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RESEARCH MEMORANDUM

THEORETICAL ANALYSIS OF THE MOTIONS OF AN AIRCRAFT STABILIZED IN ROLL BY A DISPLACEMENT-RESPONSE, FLICKER-TYPE AUTOMATIC PILOT

By

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SUMMARY

A general analysis is presented which allows the rolling motions of an aircraft using a displacement-response, flicker-type automatic pilot to be determined. It is shown that the inertia, damping, and control characteristics of the aircraft in roll and the lag time of the automatic system are sufficient to define these rolling oscillations, and charts are presented by which the amplitude and period of the resultant steady-state oscillations may be found. Because of the inherent residual oscillations, this system is not considered ideal for many stabilization problems; however, this flicker-type system may offer a simple and economical solution to those applications where steady-state oscillations may not be objectional. Current trends in pilotless-aircraft designs indicate that the topic system can possibly provide roll stabilization with small amplitude residual oscillations. The analysis permits the definition of stabilization boundaries which will reveal whether or not a specific installation may prove satisfactory. A method for finding the transient conditions leading to steady state and the case of out-of-trim moments producing roll are also considered. Close agreement existed between the theoretical results and roll-simulator tests.

INTRODUCTION

Since the beginning of investigations into the field of pilotless-aircraft research, one of the most important and difficult of the fundamental problems demanding solution has been that of roll stabilization by means of automatic control. Although considerable work has been done in the analyses of various automatic pilots, these efforts usually concerned the application of various systems to particular aircraft. This paper, however, represents an effort to study a single type of automatic pilot and its adaptability in terms of familiar aircraft parameters.

Of the numerous possible methods for obtaining automatic roll stabilization, the one which has been selected for discussion here is the

displacement-response, flicker-type system. In this system the sense of the control moment is dependent solely on the sense of the angular displacement from a zero reference. The control moment is considered to be constant in either one direction or the other at all times. It is recognized that such a system may not be ideal for many stabilization problems; however, the flicker-type system may offer a simple and economic solution to those applications where residual oscillations of small magnitude are not objectionable.

It is the purpose of this paper to determine how the aircraft and automatic-pilot parameters affect the rolling motions of an aircraft employing the topic system. This analysis includes the determination of the limits of the parameters involved in defining practical stabilization boundaries for the system. In addition, charts are presented which may be used independently from the mathematical analysis to determine the amplitude and frequency of the rolling oscillations of any given aircraft. Results of tests of a subject automatic pilot and roll simulator are included to show the close agreement of this theoretical approach with experimental results.

SYMBOLS

ϕ angle of bank, radians

p angular rolling velocity, radians per second $(d\phi/dt)$

I_x moment of inertia about longitudinal axis of aircraft, slug-feet^2

L rolling moment, foot-pounds

L_p rate of change of rolling moment with angular rolling velocity, foot-pounds per radian per second $(\partial L/\partial p)$

δ combined differential deflection of ailerons, radians

L_δ rate of change of rolling moment with aileron deflection, foot-pounds per radian $(\partial L/\partial \delta)$

δL_δ control moment, foot-pounds

L_o out-of-trim moment producing roll, foot-pounds

ϵ ratio of out-of-trim moment to control moment $\left(|L_o/\delta L_\delta|\right)$

t time, seconds

Δt time increments measured from translated time origins, seconds

τ time lag between signal reversal and instantaneous control deflection, seconds

a damping-to-inertia ratio, per second $\left(|L_p/I_x|\right)$

K automatic stabilization parameter, dimensionless $(a\tau)$

p_{max} maximum angular rolling velocity when out-of-trim and control moments are additive, radians per second (see equation (4))

p'_{max} maximum angular rolling velocity when out-of-trim and control moments are opposed, radians per second (see equation (22))

C fractions of p_{max}

c fractions of p'_{max}

C' fraction of initial rolling velocity of a cycle existing at end of first half cycle

C'' fraction of initial rolling velocity of a cycle existing at end of complete cycle

A amplitude of steady-state rolling oscillations (one-half total displacement), radians

A_o displacement of mean line of steady-state rolling oscillations due to out-of-trim conditions, radians

B dimensionless amplitude factor $\left(\delta L_\delta I_x/(L_p)^2\right)$

P period of the steady-state rolling oscillations, seconds

log natural logarithm

e base of natural logarithms (2.7183)

ϕ_c basic change of angle of bank induced by control action, radians (see equation (13))

b wing span of aircraft, feet

S wing area, square feet

ρ mass density of air, slugs per cubic foot

V velocity of aircraft, feet per second

q dynamic pressure, pounds per square foot $\left(\rho V^2/2\right)$

C_l rolling-moment coefficient (L/qSb)

$C_{l_{\frac{pb}{2V}}}$ nondimensional damping-in-roll coefficient $\left(\frac{\partial C_l}{\partial \frac{pb}{2V}}\right)$

C_{l_δ} rate of change of rolling-moment coefficient with aileron deflection $(\partial C_l/\partial \delta)$

Numerical subscripts refer to time limits. (See fig. 1.)

DESCRIPTION OF THE AUTOMATIC PILOT

The displacement-response, flicker-type automatic-pilot design considered in this paper as a roll-stabilization system consists primarily of one displacement gyroscope, a power supply, a servomotor, and an aerodynamic control surface. In general, the analysis will apply to any automatic pilot which causes the application of a constant control moment for a displacement deviation. Such a system represents one of the simpler and more economic nonlinear servomechanisms used for automatic control.

A schematic diagram of a typical system of this type is shown in figure 2. In operation, a rider arm is attached to the outer gimbal of the displacement gyroscope which provides the constant zero space reference. A commutator drum is attached to the mounting case of the gyroscope and provides the aircraft displacement reference. Thus, a deviation in the angle of bank is detected by contact of the rider arm upon the commutator drum. The commutator drum is divided into two segments, and contact of the rider arm with either of these segments completes a power supply circuit which energizes the servomotor. Energization of the servomotor causes an instantaneous deflection of the aerodynamic control and maintains this deflection until contact is made with the other commutator segment.

The intelligence of the system is such that the sense of the control moment is directed to restore the aircraft toward the zero space reference. Hence, as the aircraft rolls to the right, the rider arm is on the segment which completes the circuit calling for left control. In this analysis, controlled motions involving bank angles greater than 180° are not considered.

METHOD OF ANALYSIS

The analysis of the rolling motions of an aircraft controlled in roll by a displacement-response, flicker-type automatic pilot can be

briefly summarized in three steps. First, a description of the angle-of-bank variation during a complete cycle of a rolling oscillation is written in terms of the general aircraft parameters which define the motion. Secondly, from the general expressions of the rolling motion, the rolling velocities at the beginning and end of the cycle are related; from this, the analysis shows whether the aircraft is performing a series of steady oscillations or whether the oscillations in the transient state are growing or damping to the steady-state conditions. The third step is the determination of the general expressions which define the amplitude and period of the rolling oscillations.

Of primary importance in this analysis is the basic response of the aircraft to an instantaneously applied control moment. It is assumed in this analysis that motions in pitch or yaw do not affect the rolling behavior. Therefore, the basic response of the aircraft in roll to an abrupt application of control is given by the rolling-moment equation for one degree of freedom.

Two mathematical principles are employed in representing the motion. The first of these principles allows the basic response of the aircraft to be redefined from different origins on the time scale; this feature is termed the principle of redefinition through time translation. The second principle is that of superposition of individual effects to determine resultant motions. The use of these principles becomes clear as actual applications are made.

DERIVATION OF EQUATIONS

Basic equations of motion.- The basic differential equation of motion in roll in this analysis is the rolling-moment equation of one degree of freedom:

$$I_x \frac{d^2\phi}{dt^2} - L_p \frac{d\phi}{dt} = \delta L_\delta + L_o \tag{1}$$

The term L_o is included to account for any continuous out-of-trim moment of the aircraft that may be producing roll. The solution of this linear differential equation yields the relationships

$$\phi = \frac{\delta L_\delta + L_o}{L_p{}^2} I_x \left[e^{(L_p/I_x)t} - \left(\frac{L_p}{I_x}\right)t - 1 \right]$$

$$+ p(0) \frac{I_x}{L_p} \left[e^{(L_p/I_x)t} - 1 \right] + \phi(0)$$

$$p = \frac{\delta L_\delta + L_o}{L_p}\left[e^{(L_p/I_x)t} - 1\right] + p(0)e^{(L_p/I_x)t} \tag{3}$$

where $\phi(0)$ and $p(0)$ are, respectively, the angle of bank and rolling velocity at the time origin considered.

Equations (2) and (3) are modified in form for use in this paper. Since, in this analysis, no particular time is considered zero time because of the use of time translation, the notation in equations (2) and (3) is changed from t to Δt. In this case Δt indicates time increments from a defined origin, which is where Δt is considered equal to zero. Hence, equations will represent the variations following the translated time origins. A second change is made through the substitution of

$$a = \left|\frac{L_p}{I_x}\right|$$

where a is termed the damping-to-inertia ratio and the vertical bars indicate the absolute magnitude. A final modification is the expression of $p(0)$ as a fraction of the maximum rolling velocity. From equation (3)

$$p_{max} = -\frac{\delta L_\delta + L_o}{L_p} \tag{4}$$

and

$$p(0) \equiv Cp_{max} \tag{5}$$

where C is the fraction of p_{max} at the time origin considered.

Making these modifications, the basic equations of motion are

$$\phi = \frac{\delta L_\delta + L_o}{L_p^{\,2}} I_x\left[(1 - C)e^{-a\Delta t} + a\,\Delta t - (1 - C)\right] + \phi(0) \tag{6}$$

$$p = \frac{\delta L_\delta + L_o}{L_p}\left[1 - (1 - C)e^{-a\Delta t}\right] \tag{7}$$

First half-cycle.– In this analysis the out-of-trim moment L_o is assumed to be a constant positive moment causing roll to the right. At point 0, figure 1, the angle of bank is zero and the rate of roll is defined as $C_0 p_{max}$. With these initial conditions the angle-of-bank variation between points 0 and 1 is given by equation (6) as

$$\phi_{01} = \frac{\delta L_\delta + L_o}{L_p^2} I_x \left[(1 - C_0)e^{-a\Delta t} + a\,\Delta t - (1 - C_0)\right] \tag{8}$$

At zero angle of bank (point 0) the automatic pilot signals for a reversal of the controls. Because of the inherent lag in the operation of the physical system, the abrupt deflection of the control occurs at a finite time after the signal is given. As the aircraft is assumed to be rolling at this point, it reaches a definite angle of bank and rolling velocity at the end of this lag period (point 1, fig. 1). This lag time is considered constant for any system and is defined as τ. Substituting this value into equation (8) gives the angle of bank at point 1 as

$$\phi_1 = \frac{\delta L_\delta + L_o}{L_p^2} I_x \left[(1 - C_0)e^{-a\tau} + a\tau - (1 - C_0)\right] \tag{9}$$

The rolling velocity at point 1 is found to be

$$p_1 = C_1 p_{max}$$

where

$$C_1 = \left[1 - (1 - C_0)e^{-a\tau}\right] \tag{10}$$

The factor $a\tau$ is of prime importance in this analysis. The symbol K will be termed the automatic stabilization parameter and is defined by

$$K \equiv a\tau \equiv \left|\frac{L_p}{I_x}\right|\tau \tag{11}$$

To completely define the result of the control reversal at point 1, it is necessary to define the angle-of-bank variation that would have existed had the control not reversed. The definition of this curve, shown dashed in figure 1, is accomplished by time translation. If time increments are considered to be measured from point 1, the basic response equation of the aircraft may be used with a change in the initial constants, the new constants being the angle of bank and rolling velocity that were determined for point 1. In this way the rolling motion that would have occurred if the control had not reversed is redefined in terms of time increments from the time of the control reversal, which is considered a new time origin. This is written as

$$\phi_{012} = \frac{\delta L_\delta + L_o}{L_p^2} I_x \left[(1 - C_1)e^{-a\Delta t} + a\,\Delta t - (1 - C_1)\right] + \phi_1 \qquad (12)$$

The effect of the reversal of the control in producing roll in the opposite direction is determined by superposition. The algebraic summation of the motion that would have existed had the control not reversed with the incremental changes in angle of bank caused by the control reversal gives the resultant motion of the aircraft following the control reversal.

The incremental changes introduced by the control reversal may be expressed by the basic response equation of the aircraft. The basic rolling moments produced are due to control deflections from the neutral to the maximum positions. However, at point 1, the control reversal is from a maximum deflection in one direction to the equal value in the other direction. Consequently, the incremental changes in the angle-of-bank variation introduced by the control action at point 1 are twice the magnitude given by the original basic response equation. The constants of integration for the control effect are determined by conditions of zero angle of bank and zero rolling velocity. This control effect is written as

$$2\phi_c = 2\,\frac{\delta L_\delta}{L_p^2} I_x \left(e^{-a\Delta t} + a\,\Delta t - 1\right) \qquad (13)$$

The resultant rolling motion between points 1 and 2, ϕ_{12}, is found by the superposition of the motions expressed in equations (12) and (13). This gives

$$\phi_{12} = \frac{\delta L_\delta + L_o}{L_p^{\,2}} I_x \left[(I - C_1)e^{-a\Delta t} + a\,\Delta t - (1 - C_1)\right]$$

$$+ \phi_1 - 2\,\frac{\delta L_\delta}{L_p^{\,2}} I_x \left(e^{-a\Delta t} + a\,\Delta t - 1\right) \tag{14}$$

It is now defined that after a time t_{12} seconds from the time of the control reversal the aircraft has reached a rate of roll of $-C'C_0p_{max}$, or has reached the fraction C' of the rolling velocity existing at the beginning of the cycle. The negative sign is used since the direction of the rolling velocity is opposite that defined at the signal reversal at point 0. This substitution into the rolling-velocity expression obtained from differentiation of equation (14) gives

$$-C'C_0\left(\frac{\delta L_\delta + L_o}{L_p}\right) = \left(\frac{\delta L_\delta + L_o}{L_p}\right)\left[1 - (1 - C_1)e^{-at_{12}}\right] - 2\,\frac{\delta L_\delta}{L_p}\left(1 - e^{-at_{12}}\right) \tag{15}$$

Before solving equation (15) for t_{12} it is convenient to make the following substitution:

$$\epsilon = \left|\frac{L_o}{\delta L_\delta}\right| \tag{16}$$

where ϵ is termed the out-of-trim ratio. Substituting equation (16) into equation (15) and using equation (10) for C_1 gives t_{12} as

$$t_{12} = \frac{1}{-a}\log Y \tag{17}$$

where

$$Y = \frac{1 - C'C_0\left(\frac{1 + \epsilon}{1 - \epsilon}\right)}{1 + \left[1 - (1 - C_0)e^{-K}\right]\left(\frac{1 + \epsilon}{1 - \epsilon}\right)} \tag{18}$$

Furthermore, let t_{12} also be the time at which $\phi = 0$. (See fig. 1.) With $\phi = 0$ and $\Delta t = t_{12}$ in equation (14), and with equation (17) for t_{12}, equation (16) for L_o, and equation (10) for C_1, the following form is reduced:

$$(1 + \epsilon)\left[(1 - C_0)e^{-K}\right]Y - 2Y + (1 - \epsilon)\log Y + 2 + (1 + \epsilon)(K - 1 + C_0) = 0 \tag{19}$$

where Y is defined by expression (18).

Equation (19) is the conditional equation for the first half-cycle. This expression permits the determination of the rolling velocity at the end of the first half-cycle $C'C_0p_{max}$ when that at the start of the cycle C_0p_{max} is known; these velocities are functions of the automatic stabilization parameter K and the out-of-trim ratio ϵ which must be known or estimated for the aircraft.

Second half-cycle.- At point 2 (fig. 1) it is noted that the control moment is to the left and hence would require a negative sign. The sense of the out-of-trim moment is, of course, unchanged. In this case the result of this opposition of rolling moments is a change in the expression that is given to the basic response equations of the aircraft. These equations are rewritten as

$$\phi = \frac{-\delta L_\delta + L_o}{L_p^{\,2}} I_x\left[(1 - c)e^{-a\Delta t} + a\,\Delta t - (1 - c)\right] + \phi(0) \tag{20}$$

$$p = \frac{-\delta L_\delta + L_o}{L_p}\left[1 - (1 - c)e^{-a\Delta t}\right] \tag{21}$$

where the maximum rolling velocity when the control and out-of-trim moments are opposed takes the form

$$p'_{max} = \frac{-\delta L_\delta + L_o}{L_p} \tag{22}$$

and c is defined as the fraction of p'_{max} at the time origin considered.

The angle-of-bank variation between points 2 and 3 is now required. At point 2, $\phi = 0$ and the rolling velocity fraction c_2 for use in equations (20) and (21) takes the form

$$c_2 = C'C_0\left(\frac{1 + \epsilon}{1 - \epsilon}\right) \tag{23}$$

when equation (16) is used for L_o. The variation ϕ_{23} is now written from equation (20) as

$$\phi_{23} = \frac{-\delta L_\delta + L_o}{L_p^{\,2}} I_x\left[(1 - c_2)e^{-a\Delta t} + a\,\Delta t - (1 - c_2)\right] \tag{24}$$

The steps used in the consideration of the first half-cycle are followed for the second half-cycle with the following important relationships resulting:

$$\phi_3 = \frac{-\delta L_\delta + L_o}{L_p^{\,2}} I_x\left[(1 - c_2)e^{-K} + K - (1 - c_2)\right] \tag{25}$$

$$p_3 = c_3 p'_{max} \tag{26}$$

$$c_3 = \left[1 - (1 - c_2)e^{-K}\right] \tag{27}$$

The motion ϕ_{234} which would have resulted had the control not reversed at point 3 is given as

$$\phi_{234} = \frac{-\delta L_\delta + L_o}{L_p^{\,2}} I_x\left[(1 - c_3)e^{-a\Delta t} + a\,\Delta t - (1 - c_3)\right] + \phi_3 \tag{28}$$

and by superposition the resultant rolling motion ϕ_{34} following the control reversal takes the form

$$\phi_{34} = \frac{-\delta L_\delta + L_o}{L_p^2} I_x\left[(1 - c_3)e^{-a\Delta t} + a\,\Delta t - (1 - c_3)\right]$$

$$+ \phi_3 + 2\,\frac{\delta L_\delta}{L_p^2} I_x\left(e^{-a\Delta t} + a\,\Delta t - 1\right) \tag{29}$$

Expressing the rolling velocity after a time t_{34} seconds from the second control reversal as $C''C_0p_{max}$, or as the fraction C'' of the rolling velocity at the beginning of the cycle, allows t_{34} to be written as

$$t_{34} = \frac{1}{-a} \log X \tag{30}$$

where

$$X = \frac{1 - C''C_0}{1 + \left(1 - e^{-K}\right)\left(\frac{1 - \epsilon}{1 + \epsilon}\right) + C'C_0e^{-K}} \tag{31}$$

Finally, the conditional equation for the second half-cycle is reduced to the form

$$(1 - \epsilon)(1 - X)\left[1 - C'C_0\left(\frac{1 + \epsilon}{1 - \epsilon}\right)\right]e^{-K} - (1 - \epsilon)\left\{- \log X + K\right.$$

$$\left. + \left(e^{-K} - 1\right)\left[1 - C'C_0\left(\frac{1 + \epsilon}{1 - \epsilon}\right)\right]\right\} + 2(X - \log X - 1) = 0 \tag{32}$$

Equation (32) allows the determination of the rolling velocity at the end of the cycle $C''C_0p_{max}$ when the rolling velocity at the start of

the second half-cycle $C'C_0p_{max}$, the automatic stabilization parameter K, and the out-of-trim ratio ϵ are known.

Amplitude equation.- The amplitude of the oscillation is defined as one-half the total displacement. The total displacement is found from the maximum angles of bank for each half-cycle. Since the maximum angle of bank occurs at zero rolling velocity, the substitution of the time to reach zero rolling velocity into the angle-of-bank equation for the desired portion will give the required maximum angle. These results are written as

$$(\phi_{12})_{max} = B\left\{(1 + \epsilon)(K + C_0) + (1 - \epsilon)\log\left[\frac{1 - \epsilon}{2 - (1 + \epsilon)(1 - C_0)e^{-K}}\right]\right\} \tag{33}$$

$$(\phi_{34})_{max} = -B\left(K(1 - \epsilon) + C'C_0(1 + \epsilon) + (1 + \epsilon)\log\left\{\frac{1 + \epsilon}{2 - (1 - \epsilon)e^{-K}\left[1 - C'C_0\left(\frac{1 + \epsilon}{1 - \epsilon}\right)\right]}\right\}\right) \tag{34}$$

where B is termed the amplitude factor and is defined as

$$B \equiv \frac{\delta L_\delta I_x}{L_p^{\,2}} \tag{35}$$

The amplitude A of the oscillation is given as

$$A = \frac{1}{2}\left[(\phi_{12})_{max} - (\phi_{34})_{max}\right] \tag{36}$$

and is calculated by using equations (33) and (34).

Mean-line displacement.- Since the presence of the out-of-trim moment causes the oscillation to be unsymmetrical about the zero angle-of-bank position, the mean line of the oscillation is displaced from zero

in the direction of the out-of-trim moment. The displacement of the mean line is designated A_o and is given by the equation

$$A_o = \frac{1}{2}\left[(\phi_{12})_{max} + (\phi_{34})_{max}\right] \tag{37}$$

Period equation.- The period of the oscillation is defined as the time required to complete 1 cycle. From figure 1 it is evident that the period P may be expressed as

$$P = 2\tau + t_{12} + t_{34} \tag{38}$$

Substituting equation (17) for t_{12} and equation (30) for t_{34} allows equation (38) to be written as

$$P = \tau\left[2 - \frac{1}{K}(\log X + \log Y)\right] \tag{39}$$

where X and Y are defined by equations (31) and (18), respectively.

Angle-of-bank equation.- The angle-of-bank time equation is written in four parts. Each part is computed separately, and time increments are from $\Delta t = 0$ for each part. The equation for the resultant angle-of-bank variation is

$$\phi = \phi_{01} + \phi_{12} + \phi_{23} + \phi_{34} \tag{40}$$

where the equations are (8) for ϕ_{01}, (14) for ϕ_{12}, (24) for ϕ_{23}, and (29) for ϕ_{34}.

PRESENTATION OF RESULTS

General Remarks

Prior to the application of the derived equations to the detailed analysis, several observations of some of the actual physical properties of an aircraft-automatic pilot combination are presented. In this instance, as in nearly every other system, it is the actual physical characteristics that define the limitations of the automatic stabilization system.

In any real automatic-pilot system of the type under discussion, there is in its operation a definite time lag – the time between the sensing of an error and the application of corrective control. Considering the statement above, it is readily possible to include all forms of lag, whether they be considered mechanical, electrical, aerodynamic, or otherwise. Because of this lag the aircraft, when once disturbed, must oscillate, these oscillations never damping completely. It is only in the theoretical case of zero lag, which represents a limit of possibilities for this system rather than a possibility itself, that the induced oscillations following a disturbance will damp to zero amplitude.

In general, it may be stated that within certain ranges of various parameters the automatically controlled aircraft will, upon being disturbed, tend to oscillate at a definite amplitude and frequency. This condition is defined as the steady-state condition, and at this steady-state condition the aircraft will never exceed a definite rate of roll which is real and determinable. If the disturbance is such as to cause the rate of roll at the time of signal reversal to be greater than the steady-state value, the transient oscillations which follow will damp to the steady-state conditions; and if the disturbance is such as to cause the rate of roll to be less than the steady-state value, the resulting transient oscillations will build up to the steady-state values. This steady-state condition is a real limit which theoretically is only approached; however, for all practical purposes it is usually very closely approached in a relatively short time after the disappearing disturbance is encountered.

Transient and Steady State

The conditional equations, defined by expressions (19) and (32), present the relationships which will describe the actions of the aircraft under any prescribed condition. The prescribed condition necessarily involves the automatic stabilization parameter K and the out-of-trim factor ϵ; for a given aircraft these parameters must either be known or estimated. The other factor of the prescribed condition is the fraction C_0 of maximum rolling velocity at the time of a signal reversal.

The conditional equation for the first half-cycle will show the fraction C' of the initial rolling velocity considered $C_0 p_{max}$ that exists at the time of the second signal reversal, provided ϵ and K are given. The use of the second conditional equation presupposes that C' has been determined by use of the first conditional equation, and it reveals the rolling velocity which exists at the end of the cycle $C''C_0 p_{max}$.

Transient state.– The two conditional equations may be used to show the transient conditions occurring as the aircraft approaches the steady state. An evaluation of the two conditional equations which reveals C'' to be less than unity shows that the rolling velocities of the aircraft are decreasing, and hence, the aircraft motions are damping to the steady-state condition. The increase to the steady-state condition is shown by values of C'' greater than one.

As an example, the analysis is applied to the determination of transient conditions with the assumption of perfect trim $(\epsilon = 0)$. In this case the rolling oscillations are symmetrical about zero angle of bank. For a value of $K = 0.5$, the respective values of C' and C'' were computed for various values of C_0 by using the two conditional equations. These results are presented as figure 3 with C'' plotted against C_0. Although figure 3 is a particular case, the shape of this curve is typical of those obtained when any values of ϵ and K are used.

As an illustration of the use of this type of curve, let a value of $C_0 = 0.2$ be considered. From figure 3, the rolling velocity at the end of that cycle $C''C_0p_{max}$ is shown to be $(3.55)(0.2)p_{max} = 0.71p_{max}$. Considering $C_0 = 0.71$ for the next cycle, the rolling velocity at the end of this second cycle is given from figure 3 again as $(1.05)(0.71)p_{max} = (0.7455)p_{max}$. Continuing this method would reveal that the condition of $C'' = 1.0$ and $C_0 = 0.75$ is approached; this is the approached steady-state condition since all of the cycles become approximately identical. Assuming a value of C_0 greater than the steady-state value will also show the tendency to approach the steady-state condition. Thus, when $C'' < 1$, the damping character of the motion is shown, and the increasing action is revealed when $C'' > 1$.

Steady state.– At the steady-state condition, the value of C'' must be unity since this case assures the equivalence of every cycle. Since C' is defined when C_0 is defined (equation (19)), the steady-state conditions may be expressed solely as functions of ϵ, K, and C_0. Equations (19) and (32) have been solved for steady-state conditions and the results presented in figure 4 as K against C_0 with ϵ as a parameter. Thus, for given values of ϵ and K, the value of C_0 at steady-state is shown by figure 4.

Stabilization Boundaries

In this analysis motions involving angles of bank greater than 180° are not considered. This consideration allows limiting values of various parameters to be defined which will permit this maximum angle-of-bank condition. Indeed, limiting values can be defined for any maximum angle

of bank. These upper limits of the various factors are termed stabilization boundaries. These boundaries are presented to allow a rapid evaluation of whether or not a given aircraft employing this automatic pilot may possibly give satisfactory amplitude characteristics.

Since the effects of out-of-trim conditions in producing roll are included, the maximum angle of bank will be in the direction of the out-of-trim moment (to the right in this analysis). This maximum angle of bank has been defined in equation (33) as $(\phi_{12})_{max}$. Substituting π radians (180°) into this expression will give the relationship that must exist between the parameters to insure this condition. This expression is

$$\pi = B\left\{(1 + \epsilon)(K + C_0) + (1 - \epsilon)\log\left[\frac{1 - \epsilon}{2 - (1 + \epsilon)(1 - C_0)e^{-K}}\right]\right\} \tag{41}$$

It is noted that, at steady state, the bracketed term in equation (41) is defined when ϵ and K are defined. Figure 5 is presented as the solution of equation (41) and is expressed as a function of the amplitude factor B and the automatic stabilization parameter K for various out-of-trim ratios ϵ. Also shown on figure 5 is the boundary where the limit maximum angle of bank is $\pi/6$ radians (30°).

If the values of K and B for an aircraft are plotted as a point on this graph and the point is below the boundary, the aircraft will stabilize in roll at an amplitude less than the value shown for the boundary. These curves indicate clearly that the magnitude of these factors is rather limited.

Amplitude Equations

The amplitude of the rolling oscillations, defined by equation (36), is considered herein only for the steady state. Since values of C_0, C', and C'' are defined at steady state when ϵ and K are known, the amplitude A may be considered as a function of three parameters, ϵ, K, and B. Since the steady-state amplitude A varies directly with the amplitude factor B, it is possible to present A/B against the automatic stabilization parameter K for any given value of the out-of-trim ratio ϵ.

Figure 6 shows the A/B variation with K for the case of perfect trim ($\epsilon = 0$). (Value of A computed from this figure is in degrees.)

The consideration of out-of-trim conditions led to calculations of the steady-state amplitudes for several values of ϵ over the range

of K from 0 to 4. These calculations show that at any value of ϵ the value of A/B was always within 6 percent of the value of A/B at $\epsilon = 0$ for the same value of K. In other words, the amplitude of the steady-state rolling oscillations (one-half of the total displacement) is practically unaffected by out-of-trim moments.

In understanding this action, it is necessary to remember that K and B are constant and only ϵ is changed. With the introduction of ϵ, it is seen that the unsymmetrical character of the oscillations appears, the presence of the out-of-trim moment causing the control action to be slow in stopping roll in one direction and very quick in stopping roll in the other direction. This effect, however, results in very slight changes in the total amplitude displacement since the out-of-trim condition causes a greater maximum angle of bank in one direction but a lesser maximum value of bank in the other direction. Consequently, figure 6 may also be used satisfactorily in estimating the amplitude of the steady-state roll oscillations under out-of-trim conditions.

Mean-line displacement.- The unsymmetrical nature of the rolling motions under out-of-trim conditions may be expressed as the displacement of the mean line of the oscillations from the zero reference. Equation (37) represents this displacement which is rewritten here as

$$A_o = \frac{B}{2}\left[f_{12}(\epsilon, K) + f_{34}(\epsilon, K)\right] \tag{42}$$

where $f_{12}(\epsilon, K)$ and $f_{34}(\epsilon, K)$ are the bracketed terms of equations (33) and (34), respectively, both of which are completely defined by ϵ and K under the steady-state conditions which are under consideration here.

The solution of equation (42) for the steady state is presented in figure 7. The ratio of the mean-line displacement A_o to the amplitude factor B is expressed as a function of ϵ and K. In order to determine the extent that the mean line of the rolling oscillations has been displaced, the value of the ordinate of figure 7, at the values of K and ϵ in question, multiplied by the amplitude factor B will give the value of the displacement A_o in degrees.

Period Equations

The determination of the period of the rolling oscillations is accomplished by use of equation (39). Figure 8 presents the graphical interpretation of equation (39) for steady-state conditions, and is a plot of the ratio of the period P to the lag time τ against K for various values of ϵ. Using the value of K for the aircraft, the

P/τ value read from figure 8 multiplied by the lag time τ will give the period of the steady-state oscillations.

Although the ratio of the period to the lag time is presented, care should be taken in interpreting the significance of this ratio. The period does not vary directly with the lag time since K is also a function of τ. For example, the doubling of τ does not double the period since the value of P/τ is changed due to the resultant doubling of K.

The dotted portions of the curves of figure 8 are extrapolations. This was a result of the determination of the natural logarithm of the factor Y in equation (39) which involves a very careful consideration of the extent to which this decimal fraction is evaluated. A more exact solution was not considered justified, particularly under the more extreme out-of-trim conditions.

APPLICATION OF RESULTS

It is evident that the results of this analysis can be used to determine the steady-state amplitude and period without actually substituting into equations. This procedure may be broken down into five steps.

(1) Determine the factors K, ϵ, and B.

(2) Plot K and B as a point on figure 5.

(3) Calculate the amplitude from figure 6.

(4) Calculate the displacement of the mean line from figure 7.

(5) Calculate the period from figure 8.

Step (2) will indicate immediately whether or not the conditions might possibly satisfy some specific amplitude requirement. The three latter steps will give the actual values at steady state. If the actual bank variation is desired, equation (40) should be used.

In an effort to indicate whether or not this automatic roll-stabilization system might be of use in current problems, table I was prepared. The aircraft shown are not identical with any particular pilotless-aircraft designs, although current trends are revealed in the magnitude of the characteristics presented. (The silhouette of aircraft 5 is approximately one-half the relative size shown for the others.) In the case of aircraft 3, the effect of doubling the lag time is shown to increase the amplitude and period by a factor of about 1.7 under the conditions considered. Aircraft 1 and 3 show the possible importance of the amplitude

factor B. In both cases the values of K are comparable, but because of B the amplitude for aircraft 2 is extremely large, whereas aircraft 1 may be satisfactory for some applications.

Although no special trends can be determined from this table, it does indicate that satisfactory roll stabilization with the topic system is a definite possibility in some cases. The method presented herein allows a rapid determination of the rolling characteristics and for a definite aircraft would be of considerable aid in determining the effects of variations of individual parameters.

In the following paragraphs the present method will also be shown for one of the cases checked by roll-simulator tests. Attention will also be called to other additional factors which affect the application of this method.

Roll-Simulator Tests

In order to substantiate these theoretical results, experimental tests were run using the roll simulator developed by the Instrument Research Division of the Langley Laboratory. This instrument is a single degree of freedom system which is controlled by an actual automatic pilot mounted on the moving table. The control torque-to-inertia ratio and aircraft damping-to-inertia ratio are electromechanically simulated.

An Azon gyroscopic unit and an electrically operated servomotor were used as a typical displacement-response, flicker-type automatic pilot. In these tests the values of the torque-to-inertia ratio $(\delta L_\delta/I_x)$ and the damping-to-inertia ratio (L_p/I_x) were varied. Oscillograph records of angle-of-bank and control position (servo-operation) against time were taken. Figure 9 shows the recorded results for two of the cases tested.

It is noted that the experimental control motion is trapezoidal in shape. It is apparent, therefore, that the lag time cannot be taken as the time period between the signal reversal at zero angle of bank and the time the control reached full reverse deflection because to do so would be to neglect the effect of the control during the time it was reversing. Since the theoretical analysis is based on the instantaneous application of the maximum control torque, an equivalent square wave was substituted for the control motion. An effective lag τ was computed as the time between signal reversal (at zero angle of bank) and the time of control reversal given by the equivalent square wave control motion.

Two illustrative cases which were tested are presented herein.

Case 1.– Calculations using the present method are given for the conditions employed in case 1; the experimental results are presented in figure 9(a).

Given:

$$\frac{\delta L_\delta}{I_x} = 32.0$$

$$a = \left|\frac{L_p}{I_x}\right| = 4.0$$

Calculated:

$$B = \frac{\delta L_\delta / I_x}{a^2} = 2.0$$

$$\tau = 0.025 \text{ sec}$$

$$K = a\tau = 4.0(0.025) = 0.10$$

From figure 6, at $K = 0.10$ and $\epsilon = 0$, $\frac{A}{B} = 8.0$. Therefore,

$$A = 8.0(2.0) = 16.0^\circ$$

The average measured amplitude equals 15.7°. From figure 8, at $K = 0.10$ and $\epsilon = 0$,

$$\frac{P}{\tau} = 21.2$$

$$P = 21.2(0.025) = 0.530 \text{ sec}$$

The average measured period equals 0.511 second.

Case 2.– Case 2 represents typical conditions for an aircraft similar to number 3 (table I) at a Mach number of 0.6. Figure 9(b) shows the

simulator results. For this case values of $\frac{\delta L_\delta}{I_x} = 43.5$, $a = 9.74$, and $\tau = 0.026$ were used. These data give a calculated amplitude of 8.95°, whereas the average measured amplitude is 10.8°. The calculated period is 0.355 second as compared with an average measured period of 0.409 second.

The differences encountered in the measured and calculated values are well within the limits of accuracy afforded the simulator at the present time. Additional error is believed to have entered case 2 due to the fact that the employed torque-to-inertia and damping-to-inertia ratios are out of the linear portions of the simulator characteristics.

Additional Factors

In discussing the topic of automatic roll stabilization, there are several additional factors that should be presented in order that the applicability of the preceding analysis may be more clearly realized. These factors concern some of the included parameters, their significance and use, as well as some others not specifically included in this paper.

Effects of velocity and air density.- Two primary factors involved in the analysis of the rolling motions of an aircraft are affected considerably by changes in velocity and air density; these parameters are L_p and L_δ. Both of these derivatives are in terms of actual rolling-moment variation and hence are affected by velocity and air density. It can be shown that, for a constant damping-in-roll coefficient $C_{l_{\frac{pb}{2V}}}$, the parameter L_p varies directly as the product of the air density ρ and the velocity V. It may also be shown that the control parameter L_δ varies directly with the dynamic pressure q when the coefficient C_{l_δ} is assumed constant.

It is necessary, therefore, to emphasize that the analysis herein will pertain only to those conditions upon which the determination of L_p and L_δ are based. The effects of changes in velocity and/or air density on the automatic roll-stabilization characteristics of an aircraft require that they be investigated as desired by particular cases.

Effects of compressibility.- In the previous paragraphs it was mentioned that the coefficients $C_{l_{\frac{pb}{2V}}}$ and C_{l_δ} were considered constant in determining the effects of velocity and air density. However, the

constancy of these coefficients, particularly in the transonic speed range, is not assured due to the compressibility effects. If automatic roll stabilization is contemplated in or through this speed range, the stabilization characteristics should be based on whatever theoretical or experimental data are available concerning the coefficients.

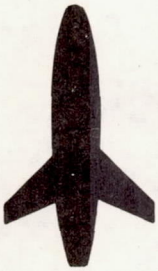
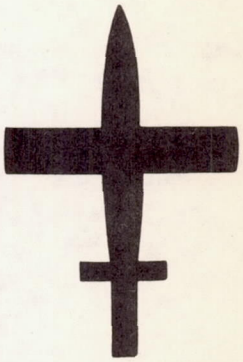
Significance of δL_δ.- In this analysis the actual moment produced by the control is expressed in aerodynamic notation δL_δ. However, the corrective control moments may not necessarily be aerodynamic; for example, the use of intermittent rocket jet blasts has been suggested as a possible control device, particularly for use on extremely high-altitude pilotless aircraft or guided missiles. It is emphasized that the term δL_δ used in this paper refers to the actual moment used in controlling the aircraft, regardless of its nature; however, care should be exercised to insure a proper application to the problem when systems other than aerodynamic controls are used.

CONCLUDING REMARKS

A general analysis of the rolling motions of an aircraft using a displacement-response, flicker-type automatic pilot has been presented. Because of the inherent residual oscillations, this system is not considered ideal for many stabilizations problems; however, this flicker-type system may possibly offer a simple and economic solution to those applications where steady-state oscillations may not be objectionable. Current trends in pilotless-aircraft designs indicate that the topic system can possibly provide roll stabilization with small amplitude residual oscillations. Limits of the parameters involved have been presented as stabilization boundaries which allow a determination of whether or not a definite installation may possibly be satisfactory. Charts have been prepared for the determination of the amplitude and period of the resultant steady-state oscillations, and these theoretical considerations have shown close agreement with roll-simulator tests. A method for finding the conditions of the transient oscillations leading to steady state has been outlined. Consideration has also been given to the case where out-of-trim moments causing roll are present, and this effect is discussed.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va.

TABLE I

						
Aircraft	1	2	3		4	5
Weight, lb	600	150	500		1200	5200
Diameter, in.	10	8	20		17	34
Length/Dia.	23	15	6.0		12	9.6
Aspect Ratio	1.5	2.5	4.6		4.0	5.1
Mach Number	1.7	1.5	1.5		0.85	0.65
δL_δ, ft-lb	218	347	930		270	600
L_p, ft-lb/rad/sec	50.5	6.58	420		17.5	3890
I_x, slug-ft^2	2.1	0.3	7.8		14.8	1665
$a = L_p/I_x$	24.0	21.9	54.0		1.18	2.34
τ, sec	0.025	0.025	0.025	0.050	0.025	0.025
$K = a\tau$	0.60	0.55	1.35	2.70	0.0295	0.058
$B = (\delta L_\delta)I_x/L_p^2$	0.18	2.41	0.041	0.041	13.0	0.066
Fig. 6 A/B	43.0	39.5	90.5	171	2.5	4.8
Fig. 7 P/τ	9.3	9.6	6.7	5.4	30.0	28.0
Amplitude, deg	7.75	95	3.71	7.0	32.5	0.315
Period, sec	0.232	0.24	0.168	0.27	0.75	0.70

NACA

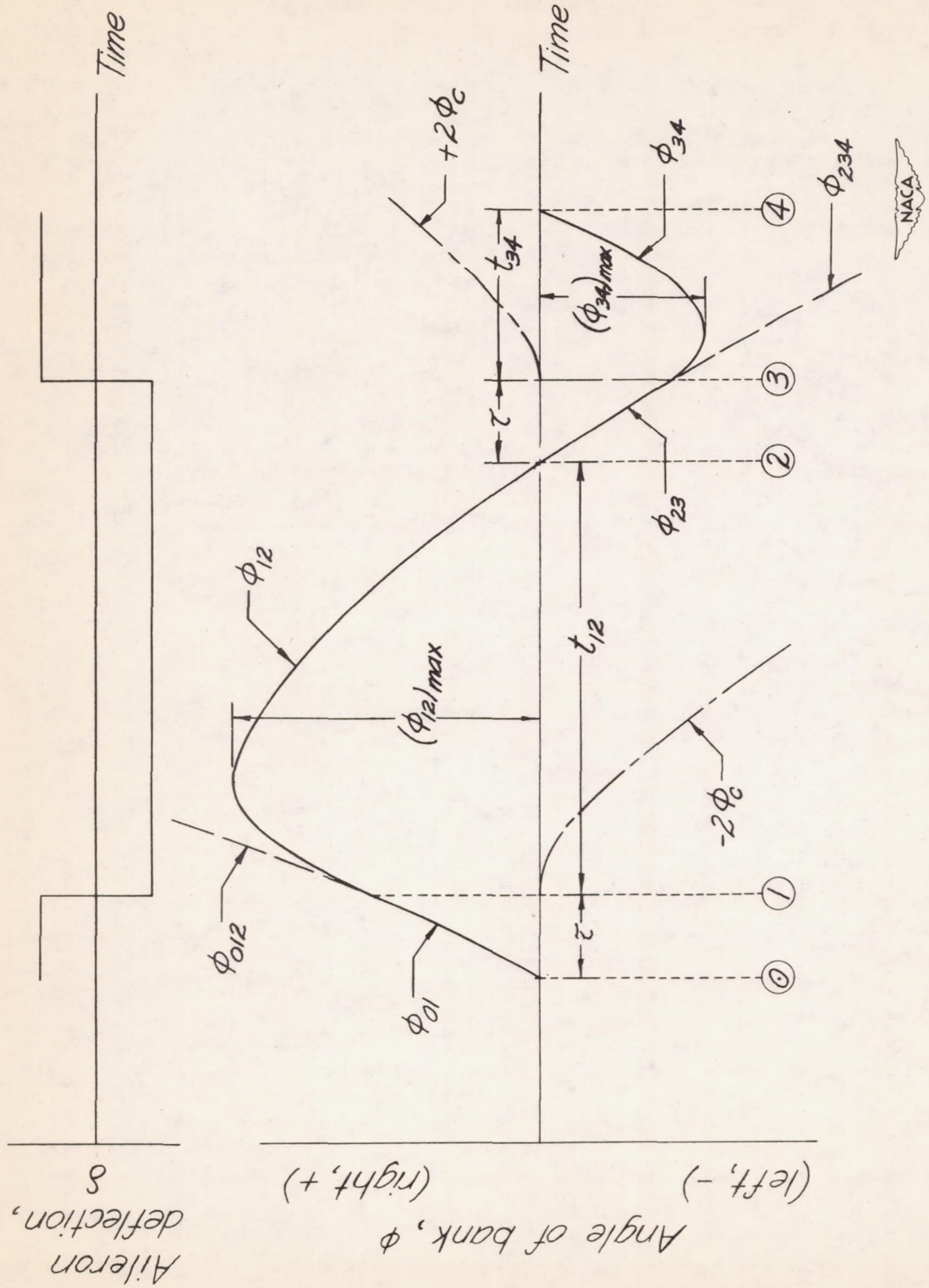


Figure 1.- Mathematical nomenclature for one cycle of roll.

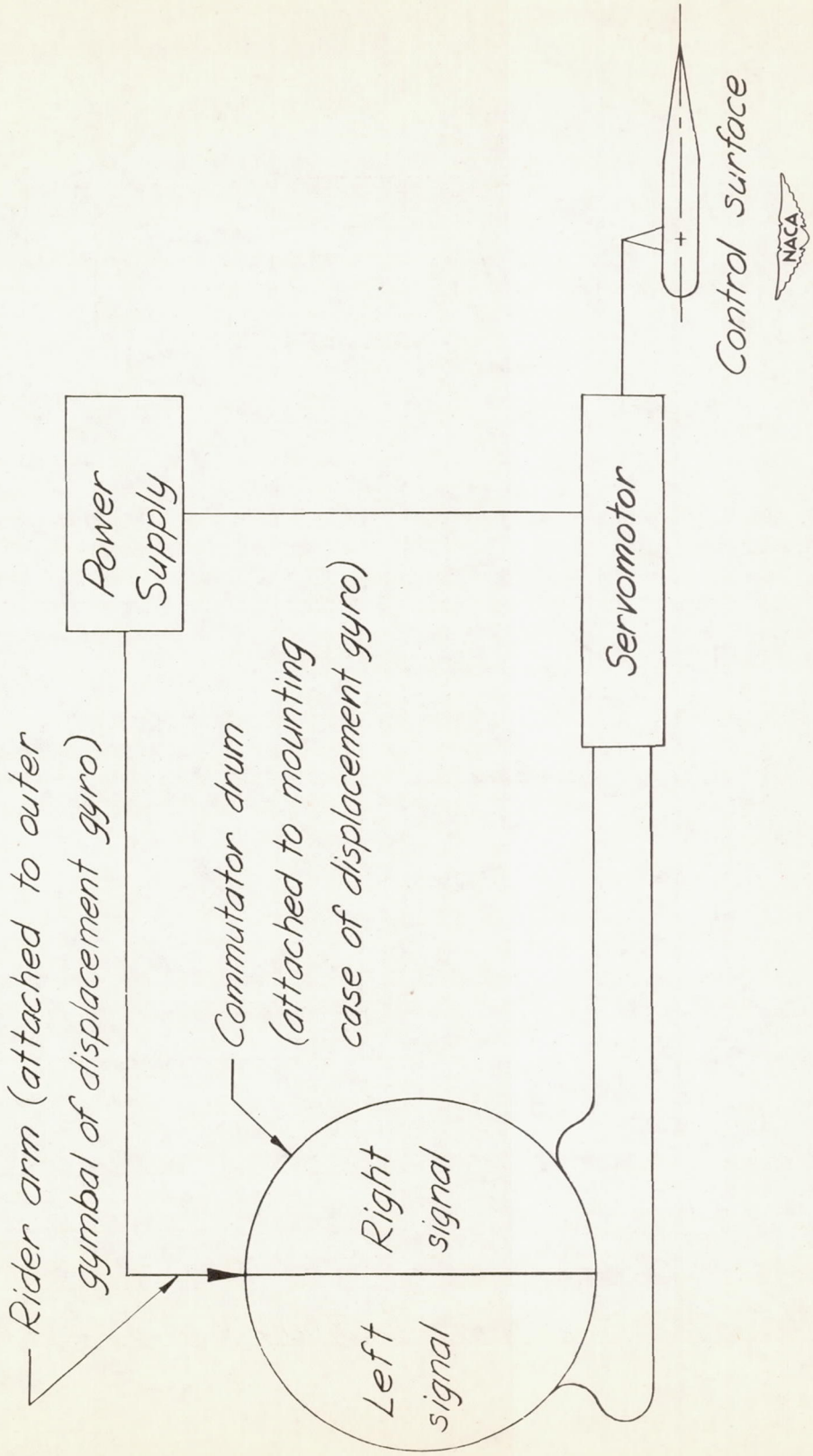


Figure 2.- Schematic diagram of a typical displacement-response, flicker-type automatic pilot.

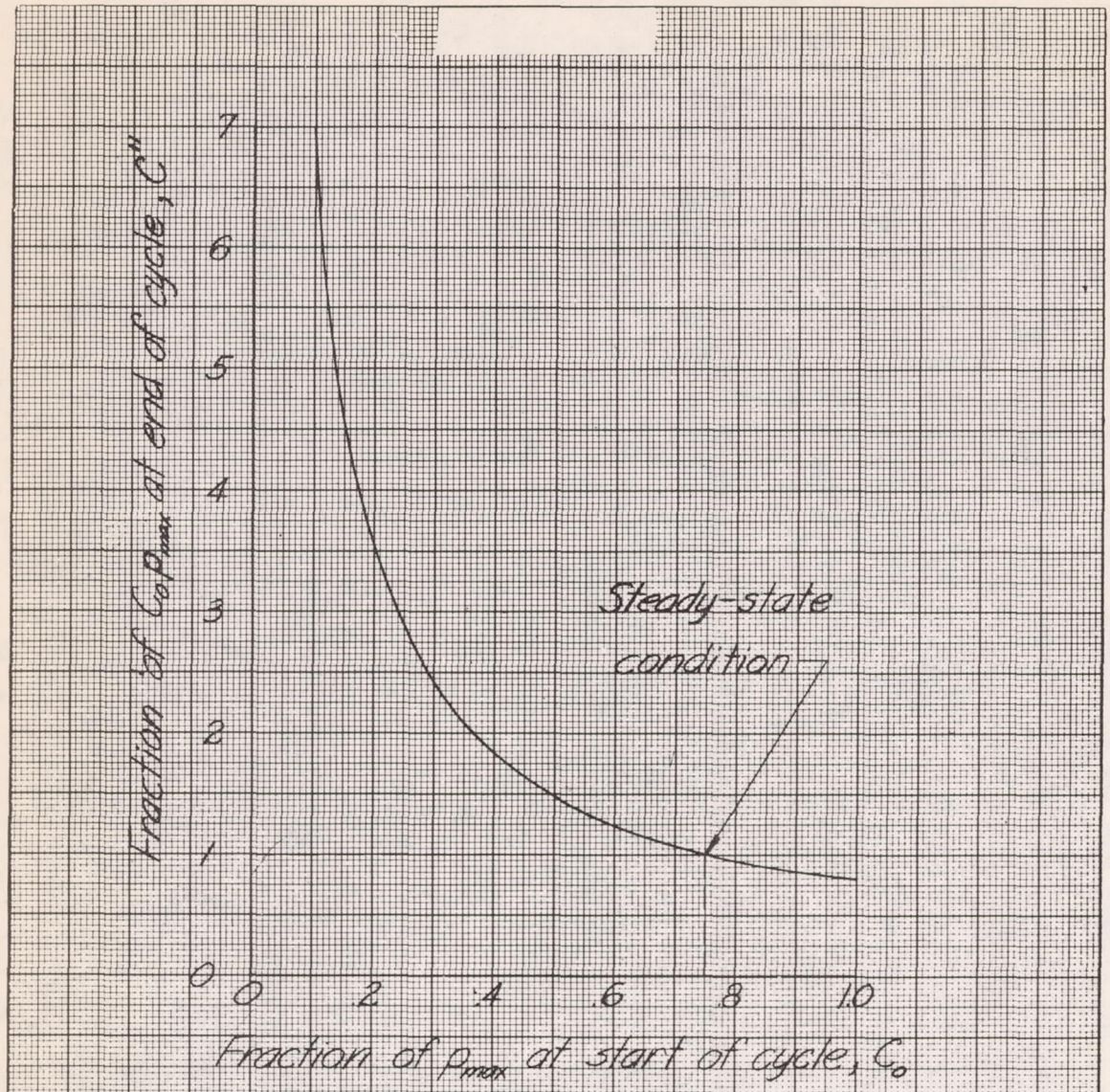


Figure 3.- Typical variation of C'' with C_0 for the transient state. Case shown is for $\epsilon = 0$ and $K = 0.5$.

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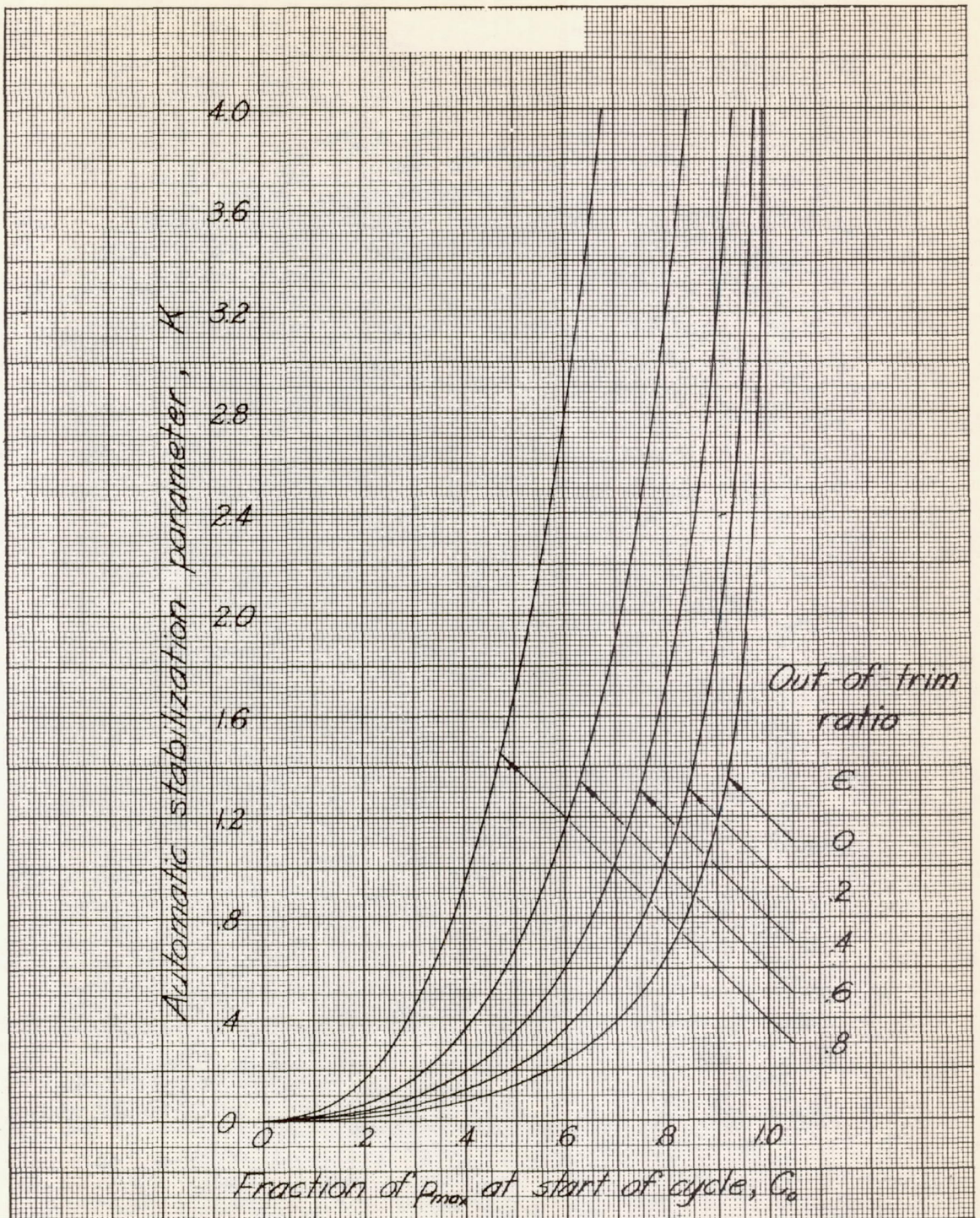

Figure 4.- The relationships of ε, K, and C_0 at steady state.

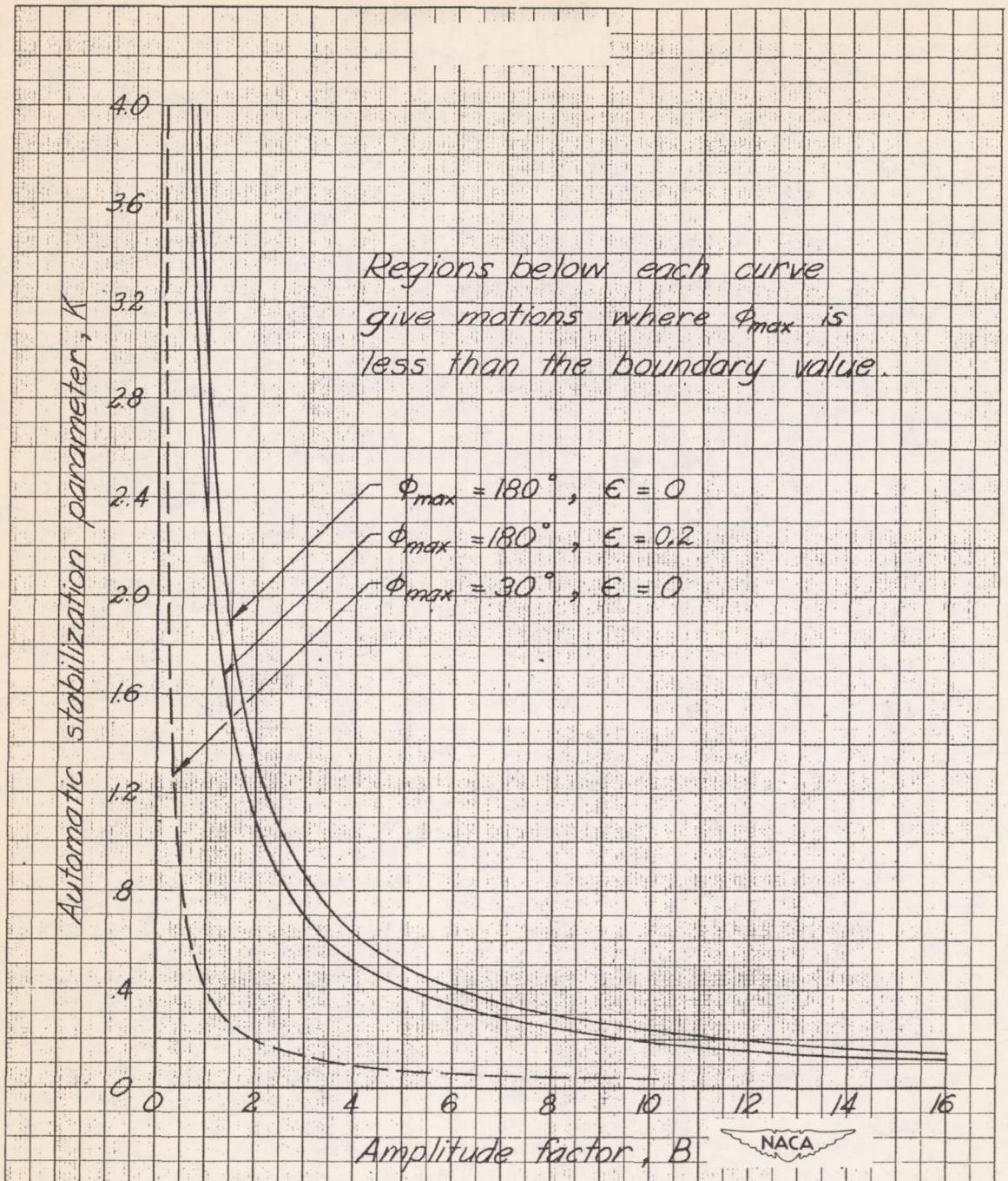

Figure 5.- Stabilization boundaries for an aircraft using the displacement-response, flicker-type automatic pilot.

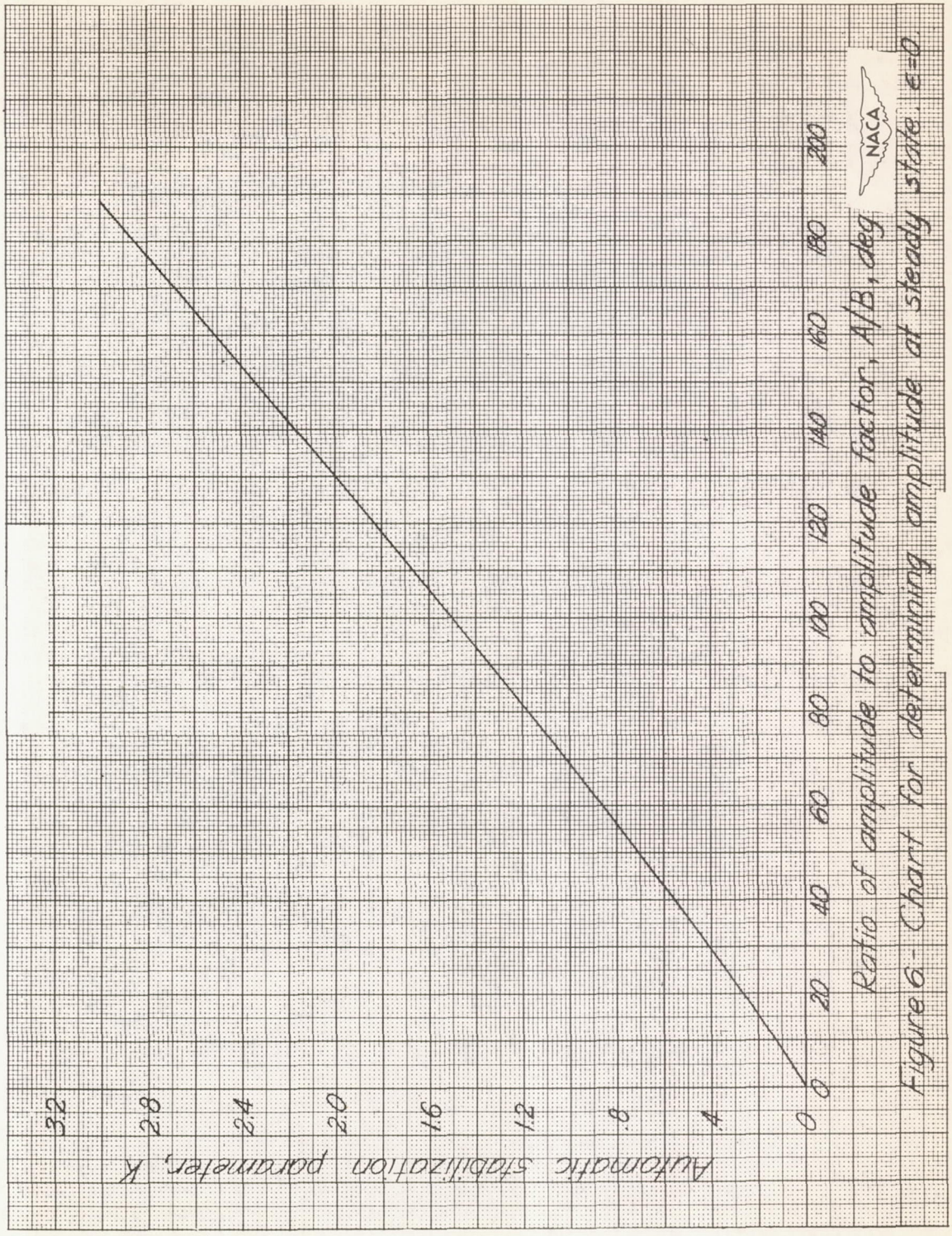

Figure 6.- Chart for determining amplitude at steady state, $\epsilon = 0$.

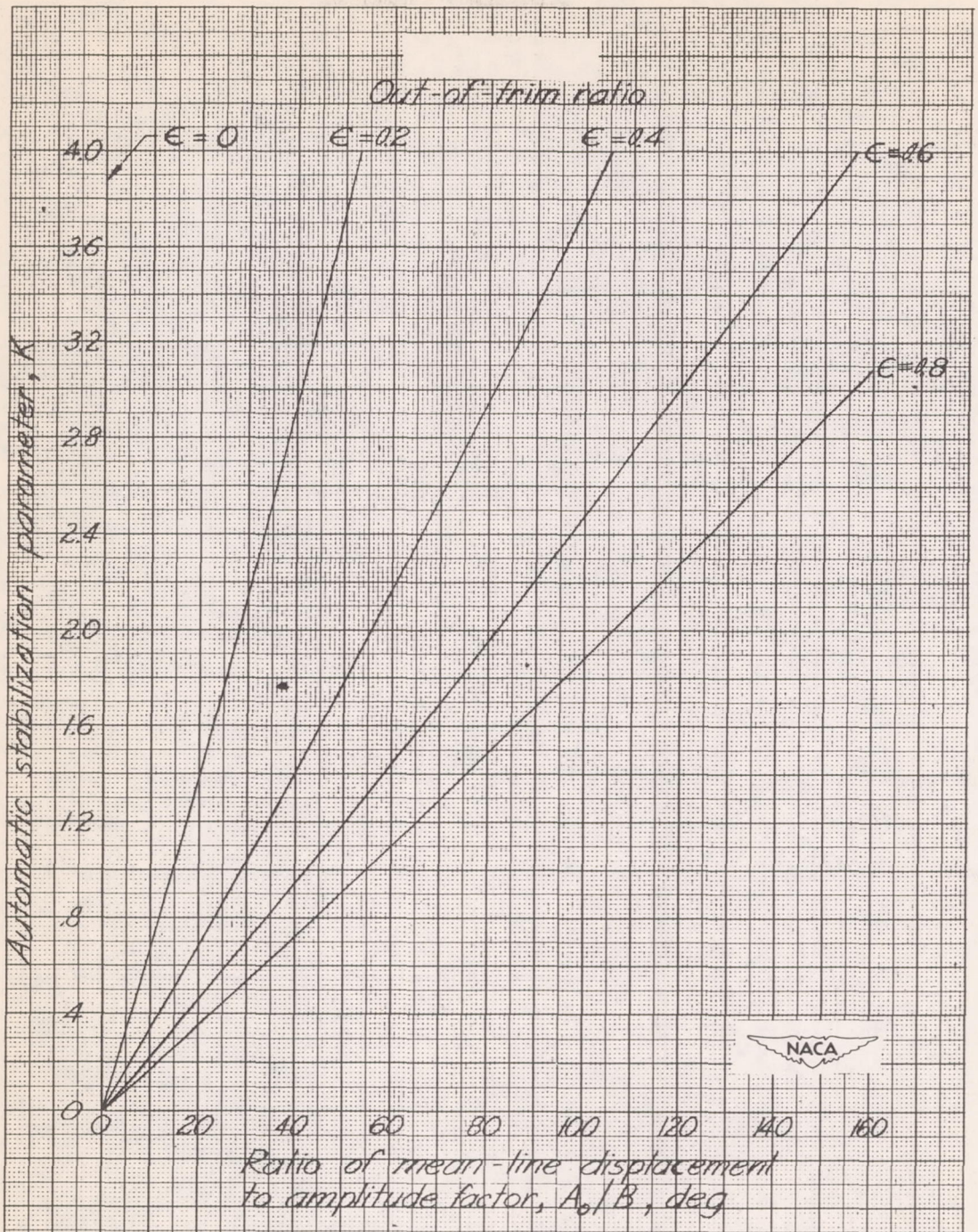

Figure 7.- Chart for determining the displacement of the mean line of steady-state roll oscillations for various out-of-trim conditions.

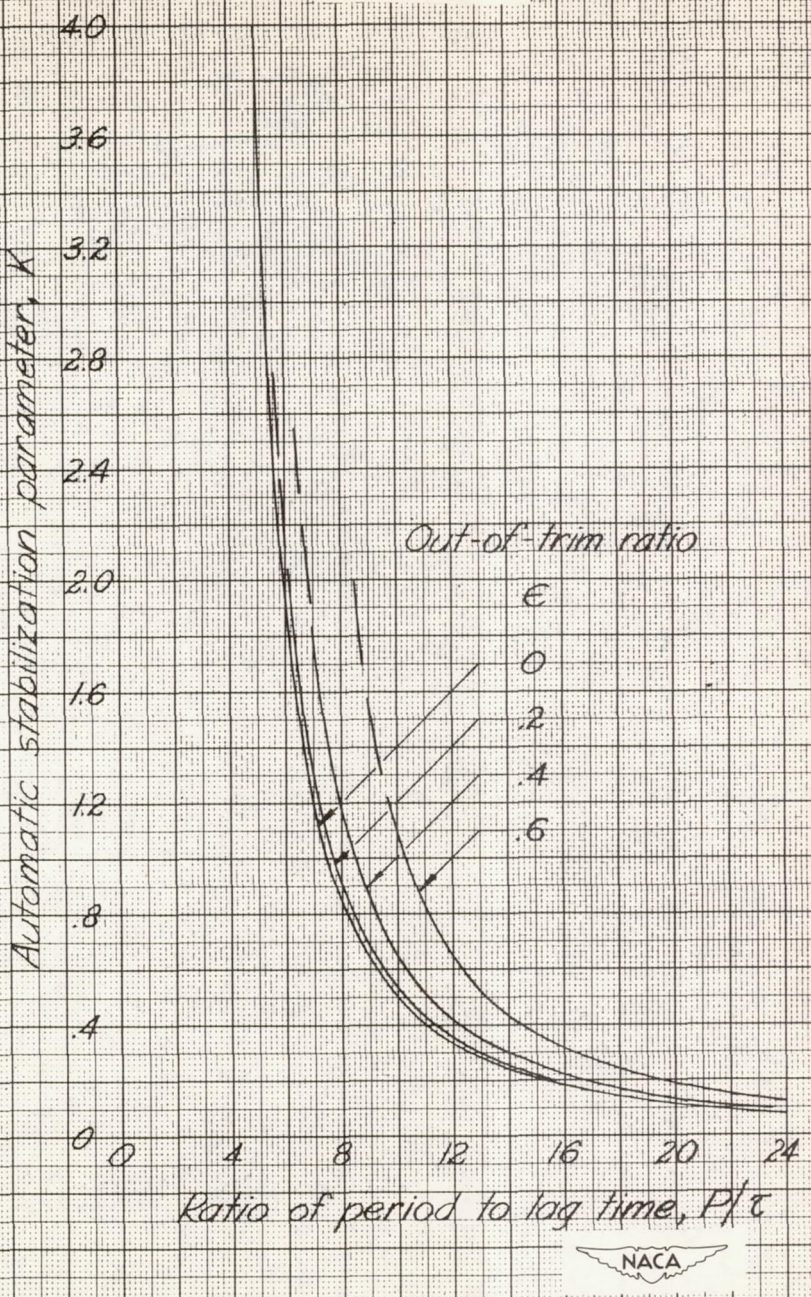

Figure 8.- Chart for determining the period of steady-state roll oscillations for various out-of-trim conditions.

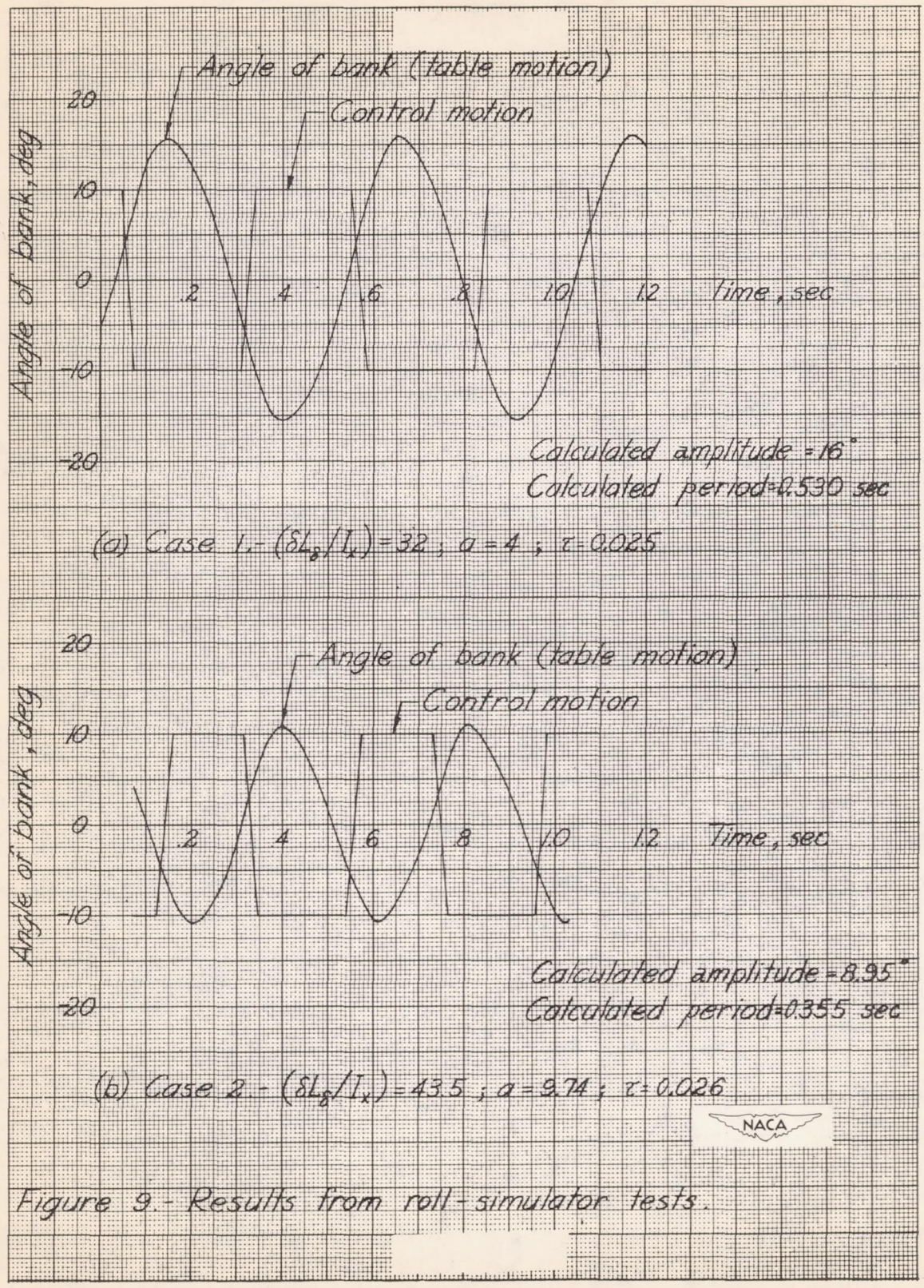

Figure 9.- Results from roll-simulator tests.