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RESEARCH MEMORANDUM

APPROXIMATE RELATIVE-TOTAL-PRESSURE LOSSES OF AN INFINITE CASCADE OF SUPERSONIC BLADES WITH FINITE LEADING-EDGE THICKNESS

By John F. Klapproth

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

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APPROXIMATE RELATIVE-TOTAL-PRESSURE LOSSES OF AN INFINITE CASCADE OF SUPERSONIC BLADES WITH FINITE LEADING-EDGE THICKNESS

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SUMMARY

By application of a hyperbolic approximation to the form of the bow waves caused by blunt leading edges on an infinite cascade of supersonic blades, the approximate losses in relative total pressure due to the external bow-wave system arising from blunt edges and subsonic axial entrance velocities were computed. The losses increase linearly with leading-edge radius for any given relative Mach number. For a relative Mach number of 1.60, leading-edge radii may be approximately 1.5 percent of the normal blade gap with a 1-percent loss of relative total pressure.

INTRODUCTION

In an effort to minimize the pressure losses through supersonic compressors, blade leading edges have been designed with a perfect wedge. On the basis of fabrication and durability, however, a knife edge is impractical; consequently, the leading edge must be given a finite thickness. The problem of estimating the losses associated with a given leading-edge thickness is then encountered.

The presence of a blunt edge on blades with a subsonic axial velocity causes the formation of a standing wave pattern, as illustrated in figure 1(a). A detached bow wave forms in front of each blade, with normal shock losses occurring immediately in front of the blunt edge. The losses decrease along the bow wave as the distance from the nose increases until the wave approaches a Mach wave and the losses become negligible.

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An investigation was conducted at the NACA Lewis laboratory to estimate the approximate losses incurred through this external wave pattern in order to determine the practical thickness that may be used at the leading edge. The losses due to the contained wave pattern (inside the blade passage) may be computed from shock relations for oblique and normal shock, and are determined by the blade design and the properties of the flow upstream of the cascade.

SYMBOLS

The following symbols are used in this report:

b number of blades

M' relative Mach number

P' relative total pressure

r radius

s normal distance between blades, $(2\pi r \cos \beta')/b$

x coordinate measured along relative free-stream direction

x_0 distance from foremost point of detached shock to intercept of its asymtote on x-axis

y coordinate measured perpendicular to relative free-stream direction

β' angle between relative flow direction and axis of rotation

γ ratio of specific heats

φ angle between shock and free-stream direction

Subscripts:

0 far upstream of rotor

1 immediately before shock

2 immediately behind shock

3 entrance to rotor passage

LE leading edge

SB point on leading edge where blade contour is inclined at wedge angle corresponding to shock detachment

ANALYSIS

The losses that occur because of the bow-wave system can be expressed in terms of the relative total pressure far upstream of the blades P'_0 and the relative total pressure P'_3 at the entrance into the rotor passage.

The relative-total-pressure loss of the air entering each passage can be considered equal to the total loss along one shock wave, integrated from the blade to infinity, which is illustrated by figure 1(a). The flow entering the passage 0-c is seen to pass through the shock wave caused by the blade at 0 between the points 1 and 2. The flow passes through the shock wave from the preceding blade between 2' and 3'; this region is seen to be identical with the region 2-3 of the shock from 0. Similarly, 3"-4" is identical with 3-4, and so forth.

Because the mass flow entering each passage suffers losses identical to the loss along one shock wave from the leading edge to infinity, the average relative-total-pressure loss can be expressed as

$$\left(1 - \frac{P'_3}{P'_0}\right) = \frac{\int_0^{\infty} \left(1 - \frac{P'_2}{P'_1}\right) dy}{\int_0^{s} dy} \tag{1}$$

The coordinate y is taken perpendicular to the mean relative velocity at the entrance.

The total-pressure recovery across the shock wave at any point is (reference 1)

$$\frac{P'_2}{P'_1} = \left(\frac{2\gamma}{\gamma+1} M'^{\,2}_1 \sin^2\varphi - \frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{1-\gamma}} \left[\frac{(\gamma-1)\, M'^{\,2}_1 \sin^2\varphi + 2}{(\gamma+1)\, M'_1 \sin^2 \varphi}\right]^{\frac{\gamma}{1-\gamma}} \tag{2}$$

The evaluation of equation (1) can be simplified if the change in relative total pressure due to the bow-wave system is small through the region 2'-3' and the preceding waves (fig. 1(a)). The relative Mach number before the shock may then be assumed constant, and the problem is reduced to that of a single blunt body in a uniform supersonic stream (fig. 1(b)). The x direction is parallel to the entrance region of the blade. The integration of equation (1) then requires only a relation between the shock angle φ and the coordinate y. (When the blade is considered as an isolated symmetrical body, the flow is at a slight angle of attack, equal to half the included wedge angle of the blade.)

By approximating the bow wave with a hyperbola asymtotic to the Mach lines (reference 2), Moeckel obtained good correlation between observed and computed shock forms. By application of this hyperbolic approximation to find the shock location and inclination, the approximate shock losses due to the bow waves may be determined from equations (1) and (2).

By following the notation of reference 2, which uses as a reference dimension the y-coordinate of the sonic point on the body y_{SB} (defined as the point on the body where the contour is inclined at the wedge angle corresponding to shock detachment), the form of the wave is expressed as

$$\frac{y}{y_{SB}} = \frac{1}{\sqrt{M'_1{}^2-1}}\sqrt{\left(\frac{x}{y_{SB}}\right)^2 - \left(\frac{x_0}{y_{SB}}\right)^2} \tag{3}$$

where x/y_{SB} is measured along the free-stream direction and x_0/y_{SB} is a constant that locates the hyperbola with respect to the leading edge and is a function only of the free-stream Mach number. The values of x_0/y_{SB} as a function of M'_1 are given in reference 2. Differentiation of equation (3) gives the slope of the shock as a function of y/y_{SB}. Then

$$\varphi = \text{arc tan}\sqrt{\frac{\left(\frac{x_0}{y_{SB}}\right)^2 + (M'_1{}^2-1)\left(\frac{y}{y_{SB}}\right)^2}{(M'_1{}^2-1)\frac{y}{y_{SB}}}} \tag{4}$$

1232

The loss in relative total pressure is determined by substitution of equations (2) and (4) in equation (1). Integrating the denominator of equation (1) in terms of the reference dimension gives the following relation:

$$\left(1 - \frac{P'_3}{P'_0}\right) = \frac{y_{SB}}{(2\pi r \cos\beta')/b} \int_0^{\infty} \left(1 - \frac{P'_2}{P'_1}\right) d\left(\frac{y}{y_{SB}}\right) \tag{5}$$

Inasmuch as the value of the integral is a function only of the relative Mach number, the losses increase linearly with y_{SB} for any given relative Mach number. Because an analytical integration was inconvenient, a numerical integration was made. The results are shown in figure 2 for relative Mach numbers of 1.40, 1.60, 1.80, and 2.00.

As a final simplification, the coordinate y_{SB} was considered the leading-edge radius. For a relative Mach number of 1.60, the coordinate y_{SB} is about 3 percent less than the leading-edge radius. Figure 2 can then be considered a plot of relative-total-pressure loss against the ratio of the leading-edge radius to the normal distance between blades at the entrance. At a relative Mach number of 1.60, the leading-edge radius may be approximately 1.5 percent of the normal blade gap with a 1-percent loss of relative total pressure. For the supersonic rotor investigated in reference 3, the blades could have a leading-edge radius of 0.007 inch or a thickness of 0.014 inch with a loss of about 1 percent, or a 0.030-inch thickness with an approximate 2-percent loss in relative total pressure because of the external bow-wave system.

CONCLUDING REMARKS

The approximate relative-total-pressure loss due to the external bow-wave system caused by blunt edges of an infinite cascade of supersonic blades, was computed. The losses for any given relative Mach number increase linearly with the leading-edge radius.

For a relative Mach number of 1.60, the leading-edge radius may be approximately 1.5 percent of the normal blade gap for a 1-percent loss of relative total pressure. 1232

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio.

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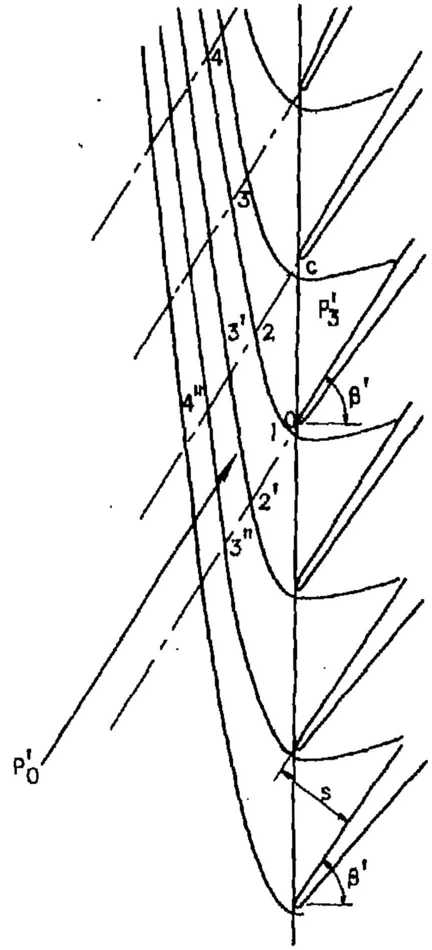
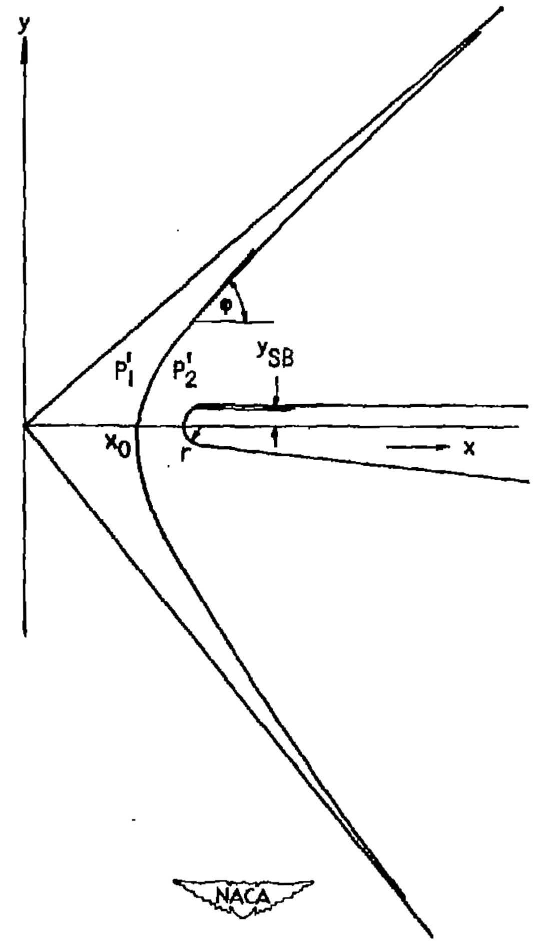


(a) Cascade.



(b) Notation diagram for single blade.

Figure 1. - Wave pattern caused by blunt leading edges on infinite cascade with subsonic axial velocities.

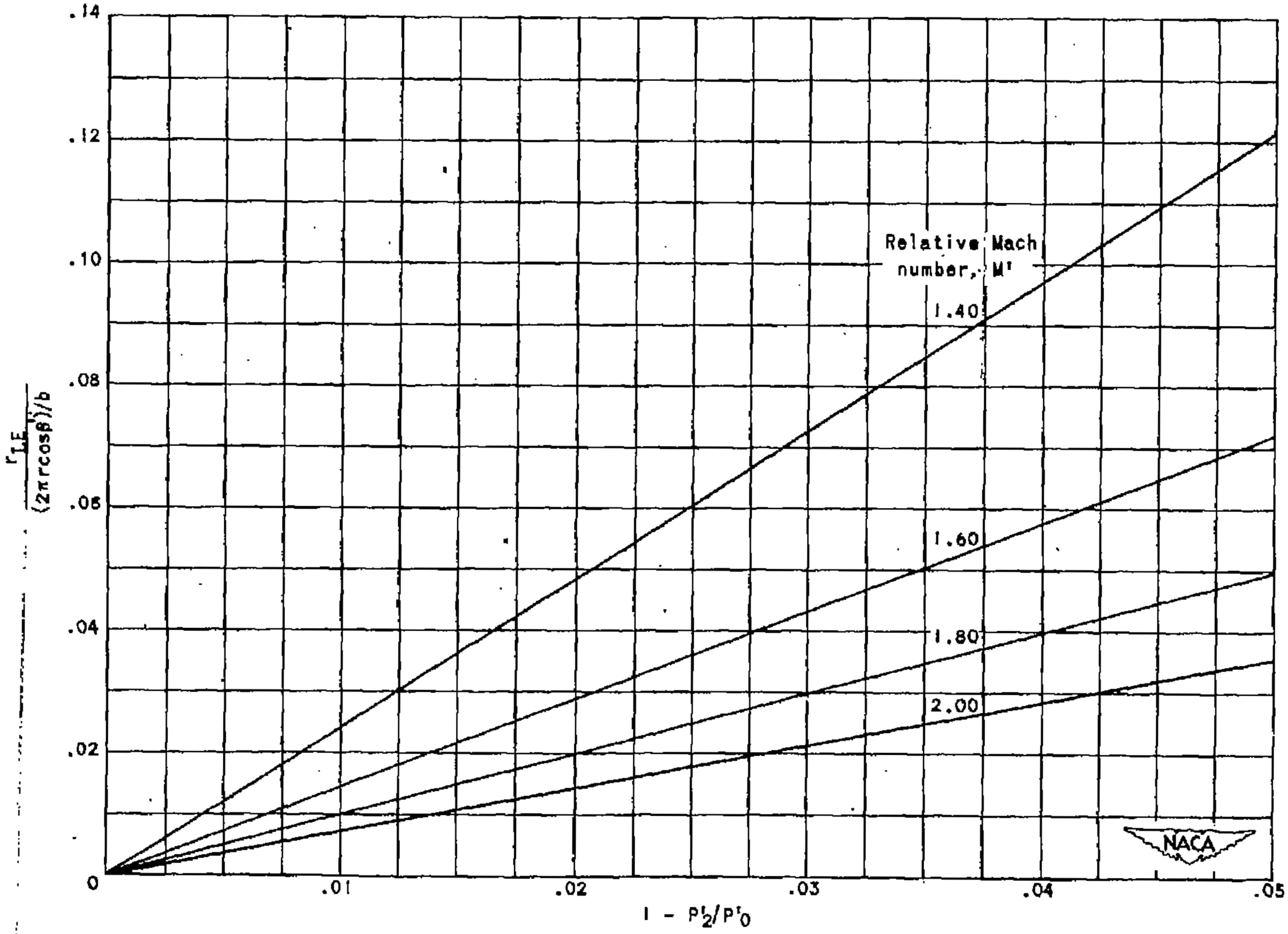


Figure 2. - Relative-total-pressure loss as function of ratio of leading-edge radius to normal distance between blades for various Mach numbers.