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RESEARCH MEMORANDUM

COOLING OF GAS TURBINES

VIII - THEORETICAL TEMPERATURE DISTRIBUTIONS THROUGH GAS TURBINE WITH SPECIAL BLADES AND COOLING FINS ON THE RIM

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SUMMARY

A theoretical analysis of the radial-temperature distribution through a turbine with specially designed turbine blades was made for a turbine with and without cooling fins on the rim. The effect on blade life and effective gas temperature of the addition of cooling fins on the rim was investigated for selected turbine operating conditions. The accuracy of the radial-temperature distribution was then determined by finding a two-dimensional temperature distribution through the turbine rim and rotor for the turbine with and without cooling fins on the rim. The two-dimensional distribution was obtained by consideration of radial- and axial-temperature gradients and by application of the relaxation method to the resulting differential equation. A three-dimensional temperature distribution for a section of the turbine rim near a blade root was then determined by use of three-dimensional relaxation.

The results showed that, for the selected conditions, the addition of cooling fins on the rim permitted only a slight increase in the effective gas temperature. The two-dimensional investigation proved the radial distribution to be sufficiently accurate for most applications. The three-dimensional study indicated the type of temperature gradients to be expected in a section of the rim surrounding a blade root and may be applied to determinations of rim thermal stresses.

INTRODUCTION

High-temperature materials now available for gas turbines limit turbine-inlet temperatures to about 1500° F; consequently, some method of turbine cooling is necessary if inlet temperatures above this value are to be used. Investigations have been conducted at the NACA Cleveland laboratory (references 1 to 9) to determine the relative merits of indirectly cooling turbine blades (by conducting heat away from the blades) and of directly cooling them (by passing air or liquids through hollow passages in the blades). Increases in gas temperatures of the order of 100° to 200° F were found possible for indirect cooling; considerably greater increases in gas temperatures were made possible by direct cooling with either air or water as the coolant.

In order to apply direct cooling using either air or water as the coolant, the turbine blades must be designed with hollow passages. The manufacturing problems involved in the design of hollow blades, however, appear to limit blade forms as compared with solid blades. Some turbine blades for indirect cooling have been designed and tested; one of these is a blade with a long, thin trailing section and with a high degree of twist.

The results of a theoretical investigation of indirect cooling for a turbine with this special blade are presented herein. Radial-temperature distributions were obtained before and after the addition of cooling fins on the rim and the increases in blade life and effective gas temperature resulting from the addition of rim fins were calculated for a chosen set of turbine operating conditions. The radial-temperature distribution was then used as a trial solution and a two-dimensional temperature distribution through the turbine rim and rotor was obtained by considering additional temperature gradients in the axial direction. A three-dimensional relaxation was then carried out in a section of the rim surrounding a blade root.

SYMBOLS

The following symbols are used in this report:

A	average cross-sectional area, (sq ft)	
B	integration constants	(ft)
C		(°F)
D		(°F)
E		(°F)
F		$(°F/ft^{1/2})$
G		$(°F/ft^{1/2})$

i	imaginary unit, $\sqrt{-1}$
J_0, $i^{1/3}J_{-1/3}$, $i^{-1/3}J_{1/3}$, $i^{-2/3}J_{2/3}$, iJ_1, iH_0, H_1	Bessel functions
k	thermal conductivity of metal, Btu/(hr)(sq ft)($^{\circ}$F/ft)
L	blade length, (ft)
l	net spacing in relaxation method
n	coordinate perpendicular to both radial and axial directions
Q	temperature residual at net point
q_i	heat-transfer coefficient between hot gas and metal, Btu/(hr)(sq ft)($^{\circ}$F)
q_o	heat-transfer coefficient between metal and cooling air, Btu/(hr)(sq ft)($^{\circ}$F)
r	radius, (ft) (also coordinate in radial direction)
T	temperature, ($^{\circ}$F)
t	average thickness, (ft)
V	tip speed, (ft/sec)
w	linear dimension on rim common with blades, (ft)
x	distance of blade point from blade tip, (ft)
y	$L - x$, (ft)

z	coordinate in axial direction
α	$(2q_i/kt_b)^{1/2}$
β	$(2q_o/kt_r)^{1/2}$
γ	$(4\pi q_o/kA_{rot})^{1/2}$
θ	T_m-T_a

Subscripts:

a	cooling air
b	blade
e	effective
m	metal
r	rim (with A denotes area exposed to gas)
rot	rotor

ANALYSIS

One-Dimensional Temperature Distribution

A radial-temperature distribution was obtained by the method developed in reference 2 for a turbine with and without cooling fins on the rim and with blades having long, thin trailing sections and a high degree of twist. The following simplifying assumptions were made:

1. Constant mean values of the area, perimeter, thermal conductivity, and heat-transfer coefficients were used for each section.

2. No temperature gradients other than radial were considered.

3. The effect of radiation was negligible.

4. Cooling on both sides of the turbine rim was considered.

5. For the turbine with cooling fins on the rim, the over-all heat-transfer coefficient was taken as 4.4 times the flat-plate value of q_o. The value 4.4 was calculated from the fin dimensions by the method of reference 10 for fins that were so designed as to withstand the estimated stresses. The flat-plate value of q_o was calculated by Reynolds analogy (reference 11, p. 162) from values of the friction coefficient appropriate to the relative velocity of the rotor and the air.

Expressions for the temperature in the blades, rim, and rotor of a turbine are derived in reference 2. These expressions were obtained by setting up heat balances for an element of the blade, the rim, and the rotor, and by solving the resulting differential equations. This process yielded the following three equations:

Blade,

$$T_e - T_m = C \cosh \alpha\ (x-B) \tag{1}$$

Rim,

$$T_m - T_a = DJ_0\ (i\beta r) + EiH_0\ (i\beta r) \tag{2}$$

Rotor,

$$T_m - T_a = r^{1/2}\ Fi^{1/3}\ J_{-1/3}\ (2/3\gamma i r^{3/2}) + Gi^{-1/3}\ J_{1/3}\ (2/3\gamma i r^{3/2}) \tag{3}$$

The metal temperatures of the blade, rim, and rotor were found for ranges of the variables from $x = 0.2116$ foot (corresponds to $r = 0.3750$ ft) to $x = 0$ feet (corresponds to $r = 0.5886$ ft), from $r = 0.2775$ to $r = 0.3750$ foot, and from $r = 0$ to $r = 0.2775$ foot, respectively.

The six integration constants in equations (1) to (3) may be evaluated as follows: From the boundary conditions, $\frac{\partial T}{\partial r}$ 0 for $r = 0$ and $\frac{\partial T}{\partial x} = 0$ for $x = 0$, it can be shown that $B = G = 0$. The other four integration constants may be found by solving the equations obtained by equating temperatures and heat flows at the junctions of the various sections. These equations were found to be

$$(0.2775)^{1/2} F i^{1/3} J_{-1/3}\left[2/3\gamma i(0.2775)^{3/2}\right] = DJ_0\left[i\beta(0.2775)\right] + EiH_0\left[i\beta(0.2775)\right] \quad (4)$$

$$0.0267\gamma(0.2775)Fi^{-2/3} J_{2/3}\left[2/3\gamma i(0.2775)^{3/2}\right] = 0.0375\beta\left\{-DiJ_1\left[i\beta(0.2775)\right] - EH_1\left[i\beta(0.2775)\right]\right\} \quad (5)$$

$$T_a + DJ_0\left[i\beta(0.3750)\right] + EiH_0\left[i\beta(0.3750)\right] = T_e - C\cosh(0.2116\alpha) \quad (6)$$

$$\beta k(0.0506)\left\{-DiJ_1\left[i\beta(0.3750)\right] - EH_1\left[i\beta(0.3750)\right]\right\} = k(0.0126)\left[C\alpha\sinh(0.2116\alpha)\right] + 0.0380q_1\left[C\cosh(0.2116\alpha)\right] \quad (7)$$

Values of the Bessel functions are given in reference 12. The foregoing analysis was applied to a specific turbine equipped with a special turbine blade. (See fig. 1.) The necessary numerical values for calculations for this turbine were:

$T_a = -19.5^\circ$ F

$T_e = 1500^\circ$ F

$q_1 = 44.2$ Btu/(hr)(sq ft)(°F) (obtained from reference 11 for a gas flow of 6 lb/sec through the turbine)

$k = 12$ Btu/(hr)(sq ft)(°F/ft)

$t_b = 0.0094$ (ft)

$$t_r = 0.1388, \text{ (ft)}$$

$$A_{rot} = 0.1722, \text{ (sq ft)}$$

$$q_o = 30 \text{ Btu/(hr)(sq ft)(}^\circ\text{F)(without fins)}$$

$$q_o = 132 \text{ Btu/(hr)(sq ft)(}^\circ\text{F)(with fins)}$$

$$\alpha = 27.89, \text{ (ft)}^{-1}$$

$$\beta = 6, \text{ (ft)}^{-1} \text{ (without fins)}$$

$$\beta = 12.56, \text{ (ft)}^{-1} \text{ (with fins)}$$

$$\gamma = 13.7, \text{ (ft)}^{-3/2}$$

The solutions of equations (4) to (7), upon substitution of these values, gave the following values for C, D, E, and F:

Without fins	With fins
$C = 2.59^\circ$ F	$C = 4.41^\circ$ F
$D = 384.60^\circ$ F	$D = 35.92^\circ$ F
$E = -71.53^\circ$ F	$E = -13,360.00^\circ$ F
$F = 880.70^\circ$ F/(ft)$^{1/2}$	$F = 117.30^\circ$ F/(ft)$^{1/2}$

The radial-temperature distributions through the turbine were then obtained (fig. 2) by the use of these calculated integration constants and equations (1) to (3).

Special attention was then given to the radial-temperature distribution through the turbine blades. Stress-to-rupture data for cast S816 high-temperature alloy, obtained from figure 3 in reference 13, were combined with the blade-stress distribution, obtained by the use of equation (3) in reference 9, and allowable blade-temperature curves were obtained for various blade-tip speeds and various blade lives. The allowable blade-temperature curves for 1000-hour blade life are plotted in figure 3 with the blade-temperature distributions for the turbine with and without cooling fins on the rim.

In order to determine the effect on blade life of the addition of cooling fins on the rim, an allowable blade-temperature-distribution curve tangent to, or just above, the blade-temperature-distribution curve for the turbine with rim fins must be obtained for a fixed blade tip speed (equal to the maximum allowable blade-tip speed for the turbine without rim fins) and for a blade life as yet unknown. A family of allowable blade-temperature-distribution curves was found for this fixed tip speed and for different blade lives. The curves for 1000-, 2000-, and 3000-hour blade life are plotted in figure 4 with the blade-temperature distributions for the turbine with and without rim fins at a tip speed of 1170 feet per second.

The effect of the addition of cooling fins on the rim on the effective gas temperature was then investigated. In order to determine this effect, an allowable blade-temperature curve was fixed for 1000-hour blade life and a blade tip speed of 1170 feet per second. A family of blade-temperature-distribution curves was then found for various effective gas temperatures. A blade-temperature-distribution curve tangent to, or just below, the fixed allowable blade-temperature-distribution curve was required. The curves so obtained are shown in figure 5.

Two-Dimensional Temperature Distribution

Two-dimensional temperature distributions were obtained through the turbine rim and rotor by considering temperature gradients in both the radial and the axial directions. Symmetry was assumed about the radius of the rotor because cooling was considered at both sides of the rim. From a heat balance for an element $2\pi r \partial r \partial z$ of the wheel, the heat-flow equation was found to be

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{\partial^2\theta}{\partial z^2} = 0 \tag{8}$$

Along the cooled edge of the turbine, the boundary condition was

$$\frac{\partial\theta}{\partial z} = \frac{q_o}{k}\,\theta$$

The boundary condition at the rim surface exposed to the hot gas was

$$\frac{\partial\theta}{\partial r} = \frac{q_1}{k}\,\theta$$

And the boundary condition at the junction of the rim and blades was

$$30kA_b \frac{\partial\theta}{\partial x} = 2\pi k r_r w \frac{\partial\theta}{\partial r} - q_1 A_r \theta$$

where account is taken of heat entering the rim by both convection and conduction. It was assumed there was no heat flow across the other boundaries.

A solution of equation (8) was found by applying the relaxation method (reference 14). The one-dimensional temperature distribution previously obtained was assumed as the radial-temperature distribution along the cooled edge of the turbine. From these values, the definition of θ, and the boundary conditions, estimated values of θ were obtained and inserted at each point of a square network of points. Temperature residuals (interior heat sinks) were then calculated at each net point and reduced to minimum values by the use of fixed relaxation patterns. From these revised values of θ, the metal temperature T_m was found at each net point, and isotherms were then drawn. (See fig. 6.)

Three-Dimensional Temperature Distribution

In order to obtain a more accurate temperature distribution around a blade root, a detailed three-dimensional relaxation was carried out for a small portion of the turbine rim. The portion selected extended from the center of a blade to a point midway between that blade and the next blade, from the center of the rim to the cooled edge of the rim, and from the rim surface 1/4 inch in a radial direction. (See fig. 7.) As before, cooling was considered on both sides of the turbine rim. Symmetry was assumed about the center of the rim, about the center of the blade, and about the line midway between the two blades. Because only a small portion of the rim was considered and the curvature of the various boundary surfaces of such a portion was slight, the curvature was disregarded. The heat-flow equation, derived by setting up a heat balance for the element $\partial x\ \partial y\ \partial z$ was found to be

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2} = 0$$

Boundary conditions were the same as those considered in the two-dimensional investigation and the solution was obtained in exactly the same way, except that a cubical network of points was required here; hence, a new relaxation pattern was employed.

RESULTS AND DISCUSSION

Three different sections of the specially designed turbine blade considered in this investigation are shown in figure 1. The root section, the central section, and the tip section are presented in order to show the high degree of twist and the long, thin trailing section of the blade, both of which preclude direct cooling.

The radial-temperature distribution through the turbine rotor, rim, and blades, obtained for the turbine with and without cooling fins on the rim by application of equations (1) to (3), is shown in figure 2. The addition of fins lowered the rotor and the rim temperatures by at least 325° F, but the reduction in blade temperatures decreased as the blade tip was approached and, for the upper half of the blade, the temperature reduction due to the addition of cooling fins on the rim was negligible.

The temperature distribution through the blade for the turbine with and without cooling fins on the rim plotted with allowable blade-temperature curves for various tip speeds and 1000-hour blade life is presented in figure 3. The critical blade point for the turbine without fins and for an effective gas temperature of 1500° F was found to be at 0.35 the distance from the blade root; at this point, the allowable blade temperature was about 1435° F and the tip speed was 1170 feet per second. The critical blade point for the turbine with fins occurred slightly farther from the blade root.

The effect on blade life of the addition of cooling fins on the rim is shown in figure 4. For a fixed blade tip speed of 1170 feet per second and an effective gas temperature of 1500° F, the addition of fins indicates an increase in blade life from 1000 to 2000 hours. In practice, such an increase would be difficult to realize because the effective gas temperature and weight flow would have to be kept uniform and the blade material would have to be homogeneous to a high degree. The results indicate that near the rupture point the life of the material is highly sensitive to small temperature changes.

The effect of the addition of cooling fins on the rim on the effective gas temperature is illustrated in figure 5. It was found that for a blade life of 1000 hours and a blade tip speed of 1170 feet per second, the addition of fins permitted an effective gas temperature of 1530° F, an increase of 30° F. It is pointed out that figure 6 in reference 3, which shows possible increases in effective gas temperature for a range of blade parameters, gives in this case a result consistent with this value.

From the preceding results, it may be observed that the addition of cooling fins on the rim resulted in doubling the blade life, although it permitted only a slight increase in effective gas temperature.

The results of a two-dimensional investigation of the temperature distribution through the turbine rim and rotor, obtained by use of the relaxation method, are shown in figure 6. The investigation was made primarily to determine the accuracy of the radial (one-dimensional) distribution previously found. The results are limited to the turbine rim and rotor because no significant changes could be found in the blade-temperature distribution without a considerable amount of calculation necessitated by a reduction in net spacing and a large increase in the number of net points used. The radial-temperature distribution proved to be accurate; the addition of cooling fins on the rim resulted in lowering the temperature at the blade root by about 300° F. Correspondingly larger increases were found nearer the turbine axis.

Although the three-dimensional temperature investigation through a section of the turbine rim near a blade root is not directly related to the other investigations reported herein, it is included because of its value for any future study of rim thermal stresses. The results of the investigation are shown by the isothermal surface in figure 7; the 1/4-inch section of the rim that was considered is divided into three layers in order to show more clearly the isothermal surfaces. A careful study of figure 7 shows the temperature changes in the various directions for the rim section that was used in the investigation.

SUMMARY OF RESULTS

The following results were obtained from a theoretical investigation of the temperature distribution through a turbine with a specially designed blade and with and without cooling fins on the rim:

1. For a fixed tip speed of 1170 feet per second and an effective gas temperature of 1500° F, the addition of rim fins permitted a slight increase in the effective gas temperature (from 1500° to 1530° F).

2. The two-dimensional temperature distribution, obtained by the relaxation method, proved the analytically determined radial distribution to be sufficiently accurate for most applications.

3. The three-dimensional temperature distribution revealed the conditions existing in a section of the rim near a blade root and may be applied to investigations of thermal stresses.

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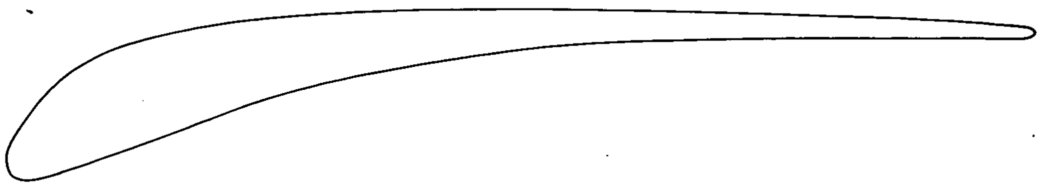
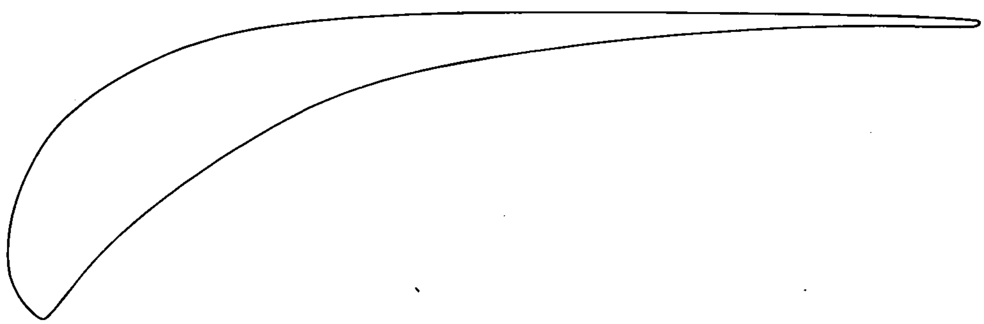
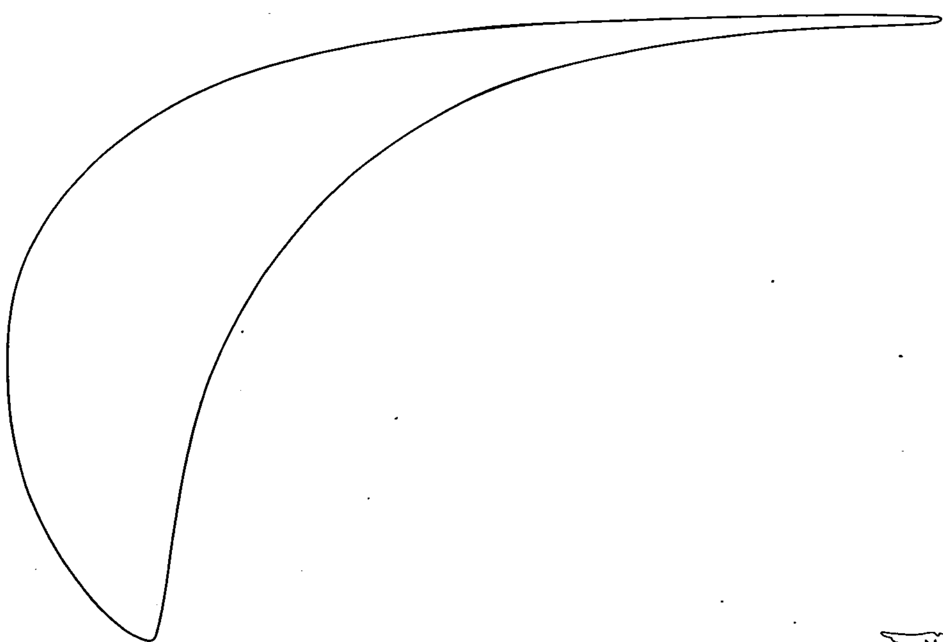


Tip section



Central section



Root section

Figure 1. - Selected sections of special turbine blade suitable for rim cooling.

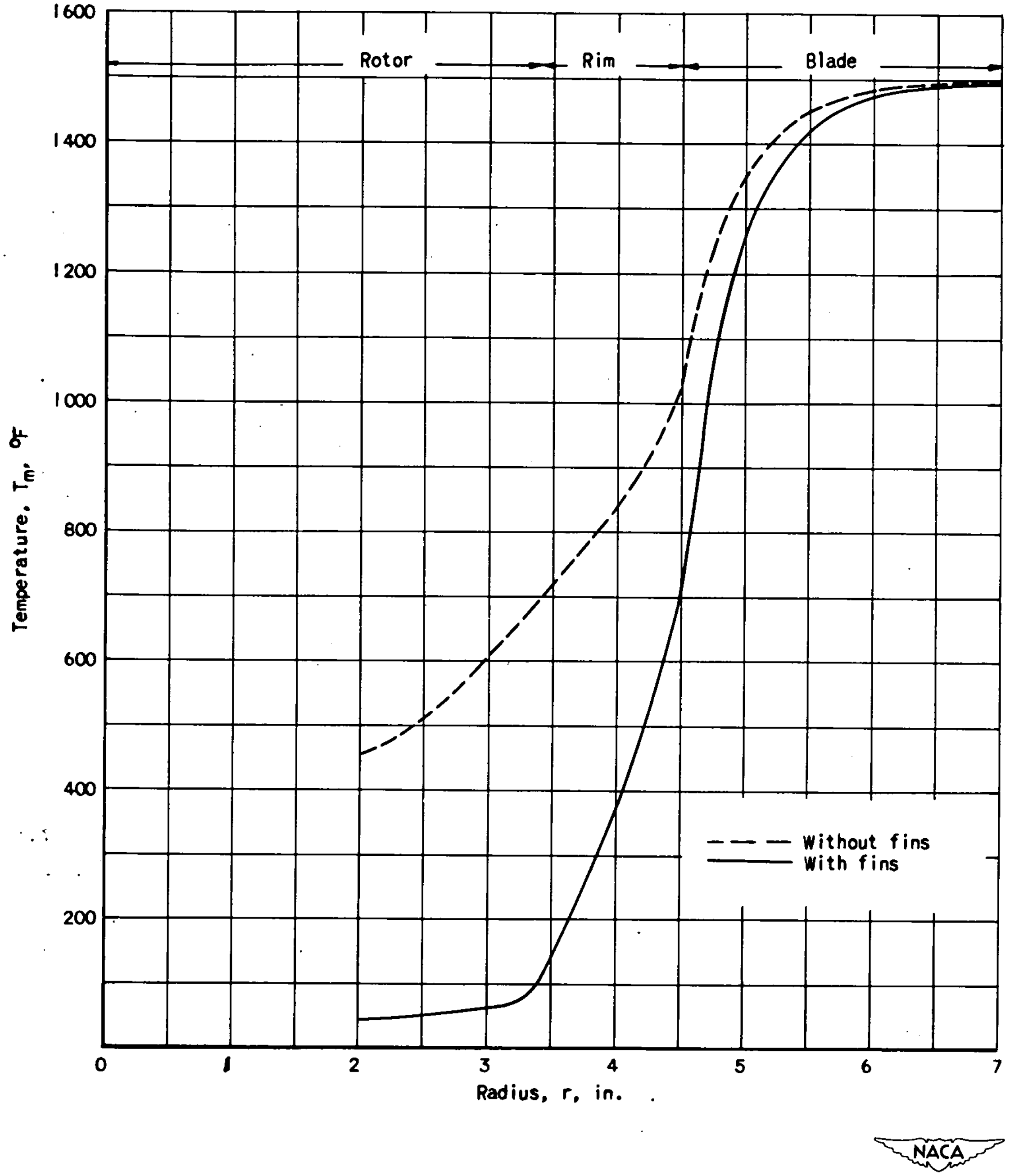


Figure 2. - Radial-temperature distribution through turbine with special turbine blades for effective gas temperature of 1500° F.

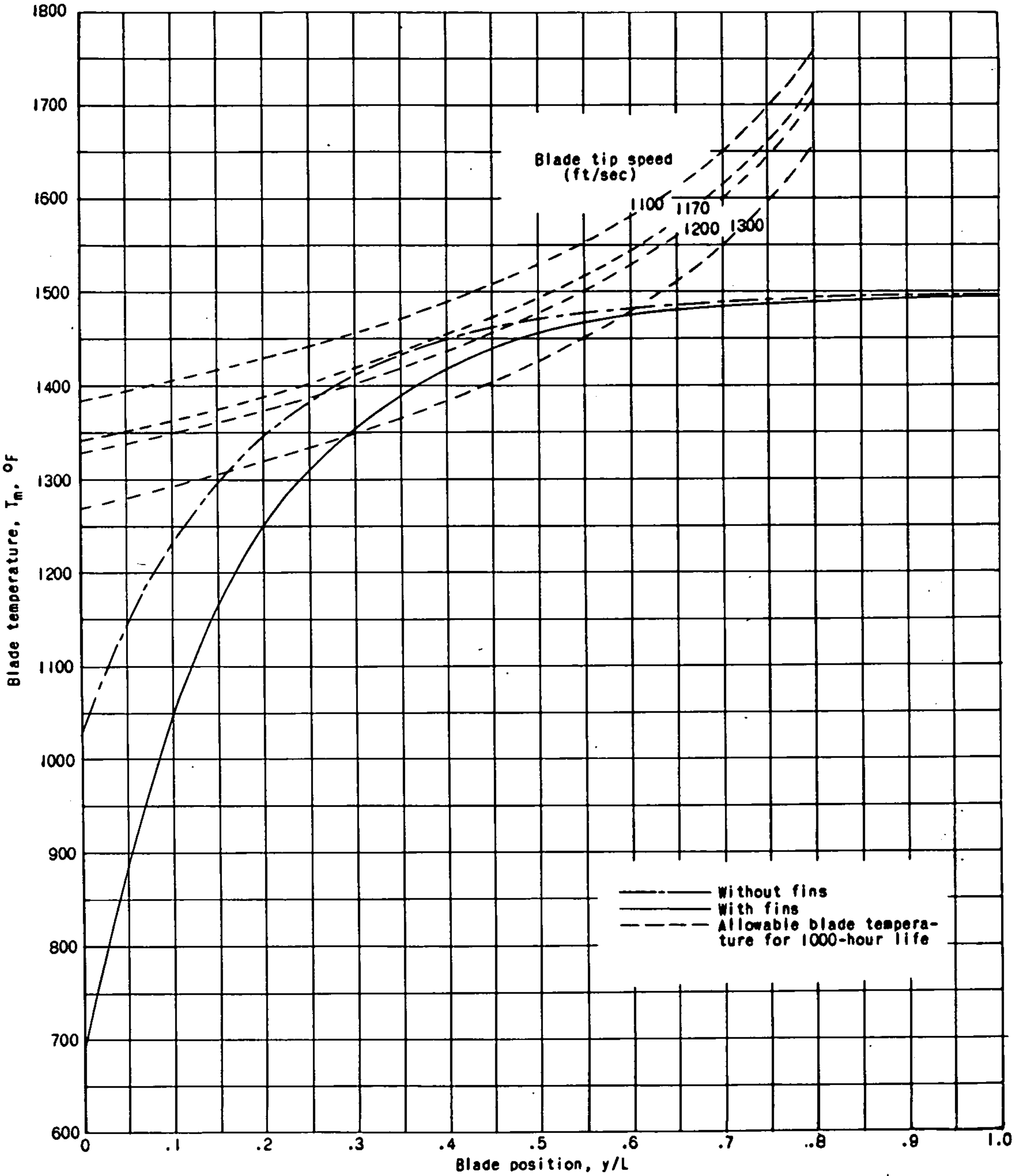


Figure 3. - Radial-temperature distribution through special turbine blade for effective gas temperature of 1500° F.

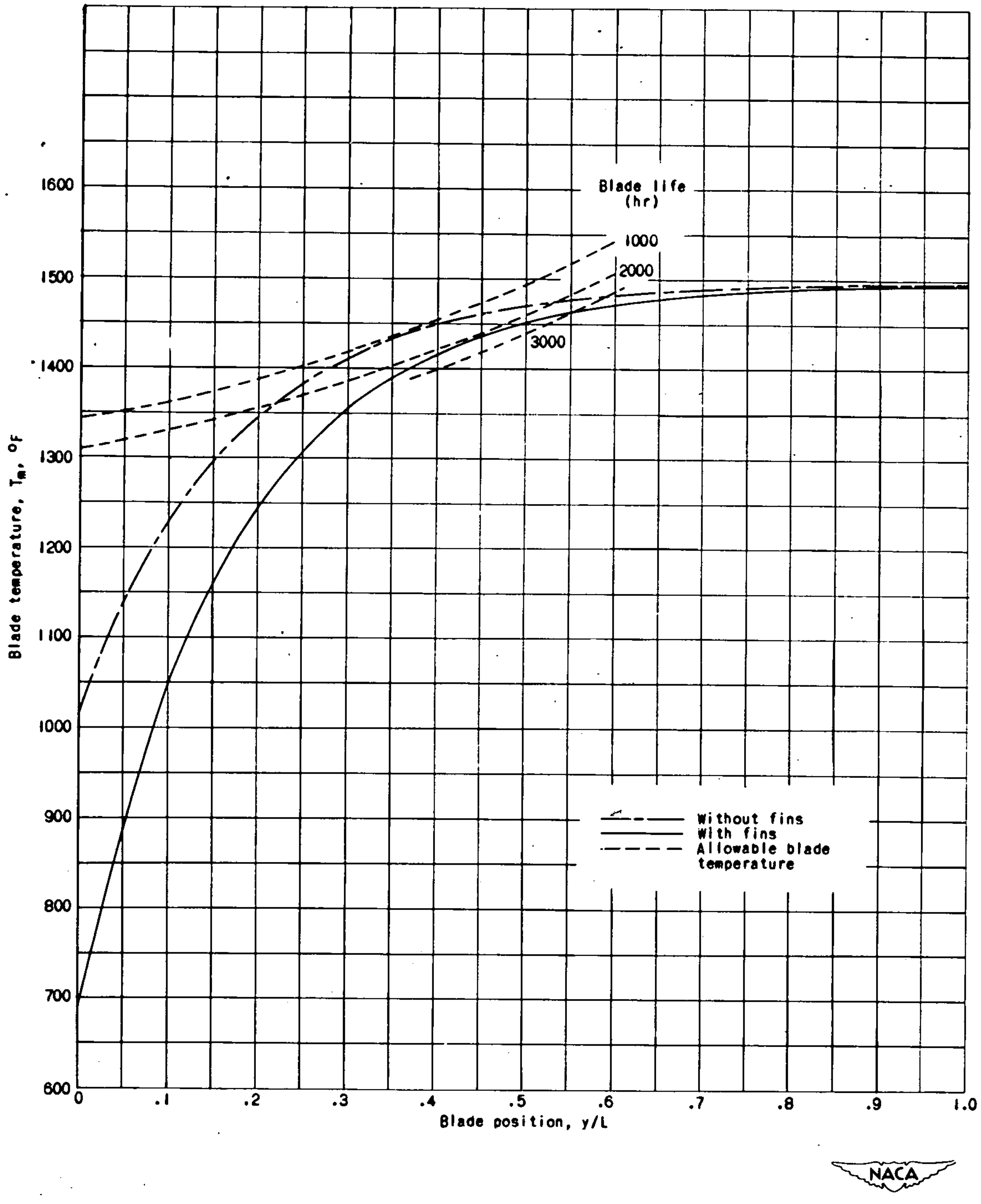


Figure 4. - Allowable increase in blade life for turbine with cooling fins on the rim for blade tip speed of 1170 feet per second and effective gas temperature of 1500° F.

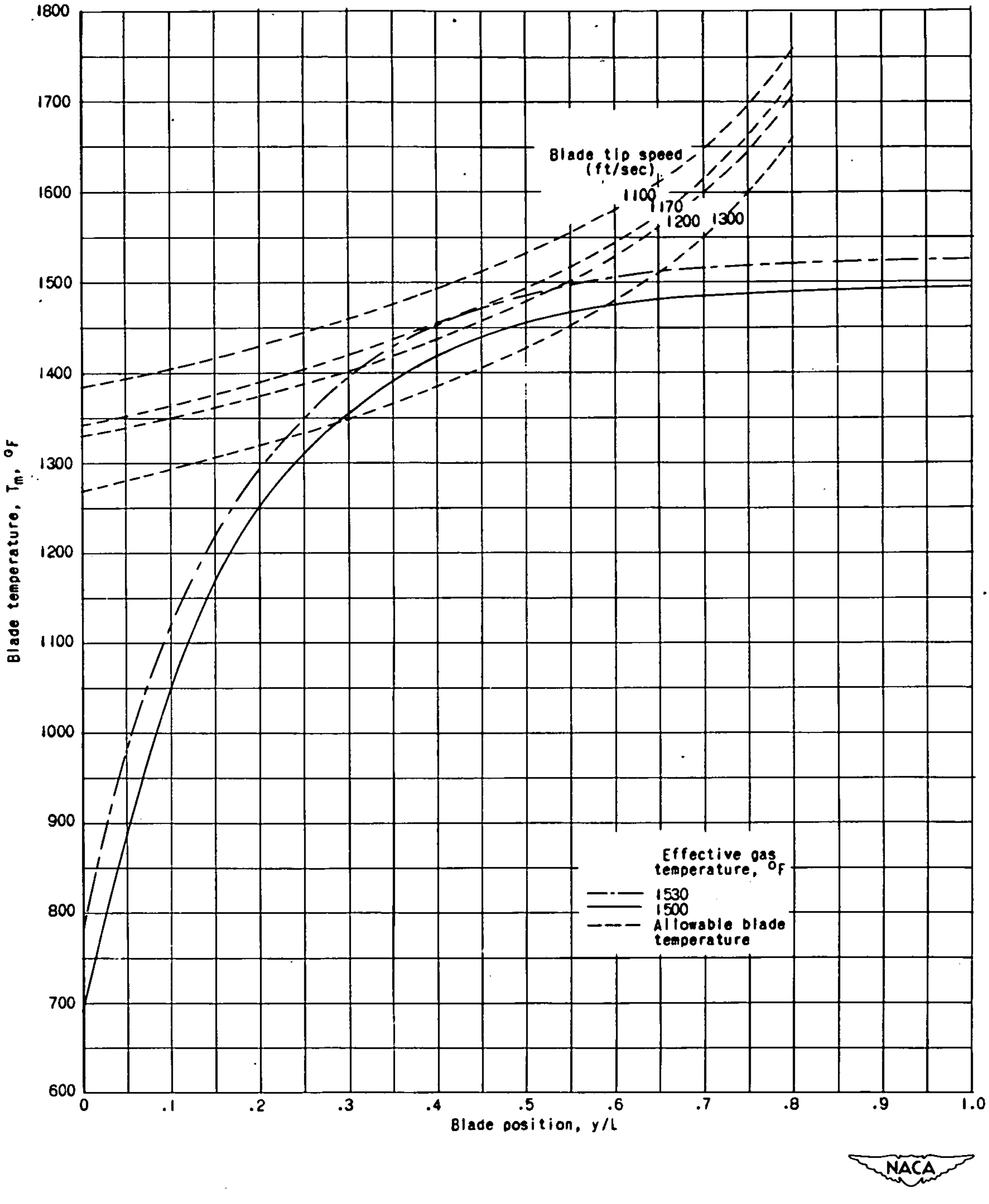
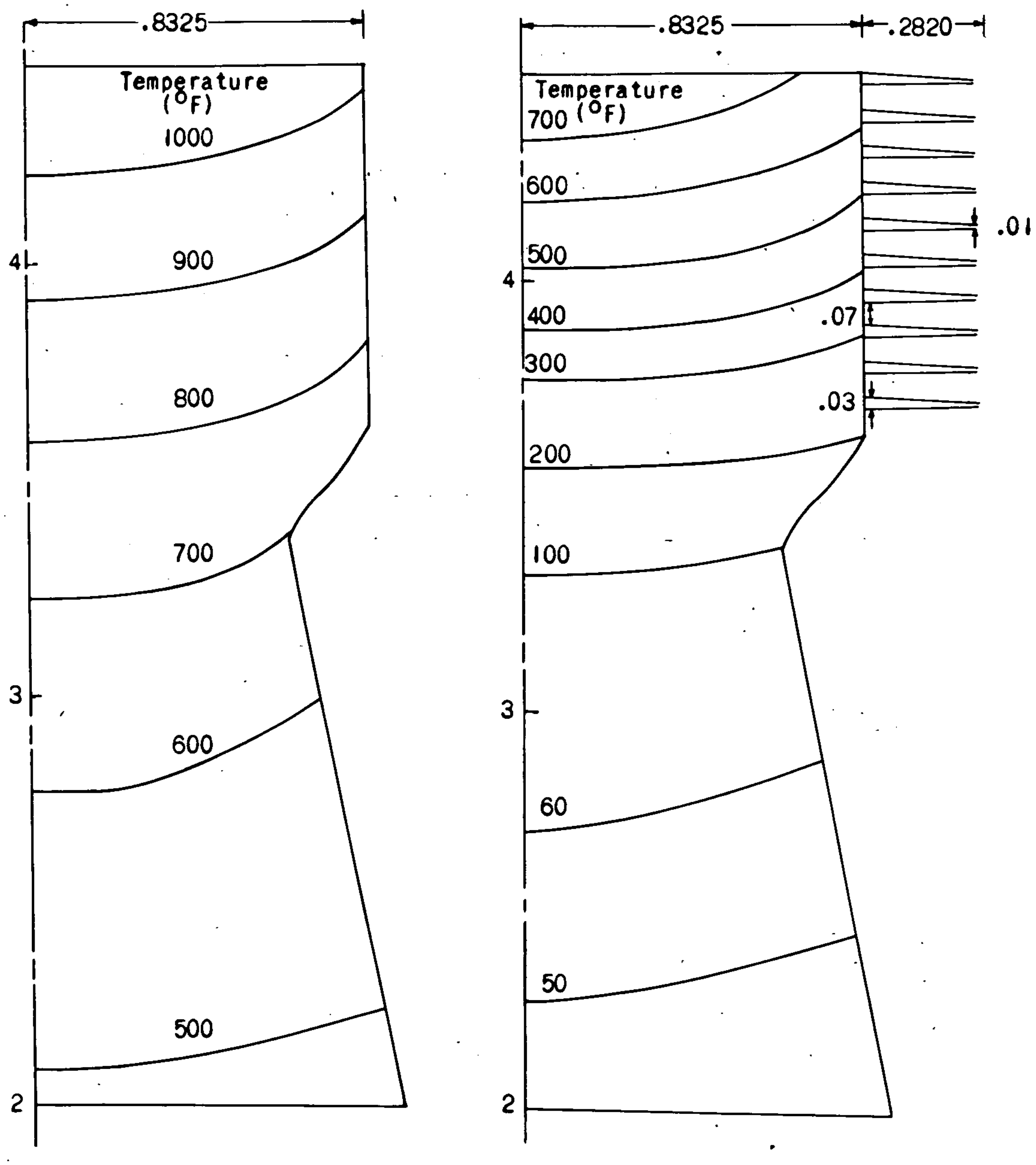


Figure 5. - Allowable increase in effective gas temperature for turbine with cooling fins on the rim. Blade tip speed, 1170 feet per second; blade life, 1000 hours.



(a) Without cooling fins on the rim.

(b) With cooling fins on the rim.

Figure 6. – Temperature distribution through cross section of turbine rim and rotor for effective gas temperature of 1500° F. (All dimensions in in.)

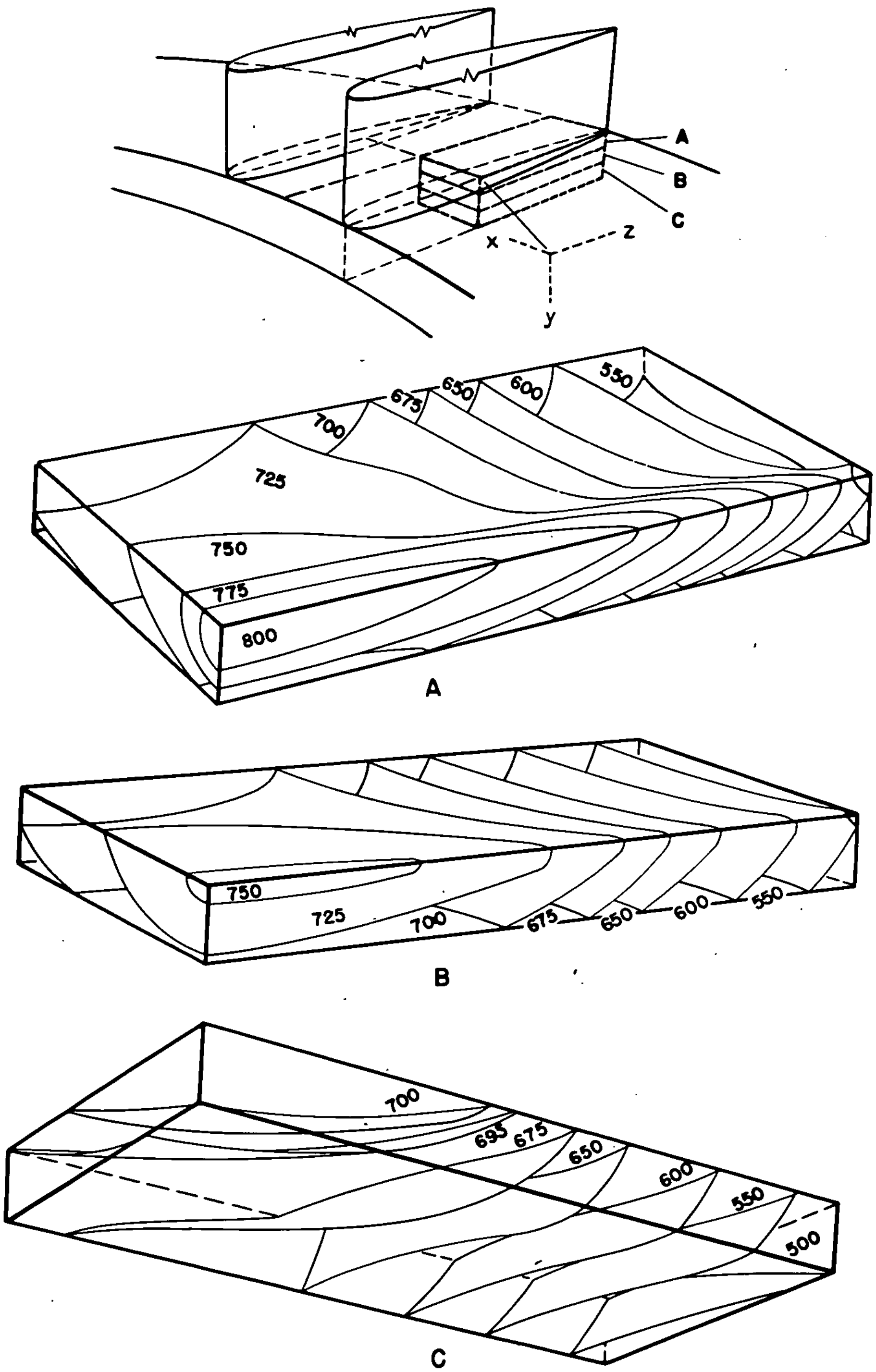


Figure 7. - Isothermal surfaces in section of turbine rim near blade root considered for three-dimensional relaxation. (Temperature in $^{\circ}$ F.)