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THERMODYNAMIC STUDY OF A ROOTS COMPRESSOR AS A SOURCE OF HIGH-TEMPERATURE AIR

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4276

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SUMMARY

A Roots compressor is analyzed to ascertain the feasibility of using the device to produce high-temperature air. The production of high temperature is associated with the leakage of the hot outlet air through the tip clearance to the inlet and the subsequent mixing with the supply air. This regenerative effect allows the compressor to operate with a much higher temperature at its inlet than is available from the supply.

The calculated performance of such a unit is very sensitive to the assumed value of Stanton number. For example, for estimated reasonable values of Stanton number of 0.002 and 0.003, the maximum calculated outlet temperatures were 7000° and 5000° R, respectively. For a net flow of 1 pound per second, the speed, rotor diameter, and clearance assume values of the order of 3000 to 4000 rpm, 3 to 4 feet, and 0.05 to 0.15 inch, respectively.

INTRODUCTION

With the advent of very high speed missiles and the likelihood of highly supersonic aircraft, a great need is apparent for new types of research equipment. Since stagnation temperatures in these new areas will be measured in thousands of degrees, the ability to produce a steady stream of air at these temperatures is of paramount importance. In an effort to provide such a stream of air, various techniques for producing hot gas are being studied at the NACA Lewis laboratory. This report presents the thermodynamic analysis of a Roots compressor, one of the methods considered. Size, power requirements, and limiting temperature are indicated. The effects of structural loads and various cooling systems on the design are not analyzed.

Adiabatic compression of a gas produces a temperature ratio that increases as the pressure ratio to the power $\frac{\gamma-1}{\gamma}$, where γ is essentially the local isentropic exponent. Thus any compression device is a potential source of high-temperature air, but its adequacy as a heater depends on how closely it can approach the adiabatic condition without damage due to overheating. Therefore, devices with a high ratio of volume to surface area and with a simple design that offers easy access for cooling are best suited for the purpose. Positive-displacement compressors, particularly of the rotary type, are in this category.

This report analyzes the thermodynamic capabilities of the simplest version of a so-called Roots compressor to deliver air at extremely high temperatures. In this machine (fig. 1) a certain volume of air is transported from the low-pressure side to the high-pressure side by the sweeping action of two multilobed intermeshing rotors. The displacement volume is transported at essentially the inlet pressure. The compression takes place when the volume containing the displaced air is opened to the high-pressure side of the blower. The high-pressure air then rushes into the displacement chamber and compresses the delivered air. In the remaining part of the cycle the lobes enmesh and return so as to avoid trapping the high-pressure air and returning it. 4276

During the entire cycle there is some leakage of high-pressure air around and between the rotors to the low-pressure side. Because of this leakage and the subsequent mixing with the supply air, the blower may operate with a much higher temperature at its inlet than is available at the supply. This regenerative effect considerably increases the calculated maximum outlet temperatures, particularly when only a low-temperature supply is available.

SYMBOLS

A	cross-sectional area, sq ft
A_d	cross-sectional area of displacement volume, sq ft
a	speed of sound, ft/sec
c_p	specific heat at constant pressure, Btu/(lb)(°R)
D	diameter of impeller, ft
g	acceleration due to gravity, 32.2 ft/sec^2
h	enthalpy, Btu/lb
K	q_l/q
l	length of impeller, ft

4276

N	number of lobes per impeller
n	polytropic exponent
$\mathcal{P}$	power, hp
p	pressure
q	heat loss (+), Btu/sec
R	gas constant, 53.3 for air, ft/°R
S	wetted surface area pertinent to heat loss, sq ft
St	Stanton number
T	temperature, °R
U	fluid velocity, ft/sec
w	weight flow, lb/sec
Z	compressibility factor, $p/\rho RT$
γ	local isentropic exponent
Δ	clearance, in.
ρ	density, lb/cu ft
Φ	$\left(\frac{\pi}{NC_3}\right)^{1/3}\left(\frac{2C_1}{C_2}\right)^{2/3} C_4$
ω	rotor speed, radians/sec

Subscripts:

av	average
d	displacement flow
f	film temperature
l	leakage flow
max	maximum
s	supply flow

tot total

w wall conditions

0 blocked exit conditions

1 inlet conditions (mixed state of leakage and supply flows)

2 outlet condition

4276

DEVELOPMENT OF EQUATIONS

Many assumptions and some limitations are required in the analysis. Some of these are in agreement with alternate calculations and existing data, as indicated in appendix A. Others remain unverified, and their effect is open to conjecture. Since the degree of validity of these assumptions probably determines the adequacy of the analysis, all assumptions are stated explicitly as they are introduced.

The present analysis assumes that the performance has reached its equilibrium status. Thus, the transients that may lead to this equilibrium are not considered, although presumably such transients might be analyzed on a quasi-steady basis.

The real cycle of interdependent and simultaneous phenomena is simulated in the analysis by a sequence of simple isolated processes as follows. An amount of air that fills the displacement volume $A_d l$ is transported from the inlet to the high-pressure side of the compressor, where it undergoes a polytropic compression to the outlet state conditions. Some of the outlet air is returned to the inlet through the leakage bypass (fig. 1(b)) after losing an amount of heat q_l. This leakage then mixes with the supply air to form the mixture referred to as the inlet air. The cycle then repeats. It should be noted that the leakage bypass as indicated in figure 1(b) is only a schematic convenience. Actually, the leakage flow is through the compressor clearances. The following sections present a thermodynamic analysis of this process.

Polytropic Exponent

The temperature ratio across the blower is given by a relation involving a polytropic exponent n:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \qquad (1)$$

where T_1 is the mixed inlet temperature. For a nonflow polytropic process occurring with a loss of heat, the exponent n is given by (see eq. (97), ref. 1)

$$n = 1 + \frac{\gamma - 1}{1 + \gamma\left[\frac{q_d}{w_d(h_2 - h_1)}\right]} \tag{2}$$

4276 where $(h_2 - h_1)$ is the enthalpy rise and q_d/w_d is the heat loss per pound during the compression (displacement heat loss). Since this equation is derived for a constant specific-heat ratio, it is necessary to use an isentropic exponent γ evaluated for average air properties between states 1 and 2.

Heat Loss

Equations (1) and (2) may be combined to give the following expression for the heat loss:

$$\frac{q_d}{w_d(h_2 - h_1)} = \frac{\gamma - 1}{\gamma} \frac{\ln(p_2/p_1)}{\ln(T_2/T_1)} - 1 \tag{3}$$

The definition of a Stanton number may also be used to express the heat loss q_d:

$$St \equiv \frac{q_d}{\rho S U c_p (T - T_w)}$$

Then, if the displacement weight flow w_d is expressed in terms of the displacement volume, the rotational speed ω, and the number of lobes per rotor N,

$$w_d = \frac{1}{\pi} N \omega A_d l \rho_1 \tag{4}$$

another expression for heat loss is obtained:

$$\frac{q_d}{w_d(h_2 - h_1)} = \frac{\pi \ St \ US\left[\rho c_p (T - T_w)\right]_{av}}{N \omega l A_d \rho_1 (h_2 - h_1)} \tag{5}$$

For the present analysis the effective fluid velocity is taken as the tip speed

$$U = \omega \frac{D}{2} \tag{6}$$

This value of velocity should yield a conservative heat-transfer result, since the actual fluid velocities are below this value. The average value of $\rho c_p(T - T_w)$ is evaluated in terms of the enthalpy at the average of the film temperatures at the inlet and exit:

$$\left[\rho c_p(T - T_w)\right]_{av} = \frac{\rho_1}{2}\left[\frac{\rho_{1,f}(h_1 - h_w) + \rho_{2,f}(h_2 - h_w)}{\rho_1}\right]$$

$$= \rho_1 T_1\left[\frac{Z_1}{Z_{1,f}}\left(\frac{h_1 - h_w}{T_1 + T_w}\right) + \frac{p_2}{p_1}\frac{Z_1}{Z_{2,f}}\left(\frac{h_2 - h_w}{T_2 + T_w}\right)\right] \tag{7}$$

The compressibility factor $Z = p/\rho RT$ (molecular-weight ratio) is included in equation (7). For the conditions considered ($T_2 \leq 7000^\circ$ R), the ratios of the compressibility factors are near 1.0. For $\frac{Z_1}{Z_{1,f}} = \frac{Z_1}{Z_{2,f}} = 1$, substitution of equation (7) into equation (5) results in

$$\frac{q_d}{w_d(h_2 - h_1)} = \frac{\pi}{2N}\,St\,\frac{S}{A_d}\,\frac{D}{l}\left(\frac{T_1}{h_2 - h_1}\right)\left[\left(\frac{h_1 - h_w}{T_1 + T_w}\right) + \frac{p_2}{p_1}\left(\frac{h_2 - h_w}{T_2 + T_w}\right)\right] \tag{8}$$

This equation and equation (3), being two independent expressions for the same quantity, must both be satisfied simultaneously.

Geometry

From dimensional considerations, the wetted area can be represented in terms of the significant dimensions impeller diameter D and impeller length l and the proportionality constants C_1 and C_2. Therefore,

$$S_{tot} = C_1D^2 + C_2Dl \tag{9}$$

where C_1 and C_2 are constants for geometrically similar blowers. However, if the total internal wetted area S_{tot} is minimized for a given displacement volume, an additional relationship relating D and l is obtained. No further study of the effect of this choice is made; however, it appears intuitively to be a favorable choice. Thus, if a class of machines is considered wherein the displacement volume is constant (i.e., D^2l = const.), minimizing S_{tot} results in

$$\frac{D}{l} = \frac{C_2}{2C_1} \tag{10}$$

With the use of equations (9) and (10), the ratio of the total internal wetted area to A_d is $S_{tot}/A_d = 3C_1/C_3$, where $C_3 = A_d/D^2$.

If it is assumed that the heat loss during the compressive part of the cycle depends only on the wetted area of the displacement volume, the expression for the pertinent area ratio is

$$\frac{S_d}{A_d} = \frac{C_1'}{C_3} + \frac{2C_2'}{C_3}\frac{C_1}{C_2}$$

where C_1' and C_2' are defined by

$$S_d = C_1'D^2 + C_2'Dl$$

The geometric relations derived in this section for a compressor with minimum total wetted area are used to obtain a representative value for the $\frac{S_d}{A_d}\frac{D}{l}$ term in equation (8). For the performance evaluation of a specific blower design, actual physical dimensions are used.

Leakage

The flow that leaks back past the rotors is based on an average clearance Δ that is assumed to exist between the rotors and at both the ends and tips of the rotors. A parameter that is convenient in the analysis is the slippage speed ω_0. This parameter, which is the speed required of the rotor to maintain a specified pressure ratio with no net pumping (i.e., $w_s = 0$), is obtained by equating the leakage flow to the displacement flow. Assuming a flow coefficient of unity, the leakage flow w_l is

$$w_l = \rho_l A_l U_l \tag{11}$$

The leakage flow area around the rotor sides and tips may be expressed in terms of the rotor length (since D/l is a known constant), the average clearances, and a geometric constant:

$$A_l = \frac{C_4}{12} l\Delta$$

If the blower pressure ratio is sufficiently high, the clearances will be choked, so that the velocity may be taken as the local sonic speed. If the stagnation conditions are taken as those of state 2, the leakage flow can be written

$$w_l = \rho_2\left(\frac{\rho_l}{\rho_2}\right)\left(\frac{a_l}{a_2}\right)a_2A_l$$

where a is the velocity of sound. Thus,

$$w_l = \frac{p_2}{RT_2}\left(\frac{\gamma + 1}{2}\right)^{-\frac{1}{\gamma-1}}\left(\frac{\gamma + 1}{2}\right)^{-1/2} \sqrt{\gamma_2 g R T_2}\, A_l$$

$$= \frac{p_2}{\sqrt{RT_2}} \sqrt{\gamma_2 g}\, \frac{C_4}{12}\, l\Delta\left(\frac{\gamma + 1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \tag{12}$$

4276

where γ is the local isentropic exponent taken at some average value between the sonic and stagnation states.

Energy Balance

The leakage flow is further determined by an energy balance between the supply flow, the leakage flow, and the displacement flow:

$$w_d h_1 = w_s h_s + w_l h_l$$

A mass balance at this point gives

$$w_d = w_s + w_l$$

In order to account for the fact that the heat loss during the compressive part of the cycle does not represent the total energy loss by cooling, the remainder of the heat loss q_l is considered to be removed from the leakage air. Thus,

$$w_l(h_2 - h_l) = q_l$$

Combining the three preceding equations results in

$$(w_l h_2 - q_l) = (w_l + w_s)h_1 - w_s h_s \tag{13}$$

or

$$\frac{w_l}{w_s} = \frac{h_1 - h_s}{h_2 - h_1} + \frac{q_l}{w_s(h_2 - h_1)} = \frac{h_1 - h_s}{h_2 - h_1} + \frac{q_l}{q_d}\,\frac{q_d}{w_d(h_2 - h_1)}\left(1 + \frac{w_l}{w_s}\right) \tag{14}$$

4276

Solving equation (14) for w_l/w_s gives

$$\frac{w_l}{w_s} = \frac{\frac{h_1 - h_s}{h_2 - h_1} + \frac{q_l}{q_d}\frac{q_d}{w_d(h_2 - h_1)}}{1 - \frac{q_l}{q_d}\frac{q_d}{w_d(h_2 - h_1)}} \tag{15}$$

The value of q_l/q_d that would accurately predict blower performance is not known. It is therefore assumed that q_l/q_d can be adequately represented by a ratio of surface areas, which will be designated K. If the total heat loss is proportional to the total internal surface area S_{tot}, and if the displacement heat loss is proportional to the wetted surface of the displacement volume S_d,

$$\frac{q_l}{q_d} = K = \frac{S_{tot}}{S_d} - 1 \tag{13a}$$

Although several alternative assumptions of this type can be made, the effects are not investigated in detail.

Speed

Equating the leakage flow (eq. (12)) to the displacement weight flow (eq. (4)) and making use of the equation of state (eq. (1)) and the geometric relations obtained in a previous section result in

$$\omega_0^{1/3} = \frac{\Phi}{12}\frac{a_1\Delta}{f(\gamma)}\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{2n}}\left(\frac{w_l}{\rho_1}\right)^{-2/3} \tag{16}$$

where

$$\Phi = \left(\frac{\pi}{NC_3}\right)^{1/3}\left(\frac{2C_1}{C_2}\right)^{2/3} C_4$$

and

$$f(\gamma) = \sqrt{\frac{\gamma_1}{\gamma_2}}\left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

For a constant pressure ratio the speed required to pump a given net flow is related to the slip speed, because the displacement flow varies directly as ω, while the leakage flow is a constant. This gives

$$\frac{\omega}{\omega_0} = \frac{w_d}{w_l} = 1 + \frac{w_s}{w_l} \tag{17}$$

4276

Power

Once the performance of the blower is established, the horsepower is determined from the following formula:

$$\mathscr{P} = \frac{w_d}{\rho_1}(p_2 - p_1) = w_d R T_1\left(\frac{p_2}{p_1} - 1\right) = w_s R T_1\left(1 + \frac{w_l}{w_s}\right)\left(\frac{p_2}{p_1} - 1\right) \tag{18}$$

Size

The physical size of the blower unit can be obtained by substituting into equation (4) the geometric relations obtained by minimizing the internal surface area:

$$D = \left(\frac{\pi}{NC_3}\frac{C_2}{2C_1}\frac{w_d}{\rho_1 \omega}\right)^{1/3} \tag{19}$$

CALCULATION PROCEDURE

The performance and characteristics of the blower are determined by the foregoing equations. Equation (8) relates heat loss to properties of state and compressor geometry. It is interesting to note that for geometrically similar blowers the term $\frac{S_d}{A_d}\frac{D}{l}$ appearing in equation (8) is independent of blower size. Assigning values of C_1, C_2, C_3, C_1', and C_2' that are representative of existing two-lobe blowers results in $\frac{\pi}{2N}\frac{S_d}{A_d}\frac{D}{l} = 12.89$. (The values of the shape constants are given in table I.) Using this value for $\frac{S_d}{A_d}\frac{D}{l}$ and equating equations (3) and (8) yield

$$\frac{\gamma - 1}{\gamma}\frac{\ln(p_2/p_1)}{\ln(T_2/T_1)} - 1 = 12.89\ St\left(\frac{T_1}{h_2 - h_1}\right)\left[\left(\frac{h_1 - h_w}{T_1 + T_w}\right) + \frac{p_2}{p_1}\left(\frac{h_2 - h_w}{T_2 + T_w}\right)\right] \tag{20}$$

Ten parameters appear in this equation: St, T_w, T_2, T_1, p_2, p_1, h_2, h_1, h_w, and γ. The air properties (such as given by a Mollier chart), however, determine the enthalpies and γ as functions of the corresponding temperatures and pressures; therefore, equation (20) may be considered to contain only six parameters St, T_w, T_2, T_1, p_2, and p_1. This equation can be used to determine the relations between these parameters that will produce a given value of outlet temperature T_2. This can be done in terms of arbitrarily selected values of St, T_w, and inlet pressure p_1. Equation (20) then reduces to a relation involving only p_2 and T_1. For convenience, the inlet temperature may be selected as the identifying parameter.

Then, for each T_1, either equation (3) or equation (8) can be used to find the displacement heat-loss term for given values of T_2, St, T_w, and p_1. Equation (15) next gives the leakage flow compared with the net (or supply) flow for each supply temperature (i.e., each h_s); and, if the unit is "sized" by the selection of a net flow w_s, equations (16) and (17) give the speed in terms of the clearance Δ, and equations (18) and (19) yield the power required and the physical size of the unit, respectively.

4276

It is clear that the calculation procedure described is not simple, nor is it convenient to draw general conclusions from examination of these equations. However, one interesting simplification can be made. The type of blower under consideration produces its maximum temperature when there is no net flow. Under these circumstances all the displacement flow returns as leakage. Then equation (15) must be replaced by its counterpart, equation (14) with $w_s = 0$. This gives

$$\frac{q_d}{w_d(h_2 - h_1)} = \frac{1}{K} \tag{21}$$

Using this value in equations (3) and (8) makes possible the calculation of a maximum Stanton number for a given outlet temperature (and assigned values of p_1 and T_w). This limit is obtained by trial as the highest value of St for which there is any value of $T_1 < T_2$ satisfying both equations (3) and (8).

THERMODYNAMIC PERFORMANCE CALCULATIONS

Maximum Allowable Stanton Numbers

In a real installation the success or failure to produce high temperature may depend critically on the Stanton number, over which there is no independent control and which at present cannot be predicted accurately. Some idea of the permissible magnitude of these Stanton numbers, though, can be obtained by use of the no-flow calculations in which a maximum allowable Stanton number is obtained. In order to establish the range of interest, the maximum tolerable Stanton number is examined as a function of outlet temperature, inlet pressure, and surface temperature.

The effect of Stanton number on outlet temperature is determined by the heat-loss equation (8). As the Stanton number is increased, the heat loss approaches a limiting value beyond which the Roots blower will not yield the desired outlet temperature. This limiting value is calculated as previously described. The variation of allowable Stanton number with inlet temperature T_1 for a typical case is shown in figure 2 for final temperatures of 6000° and 9000° R. It is interesting to note that the value of T_1 that corresponds to the maximum Stanton number is not the maximum T_1 for which a solution exists, but that for a given St the curves are double-valued in T_1. These two modes of operation, for a given T_2 and St, provide the same ratio $\frac{q_d}{w_d(h_2 - h_1)} = \frac{1}{K}$ as discussed earlier; however, the heat loss and enthalpy rise for one are higher than for the other, but in the same ratio.

Figure 3 presents the loci of the peaks of curves like those of figure 2, but the results are plotted against T_2. This defines the maximum St for which the specified T_2 can be obtained in a blower with no net flow. It also represents a greater value than can be tolerated if the blower is delivering a net flow. The figure indicates that, if St ≈ 0.002 with a surface temperature of 2000° R (K = 2.93), the maximum outlet temperature for an inlet pressure of 4 atmospheres is about 7000° R. The knee in the curve at an outlet temperature of about 9000° R indicates the possibility of obtaining temperatures considerably in excess of 7000° R if Stanton numbers as low as 0.001 or less can be realized.

Above about 7000° R the use of a constant polytropic exponent n in the calculation begins to be questionable. To verify this point, two calculations (7000° and 15,000° R) were made with a varying polytropic exponent. This was done by assuming that the compressive action takes place in four parts, for each of which a different exponent was determined. The results for 7000° R indicate that a constant n is satisfactory for this temperature level. At 15,000° R the incremental calculation indicates that the allowable St is considerably lower than that calculated with a single value of n. It was also necessary to include the compressibility factor Z in the 15,000° R calculation.

The effect of surface temperature is also shown in figure 3. If the wall temperature is maintained at 4000° R instead of at 2000° R, much less heat will be lost from the system. This, in turn, raises the maximum allowable Stanton number. Although a wall temperature of 4000° R appears very high for moving parts, it may be possible to allow the casing temperature to reach this value while maintaining the temperature of the impeller at 2000° R. The value of the maximum allowable Stanton number for such a condition would be between the values shown in the figure.

4276

Calculations for inlet pressures of 0.1, 4, and 10 atmospheres and T_2 of 7000° R indicate that the maximum allowable Stanton number decreases with increasing pressure. Decreasing the inlet pressure from 4 to 0.1 atmosphere raises the maximum Stanton number only 0.0002, while an increase from 4 to 10 atmospheres decreases the maximum Stanton number 0.0004. Thus, it appears that 3 to 4 atmospheres is a realistic choice as an inlet pressure, since at lower pressures the Stanton number advantage due to decreasing the pressure may be outweighed by an increase of machine size.

The heat loss during the compressive cycle is based on the surface area of the displacement volume. Other possibilities of evaluating this heat loss exist. If, for instance, a larger percentage of the heat loss of the unit is charged to the compressive part of the cycle ($K = 0.746$, e.g.), the maximum allowable Stanton number will be considerably reduced, as shown in figure 3. The decrease in K from 2.93 to 0.746 for $T_2 = 7000°$ R is equivalent to increasing the heat loss in the compressive part of the cycle by a factor of 2.25. However, another assumption regarding this heat loss could be made; namely, that the compressive cycle is essentially adiabatic and that heat loss occurs during the gas-transport phenomenon and during the leakage part of the cycle.

Although no calculations were made using this assumption, the results would probably indicate an increased allowable St compared with those of figure 3. Thus, the calculations of figure 3 are assumed to be realistically conservative estimates.

The maximum allowable Stanton number calculations indicate that reasonably high temperatures can be obtained in practice if the Stanton number does not exceed about 0.002. In order to determine what values of Stanton number may be expected, existing low-temperature Roots blower data were used to calculate values of Stanton number. Values obtained vary from 0.002 to 0.005. As a further estimate, boundary-layer calculations were made that yielded Stanton numbers of about 0.002 (see appendix B). Since the range of estimated values of Stanton number more or less coincides with the maximum allowable Stanton numbers for high-temperature blowers, the possibility of obtaining high temperatures with a Roots blower exists. However, extreme caution should be exercised, since there is some question as to the accuracy of the Stanton number estimates.

Hypothetical 7000° R Unit

Since estimates indicate Stanton numbers of the same order of magnitude as the allowable Stanton numbers, temperatures of the order of 7000° R may be attainable. To show the effects of the various design parameters on performance, a hypothetical 7000° R unit that delivers flow at a rate of 1 pound per second is examined.

For a wall temperature of 2000° R and p_1 of 4 atmospheres, the maximum allowable Stanton number is slightly over 0.002. However, if the actual Stanton number is close to the maximum allowable value, the required power is very large. As an example, figure 4(a) presents the increase in power required to deliver a flow of 1 pound per second at an outlet temperature of 7000° R for a change in St from 0.0014 to 0.0021. The figure shows that the power increases by a factor of 6.5 as the Stanton number is increased by a factor of 1.5. The points on the figure represent a design for the minimum power for each Stanton number, and thus require a variation in blower dimensions that will be determined only when the speed is selected. The condition of minimum power exists for much the same reason that there is a peak in the curve of St_{max} against T_1 for a given T_2 (fig. 2). This is illustrated in figure 5, which shows the variation of power and pressure ratio with T_1. At the low values of T_1, the power is large because of the high pressure ratio. At high T_1, the power is dominated by the heat loss.

4276

The available choice of blower dimensions at a given St (0.0021) with variation of speed for the minimum-power point is illustrated by figure 6. If a choice in speed is made, the effect of St on the required dimensions is illustrated (for 2000 rpm) in figure 4(b). The impeller diameter becomes quite formidable for the high Stanton numbers. A compensating factor, however, is the generous clearance allowed.

Another important comparison is the determination of the effect of St for a fixed design. The variation of power and speed is shown in figure 7 for a rotor diameter of 4 feet and a clearance of approximately 0.11 inch. These dimensions were selected to correspond to the minimum-power condition at a Stanton number of 0.0021. The power compares closely with that of figure 4(a). An alternative design possibility would be to select the blower for operation at a minimum-power point for a lower Stanton number (0.0014). However, if such a selection were made, the maximum allowable Stanton number would be less than 0.0021. This suggests that for a practical design, in the absence of a dependable prediction of St, the maximum possible power should be selected and the dimensions chosen according to the highest estimated value of St.

The effect of St on the required speed (fig. 7) shows that for the dimensions of this example the speed of the impellers increases by almost a factor of 2 when the Stanton number is increased from 0.0014 to 0.0021.

Up to this point the supply temperature and pressure have not been considered. In order to pick the supply conditions (T_s and p_1) that

correspond to the most acceptable combination of required power and the least fabrication difficulty, the theoretical performance of a particular blower is examined in more detail. Variations in speed and power as the supply temperature is changed for a specific blower ($D = 3$ ft; $\Delta = 0.036$ in.) are indicated in figure 8. The dimensions and speed are chosen so that at a supply temperature of 2000° R the power is minimized. Calculations indicate that it would be impossible to size the blower to operate most efficiently with a supply temperature of 5000° R and still operate the compressor at 2000° R. Therefore, it would be necessary to design a blower to operate at the lowest required supply temperature. However,
4276 if reduction of the power loading of the blower is desired, operation at higher supply temperatures (if they become available) would be advantageous.

Figure 9 shows the effect of varying the design inlet pressure. The upper curve indicates that for minimum power consumption the blower should not be designed for extremely high or extremely low inlet pressures. The two lower curves illustrate the trends of design diameter and clearance with variation in pressure for an arbitrarily chosen speed of 2000 rpm. The low pressures require very large rotors (with concomitant high tip speeds) but allow generous clearances.

For purposes of comparison, calculations were made for an 8000° R Roots blower assuming the same St and T_s. The power required increases to about 5 times that of the 7000° R blower, while the required speed for a given size increases by a factor of 2. A somewhat mitigating factor is the increase of allowable clearances by a ratio of about 4.

SUPPLEMENTARY CONSIDERATIONS

Since the present investigation constitutes essentially a thermodynamic analysis of a Roots blower, other aspects are not emphasized. However, since these aspects are important, they will be briefly mentioned to indicate difficulties that may possibly arise with use of such a device.

The performance for a given installation depends to a large extent on the clearances under running conditions. The clearances in turn probably depend on the surface temperatures. Thus, if these temperatures cannot be adequately controlled, either the clearances will be too low and rotor seizure may occur, or they will be too large and the expected performance at a given rotative speed will not be achieved. Excessive speeds may also cause large rotor deformations, and hence should be considered in any design analysis.

Designs requiring high speeds and steep temperature gradients result in high stress levels that may approach present metallurgical limits.

This condition is aggravated by the impact loads that are experienced by the impeller lobes when the displacement volume is released at the high-pressure side of the unit.

Considerably more power is required in a Roots blower than in comparable adiabatic compression because of the nature of the compressive phenomenon. 4276

Another difficulty is that the delivered flow will be of a pulsing nature (at a frequency of twice the product of the number of lobes per rotor and the rpm). The wave shape and the amplitude can undoubtedly be favorably modified by using twisted rotors or by changing the shape of the casing slightly, but the pulsations cannot be eliminated entirely.

Considerable difficulty may be encountered in designing a cooling system capable of maintaining reasonable wall temperatures for a given installation. High cooling rates may necessitate that all surfaces exposed to high-temperature gases be constructed of thin-walled cooling passages. Consequently, high rotative speeds of the impeller would impose severe operating conditions.

To ascertain the magnitude of these cooling problems, local cooling rates were investigated. The cooling rates were evaluated at the rotor tips, since this would seem to be the most difficult part of the machine to cool. The calculation assumed a locally sonic flow expanding from the outlet conditions, and the St referred to was based on the local (sonic) density. The adiabatic wall temperature was taken as the stagnation temperature. This seems reasonable with respect to the recovery factors near 1.0 obtained in the high-temperature calculation of reference 2. The results (fig. 10) are presented for St of 0.001; but, since the heat load is directly proportional to St, the effect of any St is easily obtained. Outlet pressure has an important effect on the heat load; but, even for values of p_2 as high as 20 atmospheres with St of 0.002 and T_2 of 7000° R, the local cooling rate is less than 4×10^6 Btu/(hr)(sq ft). Values as high as 5×10^6 have been achieved without burnouts in rocket motors with fairly conventional liquid convection schemes. With the use of pressurized coolants near their critical state, even higher cooling rates may possibly be achieved.

CONCLUDING REMARKS

A Roots blower has been analyzed to ascertain the feasibility of using the device to produce high temperatures. Since the assumptions necessary for the analysis and calculations critically affect the value of the resulting exit temperatures, assumptions were selected wherever possible such that conservative results were obtained. The ability of the Roots blower to produce high temperatures is largely associated with the leakage of the hot gases to the inlet side and the subsequent mixing with the supply air. This regenerative effect causes the blower to have a much higher temperature air at its inlet than is available at the supply.

4276

The high-temperature operation of a Roots blower is very sensitive to the Stanton number. For example, if the wall temperature is maintained at 2000° R and the inlet pressure at 3 atmospheres, the blower will produce temperatures of about 5000°, 7000°, and over 10,000° R if the Stanton number is respectively 0.003, 0.002, and 0.001.

The size of a high-temperature installation can be judged from the following example for a net flow of 1 pound per second: For an outlet temperature of 7000° R, a Stanton number of 0.0014, a wall temperature of 2000° R, a supply temperature of 4000° R, and an inlet pressure of 3 atmospheres, the rotors would be 4 feet in diameter, the clearances roughly 0.1 inch, the speed 750 rpm, and the required horsepower 9000.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, April 26, 1957

APPENDIX A

VERIFICATION OF EQUATIONS

Sample calculations appear to verify the equations for displacement and leakage flow and for the no-flow pressure ratio. For the low-temperature unit described in reference 3, the displacement and leakage flows were calculated from the present equations with $n = \gamma$ (essentially no heat loss). The test conditions were 6000 rpm, T_1 of 520° R, and p_1 of 8.87 pounds per square inch absolute. The calculated displacement flow (eq. (4)) and leakage flow (eq. (17)) are 0.466 and 0.10 pound per second, respectively, indicating a net flow of 0.366 pound per second. The reported net flow is 0.365 pound per second. For the same unit at 1100 rpm with no net flow, the calculated pressure ratio is 1.72 (eq. (16)). This value is corrected by 1 percent, since the pressure ratio is not of sufficient magnitude to choke the leakage flow through the tip and end clearances. The value reported in reference 3 is 1.65.

APPENDIX B

EVALUATION OF STANTON NUMBER

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Estimates

Blower calculations. - In order to determine what values of Stanton number St might be expected in practice, the theory presented herein was used in an inverse fashion to estimate the apparent St of several Roots blowers used for supercharging diesel engines. The method consists in calculating T_1 by use of either the speed and weight flow or the speed and horsepower. The correct St is taken as the value yielding the same result by either calculation. Because only small heat losses are involved, the calculations are subject to considerable error and can only be considered to establish order of magnitude. The following table indicates values obtained:

Blower	Rpm	Calculated St
3-Lobe	3600	0.003
	3600	.002
2-Lobe	4400	0.005
	5200	No solution

For the two-lobe blower the data are particularly inconclusive, giving a very high value (0.005) for one case and no solution for the other case.

Boundary-layer theory. - Two additional estimates of values of Stanton number were obtained from independent calculations. One calculation uses the turbulent-boundary-layer theory of reference 4. This calculation requires the estimate of a pertinent Reynolds number and flow Mach number. Reynolds number was estimated by taking the rotor tip speed as the pertinent velocity and the rotor tip diameter as the characteristic length. For a flow Mach number of zero (conservative value), a Stanton number of 0.002 was obtained for both the three-lobe blower and a hypothetical 7000° R blower.

The laminar stagnation-point boundary-layer theory of reference 5 was used to obtain another estimate of Stanton number. The problem considered was a hemispherical blunt body of diameter equal to one half the rotor diameter advancing through air at rotor tip velocity. The stream conditions are taken as blower-exit temperature and pressure. Use of this method yields a Stanton number of 0.0025 for the hypothetical 7000° R unit.

It should be emphasized that the preceding calculations are only estimates; however, they are in some agreement with the blower calculations of the previous section. More certainty in the Stanton number requires more extensive blower data than exist at present. The following section describes a means of obtaining better estimates of Stanton number with a small production blower.

Possible Measurements of Stanton Number

Values of St that are directly applicable to the present analysis 4276
can be measured by running an existing blower at the highest temperatures that can be realized and measuring sufficient performance data to calculate the Stanton number from the present theory. In order to determine the accuracy with which St can be determined by this method, calculations were made for the no-flow operation of a small production-model Roots-Connersville blower (series AF, size 22) with minor modifications. The results are shown in figure 11 for surface temperatures of 2000° and 520° R. Controlling the actual surface temperature independently is impossible, but it would probably lie between these values. The figure thus indicates that the no-flow temperature will be a measure of St if the surface temperature is reasonably well known.

Operating the same blower without a blocked exit provides a different means of evaluating St. Under these circumstances, if the exit temperature and clearance are measured, any two of the three parameters - power, speed, and weight flow - will determine the Stanton number. For example, figure 12 shows the variation of weight flow, power, and speed that determines the value of St if the exit temperature is 2000° R and the clearance is 0.002 inch. It is apparent that the blower must be operated at a high speed to obtain the most accuracy from this technique. Therefore, the three levels of St illustrated in the figure (0.0014, 0.0028, and 0.0042) should be easily distinguishable.

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2. Romig, Mary F., and Dore, F. J.: Solutions of the Compressible Laminar Boundary Layer Including the Case of a Dissociated Free Stream. Rep. No. ZA-7-012, San Diego Div., Convair, Aug. 1954.

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4. Van Driest, E. R.: Turbulent Boundary Layer in Compressible Fluids. Jour. Aero. Sci., vol. 18, no. 3, Mar. 1951, pp. 145-161.

5. Reshotko, Eli, and Cohen, Clarence B.: Heat Transfer at the Forward Stagnation Point of Blunt Bodies. NACA TN 3513, 1955.

TABLE I. - VALUES OF CONSTANTS

FOR COMPRESSOR CALCULATIONS

N	2
C_1	4.24
C_2	9.42
C_3	0.22
C_4	6.68
Φ	11.99
$\frac{S_{tot}}{A_d}$	57.8
$\frac{S_d}{A_d}$	14.77
C_1'	$2C_3$
C_2'	3.14

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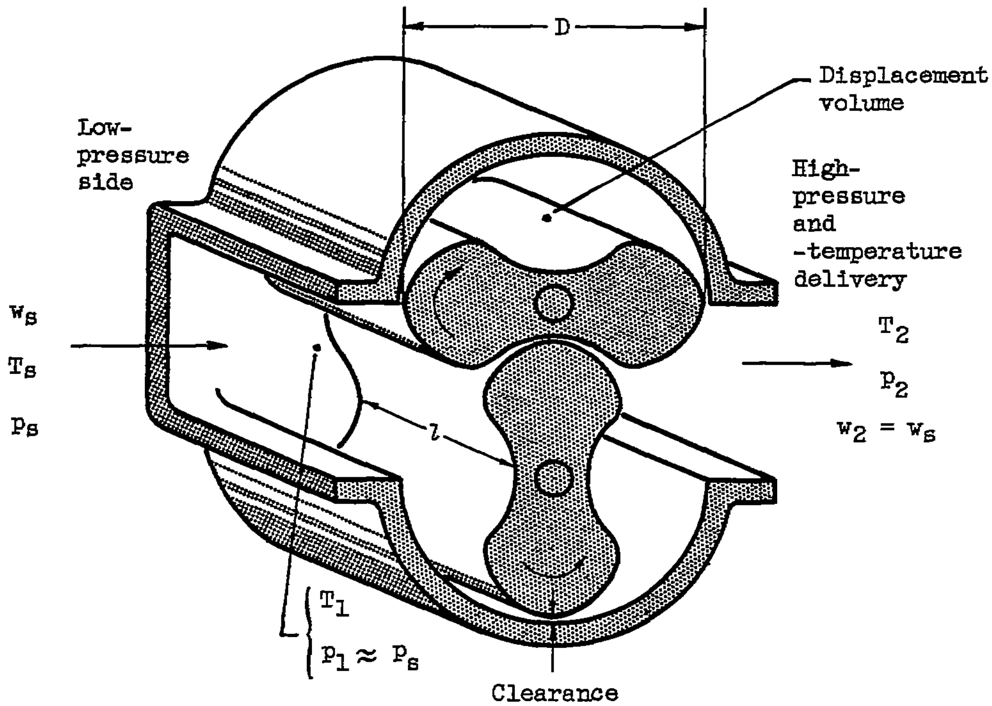
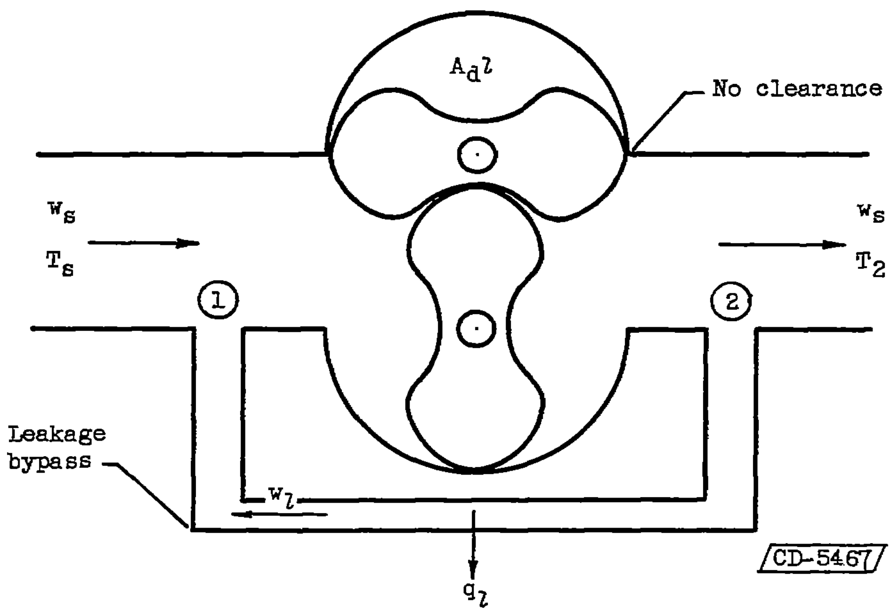


(a) Idealized two-lobe untwisted Roots blower.
Displacement volume = $A_d l$



(b) Idealized cycle with leakage bypass.

Figure 1. - Idealized Roots compressor.

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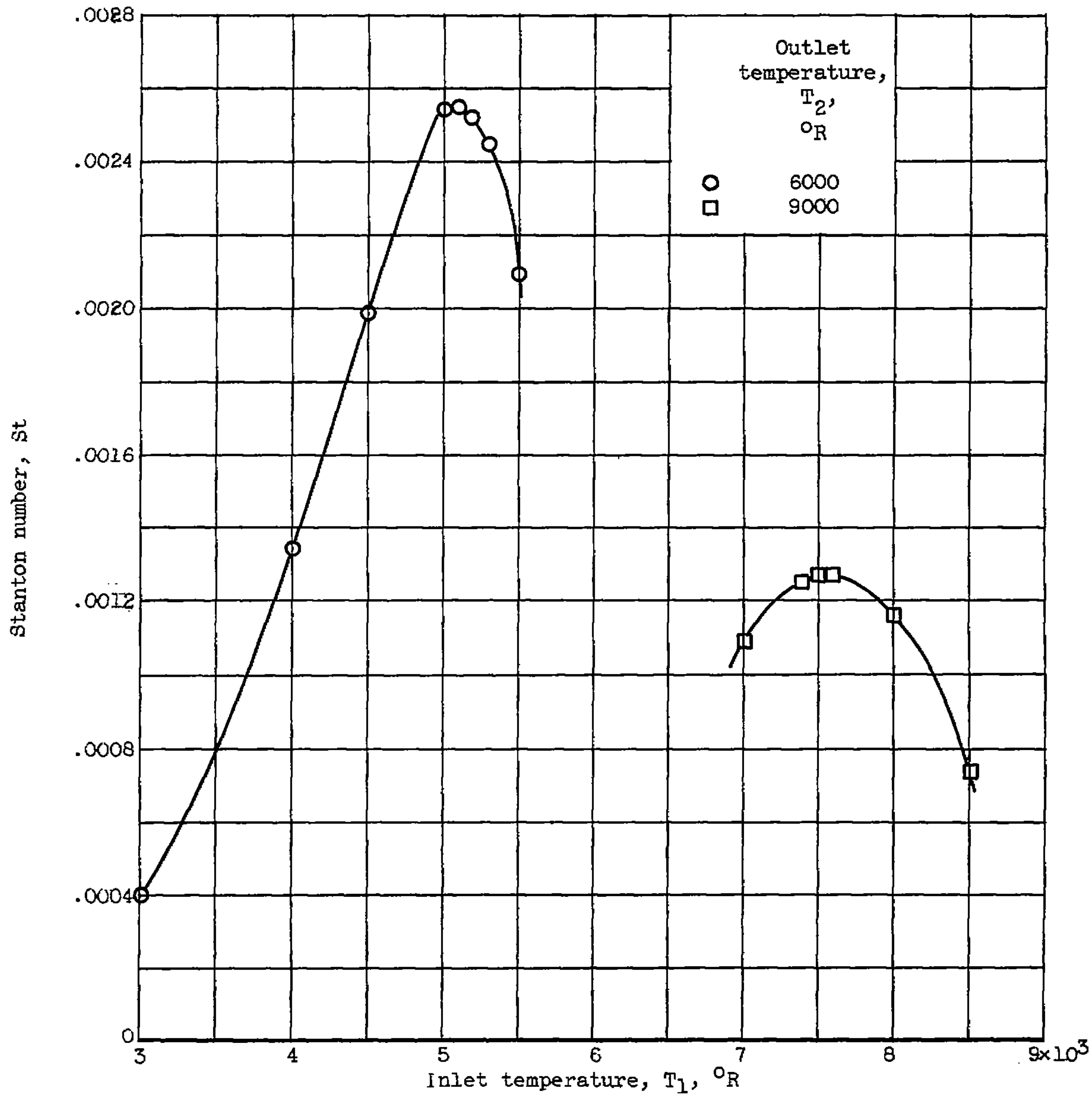

Figure 2. - Allowable Stanton number as function of inlet temperature for blocked-exit hypothetical high-temperature Roots blowers. Wall temperature, 2000° R; inlet pressure, 4 atmospheres.

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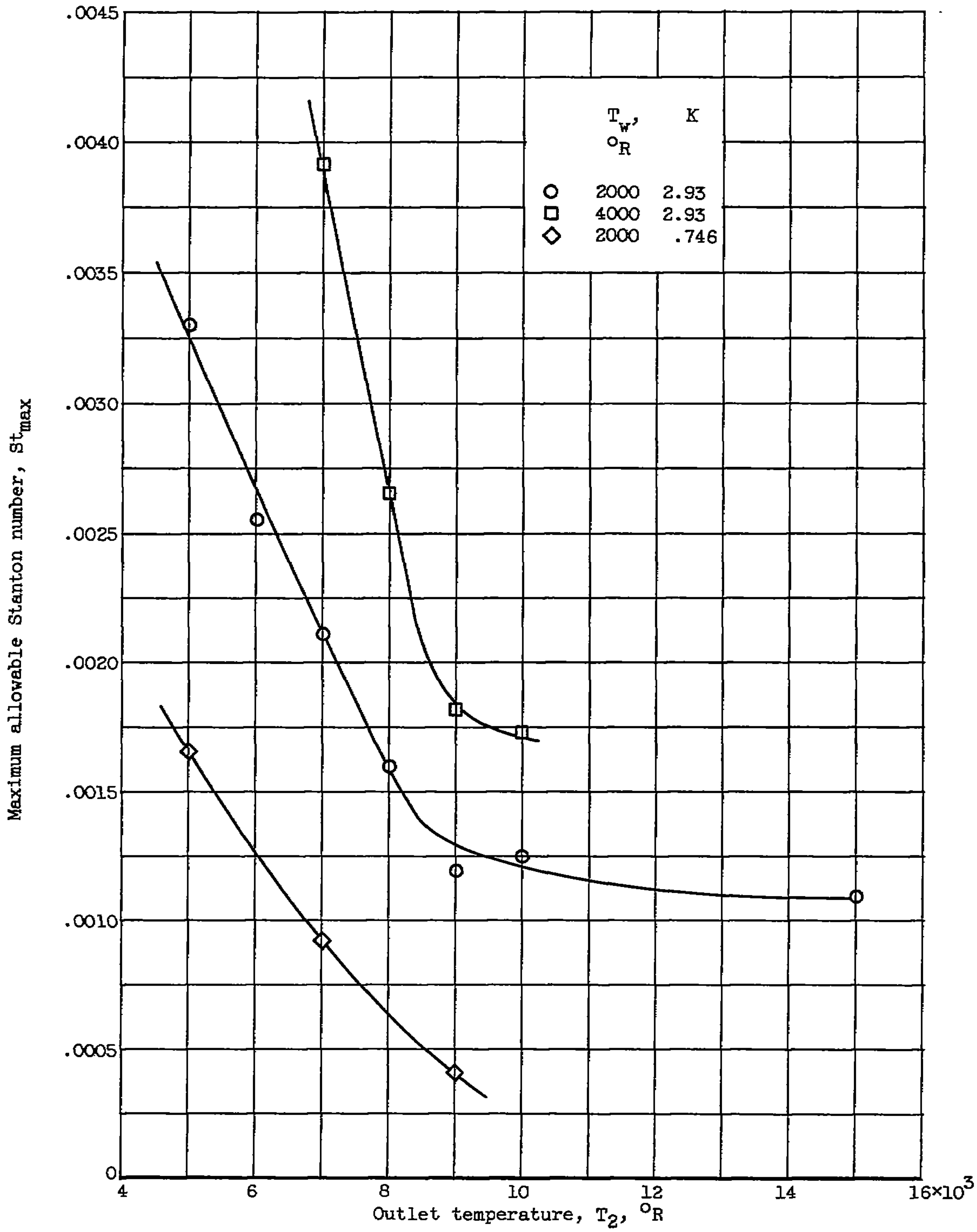
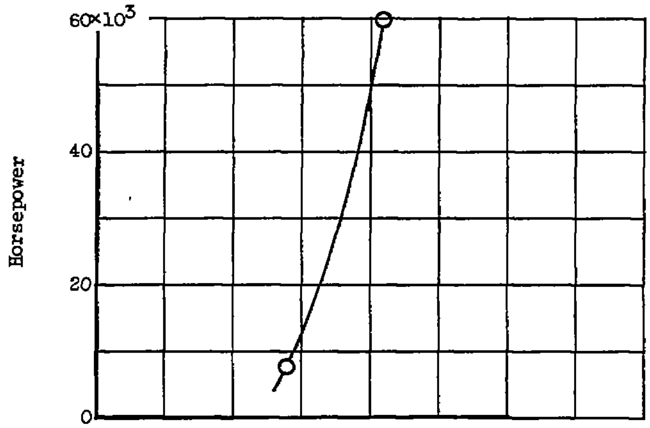
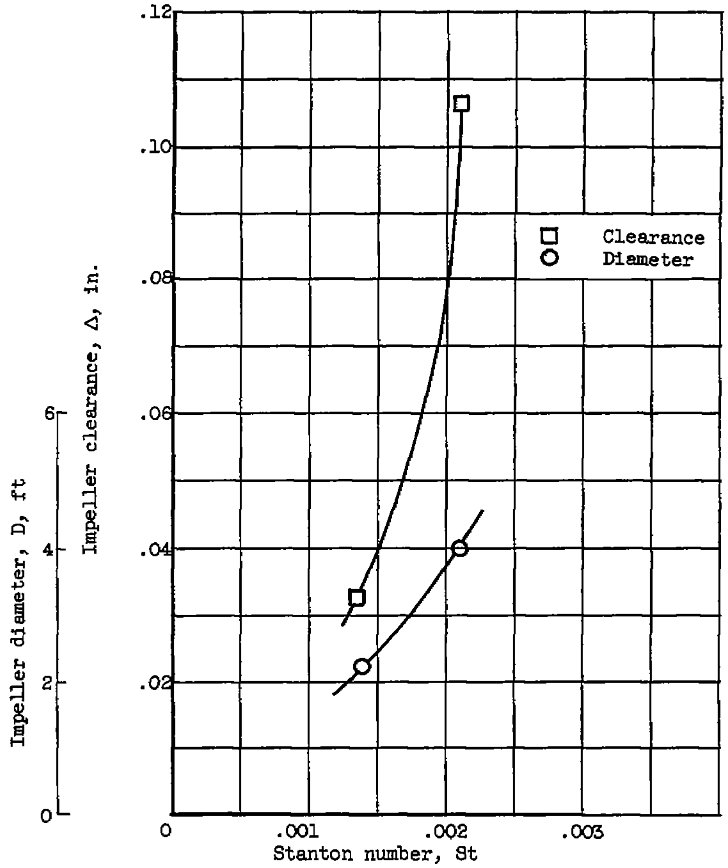


Figure 3. - Variation of maximum allowable Stanton number with outlet temperature. Inlet pressure, 4 atmospheres.



(a) Horsepower against Stanton number.



(b) Impeller diameter and clearance against Stanton number. Rotor speed, 2000 rpm.

Figure 4. - Effect of Stanton number on design parameters for hypothetical high-temperature Roots blowers. Outlet temperature, 7000° R; supply temperature, 4000° R; wall temperature, 2000° R; inlet pressure, 3 atmospheres; net weight flow, 1 pound per second.

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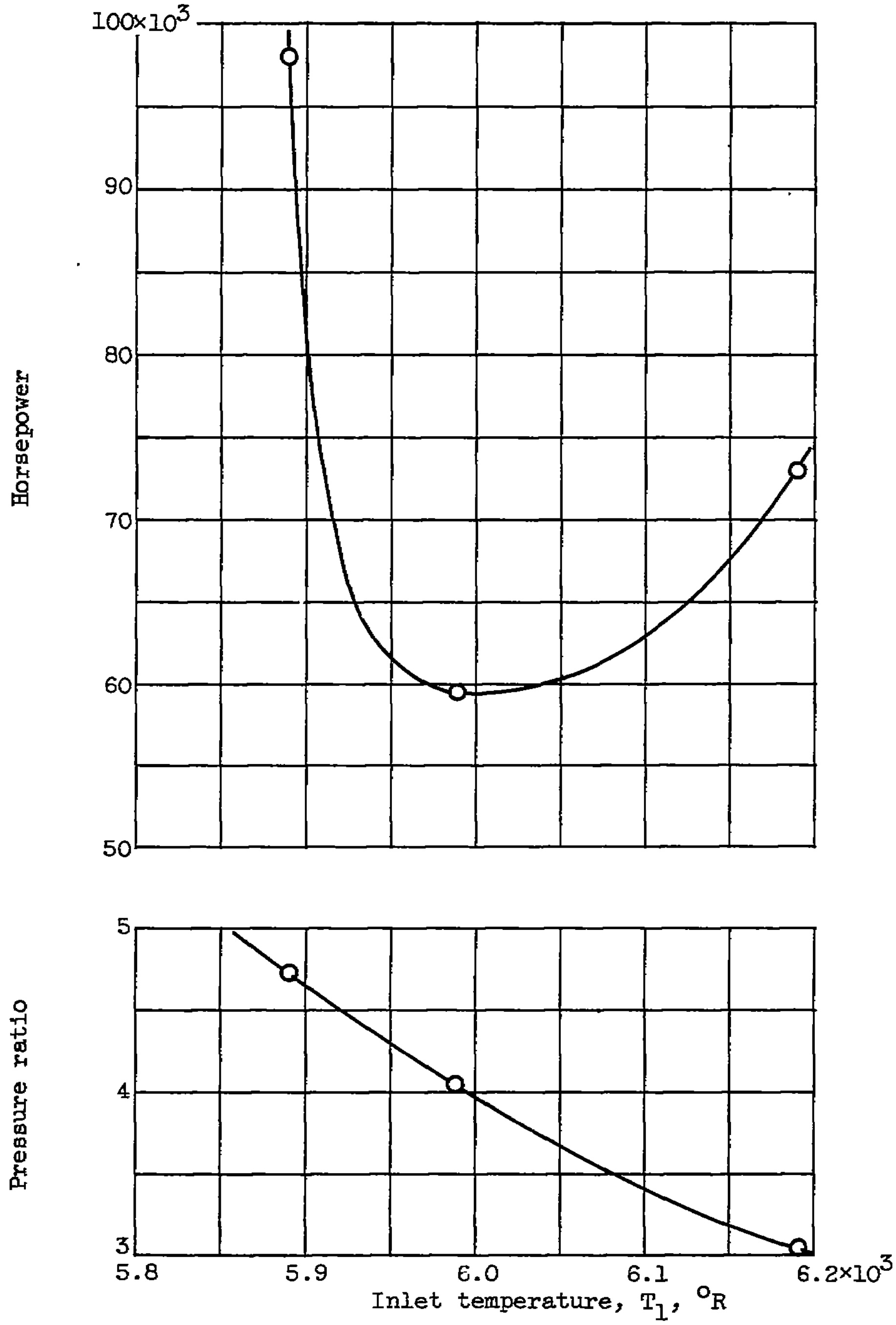

Figure 5. - Design pressure ratio and horsepower associated with various inlet temperatures for specified operating conditions of hypothetical high-temperature Roots blowers. Outlet temperature, 7000° R; supply temperature, 4000° R; wall temperature, 2000° R; inlet pressure, 3 atmospheres; net weight flow, 1 pound per second; Stanton number, 0.0021.

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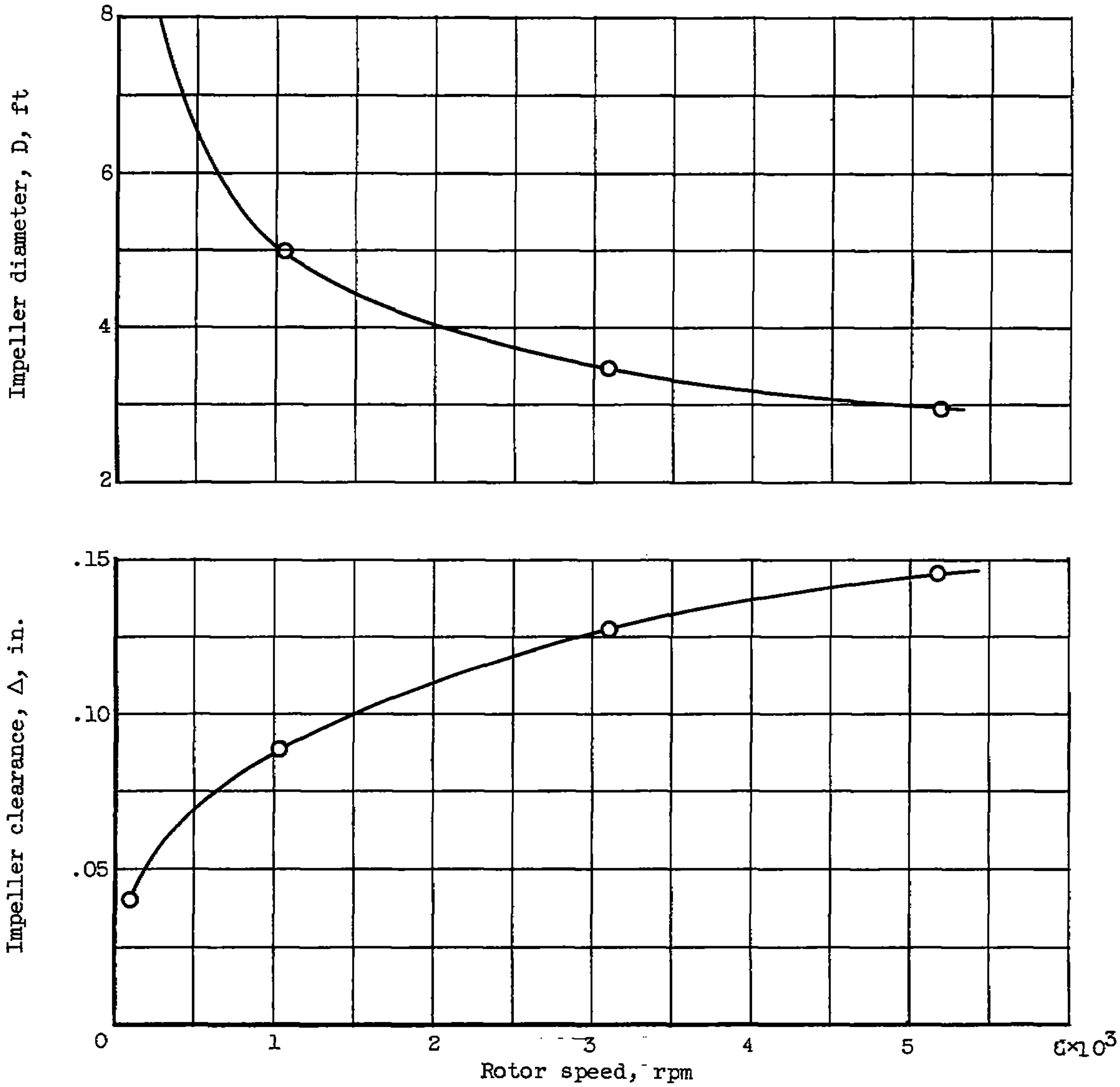

Figure 6. - Effect of rotative speed on physical dimensions of hypothetical high-temperature Roots blowers at minimum-power condition (60,000 hp). Outlet temperature, 7000° R; supply temperature, 4000° R; wall temperature, 2000° R; inlet pressure, 3 atmospheres; net weight flow, 1 pound per second; Stanton number, 0.0021.

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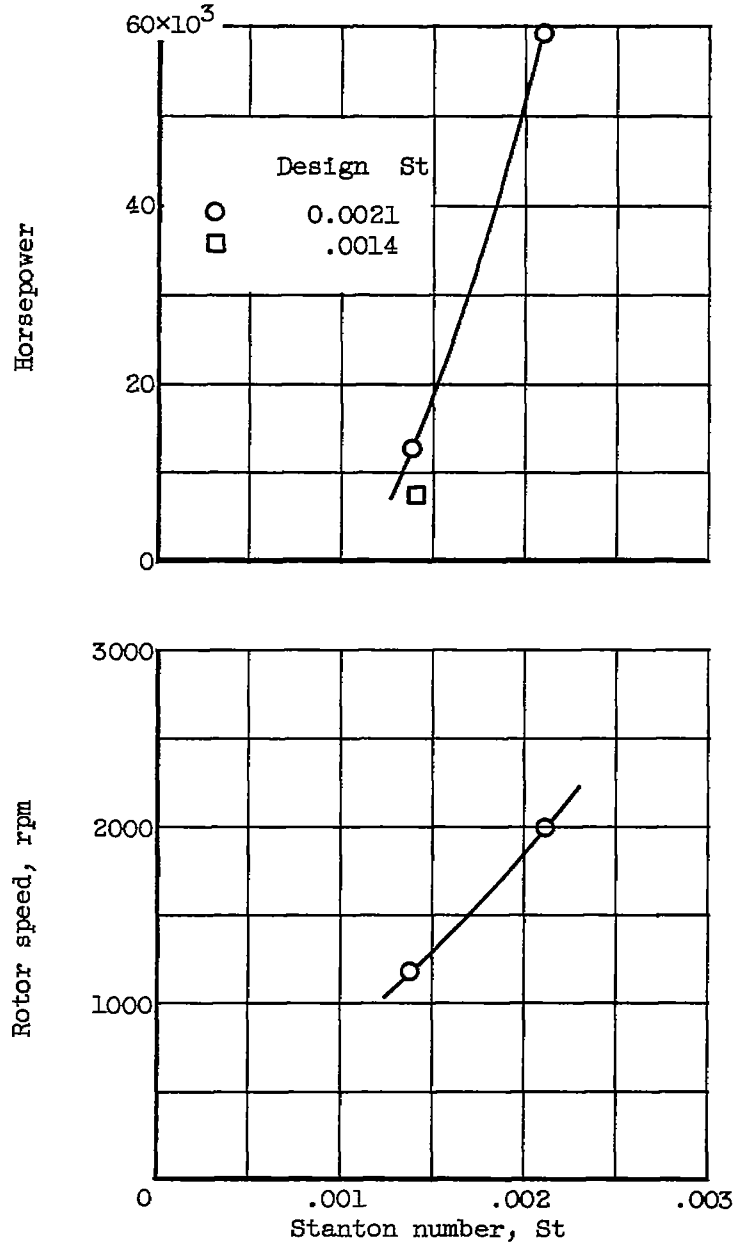

Figure 7. - Effect of Stanton number on operating horsepower and speed of specific hypothetical high-temperature Roots blower. Outlet temperature, 7000° R; supply temperature, 4000° R; wall temperature, 2000° R; inlet pressure, 3 atmospheres; net weight flow, 1 pound per second; impeller diameter, 4 feet; clearance, 0.108 inch.

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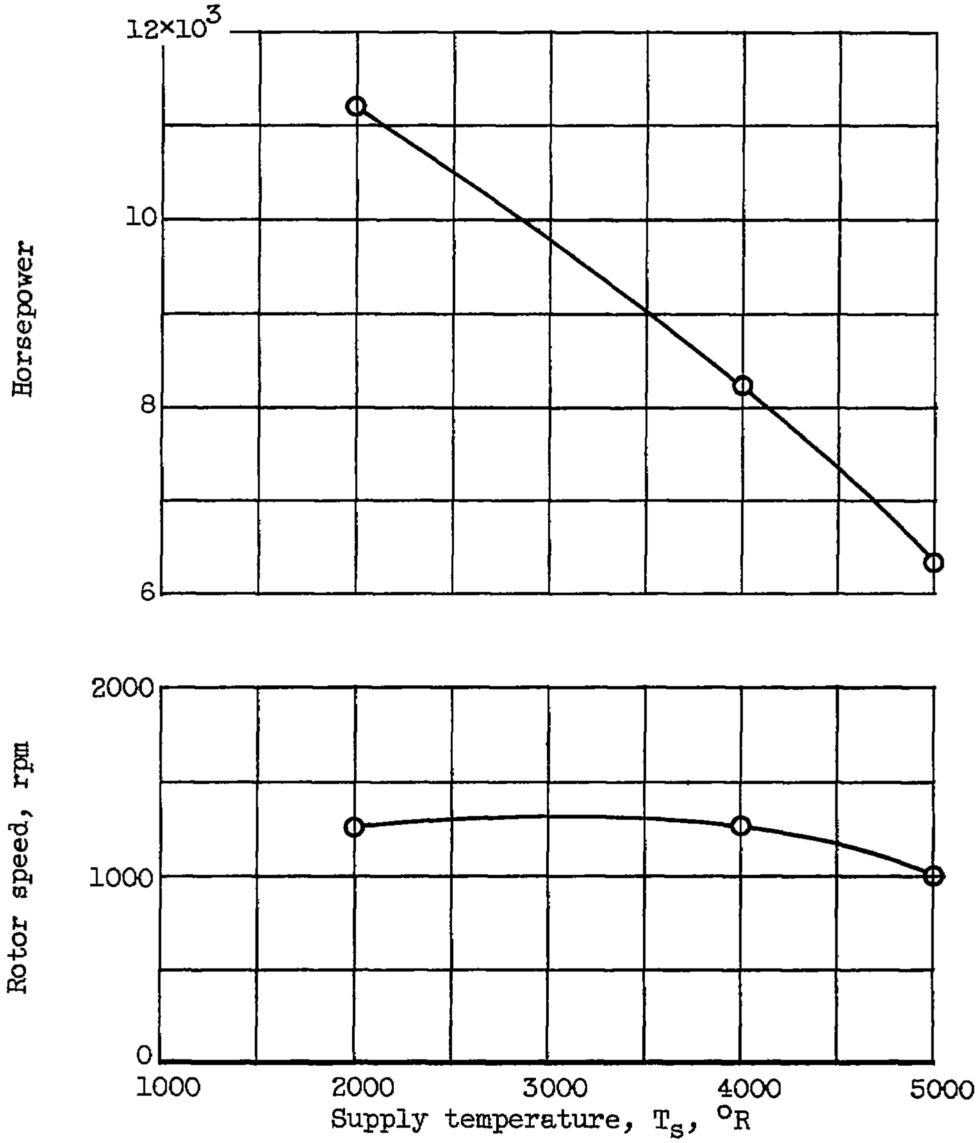

Figure 8. - Effect of supply temperature on required horsepower and speed of specific hypothetical high-temperature Roots blower. Outlet temperature, 7000° R; wall temperature, 2000° R; inlet pressure, 3 atmospheres; net weight flow, 1 pound per second; Stanton number, 0.0014; impeller diameter, 3 feet; clearance, 0.036 inch.

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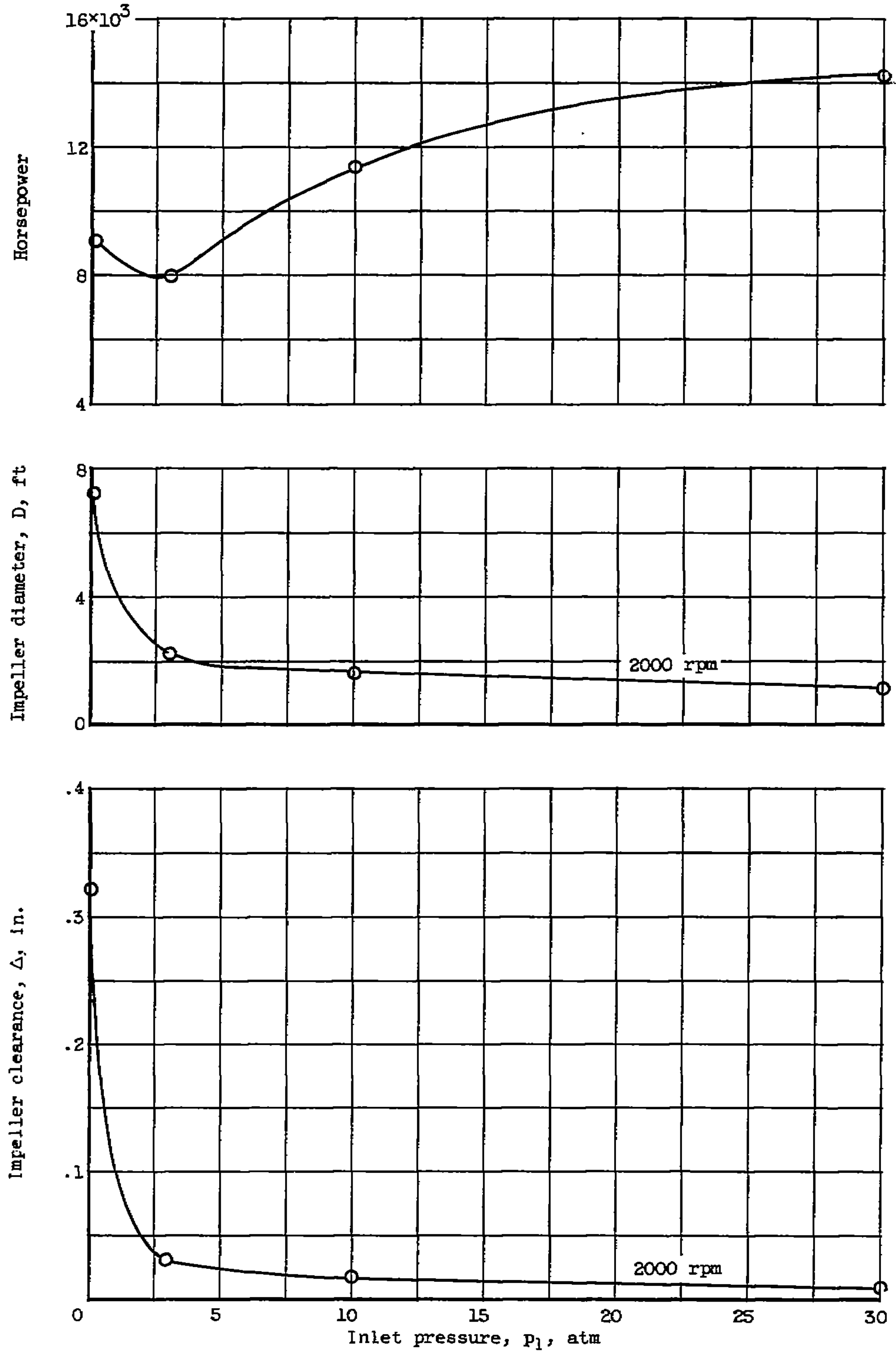

Figure 9. - Effect of inlet pressure on required horsepower, diameter, and clearance of hypothetical high-temperature Roots blowers operating at minimum-power condition. Outlet temperature, 7000° R; supply temperature, 4000° R; wall temperature, 2000° R; net weight flow, 1 pound per second; Stanton number, 0.0014.

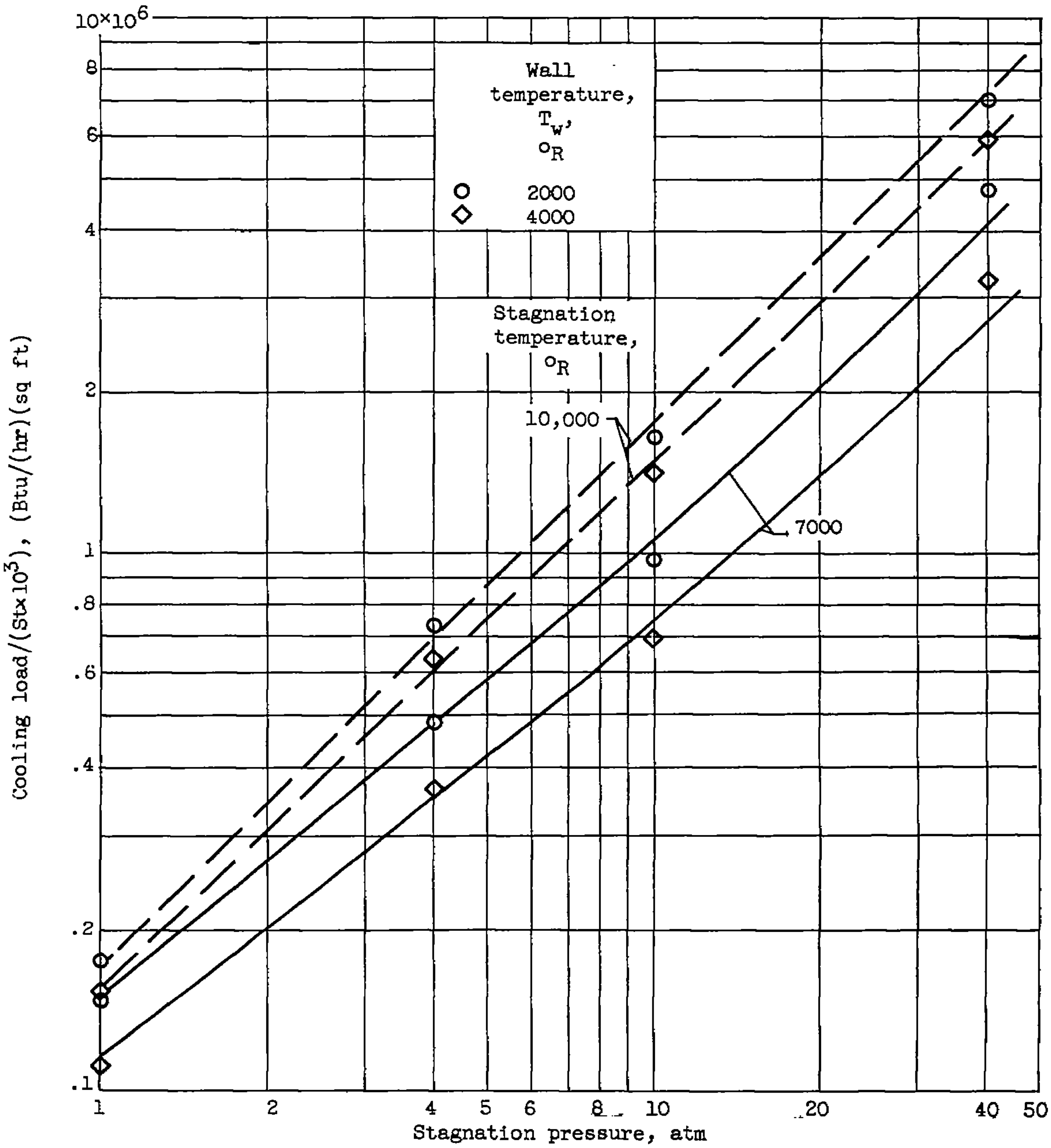

Figure 10. - Cooling load in sonic flow.

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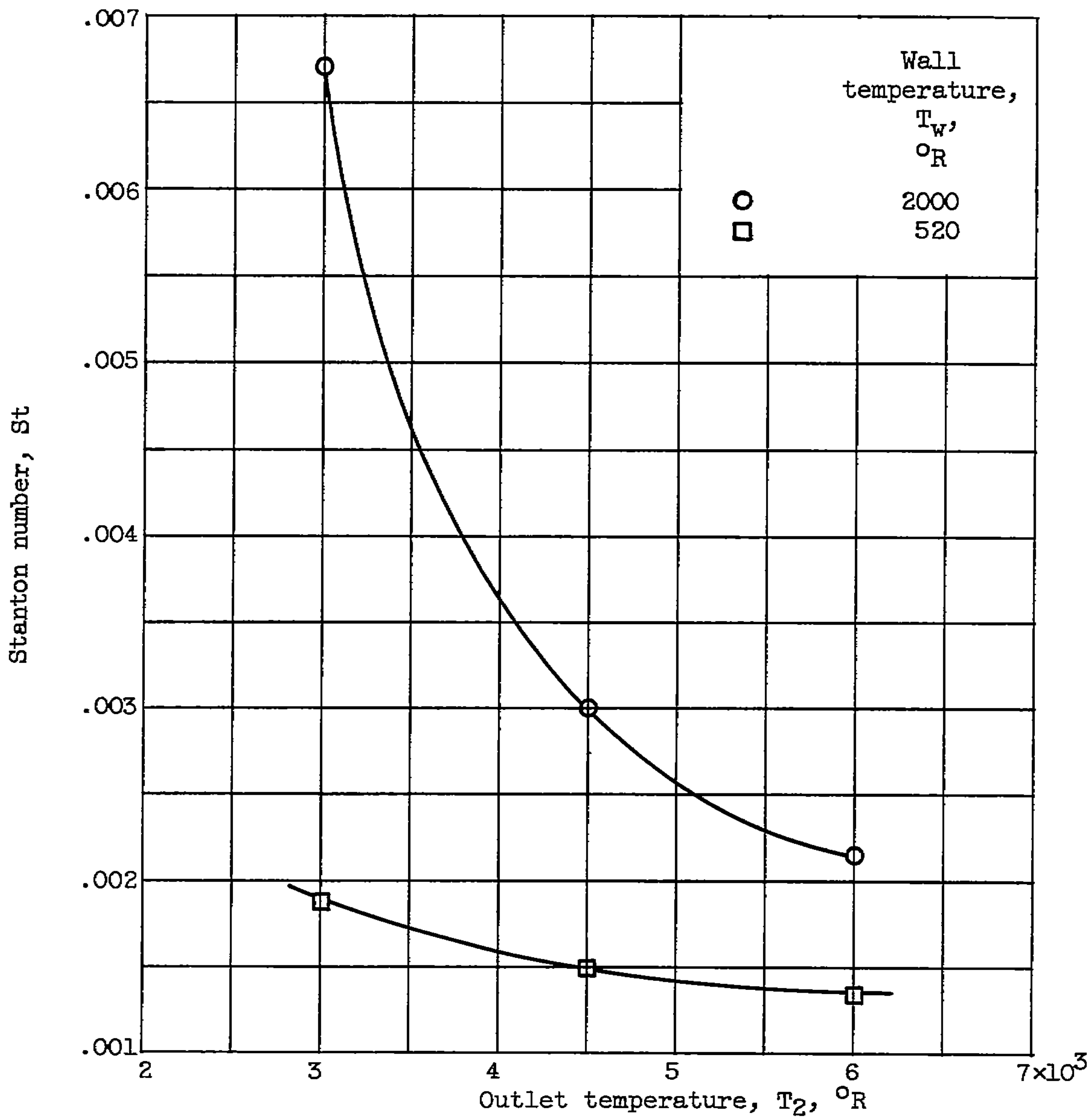

Figure 11. - Calculated Stanton number as function of outlet temperature for Roots-Connersville blower (series AF, size 22) with blocked exit. Inlet pressure, 1 atmosphere.

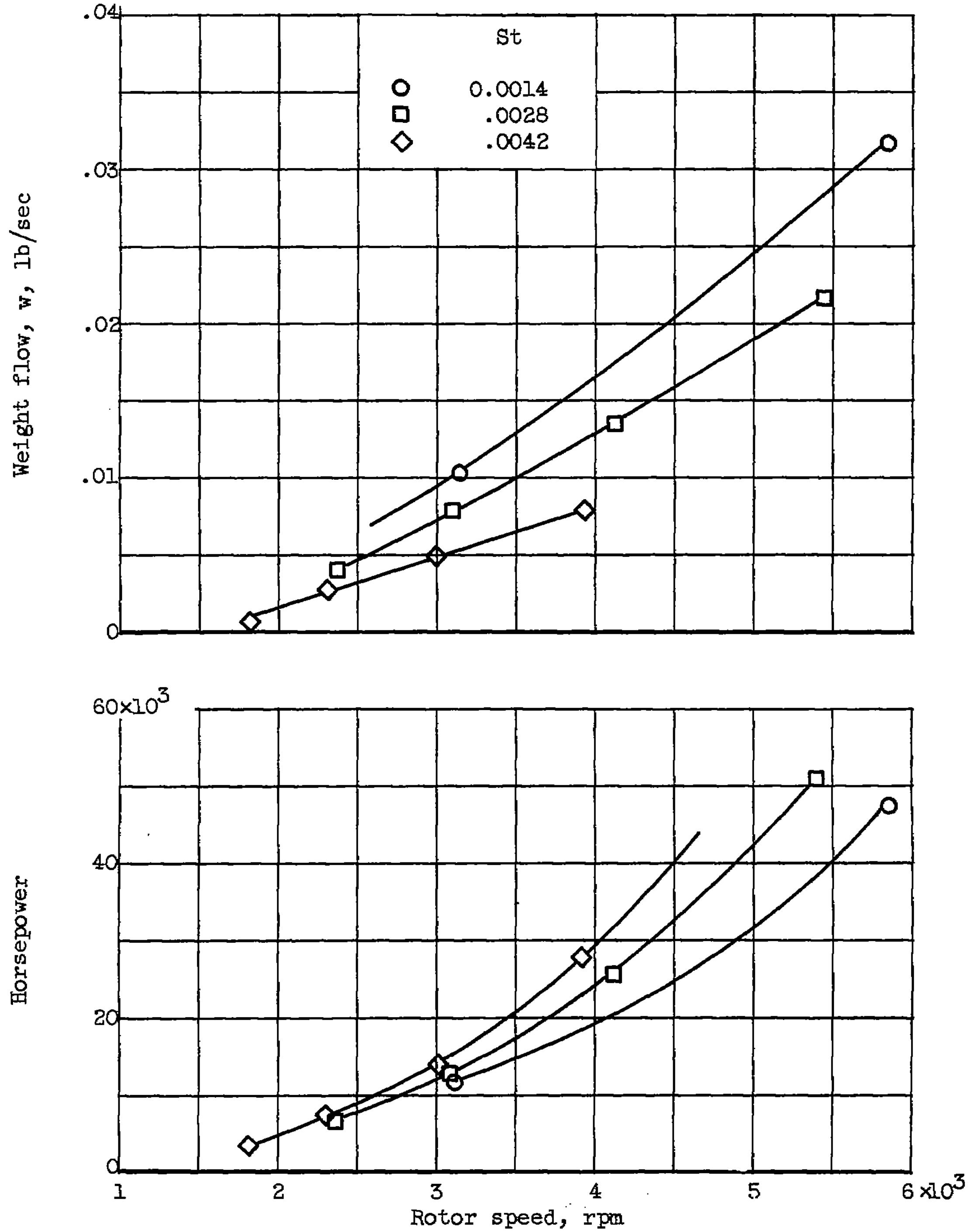

Figure 12. - Performance parameters for various Stanton numbers for Roots-Connersville blower (series AF, size 22) delivering net flow. Outlet temperature, 2000° R; supply temperature, 520° R; wall temperature $(T_s + T_2)/2$, 1260° R; inlet pressure, 1 atmosphere; clearance, 0.002 inch.

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