NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4232

A METHOD FOR THE CALCULATION OF WAVE DRAG ON SUPERSONIC-EDGED WINGS AND BIPLANES

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Washington
March 1958

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A METHOD FOR THE CALCULATION OF WAVE DRAG ON SUPERSONIC-EDGED WINGS AND BIPLANES

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SUMMARY

A method is presented for finding the lift, moment, and drag on three-dimensional wings or biplanes with supersonic edges and a straight trailing edge normal to the free stream. The minimum wave drag for fixed lift or volume is given for several special cases. Simple applications of the method may provide some measure of the degree to which more abstract methods for finding minima can be relied upon as a measure of optimum real systems.

INTRODUCTION

The importance of wave interference in supersonic flow has been clearly demonstrated, but methods for studying its effects on general configurations are quite complicated and lead usually to numerical procedures. The analysis of even the simplest interfering systems is sometimes involved. However, even though involved, such analyses can at least be carried out and results applying to more than just specific combinations can be achieved. These results are useful principally, perhaps, in providing a background of experience needed to extrapolate the meager and laborious calculations for the more practical but more complex configurations.

Two different types of interfering systems in linearized supersonic flow are considered in this report: one, two-dimensional wings in any number of planes; and the other, three-dimensional plane wings or biplanes with supersonic leading edges and trailing edges normal to the free-stream direction. Both of these cases have received previous attention (see refs. 1 through 5) but not, apparently, in the manner presented in the following.

LIST OF IMPORTANT SYMBOLS

a_n see equation (20)

c wing chord

C_D	wing drag coefficient based on plan-form area
C_L	wing lift coefficient based on plan-form area
D	drag
h	distance of wing surface from camber line
k_{mn}	$\dfrac{\binom{2n}{2m}\binom{2m}{m}}{(2\beta)^{2m}}$ where $\binom{2n}{2m}$ and $\binom{2m}{m}$ are binomial coefficients
L	lift
M	moment
M_∞	free-stream Mach number
q_∞	free-stream dynamic pressure
s	wing semispan
S	fixed space within which lift and volume elements are confined
t	distance of upper wing from $z = 0$ plane
u,w	perturbation velocities along x and z axes
V	wing volume
x,y,z	Cartesian coordinates
y_r	distance from xz plane to right edge of Mach envelope
α	angle of attack
β	$\sqrt{M_\infty{}^2-1}$
λ	wing slope
Λ_{2n}	see equation (22)
ν	conormal
σ	$\frac{\beta s}{c}$
τ	wing thickness ratio

φ perturbation velocity potential

Φ_{2n} see equation (17)

VELOCITY FIELDS IN TWO-DIMENSIONAL FLOW

The analysis of the wave equation for two-dimensional flow is extremely simple and the formal presentation given here is intended mainly to serve as an initiation to the similar analysis used in the succeeding section on three-dimensional flow.

A general solution (which follows from an application of Green's theorem to the wave equation) to the two-dimensional wave equation for the perturbation velocity potential φ, namely,

$$\beta^2\varphi_{xx}-\varphi_{zz} = 0 \tag{1}$$

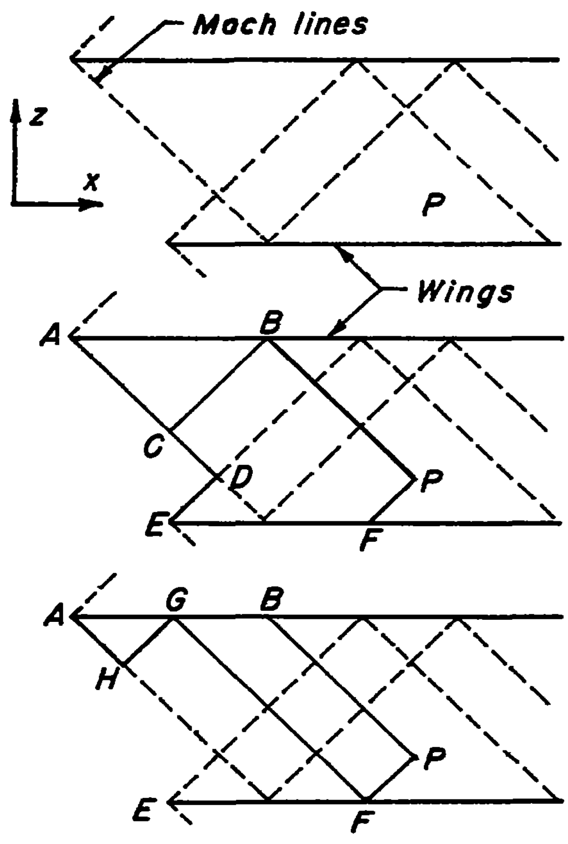
is implied by

$$\frac{1}{\beta}\oint_s \gamma \frac{\partial\varphi}{\partial\nu} |ds| = 0 \tag{2}$$

where s is the boundary of a closed area and ν is the conormal to s. For our purposes s is composed of straight line segments parallel either to the free stream - flowing in the x direction - or to characteristic (Mach) lines. The symbol γ then takes the form:

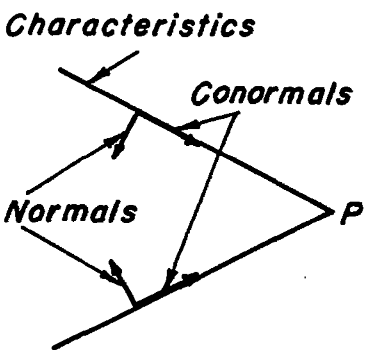
$$\gamma = \begin{cases} 1 & \text{along the free stream} \\ \beta & \text{along a characteristic line} \end{cases}$$

Let us apply equation (2) to find the velocity potential between two interfering wings - for example, at the point P in sketch (a). We can immediately write the four nonredundant equations



Sketch (a)

$$\left.\begin{aligned}
\frac{1}{\beta}\int_{\mathrm{PBADEFP}} \gamma \frac{\partial\varphi}{\partial\nu}\,|ds| = \frac{1}{\beta}\int_{\mathrm{BACB}} \gamma \frac{\partial\varphi}{\partial\nu}\,|ds| = 0 \\
\frac{1}{\beta}\int_{\mathrm{PBGFP}} \gamma \frac{\partial\varphi}{\partial\nu}\,|ds| = \frac{1}{\beta}\int_{\mathrm{GAHG}} \gamma \frac{\partial\varphi}{\partial\nu}\,|ds| = 0
\end{aligned}\right\} \tag{3}$$



Sketch (b)

which contain the four unknowns φ_P, φ_B, φ_F, and φ_G (meaning the value of the potential at the points used as subscripts). The conormal to a line parallel to the free stream is the same as the normal. The conormal to a characteristic lies along the characteristic as shown in sketch (b) (the direction is determined, essentially, by reversing the sign of the x component of the normal). Hence, the first integral in equation (3) becomes[1]

$$\int_P^B \frac{\partial\varphi}{\partial(-s)}\,|ds| + \frac{1}{\beta}\int_A^B \frac{\partial\varphi}{\partial(-z)}\,dx_1 + \int_A^D \frac{\partial\varphi}{\partial(-s)}\,|ds| + \int_D^E \frac{\partial\varphi}{\partial s}\,ds + \frac{1}{\beta}\int_E^F \frac{\partial\varphi}{\partial z}\,dx_1 + \int_F^P \frac{\partial\varphi}{\partial s}\,|ds| = 0 \tag{4}$$

Along the leading characteristics ADE the potential is zero so this equation reduces to

$$2\varphi_P - \varphi_B - \varphi_F - \frac{1}{\beta}\int_A^B w\,dx_1 + \frac{1}{\beta}\int_E^F w\,dx_1 = 0 \tag{5}$$

[1]The element $|ds|$ is always positive - fixing the positive sense of the dx integrals. The positive direction of the ds integrals is immaterial since, along a characteristic line, the sign of ds divided by its conormal is the same for either choice of positive direction.

Evaluating the other integrals and combining, one finds

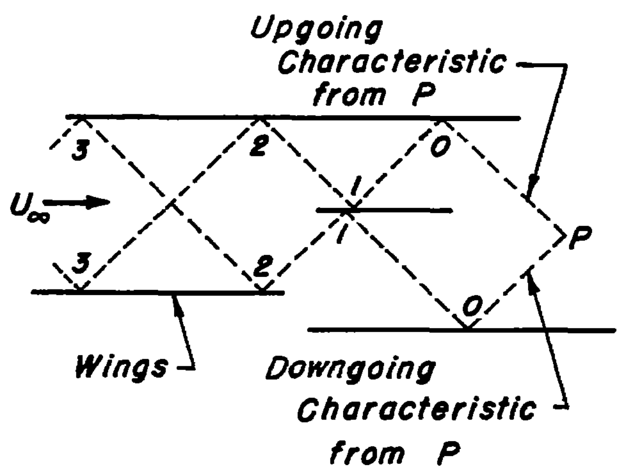
$$\varphi_P = \frac{1}{\beta}\int_A^G w\,dx_1 + \frac{1}{\beta}\int_A^B w\,dx_1 - \frac{1}{\beta}\int_E^F w\,dx_1$$

or differentiating with respect to z and x, respectively, taking into account the dependency of G, B, and F on the point $P(x,z)$

$$w_P = -w_G + w_B + w_F \tag{6}$$

$$u_P = \frac{1}{\beta}\left(w_G + w_B - w_F\right) \tag{7}$$

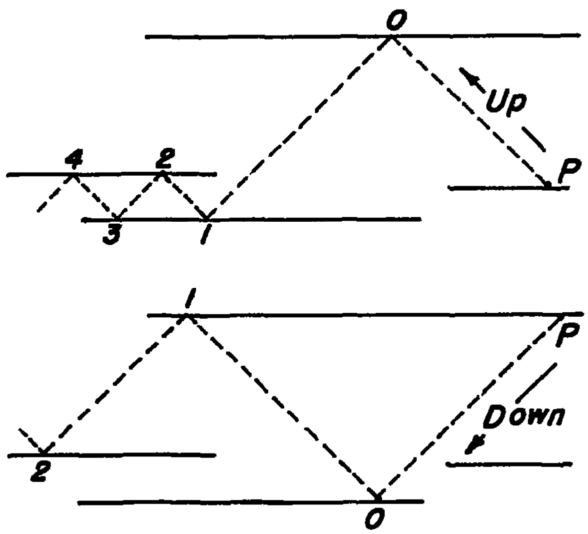
Equations (6) and (7) show that w and u at any point depend solely on the slopes (the wing slope by linearized theory is w/U_∞) of the wing surfaces at the point met by the reflecting forward characteristics from P. These equations can easily be generalized to form an expression for the induced velocities in an arbitrary two-dimensional linearized flow. Let w_{u_i} represent the value of the vertical velocity given by the wing slope at a point where the upgoing forward characteristic from P reflects from a wing surface, see sketch (c), and let w_{d_i} represent the same for the downgoing characteristic. Then the value of u and w at P are simply



Sketch (c)

$$w_P = \sum_{0}^{n}(-1)^i w_{u_i} + \sum_{0}^{m}(-1)^i w_{d_i} \tag{8}$$

$$u_P = \frac{1}{\beta}\left[\sum_{0}^{n}(-1)^i w_{u_i} - \sum_{0}^{m}(-1)^i w_{d_i}\right] \tag{9}$$



Sketch (d)

If the point P lies on the wing surface, these equations simplify. In such a case equation (8) becomes an identity and equation (9) becomes, for a point on the upper surface of a wing (see sketch (d)),

$$u_P = \frac{1}{\beta}\left[-w_P + 2\sum_{0}^{n}(-1)^i w_{u_i}\right] \tag{10a}$$

and for a point on the lower surface of a wing

$$u_P = \frac{1}{\beta}\left[w_P - 2\sum_{0}^{m}(-1)^i w_{d_i}\right] \tag{10b}$$

Since the wing slope λ is given by

$$\lambda = w/U_\infty$$

and the pressure coefficient by

$$C_p = -2u/U_\infty$$

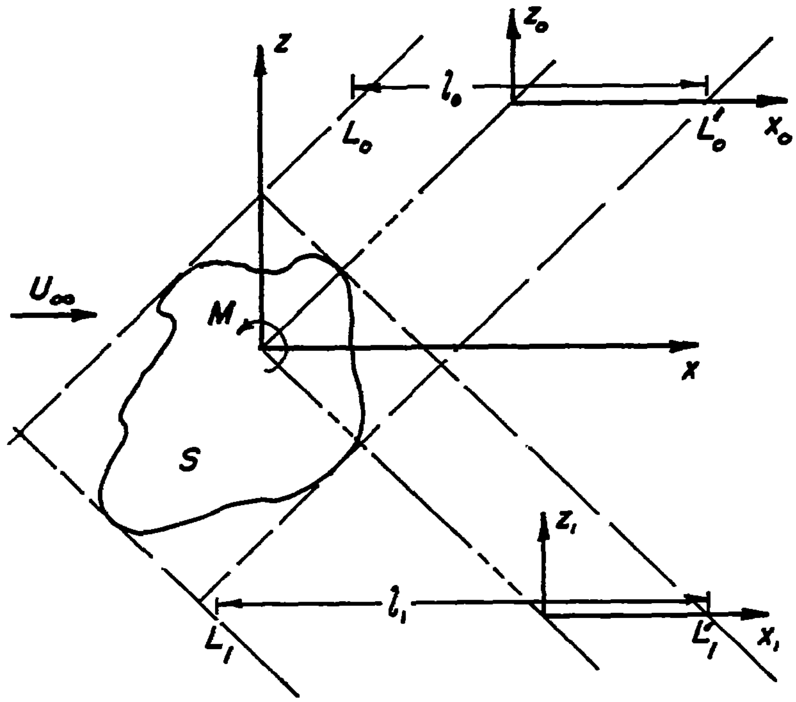
the forces and moments on any system of wings in a two-dimensional linearized flow field can be determined by the above equations.

OPTIMUM TWO-DIMENSIONAL LIFTING SYSTEMS

The conditions for optimum lifting systems in two-dimensional supersonic flow are easily obtained. By optimum systems we will mean those which have minimum values of $C_D/\beta C_L^2$ under certain restraints and neglecting friction drag.[2] One of the simplest means for expressing these optimums is to study the momentum flux across control surfaces above and below the wings.

[2]It must be emphasized that friction drag is of utmost importance in the practical consideration of biplanes. This report, however, is devoted to the study of some of the properties of nonviscous fields.

Consider a group of wings confined within a given space S, such as that shown in sketch (e), and an x,z coordinate system fixed somewhere



Sketch (e)

relative to S. Then the lift, drag, and pitching moment about x=z=0 are given to the lowest order by

$$\frac{L}{q_\infty} = \frac{2}{U_\infty}\left(\int_{L_o}^{L_o'} u_o dx_o - \int_{L_1}^{L_1'} u_1 dx_1\right) \tag{11}$$

$$\frac{M}{q_\infty} = \frac{2}{U_\infty}\left(\int_{L_o}^{L_o'} x_o u_o dx_o - \int_{L_1}^{L_1'} x_1 u_1 dx_1\right) \tag{12}$$

$$\frac{D}{q_\infty} = \frac{2\beta}{U_\infty^2}\left(\int_{L_o}^{L_o'} u_o^2 dx_o + \int_{L_1}^{L_1'} u_1^2 dx_1\right) \tag{13}$$

where the x_o, z_o and x_1, z_1 coordinate systems are above and below S, respectively, with their origins on the Mach lines from x=z=0, and u_o and u_1 are the values of u along x_o and x_1, respectively.

Employing calculus of variation techniques, we find for a minimum drag

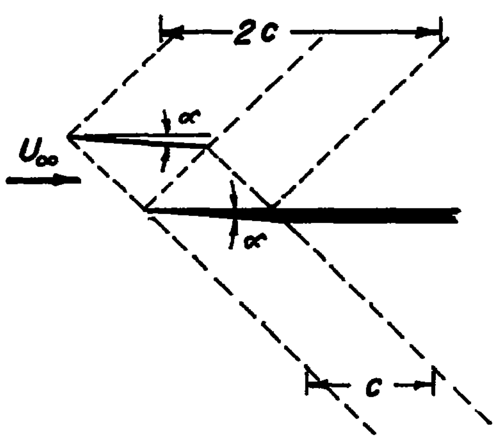
$$\left.\begin{aligned}\frac{u_0}{U_\infty} &= -\frac{1}{2\beta}\left(K_0 + x_0 K_1\right)\\[2ex]\frac{u_1}{U_\infty} &= \frac{1}{2\beta}\left(K_0 + x_1 K_1\right)\end{aligned}\right\} \tag{14}$$

where K_0 and K_1 are constants fixed by the given lift and pitching moment. If the upgoing and downgoing waves from S are equally wide and the pitching moment is to be zero about the center of their intersection, K_1 is zero and the optimum u variation is simply a constant. This is the case for the "ordinary" unstaggered Busemann biplane according to linearized theory.

In any case, by setting K_1 equal to zero, one obtains results representing the absolute minimum value of $C_D/\beta C_L{}^2$. It follows immediately that this minimum is given by

$$\frac{D/q_\infty}{\beta(L/q_\infty)^2} = \frac{1}{2(l_0 + l_1)} \tag{15}$$

where l_0 and l_1 are the widths in the free-stream direction of the waves from S.



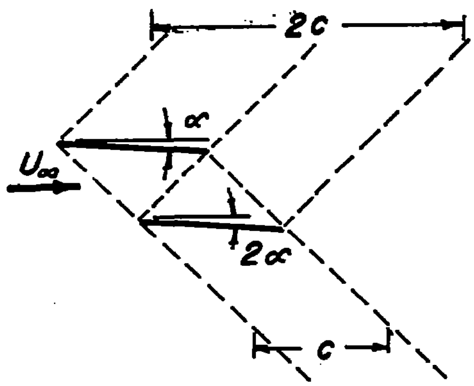
Sketch (f)

However, the potential-flow minimum given by equation (15) is not always realistic for the simple reason that it cannot always be attained by a real system of wings with finite chords. A simple example of this is the staggered biplane shown in sketch (f). For coefficients based on the wing chord, equation (15) states

$$\left.\frac{C_D}{\beta C_L{}^2}\right)_{min} = \frac{1}{6}$$

It is easily verified by means of equation (9) that this minimum cannot be attained if both wings are closed.[3] In the particular example shown, the upper wing was taken to be a flat plate, in which case the lower wing must be an open wedge.

This difficulty with closure can readily be overcome in two-dimensional flows by adding another restraint - namely, that the net mass flow through the enclosing control surface be zero. Systems of closed, two-dimensional wings having minimum $C_D/\beta C_L^2$ (unrestrained as to pitching moment and friction drag) have been thoroughly studied by Licher in reference 1. She shows that the optimum pair of closed wings in the relative positions shown in sketch (g) is composed of two flat plates, the lower one at twice the angle of attack of the upper, and the minimum value of $C_D/\beta C_L^2$ (based, again, on the wing chord) is



Sketch (g)

$$\frac{C_D}{\beta C_L^2} = \frac{3}{16}$$

The principal point to be made in the above discussion can be expressed as follows: Systems having a minimum drag under the sole restraint that the lift (and moment) be fixed may be composed of wings that

(1) Necessarily have some volume

(2) May not close

The stressing of this point may seem unnecessary since, for example, it has been known for some time that lift and volume can have favorable interference when the lift is carried above the volume. In fact, in two dimensions, many multiplanar combinations of lift and volume necessary to give minimum $C_D/\beta C_L^2$ are illustrated in reference 1. In studies of optimum fields in three-dimensional flows, however, these issues are more obscure and the statement of the problem is sometimes dominated by experience with the vast number of planar problems where lift and volume always separate. Consider, for example, the minimization techniques introduced by Ward in reference 6. Aside from the condition that the drag be minimized for a fixed lift, the added restraint of mass continuity across the control surfaces should be introduced since, in general, some volume will be required in a three-dimensional flow to obtain a minimum $C_D/\beta C_L^2$. Of course, this is only a necessary condition for existence;

[3]It can never be attained by closed wings having enclosing Mach waves of unequal widths.

a sufficient condition would require that the volume be everywhere real. Illustrations of this difficulty are given in the next section.

A study of two-dimensional flow provides little experience for the problem of "real volumes" in the general case. This is because mutually interfering multiwing systems in two dimensions always have solutions that have no net lift or wave drag but have arbitrary amounts of volume, within, always, the limits of the assumptions bounding the theory. Since these solutions all have zero momentum flux everywhere external to their enclosing Mach waves, they can be added (in linearized theory) to any other solution without affecting its lift or drag but providing, for the whole, a real system of wings. In the three-dimensional case these zero lift and drag solutions do not generally exist,[4] however, and the existence problem is much more complicated.

THREE-DIMENSIONAL WINGS WITH SUPERSONIC LEADING EDGES AND STRAIGHT UNSWEPT TRAILING EDGES

Derivation of Basic Equations

The linearized equation governing three-dimensional supersonic flow is

$$\beta^2\varphi_{xx} - \varphi_{yy} - \varphi_{zz} = 0 \tag{16}$$

Let us consider flows that are symmetrical about an xz plane and place the x and z axes in the plane of symmetry. Then we use the definition[5]

$$\Phi_{2n} = \int_0^{y_r} y^{2n}\varphi(x,y,z)\,dy \tag{17}$$

Multiplying equation (16) by y^{2n} and integrating each term from 0 to y_r, we find

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[4]A three-dimensional zero lift and drag solution for a finite space probably exists only when the space is completely enshrouded by a surface having outer boundaries parallel to the free stream.

[5]The definition can be extended to odd powers (in the sense that the whole flow field expands in the form $|y|^n$), provided the first power is not used. Incidentally the term $|y|$ is of considerable interest since it appears in the study of a delta wing. Unfortunately, however, to use this method on a flow containing directly the term $|y|$ the value of $\varphi(x,o,z)$ is required.

$$\beta^2\Phi_{2n_{xx}} - \Phi_{2n_{zz}} = \left(\frac{\partial\varphi}{\partial\nu}\right)_{y_r} y_r^{2n} + 2n(2n-1)\Phi_{2n-2}$$

where $(\partial\varphi/\partial\nu)_{y_r}$ is the conormal derivative of φ on the surface y_r. Since this is a characteristic surface, the conormal lies along the surface and the term $(\partial\varphi/\partial\nu)_{y_r}$ is zero. Hence, our basic equation reduces to

$$\beta^2\Phi_{2n_{xx}} - \Phi_{2n_{zz}} = 2n(2n-1)\Phi_{2n-2} \tag{18}$$

Let us consider one surface of a wing located in the plane $z = t$. The lift on this surface is given by

$$\frac{L}{q_\infty} = \pm 2\int_0^c dx \int_0^{y_r} \frac{2\varphi_x(x,y,t)}{U_\infty}\, dy$$

where c is the root chord and the upper sign applies if it is an upper surface and the lower sign if it is a lower surface. This becomes

$$\frac{L}{q_\infty} = \pm \frac{4}{U_\infty}\int_0^c \Phi_{o_x}(x,t)dx = \pm \frac{4}{U_\infty}\Phi_o(c,t) \tag{19}$$

Similarily, the moment and drag are given by the equations

$$\frac{M}{q_\infty} = \pm \frac{4}{U_\infty}\int_0^c x\Phi_{o_x}(x,t)dx$$

$$\frac{D}{q_\infty} = \mp 2\int_0^c dx \int_0^{y_r} \lambda(x,y)\, \frac{2\varphi_x(x,y,t)}{U_\infty}\, dy$$

If the surface slope is expanded in the form

$$\lambda = a_o(x) + a_2(x)y^2 + \ldots = \sum a_{2n}(x)y^{2n} \tag{20}$$

then

$$\frac{D}{q_\infty} = \mp \frac{4}{U_\infty} \sum \int_0^c a_{2n}(x)\Phi_{2n_x}(x,t)dx \tag{21}$$

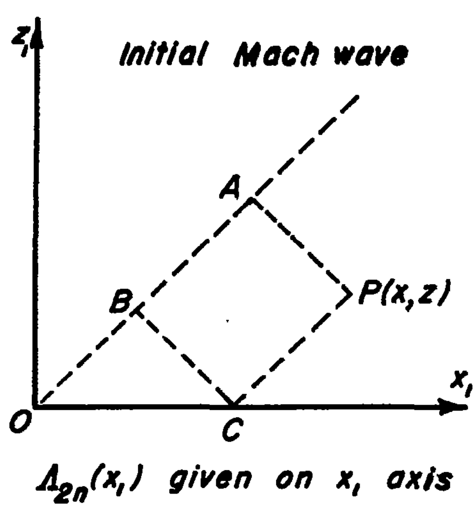
Our problem is now to find expressions for Φ_{2n} and Φ_{2n_x} in terms of $\Lambda_{2n}(x)$, where $\Lambda_{2n}(x)$ is the 2nth span moment of the wing slopes given by

$$\Lambda_{2n}(x) = \frac{1}{U_\infty}\left(\frac{\partial\Phi_{2n}}{\partial z}\right)_{z=t} = \int_0^{y_r} y^{2n}\lambda(x,y)dy \tag{22}$$

Solution of Equations

Single wing.- First let us consider the upper surface of a single wing. The general solution to equation (18) can be written in a form analogous to equation (2), thus

$$\frac{1}{\beta}\oint_s \gamma \frac{\partial\Phi_{2n}}{\partial\nu}|ds| = \frac{1}{\beta}\iint_A 2n(2n-1)\Phi_{2n-2}(x_1,z_1)dx_1dz_1 \tag{23}$$



Sketch (h)

where A is the area enclosed by the contour integral.

Proceeding as in the solution of equation (2), we apply equation (23) to the areas PABCP and BOCB in sketch (h). This yields two equations with the two unknowns Φ_{2n_P} and Φ_{2n_C}. Eliminating Φ_{2n_C} gives

$$\Phi_{2n_P} = -\frac{U_\infty}{\beta}\int_0^{x-\beta z}\Lambda_{2n}(x_1)dx_1 +$$

$$\frac{2n(2n-1)}{2\beta}\left(\iint_{PAOCP}\Phi_{2n-2}dx_1dz_1 + \iint_{BOCB}\Phi_{2n-2}dx_1dz_1\right) \tag{24}$$

Equation (24) can be greatly simplified. Consider, for example, the cases $n = 0$ and $n = 1$, thus

$$\Phi_0 = -\frac{U_\infty}{\beta}\int_0^{x-\beta z}\Lambda_0(x_1)dx_1$$

$$\Phi_2 = -\frac{U_\infty}{\beta}\int_0^{x-\beta z}\Lambda_2(x_1)dx_1 +$$

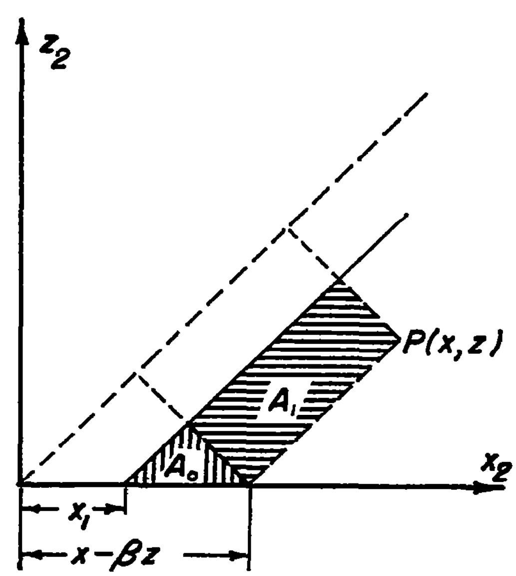
$$\frac{1}{\beta}\left[\iint_{PAOCP}dx_2dz_2\int_0^{x_2-\beta z_2} -\frac{U_\infty}{\beta}\Lambda_0(x_1)dx_1 + \iint_{BOCB}dx_2dz_2\int_0^{x_2-\beta z_2} -\frac{U_\infty}{\beta}\Lambda_0(x_1)dx_1\right]$$

Reversing the order of integration of the last two integrals gives

$$\Phi_2 = -\frac{U_\infty}{\beta}\int_0^{x-\beta z}\Lambda_2(x_1)dx_1 -$$

$$\frac{U_\infty}{\beta^2}\int_0^{x-\beta z}\Lambda_0(x_1)(A_1+2A_0)dx_1$$

where A_1 and A_0 are the areas defined in sketch (i). One can readily show that



Sketch (i)

$$A_1 + 2A_o = \frac{1}{2\beta}\left[(x-x_1)^2-\beta^2z^2\right]$$

Hence,

$$\Phi_2 = -\frac{U_\infty}{\beta}\left\{\int_o^{x-\beta z}\Lambda_2(x_1)dx_1 + \frac{1}{2\beta^2}\int_o^{x-\beta z}\left[(x-x_1)^2-\beta^2z^2\right]\Lambda_o(x_1)dx_1\right\} \quad (25)$$

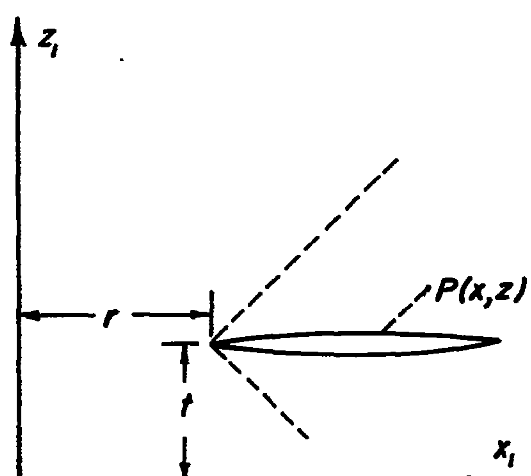
Let

$$k_{mn} = \frac{\binom{2n}{2m}\binom{2m}{m}}{(2\beta)^{2m}}$$

and equation (25) can be generalized to the form

$$\Phi_{2n} = -\frac{U_\infty}{\beta}\sum_{m=o}^{n} k_{mn}\int_r^{x-\beta(z-t)}\left[(x-x_1)^2-\beta^2(z-t)^2\right]^m\Lambda_{2n-2m}(x_1)dx_1 \quad (26a)$$

when $P(x,z)$ is above the wing as in sketch (j) or

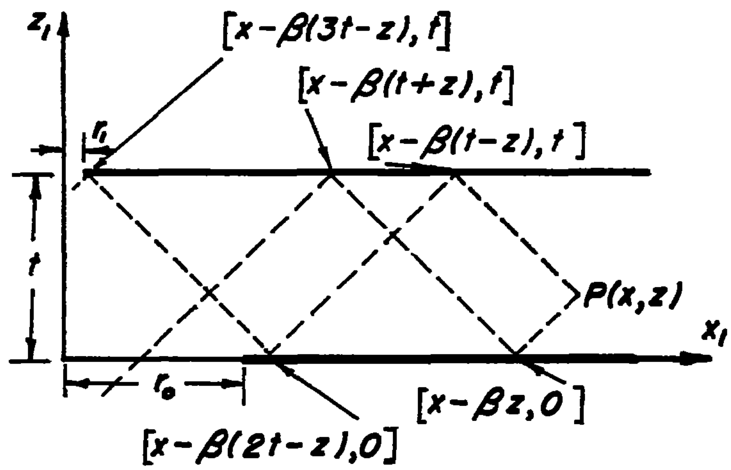


Sketch (j)

$$\Phi_{2n} = \frac{U_\infty}{\beta}\sum_{m=0}^{n} k_{mn}\int_{r}^{x-\beta(t-z)}\left[(x-x_1)^2-\beta^2(t-z)^2\right]^m \Lambda_{2n-2m}(x_1)\,dx_1 \qquad (26b)$$

when P(x,z) is below the wing.

Biplane.- A solution to equation (18) for the flow about a supersonic-edged biplane can also be derived. This solution was determined by means of equation (23) but the details of the derivation will be omitted since the derivation can readily be checked by showing that it satisfies the original equation. Returning to the notation introduced in sketch (c) for upgoing and downgoing characteristics from the point P (see sketch (k)), one can show



Sketch (k)

$$\Phi_{2n} = -\frac{U_\infty}{\beta}\sum_{i=0}^{\infty}\sum_{m=0}^{n}(-1)^i k_{mn}\int_{r_1 \text{ or } r_0}^{x-\beta(z-it)}\left[(x-x_1)^2-\beta^2(z-it)^2\right]^m \Lambda_{u_{2n-2m}}(x_1)\,dx_1 +$$

$$\frac{U_\infty}{\beta}\sum_{i=0}^{\infty}\sum_{m=0}^{n}(-1)^i k_{mn}\int_{r_1 \text{ or } r_0}^{x-\beta(t+it-z)}\left[(x-x_1)^2-\beta^2(t+it-z)^2\right]^m \Lambda_{d_{2n-2m}}(x_1)\,dx_1$$

(27)

where Λ_u and Λ_d are the values of Λ at the points where the upgoing and downgoing characteristics from P reflect from the wings and the summation with respect to i is continued until the reflecting characteristic passes out the front of the biplane.

Drag for Fixed Volume

Single wing.- The drag can be expressed in terms of the wing geometry by substituting equation (26) into equation (21). Assuming the wing lies in the $z = 0$ plane with its nose at the origin ($r=t=0$ in sketch (j)), we can simplify equation (26a) to the form

$$\Phi_{2n} = -\frac{U_\infty}{\beta}\sum_{m=0}^{n} k_{mn}\int_0^x (x-x_1)^{2m}\Lambda_{2n-2m}(x_1)\,dx_1 \tag{28}$$

Now if the slope of the upper surface is given by

$$\lambda(x,y) = \sum_{k=0}^{\infty} a_{2k}(x)y^{2k} \tag{29}$$

then

$$\Lambda_{2n} = \sum_{k=0}^{\infty}\frac{y_r^{2n+2k+1}}{2n+2k+1}a_{2k}(x) \tag{30}$$

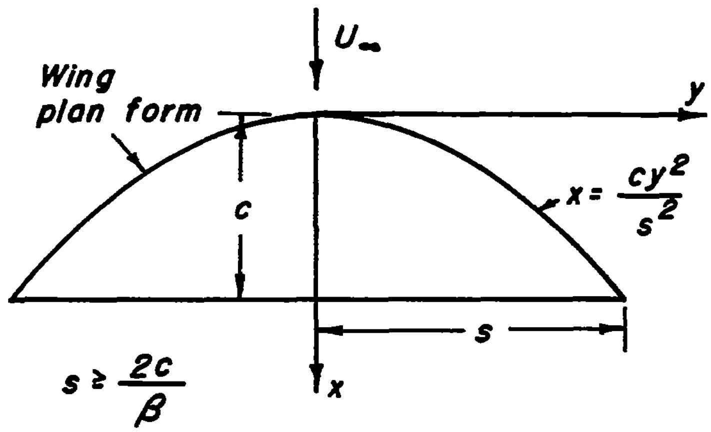
and the drag of the upper surface can be expressed in terms of the a's by

$$\frac{D}{q_\infty} = \frac{4}{\beta}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{n}\frac{k_{mn}}{2n+2k+1-2m}\int_0^c a_{2n}(x)\,dx\,\frac{\partial}{\partial x}\int_0^x (x-x_1)^{2m}y_r^{2n+2k+1-2m}a_{2k}(x_1)\,dx_1 \tag{31}$$

or, alternatively, by

$$\frac{D}{q_\infty} = \frac{4}{\beta}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\left[\frac{1}{2n+2k+1}\int_0^c a_{2n}(x)a_{2k}(x)y_r^{2n+2k+1}\,dx + \sum_{m=1}^{n}\frac{2mk_{mn}}{2n+2k+1-2m}\int_0^c a_{2n}(x)\,dx\int_0^x y_r^{2n+2k+1-2m}(x-x_1)^{2m-1}a_{2k}(x_1)\,dx_1\right] \tag{32}$$

As a specific example, let the wing leading edge have the parabolic shape shown in sketch (l). If $h(x,y)$ represents the height of the upper surface above the $z = 0$ plane, one can set



Sketch (l)

$$h = c\left(1 - \frac{x}{c}\right)\left[\frac{x}{c} - \left(\frac{y}{s}\right)^2\right] \sum \sum A_{in} \left(\frac{x}{c}\right)^i \left(\frac{y}{c}\right)^{2n} \qquad (33)$$

With this representation the upper surface must lie in the $z = 0$ plane along the leading and trailing edges.

As the simplest case one can seek to find the shape which will have the least drag for a given wing volume when $n = 0$; that is, when the only freedom in section variation is in the x direction. In this case

$$a_0 = \sum_0^\infty A_i \left[(1+i) - \frac{x}{c}(2+i)\right]\left(\frac{x}{c}\right)^i$$

$$a_2 = \frac{1}{s^2} \sum_0^\infty A_i \left[-i + \frac{x}{c}(1+i)\right]\left(\frac{x}{c}\right)^{i-1}$$

and one can show

$$\frac{D}{\beta q_\infty c^2} = \sum_0^\infty \sum_0^\infty A_i A_j K_{ij} \qquad (34)$$

where if

$$\sigma = \frac{\beta s}{c} \tag{35}$$

and

$$m_o = i+j$$

$$n_o = ij$$

$$K_{ij} = \frac{64\sigma}{15}\left[\frac{15+10m_o+8n_o}{(2m_o+3)(2m_o+5)(2m_o+7)}\right] + \frac{128}{3\sigma}\left[\frac{1}{(2m_o+5)(2m_o+7)(2m_o+9)}\right] \tag{36}$$

Since the volume of the wing above the $z = 0$ plane can be expressed as

$$V = \frac{16}{3} c^2 s \sum_{0}^{\infty} \frac{A_i}{(2i+5)(2i+7)} \tag{37}$$

the set of simultaneous equations which minimize the drag for a fixed volume can easily be derived. Carrying out these calculations, one has for the minimum drag of a wing having sonic tips (i.e., $\sigma = \beta s/c = 2$)

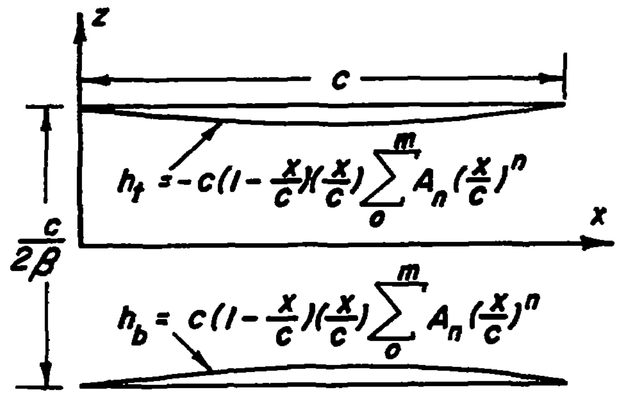
$$\frac{D}{q_\infty\left(\frac{\beta^3 V^2}{c^4}\right)} = 13.85 \tag{38}$$

Actually, the series converges so rapidly that this is the value given by the first term. Hence, with no freedom permitted y, the equation

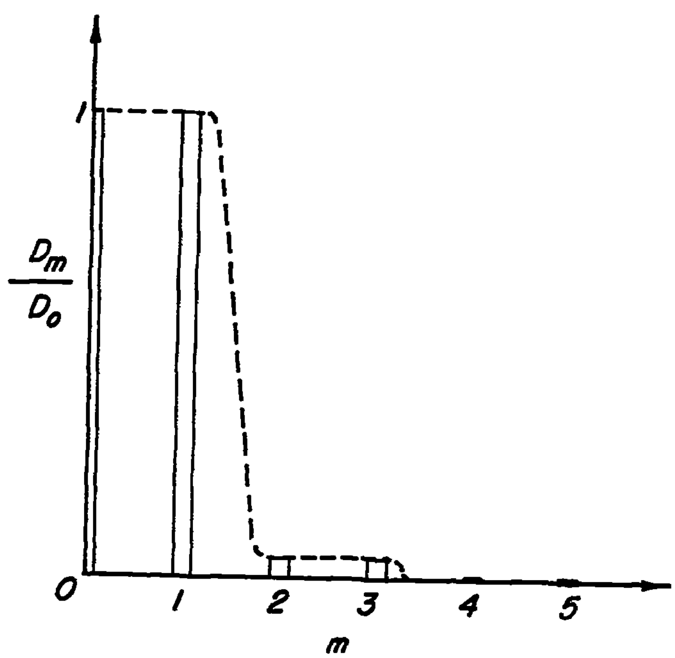
$$h = c\left(1 - \frac{x}{c}\right)\left[\frac{x}{c} - \left(\frac{y}{s}\right)^2\right]\frac{\beta V}{c^3}\left(\frac{105}{32}\right)$$

is, practically speaking, the equation of the optimum shape for a fixed volume above the plan form shown in sketch (l). This is not, perhaps, surprising since it represents a wing having a biconvex section at all span stations and this is the optimum section, for a fixed volume, on a two-dimensional supersonic wing.

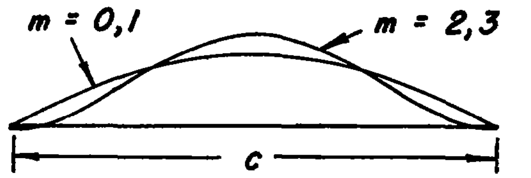
Biplane.- To initiate the study of the biplane, the two-dimensional case was studied. Of course, it is well known that, according to linearized theory, a two-dimensional biplane can carry an arbitrary amount of volume with no wave drag. However, if the wing sections are constrained to those which can be described only by polynomials of finite degree, a small amount of wave drag can persist. Consider the two-dimensional biplane composed of the section given in sketch (m). The equation for the drag can be constructed using equations (9) and (13). Imposing the condition that the volume be fixed, one finds a set of $m + 1$ simultaneous equations. Let D_o be the wave drag given for $m = 0$ (i.e., for a biplane with biconvex section), then D_m/D_o is shown in sketch (n).



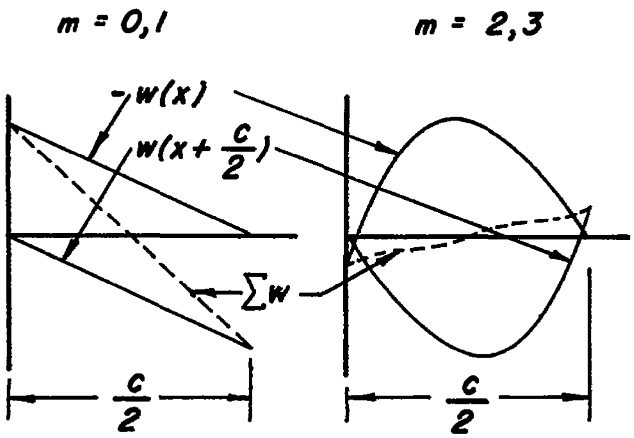
Sketch (m)



Sketch (n)



Section shapes of lower wing



Drag proportional to squared ordinates of dashed curves

Sketch (o)

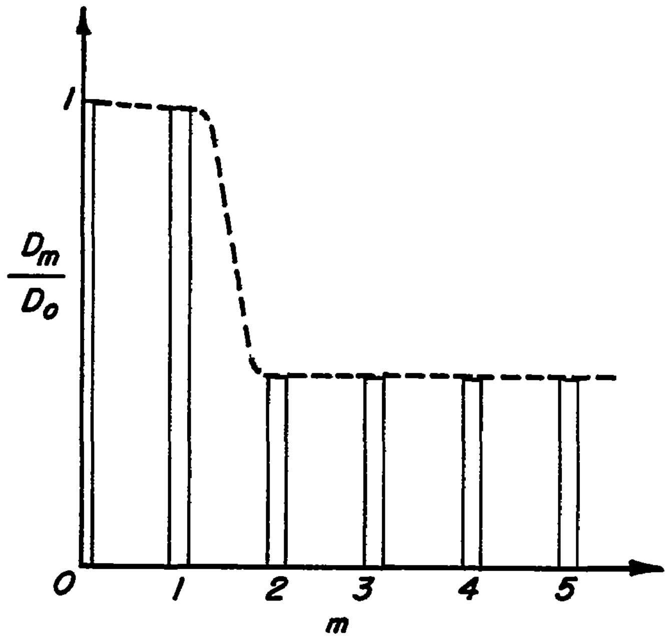
Notice the large reduction in drag brought about by increasing m from 1 to 2. The corresponding shape change is shown in sketch (o) and the reason for the large reduction is made clear by studying the superimposed section slopes (e.g., the negative slopes of the forward portion of the upper wing and the positive slopes of the aft portion of the lower, according to eq. (9)) shown in the lower part of the sketch. In the range m = 5 and 6 the value of D_m/D_o vanished, practically speaking (was equal to 1/17479).

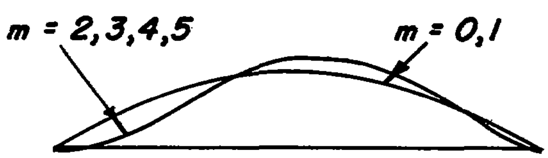
We must be careful to realize that the wave drag shown in sketch (o) is the drag of the interior part of the biplane in sketch (m). If the exterior surfaces can be made straight and parallel to the free stream, this is the total wave drag of the biplane. However, for the m = 2 or 3 case the exterior surfaces cannot be straight since the starting wing slopes are negative (or positive for the upper wing). Although the additional wave drag incurred by making the outer surfaces real would be small, this illustrates how care should be used in estimating the minimum drag of real systems from mathematical minima.

The interior drag of a three-dimensional biplane with the plan form shown in sketch (l), having sonic tips ($2c/\beta = s$), a gap to chord ratio of $1/2\beta$, and sections given by

$$h = \mp c\left(1 - \frac{x}{c}\right)\left[\frac{x}{c} - \left(\frac{\beta y}{2c}\right)^2\right]\sum_0^m A_n\left(\frac{x}{c}\right)^n$$

was also studied. The results were what one might expect from a consideration of the two-dimensional biplane. Here, of course, in contrast to the two-dimensional case, there is a nonzero lower bound to the wave drag for a fixed volume. Values of D_m/D_o up to $m = 5$ are shown in sketch (p) and the section shapes at the root chord are also given. Since no significant variation could be detected for $2 \leq m \leq 5$, the results suggest that this is close to the final minimum. Notice, again, that the initial slope of the lower wing is slightly negative.





Section shape at wing root

Sketch (p)

Drag for Fixed Lift

As a final example, a three-dimensional wing with the plan form shown in sketch (l) (again sonic tips and a gap-chord ratio of $1/2\beta$) was studied for the condition of minimum drag for a fixed lift. The section shapes of the two wings were taken to be

$$h_u = -c\tau_u \left[\frac{x}{c} - \left(\frac{\beta y}{2c}\right)^2\right] \sum_{0}^{m} A_{n_u} \left(\frac{x}{c}\right)^n - \alpha x \; ; \qquad 0 \leq x \leq \frac{c}{2}$$

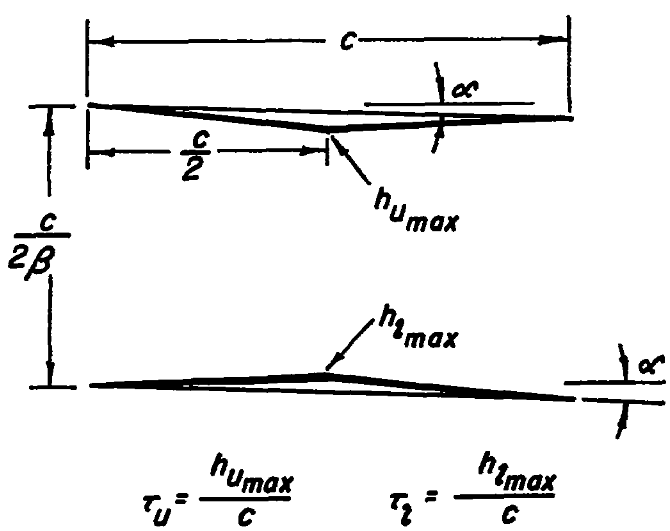
$$h_u = -c\tau_u \left[\frac{x}{c} - \left(\frac{\beta y}{2c}\right)^2\right] \frac{1-x/c}{x/c} \sum_{0}^{m} B_{n_u} \left(\frac{x}{c}\right)^n - \alpha x \; ; \qquad \frac{c}{2} \leq x \leq c$$

$$h_l = c\tau_l \left[\frac{x}{c} - \left(\frac{\beta y}{2c}\right)^2\right] \sum_{0}^{m} A_{n_l} \left(\frac{x}{c}\right)^n - \alpha x \; ; \qquad 0 \leq x \leq \frac{c}{2}$$

$$h_l = c\tau_l \left[\frac{x}{c} - \left(\frac{\beta y}{2c}\right)^2\right] \sum_{0}^{m} B_{n_l} \left(\frac{x}{c}\right)^n - \alpha x \; ; \qquad \frac{c}{2} \leq x \leq c$$

for

$$\sum_{0}^{m} A_{n_u} \left(\frac{1}{2}\right)^n = \sum_{0}^{m} B_{n_u} \left(\frac{1}{2}\right)^n = \sum_{0}^{m} A_{n_l} \left(\frac{1}{2}\right)^n = \sum_{0}^{m} B_{n_l} \left(\frac{1}{2}\right)^n = 2$$



Sketch (q)

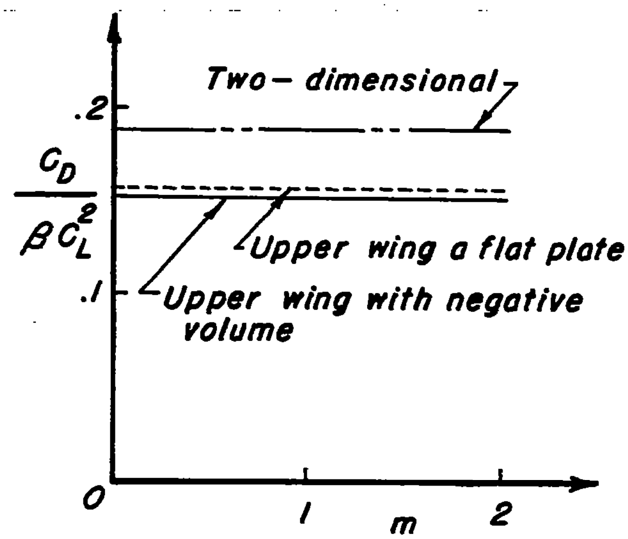
which at the root section for $m = 0$ represent the double wedge section shown in sketch (q).

Two-dimensional wings with sections like those shown have a minimum value of $C_D/\beta C_L{}^2$ equal to 3/16 which occurs when

$$\tau_l - \tau_u = \frac{1}{2}\alpha$$

and is independent of $\tau_l + \tau_u$. We recall from the discussion in the first section on the linearized version of the two-dimensional biplane that any amount of volume can be carried by real closed wings to obtain the value $C_D/\beta C_L{}^2 = 3/16$.

The situation appears to be quite different for the three-dimensional biplane with the plan form and sections described above. One can show that the lift is independent of the values of the A_n's and B_n's. When these are optimized on the basis of making the complete (interior and exterior) wave drag a minimum, the resulting equations show the lowest $C_D/\beta C_L^2$ occurs when $\tau_l - \tau_u$ is some function of α and when $\tau_l + \tau_u = 0$. But this describes in every case an unreal wing for any given volume. In all cases for the optimum, the upper wing had negative thickness equal in magnitude to the positive thickness of the lower wing. The minimum values of drag are shown in sketch (r) for m = 0, 1, 2.



Sketch (r)

With the further restraint that the wings be real, the minimum values of drag changed by the amount shown in the sketch. In this latter case the upper wing was always a flat plate. If spanwise variations were to have negligible effect, the solid line in the drag curve would apparently be near Ward's minimum (ref. 6), while the dashed line would be near the minimum for real wings.

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National Advisory Committee for Aeronautics
Moffett Field, Calif., Dec. 20, 1957

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