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RESEARCH MEMORANDUM

PRELIMINARY ANALYSIS OF PROBLEM OF DETERMINING EXPERIMENTAL PERFORMANCE OF AIR-COOLED TURBINE

III - METHODS FOR DETERMINING POWER AND EFFICIENCY

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Auth. J. W. Crowley Date 12/14/53
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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WASHINGTON

August 2, 1950

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SUMMARY

Methods are presented for determining such air-cooled turbine-performance characteristics as power, efficiency, and so forth from experimental data obtained from specific investigations. Suggested formulas are given for determining the characteristics as functions of certain parameters; the functions are generally unknown. Methods for determining the functions from the experimental investigations are suggested. Special plotting methods for isolating the effect of each parameter are outlined. These proposed methods constitute only one way of analyzing the results of the turbine investigations. All the analysis is unchecked by actual investigations and is presented solely as a guide to others in the field.

INTRODUCTION

At present, although much effort by research, design, and development groups is being concentrated on the application of air cooling to turbines, very little published material is available. A series of reports on the problem of determining the experimental performance of air-cooled turbines has therefore been prepared at the NACA Lewis laboratory. These reports should not be considered a final treatise on the subject, but rather a basis upon which work may proceed.

Formulas that are required for the evaluation of the heat-transfer and the cooling-air flow characteristics are suggested in references 1 and 2, respectively. These suggested formulas are so arranged that certain dependent parameters are functions of independent parameters. Suggested methods of experimental investigation to determine the unknown functions are presented.

This report, the last of the series, presents suggested formulas, based on present knowledge, for air-cooled turbine-performance characteristics such as power, power losses, and efficiency as functions of certain parameters. Power and efficiency formulas for the uncooled turbine are discussed in order to establish a basis for the discussion of the additional characteristics that are involved in the evaluation of the cooled turbine. Methods for analyzing experimental data in order to evaluate these functions are described. 1328

Methods for establishing the effects of the parameters are by no means restricted to those outlined herein. Improvements in techniques are expected as data on air-cooled turbines are accumulated. Because of the urgent need for published material on some feasible methods, time has not been taken to develop other, and possibly better, methods for analyzing the turbine results.

The formulas for blade power and efficiency of uncooled turbines as functions of several parameters, used herein, are based on unpublished derivations. Acknowledgement is made to Mr. Arthur W. Goldstein of the Lewis staff, who made the original derivations.

ANALYSIS

Typical Flow Path of Turbine-Blade Cooling Air

A clear concept of the path the blade cooling air follows through the engine is advantageous prior to the actual analysis of the performance characteristics of a cooled turbine. The typical flow path, illustrated in figure 1, is only one of several that could be used. All other arrangements, however, would involve similar problems.

In the arrangement shown in figure 1, the cooling air is bled from some stage of the compressor. Air for cooling the rotor blades is ducted to a point near the rotor center line and is pumped by a centrifugal impeller, which is attached to the upstream face of the turbine disk, through individual holes at the base of each blade. The air passes through the blades and out the tips and then mixes with the combustion gases. The pressure rise through the centrifugal impeller plus the compressor pressure at the bleed point must satisfy the pressure requirements at the blade roots and insure positive flow through the blades. If, in satisfying these pressure requirements, the temperature at the blade roots is too high for adequate heat transfer, then a heat exchanger should be placed in the cooling-air system. The cooling air for the stator blades is handled in a similar manner although the bleedoff point on the compressor may vary from that of the rotor blades; in figure 1, air is shown being bled off at the same point. The cooling air passing through the stators can be directed around the tail pipe and cool this engine part before being discharged. Other arrangements may be possible for reintroducing this air into the combustion-gas stream downstream of the rotor blades.

The turbine station numbers used herein are shown in figure 1. Station 4 is the position where the combustion gas and the cooling air issuing from the rotor blades have been thoroughly mixed.

Temperature-Entropy Diagrams of Turbojet Engine with Cooled Blades

The power formulas and power components presented subsequently are more easily understood if a study is made of the temperature-entropy diagrams of turbojet engines. The cycle, which is the basic ideal cycle of almost all jet-propulsion systems, is composed of four phases: isentropic compression, constant-pressure addition of heat, isentropic expansion, and constant-pressure rejection of heat. This cycle, called the Brayton cycle, is thermodynamically illustrated by an ideal temperature-entropy diagram in figure 2(a). All cycle diagrams given in figure 2 are based on 1 pound of the working fluid.

Point B in figure 2(a) represents conditions at the engine inlet. The air is isentropically compressed both in the inlet diffuser and in the compressor to reach state point C in the diagram. The air then enters the combustion chambers and addition of heat at constant pressure occurs until state point 1 (which corresponds to station 1 at turbine inlet in fig. 1) is reached. The combustion gas then isentropically expands through the turbine to reach state point 3 (corresponds to station 3 in fig. 1). When no change in conditions between stations 3 and 4 is assumed (fig. 1), state point 4 (fig. 2(a)) is the same as state point 3. The gas then expands isentropically from station 4 to station 5 (fig. 1) to reach state point 5, which is at atmospheric pressure. Heat rejection at constant pressure occurs along the line from 5 to B.

The heat supplied is represented by area AC1D; the heat rejected, by area AB5D; and the heat available for mechanical work by the engine to provide thrust, by the area BC15 in figure 2(a). The ideal work of the turbine is represented by the expansion from state point 1 to state point 3.

The actual cycle for an engine with uncooled turbine blades is illustrated in figure 2(b). The compressor and the turbine, because of inefficiencies, do not isentropically compress and expand the fluid; consequently, state points C and 3 have higher entropies than in the ideal cycle. In figure 2(b), the heat addition in the combustion chambers is assumed to occur at constant pressure and state point 4 is assumed to be the same as state point 3. These are rational assumptions and their use will not detract from the validity of the methods and formulas presented herein.

The heat supplied is the area under line C1, but the heat rejected is increased by the area ABB'A', which represents the compressor inefficiency, and by the area D'5'5D, which represents the turbine inefficiency (reference 3). The heat convertible to mechanical work for thrust purposes is then the difference between area A'C1D' and the area AB5D, which is the total heat rejected. The turbine work or the reaction work of the combustion gases on the turbine wheel, which results from the expansion between state points 1 and 3 in figure 2(b), is called the blade power P_B. This power is less than the ideal power P_{id}, which is the result of expansion between state points 1 and 4' in figure 2(b).

For the case of the engine with cooled turbine blades, two diagrams must be considered: one for the air passing through the engine in the customary manner (that is, flows across the turbine blades) and another for the air that is bled off from the compressor, passed through the rotor blades for cooling purposes, and then reintroduced into the main air stream. Figure 2(c) illustrates the cycle for the air passing through the engine in the customary manner and figure 2(d) is applicable to the rotor-blade cooling air.

The diagram shown in figure 2(c) for the working fluid is the same as that in figure 2(b) for the uncooled turbine up to state point 1. During the expansion process through the turbine, however, heat is given up to the rotor blades, which are cooled, and to the stators if they are also cooled. This heat extraction causes the expansion line to be as shown, state point 1 to state point 3, instead of that of the uncooled turbine indicated by the dashed line from state point 1 to state point 3' in figure 2(c). One theory, however, is that the expansion from state point 1 to state point 3 for the cooled turbine is along the same line as for the uncooled turbine but drops to the same pressure level as for the case shown here. The heat extraction occurring in the stator and rotor blades is believed to affect only the boundary layer around the blades and not the main gas stream. Downstream of station 3, mixing of the working

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fluid with the cooling air issuing from the rotor blades and with the cooling air from the stators if it is reintroduced into the main body of fluid occurs at constant pressure causing a cooling of the working fluid. This mixing is indicated by the line between state points 3 and 4 in figure 2(c). State point 4 to state point 5 indicates expansion of the working fluid in the jet nozzle as in figure 2(b).

If state point 3 could be accurately determined in figure 2(c), which is the state before the working fluid mixes with the cooling air, then the blade power P_B provided by the working fluid could be determined by the expansion from state point 1 to state point 3. Experimentally measuring this state, however, is very difficult. State point 3 in figure 2(c) is at a lower pressure than state point 3', which is the condition that would prevail in the uncooled turbine. This greater expansion is required across the cooled turbine because of the heat-transfer effects and the power required to pump the cooling air.

The cooling air, bled off the compressor for the rotor blades, is compressed in the inlet diffuser and partly in the compressor in the same manner as the working fluid. This compression is indicated by the line BC' in figure 2(d). The pressure at C', however, is usually lower than the pressure at C (fig. 2(c)) because the cooling air does not usually have to be compressed to as high a pressure as the working fluid. State point C' at the compressor bleedoff is assumed to be applicable to the point of admission of the air at the rotor cooling-air inlet near the center line of the shaft (fig. 1). The losses in ducting between the compressor and the rotor are neglected. Compression of the air occurs in the impeller passage on the upstream face of the rotor disk, which is indicated by the change in state from C' to h (blade-root state point) in figure 2(d). Near the blade root, the pressure and the temperature of the cooling air both increase, but near the blade tip the pressure usually decreases while the temperature continues to increase. These changes are indicated by the state line between points h and T, or 3, in figure 2(d). The heat that is given up by the working fluid and picked up by the cooling air is reflected in the shape of this line. The cooling air at the blade tip, state point T or 3, has the same pressure as the working fluid at station 3. Mixing of the cooling air with the working fluid then occurs at constant pressure as in figure 2(c), which causes the cooling air to heat up along the line between state point 3 and state point 4 in figure 2(d). The conditions at 4 in figure 2(d) are the same as those at the same point in figure 2(c). Expansion of the cooling air as a mixture with the working fluid then occurs from state point 4 to state point 5, which corresponds to the expansion in the jet nozzle.

In order to determine the heat available for providing thrust for a cooled turbine, consideration of figures 2(c) and 2(d) is necessary. The heat supplied will be the area A'ClD' (fig. 2(c)) multiplied by the weight flow of the working fluid. The heat rejected will be the weight flow of working fluid (combustion gases) multiplied by the area AB543D (fig. 2(c)) plus the product of the weight flow of cooling air through the rotor blades and the area AB5D (fig. 2(d)). Then the available thrust will be the difference between the total heat supplied and the total heat rejected. If, however, the stators are cooled, there will be an additional heat rejection that will further reduce the available thrust for a given cycle. 1328

Because only the turbine component is considered herein, the total pumping power is defined as the energy required to pump the rotor-blade cooling air from the point near the shaft where it enters the turbine rotor to the tip of the blade, that is, between state points C' and T or 3 (fig. 2(d)); ducting losses are neglected.

From the foregoing discussion of the temperature-entropy diagrams, the net power required from the turbine to compress the working fluid (neglecting mechanical losses, which are discussed later) is the difference between the blade power P_B and the power required to pump the cooling air through the blades P_P. The total pumping power $P_P + P_{PR}$ considered herein is only that power extraction required to transfer the coolant from the entrance to the turbine rotor to the blade tip and the effect of bleedoff of compressor air for cooling on engine performance is not discussed. (All symbols are defined in general terms in the appendix.)

Cooled Turbine Power Concepts

In the foregoing discussion of the temperature-entropy diagrams, the concept of ideal and net blade power of a cooled turbine is brought out. Figure 3 is a summary in schematic form of all the power components of an air-cooled turbine as considered herein. The net blade power $P_B - P_P$, caused by the forces on both the outside and the inside of the blade, is the difference between the ideal power P_{id} and the sum of blading losses P_{SL} and P_{BL}, leakage

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losses P_{TL}, and coolant pumping losses from blade root to tip P_{aL}. The shaft power P_{sh} that is available for driving the compressor and accessories is found by subtracting all the turbine mechanical losses and the coolant pumping power from the entrance to the turbine rotor to the blade root from the net blade power.

Uncooled Turbine Power Components

Before discussing the cooled turbine, a brief review of the generation of useful power and the dissipation of power through losses in an uncooled turbine is given because these power additions and extractions determine the net output of the turbine. The review can easily be extended to account for the additional effects of other power components that are introduced when cooling is used. The following sections dealing with the uncooled turbine are based on the very complete discussions and investigations of a turbine in which a complete analysis of the useful and nonuseful work of a turbine is given (reference 4).

Ideal power. - The ideal work (fig. 3) represents the maximum attainable for expansion of gas between specified total pressures at the stator inlet and the rotor outlet. Ideal work is based on a reversible process involving no heat transfer and consequently no entropy change. The total-pressure ratio is used as a basis for ideal power and any velocity energy remaining in the gas stream after expansion is not chargeable to the turbine because it is available for producing engine thrust.

$$E_s = h'_{g,1} - h'_{g,3,s} = \Delta_s h'_g \tag{1}$$

where

E_s maximum attainable energy or ideal energy, (Btu/lb)

$h'_{g,1}$ total enthalpy of gas at stator inlet, (Btu/lb)

$h'_{g,3,s}$ final total enthalpy of gas attained at rotor outlet with the process described, (Btu/lb)

The ideal or maximum attainable energy can also be computed from

$$E_s = h'_{g,1}\left[1 - \left(\frac{p'_{g,3}}{p'_{g,1}}\right)^{\frac{\gamma_g-1}{\gamma_g}}\right] \qquad (2)$$

for a γ_g assumed constant or average across the turbine stator and rotor

where

$p'_{g,1}$ average total pressure of gas at stator inlet, (lb/sq ft)

$p'_{g,3}$ average total pressure of gas at rotor outlet, (lb/sq ft)

γ_g ratio of specific heats of combustion gas

Use of equation (2) is tedious because of the determination of γ_g required, although this problem has been somewhat simplified by the charts in reference 5. Equation (1) is recommended for uncooled turbines. A subsequent section includes methods of evaluating various factors discussed throughout the report of which $\Delta_s h'_g$ is an example.

The ideal power determined from equation (1) and the gas flow rate is

$$P_{id} = w_g \Delta_s h'_g J/550 \qquad (3)$$

where

J mechanical equivalent of heat, (ft-lb/Btu)

P_{id} ideal power, (hp)

w_g combustion-gas flow rate, (lb/sec)

Blade power. - In passing across the turbine stator and rotor blades, the ideal power is decreased because of stator- and rotor-blade losses, (friction, separation, and other viscous effects). Also, because of the clearance between the rotor-blade tips and the shroud, a rotor-tip leakage loss may occur. Thus the power delivered to the turbine wheel, hereinafter called blade power, is equal to

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$$P_B = P_{id} - P_{SL} - P_{BL} - P_{TL} \tag{4}$$

where

P_B useful power delivered by blades to rotor, (hp)

P_{BL} rotor-blade loss, (hp)

P_{SL} stator loss, (hp)

P_{TL} rotor-tip leakage loss, (hp)

When no heat loss from the shroud and other turbine parts is assumed in an uncooled turbine, the energy equation is

$$h'_{g,1} - E_B = h'_{g,3} \tag{5}$$

where

E_B mechanical energy delivered to blades, (Btu/lb)

$h'_{g,3}$ total enthalpy of gas at rotor outlet, (Btu/lb)

or

$$E_B = h'_{g,1} - h'_{g,3} = \Delta h'_g \tag{6}$$

Then

$$P_B = \frac{w_g \Delta h'_g J}{550} \tag{7}$$

Because the stator- and rotor-blade losses are practically impossible to isolate and to quantitatively evaluate from over-all performance data, these losses are considered as a single loss and are determined as the difference between the ideal power and the blade power plus the accountable losses.

The rotor-tip leakage loss, principally encountered in reaction-type turbines, can be estimated from (reference 4)

$$P_{TL} = \frac{100\ C_R w_g \Delta_s h'_g J}{\left[b\sigma \sin (180 - \beta_3) + C_R\right] 550} \qquad (8)$$

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where

b blade length, (ft)

C_R rotor-tip clearance, (ft)

β_3 outlet angle of blade relative to plane of rotor disk (fig. 4), (deg)

σ thickness coefficient (unity for most reaction turbines)

Shaft power. - Other losses occurring in a turbine cause the power delivered to the turbine shaft to be less than the blade power. These are mechanical losses and include the rotor-disk frictional drag loss and the bearing and gear losses. In an experimental setup, other accessory losses may be present such as tachometer drive loss and so forth, which are grouped as miscellaneous losses. Thus, the shaft power is

$$P_{sh} = P_B - P_{RF} - P_{BE} - P_M \qquad (9)$$

where

P_{BE} bearing and gear losses, (hp)

P_M miscellaneous losses inherent in setup, (hp)

P_{RF} rotor-disk friction loss, (hp)

P_{sh} shaft power, (hp)

The rotor friction loss is the power required to rotate the rotor disk without blades against the frictional drag of the relatively stagnant gases in the clearance space on either side of the rotor disk. This power loss may be expressed as (references 4 and 6)

$$P_{RF} = 1.272 \left(\frac{\rho_g D_{R,h} u_{R,h}}{\mu_g}\right)^{-0.12} \left(\frac{N}{1000}\right)^3 D_{R,h}^{\ 5} \rho_g \qquad (10)$$

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where

$D_{R,h}$ diameter of rotor disk at blade root, (ft)

N rotor speed, rpm

$u_{R,h}$ peripheral speed of rotor disk, (ft/sec)

ρ_g density of gases surrounding disk, (slugs/cu ft)

μ_g viscosity of gases surrounding disk, (slugs/(sec)(ft))

The bearing and gear losses may be evaluated by measuring the rate of oil flow to the bearings and the temperature rise of the oil. In order to avoid heat-transfer effects, the calibration of these losses should be made with air flowing through the turbine at a temperature approximating that of the oil temperature. The pressures and the temperatures in the lubricating system should be maintained close to full-scale operating conditions. The oil-temperature thermocouples should be located as closely as possible to the bearing-oil inlet and outlet. The power loss is

$$P_{BE} = \frac{c_{p,L} w_L J(\Delta T)_L}{550} \tag{11}$$

where

$c_{p,L}$ specific heat of lubricant (oil), (Btu/(lb)($^{\circ}$F))

w_L oil flow rate, (lb/sec)

$(\Delta T)_L$ temperature rise of oil through bearings, ($^{\circ}$F)

For a given oil-inlet temperature and oil flow rate, this power loss is proportional to some power of the turbine speed, that is,

$$P_{BE} = K_{BE} N^n \tag{12}$$

where K_{BE} and n are constants for each oil flow rate and oil-inlet temperature. The values of power loss against turbine speed can be plotted on log-log coordinates for each flow rate and oil temperature. The resulting graph should appear as a straight line.

The rotor-disk friction and bearing losses may be better evaluated by another method that requires motoring of the turbine. The turbine disk without blades or a dummy wheel of the same size and shape is motored at various speeds in air at various densities while the power required for motoring is measured. If the turbine disk is used, the blade-root slots must be filled or covered to obtain a smooth rim. The static pressure and temperature are measured in the relatively stagnant air space on both sides of the turbine disk and at the rim, and the densities of the air at these locations are determined. Then, the power required for motoring, which includes bearing, gear, and rotor-disk friction losses, is plotted against the mean density of air, with turbine speed as the third variable. The value of this power, as determined by extrapolation of the curve to zero density, is the bearing and gear losses and varies only with turbine speed because the oil flow rate is a function of speed if oil-inlet temperature is maintained constant. The disk friction loss is obtained by subtracting the bearing, gear, and rim friction losses from the total motoring requirements. In considering the bearing losses obtained by the motoring investigations, the additional shaft unbalanced thrust forces encountered in actual operation are probably minor in relation to the over-all power. Hence for a given turbine, the evaluation of the disk friction loss with this method can be used to fix the value of the constant (1.272 in equation (10)) in the rotor-disk power loss equation.

The miscellaneous losses P_M in equation (9) depend on the experimental setup and vary from one setup to another. Consequently, no method of evaluation can be given. If a check on the shaft power is required, methods for evaluating the losses for each setup must be devised by the individual investigator.

Air-Cooled Turbine Power Components

No analysis has evidently been made of the useful power and power losses similar to that of reference 4 for a cooled turbine when the coolant is mixed with the combustion gas or when it is unmixed. It is difficult to separate turbine losses when two streams of fluids are mixed as is the case to be considered herein (cooling air flowing out of the rotor-blade tips and mixing with the combustion gas). This cooled turbine configuration is hereinafter called a mixed-flow turbine and should not be confused with the mixed-flow-type impeller. A system of power components, which parallels that given for the uncooled turbine, is considered wherever possible in the following analysis.

Blade power. - In an air-cooled turbine (considering only the blades), the combustion gas flows across the blades and the cooling air generally flows through the blades, almost perpendicular to the gas flow, and emerges from the blade tips. On emergence from the blades, the cooling air mixes with the combustion-gas stream downstream of the rotor. The mixed-flow case is the only one considered herein. The combustion gas contains a certain quantity of energy upstream of the turbine stators. If the stators are cooled, some of this initial energy is lost through heat transfer to the stator coolant. As considered in this analysis, the stator coolant does not mix with the combustion gas, which is probably the case for most turbines employing stator cooling. In passing across the rotor blades, the combustion-gas energy is further dissipated as heat loss to the rotor-blade coolant and as mechanical work performed by the gas on the blades. The cooling air, entering the blade roots with some initial energy, receives energy from the heat addition and from the work performed on it by the centrifugal action of the blades (pumping work). The difference between the forces exerted by the combustion gas on the outside of the blade and the forces exerted by the blades on the cooling air is the net reaction force on the blades and provides a torque to the rotor.

The energy equation for the two fluids for a turbine with cooled stator and air-cooled rotor blades, with the assumption of no heat loss from the turbine parts such as the shroud and with the assumptions that all heat transferred to the cooling air occurs in the blades, can be represented as follows:

$$w_g h'_{g,1} J - 550\, P_B - Q_S J - Q_a J + w_a h'_{a,h} J + Q_a J + 550\, P_P$$
$$= (w_a + w_g) h'_{m,4} J \qquad (13)$$

where

$h'_{a,h}$ total enthalpy of cooling air at blade root, (Btu/lb)

$h'_{m,4}$ total enthalpy of cooling-air and combustion-gas mixture downstream of turbine, (Btu/lb)

P_P power to pump cooling air through blades, (hp)

Q_a heat loss from combustion gas or heat gain by cooling air in rotor blades, (Btu/sec)

Q_S heat loss to stator, (Btu/sec)

w_a cooling-air flow rate through rotor blades, (lb/sec)

The net power transmitted to the rotor is (from equation (13))

$$P_B - P_P = \frac{\left(w_g h'_{g,1} J + w_a h'_{a,h} J\right) - \left(w_g + w_a\right) h'_{m,4} J - Q_S J}{550} \tag{14}$$

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In equations (13) and (14), the total enthalpy of the cooling air at the blade root is an absolute value. When determining $P_B - P_P$ by use of equation (14) from specific measurements, it is impossible to determine $h'_{a,h}$ directly. A total enthalpy of the cooling air relative to the moving blades $h''_{a,h}$, however, can be determined from instruments located in the cooling-air passages. Then, if these enthalpies are defined as

$$h'_{a,h} = h_{a,h} + \frac{V_{a,h}^2}{2Jg}$$

and

$$h''_{a,h} = h_{a,h} + \frac{W_{a,h}^2}{2Jg}$$

where

$h_{a,h}$ enthalpy of cooling air at blade root, (Btu/lb)

$V_{a,h}$ absolute velocity of cooling air at blade root, (ft/sec)

$W_{a,h}$ relative velocity of cooling air at blade root, (ft/sec)

and if the following relation is assumed to exist between these velocities

$$W_{a,h}^2 + u_{a,h}^2 = V_{a,h}^2$$

then equation (14) can be rewritten as

$$P_B - P_P = \frac{w_g h'_{g,1} J + w_a \left(h''_{a,h} + \frac{u_{a,h}^2}{2Jg}\right) J - \left(w_g + w_a\right) h'_{m,4} J - Q_S J}{550} \tag{15}$$

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If the stators are uncooled, the term $Q_S J$ is deleted. Equation (15 corresponds and reduces to equation (7) for the uncooled turbine if w_a, Q_S, and P_P are set equal to zero, which is the case for the uncooled turbine (assuming $\Delta h_{3-4} = 0$).

Shaft power. - Losses similar to those for the uncooled turbine that dissipate some of the power received by the rotor blades are present in the cooled turbine so that the shaft power is less than blade reaction power. For the cooled turbine, an equation similar to equation (9) can be set up, thus

$$P_{sh} = \left(P_B - P_P\right) - P_{RF} - P_{BE} - P_{PR} - P_M \tag{16}$$

and $P_B - P_P$ is the reaction power transmitted to the rotor as determined by equation (15) rather than P_B as in the uncooled turbine, and P_{PR} is the power required to pump the cooling air from its point of entry in the turbine disk to the blade root. The disk friction, bearing and gear, and miscellaneous losses are determined as in the case of the uncooled turbine.

Ideal power. - The ideal power of the cooled turbine where the cooling air and combustion gas mix downstream, considering an isentropic process (no change in entropy), is assumed to be that obtainable from both fluids provided that the total pressures at the stator inlet, the rotor-blade root in the coolant passage, and the final mixing station are specified. The maximum work is obtained, as in the uncooled turbine, with a reversible process but with the additional consideration of the simultaneous flow of both fluids. Then the equation for the combustion gas, paralleling equation (2), is

$$E_{g,s} = h'_{g,1}\left[1 - \left(\frac{p'_{m,4}}{p'_{g,1}}\right)^{\frac{\gamma_g - 1}{\gamma_g}}\right] \tag{17}$$

and, for the cooling air,

$$E_{a,s} = h'_{a,h}\left[1 - \left(\frac{p'_{m,4}}{p'_{a,h}}\right)^{\frac{\gamma_a - 1}{\gamma_a}}\right] \tag{18}$$

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where

$E_{a,s}$ ideal energy of cooling air, (Btu/lb)

$p'_{a,h}$ absolute total pressure of cooling air at rotor-blade root, (lb/sq ft)

$p'_{m,4}$ total pressure of mixture of cooling air and combustion gas dowstream of turbine, (lb/sq ft)

γ_a ratio of specific heats of cooling air

On the basis of equations (17) and (18), the ideal power is

$$P_{id} = \frac{w_g J h'_{g,1}\left[1 - \left(\frac{p'_{m,4}}{p'_{g,1}}\right)^{\frac{\gamma_g - 1}{\gamma_g}}\right] + w_a J h'_{a,h}\left[1 - \left(\frac{p'_{m,4}}{p'_{a,h}}\right)^{\frac{\gamma_a - 1}{\gamma_a}}\right]}{550} \tag{19}$$

The values of $h'_{a,h}$ and $p'_{a,h}$ are obtained indirectly from the measured relative values of $h''_{a,h}$ and $p''_{a,h}$, respectively. The absolute and relative values of enthalpy and pressure are relateó as shown by the following expressions:

$$h'_{a,h} = h''_{a,h} + \frac{u_{a,h}^2}{2Jg}$$

and

$$p'_{a,h} = p''_{a,h}\left[1 + \frac{u_{a,h}^2}{2T''_{a,h} \, Jgc_{p,a}}\right]^{\frac{\gamma_a}{\gamma_a - 1}}$$

where

$c_{p,a}$ specific heat of cooling air at constant pressure, (Btu/(lb)(°F))

$T''_{a,h}$ relative total temperature of cooling air at blade root, (°R)

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When the absolute values, as determined from the preceding expressions, are substituted in equation (19), the ideal power can be evaluated.

The ideal power for the cooled turbine is decreased because of stator- and rotor-blade losses, such as friction, separation, and other viscous effects, and possibly because of a leakage loss due to the clearance between the rotor-blade tips and the shroud. The word "possibly" is used because cooling air discharged into the clearance space may alter the leakage loss as defined in an uncooled turbine. In this case, equation (8) is believed not to apply and no substitute formula is available at present. Losses due to separation, friction, and so forth subtract from the energy of the cooling air. Then the ideal power available from both fluids in the cooled turbine may be expressed in terms of blade power, pumping power, and power losses, as

$$P_{id} = P_B + P_{SL} + P_{BL} + P_{TL} - \left(P_P - P_{aL}\right)$$

or, corresponding to equation (4), as

$$P_B - P_P = P_{id} - P_{SL} - P_{BL} - P_{TL} - P_{aL} \tag{20}$$

where

P_{aL} coolant pumping losses (friction, separation, and so forth), (hp)

$P_B - P_P$ net power delivered to turbine rotor, (hp)

The other terms have been defined and are the same as for the uncooled turbine.

The decrease in available energy due to heat transferred to the stator coolant does not appear in equation (20). This omission can be explained by referring to figure 2(c) where the expansion for an uncooled turbine is indicated by the line between state points 1 and 3'. As cooling takes place, the expansion line moves to the left and to a lower back pressure, that is, between state points 1 and 3. Thus, because of cooling, both ideal power and blade power of the cooled turbine have been changed from the uncooled case. Because these powers change, the effect of stator cooling is accounted for in the values of P_B and P_{id} in equation (20).

As in the uncooled turbine, the stator- and rotor-blade losses and the cooling-air losses are practically impossible to evaluate from cooled-turbine operational data. Knowledge of blade power, coolant pumping power, and ideal power, however, is quite sufficient for a study of turbine performance. The separation of the power components is given merely as a general outline of the factors decreasing the available energy.

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Coolant pumping power. - Pumping power is defined as the power required from the turbine to pump the cooling air through the rotor and is considered a loss. The term "rotor" includes the rotor blades and whatever portion of the disk through which the cooling air passes.

The energy input to the coolant flowing between two stations, such as the blade root and blade tip due to pumping action, is determined from consideration of the rotor torque, which is equal to the rate of change in the moment of momentum of the cooling air. The change in the moment of momentum through the blades per pound mass of cooling air is

$$r_T V_{u,a,T} - r_h V_{u,a,h}$$

where

r_h radius from shaft center line to blade root, (ft)

r_T radius from shaft center line to blade tip, (ft)

$V_{u,a,h}$ tangential component of absolute velocity of cooling air at blade root, (ft/sec)

$V_{u,a,T}$ tangential component of absolute velocity of cooling air at blade tip, (ft/sec)

The torque is then equal to

$$\frac{w_a}{g}\left(r_T V_{u,a,T} - r_h V_{u,a,h}\right)$$

The power required to pump the coolant through the blades is therefore the Euler equation,

$$P_P = \frac{w_a \omega}{g550}\left(r_T V_{u,a,T} - r_h V_{u,a,h}\right) \tag{21}$$

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where

ω angular velocity of rotation of rotor, (radians/sec)

In the derivation of the equation for the heat transferred to the cooling air (reference 1), the tangential component of the absolute velocity of cooling air at the radius is

$$V_{u,a} = f_s u = f_s \omega r \tag{22}$$

where

f_s slip factor

u tangential velocity of rotor at radius r, (ft/sec)

Also shown in reference 1 on the basis of reference 7 is that, for the dimensions of the cooling-air passage in the blades, f_s is practically equal to 1. Thus, ωr can be substituted for $V_{u,a}$ in equation (21) with negligible error so that the power for pumping the cooling air through the blades finally becomes

$$P_P = \frac{w_a \omega^2}{g550}\left(r_T^2 - r_h^2\right) \tag{23}$$

If the cooling air is introduced through the shaft and passes through the disk, the additional power for pumping the air through the disk (P_{PR} in equation (16)) is obtained using equation (23) but replacing the term $r_T^2 - r_h^2$ in this equation with $r_h^2 - r^2$, where r denotes a mean radius at which the air is introduced in the disk. Experiments (reference 8) conducted on a turbine to reduce the total pumping power losses show that the use of equation (23), but with r_h replaced by 0 for the case of air flowing from the shaft and out the blade tips gave a close approximation to the actual total pumping losses. Equation (23) was used in investigations described in reference 9 and good agreement with the measured values of pumping power was obtained.

Parameters Affecting Blade Power

Evaluations of turbine-performance relations. - Expressing the performance of jet-propulsion engines, turbines, and compressors in terms of generalized parameters derived by dimensional analysis is common practice. The grouping of variables into such parameters is useful because the number of variables involved in experiment or analysis is effectively reduced. Generalization of the data by the use of these parameters permits the experimental results obtained during operation with one set of experimental conditions to be used for estimating performance at other operating conditions. 1328

A derivation of performance parameters for jet engines showing the methods used is given in reference 10; a similar derivation, but including factors not considered in reference 10, is presented in reference 11. Typical results for turbines from wind-tunnel investigations employing similar derivations are given in references 12 and 13 in which the verification of the parameters used is obtained.

Turbine power from simple velocity diagram. - In the velocity diagram at the stator outlet (fig. 4), β_2 is the angle of attack of the fluid with respect to the rotor blade. Now

$$\tan\left(90 - \beta_2\right) = \frac{V_{2,u} - u_{x = b/2}}{V_{2,ax}} = \frac{V_2 \sin\left(90 - \alpha_2\right) - u_{x = b/2}}{V_2 \cos\left(90 - \alpha_2\right)}$$

or

$$\tan\left(90 - \beta_2\right) = \tan\left(90 - \alpha_2\right) - \frac{u_{x = b/2}}{V_2} \sec\left(90 - \alpha_2\right) \quad (24)$$

where

$u_{x = b/2}$ tangential velocity of rotor blade at midspan, (ft/sec)

V_2 stator-outlet velocity, (ft/sec)

$V_{2,ax}$ axial component of stator-outlet velocity, (ft/sec)

$V_{2,u}$ tangential component of stator-outlet velocity, (ft/sec)

α_2 angle of stator-outlet velocity (fig. 4), (deg)

When no jet deflection is assumed, α_2 is constant. Consequently, the angle of attack on which the blade power depends varies with the parameter $\frac{u_{x = b/2}}{V_2}$

or

$$P_B = f^{I}\,\frac{u_{x = b/2}}{V_2} \tag{25}$$

where

f unknown function (Superscripts differentiate unknown functions.)

Equation (25) is for the simple case of an incompressible and inviscid fluid, no heat transfer from turbine shell to oil or surrounding atmosphere, no heat transfer to cooling air flowing through blades, fixed inlet conditions, and no heat-capaicity relaxation time or lag of the fluid.

<u>Factors affecting power.</u> - In the derivation of blade power, however, all the independent variables should be considered. Compressibility considerations require the introduction of the density ratio across the turbine; viscosity requires the introduction of Reynolds and Prandtl numbers; a dimensionless parameter involving some dimension of the turbine is needed; the properties of the fluid (specific heat, conductivity, and heat-capacity lag) require the introduction of Prandtl number; and the speed of sound in the fluid at the turbine inlet must be considered. Derivation of formulas that include all the foregoing terms and that express blade power directly from data is impossible at present; consequently, recourse is made to dimensional analysis as in references 10 and 11.

<u>Turbine-blade power formula from dimensional analysis.</u> - Formulas for blade power have been derived using dimensional analysis; the derivations were based on notes, neither the notes nor the derivations being published. The derivations resulted in end equations of a form such that certain terms, such as heat-capacity lag, are omitted because of their negligible effect on turbine power. Thus for the uncooled turbine, when the heat loss from the shroud and the variation of the ratio of specific heats are neglected, the blade power of a specific turbine was shown to be a function of the following parameters:

$$\frac{P_B}{a'_{g,1}p'_{g,1}\gamma_{g,1}A_1} = f^{II}\left[\frac{p'_{g,3}}{p'_{g,1}},\ \frac{N}{a'_{g,1}},\ Re\right] \tag{26}$$

where

A_1 cross-sectional area at stator inlet, (sq ft)

$a'_{g,1}$ speed of sound in combustion gas based on total temperature at stator inlet, (ft/sec)

$p'_{g,1}$ total pressure at stator inlet, (lb/sq ft)

$p'_{g,3}$ total pressure at rotor outlet, (lb/sq ft)

Re Reynolds number

$\gamma_{g,1}$ ratio of specific heats of combustion gas at stator-inlet temperature

By methods given in reference 11 and expanded in the unpublished derivations and by removing the constant A_1 and moving it into the function f, the equation for power (equation (26)) can be corrected to standard turbine-inlet conditions. The resulting equation is

$$\frac{P_B}{\delta_1\sqrt{\theta_1}} = f^{III}\left[\frac{p'_{g,3}}{p'_{g,1}}, \frac{N}{\sqrt{\theta_1}}, Re\right] \tag{27}$$

where

p_0 standard NACA sea-level pressure, 2116.8 (lb/sq ft)

T_0 standard NACA sea-level temperature, 518.4 ($^{\circ}$R)

γ_0 ratio of specific heats of gas at standard NACA sea-level temperature

δ_1 $\gamma_{g,1}p'_{g,1}/\gamma_0 p_0$

θ_1 $\gamma_{g,1}T'_{g,1}/\gamma_0 T_0$

In most turbines the Reynolds number effect has been negligible, so that eliminating it from equation (27) causes this equation to become the familiar turbine-blade power formula for uncooled turbines. For pressure ratios less than critical, the pressure ratio $p'_{g,3}/p'_{g,1}$ in equation (27) can be replaced by the corrected gas weight flow $w_g\sqrt{\theta_1}/\delta_1$.

The net blade power in an air-cooled turbine is the difference between the power due to the gas forces on the outside of the blade P_B and the coolant pumping power P_P. More parameters than those given in equation (27) are necessary to obtain a corrected net blade power formula similar to equation (27).

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Coolant pumping power is a function of turbine speed and cooling-air flow rate as shown by equation (23). Pumping power, however, must be a function of the corrected speed and the corrected combustion-gas flow rate before it can be included in an equation similar to equation (27); then equation (23) can be rewritten as

$$P_P = \frac{(2\pi N)^2 w_a}{g550}\left(r_T^2 - r_h^2\right)$$

so that

$$\frac{P_P}{\delta_1\sqrt{\theta_1}} = \frac{2\pi N^2}{\delta_1\sqrt{\theta_1}\sqrt{\theta_1}}\,\frac{w_a\sqrt{\theta_1}}{g550}\left(r_T^2 - r_h^2\right)$$

and

$$\frac{P_P}{\delta_1\sqrt{\theta_1}} = \frac{w_a}{w_g}\,2\pi\left(\frac{N}{\sqrt{\theta_1}}\right)^2\frac{w_g\sqrt{\theta_1}}{\delta_1 g550}\left(r_T^2 - r_h^2\right)$$

Thus

$$\frac{P_P\,\dfrac{w_g}{w_a}}{\delta_1\sqrt{\theta_1}} = f^{IV}\left(\frac{N}{\sqrt{\theta_1}},\ \frac{w_g\sqrt{\theta_1}}{\delta_1}\right)$$

or

$$\frac{P_P\,\dfrac{w_g}{w_a}}{\delta_1\sqrt{\theta_1}} = f^{V}\left(\frac{N}{\sqrt{\theta_1}},\ \frac{p'_{m,4}}{p'_{g,1}}\right) \qquad (28)$$

With reference to equation (13), the terms involving Q_a cancel each other. Because of the irreversible nature of the mixing of the cooling air with the combustion gas, the heat transfer affects the available mixture entropy and thus affects the net blade power. Consequently, a parameter involving Q_a should appear in the equation for the power output of the air-cooled mixed-flow turbine, similar to equation (27).

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On the basis of the foregoing reasoning, the following equation is applicable to the air-cooled turbine considered:

$$\frac{P_B - P_P \dfrac{w_g}{w_a}}{\delta_1\sqrt{\theta_1}} = f^{VI}\left(\frac{p'_{m,4}}{p'_{g,1}}, \frac{N}{\sqrt{\theta_1}}, Re, \frac{Q_a}{\delta_1\sqrt{\theta_1}}\right) \qquad (29)$$

If the stators are cooled, another term $Q_S/\delta_1\sqrt{\theta_1}$ must be included in the right member of equation (29).

Direct determination of the function in equation (29) is, however, impossible if the data are evaluated using equation (14), which gives $P_B - P_P$. Recourse is therefore made to equation (23) for evaluating the coolant pumping power P_P. When equations (14) and (23) are combined, the blade power of an air-cooled turbine can be determined. This blade power can also be presented in dimensionless form. If equation (28) is rearranged,

$$\frac{P_P}{\delta_1\sqrt{\theta_1}} = f^{VII}\left(\frac{p'_{m,4}}{p'_{g,1}}, \frac{w_a\sqrt{\theta_1}}{\delta_1}, \frac{N}{\sqrt{\theta_1}}\right)$$

Therefore

$$\frac{P_B - P_P}{\delta_1\sqrt{\theta_1}} = f^{VIII}\left(\frac{p'_{m,4}}{p'_{g,1}}, \frac{w_a\sqrt{\theta_1}}{\delta_1}, \frac{N}{\sqrt{\theta_1}}, Re, \frac{Q_a}{\delta_1\sqrt{\theta_1}}\right) \qquad (30)$$

Because of the five dependent variables, determining f^{VIII} in equation (30) would be extremely complex. Inasmuch as P_P is obtained from equation (23), then in dimensionless form

$$\frac{P_B}{\delta_1\sqrt{\theta_1}} = f^{IX}\left(\frac{p'_{m,4}}{p'_{g,1}}, \frac{N}{\sqrt{\theta_1}}, Re, \frac{Q_a}{\delta_1\sqrt{\theta_1}}\right) \qquad (31)$$

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Investigations of an air-cooled turbine are required to establish the relation between the left side of equation (31) and the parameters in the right member.

Efficiency

In addition to the power components (losses and useful power), the efficiency is required for turbine-performance evaluation. The efficiency of an uncooled turbine, usually called the adiabatic efficiency η_{ad}, is defined as the ratio of the blade power (equation (7)) to the ideal power (equation (3)). Thus, the efficiency is given by the formula

$$\eta_{ad} = \frac{\Delta h'_g}{\Delta_s h'_g} \tag{32}$$

For the cooled turbine, a similar efficiency would be the ratio of the net power on the blade due to the action of the combustion gases and the cooling air $P_B - P_P$ (equation (14)) to the ideal power (equation (19)). Thus, both fluids are considered in the numerator and the denominator of the ratio. The efficiency formula reduces to

$$\eta_{ad} = \frac{\left(w_a h'_{g,1} + w_a h'_{a,h}\right) - \left(w_g + w_a\right)h'_{m,4} - Q_S}{w_g h'_{g,1}\left[1 - \left(\frac{p'_{m,4}}{p'_{g,1}}\right)^{\frac{\gamma_g - 1}{\gamma_g}}\right] + w_a h'_{a,h}\left[1 - \left(\frac{p'_{m,4}}{p'_{a,h}}\right)^{\frac{\gamma_a - 1}{\gamma_a}}\right]} \tag{33}$$

or

$$\eta_{ad} = \frac{\left(w_g h'_{g,1} + w_a h'_{a,h}\right) - \left(w_g + w_a\right)h'_{m,4} - Q_S}{w_g\left(h'_{g,1} - h'_{g,4,s}\right) + w_a\left(h'_{a,h} - h'_{a,4,s}\right)} \tag{34}$$

where

$h'_{a,4,s}$ final total enthalpy of cooling air with process described, (Btu/lb)

$h'_{g,4,s}$ final total enthalpy of combustion gas attained with process described, (Btu/lb)

Use of equation (33) is tedious because of the determination of γ_g and γ_a; equation (34) is therefore recommended. Methods for obtaining $h'_{a,4,s}$ and $h'_{g,4,s}$ are described in reference 14. Equation (34) reverts to equation (32) when the turbine is uncooled (that is, $w_a = 0$ and $Q_S = 0$). 1328

The efficiency, as defined by equation (34), involves both aerodynamic and cooling losses and objections may be raised to use of such an efficiency. Work is being done to break up the efficiency so defined into its component efficiencies. A certain parallelism does exist, however, between the efficiencies defined by equation (32) for the uncooled turbine and by equation (34) for the cooled turbine. Both are ratios of work done on the blades to the ideal isentropic work and, as such, equation (34) seems to have definite meaning.

Parameters Affecting Efficiency

If the principal factor influencing the efficiency of an uncooled turbine is assumed to be the correct angle of attack on the blades, methods similar to those developed for determining the parameters affecting blade power can be used to show that the same parameters affect efficiency. Then, for the uncooled turbine

$$\eta_{ad} = f^{X}\left(\frac{p'_{g,3}}{p'_{g,1}}, \frac{N}{\sqrt{\theta}}, Re\right) \tag{35}$$

For the cooled turbine

$$\eta_{ad} = \frac{P_B - P_P}{P_{id}} \tag{36}$$

and from equation (36) and equations (30), (33), and (35), it can be analytically shown that the efficiency should be represented by the formula

$$\eta_{ad} = f^{XI}\left(\frac{p'_{m,4}}{p'_{g,1}}, \frac{w_a\sqrt{\theta_1}}{\delta_1}, \frac{N}{\sqrt{\theta_1}}, Re, \frac{Q_a}{\delta_1\sqrt{\theta_1}}, \frac{Q_S}{\delta_1\sqrt{\theta_1}}\right) \tag{37}$$

The term $Q_S/\delta_1\sqrt{\theta_1}$ is omitted if the nozzles are uncooled.

When experimental results are evaluated, parameters deduced by dimensional theory do not always maintain the importance attached to them in the analysis; consequently, emphasis is placed on the lack of experimental verification of equations (31) and (37) at the present time. Experimental results may invalidate certain assumptions made in the analysis that were used to delete other terms. The cooled turbine-performance equations should therefore be used with some caution. Consideration of the parameters that have been included, however, results in the establishment of first-order effects.

Pressure Ratio across Turbine

After the turbine has been investigated, one of the most important uses of the data is in the evaluation of the performance of a jet engine under various flight conditions. Determination of the pressure and the temperature at the turbine inlet is usually possible from flight conditions, together with operational data on the engine inlet diffuser, the compressor, and the combustor. These determined turbine-inlet conditions may then be used in conjunction with the turbine-performance data to calculate flow conditions downstream of the turbine. The turbine-outlet conditions, together with the tail-pipe data and flight speed, are sufficient to determine engine thrust. One requirement is the downstream pressure of the turbine.

If the function f^{VIII} is known, the pressure $p'_{m,4}$ downstream of the cooled turbine could be tediously determined from equation (30); it would, however, be more advantageous if $p'_{m,4}$ could be determined directly from an equation similar to equation (30). The direct method is used in wind-tunnel investigations of engines (references 12 and 13). For the case where the flow is less than critical, the pressure ratio can be determined from the formula

$$\frac{p'_{m,4}}{p'_{g,1}} = f^{XII}\left(\frac{w_g\sqrt{\theta_1}}{\delta_1}, \frac{N}{\sqrt{\theta_1}}, Re, \frac{Q_a}{\delta_1\sqrt{\theta_1}}, \frac{Q_S}{\delta_1\sqrt{\theta_1}}\right) \qquad (38)$$

The indications that $w_a\sqrt{\theta_1}/\delta_1$ should possibly be included as a parameter in the right side of the equation will have to be verified by investigations.

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Reynolds Number

Consideration of Reynolds number is usually neglected in uncooled turbine-performance plots because of the difficulty of ascertaining Reynolds number effect on over-all performance. Little knowledge and considerable controversy exist as to what is the effective Reynolds number of an uncooled turbine.

The Reynolds number effect on an uncooled turbine is treated in reference 15 as follows: The Reynolds number is

$$Re_{g,2} = \frac{\rho_{g,2} V_2 D}{\mu_{g,2}} \qquad (39)$$

where

D	characteristic dimension of turbine, (ft)
$Re_{g,2}$	Reynolds number of combustion gas at stator outlet
$\rho_{g,2}$	theoretical density of gas corresponding to theoretical jet velocity at stator outlet, (slugs/cu ft)
$\mu_{g,2}$	viscosity of gas at stator outlet, (slugs/(ft)(sec))

Now

$$\frac{\rho_{g,2}}{\rho'_{g,2}} = \left(\frac{p_{g,2}}{p'_{g,2}}\right)^{\frac{1}{\gamma_g}} = \left(\frac{p_{g,3}}{p'_{g,1}}\right)^{\frac{1}{\gamma_g}} \qquad (40)$$

where

$\rho'_{g,2}$	mass density of combustion gas at stator outlet based on total pressure and total temperature, (slugs/cu ft)

In equation (40), the total pressure at the stator outlet $p'_{g,2}$ is assumed equal to the total pressure at the stator inlet $p'_{g,1}$ and the static pressure at the stator outlet $p_{g,2}$ equal to the static pressure at the rotor outlet $p_{g,3}$. The assumption that

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$p_{g,2} = p_{g,3}$ is more valid for impulse turbines. On the basis of the total temperature at the stator outlet $T'_{g,2}$ equal to the total temperature at the turbine inlet $T'_{g,1}$ (no heat loss to the stators $Q_s = 0$) and the foregoing suppositions,

$$V_2 = \sqrt{2g \frac{\gamma_g}{\gamma_g - 1} R_g T'_{g,1} \left[1 - \left(\frac{p_{g,3}}{p'_{g,1}}\right)^{\frac{\gamma_g - 1}{\gamma_g}}\right]} \tag{41}$$

where

R_g gas constant, (ft-lb/(lb)(°F))

If the assumption is made that $\mu_{g,2}$ is proportional to $\left(T_{g,2}\right)^n$ (where n is an exponent determined from viscosity curves for combustion gases), then because

$$T_{g,2} = T'_{g,2} \left(\frac{p_{g,2}}{p'_{g,2}}\right)^{\frac{\gamma_g - 1}{\gamma_g}} = T'_{g,1} \left(\frac{p_{g,3}}{p'_{g,1}}\right)^{\frac{\gamma_g - 1}{\gamma_g}} \tag{42}$$

where

$T_{g,2}$ static temperature at stator outlet, (°R)

the viscosity equation becomes

$$\mu_{g,2} = K\left(T'_{g,1}\right)^n \left(\frac{p_{g,3}}{p'_{g,1}}\right)^{\frac{\gamma_g - 1}{\gamma_g} n} \tag{43}$$

where K is a proportionality constant.

With the use of equations (40), (41), (43), and the gas law $p/\rho = gRT$, equation (39) becomes

$$Re_{g,2} = \frac{Dp'_{g,1}}{K\left(T'_{g,1}\right)^{n} g R_g T'_{g,1}} \left(\frac{p_{g,3}}{p'_{g,1}}\right)^{\frac{1+n-n\gamma_g}{\gamma_g}} \sqrt{2g \frac{\gamma_g}{\gamma_g - 1} R_g T'_{g,1} \left[1-\left(\frac{p_{g,3}}{p'_{g,1}}\right)^{\frac{\gamma_g-1}{\gamma_g}}\right]} \tag{44}$$

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If the variations in γ_g and R_g are neglected, which is possible if the parameters previously set forth are used, equation (44) for a given turbine and fluid becomes

$$Re_{g,2} = \frac{p'_{g,1}}{\left(T'_{g,1}\right)^{n+1/2}} f^{XIII}\left(\frac{p'_{g,1}}{p_{g,3}}\right) \tag{45}$$

From foregoing equations, $p'_{g,1}/p_{g,3}$ can be shown as a function of parameters already listed in the corrected basic equations for the uncooled turbine. The only new variable added by the Reynolds number is the ratio $p'_{g,1}/\left(T'_{g,1}\right)^{n+1/2}$, which replaces Re in equations (27) and (35) if this method of defining Reynolds number is used. The ratio $p'_{g,1}/\left(T'_{g,1}\right)^{n+1/2}$, which is substituted for Re, is not the total Reynolds number effect because the pressure ratio in equation (45), which is represented by other parameters in the basic equations, is also part of the Reynolds number effect. Data in references 15 and 16 support the usage of the ratio derived for the uncooled impulse-type turbine.

On the basis of equation (39) and the preceding discussion, the Reynolds number, based on pressures and temperatures at the stator outlet and neglecting γ_g and R_g changes as in the uncooled turbine equation development, is

$$Re_{g,2} = \frac{p'_{g,2}}{\left(T'_{g,2}\right)^{n+1/2}} f^{XIV}\left(\frac{p_{g,2}}{p'_{g,2}}\right) \tag{46}$$

For the case of $Q_S = 0$, equation (46) is applicable to both the cooled and the uncooled turbine. If the stators are cooled, a drop in total temperature across the stators is obtained. This temperature drop may be written as

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$$T'_{g,1} - T'_{g,2} = \frac{Q_S}{w_g c_{p,g}} \tag{47}$$

where

$c_{p,g}$ specific heat of combustion gas at constant pressure, (Btu/(lb)(°F))

From equation (47), it can be shown that

$$T'_{g,2} = T'_{g,1}\, f^{XV}\left(\frac{Q_S}{\delta_1\sqrt{\theta_1}}, \frac{w_g\sqrt{\theta_1}}{\delta_1}\right) \tag{48}$$

At equivalent operating conditions, the boundary layer should be more stable around a cooled stator blade than around an uncooled stator blade; consequently, the losses should be less for the cooled stators. The assumption that $p'_{g,2}$ is equal to $p'_{g,1}$ for the turbine with cooled stators should therefore be more accurate than for the uncooled turbine.

The parameters affecting the ratio $p_{g,2}/p'_{g,2}$ (equation (46)) are believed to exist in the cooled turbine-performance equations. As a consequence, the only parameter affecting Reynolds number that does not appear in the performance equations is $p'_{g,1}/\left(T'_{g,1}\right)^{n+1/2}$ as was shown to be the case of the uncooled turbine. In determining cooled turbine performance, it is therefore suggested that this ratio be substituted for Re in equations (31), (37), and (38).

RECOMMENDED METHODS FOR PERFORMANCE EVALUATION

Blade Power

In order to evaluate the effect of the various performance parameters on the blade power P_B, equation (31) is recommended. The function f^{IX} must be established by conducting experiments to obtain data from which families of curves can be constructed in order to determine the effect of $p'_{m,4}/p'_{g,1}$, $N/\sqrt{\theta_1}$, Re $\left[\text{or } p'_{g,1}/\left(T'_{g,1}\right)^{n+1/2}\right]$, $Q_a/\delta_1\sqrt{\theta_1}$, and if the stators are cooled $Q_S/\delta_1\sqrt{\theta_1}$ on $P_B/\delta_1\sqrt{\theta_1}$. Conducting tests in which one parameter is varied and four are held constant is difficult, if not impossible. Constant gas temperature and pressure at the turbine inlet and constant wheel speed fix $N/\sqrt{\theta_1}$ and $p'_{g,1}/(T'_{g,1})^{n+1/2}$. The rotor-blade cooling-air inlet temperature and the cooling-air flow rate can be kept constant and combustion-gas flow rate varied. In order to eliminate the effect of $Q_S/\delta_1\sqrt{\theta_1}$ on the blade power, the stators can be uncooled for these determinations in which $p'_{m,4}/p'_{g,1}$ is varied. The data for such experiments can be plotted as shown in curve 1 of figure 5(a). Because the pressure ratios considered are less than critical, the corrected weight flow of combustion gases $w_g\sqrt{\theta_1}/\delta_1$ is substituted for the pressure ratio $p'_{m,4}/p'_{g,1}$ in figures 5 to 9. As the combustion-gas flow varies, the parameter $p'_{m,4}/p'_{g,1}$ varies; and because variation in combustion-gas flow rate causes the outside heat-transfer coefficient to change, $Q_a/\delta_1\sqrt{\theta_1}$ also varies. For each point on the curve (points a, b, c, and so forth, fig. 5(a)) the value of $Q_a/\delta_1\sqrt{\theta_1}$ is somewhat different.

The cooling-air temperature or cooling-air flow rate can be then adjusted to another set of values in such a way as to cause an appreciable change in $Q_a/\delta_1\sqrt{\theta_1}$ and the determinations repeated to obtain another curve (curve 2) similar to curve 1. The average value of $Q_a/\delta_1\sqrt{\theta_1}$ for curve 2 can be less or greater than that for curve 1. This procedure is repeated until the number of curves obtained is sufficient for the desired range and interval of the parameter $Q_a/\delta_1\sqrt{\theta_1}$.

The next step is to plot the actual values of $Q_a/\delta_1\sqrt{\theta_1}$ obtained at points a, b, c, d, and e on curve 1 (fig. 5(a)) against the corresponding x, y, and z values of $w_g\sqrt{\theta_1}/\delta_1$ (curve 1, fig. 5(b)). The procedure is repeated for the other curves in figure 5(a). Through the resulting family of curves (1, 2, 3, . . ., fig. 5(b)), lines X, Y, and Z, which correspond to constant values of $Q_a/\delta_1\sqrt{\theta_1}$, are drawn. Then the values of the abscissa $w_g\sqrt{\theta_1}/\delta_1$ (x', y', and z') at the intersections of X, Y, and Z with the curves 1, 2, . . . are noted. When the value of $P_B/\delta_1\sqrt{\theta_1}$ corresponding to the x' value of $w_g\sqrt{\theta_1}/\delta_1$ is determined from curve 1 of figure 5(a) and is plotted as in figure 5(c), the point thus determined on a curve represents the value of $Q_a/\delta_1\sqrt{\theta_1}$ equal

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to the X value of figure 5(b). Repeating the procedure with curve 2 (fig. 5(a)) and y' values of $w_a\sqrt{\theta_1}/\delta_1$ (fig. 5(b)) determines a second point on the curve in figure 5(c) having a value of $Q_a/\delta_1\sqrt{\theta_1} = X$. This procedure is repeated until the desired number of curves, each for a constant value of $Q_a/\delta_1\sqrt{\theta_1}$, are obtained (fig. 5(c)).

The experiments and procedures for determining figure 5(c) can be repeated for other constant values of turbine speed $N/\sqrt{\theta_1}$ until a series of curves as shown in figure 6 is obtained.

Satisfactory evaluation of the effect of $Q_S/\delta_1\sqrt{\theta_1}$ for all conditions is expected if, for several fixed values of $Q_S/\delta_1\sqrt{\theta_1}$, the previously described determinations are repeated for only the highest and lowest values of $N/\sqrt{\theta_1}$ and $Q_a/\delta_1\sqrt{\theta_1}$; the resulting curves are shown in figure 7. The effect of $Q_S/\delta_1\sqrt{\theta_1}$ at other values of these two parameters can probably be determined by interpolation of the curves shown in figures 6 and 7. Because variation of the parameter $Q_S/\delta_1\sqrt{\theta_1}$ is related to changes in w_g, adjustment of stator coolant flow or coolant temperature is necessary in order to keep the parameter constant so as to obtain curves as shown in figure 7 without a great amount of cross-plotting.

For the turbine with cooled stators, a possible means of eliminating the parameter $Q_S/\delta_1\sqrt{\theta_1}$ from equation (37) is to substitute δ_2 and θ_2 for δ_1 and θ_1, respectively. The total temperature at the stator outlet, which is used to calculate θ_2, can be determined from equation (47). Inasmuch as calculating $p'_{g,2}$ upon which δ_2 depends and measuring between the stators and the rotor are very difficult, it would be desirable if δ_1 could be used. As previously stated, heat pickup from the gases by the stators should stabilize the stator boundary layer and result in smaller losses in total pressure across the stators than if they were uncooled. Hence as Q_S is increased, the total-pressure drop across the stators is expected to decrease. Because the drop is small in any case, the effect of $Q_S/\delta_1\sqrt{\theta_1}$ on the dependent and independent parameters is probably negligible even if δ_1 is used. Substitution of θ_2 for θ_1 in the parameters in figures 5 and 6 is therefore recommended and as a result, a series of curves similar to those of figure 6 that will be applicable for any $Q_S/\delta_1\sqrt{\theta_1}$ value is expected.

This proposal remains unchecked until such time as investigations of the type shown in figure 7 have been conducted.

The final parameter to be evaluated is $p'_{g,1}/(T'_{g,1})^{n+1/2}$, which replaces the Reynolds number parameter Re in the equations. 1328 Usually the Reynolds number effect is small and, as in the investigations for determining the effect of $Q_S/\delta_1\sqrt{\theta_1}$, it is thought that if investigations are conducted varying the parameter $p'_{g,1}/(T'_{g,1})^{n+1/2}$ for highest and lowest values of $N/\sqrt{\theta}$, $w_g\sqrt{\theta_1}/\delta_1$, and $Q_a/\delta_1\sqrt{\theta_1}$, the results can be interpolated to find the effect of $p'_{g,1}/(T'_{g,1})^{n+1/2}$ on the net blade power at other values of these three parameters. A set of curves determined by the method just described is shown in figure 8, where θ_2 is used on the supposition that such usage eliminates the stator heat-loss parameter from consideration.

The shapes of curves that result from investigations are unknown and the curves of figures 5 to 8 are merely representative of the method described and have no relation to actual results. Although experimental investigations have not yet been conducted by the NACA or by other research groups (as far as is known), it is, however, expected on the basis of uncooled turbine data that for a given value of $Q_a/\delta_1\sqrt{\theta_2}$ such as X the curves of figure 6 may look like those in figure 9.

For the experiments required to construct figures 5 to 9, $P_B - P_P$ is calculated using equation (15) and then P_B is determined by adding the coolant pumping power P_P, as determined by equation (23), to equation (15).

Shaft Power

Determination of the sources of large losses is necessary if the shaft power is appreciably different from the blade power. The more common losses that must be determined are the rotor-disk friction, bearing and gear, and miscellaneous losses, which have been described.

Disk friction losses. - The motoring investigation method fully described previously is recommended for determining the rotor-disk

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friction losses. The type of curve to draw after the losses are determined can be deduced from equation (10), the theoretical equation for these losses. From equation (10),

$$P_{RF} = C\left(\mu_g^{0.12} N^{2.88} \rho_g^{0.88}\right) \tag{49}$$

where

C constant

or

$$\frac{P_{RF}}{\mu_g^{0.12} \rho_g^{0.88}} = CN^{2.88} \tag{50}$$

Consequently if the motoring test values of $P_{RF}/\mu_g^{0.12}\rho_g^{0.88}$ are plotted against the turbine speed N on log-log coordinates, a straight line should theoretically result. From the experimental curve, the value of the constant can be obtained as a check against the theoretical constant in equation (49).

Bearing and gear losses. - The bearing and gear losses should be obtained by both methods, heat to oil and motoring investigations previously discussed. The types of plot to make can be easily inferred from the discussion of the parameters affecting these losses (that is, turbine speed, oil flow rate, and oil-inlet temperature).

Miscellaneous losses. - These losses depend on the accessories being driven by the turbine and, inasmuch as the type and number may be different from setup to setup, the investigations required or the representation of the losses in plots are therefore omitted.

Efficiency

The effect of the various performance parameters on the efficiency (equation (37)) can be evaluated from the same data and in a manner similar to that described for the evaluation of blade power. Equation (34) is used to calculate the efficiency. Plots similar to those of figures 5 to 9 are made with efficiency as the dependent parameter involved instead of $P_B/\delta_1\sqrt{\theta_2}$.

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Pressure Ratio

In addition to the determination of blade power and efficiency from investigations, evaluation of the effect of the pressure ratio as shown by equation (38) on the performance parameters is recommended. The usefulness of this type of evaluation of pressure ratio can be demonstrated in the calculations of engine thrust. The data obtained for the determination of power and efficiency can be used to evaluate the pressure ratio, the working plots and final plots being similar to figures 5 to 9.

EVALUATION OF FACTORS IN FORMULAS

Heat Loss from Combustion Gases

The combustion gases passing over the stator and rotor blades lose heat to the cooling air passing through these turbine parts. The method of determining the heat loss to the rotor blades Q_a is fully discussed in reference 1. The heat loss to the cooling air passing through the stators can be calculated from the simple formula

$$Q_S = w_{a,S} c_{p,a} \Delta T'_{a,S} \tag{51}$$

where

$w_{a,S}$ cooling-air flow rate through stators, (lb/sec)

$\Delta T'_{a,S}$ rise in total temperature of cooling air passing through stators, ($^{\circ}$F)

Thus, measurements of the rise in total temperature and the flow rate of the cooling air are required.

Enthalpy of Combustion Gases

In several equations, the enthalpy $h'_{g,1}$ based on total temperature at the turbine inlet $T'_{g,1}$ occurs. In order to evaluate this factor, the fuel-air ratio and the hydrogen-carbon ratio of the fuel must be known in addition to the temperature. The method of determining the enthalpy, when the values of $T'_{g,1}$, f/a, and H/C are known, is given in reference 14.

The total enthalpy of the mixture of combustion gases and cooling air downstream of the turbine $h'_{m,4}$ is determined in like manner from the total temperature of the mixture $T'_{m,4}$ and from a fuel-air ratio based on the ratio of the weight flow rate of fuel to the sum of the weight flow rates of combustion gases and cooling air through the rotor; the hydrogen-carbon ratio remains unchanged.

With the total temperature of the cooling air at the blade inlet or wherever the air is introduced into the wheel known, the total enthalpy of the cooling air h''_a can be determined by the charts in reference 14.

Two other factors must be known: the ideal enthalpy change of the combustion gases across the turbine $\Delta_s h'_g$ and the ideal enthalpy change of the cooling air from the point of entrance to the rotor to the point of mixing with the gases downstream of the rotor $\Delta_s h'_a$. In addition to the temperature, fuel-air ratio, and hydrogen-carbon ratio required for determining the initial enthalpy states, the total-pressure ratio across the turbine $p'_{g,1}/p'_{m,4}$ must be known to evaluate $\Delta_s h'_g$; similarly the pressure ratio $p'_{a,h}/p'_{m,4}$ must be known to evaluate $\Delta_s h'_a$.

Gas Properties

The viscosity of the gases μ_g surrounding the disk must be known in determining the rotor-disk friction loss. In order to determine the viscosity, the static temperature and the fuel-air ratio of the gases around the disk and hydrogen-carbon ratio of the fuel must be known. The error will be small, however, if the fuel-air ratio of the gases passing across the blades is used. The viscosity of combustion gases can be found in reference 17 and values of R_g are given in reference 5.

The terms θ and δ appearing in the formulas are what might be called correlating factors. In both θ and δ, the terms γ_g and γ_0 must be known. Values of γ_g can be determined from the total temperature, fuel-air ratio, and hydrogen-carbon ratio at the location in question or can be determined by using reference 18. The standard ratio of specific heats $(\gamma_0 = 1.40)$ is for standard sea-level air.

The exponent n used in the term substituted for Reynolds number appears in the expression by virtue of the proportionality that exists between the viscosity of the combustion gases and the gas temperature raised to the nth power. This exponent can be determined from curves of viscosity of gases against temperature for several fuel-air ratios, which in the working range of temperatures (from 1000° to 3000° R) can be closely approximated by parallel straight lines having a common slope of about 0.65. A value for n of about 0.65 is therefore recommended for use in the expression replacing the Reynolds number.

Pressures and Temperatures of Gases surrounding Rotor

The density of the gases surrounding the disk surfaces appear as a parameter in the plots of disk-friction power loss against rotational speed. If the rotor-disk friction losses, presumably obtained by investigations, are required for turbine operating conditions other than those maintained for the investigations, some means must be provided for computing the density. This computation necessitates setting up relations so that the static temperatures and pressures in the region of the disk can be determined from other operating conditions, possibly the conditions of the gases upstream and downstream of the turbine. These relations probably differ for each turbine and consequently no general formula can be cited. An attempt should be made to establish empirical relations between the gas conditions at the upstream face of the disk with those of the gases upstream of the turbine. In like manner, relations should also be established, if possible, between the gas conditions at the downstream face of the disk with those of the gas flow downstream of the turbine.

SUMMARIZING DISCUSSION

For the uncooled turbine, general plots of turbine performance have been developed that incorporate such variables as power, pressure ratio, efficiency, and so forth in a single performance map. Each such general plot has a particular significance. For example, when matching of compressor and turbine is required, reference 19 gives recommended plots. Another example is illustrated in reference 4.

Until cooled turbine-performance data are obtained and plotted in some manner similar to that illustrated in figures 5 to 9, from

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which curve shapes can be established, visualizing a generalized plot that represents all the performance characteristics of a cooled turbine is difficult. Because of the many parameters, which on the basis of analysis appear to affect the performance, construction of a single plot showing all the effects on every characteristic may be either impossible or even undesirable. Nevertheless, adaptation of uncooled turbine-performance plotting schemes to the analysis of cooled turbines for reasons of consistency, convenience, and correlation is recommended.

In addition to the general plots, each investigator may find it useful to construct additional plots that give information of a specific nature. No attempt has been made herein to anticipate the nature of such specific plots. A useful basic set of curves consists of the maximum net power (the gross power minus the pumping losses) and the power required for compressing the cooling air to the pressure required at the point of entrance in the turbine wheel plotted against coolant flow rate for several fixed sets of turbine conditions. Maximum net power represents the case for which blade temperatures are at the allowable limiting values. Such performance curves require the use of practically all the characteristic curves established from data and described in references 1 and 2.

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APPENDIX

SYMBOLS

The following symbols are used in this report:

A	cross-sectional area, sq ft
a	speed of sound, ft/sec
b	length of blade (span), ft
C	constant
C_R	rotor-tip clearance, ft
c_p	specific heat at constant pressure, Btu/(lb)(°F)
D	characteristic dimensions of turbine, ft; diameter, ft
E	mechanical energy, Btu/lb
f	unknown function (Superscripts differentiate unknown functions.)
f/a	fuel-air ratio
f_s	slip factor
g	ratio of absolute to gravitational unit of mass, lb/slug; acceleration due to gravity, ft/sec^2
H/C	hydrogen-carbon ratio
h	enthalpy, Btu/lb
h'	enthalpy based on total temperature, Btu/lb
h''	enthalpy based on total temperature relative to moving blades, Btu/lb
J	mechanical equivalent of heat, 778.3 ft-lb/Btu
K	constant
N	speed of rotation, rpm

n	exponent
P	power, hp
p	static pressure, lb/sq ft absolute
p'	total pressure, lb/sq ft absolute
p''	total pressure relative to moving blades, lb/sq ft absolute
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R	gas constant, ft-lb/(lb)($^{\circ}$F)
Re	Reynolds number
r	radius, ft
T	static temperature, $^{\circ}$R
T'	total temperature, $^{\circ}$R
T''	total temperature relative to moving blades, $^{\circ}$R
u	tangential velocity, ft/sec
V	absolute velocity, ft/sec
W	velocity relative to moving blades, ft/sec
w	weight flow rate, lb/sec
α	angle of absolute velocity (fig. 4), deg
β	angle of relative velocity, deg
γ	ratio of specific heats
Δ	prefix to indicate change
δ	pressure correction ratio, $\gamma p'/\gamma_0 p_0$
η	efficiency
θ	temperature correction ratio, $\gamma T'/\gamma_0 T_0$

μ	absolute viscosity, slugs/(sec)(ft)
ρ	mass density, slugs/cu ft
ρ'	mass density based on total conditions, slugs/cu ft
σ	thickness coefficient (unity for reaction turbines)
ω	angular velocity, radians/sec

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Subscripts:

a	cooling air
ad	adiabatic
aL	cooling-air losses
ax	axial
B	blade
BE	bearing and gear losses
BL	rotor-blade loss
g	combustion gas
h	station at blade root
id	ideal
L	lubricant (oil)
M	miscellaneous losses
m	mixture (referring to mixing of combustion gases and cooling air)
O	NACA sea-level air
P	pumping power from blade root to tip when used with P
PR	pumping power from center line of engine to blade root when used with P
R	rotor

RF rotor-disk friction loss

S stator

SL stator loss

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s isentropic process

sh shaft

T station at blade tip

TL rotor-tip leakage loss

u tangential component

x radial distance from blade root to point on blade span considered, ft

Stations through turbine

1 stator inlet

2 stator outlet or rotor-blade inlet

3 rotor-blade outlet

4 downstream of turbine where complete mixing of combustion gases and cooling air occurs

5 jet-nozzle outlet

$\left.\begin{array}{l} A,A' \\ B,B' \\ C,C' \\ D,D' \\ 3' \\ 4' \\ 5' \end{array}\right\}$ State points on temperature-entropy diagram

The quantities related to absolute total conditions are designated by a prime. The quantities related to relative total conditions are designated by a double prime.

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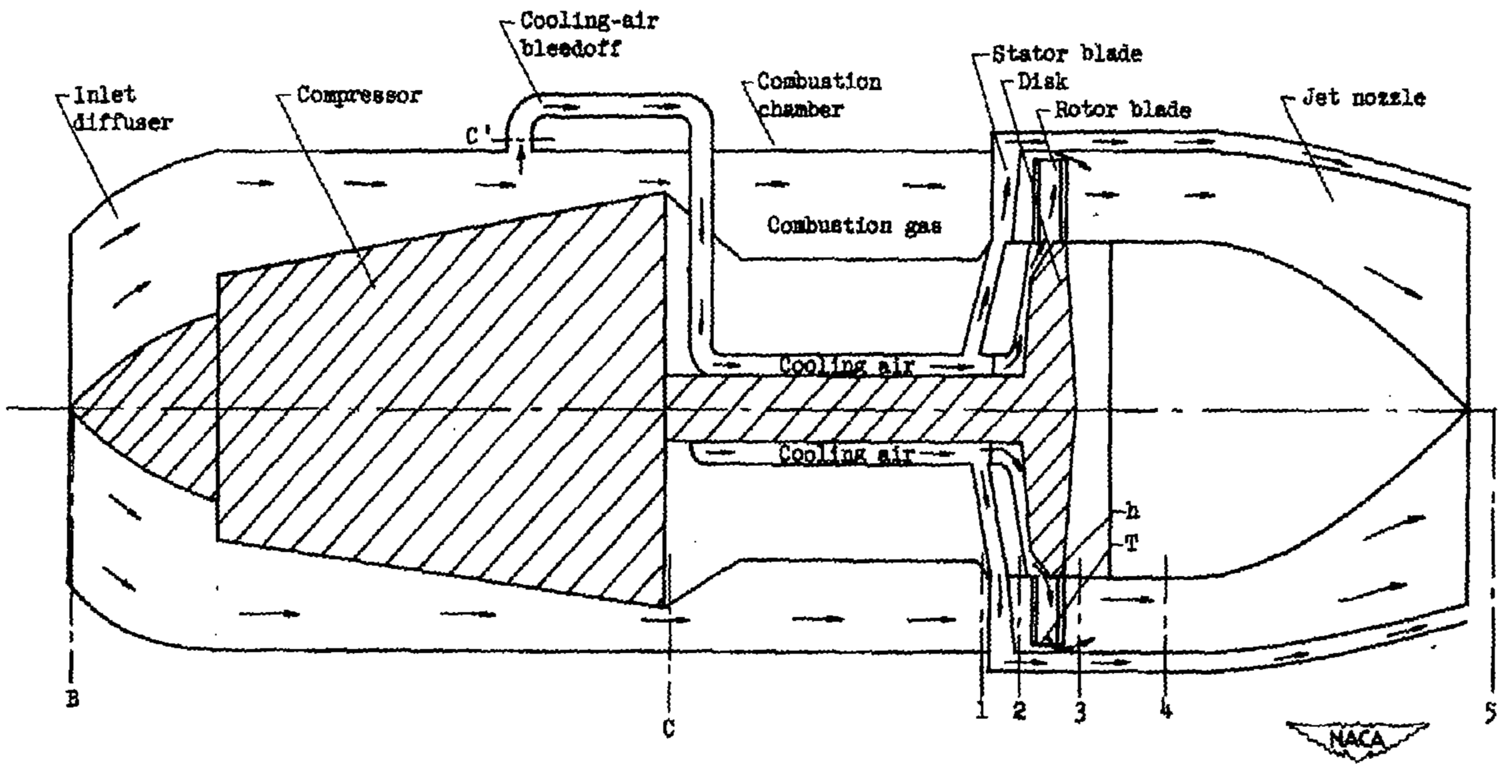

Figure 1. - Schematic diagram of turbojet engine showing turbine cooling-air flow path and suggested measuring stations for investigations of air-cooled turbines.

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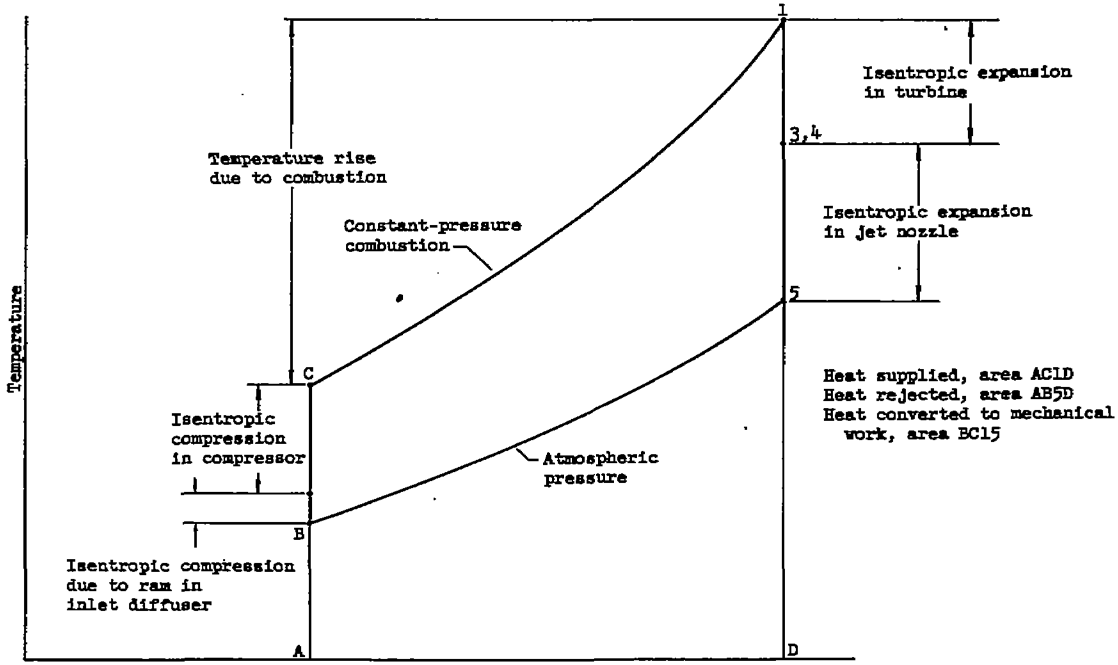
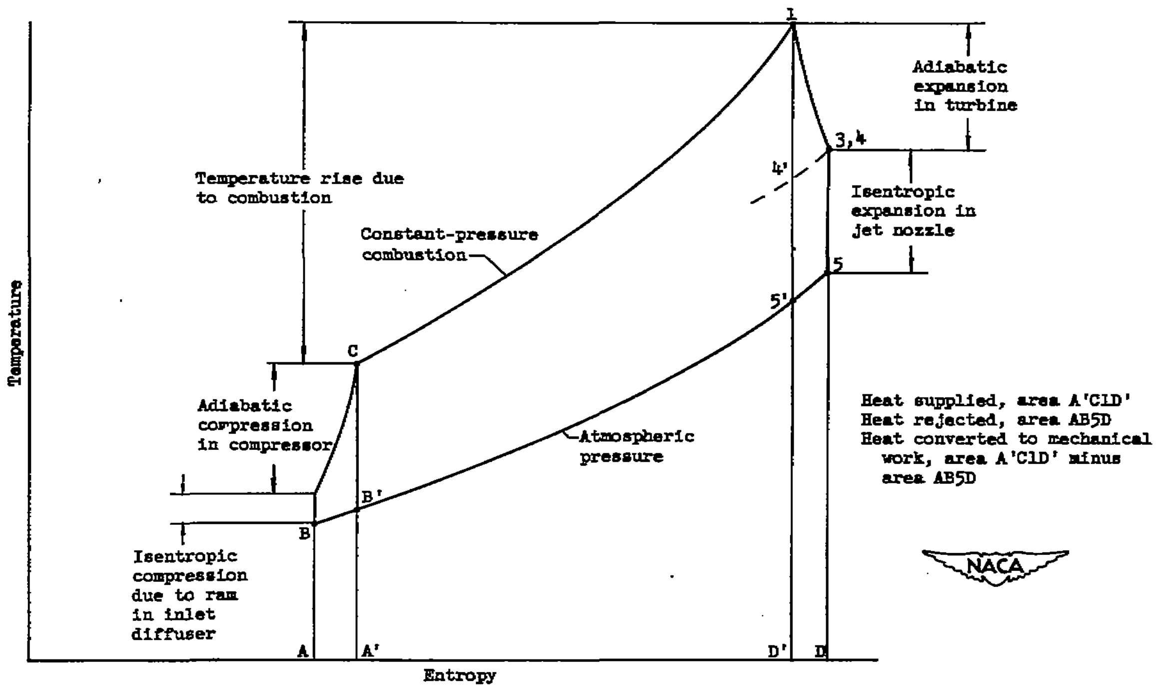
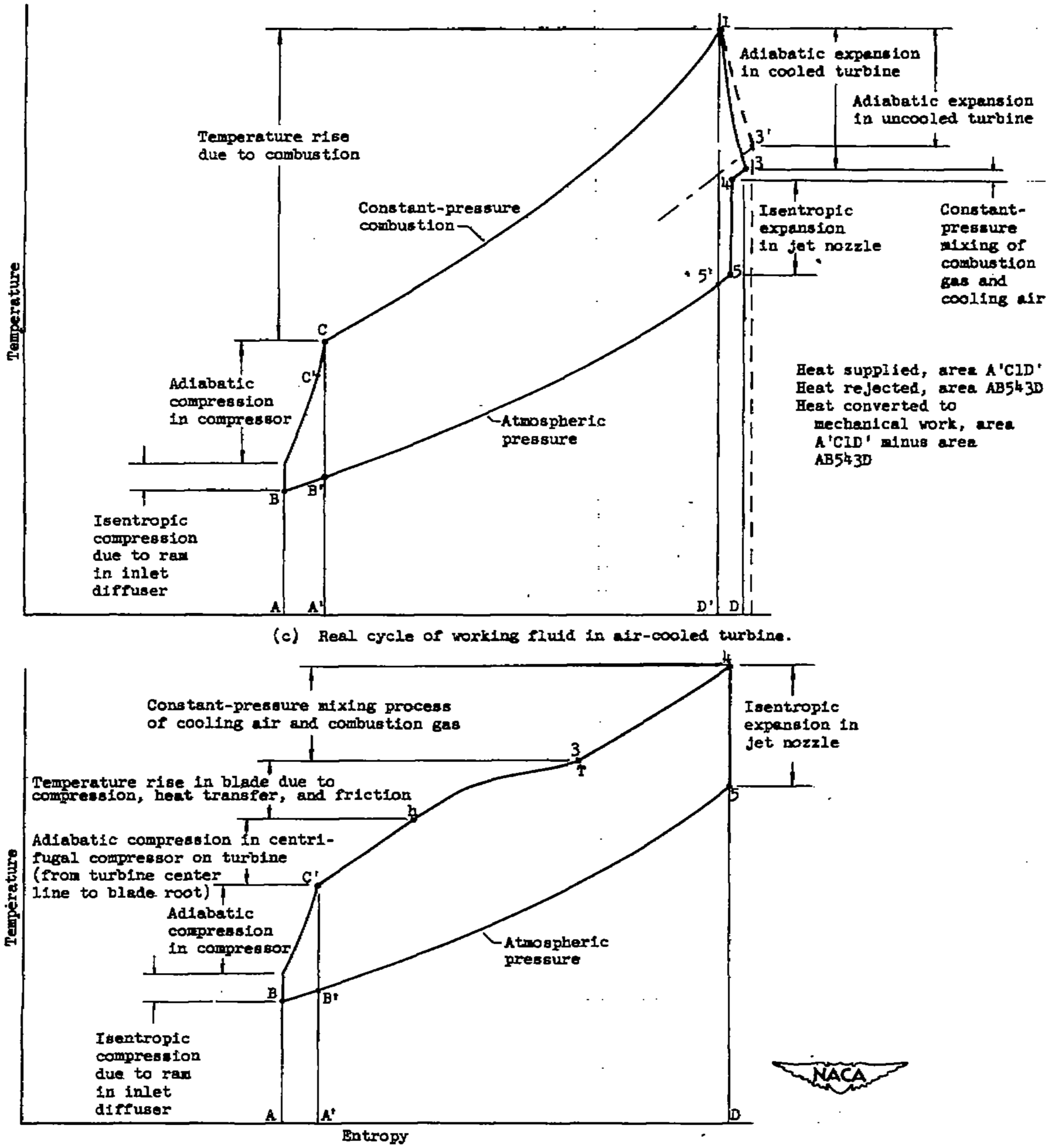


(a) Ideal cycle of working fluid (Brayton cycle).



(b) Real cycle of working fluid in uncooled turbine.

Figure 2. - Temperature-entropy diagram for turbojet engine.



(c) Real cycle of working fluid in air-cooled turbine.

(d) Real cycle of rotor-blade cooling air in air-cooled turbine.

Figure 2. - Concluded. Temperature-entropy diagram for turbojet engine.

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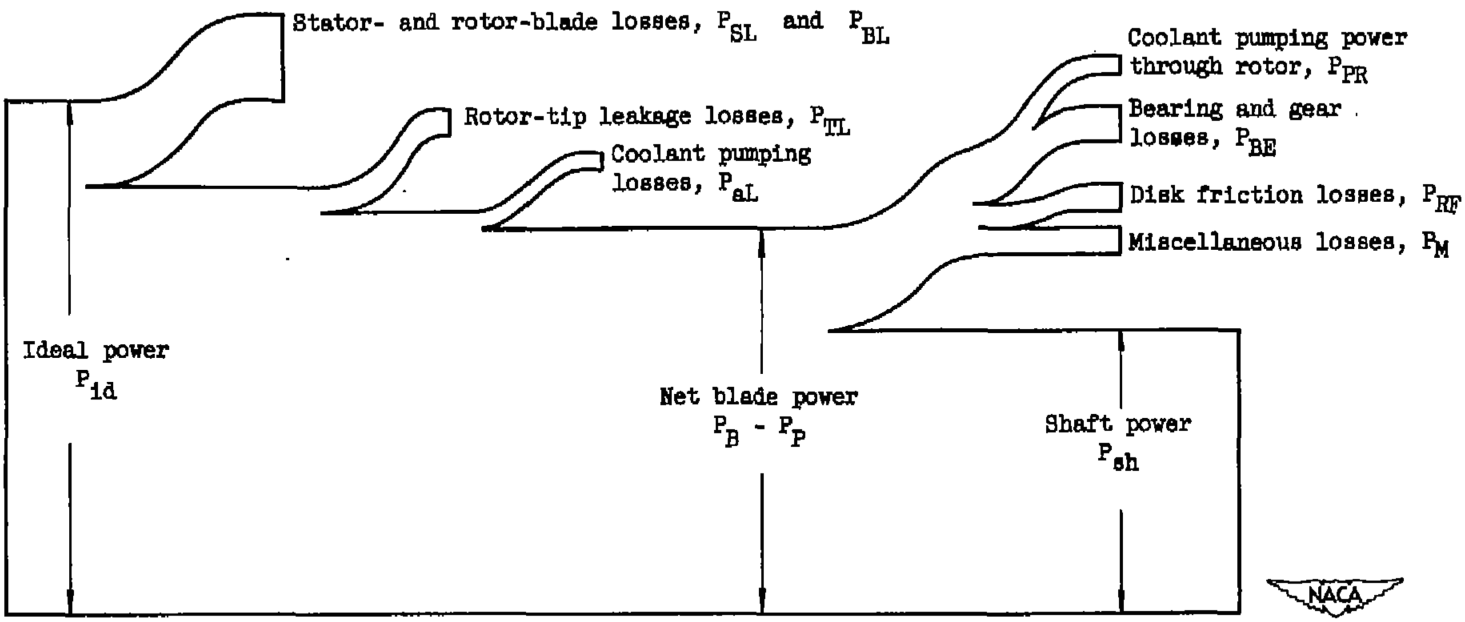

Figure 3. - Schematic relation of power concepts of cooled turbine showing power-loss dissipation.

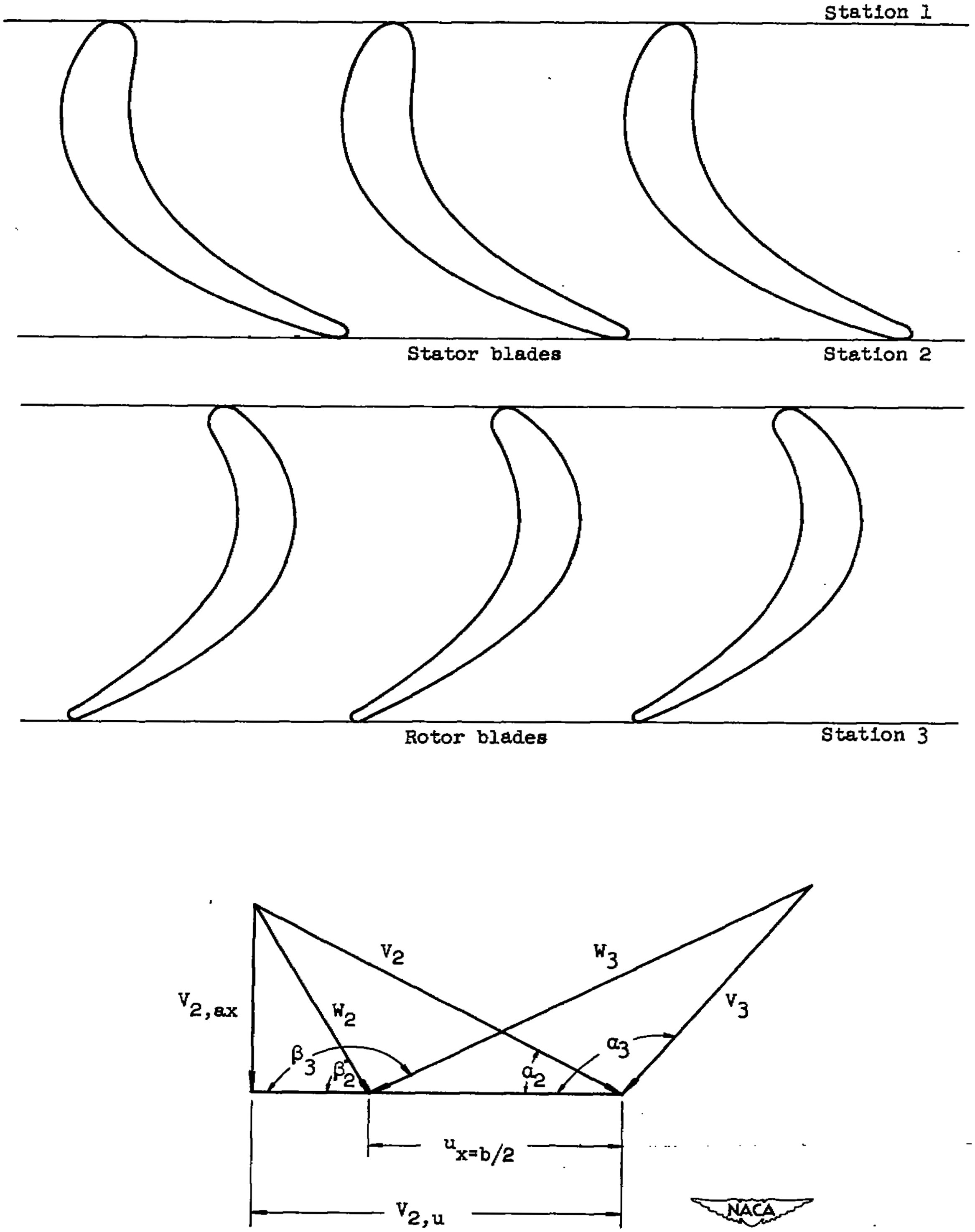

Figure 4. - Velocity diagram for stator and rotor blades.

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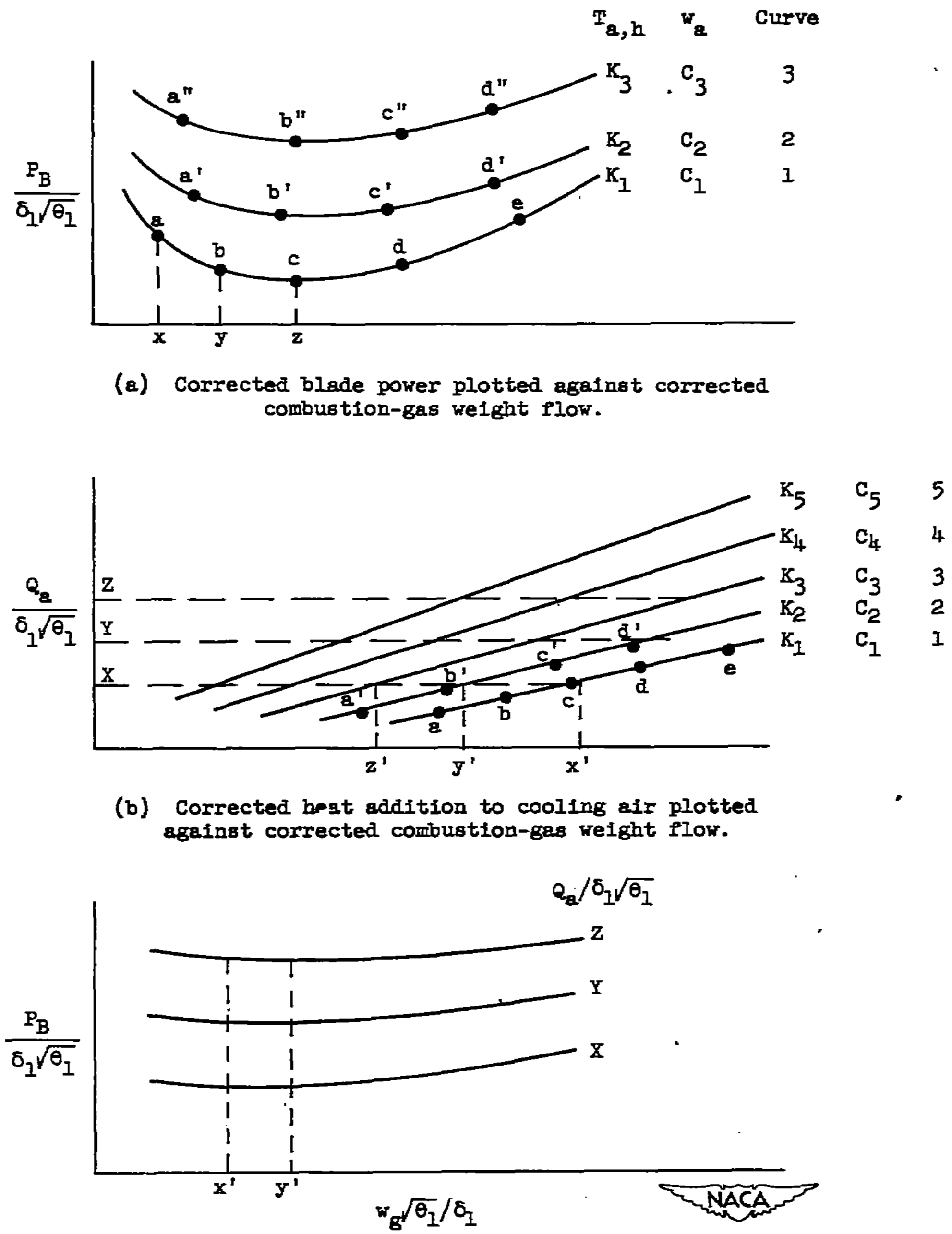
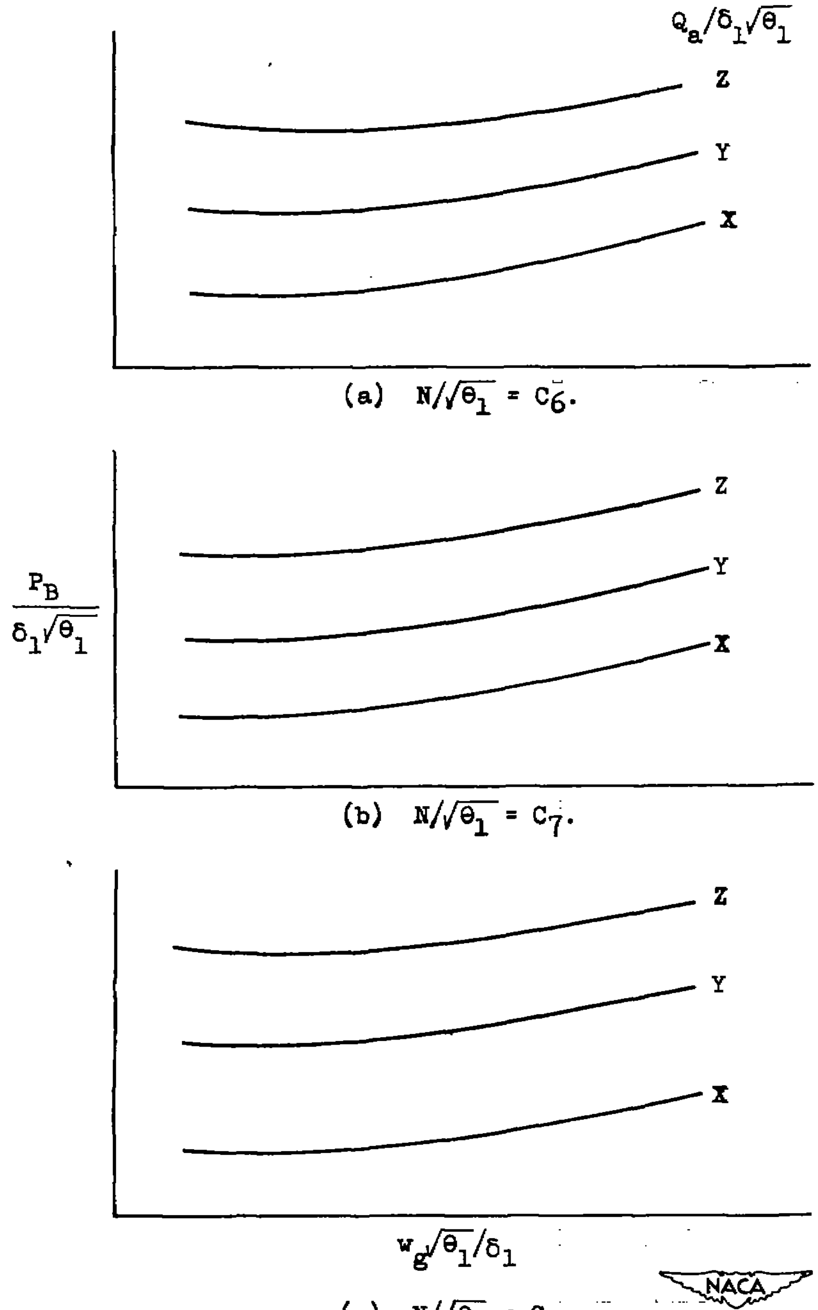


(a) Corrected blade power plotted against corrected combustion-gas weight flow.

(b) Corrected heat addition to cooling air plotted against corrected combustion-gas weight flow.

(c) Corrected blade power plotted against corrected combustion-gas weight flow for constant values of corrected heat addition to cooling air.

Figure 5. - Plotting methods used to obtain corrected blade power as function of corrected combustion-gas weight flow and corrected heat addition to cooling air. $Q_S/\delta_1\sqrt{\theta_1}$, 0; $N/\sqrt{\theta_1}$, constant; $p'_{g,1}/(T'_{g,1})^{n+1/2}$, constant; and $p'_{g,1}$, constant.



(c) $N/\sqrt{\theta_1} = C_8$.

Figure 6. - Variation of corrected blade power with corrected combustion-gas weight flow for several corrected heat additions to cooling air and three corrected turbine speeds. $Q_S/\delta_1\sqrt{\theta_1}$, 0; $p'_{g,1}/(T'_{g,1})^{n+1/2}$, constant; and $p'_{g,1}$, constant.

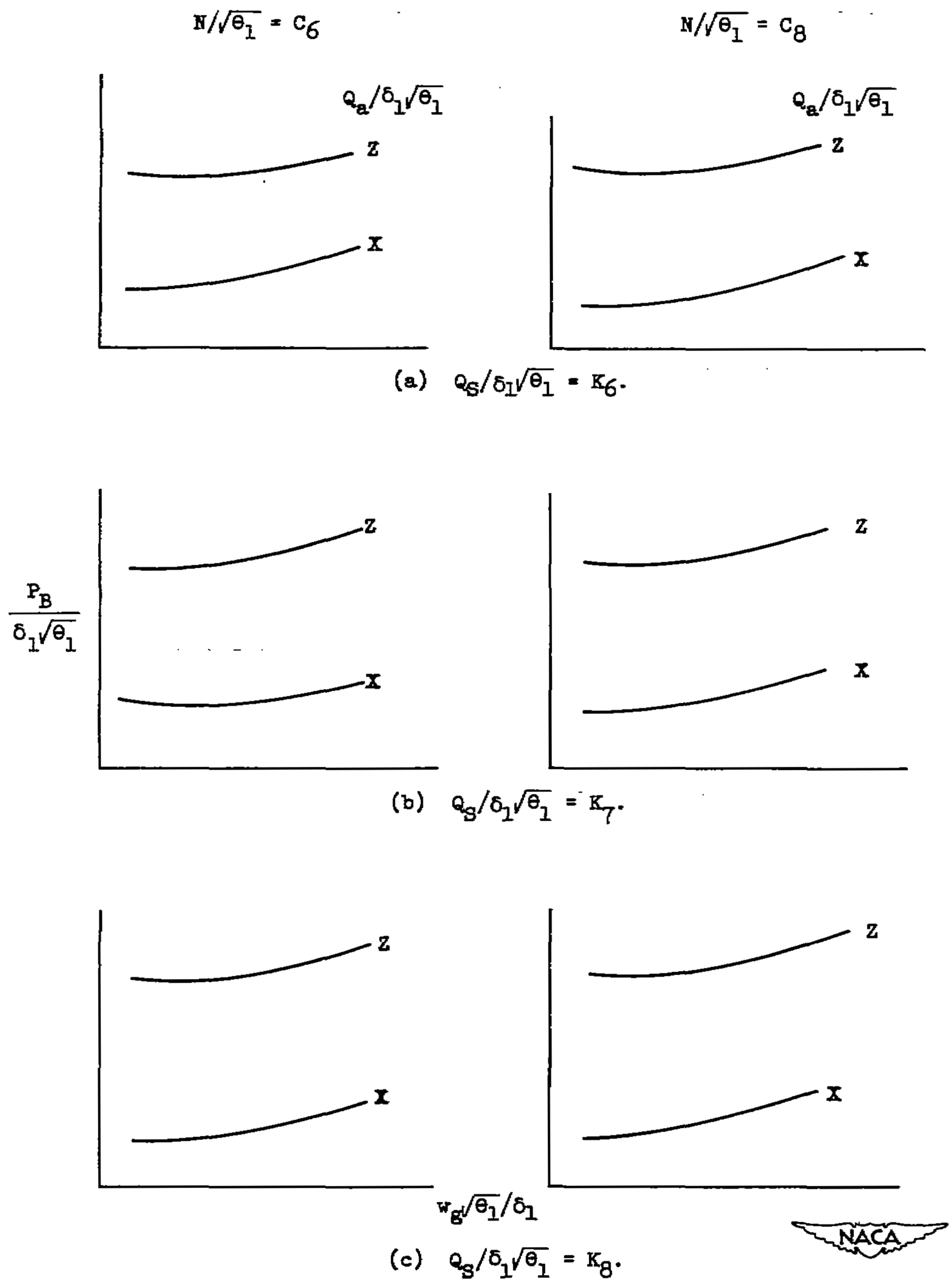
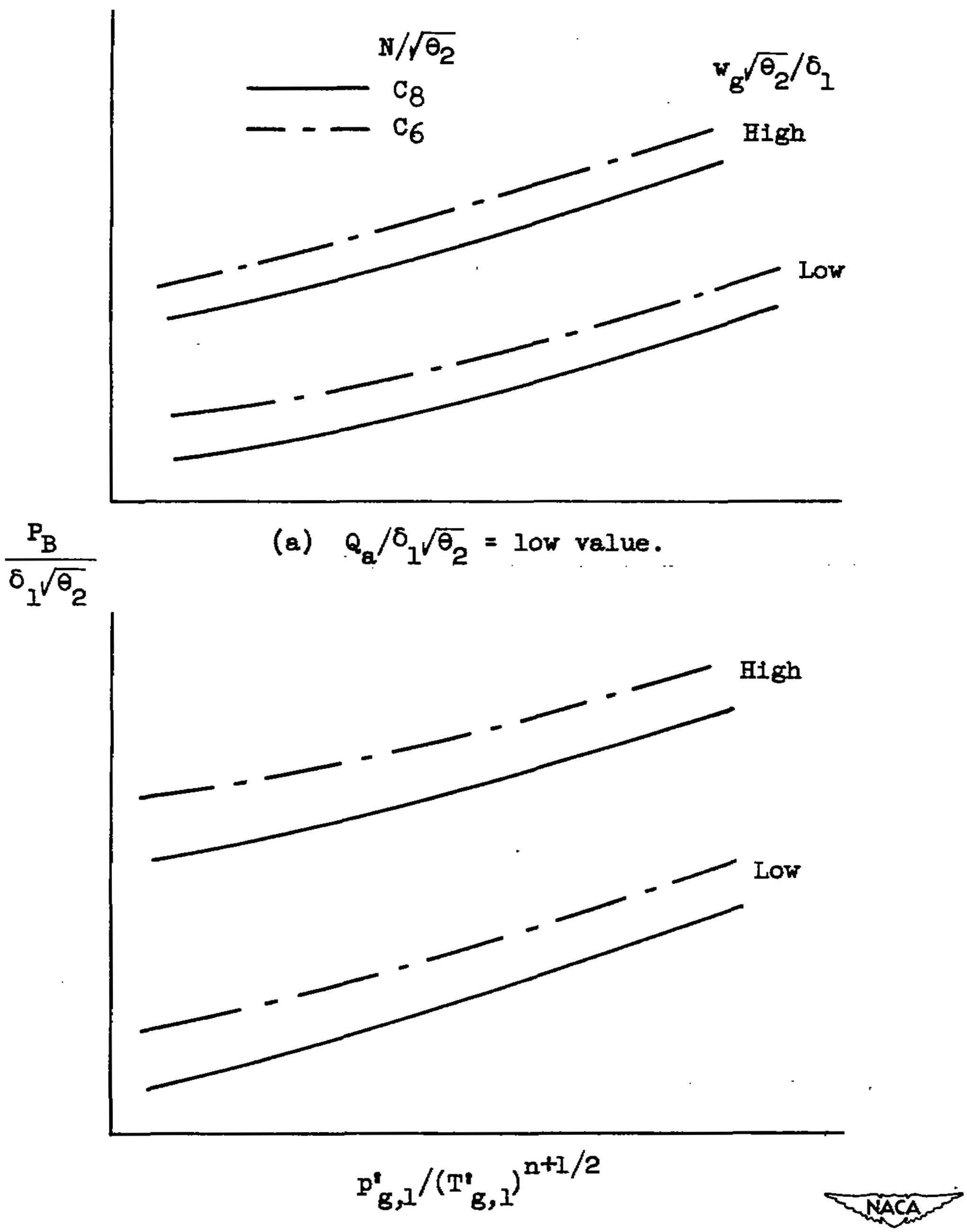


Figure 7. - Variation of corrected blade power with corrected combustion-gas weight flow for two values of corrected heat addition to cooling air, two values of corrected turbine speed, and several values of corrected heat addition to stators.



(b) $Q_a/\delta_1\sqrt{\theta_2}$ = high value.

Figure 8. - Variation of corrected blade power with Reynolds number parameter for several values of corrected turbine speed, corrected combustion-gas weight flow, and corrected heat addition to cooling air.

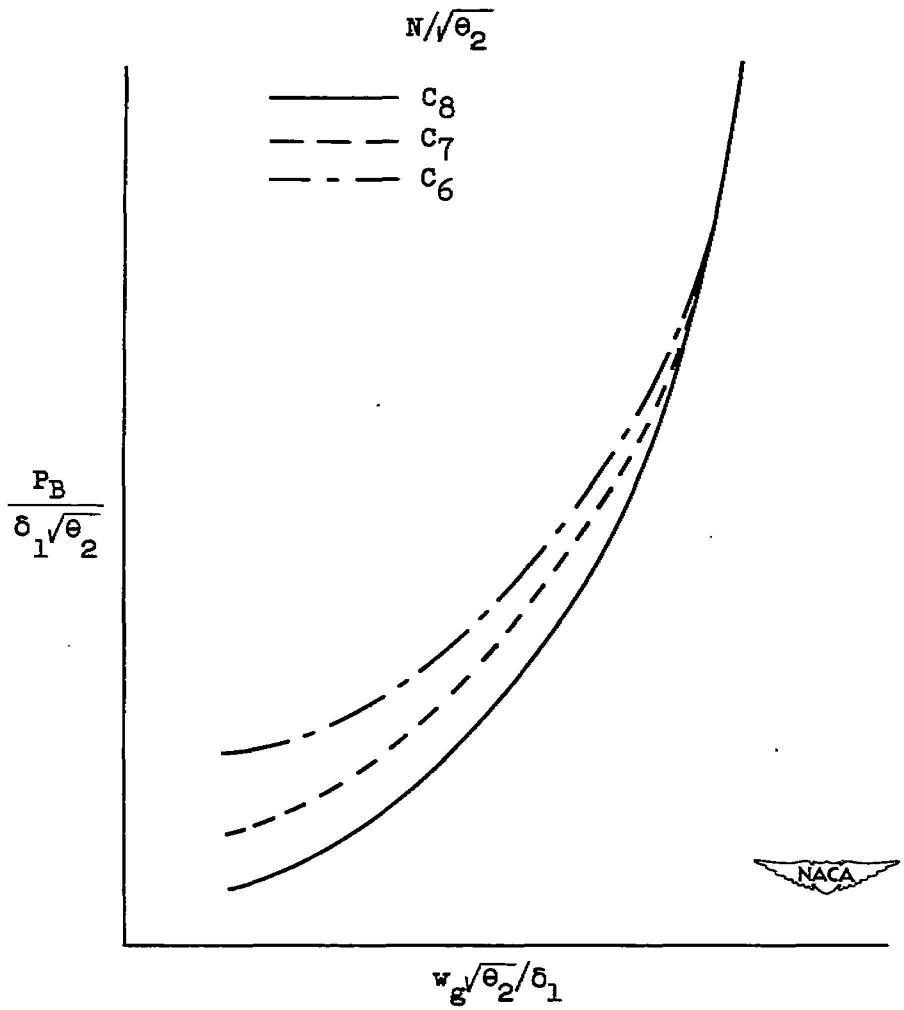


Figure 9. - Variation of corrected blade power with corrected combustion-gas weight flow for three corrected turbine speeds at constant corrected heat addition to cooling air.