NACA RM L6K14 (c.203) c.1

Copy No. 203

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RM No. L6K14

RESEARCH MEMORANDUM

A DEVICE FOR MEASURING SONIC VELOCITY AND COMPRESSOR MACH NUMBER

By

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS
WASHINGTON
January 16, 1947

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RM L6K14

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A DEVICE FOR MEASURING SONIC VELOCITY AND COMPRESSOR MACH NUMBER

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SUMMARY

A device has been developed which measures the velocity of sound in fluids at stagnation and is especially adaptable to turbine and compressor testing for which the constituency of the working fluid may be in doubt. By utilizing the shaft frequency of a rotary compressor, the instrument can also be used to provide a direct measurement of the compressor Mach number (ratio of blade-tip velocity to inlet velocity of sound at stagnation). A Helmholtz resonator is employed in the measurement of the sound velocity.

Viscous effects in the orifice of the Helmholtz resonator are shown to be important and can be taken into account with the help of a parameter obtained from Stokes solution of the flow near an oscillating wall. This parameter includes the kinematic viscosity of the fluid and the frequency of sound in the resonator. When these effects are recognized, the resonator can be calibrated to measure velocity of sound or compressor Mach number to an accuracy of better than 0.5 percent.

INTRODUCTION

One of the important parameters used in the evaluation of rotary compressor (and turbine) performance is the ratio of the blade-tip velocity to the inlet stagnation velocity of sound. This ratio is usually referred to as compressor Mach number.

In testing high-speed compressors, it is frequently desirable to reduce structural loads by substituting a gas which has a lower velocity of sound than air. (See reference 1.) In this case, for which the constituency of the gas may also be in doubt because of air leakage, information about the velocity of sound in the mixture is necessary. The present paper describes an accurate sonic-velocity measuring instrument which can utilize the shaft frequency to provide a direct measurement of the compressor Mach number.

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Because the instrument described herein must utilize the shaft frequency of the compressor, a Pierce interferometer (reference 2) cannot be used although the interferometer will provide accurate measurements of sonic velocity.

SYMBOLS

M_c	compressor Mach number (V_t/a)
V_t	blade-tip velocity
a	velocity of sound in compressor working fluid
λ	wave length of sound
D	diameter of compressor at blade tips
d	diameter of orifice of resonator
Q	volume of resonator
K	resonator-orifice constant
u	velocity in direction of motion of wall
y	distance from wall
t	time
$\frac{n}{2\pi}$	frequency of oscillation
$\nu_0 \rightarrow$ n	frequency of sound in resonator, cycles per second
υ	kinematic viscosity of gas
γ	ratio of specific heats

DESCRIPTION OF APPARATUS

A schematic drawing of the apparatus incorporated in the instrument is shown as figures 1 and 2. This instrument consists of a Helmholtz resonator and earphone enclosed in a chamber containing a sample of the compressor working fluid at the inlet conditions. The earphone is driven at the same frequency as the compressor by means of a small electrical pickup from the compressor shaft. Then the wave length of sound λ emitted by the earphone is

$$\lambda = \frac{a}{n} \tag{1}$$

where a is the velocity of sound in the compressor working fluid and n is the rotational speed in revolutions per second. The velocity of the blade tips V_t is

$$V_t = \pi D n \tag{2}$$

where D is the diameter of the compressor at the blade tips. The ratio of the blade-tip velocity to inlet velocity of sound M_c is then

$$M_c = \frac{V_t}{a} = \frac{\pi D}{\lambda} \tag{3}$$

It is therefore necessary to measure only the wave length of sound emitted by the earphone to obtain compressor Mach number. In order to measure the velocity of sound in the working fluid, the rotational speed must also be measured or, more accurately, the earphone must be switched to an oscillator of known frequency. According to Lord Rayleigh (reference 3, p. 174) the wave length of sound to which a Helmholtz resonator will respond is,

$$\lambda = 2\pi\sqrt{\frac{Q}{K}} \tag{4}$$

where Q is the volume of the resonator and K (reference 3, p. 172) is equal to a constant which depends only on the dimensions of the resonator orifice. If K is predetermined (the volume of the Helmholtz resonator is variable), the measurement of the wave length, and hence of M_c, should depend only on measurement of the resonator volume. The volume of the resonator (the only accurate measurement required) is measured, as shown in figure 1, by means of a mercury manometer. The compressor Mach number is measured without accurate knowledge of the constituency of the working fluid or of the speed of compressor rotation.

A sample of the working fluid is housed in the leak-tight device shown in figure 1, and this fluid is at approximately the same temperature and the same pressure as the temperature and pressure at the inlet of the compressor. Since the constituency of the working fluid might change with time, the fluid is continuously circulated through the device except when a measurement is being taken. Measurements are taken at intervals which depend upon the rate at which the constituency changes. The fluid enters the device through the sump and manometer glass into the resonator and then into the chamber through the resonator orifice; thus, all the gas can be exchanged. The gas is exhausted to atmosphere through a small vacuum pump. If a temperature difference exists between the resonator and inlet to the compressor, it is measured with a differential thermopile at these two points and corrected by multiplying the measured velocity of sound or Mach number by the square root of the temperature ratio. This method is valid except when the temperature difference is great enough to produce noticeable changes in the ratio of the specific heats γ (of the order of 75° F). The Helmholtz resonator is a steel tube with a cross-sectional area of 22.25 square centimeters and a height of 9 centimeters and its volume is varied by means of a mercury inlet at the bottom connected to the sump and manometer. The sump can be raised and lowered, and the level of mercury in the resonator is read at the manometer by means of a microscope which is raised or lowered to focus on the meniscus. The microscope stand is equipped with a vernier scale which can easily be read to 0.01 centimeter. The fluid above the manometer and resonator flows to or from the sump when the mercury level is changed, so that there is little flow through the resonator orifice when its volume is changed. The resonator orifice consists of a $\frac{1}{4}$-inch-diameter hole in a $\frac{1}{8}$-inch Plexiglas plate at the top of the resonator. Two very thin steel needles pressed through the Plexiglas support a 0.0001-inch-diameter platinum hot wire $\frac{1}{8}$-inch long in the center of the orifice. With no signal the hot wire is operated at a constant temperature which is somewhat below that for red heat. The hot-wire driving current is dependent upon the pressure and heat capacity of the working fluid. When the Helmholtz resonator is at resonance with the sound emitted by the earphone, the mass of gas in the resonator orifice is vibrating at maximum amplitude. The hot wire is part of a sensitive Wheatstone bridge and indicates a condition of resonance when its resistance change, which is caused by the cooling effect of the moving gas in the orifice, is at a maximum. The condition of resonance is then noted by a sensitive galvanometer (0 to 200 microammeter) in the bridge circuit. The earphone was located about $\frac{1}{2}$-inch above the orifice and its position was securely fixed by means of brackets. Electrical leads from the earphone, hot wire, and thermocouple were carried through the bottom of the chamber through a Bakelite terminal strip which was held by screws and made leak tight with a rubber gasket. The electrical pickup, located on the compressor shaft, consisted of a two-pole permanently magnetized metal ring slipped over the shaft and a coil wound on a C-shape laminated iron form. This form was not magnetized and its faces cleared the magnetized ring of the shaft by about $\frac{1}{64}$ inch. The metal ring was made of Paragon steel hardened at 1500° F without drawing and was machined and keyed to fit the shaft and permanently magnetized before installation. No decrease in its magnetic field was noted after a number of hours at the maximum compressor speed. This pickup served as a source of alternating current at the same frequency as the compressor rotation. The alternating current was fed (sometimes through an audio amplifier) into the earphone. When the velocity of sound was being measured, the earphone was driven with a variable-frequency oscillator.

METHOD OF CALIBRATION

In order to use the instrument for measuring compressor Mach number, the volume of the resonator at any mercury level and the value of the orifice constant K of the resonator must be known. The resonator volume was determined by the addition of a measured amount of mercury to the manometer glass and resonator and measurement of the change of the mercury level. In order to determine the value of the orifice constant, a General Radio Company beat-frequency oscillator was used as a source of variable frequency. The oscillator was calibrated against a 1000-cycle vacuum-tube tuning-fork oscillator accurate to 0.2 percent by means of Lissajous figures. The oscillator frequency was adjusted until the galvanometer in the bridge circuit indicated maximum deflection. This point of maximum galvanometer deflection could be determined and reproduced to an accuracy of better than 0.5 percent of frequency. It should be mentioned that the large chamber must be of such dimensions that it will not resonate anywhere in the range of working frequencies and thereby introduce erroneous galvanometer deflections which have no connection with the volume setting of the resonator. Air dried with calcium chloride was used in calibration tests since the velocity of sound in dry air is accurately known. The device was evacuated by means of a vacuum pump to a pressure below 1 millimeter of mercury and dry air was admitted through the tube which runs to the compressor inlet. This tube was not connected to the compressor inlet until after calibration. The pressure in the device was measured by means of the mercury manometer. From measurements of frequency and resonator volume, with the use of the known velocity of sound, the value of K in equation (4) could be obtained. As was mentioned previously, the value of K, according to perfect-fluid theory, should be a constant depending only on the dimensions of the resonator orifice. This perfect-fluid assumption would mean that, provided that a condition of resonance is obtained, the value of K should not depend on the volume of the resonator, the velocity of sound in the gas, or the frequency of the sound. This assumption also means that the pressure and temperature of the gas do not affect the value of K if they are taken into account when the velocity of sound is calculated. Also, according to reference 3, page 174, the velocity of sound used to obtain λ in equation (4) is the velocity of sound in the orifice of the resonator, whereas the velocity of sound in the large part of the resonator does not affect this equation.

Since the resonant frequency depends only on the velocity of sound in the orifice of the resonator, the effects of the heating of this small mass of gas due to the hot wire must be recognized and eliminated. With larger wires a variation of K of as much as 3.5 percent could be obtained by variation of the hot-wire heating current, whereas with a 0.0001-inch hot wire, no change of K could be noted when the heating current was varied. (The hot wire was then not heating the mass of gas in the orifice to a measurable degree.)

In order to find the value of K, measurements were made with the use of gases in which the velocity of sound is known and widely different at pressures from 0.1 to 1 atmosphere and at various volume settings corresponding to a wide range of frequency. By repeated and careful experiments, a variation of K of the order of 15 percent was noted when any or all the aforementioned conditions were substantially varied. No variation of K was noted with a change of sound amplitude. It became apparent, therefore, that before an accurate and reliable calibration could be obtained, effects other than perfect-fluid effects would have to be taken into account. The action of viscosity in the layer of gas near the boundary of the orifice would reduce the effective diameter and, therefore, could produce the changes in the value of K which the experiment showed.

Stokes (reference 4) solved a related, idealized problem; namely, the viscous motion in the neighborhood of a wall which executes oscillations in its own plane. The velocity of the gas near the wall is shown to be given by

$$u = Ae^{-y\sqrt{\frac{n}{2\upsilon}}} \cos\left(nt - y\sqrt{\frac{n}{2\upsilon}}\right) \tag{5}$$

where u is the velocity in the direction of the motion of the wall, y is the distance from the wall, A is an arbitrary constant, t is time, $\frac{n}{2\pi}$ is frequency of oscillation, and υ is the kinematic viscosity of the gas. This solution is not meant to give a quantitative answer for the viscous effects in question, but it can be used to infer that the restriction of the orifice of a Helmholtz resonator by the action of viscosity should depend upon the ratio of the length $\sqrt{\upsilon/n}$ to the diameter d of the orifice. In figure 3, the results of a series of measurements of K/d are plotted against the number $\sqrt{\frac{\upsilon/n}{d}}$. When the results are plotted against this parameter all the variations of K which were found are included. A few points were taken with the use of a $\frac{1}{2}$-inch-diameter orifice, $\frac{1}{4}$-inch thick and roughly similar to the small orifice. Since the sensitivity of the hot wire was reduced by a factor of about 4 because of the larger orifice, and since the larger orifice was not made with enough care to be accurately similar, the agreement is considered satisfactory. Also, the error is possibly due to the larger needles necessary to support the hot wire. In any case, these data obtained with the $\frac{1}{2}$-inch orifice are not meant to be conclusive. The value of K/d, calculated from Lord Rayleigh's perfect-fluid case with the use of only the dimensions of the orifices (reference 3, p. 181), has also been plotted. This point appears to lie close to the curve in figure 3 when the curve is extrapolated to zero viscosity. It appears, therefore, that if viscous effects are taken into account accurate measurements of the velocity of sound or related parameters can be made with a Helmholtz resonator.

The maximum scatter from the curve in figure 3 is seen to be approximately 1 percent of K (disregarding the $\frac{1}{2}$-inch orifice) which would be a scatter of 0.5 percent of λ, whereas most of the points are much closer. As was stated previously, the value of the resonant frequency, as noted by the galvanometer, could be a maximum of 0.5 percent in error. The peaks of galvanometer deflection became broader at the high values of $\sqrt{\upsilon/n}$, so that measurements here are least accurate. Accuracy better than 0.5 percent was not needed in this work. From the slope of the curve of figure 3 an error in the value of υ/n of approximately 20 percent would be necessary to produce an error of 0.5 percent on λ, which justifies the statement that an accurate knowledge of the working fluid is not necessary to measure the velocity of sound or compressor Mach number accurately with this apparatus. Neither is an accurate measurement of rotational speed necessary to measure compressor Mach number. An accurate knowledge of the working fluid can be obtained however from measurements of velocity of sound, temperature, and pressure. (See reference 1.)

CONCLUDING REMARKS

From the results of an investigation which employs a Helmholtz resonator in an instrument for measuring sonic velocity and compressor Mach number the following conclusions were drawn:

1. It has been shown that a Helmholtz resonator can be calibrated to provide accurate measurements of velocity of sound and can be employed in an instrument utilizing shaft frequency of a rotary compressor to measure compressor Mach number. With the instrument described, the velocity of sound or the compressor Mach number can be measured to an accuracy of better than 0.5 percent, even though the constituency of the working fluid is not accurately known.

2. Calibration of the Helmholtz resonator to include the action of viscosity at the orifice is accomplished by the use of a parameter obtained from Stokes solution of the flow near an oscillating wall, which includes the kinematic viscosity of the gas and the frequency of sound in the resonator.

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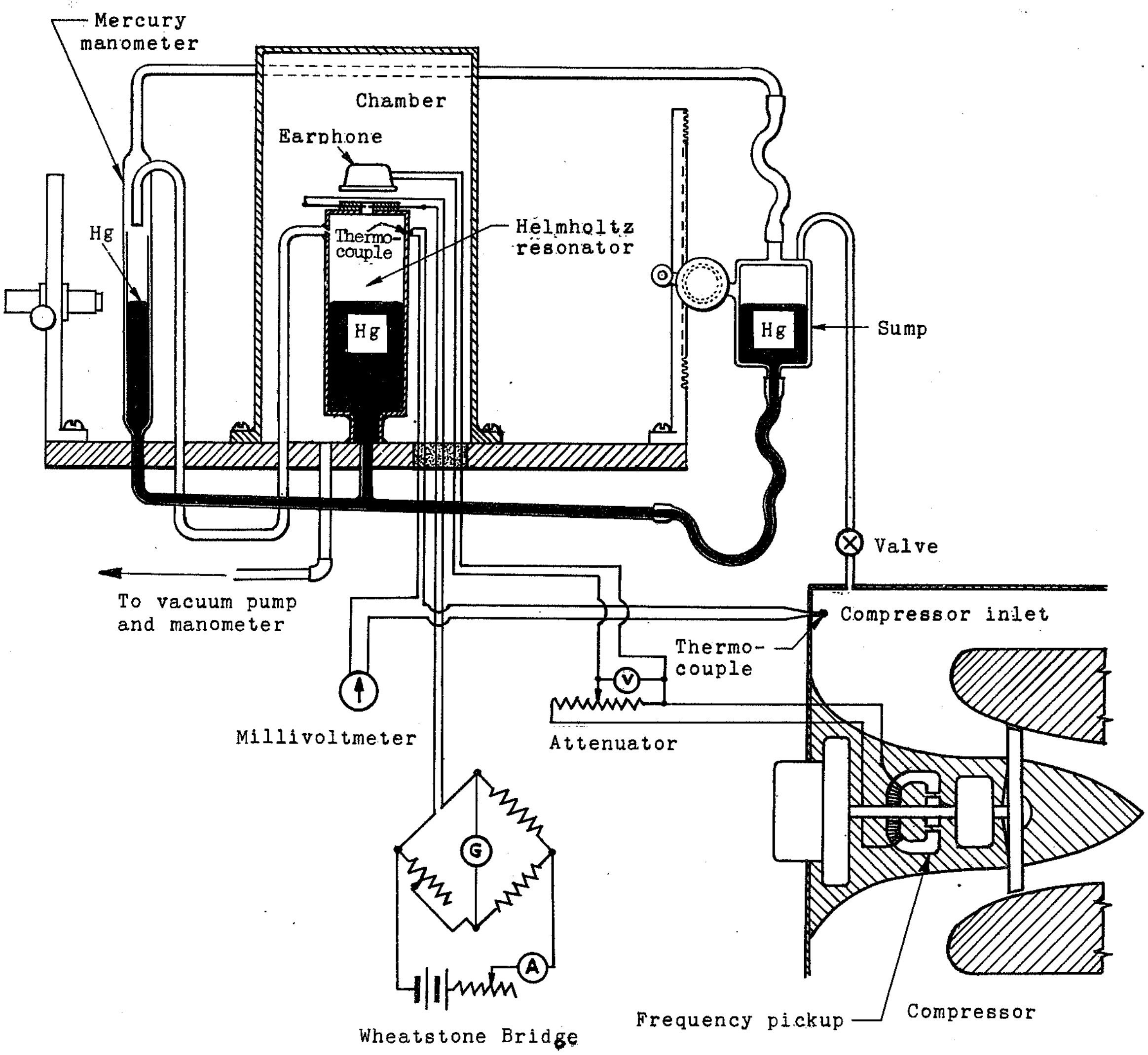

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Figure 1.- Schematic arrangement of instrument for measuring sonic velocity and compressor Mach number.

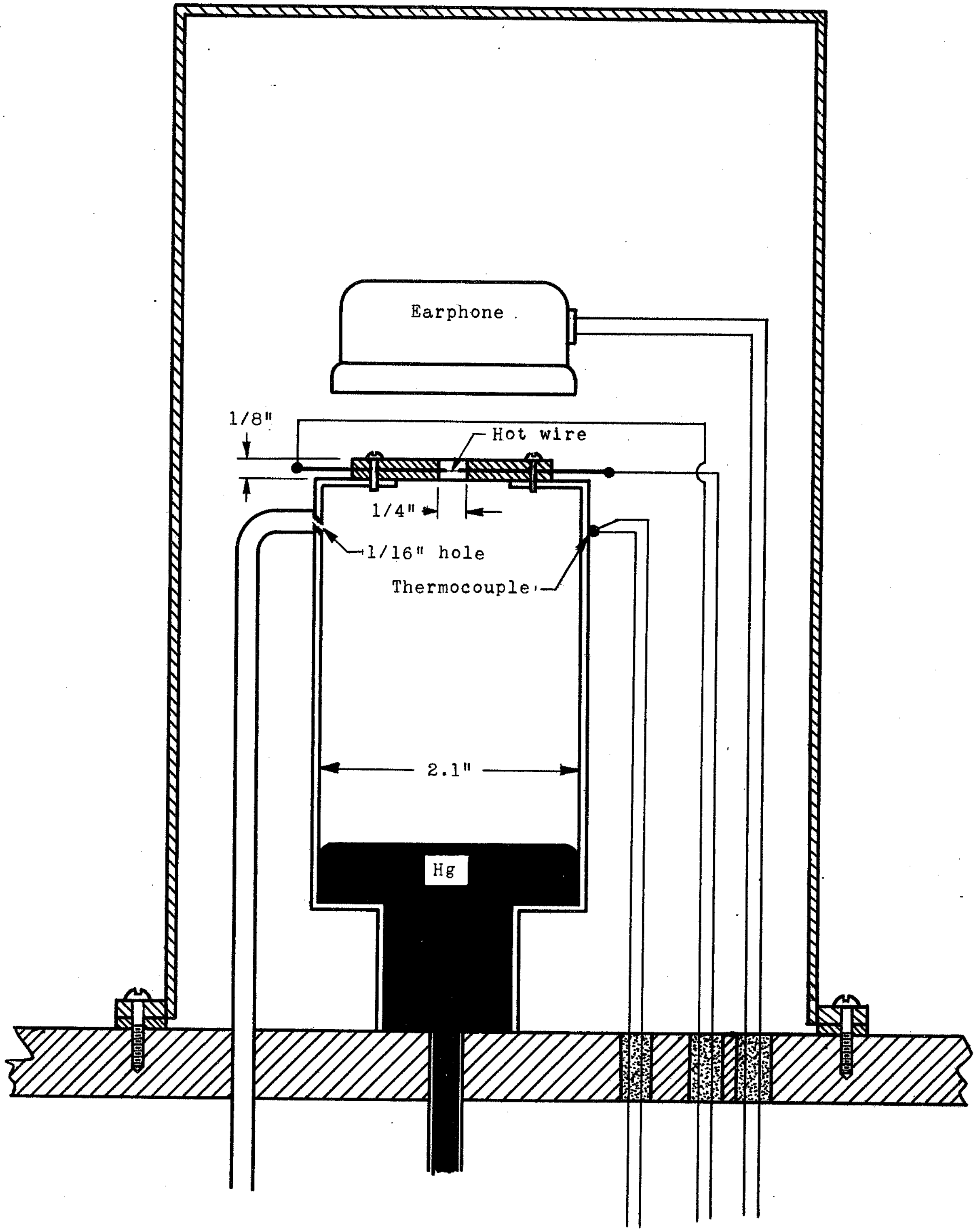

Figure 2.- Helmholtz resonator and chamber.

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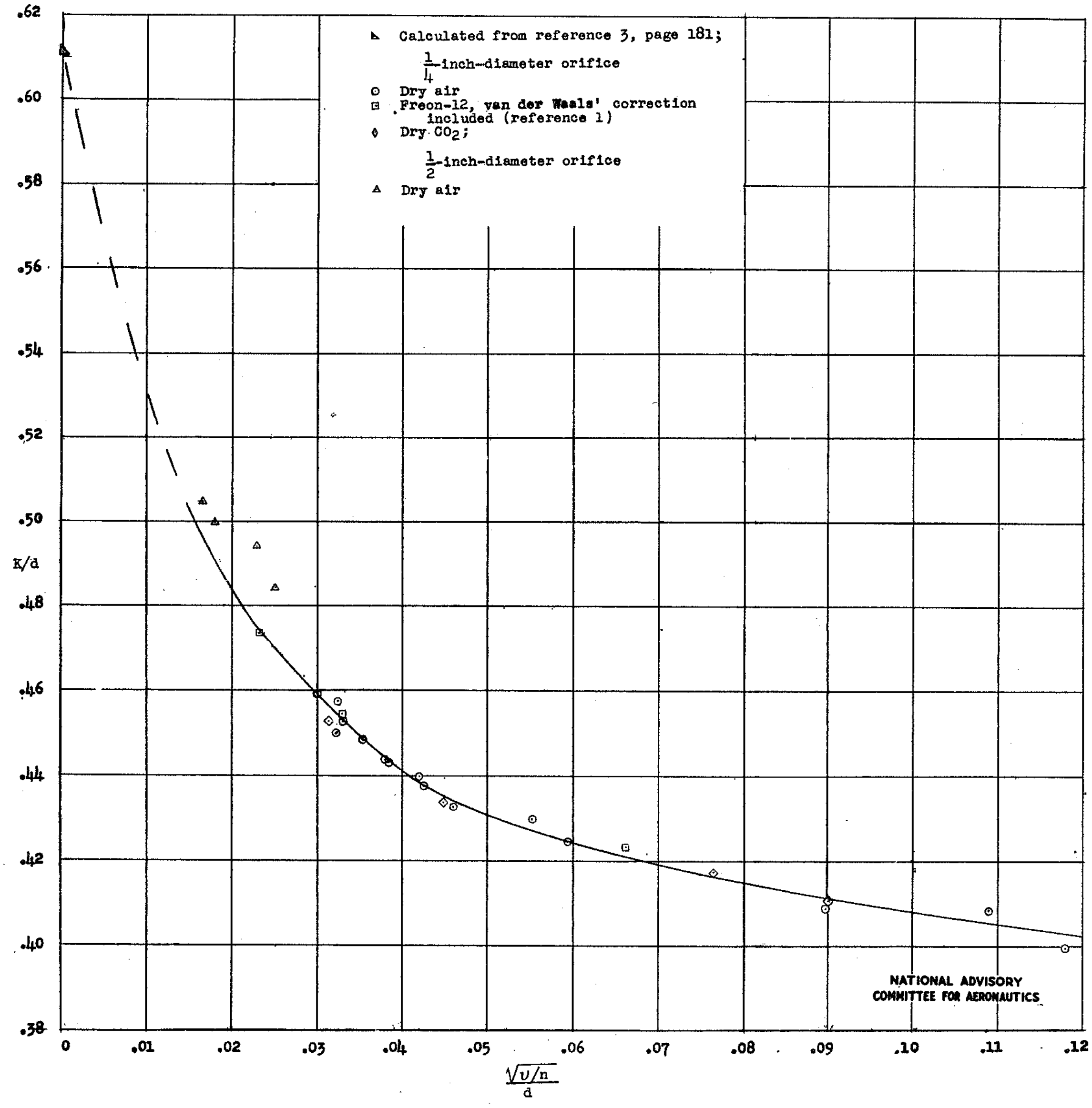

Figure 3.- Effect of the variation of kinematic viscosity and sound frequency in a Helmholtz resonator on the parameter K/d for the resonator orifice. Measurements on each gas were taken over a range of pressures from 0.1 to 1 atmosphere and over a range of frequencies of 200 to 400 cycles per second.