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RESEARCH MEMORANDUM

THEORETICAL INVESTIGATION OF THRUST AUGMENTATION OF TURBOJET ENGINES BY TAIL-PIPE BURNING

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SUMMARY

A theoretical analysis was made of thrust augmentation of turbojet engines by tail-pipe burning and charts are presented from which the thrust augmentation produced may be evaluated from the normal engine data and the performance of the tail-pipe burner. Curves are also given from which the friction and momentum total-pressure losses occurring in the tail-pipe burner may be calculated for any set of design and operating conditions. With the use of the charts, illustrative cases are calculated and curves are presented showing the effects of the principal design and operating variables on thrust augmentation. When practical values of burner-design variables and a burner-exit temperature of 3600° R were assumed, calculations indicated that it is possible to augment the static sea-level thrust of a current turbojet engine 42 percent and the thrust produced at 700 miles per hour 96 percent. The computations indicated that the augmentation that would be produced for a given set of conditions would be approximately doubled by increasing the airplane velocity from zero to one-half normal jet velocity. The effect of altitude on the thrust augmentation produced for a given set of conditions was shown to be slight.

INTRODUCTION

The take-off thrust of turbojet engines is considerably lower than that of conventional engine-propeller combinations due to the low propulsive efficiency of turbojet engines operating at low airplane velocities. In an attempt to improve the take-off, the climb, and the high-speed performance of airplanes powered by turbojet engines, an investigation of various methods of augmenting the thrust produced by this type of engine is being conducted at the NACA Cleveland laboratory.

One of the methods being investigated is tail-pipe burning, which consists in providing a tail-pipe burner between the turbine discharge and the exhaust-nozzle inlet of the turbojet engine. The tail-pipe burner, which is located downstream of the turbine and therefore does not affect the turbine operating temperature, heats the turbine exhaust gases to a temperature considerably higher than would be possible ahead of the turbine because of the temperature limit imposed by the strength characteristics of the turbine materials. The increased temperature of the gases at the exhaust-nozzle inlet results in an increased jet velocity and therefore greater thrust. The addition of a tail-pipe burner results in a decreased total pressure at the exhaust-nozzle inlet caused by friction losses and momentum pressure loss due to burning. This decreased total pressure tends to reduce the jet velocity and therefore to reduce the thrust increase produced by the increased temperature.

An analysis of this method of augmentation was made to provide charts from which the performance of a turbojet engine operating with tail-pipe burning could be conveniently estimated. The charts and the analysis presented in this report enable the prediction of the thrust augmentation produced when the normal jet velocity, the tail-pipe-burner temperature ratio, and the tail-pipe-burner pressure loss are known. Additional curves are presented for evaluating the friction pressure losses and momentum pressure loss due to tail-pipe burning. With the use of the charts, illustrative cases are calculated and curves that show the effects of airplane velocity, tail-pipe-burner inlet velocity, tail-pipe-diffuser efficiency, and burner drag coefficient on thrust augmentation are presented. The effect of tail-pipe burning on the thrust and specific fuel consumption of a turbojet engine operating at various airplane velocities and for altitudes of sea level and 30,000 feet is computed for a representative case. The specific fuel consumption of thrust augmentation (ratio of change in fuel consumption to change in thrust) is also presented.

SYMBOLS

The following symbols are used in the theoretical analysis:

A cross-sectional area, (sq ft)

C_A nozzle-throat area coefficient

C_D tail-pipe-burner drag coefficient, $\dfrac{\Delta P_{f,b}}{\frac{\rho_6}{2} V_6^{\,2}}$

C_V exhaust-nozzle velocity coefficient

E friction energy losses, (ft-lb)/(slug)

F net thrust of normal engine with conventional tail pipe, (lb)

F_a net thrust of augmented engine, (lb)

F_j jet thrust, (lb)

f_b fuel-air ratio of tail-pipe burner (ratio of fuel introduced in tail-pipe burner to air inducted by turbojet engine)

f_e fuel-air ratio of engine combustion chambers

g acceleration due to gravity, 32.17 (ft)/(sec^2)

J mechanical equivalent of heat, 778 (ft-lb)/(Btu)

K dimensionless factor that accounts for effect of pressure losses on thrust augmentation

M mass rate of air flow, (slug/sec)

P total pressure, (lb)/(sq ft)

p_s static pressure, (lb)/(sq ft)

R average value of gas constant over augmentation temperature range, 1715 (ft-lb)/(slug)(°R)

T total temperature, (°R)

T_a mass average total temperature at tail-pipe-burner exit during augmented operation, (°R)

t static temperature, (°R)

V velocities relative to airplane, (ft)/(sec)

V_0 airplane velocity, (ft)/(sec)

W weight flow of fuel, (lb/hr)

ΔP total-pressure loss in tail pipe, $\Delta P_f + \Delta P_m$, (lb/sq ft)

ΔP_b total-pressure loss due to friction in tail-pipe burner, (lb)/(sq ft)

ΔP_f total-pressure loss due to friction in tail-pipe burner and diffuser, (lb)/(sq ft)

ΔP_m total-pressure loss due to momentum increase, (lb)/(sq ft)

γ average ratio of specific heat at constant pressure to specific heat at constant volume over augmentation temperature range, 1.30

η_c over-all combustion efficiency of engine and tail-pipe burner

η_d efficiency of tail-pipe diffuser (ratio of increase in potential energy of gas in passing through diffuser to decrease in kinetic energy), $\dfrac{\dfrac{R\gamma}{\gamma - 1} t_5 \left[\left(\dfrac{p_{s,6}}{p_{s,5}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{\frac{1}{2} V_5^2 - \frac{1}{2} V_6^2}$

ρ mass density, (slug)/(cu ft)

Subscripts:

a augmented

b tail-pipe burner

d tail-pipe diffuser

j jet

n exhaust-nozzle throat

0 ambient atmospheric conditions

5 exhaust-cone exit

6 tail-pipe-burner inlet

7 tail-pipe-burner exit

ANALYSIS

The tail-pipe modifications necessary to equip a turbojet engine for thrust augmentation by tail-pipe burning are illustrated in figure 1. Figure 1(a) shows the engine with a conventional tail pipe and figure 1(b) the engine with the tail pipe modified for thrust augmentation by tail-pipe burning. The chief difference is the addition of a diffuser section between the exhaust-cone exit (station 5) and the tail-pipe-burner inlet (station 6), and the installation of the tail-pipe burner section. The diffuser section is generally necessary to reduce burner-inlet velocities to a value sufficiently low to keep burner pressure losses at a low value. Because the exhaust-nozzle area required for the augmented operation is greater than that for normal operation owing to the decreased density of the exhaust gases during combustion, the engine must be provided with some means of varying the exhaust-nozzle area.

The analysis of thrust augmentation by tail-pipe burning can be conveniently divided into three parts: thrust augmentation, pressure losses occurring in the tail-pipe burner, and required changes in exhaust-nozzle area.

The thrust and the jet velocity of the engine equipped with a conventional tail pipe are designated the normal thrust and the normal jet velocity. The exhaust-cone-exit temperature is unchanged by tail-pipe burning and the total temperature at the tail-pipe-burner inlet is equal to the total temperature at the exhaust-cone exit.

Augmentation

For convenience in derivation and application, the ratio of the thrust of the augmented turbojet engine F_a at any given operating condition to the thrust F of the normal engine (no tail-pipe burning and with original tail pipe) at the same operating conditions of altitude, airplane velocity, engine speed, and turbine-inlet temperature is used as a primary variable. With equality of these conditions, the turbine-discharge temperature and the mass flow of air through the engine will also be equal for the two configurations. It is presupposed that in each case the exhaust-nozzle area is adjusted to give this equality of conditions.

In any turbojet engine the thrust is determined by the mass air flow through the engine, the fuel-air ratio, the total pressure and temperature at the exhaust-nozzle inlet, the ambient-air pressure, the exhaust-nozzle velocity coefficient, and the airplane velocity. During thrust augmentation with a tail-pipe burner, the exhaust-nozzle-inlet total temperature will be increased, which will tend to increase the thrust, but the inlet total pressure will be decreased. This pressure loss imposed by the tail-pipe burner will tend to decrease the gain anticipated from the increased temperature.

When the equations for normal and augmented net thrust are developed, it is possible to find the ratio of augmented net thrust to normal net thrust F_a/F in terms of the ratio of tail-pipe-burner exit temperature to the exhaust-cone exit temperature (tail-pipe-burner inlet) T_a/T_5, the ratio of total-pressure loss in the tail pipe to exhaust-cone-exit total pressure $\Delta P/P_5$, the ratio of airplane velocity to normal jet velocity V_0/V_j, and the engine and tail-pipe burner fuel-air ratios f_e and f_b. The equations for the ratio of augmented thrust to normal thrust are derived in appendix A subject to the assumption that exhaust gas behaves as a perfect gas with constant thermodynamic properties.

The errors due to this assumption are quite small. Actually, the gas constant R changes very little over the range of fuel-air ratios considered. Although the ratio of specific heats γ varies from 1.27 to 1.33 over the temperature range involved, it enters the jet-velocity equation (equation (2), appendix A) twice with compensating effects; the maximum error involved by use of an average value of 1.30 is less than 0.4 percent.

The final equation for the net-thrust ratio, which is derived in appendix A, is

$$\frac{F_a}{F} = \frac{\sqrt{K\left(\frac{T_a}{T_5}\right)(1 + 2\,f_b)} - \frac{V_0}{V_j}\,(1 - f_e)}{1 - \frac{V_0}{V_j}\,(1 - f_e)} \tag{12}$$

where

$$K = \left(\frac{2\gamma R}{\gamma - 1}\,\frac{C_V{}^2 T_5}{V_j{}^2}\right)\left[1 - \left(1 - \frac{\gamma - 1}{2\gamma R}\,\frac{V_j{}^2}{C_V{}^2 T_5}\right)\left(1 - \frac{\Delta P}{P_5}\right)^{\frac{1-\gamma}{\gamma}}\right] \tag{7}$$

(The equation numbers correspond to the equation numbers in the appendixes.) The factor K accounts for the loss in total pressure due to the introduction of the tail-pipe burner. A value of K equal to unity represents an ideal case for which no pressure losses are incurred.

The terms involving fuel-air ratio in equation (12) are necessary to account for the difference in exhaust-gas mass flow between the normal and augmented cases, which is equal to the fuel added in the tail-pipe burner.

Burner Pressure Losses

The total-pressure loss occurring in the tail-pipe burner, which must be known in order to evaluate augmented performance, can be estimated by considering that the pressure loss results from two causes: (1) friction losses caused by both the inefficiency of the tail-pipe diffuser and the drag of the burner itself; and (2) momentum loss caused by the increased velocity of the burner-exit gases during burning. The friction pressure loss will be present regardless of burning and therefore will tend to penalize the performance of an engine equipped for tail-pipe burning when the burner is not in use.

The friction pressure loss is determined in terms of the diffuser efficiency η_d, the diffuser velocity ratio V_5/V_6, and the burner drag coefficient C_D. The exhaust-cone-exit conditions for the modified configuration (T_5, P_5, and V_5) are assumed to be equal to those existing at the exhaust-cone exit on the normal engine. If the exhaust cone is modified for thrust augmentation, station 5 for the augmented case will be that point for which the conditions are equal to those at the exhaust-cone exit of the normal engine.

The momentum pressure loss is caused by the increased velocity of the gases at the burner exit when the tail-pipe burner is in operation. This increased velocity results in a loss in total as well as static pressure. Because the jet velocity and therefore the thrust are dependent on the total pressure at the exhaust-nozzle inlet, it is the loss in total pressure that is of interest in this analysis.

Based on the following assumptions, the expressions for friction pressure loss and momentum pressure loss due to burning are derived in appendix B:

(1) Combustion occurs in a duct having constant cross-sectional area.

(2) Gas conditions of velocity, pressure, and temperature are constant throughout any cross section.

(3) Exhaust gas behaves as a perfect gas having constant thermodynamic properties.

(4) The effect of fuel added is neglected.

The final expression for friction pressure drop, which is derived in appendix B, is

$$\frac{\Delta P_f}{P_5} = 1 - \left\{1 - \frac{\gamma - 1}{2\gamma R}\frac{V_6^2}{T_5}\left[C_D + (1 - \eta_d)\left(\frac{V_5^2}{V_6^2} - 1\right)\right]\right\}^{\frac{\gamma}{\gamma-1}} \tag{18}$$

and that for momentum pressure drop is

$$\frac{\Delta P_m}{P_6} = 1 - \frac{\left(1 - \frac{\gamma - 1}{2\gamma R}\frac{V_6^2}{T_5}\right)^{\frac{\gamma}{\gamma-1}}\left(1 + \frac{\frac{V_6^2}{T_5}}{R - \frac{\gamma - 1}{2\gamma}\frac{V_6^2}{T_5}}\right)}{\left(1 - \frac{\gamma - 1}{2\gamma R}\frac{V_7^2}{T_a}\right)^{\frac{\gamma}{\gamma-1}}\left(1 + \frac{\frac{V_7^2}{T_a}}{R - \frac{\gamma - 1}{2\gamma}\frac{V_7^2}{T_a}}\right)} \tag{33}$$

where

$$\frac{V_7}{\sqrt{T_a}} = \frac{\sqrt{\frac{T_5}{T_a}}\left(R\frac{\sqrt{T_5}}{V_6} + \frac{\gamma+1}{2\gamma}\frac{V_6}{\sqrt{T_5}}\right) - \sqrt{\frac{T_5}{T_a}\left(R\frac{\sqrt{T_5}}{V_6} + \frac{\gamma+1}{2\gamma}\frac{V_6}{\sqrt{T_5}}\right)^2 - 2R\left(\frac{\gamma+1}{\gamma}\right)}}{\frac{\gamma + 1}{\gamma}} \tag{32}$$

In equation (32) the value of the radical decreases as T_a/T_5 increases and an upper limit for T_a/T_5 is reached at which the value of the radical becomes zero. Any higher value of T_a/T_5

would result in a negative value for the radical and would indicate an imaginary value for $V_7/\sqrt{T_a}$. The physical significance of this limiting value to velocity is that the gas velocity at the tail-pipe-burner exit has reached the local speed of sound and no further increase is possible. This phenomenon of "thermal choking" is discussed in reference 1.

The total-pressure loss across the burner is the sum of the friction pressure loss and the momentum pressure loss. For the purpose of computing the total-pressure loss across the burner, it is sufficiently accurate to use the following approximation, which is derived in appendix B:

$$\frac{\Delta P}{P_5} = \frac{\Delta P_f}{P_5} + \frac{\Delta P_m}{P_6} \tag{37}$$

where $\frac{\Delta P_f}{P_5}$ and $\frac{\Delta P_m}{P_6}$ are defined by equations (18) and (33), respectively.

Required Exhaust-Nozzle Area Ratios

The temperature rise and the total-pressure loss occurring at the inlet to the exhaust nozzle during augmented operation increase the area required over that for normal operation. For the case in which supersonic flow exists in the exhaust nozzle for both the normal and augmented configurations, the expression for the effective area ratio for convergent nozzles, as derived in appendix B, is

$$\frac{C_{A,a}\ A_{n,a}}{C_A\ A_n} = \left(\frac{1 + f_b}{1 - \frac{\Delta P}{P_5}}\right) \sqrt{\frac{T_a}{T_5}} \tag{45}$$

For the case in which the velocity in the exhaust-nozzle throat is less than sonic for both configurations, the expression is

$$\frac{C_{A,a}\ A_{n,a}}{C_A\ A_n} = (1 + f_b) \sqrt{\frac{1}{K}\frac{T_a}{T_5}} \tag{47}$$

Because any jet engine equipped for thrust augmentation by tail-pipe burning will be equipped with an adjustable-area exhaust nozzle, equations (45) and (47) will be of value in determining the approximate area range required of the nozzle. The area coefficients $C_{A,a}$ and C_A may change appreciably between the augmented and normal cases depending on the design of the adjustable-area nozzle.

For convenience of application, the foregoing relations required to compute the thrust augmentation, equations (12), (7), (18), and (33) are presented in graph form.

RESULTS AND DISCUSSION

General Curves

The ratio of augmented net thrust to normal net thrust F_a/F, as a function of effective burner-exit temperature factor $KT_a\left(\frac{1600}{T_5}\right)(1+2f_b)$ for various values of $\frac{V_0}{V_j}(1-f_e)$, as expressed in equation (12) is graphically presented in figure 2. The effect of nonuniform tail-pipe-burner exit temperature is discussed in appendix C. Because strength characteristics of turbine materials limit the turbine-exit temperature of current turbojet engines to approximately 1600° R, the value 1600° R was introduced into the effective-temperature factor in order that the value of the effective tail-pipe-burner temperature factor in figure 2 would lie in the vicinity of the actual burner-discharge temperature. The factor K has a maximum value of unity, which represents an ideal system in which no pressure losses are incurred. Figure 2 shows that over the range of tail-pipe-burner exit temperatures under consideration, for a given ratio of airplane velocity to normal jet velocity V_0/V_j, the ratio of augmented net thrust to normal net thrust F_a/F varies almost linearly with effective burner-exit temperature factor. Figure 2 indicates that increased airplane velocity has a beneficial effect on augmentation produced at a given effective burner-exit temperature, the augmentation being doubled as airplane velocity increases from zero to one-half normal jet velocity.

The variation of the factor K, which accounts for pressure losses occurring in the tail-pipe burner, with normal jet-velocity factor $\frac{V_j}{C_V}\sqrt{\frac{1600}{T_5}}$ for various ratios of tail-pipe-burner total-pressure loss to exhaust-cone-exit total pressure $\Delta P/P_5$ is shown in figure 3. This figure is a graphical representation of equation (7). The ratio $1600/T_5$ is introduced in the normal jet-velocity factor to enable rapid use of the curve, as previously mentioned. Figure 3 indicates the desirability of keeping the pressure losses in the tail-pipe burner at a minimum, especially at low values of the jet-velocity factor $\frac{V_j}{C_V}\sqrt{\frac{1600}{T_5}}$. At a value of $\frac{V_j}{C_V}\sqrt{\frac{1600}{T_5}}$ of 1600, increasing the total-pressure losses in the tail-pipe burner from 5 to 15 percent of the burner-inlet total pressure reduces the value of K from 0.901 to 0.681. At a burner-exit temperature T_a of 3600° R, this decrease in the value of K results in an 18-percent reduction in static-thrust augmentation and a 13-percent decrease in thrust without tail-pipe burning. Figure 3 also shows a beneficial effect of increased airplane velocity on augmentation in addition to that discussed in connection with figure 2. At the increased jet velocities that accompany increased airplane velocities, the value of K increases and thus raises the effective tail-pipe-burner exit temperature factor $KT_a\left(\frac{1600}{T_5}\right)(1+2f_b)$.

The ratio of friction total-pressure loss to exhaust-cone-exit total pressure $\Delta P_f/P_5$ and the ratio of momentum total-pressure loss to burner-inlet total pressure $\Delta P_m/P_6$ can be determined from figures 4 and 5, respectively. The value of $\Delta P/P_5$ used in finding K from figure 3 is simply the sum of $\Delta P_f/P_5$ and $\Delta P_m/P_6$.

In figure 4 the ratio of friction total-pressure loss to exhaust-cone-exit total pressure $\Delta P_f/P_5$ is plotted against burner-inlet velocity factor $V_6\sqrt{\frac{1600}{T_5}}$ for various values of the over-all drag factor $C_D + (1-\eta_d)\left(\frac{V_5^2}{V_6^2} - 1\right)$. This figure, which is plotted from equation (18), indicates the large friction total-pressure losses incurred at high values of the velocity factor $V_6\sqrt{\frac{1600}{T_5}}$ and of the drag factor $C_D + (1-\eta_d)\left(\frac{V_5^2}{V_6^2} - 1\right)$.

An examination of equation (18) indicates that the conditions for which the friction pressure losses in the tail-pipe burner are a minimum are determined by the sum of the diffuser efficiency η_d and the burner drag coefficient C_D. If the sum of η_d and C_D is greater than 1, the minimum friction pressure losses occur for an infinite diffuser velocity ratio V_5/V_6; whereas if the sum of η_d and C_D is less than 1, the minimum losses occur at a diffuser velocity ratio of 1. In an actual design, this criterion for minimum friction pressure losses must be considered in conjunction with momentum pressure losses during burner operation and practical considerations of size and burner-combustion characteristics.

The ratio of momentum total-pressure loss to burner-inlet total pressure $\Delta P_m/P_6$, which is calculated from equations (32) and (33), is shown as a function of burner-inlet velocity factor $V_6\sqrt{\frac{1600}{T_5}}$ for various values of tail-pipe-burner temperature ratio T_a/T_5 in figure 5. The rapid increase in momentum pressure losses with increased burner-inlet velocity for a given temperature ratio across the tail-pipe burner indicates the desirability of providing a diffuser section before the tail-pipe burner. Figure 5 also indicates that for any burner-inlet velocity there is a limiting temperature ratio that can be attained. This limiting temperature ratio is reached when the gas velocity at the burner exit reaches sonic velocity and increases as the burner-inlet velocity decreases.

A sample calculation, which illustrates the method of using figures 2 to 5 to determine the augmentation for a specific turbojet engine, is given in appendix D.

Illustrative Cases

With the use of the curves previously discussed, illustrative cases were calculated to show the effect of the various design variables on thrust augmentation of a typical current turbojet engine. The results are presented in figures 6 to 9. The curves presented in figures 6 to 8 are for sea-level altitude and it was assumed that the jet-velocity factor $\frac{V_j}{C_V}\sqrt{\frac{1600}{T_5}}$ was 1550 feet per second for static conditions and 2010 feet per second for an airplane velocity of 1026 feet per second (700 mph). For both conditions the normal tail-pipe total temperature T_5 was assumed to be 1650° R. The calculations for figures 6 to 8 did not include the effect of the added mass flow supplied by the engine and tail-pipe-burner fuel flows and therefore result in slightly conservative values of augmentation. In figures 6 to 8 the uppermost curve represents an ideal case in which no friction nor momentum pressure losses are incurred. A vertical line is drawn on each figure at the normal tail-pipe gas temperature of 1650° R and the point at which each curve intersects this line shows the loss in thrust produced by the installation of the burner when no burning occurs.

In figures 6(a) and 6(b) the ratios of augmented net thrust to normal net thrust F_a/F as a function of tail-pipe-burner exit temperature T_a for various values of burner-inlet velocity V_6 are presented for airplane velocities of 0 and 700 miles per hour, respectively. An exhaust-cone-exit velocity V_5 of 750 feet per second, a burner drag coefficient C_D of 1.0, a tail-pipe diffuser efficiency η_d of 0.8, and an exhaust-nozzle velocity coefficient C_V of 0.975 were assumed for both static and high-speed conditions. Figure 6 indicates that the lower the burner-inlet velocity, the greater the augmentation that can be obtained for a given burner-exit temperature. For example, an engine having a burner drag coefficient C_D of 1.0 and a burner-inlet velocity V_6 of 700 feet per second will produce a maximum static-thrust augmentation of 9.5 percent for an optimum value of burner-exit temperature compared with 36.5 percent with a burner-inlet velocity V_6 of 400 feet per second and a burner-exit temperature T_a of 3600° R (fig. 6(a)). The improved performance at the low burner-inlet velocity is due to the fact that losses that are incurred in the diffusion process are more than compensated for by the decreased burner pressure losses resulting from low burner-inlet velocities. Because decreased burner-inlet velocities require larger tail pipes, the amount of diffusion may be limited by airplane-installation considerations.

For this case the diffuser-exit diameter required to give burner-inlet velocities of 600, 400, and 200 feet per second are 1.107, 1.344, and 1.885 times the diffuser-inlet diameter, respectively. A comparison of figures 6(a) and 6(b) shows the beneficial effect of high airplane velocity on thrust augmentation. At a burner-inlet velocity of 400 feet per second and a burner-exit temperature of 3200° R, the thrust augmentation increases from 30 percent for static conditions (fig. 6(a)) to 69 percent for an airplane velocity of 700 miles per hour (fig. 6(b)). This effect is even more pronounced in burners having a high inlet velocity and correspondingly greater pressure losses. For a burner-inlet velocity of 750 feet per second and a burner-exit temperature of 3200° R, the augmentation increases from 2.5 percent for static conditions (fig. 6(a)) to 35 percent for an airplane velocity of 700 miles per hour (fig. 6(b)).

The effect of burner drag coefficient on thrust augmentation for airplane velocities of 0 and 700 miles per hour is shown in figures 7(a) and 7(b), respectively. Curves are included for burner-inlet velocities of 400 and 700 feet per second. For both the static and high-speed conditions, an exhaust-cone-exit velocity V_5 of 750 feet per second, a diffuser efficiency η_d of 0.8, and an exhaust-nozzle velocity coefficient C_V of 0.975 were assumed. This figure indicates that the thrust augmentation available is very sensitive to burner drag coefficient especially at high burner-inlet velocities. For example, an engine having a burner-inlet velocity V_6 of 700 feet per second and a burner drag coefficient C_D of 2.0 will produce only 93 percent of the normal thrust (7-percent loss) at static conditions for the optimum burner-exit temperature and 82 percent of the normal thrust (18-percent loss) without burning (fig. 7(a)). At an airplane velocity of 1026 feet per second (700 mph) (fig. 7(b)), the same burner with an exit temperature of 3200° R will produce a thrust augmentation of 23.5 percent but without burning will cause a loss in thrust of 22.5 percent.

Figures 8(a) and 8(b) show the effect of diffuser efficiency on thrust augmentation for airplane velocities of 0 and 1026 feet per second (700 mph), respectively. A burner-inlet velocity V_6 of 400 feet per second, a burner drag coefficient C_D of 1.0, and an exhaust-nozzle velocity coefficient C_V of 0.975 were assumed and curves are presented for exhaust-cone-exit velocities of 750 and 1050 feet per second. Figure 8 shows that for a diffuser efficiency of 1.0 the thrust augmentation is independent of diffuser-inlet velocity; however, as the diffuser efficiency decreases the effect of increasing exhaust-cone-exit velocity is to decrease the possible thrust augmentation. For example, at a tail-pipe-burner exit temperature of 3200° R, by reducing the diffuser efficiency from 1.0 to 0.6 the value of the augmented thrust at zero airplane speed is reduced approximately 4 and 10 percent of the normal thrust for diffuser-inlet velocities of 750 and 1050 feet per second, respectively. A similar decrease in thrust augmentation is obtained at an airplane velocity of 700 miles per hour for the same conditions.

In figure 9 the net-thrust ratio F_a/F, the specific fuel consumption for normal engine operation $3600\ W/F$, the augmented specific fuel consumption $3600\ W_a/F_a$, and the specific fuel consumption of the thrust augmentation $3600\ (W_a-W)/(F_a-F)$ (increase in fuel consumption divided by increase in thrust) are plotted against airplane velocity V_0 for altitudes of sea level and 30,000 feet. For the calculations of this figure the effect of the tail-pipe-burner fuel flow on mass flow was included and the following conditions were assumed: exhaust-cone-exit temperature T_5, 1650° R; burner-exit temperature T_a, 3600° R; exhaust-cone-exit velocity V_5, 750 feet per second; burner-inlet velocity V_6, 400 feet per second; burner drag coefficient C_D, 1.0; tail-pipe diffuser efficiency η_d, 0.8; and over-all combustion efficiency of the engine and tail-pipe burner η_c, 0.85. For the calculations of the data presented in figure 9, the over-all combustion efficiency η_c is defined as the combined combustion efficiency of the engine and tail-pipe burner. The efficiency of the engine combustion chambers was calculated to be approximately 0.95 and the efficiency of the tail-pipe burner was chosen so as to give an over-all efficiency of 0.85. For all conditions of airplane velocity and altitude, the resulting combustion efficiency of the tail-pipe burner was approximately 0.80. The jet-engine performance data used in calculating the curves are representative of current turbojet engines. The fuel flows required for tail-pipe burning were calculated from the data of reference 2, which considers the effect of variation in specific heat.

Figure 9 indicates the increase in augmentation accompanying increased airplane velocity. For example, at sea level, when the airplane velocity is increased from 0 to 700 miles per hour, the thrust augmentation is increased from 42 to 96 percent. The tail-pipe burner for this case will cause a loss in thrust without burning (fig. 6) of 4 percent of the normal thrust under static conditions and 5 percent at an airplane velocity of 700 miles per hour. For the particular set of conditions chosen, figure 9 indicates that the amount of thrust augmentation produced does not markedly change with a change in altitude from sea level to 30,000 feet, the low altitude being slightly better at high airplane velocities and the high altitude being somewhat better at low airplane velocities. The specific fuel consumption of augmentation is somewhat less at an altitude of 30,000 feet than at sea level, and the effect of increased airplane velocity is to decrease the specific fuel consumption at both altitudes.

SUMMARY OF RESULTS

Based on a theoretical analysis of thrust augmentation of turbojet engines by tail-pipe burning, the following conclusions are indicated:

1. By use of practical values for burner-design variables and an effective tail-pipe-burner exit temperature of 3600° R, calculations indicated that it was possible to augment the sea-level thrust of a typical current turbojet engine 42 percent at static conditions and 96 percent at an airplane velocity of 700 miles per hour. The analysis indicated that the friction pressure losses occurring in the tail-pipe-burner installation would result in a loss in thrust of 4 percent for static conditions and 5 percent at 700 miles per hour without burning.

2. For a given set of conditions, the calculated thrust augmentation produced was relatively insensitive to altitude; the low altitudes were slightly more favorable at high airplane velocities and the high altitudes were somewhat better at low airplane velocities.

3. On current turbojet engines having high gas velocities at the exhaust-cone exit it is desirable in modifying the engine for thrust augmentation by tail-pipe burning to install an additional diffuser section ahead of the burner to reduce the gas velocities and therefore the momentum and friction pressure losses occurring in the tail-pipe burner.

Aircraft Engine Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio.

APPENDIX A

DERIVATION OF THRUST EQUATIONS

The jet velocity of a turbojet engine is determined by the total temperature and total pressure of the gas at the exhaust-nozzle inlet and atmospheric static pressure according to the equation

$$V_j = \sqrt{C_V^2 \frac{2\gamma R}{\gamma - 1} T_5 \left[1 - \left(\frac{p_{s,0}}{P_5}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{1}$$

When burning occurs in the tail pipe, the total temperature and pressure at the exhaust-nozzle inlet are changed and the augmented jet velocity becomes

$$V_{j,a} = \sqrt{C_V^2 \frac{2\gamma R}{\gamma - 1} T_a \left[1 - \left(\frac{p_{s,0}}{P_7}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{2}$$

When equation (2) is divided by equation (1), the ratio of the augmented to the normal jet velocity is

$$\frac{V_{j,a}}{V_j} = \sqrt{\frac{T_a}{T_5}} \sqrt{\frac{1 - \left(\frac{p_{s,0}}{P_5}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{P_5}{P_7}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{p_{s,0}}{P_5}\right)^{\frac{\gamma-1}{\gamma}}}} \tag{3}$$

When the value under the second radical of equation (3), which is the factor that accounts for the effect of the pressure loss occurring due to tail-pipe burning and burner drag on jet velocity, is denoted K, equation (3) may be written as

$$\frac{V_{j,a}}{V_j} = \sqrt{K \frac{T_a}{T_5}} \tag{4}$$

where K is

$$K = \frac{1 - \left(\frac{p_{s,0}}{P_5}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{P_5}{P_7}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{p_{s,0}}{P_5}\right)^{\frac{\gamma-1}{\gamma}}} \tag{5}$$

When the value of $\left(\frac{p_{s,0}}{P_5}\right)^{\frac{\gamma-1}{\gamma}}$ obtained from equation (1) is substituted in equation (5) and the loss in total pressure occurring due to tail-pipe burning is taken as $\Delta P = P_5 - P_7$, the expression for K becomes

$$K = \frac{1 - \left(1 - \frac{\gamma - 1}{2\gamma R} \frac{V_j^2}{C_V^2 \ T_5}\right) \left(\frac{P_5}{P_5 - \Delta P}\right)^{\frac{\gamma-1}{\gamma}}}{\frac{\gamma - 1}{2\gamma R} \frac{V_j^2}{C_V^2 \ T_5}} \tag{6}$$

By rearrangement equation (6) becomes

$$K = \left(\frac{2\gamma R}{\gamma - 1} \frac{C_V^2 \ T_5}{V_j^2}\right) \left[1 - \left(1 - \frac{\gamma - 1}{2\gamma R} \frac{V_j^2}{C_V^2 \ T_5}\right)\left(1 - \frac{\Delta P}{P_5}\right)^{\frac{1-\gamma}{\gamma}}\right] \tag{7}$$

The normal net thrust of a turbojet engine is equal to

$$F = M\,(V_j - V_0) + Mf_eV_j \tag{8}$$

In the same manner, the net thrust produced with tail-pipe burning is

$$F_a = M\,(V_{j,a} - V_0) + M\,(f_e + f_b)\,V_{j,a} \tag{9}$$

When equation (9) is divided by equation (8), the ratio of the augmented to the normal net thrust is

$$\frac{F_a}{F} = \frac{V_{j,a}\,(1 + f_e + f_b) - V_0}{V_j\,(1 + f_e) - V_0} \tag{10}$$

When both numerator and denominator of equation (10) are divided by $V_j\,(1 + f_e)$ and the value obtained from equation (4) is substituted for $V_{j,a}/V_j$, the ratio of the augmented net thrust to normal net thrust is

$$\frac{F_a}{F} = \frac{\sqrt{K\,\frac{T_a}{T_5}}\left(\frac{1 + f_e + f_b}{1 + f_e}\right) - \frac{V_0}{V_j}\left(\frac{1}{1 + f_e}\right)}{1 - \frac{V_0}{V_j}\left(\frac{1}{1 + f_e}\right)} \tag{11}$$

By including the fuel-air-ratio terms under the radical and performing the indicated divisions, equation (11), neglecting second- and higher-order terms, becomes

$$\frac{F_a}{F} = \frac{\sqrt{K\,\frac{T_a}{T_5}\,(1 + 2f_b)} - \frac{V_0}{V_j}\,(1 - f_e)}{1 - \frac{V_0}{V_j}\,(1 - f_e)} \tag{12}$$

APPENDIX B

DERIVATION OF BURNER PRESSURE-LOSS AND EXHAUST-NOZZLE-AREA EQUATIONS

Burner Pressure Loss

The loss in total pressure when burning occurs in a constant cross-sectional-area tail pipe can be conveniently divided into two parts: The first loss is due to additional friction and turbulence and is accounted for by the diffuser efficiency and the burner drag coefficient. The second loss is the momentum pressure loss due to burning and is determined by the ratio of the outlet to the inlet gas temperature of the burner.

By definition, the diffuser efficiency is the increase in isentropically available static-pressure energy divided by the reduction in kinetic energy. Thus, the energy loss in a diffuser is the difference between the reduction in kinetic energy $\left(\frac{V_5^2}{2} - \frac{V_6^2}{2}\right)$ and the increase in pressure energy $\eta_d\left(\frac{V_5^2}{2} - \frac{V_6^2}{2}\right)$:

$$E_d = (1 - \eta_d)\left(\frac{V_5^2}{2} - \frac{V_6^2}{2}\right) \tag{13}$$

By definition, the burner drag coefficient is

$$C_D = \frac{\Delta P_{f,b}}{\frac{\rho}{2} V_6^2} \tag{14}$$

Inasmuch as the energy loss is equal to $\Delta P/\rho$ for small pressure losses, the energy loss due to burner drag is

$$E_b = C_D \frac{V_6^2}{2} \tag{15}$$

The total energy loss due to friction and turbulence in the diffuser and burner $(E_d + E_b)$ are summed and equated to the energy released by the expansion through the pressure ratio (P_6/P_5):

$$(1 - \eta_d)\left(\frac{V_5^2}{2} - \frac{V_6^2}{2}\right) + C_D \frac{V_6^2}{2} = \frac{\gamma R}{\gamma - 1} T_5 \left[1 - \left(\frac{P_6}{P_5}\right)^{\frac{\gamma-1}{\gamma}}\right]. \tag{16}$$

By rearrangement equation (16) becomes

$$\frac{P_6}{P_5} = \left\{1 - \frac{\gamma - 1}{\gamma RT}\left[C_D \frac{V_6^2}{2} + (1 - \eta_d)\left(\frac{V_5^2}{2} - \frac{V_6^2}{2}\right)\right]\right\}^{\frac{\gamma}{\gamma-1}} \tag{17}$$

The pressure P_6 has been so chosen that the pressure losses between P_5 and P_6 will include the burner as well as the diffuser friction pressure loss. The friction pressure loss ΔP_f is then equal to $(P_5 - P_6)$ and by solving equation (17) for this term and simplifying

$$\frac{\Delta P_f}{P_5} = 1 - \left\{1 - \frac{\gamma - 1}{2\gamma R} \frac{V_6^2}{T_5}\left[C_D + (1 - \eta_d)\left(\frac{V_5^2}{V_6^2} - 1\right)\right]\right\}^{\frac{\gamma}{\gamma-1}} \tag{18}$$

The momentum pressure loss occurs due to the increase in velocity of the gas when it is heated and consequently has its density reduced. In a constant-area pipe, the force necessary to produce this acceleration must come from a reduction in static pressure. A loss also occurs in total pressure and it is this loss that must be evaluated inasmuch as the jet velocity depends upon the total pressure.

From the conservation of momentum between stations 6 and 7, the following equation can be written:

$$Ap_{s,6} + MV_6 = Ap_{s,7} + MV_7 \tag{19}$$

Inasmuch as the mass flows past stations 6 and 7 are equal, the following is true:

$$M = \frac{p_{s,6}}{Rt_6} AV_6 = \frac{p_{s,7}}{Rt_7} AV_7 \tag{20}$$

When the values of M from equation (20) are substituted in equation (19),

$$p_{s,6}\left(1 + \frac{V_6^2}{Rt_6}\right) = p_{s,7}\left(1 + \frac{V_7^2}{Rt_7}\right) \tag{21}$$

The total and static states at stations 6 and 7 are related by the following equations:

$$T_6 = T_5 = t_6 + \frac{\gamma - 1}{2\gamma R} V_6^2 \tag{22}$$

$$T_7 = T_a = t_7 + \frac{\gamma - 1}{2\gamma R} V_7^2 \tag{23}$$

$$P_6 = p_{s,6}\left(\frac{T_5}{t_6}\right)^{\frac{\gamma}{\gamma-1}} \tag{24}$$

$$P_7 = p_{s,7}\left(\frac{T_a}{t_7}\right)^{\frac{\gamma}{\gamma-1}} \tag{25}$$

By converting to total temperature and pressure, equation (21) becomes

$$P_6\left(1 - \frac{\gamma - 1}{2\gamma R}\frac{V_6^2}{T_5}\right)^{\frac{\gamma}{\gamma-1}}\left[1 + \frac{V_6^2}{R\left(T_5 - \frac{\gamma - 1}{2\gamma R} V_6^2\right)}\right] = P_7\left(1 - \frac{\gamma - 1}{2\gamma R}\frac{V_7^2}{T_a}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\left[1 + \frac{V_7^2}{R\left(T_a - \frac{\gamma - 1}{2\gamma R} V_7^2\right)}\right] \tag{26}$$

When equation (26) is rearranged, the expression for the ratio of the total pressure at the tail-pipe-burner exit to the total pressure at the tail-pipe-burner inlet is

$$\frac{P_7}{P_6} = \frac{\left(1 - \frac{\gamma - 1}{2\gamma R}\frac{V_6^{\,2}}{T_5}\right)^{\frac{\gamma}{\gamma-1}} \left(1 + \frac{\frac{V_6^{\,2}}{T_5}}{R - \frac{\gamma - 1}{2\gamma}\frac{V_6^{\,2}}{T_5}}\right)}{\left(1 - \frac{\gamma - 1}{2\gamma R}\frac{V_7^{\,2}}{T_a}\right)^{\frac{\gamma}{\gamma-1}} \left(1 + \frac{\frac{V_7^{\,2}}{T_a}}{R - \frac{\gamma - 1}{2\gamma}\frac{V_7^{\,2}}{T_a}}\right)} \tag{27}$$

In order to determine $V_7/\sqrt{T_a}$, which must be found before equation (27) can be solved, substitute for $p_{s,6}$ and $p_{s,7}$ in equation (19) the values determined in equation (20),

$$\frac{Rt_6}{V_6} + V_6 = \frac{Rt_7}{V_7} + V_7 \tag{28}$$

When converted to total temperature, equation (28) becomes

$$\frac{RT_6}{V_6} - \frac{\gamma - 1}{2\gamma} V_6 + V_6 = \frac{RT_a}{V_7} - \frac{\gamma - 1}{2\gamma} V_7 + V_7 \tag{29}$$

Simplifying equation (29) and because $T_6 = T_5$

$$\frac{RT_5}{V_5} + V_6\left(\frac{\gamma + 1}{2\gamma}\right) = \frac{RT_a}{V_7} + V_7\left(\frac{\gamma + 1}{2\gamma}\right) \tag{30}$$

Equation (30) can be rearranged as a quadratic in $V_7/\sqrt{T_a}$,

$$\frac{V_7^{\,2}}{T_a}\,\frac{\gamma+1}{2\gamma} - \frac{V_7}{\sqrt{T_a}}\left[\frac{R\dfrac{\sqrt{T_5}}{\sqrt{T_a}}}{\dfrac{V_6}{\sqrt{T_5}}} + \frac{V_6}{\sqrt{T_5}}\ \sqrt{\frac{T_5}{T_a}}\left(\frac{\gamma+1}{2\gamma}\right)\right] + R = 0 \tag{31}$$

When equation (31) is solved for $V_7/\sqrt{T_a}$

$$\frac{V_7}{\sqrt{T_a}} = \frac{\sqrt{\dfrac{T_5}{T_a}}\left(R\,\dfrac{\sqrt{T_5}}{V_6} + \dfrac{\gamma+1}{2\gamma}\,\dfrac{V_6}{\sqrt{T_5}}\right) - \sqrt{\dfrac{T_5}{T_a}\left(R\,\dfrac{\sqrt{T_5}}{V_6} + \dfrac{\gamma+1}{2\gamma}\,\dfrac{V_6}{\sqrt{T_5}}\right)^2 - 2R\left(\dfrac{\gamma+1}{\gamma}\right)}}{\dfrac{\gamma+1}{\gamma}} \tag{32}$$

Equation (32) gives a value for $V_7/\sqrt{T_a}$ in terms of $V_6/\sqrt{T_5}$ and T_a/T_5 and when combined with equation (27) gives a value for P_7/P_6 in terms of the same variables. Equation (27) can be rewritten to express the momentum pressure loss $(\Delta P_m = P_6 - P_7)$ as a ratio to P_6

$$\frac{\Delta P_m}{P_6} = 1 - \frac{\left(1 - \dfrac{\gamma-1}{2\gamma R}\,\dfrac{V_6^{\,2}}{T_5}\right)^{\frac{\gamma}{\gamma-1}}\left(1 + \dfrac{\dfrac{V_6^{\,2}}{T_5}}{R - \dfrac{\gamma-1}{2\gamma}\,\dfrac{V_6^{\,2}}{T_5}}\right)}{\left(1 - \dfrac{\gamma-1}{2\gamma R}\,\dfrac{V_7^{\,2}}{T_a}\right)^{\frac{\gamma}{\gamma-1}}\left(1 + \dfrac{\dfrac{V_7^{\,2}}{T_a}}{R - \dfrac{\gamma-1}{2\gamma}\,\dfrac{V_7^{\,2}}{T_a}}\right)} \tag{33}$$

The total-pressure loss due to both friction and momentum can now be found from the separate pressure losses evaluated by equations (18) and (33). The total-pressure ratio across the burner is

$$\frac{P_7}{P_5} = \left(\frac{P_6}{P_5}\right)\left(\frac{P_7}{P_6}\right) \tag{34}$$

When these ratios are expressed in terms of the respective pressure losses

$$1 - \frac{\Delta P}{P_5} = \left(1 - \frac{\Delta P_f}{P_5}\right)\left(1 - \frac{\Delta P_m}{P_6}\right) \tag{35}$$

When simplified, equation (35) becomes

$$\frac{\Delta P}{P_5} = \frac{\Delta P_f}{P_5} + \frac{\Delta P_m}{P_6} - \frac{\Delta P_f}{P_5}\frac{\Delta P_m}{P_6} \tag{36}$$

The third term on the right-hand side of equation (36), which is quite small compared with the other two, can be neglected without appreciable error. Neglecting this term is equivalent to assuming that $P_6 = P_5$ and results in the total-pressure loss being expressed as

$$\frac{\Delta P}{P_5} = \frac{\Delta P_f}{P_5} + \frac{\Delta P_m}{P_6} \tag{37}$$

Exhaust-Nozzle Area

Expressions for the exhaust-nozzle throat-area ratios required for the augmented and normal cases can be derived in the following manner:

From the conservation of mass, for the normal case

$$M = \frac{\rho_n \, C_A \, A_n \, V_n}{1 + f_e} \tag{38}$$

and for the augmented case

$$M = \frac{\rho_{n,a} \, C_{A,a} \, A_{n,a} \, V_{n,a}}{1 + f_e + f_b} \tag{39}$$

By equating and solving equations (38) and (39) for the effective area ratio, the following expression is obtained:

$$\frac{C_{A,a}\ A_{n,a}}{C_A\ A_n} = \left(\frac{1 + f_e + f_b}{1 + f_e}\right)\left(\frac{\rho_n}{\rho_{n,a}}\right)\left(\frac{V_n}{V_{n,a}}\right) \tag{40}$$

From the equation of state

$$\frac{\rho_n}{\rho_{n,a}} = \frac{p_{s,n}}{p_{s,n,a}}\ \frac{t_{n,a}}{t_n} \tag{41}$$

In cases where the exhaust-nozzle-throat velocity is sonic, the ratio of the augmented to normal throat velocities depends only on the square root of the augmented to normal total-temperature ratio. The ratio of augmented to normal nozzle-throat static pressures is directly proportional to the ratio of the augmented to normal total pressures and the ratio of static temperatures is equal to the ratio of total temperatures. For this case, the ratio of the densities is

$$\frac{\rho_n}{\rho_{n,a}} = \frac{P_5}{P_7}\ \frac{T_a}{T_5} \tag{42}$$

$$\frac{\rho_n}{\rho_{n,a}} = \frac{1}{1 - \frac{\Delta P}{P_5}}\ \frac{T_a}{T_5} \tag{43}$$

Accordingly, equation (40) becomes

$$\frac{C_{A,a}\ A_{n,a}}{C_A\ A_n} = \left(1 + \frac{f_b}{1 + f_e}\right)\left(\frac{1}{1 - \frac{\Delta P}{P_5}}\right)\left(\frac{T_a}{T_5}\right)\left(\sqrt{\frac{T_5}{T_a}}\right) \tag{44}$$

When it is assumed that $f_b/(1 + f_e)$ is approximately equal to f_b, equation (44) can be reduced to

$$\frac{C_{A,a}\ A_{n,a}}{C_A\ A_n} = \left(\frac{1 + f_b}{1 - \frac{\Delta P}{P_5}}\right)\sqrt{\frac{T_a}{T_5}} \tag{45}$$

For the case where the exhaust-nozzle-throat velocity is subsonic, the nozzle-throat pressure is atmospheric static pressure and the ratio of augmented to normal jet velocities is $\sqrt{K \frac{T_a}{T_5}}$. (See appendix A, equation (4).) The ratio of augmented to normal nozzle-throat static temperature is, neglecting the effect of pressure losses, equal to the ratio of augmented to normal total temperatures. By assuming again that $f_b/(1 + f_e)$ is approximately equal to f_b, equation (40) becomes

$$\frac{C_{A,a}\,A_{n,a}}{C_A\,A_n} = (1 + f_b)\,\frac{T_a}{T_5}\,\sqrt{\frac{1}{K}\,\frac{T_5}{T_a}} \tag{46}$$

simplifying

$$\frac{C_{A,a}\,A_{n,a}}{C_A\,A_n} = (1 + f_b)\sqrt{\frac{1}{K}\,\frac{T_a}{T_5}} \tag{47}$$

It is assumed in equation (47) that the nozzle-velocity coefficient is unchanged between normal and augmented operation.

APPENDIX C

EFFECTIVE TAIL-PIPE-BURNER EXIT TEMPERATURE

In the normal turbojet engine the turbine and long tail pipe tend to mix the exhaust gases sufficiently to produce a relatively uniform temperature distribution and the jet velocity can be calculated using this temperature. If the engine is equipped with a tail-pipe burner, however, the temperature distribution may be very uneven either due to poor fuel distribution or to a design in which the fuel is so distributed as to give a hot core of gases in the center of the tail pipe and a relatively cool layer next to the tail-pipe wall for cooling purposes. In either event some method must be devised for evaluating the effective temperature of the exhaust gases in order to calculate the jet velocity.

The effective gas temperature can be determined by assuming that the total and static pressures are uniform throughout the cross-sectional area of the jet. When the jet thrust is considered as the sum of the momentum increase along each stream tube where each stream tube handles the same mass flow, the thrust is

$$F_{j,a} = M\left(V_{j,a}\right) = M_1 V_{j,a,1} + M_2 V_{j,a,2} + \cdot \cdot \cdot + M_n V_{j,a,n}$$

where the subscripts 1, 2, . . ., and n indicate individual stream tubes.

The jet velocity $V_{j,a}$ (appendix A, equation (2)) is given by the expression

$$V_{j,a} = \sqrt{C_V^2 \frac{2\gamma R}{\gamma - 1} T_a \left[1 - \left(\frac{p_{0,0}}{P_7}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

and by writing similar equations for the velocity along each individual stream tube and eliminating the constants the expression for jet thrust becomes

$$F_j \propto M\sqrt{T_a} = M_1\sqrt{T_{a,1}} + M_2\sqrt{T_{a,2}} + \cdot \cdot \cdot + M_n\sqrt{T_{a,n}} \qquad (48)$$

and because $M = M_1 + M_2 + \cdot \cdot \cdot M_n$, the effective temperature is

$$T_a = \left(\frac{M_1 \sqrt{T_{a,1}} + M_2 \sqrt{T_{a,2}} + \ldots M_n \sqrt{T_{a,n}}}{M_1 + M_2 + \ldots M_n} \right)^2 \tag{49}$$

Equation (49) is general but in order to be useful both the temperature and mass-flow distribution must be known.

For the special case when $T_{a,1}, T_{a,2} \ldots T_{a,n}$ correspond to equal mass flows $(M_1 = M_2 = \ldots M_n)$, equation (49) can be simplified to

$$T_a = \left(\frac{\sqrt{T_{a,1}} + \sqrt{T_{a,2}} + \ldots \sqrt{T_{a,n}}}{n} \right)^2 \tag{50}$$

Usually, the mass-flow distribution will not be known, but for the case when temperature measurements are taken at the center of equal flow areas, the mass flow through each area can be found and the effective temperature distribution determined as follows:

Let $M_1, M_2, \ldots M_n$ be the mass flows for the equal flow areas. The mass flow in a stream tube can be expressed as

$$M_1 = \rho_1 A_1 V_1 \tag{51}$$

If the exhaust-nozzle velocity coefficient is assumed to be equal to 1, the static temperature in the jet can be expressed as

$$t_{a,1} = T_{a,1} \left(\frac{p_{s,0}}{P_7} \right)^{\frac{\gamma-1}{\gamma}} \tag{52}$$

and the jet velocity as

$$V_j = \sqrt{\frac{2\gamma R}{\gamma - 1} T_{a,1} \left[1 - \left(\frac{p_{s,0}}{P_7} \right)^{\frac{\gamma-1}{\gamma}} \right]} \tag{53}$$

The mass flow in the stream tube is then

$$M_1 = \frac{p_{s,0}}{RT_{a,1}\left(\frac{p_{s,0}}{P_7}\right)^{\frac{\gamma-1}{\gamma}}} A_1 \sqrt{\frac{2\gamma R}{\gamma - 1} T_{a,1}\left[1 - \left(\frac{p_{s,0}}{P_7}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad (54)$$

By writing similar equations for $M_2, \ldots, M_n$, substituting in equation (39) and eliminating the constants, the expression for effective temperature becomes

$$T_a = \left(\frac{A_1 + A_2 + \ldots A_n}{\frac{A_1}{\sqrt{T_{a,1}}} + \frac{A_2}{\sqrt{T_{a,2}}} + \ldots \frac{A_n}{\sqrt{T_{a,n}}}}\right)^2 \qquad (55)$$

Inasmuch as $A_1 = A_2 = \ldots A_n$, the expression for effective burner-exit temperature T_a becomes

$$T_a = \left(\frac{n}{\frac{1}{\sqrt{T_{a,1}}} + \frac{1}{\sqrt{T_{a,2}}} + \ldots \frac{1}{\sqrt{T_{a,n}}}}\right)^2 \qquad (56)$$

In order to illustrate the effect of this type of an average, consider a tail pipe in which the central one-half area of the tail pipe has a temperature of 3600° R and the half near the circumference has a temperature of 1600° R. The effective temperature is not the arithmetic average of 2600° R but from equation (56) is

$$T_a = \left(\frac{2}{\frac{1}{\sqrt{1600}} + \frac{1}{\sqrt{3600}}}\right)^2 = \left(\frac{2}{\frac{1}{40} + \frac{1}{60}}\right)^2 = 48^2 = 2304^\circ \text{ R}$$

APPENDIX D

SAMPLE CALCULATIONS

The following sample calculations illustrate the use of the figures to evaluate (1) augmented performance from normal performance data and (2) total-pressure loss occurring in a tail-pipe burner and diffuser operating at a specified set of conditions.

Augmentation

The following example will illustrate the use of figures 2 and 3 in finding the thrust augmentation produced by tail-pipe burning for a turbojet engine operating at the following set of conditions:

Airplane velocity, V_0, ft/sec	733
Air flow, M, slugs/sec	0.889
Fuel flow, W, lb/hr	1850
Burner-inlet total temperature, T_5, $^{\circ}R$	1680
Net thrust, F, lb	1425
Ratio of total-pressure loss across tail-pipe burner and diffuser to exhaust-cone-exit total pressure, $\Delta P/P_5$	0.10
Burner-exit total temperature, T_a, $^{\circ}R$	2960
Exhaust-nozzle velocity coefficient, C_V	0.975

The jet thrust is given by the following expression:

$$F_j = F + MV_0$$

$$= 1425 + 0.889 \times 733$$

$$= 1425 + 651 = 2076 \text{ lb}$$

The jet velocity can be found from the jet thrust and mass flow of exhaust gas:

$$V_j = \frac{F_j}{M + \frac{W}{3600 \times 32.17}}$$

$$= \frac{2076}{0.889 + 0.016} = 2293 \text{ ft/sec}$$

The jet-velocity factor, which is used in figure 3 to determine K, can now be found:

$$\frac{V_j}{C_V} \sqrt{\frac{1600}{T_5}} = \frac{2293}{0.975} \times \sqrt{\frac{1600}{1680}} = 2294 \text{ ft/sec}$$

By use of figure 3, the jet-velocity factor and the ratio of loss in total pressure across the tail-pipe burner and diffuser to exhaust-cone-exit total pressure, the value of K can be found:

$$K = 0.915$$

When the fuel flows are neglected, the effective burner-exit temperature factor for use in figure 2 is evaluated as

$$KT_a \frac{1600}{T} = 0.915 \times 2960 \times \frac{1600}{1680} = 2580^\circ \text{ R}$$

and the ratio of airplane velocity to jet velocity is

$$\frac{V_0}{V_j} = \frac{733}{2293} = 0.319$$

When figure 2 is used with the effective burner-exit temperature and the ratio of airplane velocity to jet velocity, the ratio of augmented net thrust to normal net thrust is found to be

$$\frac{F_a}{F} = 1.40$$

Thus for the particular engine and conditions chosen it is possible to augment the thrust produced by 40 percent. If the weights of the fuel flows had been considered, the effect would have been a slightly higher thrust augmentation because the effective burner-exit temperature factor would have been increased by the factor $(1 + 2f_b)$ although the ratio of airplane to jet velocity would have been decreased by $(1 - f_e)$.

Burner Pressure Loss

The following example illustrates the use of figures 4 and 5 in finding the loss in total pressure occurring in a tail-pipe burner operating under the following set of conditions:

Burner-inlet velocity, V_6, ft/sec	600
Exhaust-cone-exit velocity, V_5, ft/sec	1000
Burner-inlet total temperature, T_5, °R	1680
Burner-exit total temperature, T_a, °R	2960
Burner drag coefficient, C_D	0.8
Diffuser efficiency, η_d	0.85

The nominal burner-inlet velocity for use in figure 4 is

$$V_6 \sqrt{\frac{1600}{T_5}} = 600 \sqrt{\frac{1600}{1680}} = 585.5 \text{ ft/sec}$$

and the total friction loss factor is

$$C_D + (1 - \eta_d)\left(\frac{V_5^2}{V_6^2} - 1\right) = 0.8 + (1 - 0.85)\left(\frac{1000^2}{600^2} - 1\right)$$

$$= 0.8 + 0.267 = 1.067$$

By use of the nominal burner-inlet velocity, the total friction loss factor, and figure 4, the ratio of the friction total-pressure loss to the exhaust-cone-exit total pressure is

$$\frac{\Delta P_f}{P_5} = 0.061$$

The ratio of the tail-pipe-burner exit temperature to inlet temperature is

$$\frac{T_a}{T_5} = \frac{2960}{1680} = 1.762$$

By using this value, the nominal burner-inlet velocity previously determined, and figure 5, the ratio of momentum total-pressure loss to exhaust-cone-exit total pressure is found to be

$$\frac{\Delta P_m}{P_6} = 0.052$$

The over-all loss in total pressure is equal to the sum of the friction total-pressure loss and the momentum total-pressure loss

$$\frac{\Delta P}{P_5} = \frac{\Delta P_f}{P_5} + \frac{\Delta P_m}{P_6}$$

$$= 0.061 + 0.052 = 0.113$$

For this particular set of conditions the loss in total pressure is equal to 11.3 percent of the exhaust-cone-exit total pressure.

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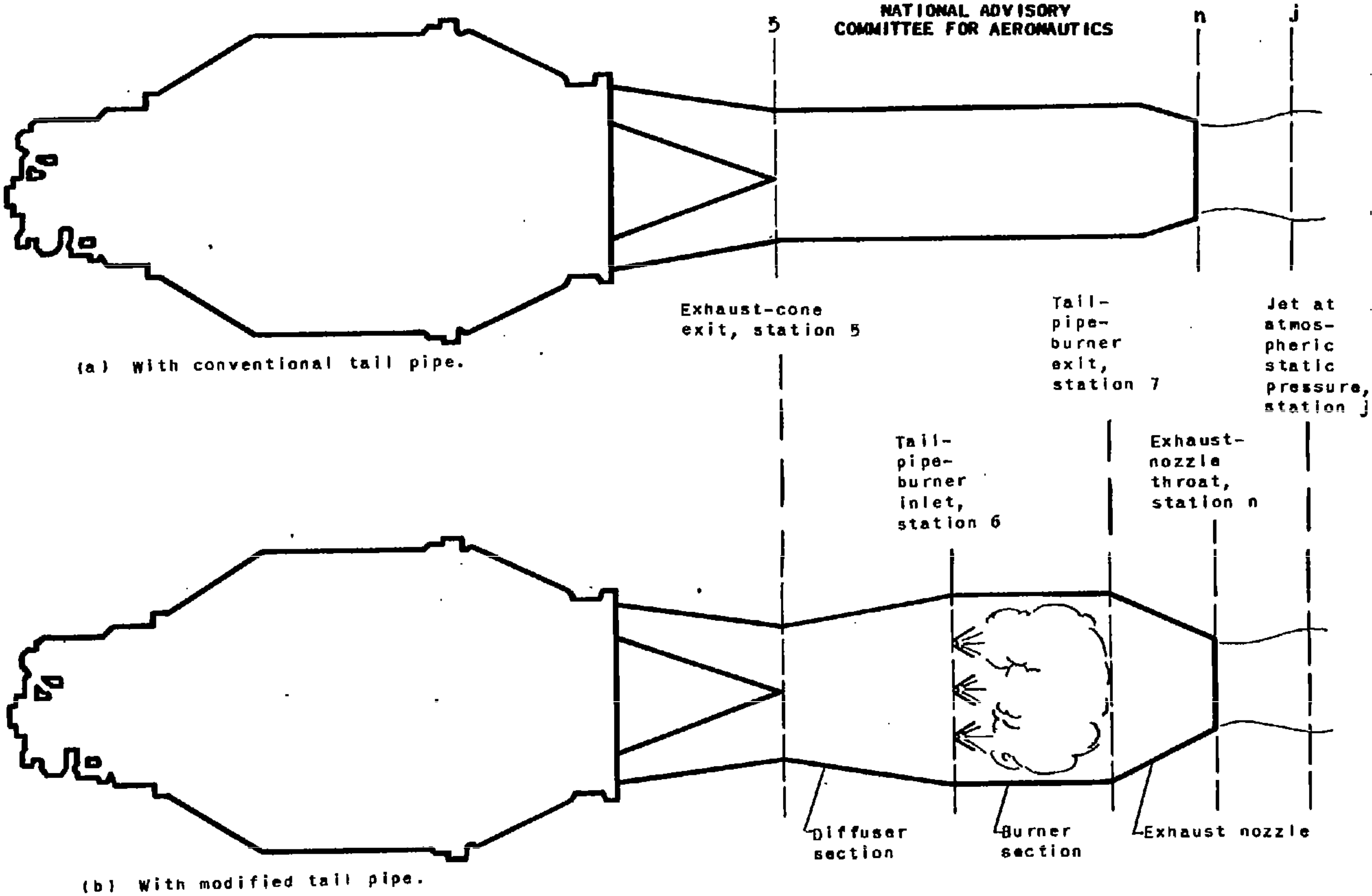


Figure 1. - Comparison of turbojet engines with conventional tail pipe and with tail pipe modified for thrust augmentation by tail-pipe burning.

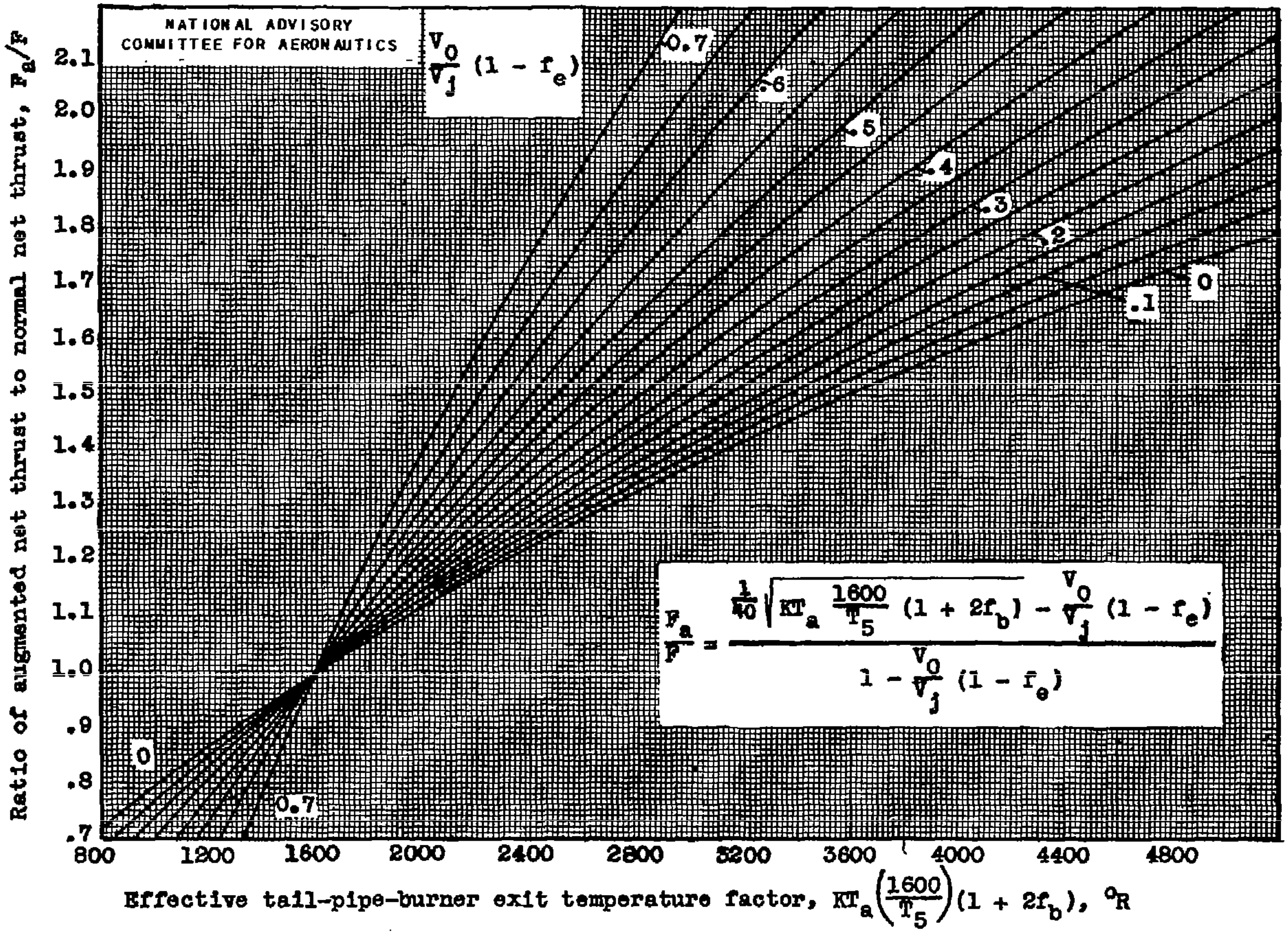


Figure 2. - Variation of augmented-to-normal net-thrust ratio with tail-pipe-burner exit temperature factor for various airplane-to-jet velocity ratios.

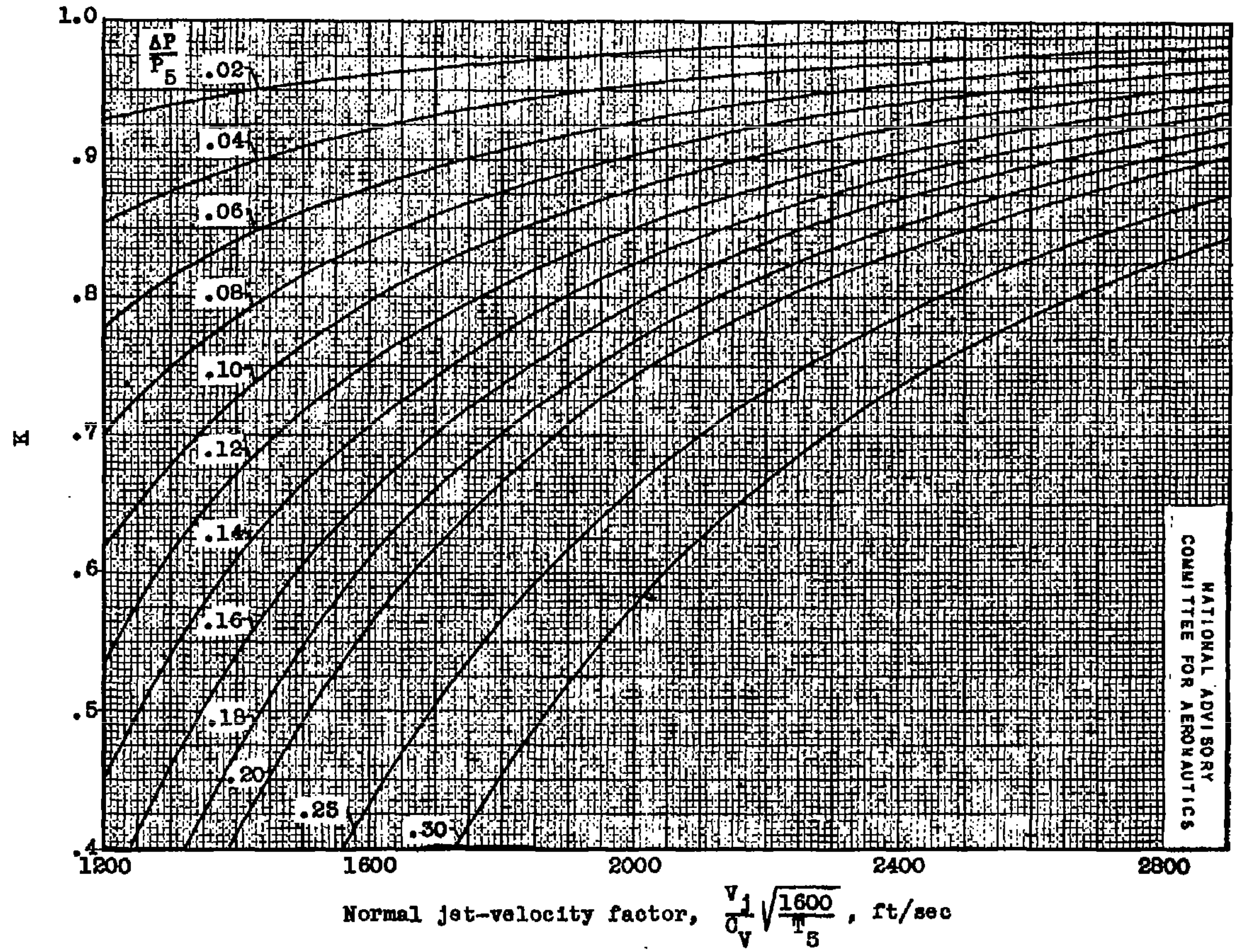


Figure 3. - Variation of K with normal jet-velocity factor for various ratios of tail-pipe-burner total-pressure loss to exhaust-cone-exit total pressure $\Delta P/P_5$.

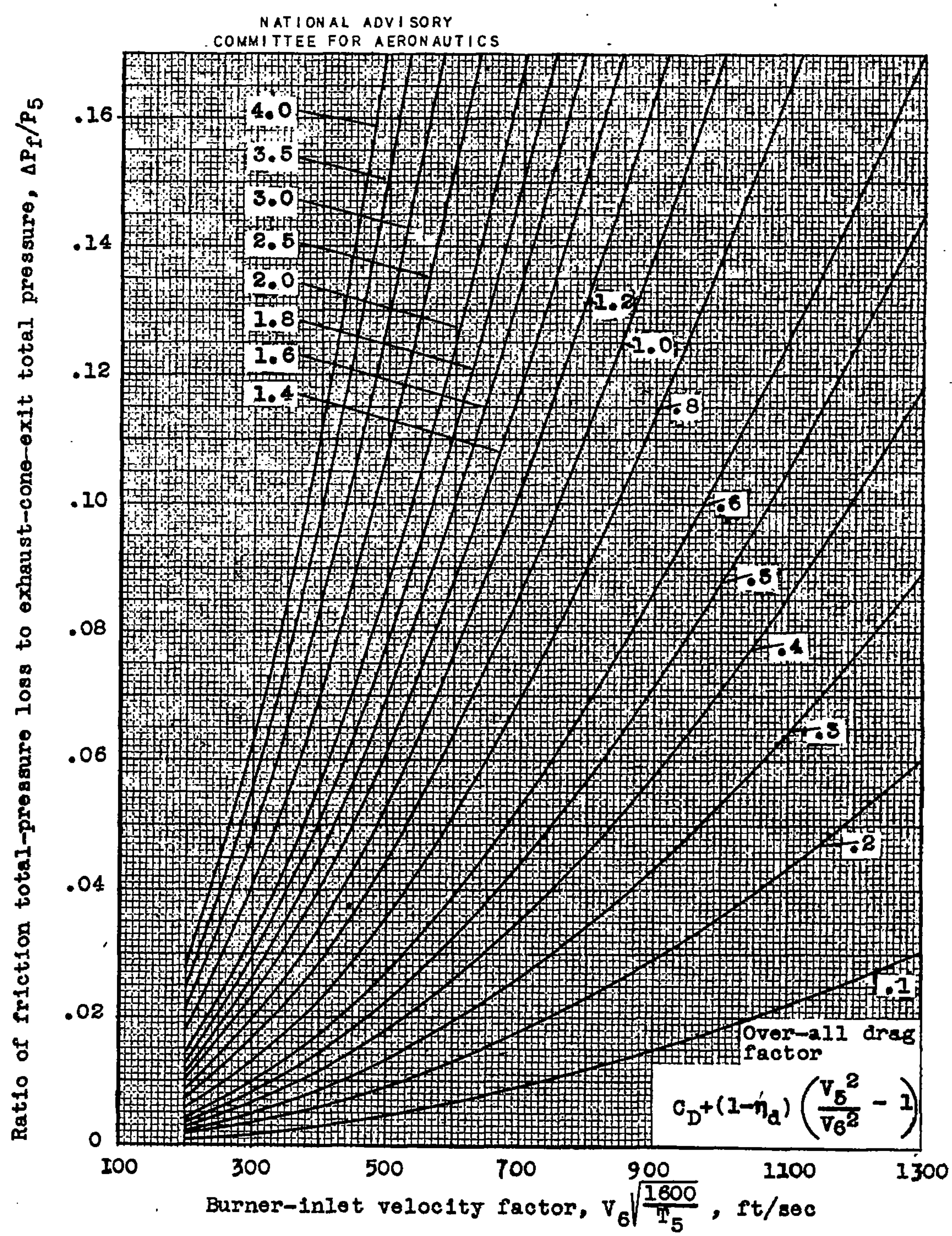


Figure 4. - Variation of ratio of friction total-pressure loss to exhaust-cone-exit total pressure with burner-inlet velocity factor for various burner drag coefficients C_D, diffuser efficiencies η_d, and diffuser velocity ratio V_5/V_6.

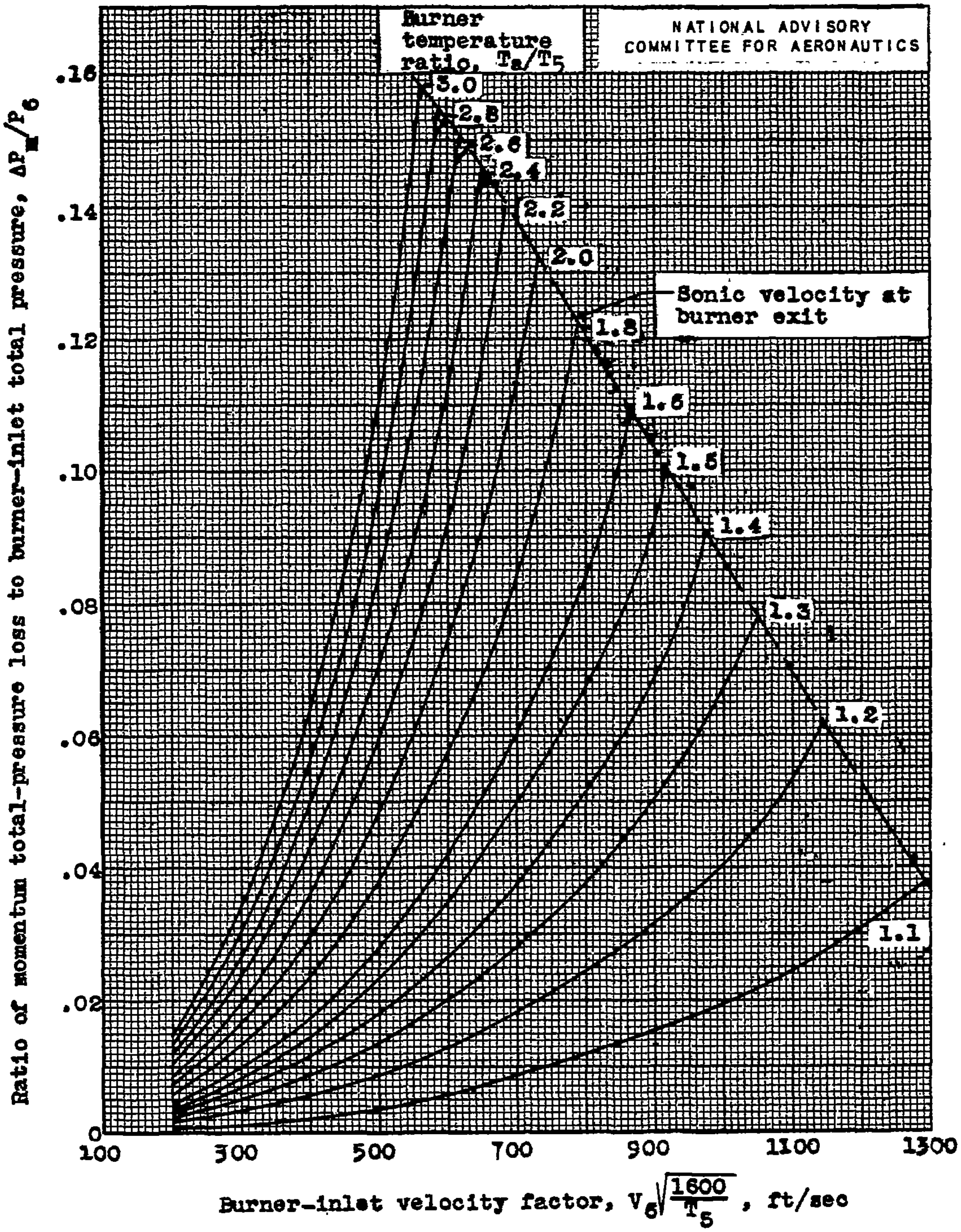
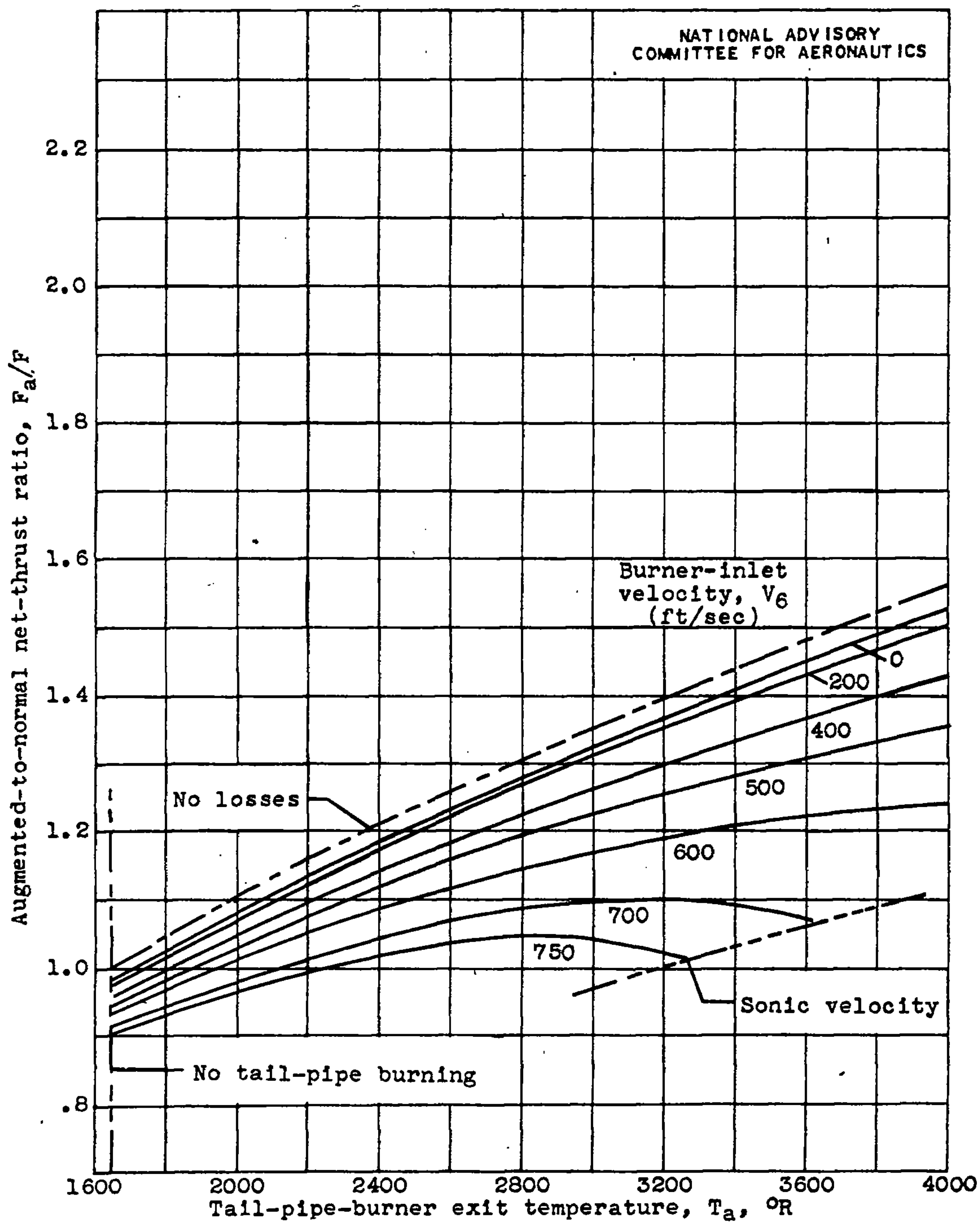
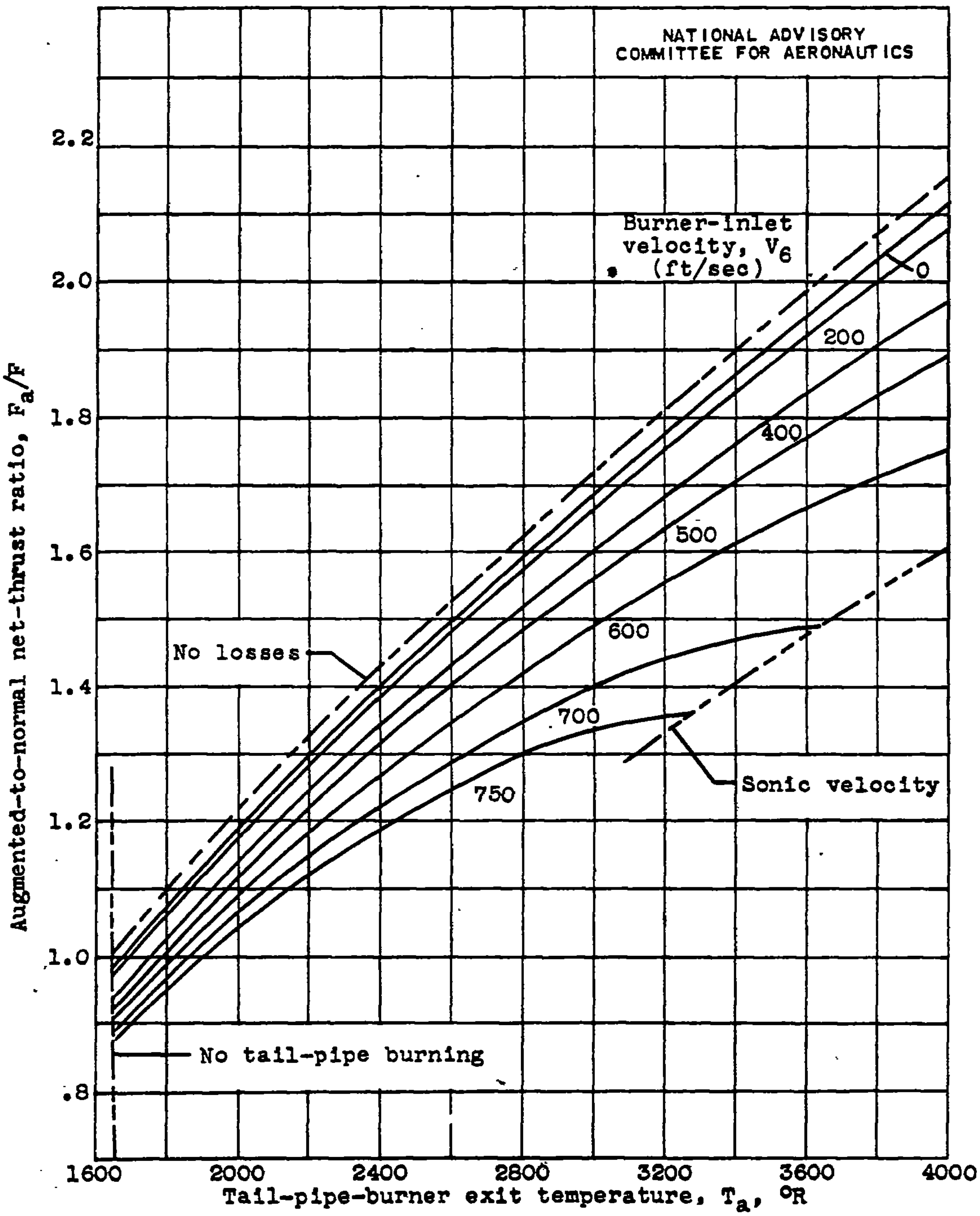


Figure 5. - Variation of ratio of momentum total-pressure loss to burner-inlet total pressure with burner-inlet velocity factor for various tail-pipe-burner temperature ratios.



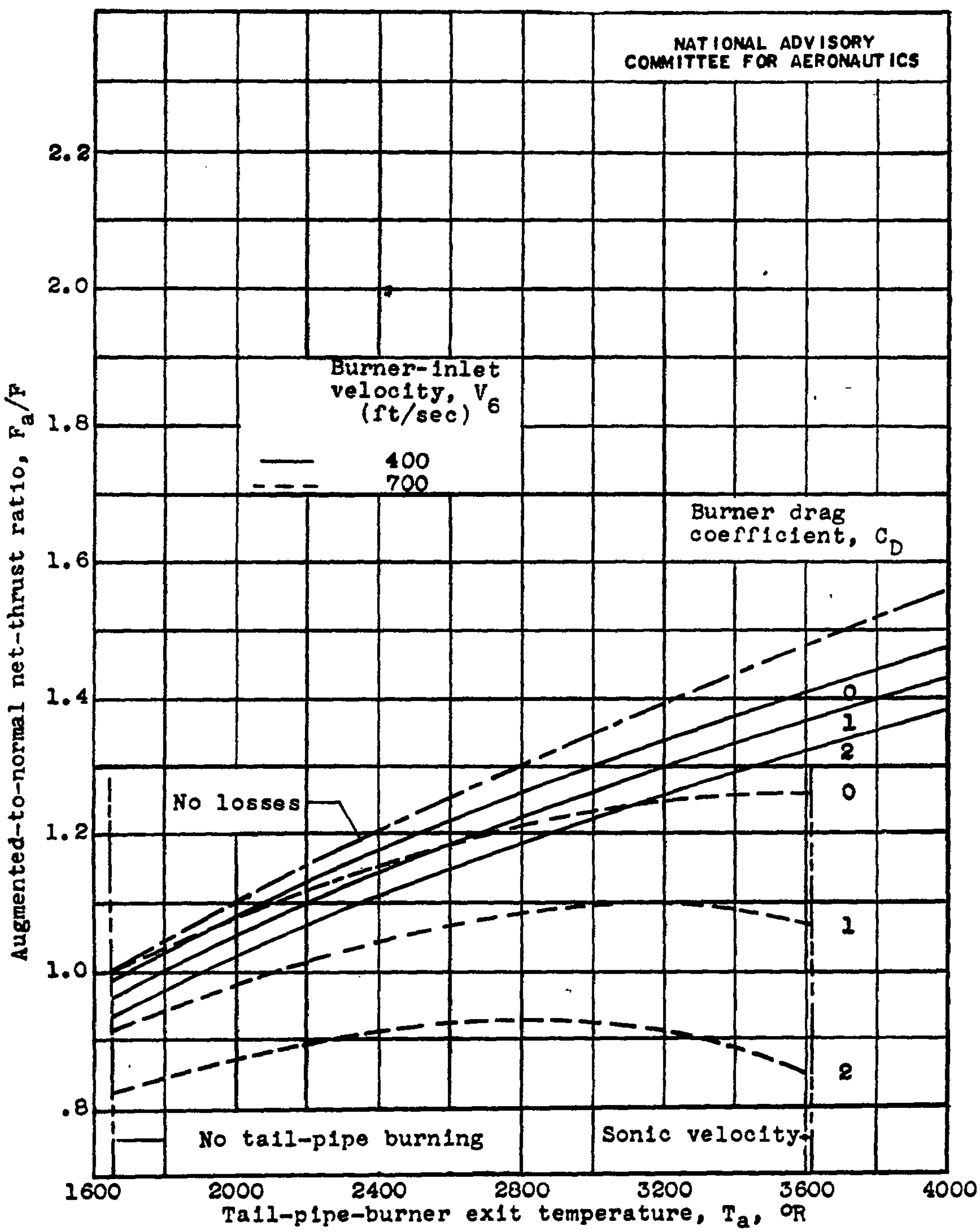
(a) Static sea-level conditions: V_0, 0 feet per second; V_j, 1635 feet per second.

Figure 6. - Variation of augmented-to-normal net-thrust ratio with tail-pipe-burner exit temperature for various burner-inlet velocities. V_5, 750 feet per second; T_5, 1650° R; C_D, 1.0; η_d, 0.8; C_V, 0.975.



(b) Sea-level high-speed conditions; V_0, 1026 feet per second; V_j, 1990 feet per second.

Figure 6. - Concluded. Variation of augmented-to-normal net-thrust ratio with tail-pipe-burner exit temperature for various burner-inlet velocities. V_5, 750 feet per second; T_5, 1650° R; C_D, 1.0; η_d, 0.8; C_V, 0.975.



(a) Static sea-level conditions: V_0, 0 feet per second; V_j, 1635 feet per second.

Figure 7. - Variation of augmented-to-normal net-thrust ratio with tail-pipe-burner exit temperature for various burner drag coefficients and burner-inlet velocities. V_5, 750 feet per second; T_5, 1650° R; η_d, 0.8; C_V, 0.975.

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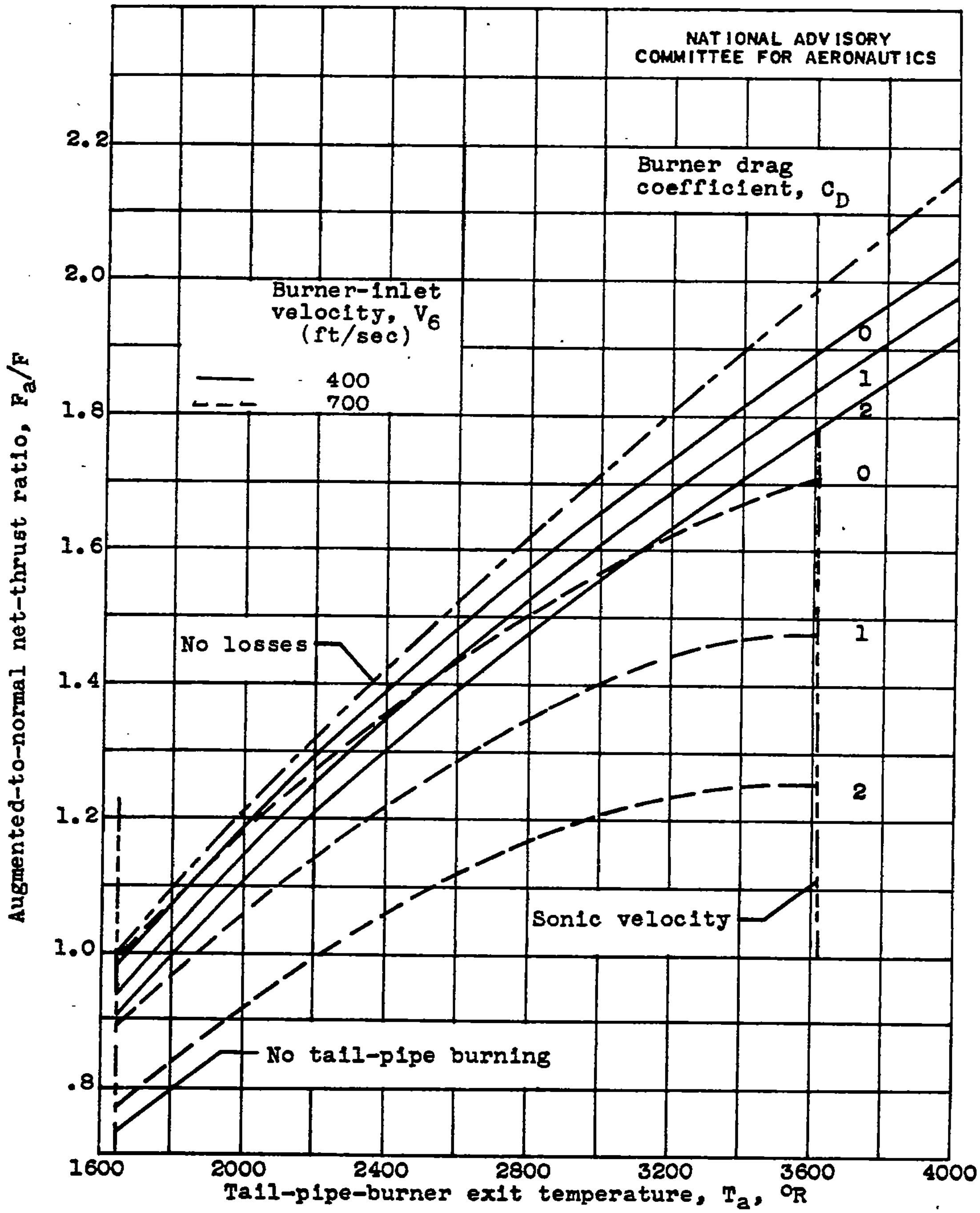
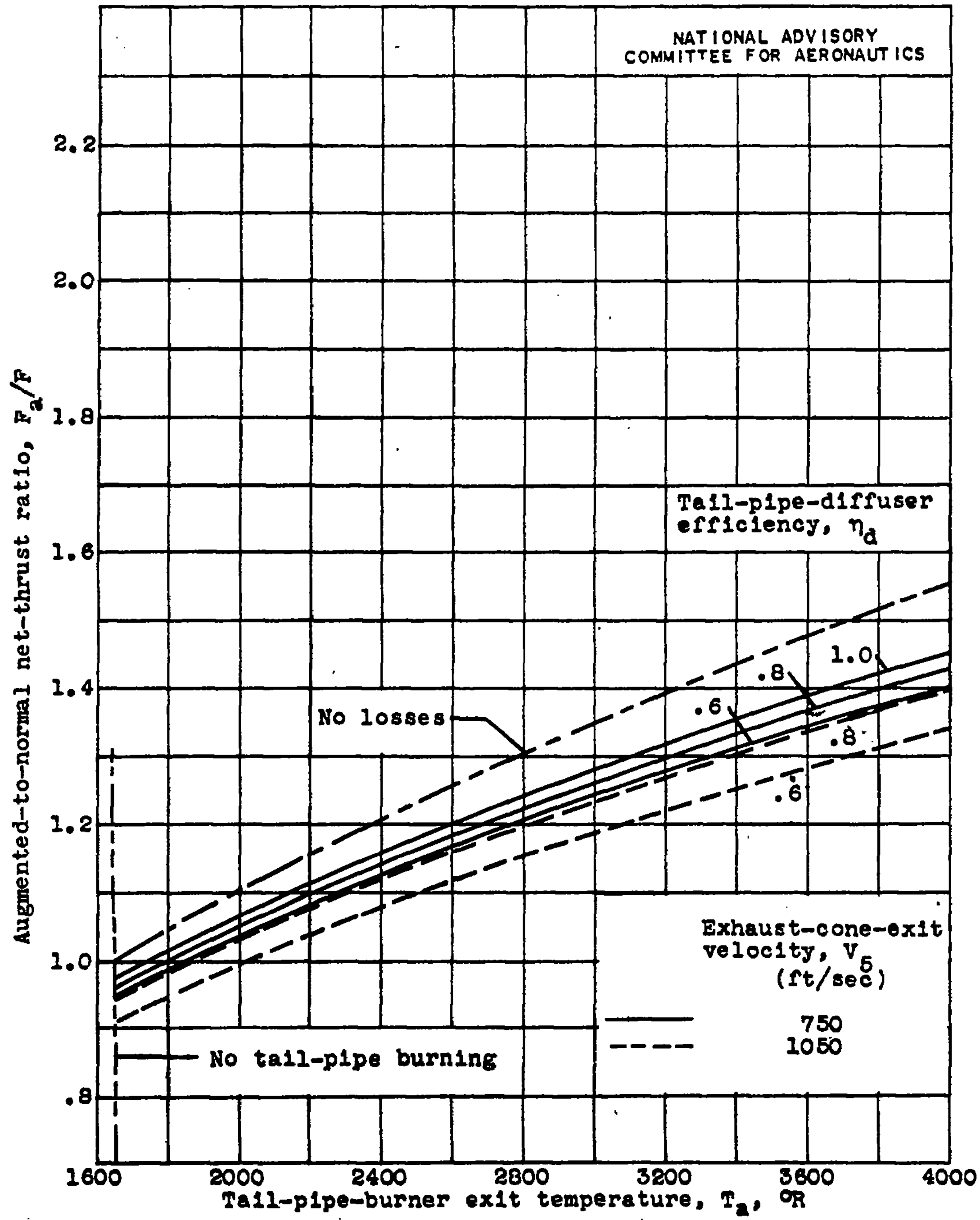


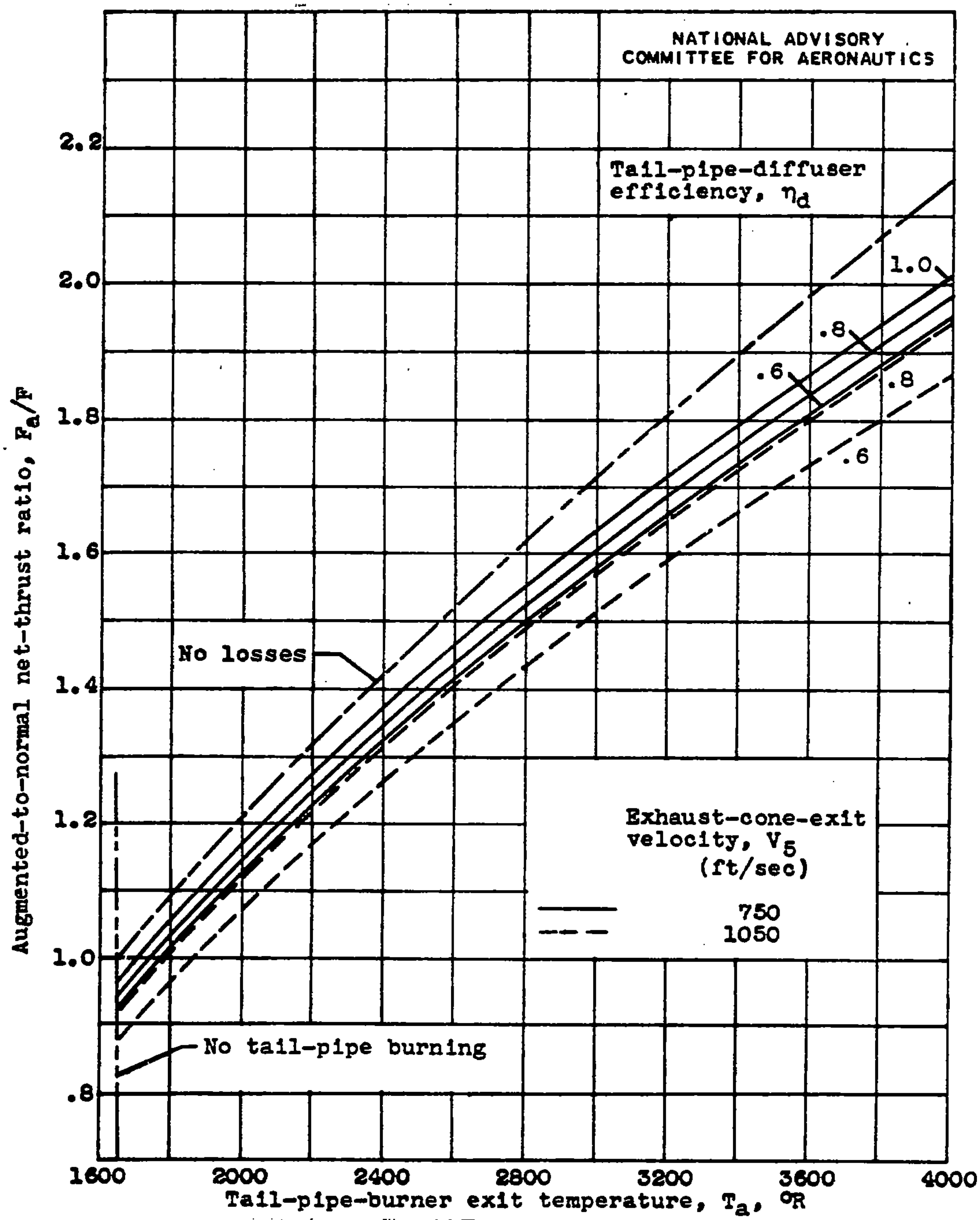
(b) Sea-level high-speed conditions; V_0, 1026 feet per second; V_j, 1990 feet per second.

Figure 7. - Concluded. Variation of augmented-to-normal net-thrust ratio with tail-pipe-burner exit temperature for various burner drag coefficients and burner-inlet velocities. V_5, 750 feet per second; T_5, 1650° R; η_d, 0.8; C_V, 0.975.



(a) Static sea-level conditions; V_0, 0 feet per second; V_j, 1635 feet per second.

Figure 8. - Variation of augmented-to-normal net-thrust ratio with tail-pipe-burner exit temperature for various tail-pipe-diffuser efficiencies and exhaust-cone-exit velocities. T_5, 1650° R; V_6, 400 feet per second; C_D, 1.0; C_V, 0.975.



(b) Sea-level high-speed conditions: V_0, 1026 feet per second; V_j, 1990 feet per second.

Figure 8. - Concluded. Variation of augmented-to-normal net-thrust ratio with tail-pipe-burner exit temperature for various tail-pipe-diffuser efficiencies and exhaust-cone-exit velocities. T_5, 1650° R; V_6, 400 feet per second; C_D, 1.0; C_V, 0.975.

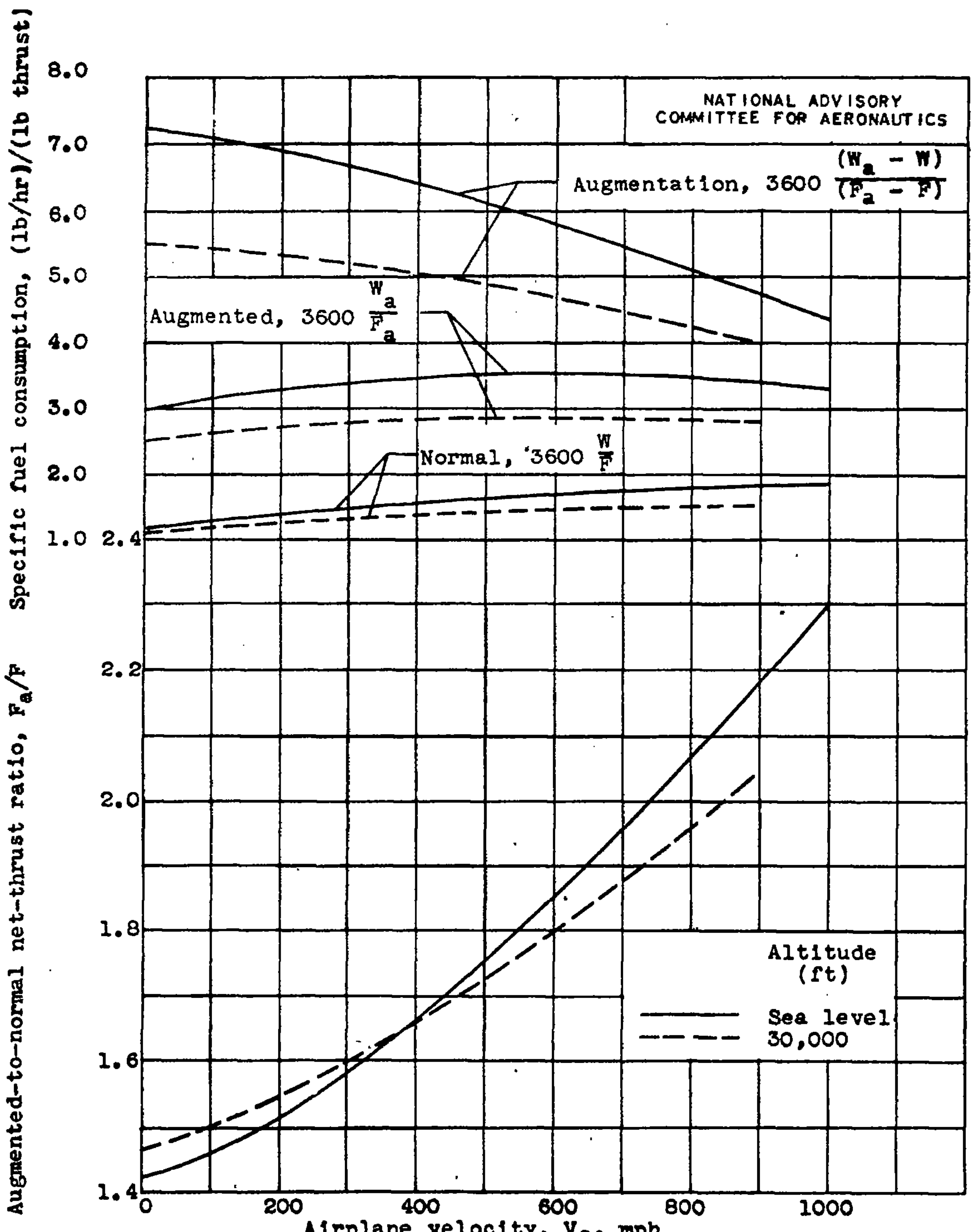


Figure 9. - Variation of augmented-to-normal net-thrust ratio and specific fuel consumptions with airplane velocity and altitude. V_5, 750 feet per second; T_5, 1650° R; V_6, 400 feet per second; T_a, 3600° R; η_d, 0.8; η_c, 0.85; C_D, 1.0; C_V, 0.975.