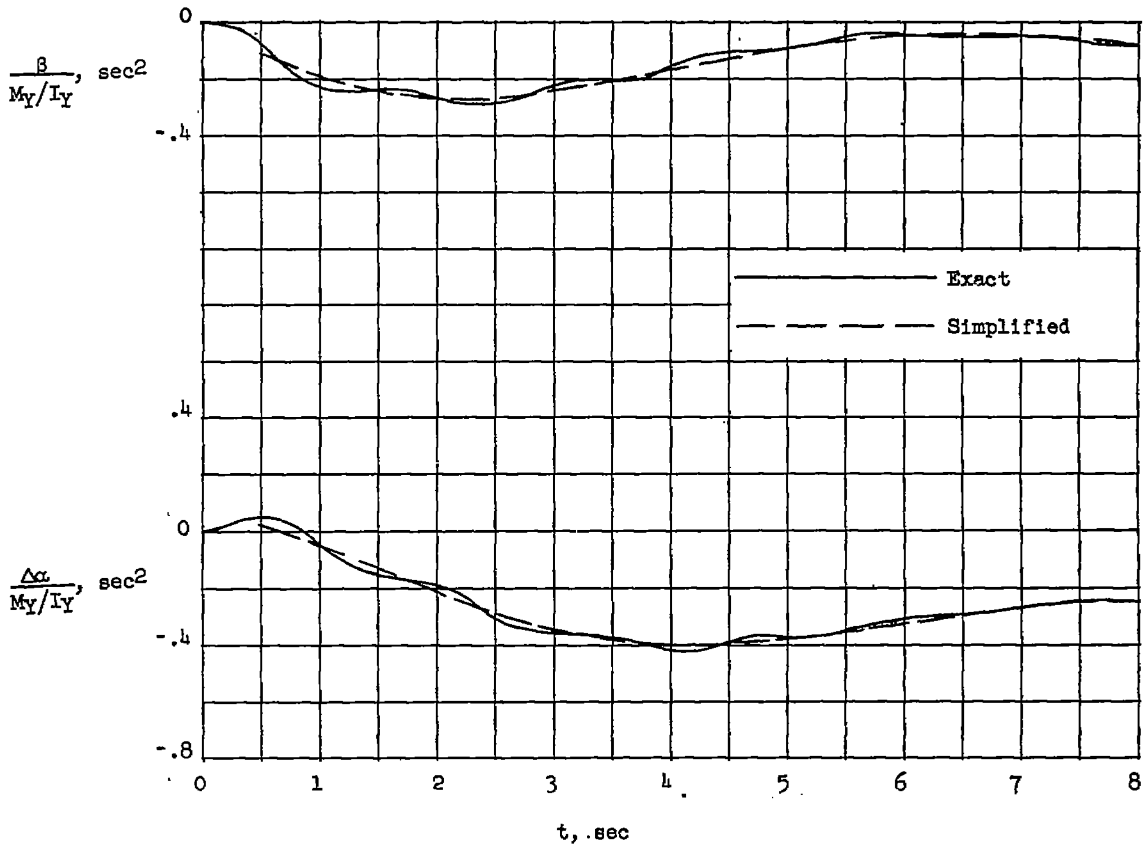
(e) p_o = -2.33 radians/sec.

Figure 3.- Continued.



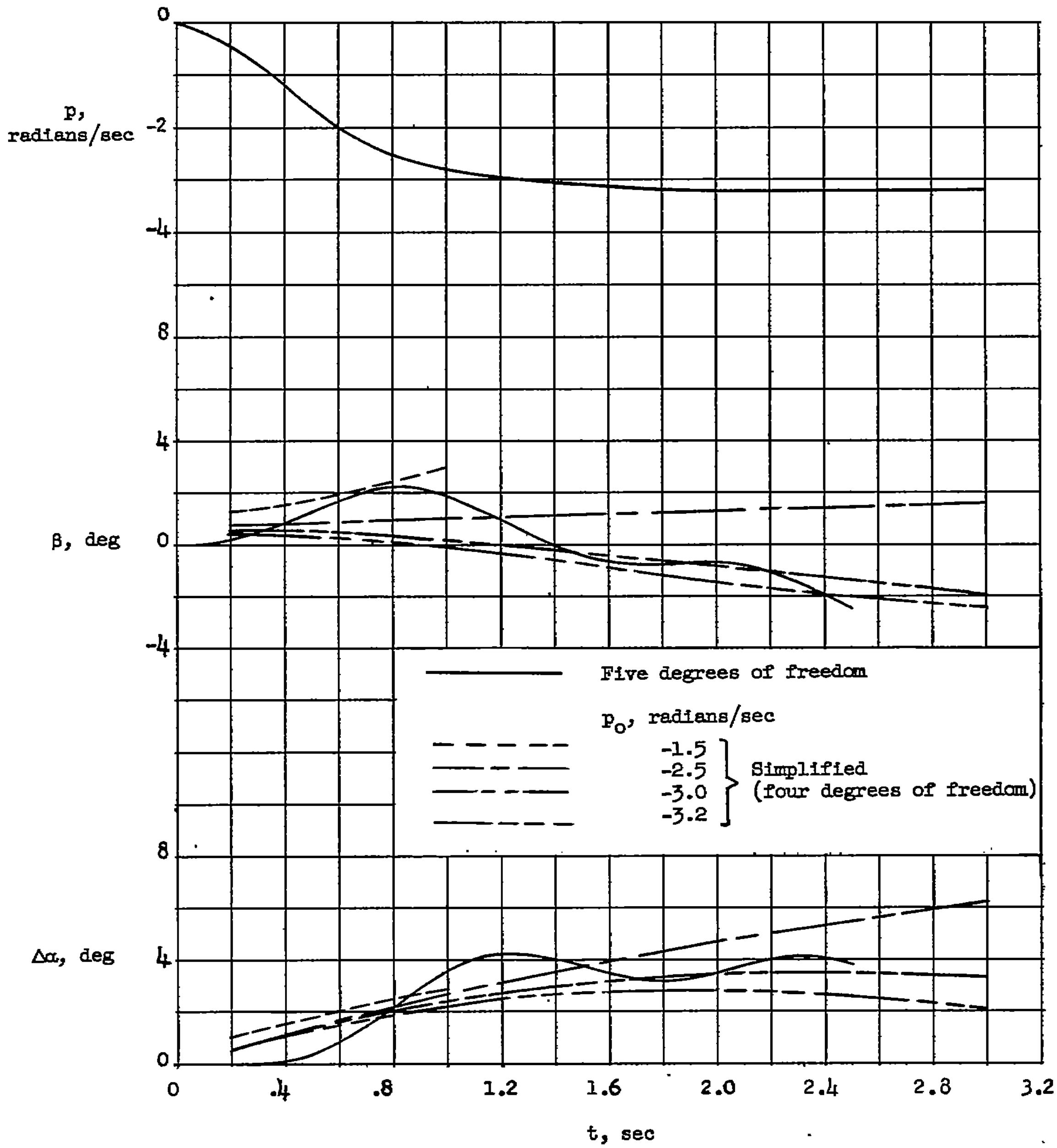
(f) $p_o = -2.5$ radians/sec.

Figure 3.- Continued.



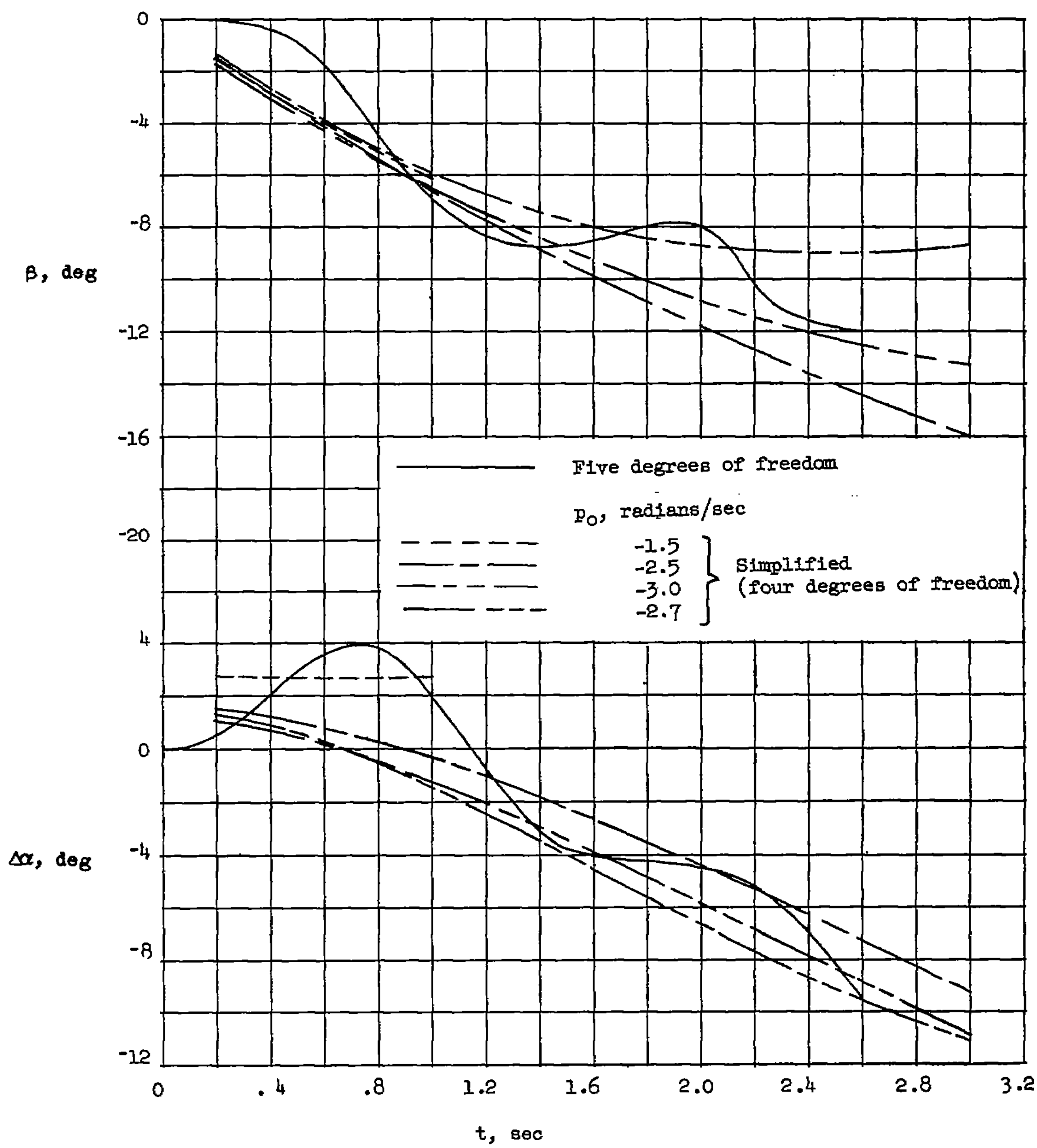
(g) p_o = -3.0 radians/sec.

Figure 3.- Concluded.



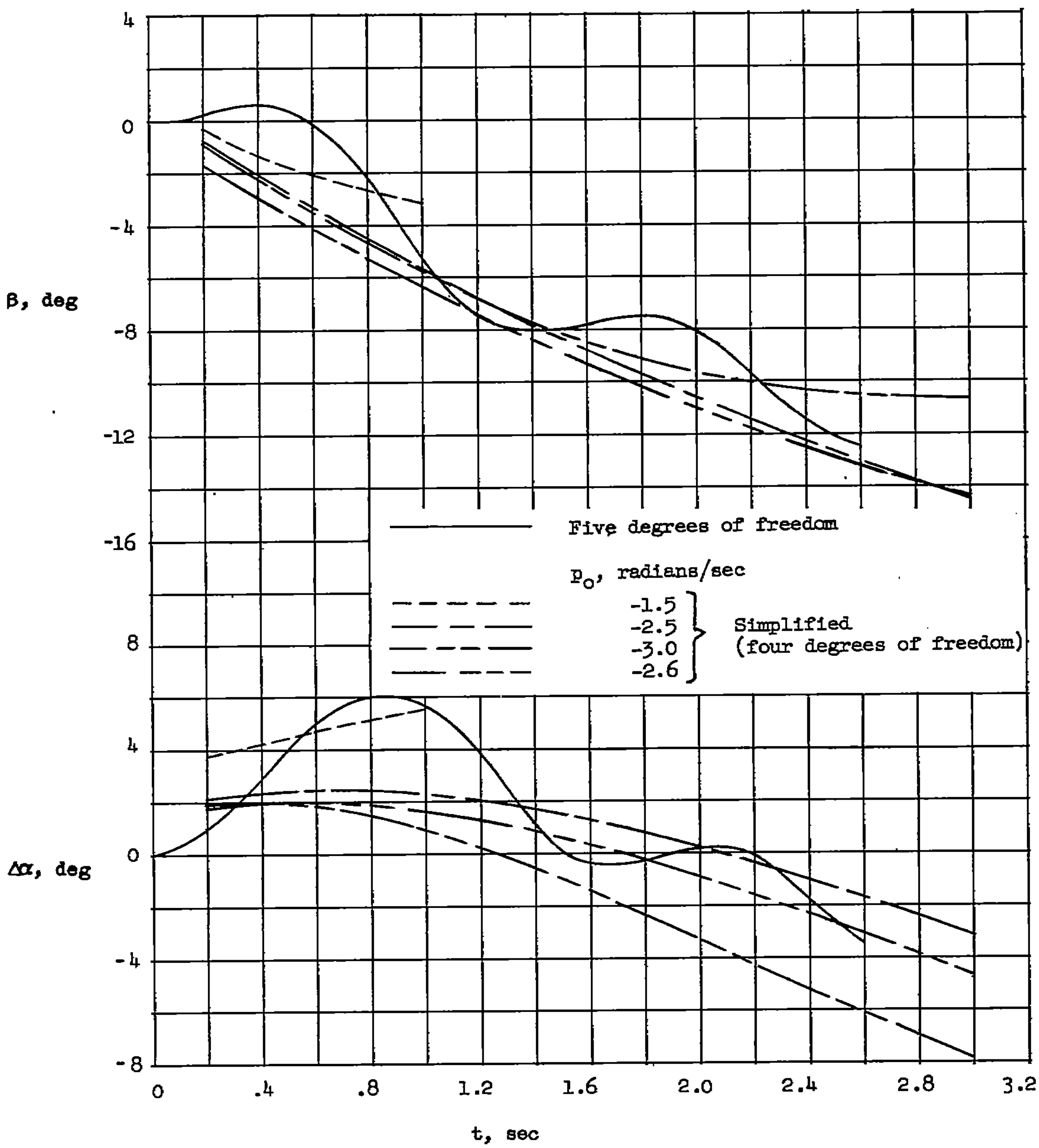
(a) Yawing-moment disturbance, $\frac{M_Z}{I_Z} = -0.2092$.

Figure 4.- Time histories of angles of attack and sideslip in rolling flight.



(b) Pitching-moment disturbance, $\frac{M_Y}{I_Y} = 0.5879$.

Figure 4.- Continued.



(c) Combined yawing- and pitching-moment disturbances, $\frac{M_Y}{I_Y} = 0.5879$, $\frac{M_Z}{I_Z} = -0.2092$.

Figure 4.- Concluded.

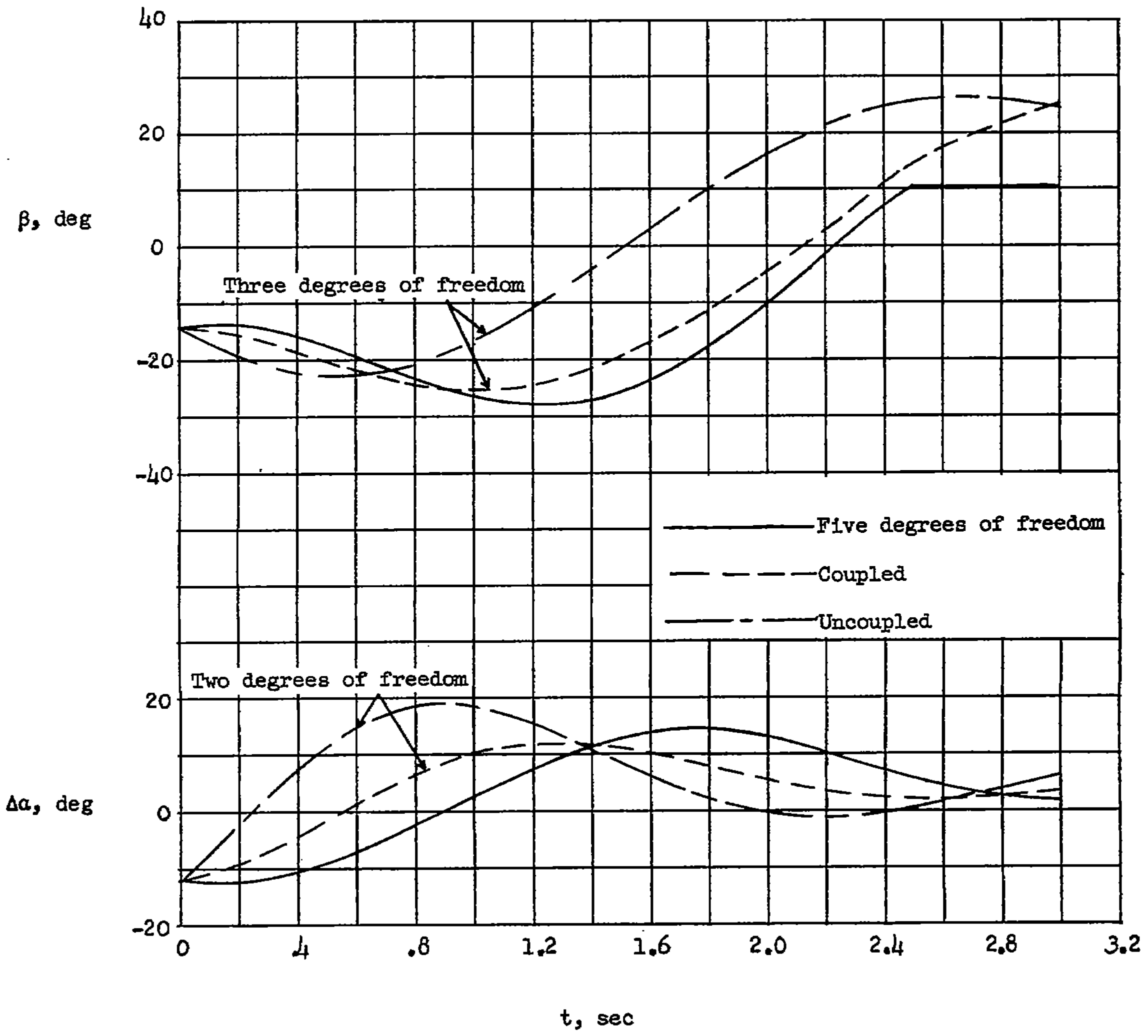


Figure 5.- Motions in angles of attack and sideslip during roll recovery.

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