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RESEARCH MEMORANDUM

AN ANALYSIS OF THE EFFECTS OF WING ASPECT RATIO AND TAIL LOCATION ON STATIC LONGITUDINAL STABILITY BELOW THE MACH NUMBER OF LIFT DIVERGENCE

By

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SUMMARY

An analysis is made of the influence of wing aspect ratio and tail location on the effects of compressibility on static longitudinal stability. The use of reduced wing aspect ratios or short tail lengths is shown to lead to serious reductions in high-speed stability and the possibility of high-speed instability.

INTRODUCTION

High-speed airplanes generally exhibit a reduction in static longitudinal stability as the Mach number is increased up to the Mach number of lift divergence. The trend toward the use of wings of reduced aspect ratio and high critical Mach numbers for high-speed airplanes attaches increasing importance to the problem of obtaining satisfactory stability at both low and high speeds. The present analysis illustrates how changes in wing aspect ratio and tail location affect the static longitudinal stability of airplanes throughout the Mach number range below the Mach number for lift divergence.

SYMBOLS

The symbols used in the analysis are defined as follows:

C_m pitching-moment coefficient $\left(\frac{\text{pitching moment}}{q\,S\,c}\right)$

C_L lift coefficient $\left(\frac{\text{lift}}{qS}\right)$

q	free-stream dynamic pressure $(\frac{1}{2}\rho V^2)$, pounds per square foot
ρ	free-stream mass density, slugs per cubic foot
V	free-stream velocity, feet per second
S	surface area, square feet
c	mean aerodynamic chord, feet
c_{av}	average chord, feet
α	angle of attack, degrees
ϵ	downwash angle, degrees
M	free-stream Mach number
β	$\sqrt{1-M^2}$
A	aspect ratio $\left(\frac{b^2}{S}\right)$
b	span, feet
l_t	longitudinal distance between the quarter-chord points of the mean aerodynamic chords of the wing and the horizontal tail, feet
h_t	vertical distance between the planes of the mean aerodynamic chords of the wing and the horizontal tail, feet
a	lift-curve slope
η	airfoil efficiency factor (approximately 0.9)
K	constant of proportionality

Subscripts

i	incompressible
c	compressible
o	two dimensional
w	wing
t	horizontal tail

DISCUSSION

Lift-curve Slope

Before considering static longitudinal stability, the effects of compressibility on lift-curve slope will be reviewed. By the use of potential theory the following expression is derived in reference 1 for the effect of compressibility on the lift-curve slope of a wing of finite aspect ratio

$$\frac{a_c}{a_i} = \frac{\pi A + a_{i_o}}{\beta \pi A + a_{i_o}} \tag{1}$$

A similar expression in terms of a_i, the incompressible lift-curve slope for a finite aspect ratio, instead of the two-dimensional incompressible lift-curve slope a_{i_o} may be derived using the the Glauert factor for the effect of compressibility on the two-dimensional lift-curve slope

$$a_{c_o} = \frac{a_{i_o}}{\beta}$$

and the fundamental relation

$$\frac{1}{a_i} = \frac{1}{a_{i_o}} + \frac{1}{\pi A}$$

The preceding expression may be readily reduced to

$$a_i = \frac{a_{i_o}}{1 + \dfrac{a_{i_o}}{\pi A}}$$

and

$$a_{i_o} = \frac{a_i}{1 - \frac{a_i}{\pi A}}$$

Rewriting for the compressible case,

$$a_c = \frac{a_{c_o}}{1 + \frac{a_{c_o}}{\pi A}} = \frac{a_{i_o}/\beta}{1 + \frac{a_{i_o}}{\beta \pi A}} = \frac{a_{i_o}}{\beta + \frac{a_{i_o}}{\pi A}}$$

$$= \left(\frac{a_i}{1 - \frac{a_i}{\pi A}}\right) \left(\frac{1}{\beta + \frac{a_i}{\pi A - a_i}}\right) = \frac{\pi A \; a_i}{\beta \pi A + a_i (1-\beta)}$$

from which

$$\frac{a_c}{a_i} = \frac{\pi A}{\beta \pi A + a_i \; (1-\beta)} \tag{2}$$

Figure 1 presents the calculated variation of a_c/a_i with Mach number for several aspect ratios, the two-dimensional incompressible lift-curve slope a_{i_o} being assumed to have a value of $2\pi\eta$ where η equals 0.9. For a rigorous analysis of the variation with Mach number of the lift-curve slope of a complete airplane, the interference and end-plate effects of fuselage and nacelles must also be considered. However, in most cases, the variation of a_c/a_i indicated by equation (2) agrees reasonably well with experimental results for wings at Mach numbers below that for lift divergence.

Static Longitudinal Stability

Considering only the pitching-moment components of the wing and horizontal tail, the static longitudinal stability of an airplane may be expressed as

$$-\frac{dC_m}{dC_L} = -\frac{dC_{m_w}}{dC_L} - \frac{dC_{m_t}}{dC_L}\frac{S_t}{S_w}\frac{c_t}{c_w} + \frac{S_t}{S_w}\frac{l_t}{c_w}\frac{a_t}{a_w}\left(1 - \frac{d\epsilon}{d\alpha}\right) \qquad (3)$$

with the center of gravity assumed at the aerodynamic center of the wing and the dynamic pressures at the wing and tail assumed equal. For other locations of the center of gravity, the equation should include another term expressing the ratio of the distance between the aerodynamic center of the wing and the center of gravity to the mean aerodynamic chord. This ratio is constant for any fixed location of the center of gravity and is independent of Mach number. For convenience, it is omitted from this analysis. The changes in the first two terms of equation (3) due to compressibility effects below the Mach number of lift divergence can be neglected as discussed in references 2 and 3. Therefore, the changes in stability due to compressibility effects may be analyzed by considering the third term:

$$-\left(\frac{\partial C_m}{\partial C_L}\right)_i = \frac{S_t}{S_w}\frac{l_t}{c_w}\frac{a_{t_i}}{a_{w_i}}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right) \qquad (4)$$

The terms in equation (4) affected by compressibility are the tail and wing lift-curve slopes and the rate of change of downwash with angle of attack. Changes in the tail and wing lift-curve slopes may be accounted for by equation (2). The changes in the downwash are assumed proportional to the changes in the wing lift-curve slope throughout the range of subcritical Mach numbers. Equation (4) may now be written as follows:

$$-\left(\frac{\partial C_m}{\partial C_L}\right)_M = \frac{S_t l_t a_{t_i}}{S_w c_w a_{w_i}}\frac{(a_c/a_i)_t}{(a_c/a_i)_w}\left[1 - \frac{\partial\epsilon}{\partial\alpha}\left(\frac{a_c}{a_i}\right)_w\right] \qquad (5)$$

To illustrate the manner in which wing aspect ratio and tail location effect the variation of static longitudinal stability with Mach number, several hypothetical airplanes will be considered under the following conditions:

1. The wings are unswept and have a taper ratio of 2 so that the downwash predictions of reference 4 may be used.

2. The lift distributions are assumed to be elliptical.

3. The angles of attack are assumed to be within the range of linear variation of lift coefficient with angle of attack.

4. The aspect ratio of the horizontal tails is fixed.

The results of figures 4, 5, and 6 of reference 4 indicate the following relation between downwash and wing aspect ratio for various tail locations expressed in terms of the wing semispan:

$$\left(\frac{\partial\epsilon}{\partial C_L}\right)_i = \frac{K}{A}$$

The constant of proportionality has the following values:

	K		
$\frac{l_t}{b/2}$	0.6	0.9	1.2
$\frac{h_t}{b/2} = 0.1$	44	41	39
$\frac{h_t}{b/2} = 0.2$	38	36	33

The factor $\partial\epsilon/\partial\alpha$ in equation (4) may be obtained in terms of the aspect ratio

$$\left(\frac{\partial\epsilon}{\partial\alpha}\right)_i = \left(\frac{\partial\epsilon}{\partial C_L}\right)_i \left(\frac{\partial C_L}{\partial\alpha}\right)_i = \frac{K}{A}\left(\frac{2\pi\eta}{1+\frac{2\eta}{A}}\right)\left(\frac{1}{57.3}\right) = \frac{0.0986K}{A+1.8}$$

With increasing Mach number below that for lift divergence, the factor $\partial\epsilon/\partial C_L$ may be regarded as constant, but the wing lift-curve slope $\partial C_L/\partial\alpha$ increases according to equation (2). Therefore, the downwash parameter at a given Mach number is:

$$\left(\frac{\partial\epsilon}{\partial\alpha}\right)_M = \left(\frac{\partial\epsilon}{\partial C_L}\right)_M \left(\frac{\partial C_L}{\partial\alpha}\right)_M = \left(\frac{\partial\epsilon}{\partial\alpha}\right)_i \left(\frac{a_c}{a_i}\right)$$

where the quantity a_c/a_i may be taken from figure 1. The variations of $\partial\epsilon/\partial\alpha$ with wing aspect ratio for zero and 0.9 Mach numbers are shown in figure 2 for three tail locations; two expressed in terms of the semispan of the wing and one defined as a constant linear distance assuming constant wing area. When $\partial\epsilon/\partial\alpha$ exceeds unity, the horizontal tail becomes destabilizing. It may be seen in figure 2 that an increase of Mach number or a reduction in wing aspect ratio or tail length increases $\partial\epsilon/\partial\alpha$.

The variation with wing aspect ratio of the quantity $\frac{1-(\partial\epsilon/\partial\alpha)}{a_w}$ appearing in equation (4) is shown in figure 3(a). Figure 3(b) presents the ratio of the value of this factor at 0.9 Mach number to its value at zero Mach number. It can be seen that reducing the wing aspect ratio or the tail length decreases the ratio.

Using the expression for $\partial\epsilon/\partial\alpha$ in terms of wing aspect ratio, the following relation for the mean aerodynamic chord of wings having a 2:1 taper ratio,

$$c = 1.037\, c_{av} = 1.037\sqrt{S/A}$$

and assuming an aspect ratio of 4 for the horizontal tail, and a two-dimensional, incompressible lift-curve slope of 1.8π equation (5), may be written

$$-\left(\frac{\partial C_M}{\partial C_L}\right)_M = \frac{S_t\, l_t\, (a_o/a_i)_t}{1.50\, S_w^{3/2}\, A^{1/2}\, (a_o/a_i)_w}\left[A + 1.8 - 0.0986K\, (a_o/a_i)_w\right] \tag{6}$$

The values of this expression at low speed may be made equal for the assumed combinations of wing aspect ratio and tail location by the appropriate choice of tail area. The calculated reductions in the components of stability from the tail with increasing Mach number for various wing aspect ratios and tail positions is shown in figure 4. The results indicate that at Mach numbers approaching that for lift divergence, the stabilizing action of the horizontal tail is greatly reduced. The reduction becomes more pronounced as the wing aspect ratio, tail length, or tail height above the plane of the wing are reduced. Thus, the use of short tail lengths or wings of low aspect ratio complicates the problem of obtaining adequate static longitudinal stability over a wide range of Mach numbers and increases the possibility of excessive stability at low speed or instability at high Mach numbers approaching that for lift divergence.

The theoretical results are compared in figure 5 with experimental results from wind-tunnel tests of models of two conventional, single-engine, low-wing, fighter-type airplanes. The predicted changes in the components of stability due to the horizontal tail agree reasonably well with the experimental results and also provide an indication of the changes in the static longitudinal stability of the complete models.

The analysis has involved unswept wings, but it would be possible to make a similar study of swept wings, some downwash results being available in reference 5 from low-speed tests of small-scale swept-back wings. The reductions in static longitudinal stability have been shown to occur below the Mach number of lift divergence. In the subsonic Mach number range above the Mach number for lift divergence, an increase in static longitudinal stability generally occurs as discussed in references 2 and 3.

CONCLUSIONS

The preceding analysis of the effects of wing aspect ratio and tail location on static longitudinal stability indicates the following:

1. Although a smaller variation of wing lift-curve slope with Mach number occurs when the wing aspect ratio of an airplane is reduced, there results a greater reduction in static longitudinal stability in the range of Mach numbers below that for lift divergence owing to compressibility effects on the downwash.

2. As the tail length of an airplane is reduced, the effects of compressibility on the static longitudinal stability become more pronounced.

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REFERENCES

1. Young, A. D.: Note on the Effect of Compressibility on the Lift Curve Slope of a Wing of Finite Span. Rep. No. Tech. Note Aero 1250, R.A.E. (British/U.S. Restricted), Aug. 1943.

2. Hood, Manley J., and Allen, H. Julian: The Problem of Longitudinal Stability and Control at High Speeds. NACA Rep. No. 767, 1943.

3. Axelson, John A.: Longitudinal Stability and Control of High-Speed Airplanes with Particular Reference to Dive Recovery. NACA RRM No. A7C24, 1947.

4. Silverstein, Abe, and Katzoff, S.: Design Charts for Predicting Downwash Angles and Wake Characteristics Behind Plain and Flapped Wings. NACA Rep. No. 648, 1939.

5. Purser, Paul E., Spearman, M. Leroy, and Bates, William R.: Preliminary Investigation at Low Speed of Downwash Characteristics of Small-Scale Swept-back Wings. NACA TN No. 1378, 1947.

$$\frac{a_c}{a_i} = \frac{\pi A}{\beta \pi A + a_i (1-\beta)} = \frac{A+1.8}{\beta A + 1.8}$$

$$a_{i_0} = 1.8\pi$$

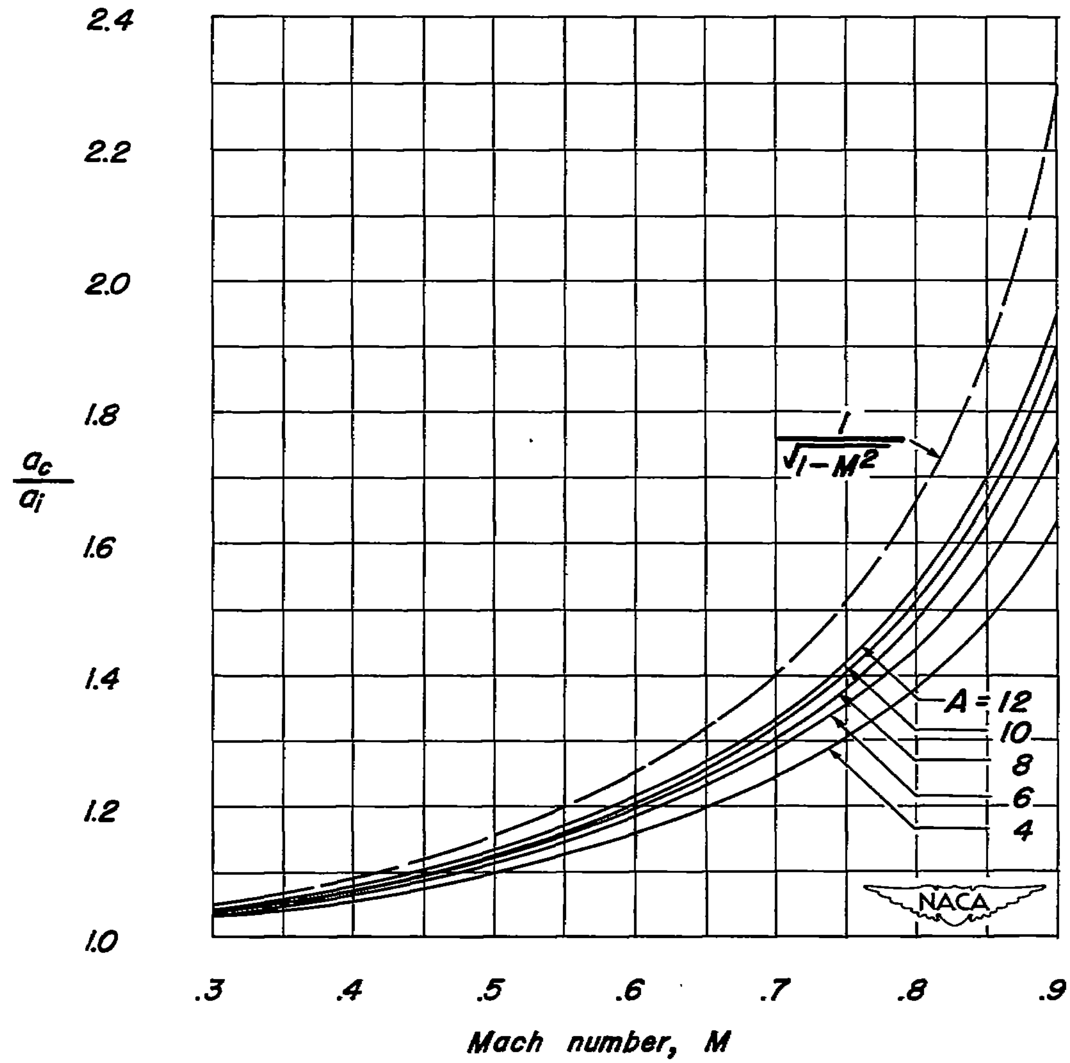

Figure 1. - Variation of lift-curve slope with Mach number for various aspect ratios.

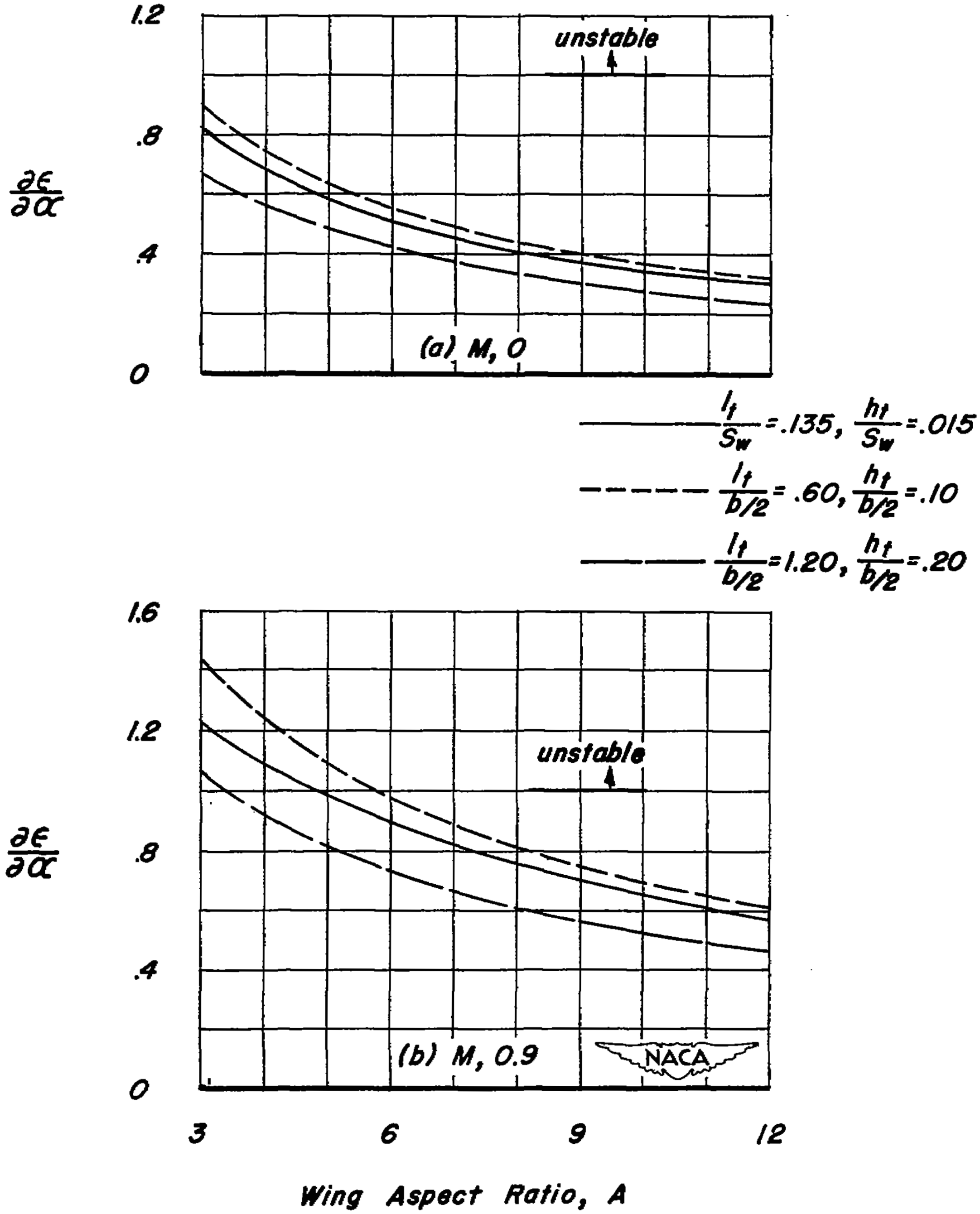

Figure 2.- Effects of tail location and Mach number on the variation of the downwash parameter $\frac{\partial \epsilon}{\partial \alpha}$ with wing aspect ratio.

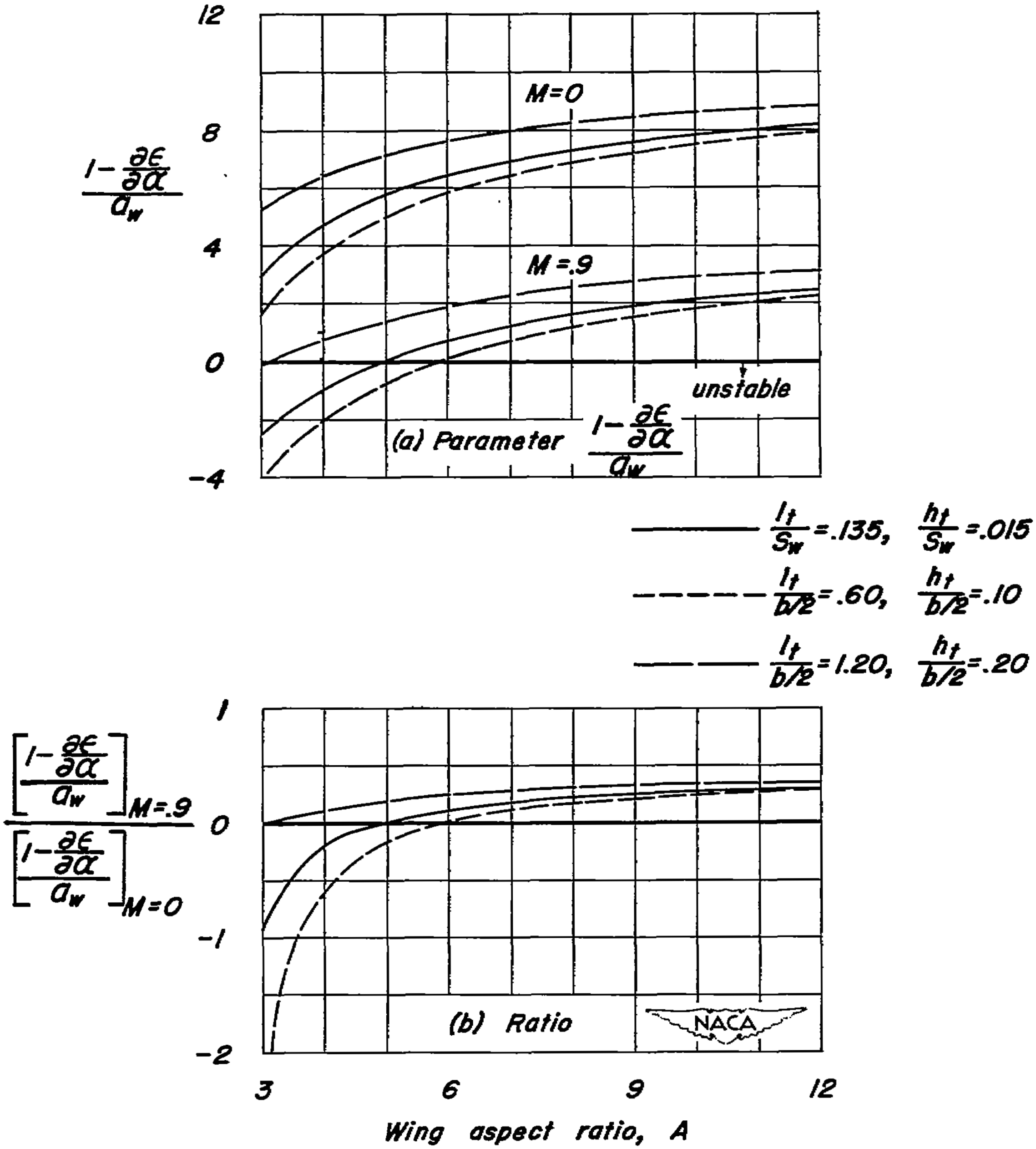

Figure 3.— Variation with wing aspect ratio of the parameter $\dfrac{1-\frac{\partial\epsilon}{\partial\alpha}}{a_w}$ *and the ratio of the high-speed to low-speed values for three tail locations.*

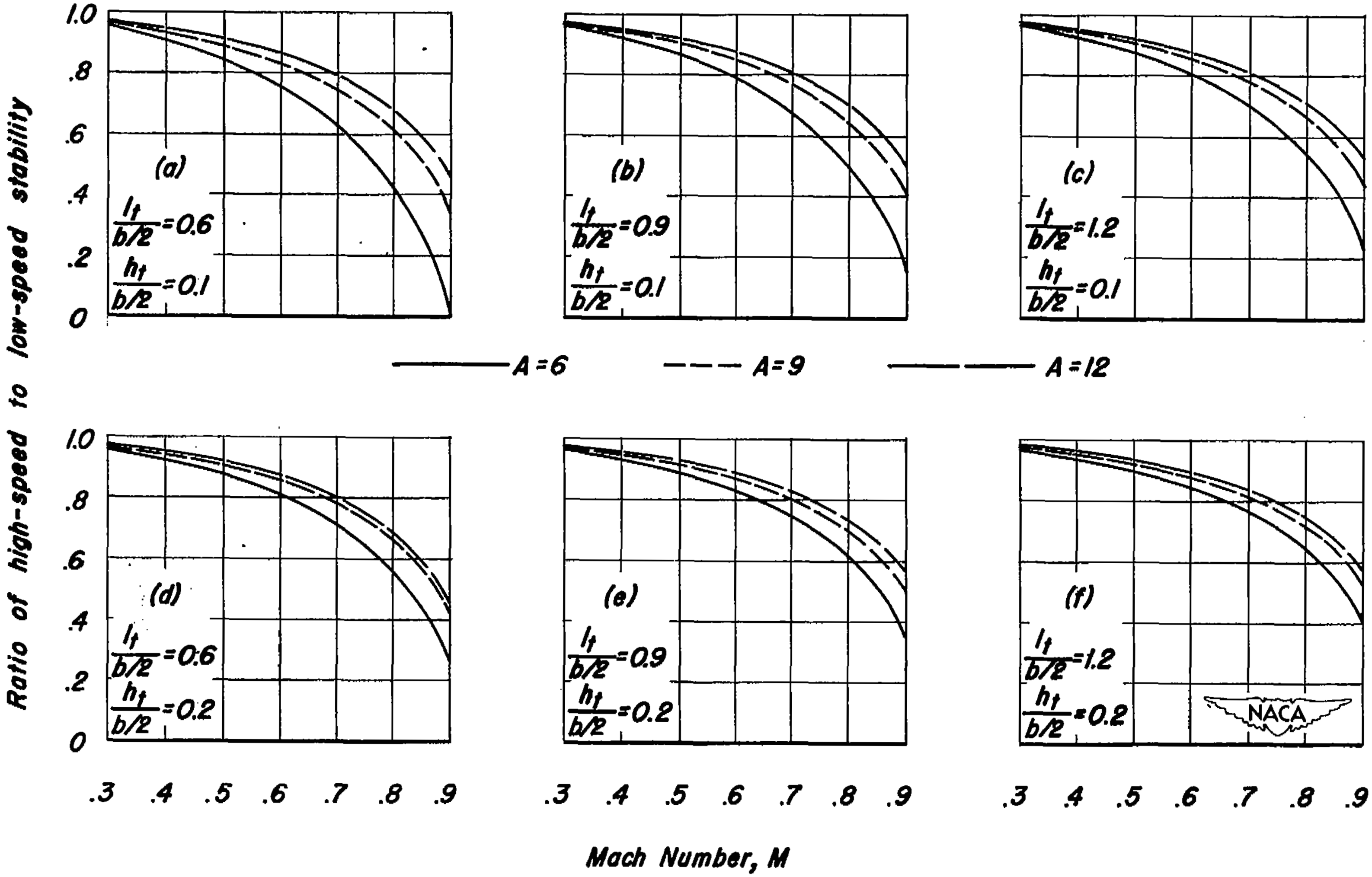

Figure 4. – Effects of tail location and wing aspect ratio on the variation with Mach number of the component of static longitudinal stability due to the tail for several hypothetical airplanes; tail aspect ratio, 4.

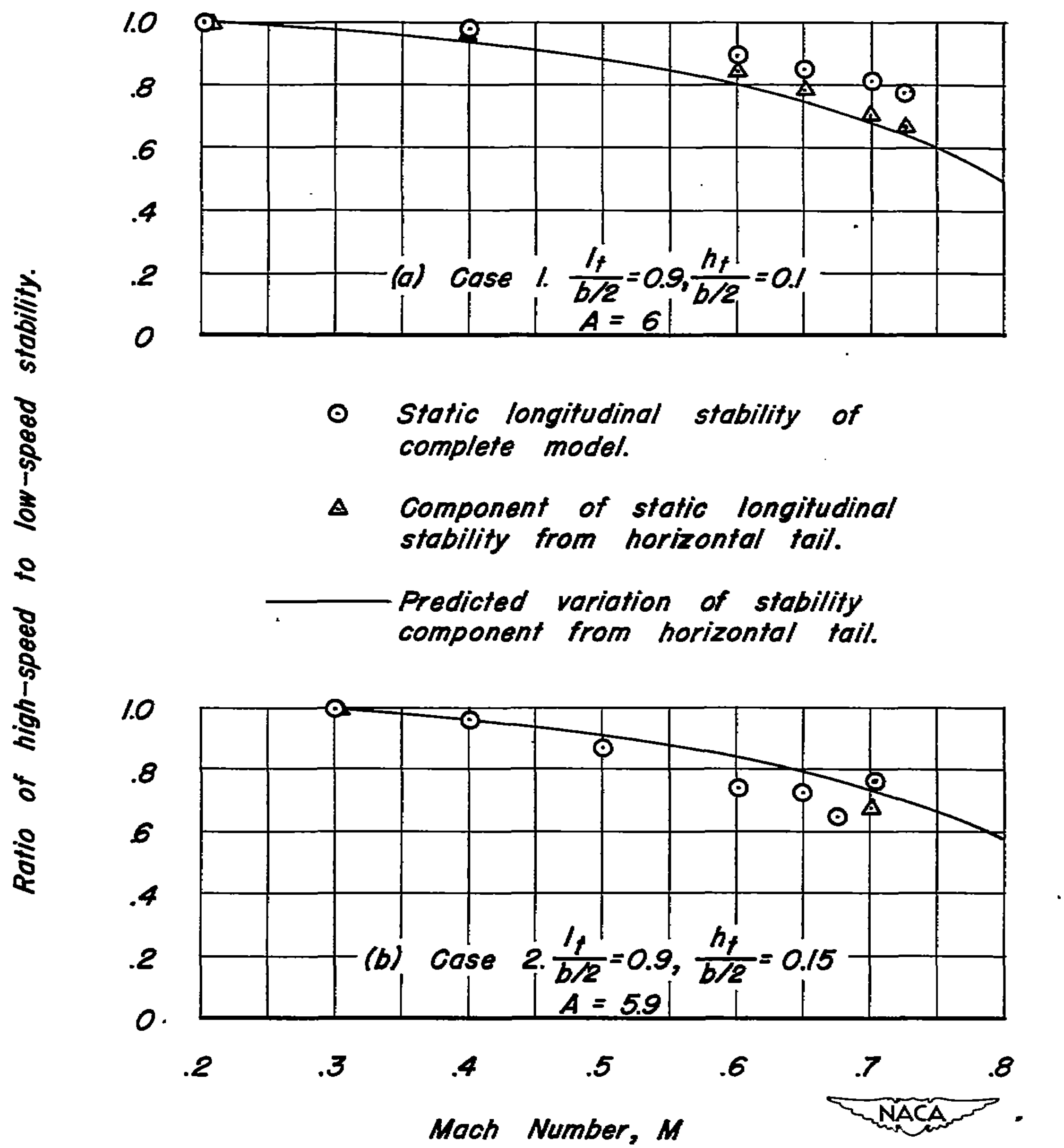

Figure 5.—Comparison of experimental results from wind-tunnel tests with predicted variation of static longitudinal stability with Mach number.