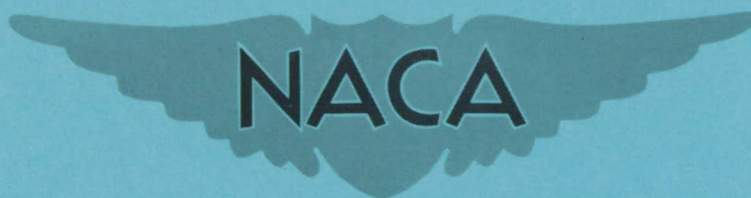


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RESEARCH MEMORANDUM

SURVEY OF ADVANTAGES AND PROBLEMS ASSOCIATED WITH
TRANSPIRATION COOLING AND FILM COOLING OF
GAS-TURBINE BLADES

By E. R. G. Eckert and Jack B. Esgar

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

NATIONAL ADVISORY COMMITTEE
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SUMMARY

Transpiration and film cooling promise to be effective methods of cooling gas-turbine blades; consequently, analytical and experimental investigations are being conducted to obtain a better understanding of these processes. This report serves as an introduction to these cooling methods, explains the physical processes, and surveys the information available for predicting blade temperatures and heat-transfer rates. In addition, the difficulties encountered in obtaining a uniform blade temperature are discussed, and the possibilities of correcting these difficulties are indicated. Air is the only coolant considered in the application of these cooling methods.

INTRODUCTION

Gas-turbine blades are usually cooled by forcing air through the hollow interior of the blade. The possibilities of this cooling method are limited by the heat-transfer rates from the blade to the coolant. Because of this limitation, other blade cooling methods are being investigated. Transpiration cooling, also called sweat or porous-wall cooling, promises to be a very effective means of cooling objects such as rocket nozzles (reference 1) that must be in contact with high-temperature, high-velocity gas streams. This method of cooling in which the coolant is forced through a porous wall to form an insulating layer of fluid between the wall and the gas stream is now being considered for use in gas-turbine stator and rotor blades. Film cooling is another means of cooling surfaces that is similar to transpiration cooling, although somewhat less effective. With film cooling the insulating fluid layer or film is formed by allowing a coolant to flow through slots or holes and then over the surface. References 2 and 3 have shown that film cooling with air as the coolant can be used effectively to decrease the temperatures of gas-turbine blades.

Since transpiration and film cooling show considerable promise as methods of turbine-blade cooling, experimental and analytical investigations are being conducted at the NACA Lewis laboratory to gain more knowledge and to expand the application of these cooling methods. This report serves as an introduction to the basic principles of transpiration and film cooling and reviews briefly the status of methods for calculating heat transfer and temperatures in blades cooled by these means. It deals only with the thermodynamic aspect of the cooling methods. Neither stresses, a very important item for both methods, nor fabrication techniques are considered, but problems that have to be solved in the design and fabrication of transpiration and film-cooled blades are discussed.

SYMBOLS

The following symbols are used in this report:

| | |
|----------------|---|
| A | area, sq ft |
| C_1 to C_4 | constants |
| c_p | specific heat at constant pressure, Btu/(lb)(°F) |
| d | average width of pore in porous metal, ft |
| F | factor in equation (11) |
| g | acceleration due to gravity, ft/sec ² |
| H | gas-to-surface heat-transfer coefficient, for porous or film-cooled surface, Btu/(°F)(sq ft)(sec) |
| H' | gas-to-surface heat-transfer coefficient for solid surface, Btu/(°F)(sq ft)(sec) |
| k | thermal conductivity, Btu/(°F)(ft)(sec) |
| L | length, ft |
| m | exponent |
| Nu | Nusselt number, $\frac{Hx}{k_g}$ |
| p | pressure, lb/sq ft |

| | |
|----------|--|
| Pr | Prandtl number, $\frac{c_p \mu_g g}{k_g}$ |
| Q | heat flow rate, Btu/sec |
| r | ratio of velocities in boundary layer (See equation (17)) |
| Re | Reynolds number, $\frac{V_g x}{\nu_g}$ |
| s | slot width, ft |
| St | Stanton number, $\frac{Nu}{RePr}$ or $\frac{H}{\rho_g c_p V_g}$ |
| T | temperature, °F |
| v | flow volume per unit area and time, ft/sec |
| V | velocity, ft/sec |
| w | flow rate per unit area, lb/(sec)(sq ft) |
| W | flow rate, lb/sec |
| x | distance, ft |
| y | distance from center line, ft (See fig. 3) |
| Y | distance from center line where $\frac{\theta}{\theta_{max}} = 0.5$ (See fig. 3) |
| δ | boundary-layer thickness, ft |
| θ | temperature difference, °F |
| μ | viscosity, (lb)(sec)/sq ft |
| ν | kinematic viscosity, $\frac{\mu_g}{\rho}$, sq ft/sec |
| ρ | density, lb/cu ft |

τ thickness, ft

$\phi = \frac{\rho c c_p v_c}{H'}$

Subscripts:

ad adiabatic

c coolant

e effective

g gas

L laminar sublayer

m mean

max maximum

w wall

1,2 designate locations

PHYSICAL NATURE OF FILM AND TRANSPIRATION COOLING PROCESSES

Internal Cooling

The conventional method of cooling turbine blades is to make the blades hollow and force cooling air through the blade interior. Usually, the cooling air enters the blade at the root and discharges into the gas stream at the tip. To make this cooling method more effective, inserts can be provided within the blade to direct the cooling air along the inner wall, or the inner surface area can be increased by the addition of fins (fig. 1(a)). An inherent disadvantage of this method is that the gas-to-blade heat-transfer coefficient is very high because of the high gas velocities. It is difficult to increase the blade-to-coolant heat-transfer coefficient to a value higher than the gas-to-blade coefficient. When both heat-transfer coefficients are equal on a hollow or an insert blade, the blade wall temperature lies half way between the gas and the coolant temperatures. It would be possible to decrease effectively the blade wall temperature by placing some thermally insulating material on the outside surface of the blade. Such an insulation has been tried in the form of ceramic coatings, but coatings of sufficient

thickness for insulating purposes lack the required structural strength for gas-turbine operation. Some method of coating the blade with a layer of air would insulate the blade better than any ceramic coating because of the much lower thermal conductivity of air.

Film Cooling

The method of film cooling uses some gas or liquid to build an insulating layer between the blade wall and the hot gases as shown in figure 1(b). The coolant is discharged by slots to the outside of the surface and is carried downstream by the outside gas flow. In this way an insulating film is built along the surface. This film is gradually destroyed on its way downstream by turbulent mixing, which allows the wall temperature to rise as shown in figure 2. The coolant film has to be renewed at a certain distance by blowing new coolant through additional slots, or the wall temperature would eventually approach gas temperature. The insulating effect of a gas layer is very good because gases have a lower thermal conductivity than any known insulating material, but a liquid layer is even better because the liquid coolant is evaporated by the heat flowing from the hot gases into the coolant film. The heat required for evaporation keeps the temperatures of the liquid film and the blade surface down to the evaporation temperature until a point downstream from the slot is reached where the liquid is completely evaporated (reference 4). Inasmuch as the heat absorption by the evaporation is the determining factor in this cooling method, it may be called evaporative film cooling. For aircraft gas turbines, air is the most readily available coolant; other gases or liquids would have to be carried as extra weight in the airplane. The combustion gases flowing around the turbine blades have physical properties that differ only slightly from the properties of air; therefore, film cooling will be considered for the case where the coolant is the same gas as the combustion gases in this report.

A simple physical process that is similar to film cooling and therefore helps to explain it is shown in figure 3. A line source of heat S is placed in a uniform gas stream. This heat source may be realized, for instance, by a heated wire. The heat that is transferred from the wire to the gas is carried downstream by convection and, at the same time, diffused by turbulent mixing. The temperature profiles in the gas flow downstream of the heat source have the shapes indicated in the figure for two distances from the heat source. An investigation by Corrsin and Uberoi (reference 5) shows that the temperature profiles at each location are similar in shape. They can be superimposed into a single curve by an appropriate change in the scale of the coordinates, as shown in the lower part of figure 3. The abscissa of this curve is

the ratio y/Y , where y is the distance from the center line I-I and Y is the distance where the temperature difference as measured against the upstream temperature reaches the value of 0.5 of the temperature difference on the center line I-I. The ordinate is the ratio θ/θ_{\max} of the temperature difference θ between the upstream temperature and the temperature at y , and θ_{\max} , the same temperature difference measured on the center line I-I (for $y = 0$). The investigations in reference 5 show that Y increases linearly with the distance x from the heat source. The area under all the temperature profiles at different distances has to be the same for continuity reasons when the loss in momentum of the fluid by the heat source can be neglected. In this case the temperature difference θ_{\max} on the center line decreased inversely proportional to the distance x . Conditions in the flow on a film-cooled surface would be identical to the conditions considered in figure 3 when there are no frictional forces within the flowing gas and when the neighborhood of the slot is excluded. In this case the center line I-I represents the wall surface and the line source S represents the slot. In reality the flow is retarded by viscous forces in the neighborhood of the wall, which causes the convective heatflow and the intensity of turbulence to be somewhat changed. The temperature distribution measured by Wieghardt (reference 6) on a film-cooled surface is also shown in figure 3 in order to give an indication of the order of magnitude of these changes. The same investigation shows that the difference between the wall temperature and the gas temperature decreases inversely proportional to $x^{0.8}$ as opposed to the decrease of θ_{\max} inversely proportional to x for the line heat source. The investigations of Wieghardt will be subsequently discussed in more detail.

Transpiration Cooling

It was previously stated that, in the method of film cooling, the air film is gradually destroyed by turbulent mixing with the hot gases. In this way, the effectiveness of the film decreases in the downstream direction from the point where it leaves the slot. This disadvantage is avoided in the transpiration-cooling method where the cooling film is continuously renewed along the blade surface. The blade is fabricated out of some porous material and the coolant is forced through the porous blade wall as shown in figure 1(c). A liquid coolant is again more effective than a gas coolant because considerable heat is absorbed by the evaporation process. Cooling with a liquid may be distinguished from cooling with a gas by calling it evaporative-transpiration or sweat cooling. The most important coolant for aircraft power plants is air for the reasons mentioned previously and, therefore, only transpiration

cooling with air will be considered in this report. Because the cooling film is continuously renewed in the transpiration-cooling method, the blade temperatures will be more uniform along the blade circumference than they will with film cooling. An additional advantage of transpiration cooling that tends to decrease the heat transfer from the hot gases to the blade surface is the fact that there is a continuous slow movement of the cooling air away from the blade surface because new cooling air is continuously forced through the pores and then leaves the surface. A counterflow is thus created between the heat flowing from the hot gases towards the blade surface and the cooling air flowing away from the blade surface. The cooling air continuously carries heat away from the surface by convection and in this way decreases the overall heat transfer from the hot gas to the surface.

This principle is explained in more detail in the simple configuration in figure 4 for a duct with the cross-sectional area constant along its length and with air at a constant, uniform velocity flowing through it. Two screens separated by the distance L are placed in the duct. These screens are kept at two different temperatures T_1 and T_2 . First, the air velocity is considered to be zero. In this case when free-convection currents are neglected, heat is conducted from the screen with the higher temperature T_2 through the stagnant air between the screens to the screen with the temperature T_1 . This heat-conduction process is described by the familiar equation

$$Q = -Ak \frac{dT}{dx} \quad (1)$$

in which Q is the heat flow per unit time, k is the thermal conductivity of the air that will be assumed independent of temperature, and T is the air temperature at the distance x from the screen having the temperature T_1 . In a steady state, the heat flow Q has to be constant along x when heat losses through the duct walls are neglected. An integration of equation (1) gives the well-known relation

$$T - T_1 = C_1 x \quad (2)$$

stating that the temperature increases linearly from one screen to the other. The heat flow is then determined by the equation

$$Q = -Ak \frac{T_2 - T_1}{L} \quad (3)$$

If the air moves with a finite velocity producing a weight flow w per unit time and cross-sectional area, the heat flow through any cross section in the duct is now composed of two parts: (1) the heat that is conducted in the direction opposite to the air stream and (2) the heat that is carried along by the air stream. Therefore, the heat flow is represented by

$$Q = -Ak \frac{dT}{dx} + c_p w A (T - T_1) \quad (4)$$

where the specific heat of the air c_p will be assumed independent of temperature. The second term of this equation contains the temperature difference $T - T_1$ because it is usual to disregard the heat that is carried along by an unheated air stream (of temperature T_1). Again, the heat flow Q has to be constant along x in a steady-state process. An integration of equation (4) together with the boundary condition $T = T_1$ when $x = 0$ yields

$$T - T_1 = C_2 \left(e^{\frac{c_p w}{k} x} - 1 \right) \quad (5)$$

With the boundary condition $T = T_2$ for $x = L$, the equation becomes

$$T - T_1 = (T_2 - T_1) \frac{e^{\frac{c_p w}{k} x} - 1}{e^{\frac{c_p w}{k} L} - 1} \quad (6)$$

The resultant heat flow Q from the screen with the temperature T_2 to the screen with temperature T_1 is

$$Q = -Ak \left(\frac{dT}{dx} \right)_{x=0} = -c_p w A \frac{T_2 - T_1}{e^{\frac{c_p w}{k} L} - 1} \quad (7)$$

For $w = 0$ the numerator and denominator in this equation are zero. However, it can be determined that the value for this indeterminate form is the same as given by equation (3). In figure 4 the air temperatures in the space between the two screens are plotted with the value $c_p w/k$ that characterizes the air velocity as the parameter.

For zero air velocity a straight line represents the air temperatures. For finite velocity the temperature profiles are curved. A negative value of the parameter $c_p w/k$ describes a flow of the air from the screen T_2 towards the screen T_1 . The gradient of the lines at $x = 0$ determines the amount of heat that flows from one screen to the other because, for $x = 0$, the second term on the right side of equation (4) is zero. It can be seen that the heat flow decreases with increasing velocities in the positive direction and increases for velocities in the negative direction. The decrease in heat flow for positive velocities is a consequence of the fact that part of the heat which leaves the screen with the temperature T_2 by conduction in a direction opposite to the air velocity is picked up by the air stream and carried back towards the screen with temperature T_2 and only part reaches the screen with the temperature T_1 .

This process by which the heat conduction flow is continuously decreased on its way by the counterflowing air takes place within a transpiration-cooled wall as well as within the boundary layers that are built up on both surfaces of the wall. Figure 5(a) shows how the temperature varies throughout the wall and through the boundary layers. Figure 5(b) indicates the temperature variation within the wall in an enlarged scale. The transpiration-cooled wall is represented in cross section. The cooling air flows through the wall from left to right. Hot gases with a temperature of 1000° F flow along the right surface and build up a boundary layer that is usually turbulent. Within this boundary layer, the temperature drops to the value on the outside wall surface. Because the surface area in contact with the cooling air is very great in a porous material, there are practically no temperature differences between the cooling air on its flow through the pores and the wall. The temperature difference between the air and the wall shown in figure 5(b) is exaggerated. The cooling air therefore leaves the porous wall with the wall surface temperature $T_{w,g}$. It leaves the wall with a small velocity normal to the surface. In passing away from the surface, the cooling air picks up momentum from the gas flow until it finally reaches the outside gas velocity. At the same time its temperature increases either by conduction or by turbulent mixing until at some distance the gas temperature T_g is reached. The condition within the coolant-air film in the immediate vicinity of the wall surface corresponds principally to the condition in the air flow discussed in

connection with figure 4. The temperature T_1 corresponds to the wall surface temperature $T_{w,g}$ and the temperature T_2 to the gas temperature T_g . The only difference is that the location of the plane in the cooling-air film that corresponds to the location of the screen with the temperature T_2 is somewhat indeterminate because the turbulent mixing process occurs over some finite distance. Essentially, the same process also takes place within the wall itself, the only difference being that the heat is conducted not only within the cooling air but also within the porous wall. The thermal conductivity k in equations (4) to (7) must be adjusted to take this process into account. In addition, a heat-transfer process from the porous wall material to the cooling air takes place as the air flows through the pores. This process determines the temperature difference between the cooling air and the solid material within the wall. A mathematical investigation of this process by Weinbaum and Wheeler (reference 7) shows that this temperature difference is very small except within a very narrow range near that part of the wall surface where the cooling air enters. This temperature difference may therefore be neglected in calculating the wall temperatures. There is a boundary layer on the coolant entrance side of the wall within which the air temperature increases from the initial value T_c to the temperature with which the air enters the pores. The thickness of this boundary layer and the temperature increase within it, however, are much smaller than on the hot side of the wall. The shape of the temperature curve within this boundary layer corresponds principally to the shape of the temperature profiles in figure 4 near the screen with the temperature T_2 . The temperatures T_c , $T_{w,m}$, and T_g shown in figure 5 were measured on a transpiration-cooled turbine blade. The temperatures within the boundary layers are only approximate shapes deduced from boundary-layer calculations. The temperatures within the wall were determined using equation (6) and measured values for the thermal conductivity of porous steel (fig. 6). The notation δ_L in figure 5 will be explained later.

A relation between the outside gas-to-surface heat-transfer coefficient and the wall temperature on the outside of the turbine blade can be easily derived in connection with figure 5. If there is no heat flow along the wall within the boundary layer on the cooling-air side, which quite often is the case, then the total heat conducted from the inner wall surface into the cooling-air boundary layer is picked up by the cooling air and is carried back into the wall by convection. In other words the resultant heat flow Q in equation (4) at any place on the wall is zero. In a steady state, the heat flow is zero at any cross section throughout the wall. On the surface of the wall facing the gas stream, the temperature of the cooling air is $T_{w,g}$. The heat that is

carried with the cooling air through the element dA of this surface is therefore

$$w c_p dA (T_{w,g} - T_c) \quad (8)$$

where w denotes the weight flow per unit area and time through dA and c_p is the specific heat of the cooling air at constant pressure. On the other hand, the heat that is transported through the gas boundary layer to the surface may be defined with a heat-transfer coefficient by the expression

$$H dA (T_g - T_{w,g}) \quad (9)$$

At high velocities in the gas stream, the effective temperature has to be introduced as gas temperature into this expression. Because the total heat flow through the area dA is zero, expressions (8) and (9) have to be equal. Therefore,

$$H (T_g - T_{w,g}) = w c_p (T_{w,g} - T_c) \quad (10)$$

This relation determines the gas-to-surface heat-transfer coefficient as soon as the effective gas temperature, the coolant temperature, and the wall temperature on the surface facing the gas stream are known from measurements; or inversely, it allows for the calculation of the blade-surface temperature from the gas-to-surface heat-transfer coefficient.

GAS-TO-SURFACE HEAT-TRANSFER COEFFICIENT

Extent of Laminar and Turbulent Boundary Layers

It is well known that gases flowing through a turbine blade grid build boundary layers along the surface of the blades. The action of the viscous forces within the flowing gases is practically confined to this region and to the wake downstream of the blades. Sometimes the flow separates from the blade surface on the convex or suction side of the blade near the trailing edge. The boundary layers may be laminar or turbulent.

When the blade is cooled, a temperature boundary layer is built up around the blade, which, for gases, has a thickness of the same order of magnitude as the flow boundary layer. The gas temperature is influenced by the cooling process only within this boundary layer and within the wake behind the turbine blade. If separation occurs, then the region behind this separation has to be included. The remainder of the gas flow is not influenced by the cooling process. The heat transfer from the blade surface into this boundary layer depends very much on whether the flow within this boundary layer is laminar or turbulent. Before a calculation of the heat transfer can be started, it must therefore be known in which places along the blade surface the boundary layer is laminar or turbulent. Consequently, a short examination as to what kind of boundary layer is to be expected on a porous blade is required.

At some point on the nose of the turbine blade, the gas flow divides into the part that flows along the suction side and the part that flows along the pressure side of the blade. This point is called the stagnation point and in this region the boundary layer is always laminar. At some distance downstream of this point, the flow within the boundary layer becomes turbulent. The location of this transition point mainly depends on the local pressure gradient and the thickness of the boundary layer at that point. The pressure distribution around a turbine blade on the pressure and suction side is determined by the outside gas flow. The pressure decreases at first with increasing distance from the stagnation point. Usually, it reaches a minimum somewhere along the blade on both sides and then increases. The transition to turbulent flow within the boundary layer probably occurs on turbine blades near the point where the pressure minimum is reached. The exact location of the point of transition cannot be predicted yet by calculation although great advances have been made in recent years in understanding the transition process. The factors influencing transition are now fairly well known. Some information also exists on their mutual importance. The investigations in this field that are of special importance for the prediction of the transition point on turbine blades will be mentioned briefly.

Tollmien and Schlichting (references 8 and 9) show that on a solid surface the transition is caused by the fact that the boundary layer becomes unstable against small fluctuations under the influences of the outside pressure distribution along the surface and of the boundary-layer thickness. A pressure decrease in flow direction stabilizes, and a pressure increase destabilizes, the boundary layer. Outside disturbances that may be present to a considerable degree in the flow in a gas turbine move the transition point forward. A theory of this process is given by Taylor in reference 10. It was found that increasing curvature decreases the stability on a concave surface (reference 11). Temperature differences within the boundary layer increase the stability when the blade is

cooler than the gas (reference 12). A comparison of average gas-to-blade heat-transfer coefficients calculated by Brown and Donoughe (reference 13) with measured data indicates that, in internally-cooled turbine blades, the transition usually occurs near the point where the pressure minimum is reached along the surface.

All these data referred to boundary layers on a solid blade. By the transpiration-cooling process, the stability of the boundary layer is decreased. Some information on this process is found in reference 14. No experimental data concerning the point of transition on turbine blades are available; however, it may be expected that laminar boundary layers will not be present downstream of the minimum-pressure point. Laminar boundary layers, therefore, will usually be confined to a region near the blade nose, and along the rest of the surface the boundary layer will be turbulent. It will be discussed later why it is well worth while to study the question of how the laminar part can be extended over a larger part of the blade surface.

Heat Transfer in Transpiration-Cooled Laminar Boundary Layer

The heat transfer in the laminar boundary layer can be analytically investigated. All the available information stems from calculations. The essential features of this type of heat transfer can be learned by a discussion of the conditions on a flat plate.

Figure 7 shows the sketch of such a porous plate in a gas stream of uniform velocity V_g . The plate is fabricated from porous material and cooling air is blown through it from a box attached to the back of the plate. The cooling air in this box has a temperature T_c and the outside gas temperature is T_g . The distribution of the cooling air along the plate surface depends on the local distribution of the porosity. Two important conditions may be considered. The first one is to find a distribution of the cooling air that keeps the outside surface temperature of the plate $T_{w,g}$ constant over the whole plate length. The amount of coolant passing through the porous plate may be characterized by a velocity v_c with which the cooling air moves away from the plate surface. This velocity v_c is not the actual velocity of the cooling air coming out of the pores, but rather it is the volume flow through a unit surface of the plate. The cooling air has this velocity when all the jets coming out of the pores combine into a continuous flow and the velocity differences are equalized. It is assumed in all the calculations for transpiration cooling that these conditions occur very close to the plate surface. The distribution of the velocity v_c along the plate that gives a constant wall temperature when equation (10) is valid

is $v_c \sim \frac{1}{\sqrt{x}}$ where x denotes the distance along the plate from the leading edge (reference 15). In this case the partial differential equations describing the flow and the heat transfer within the boundary layer can be transformed into total differential equations and solved exactly. Such a calculation shows that the boundary-layer thickness along the wall increases proportional to the square root of x and the local heat-transfer coefficient H decreases inversely proportional to the square root of x . The heat-transfer coefficient is obtained from the dimensionless equation

$$Nu = F\sqrt{Re} \quad (11)$$

where

$$Nu = \frac{Hx}{k}, \quad Re = \frac{V_g x}{\nu_g}$$

The factor F is a function of the coolant-flow velocity v_c and the Prandtl number Pr of the gas. The factor F is presented in figure 8 for a gas with a Prandtl number of one with the assumptions that the property values are constant through the boundary layer and the Mach number is small. Negative values of the abscissa indicate suction of the gas into the wall. The flow boundary layer for this case was calculated by Schlichting (reference 16). The temperature boundary layer and therefore the heat-transfer coefficient for $Pr = 1$ follow immediately from these calculations considering the similarity between velocity and temperature profile. For the case investigated, the porosity of the plate has to be varied in such a way that the velocity v_c decreases inversely proportional to the square root of x .

The second important case is a plate where the velocity v_c is constant. This case can be obtained with a constant wall thickness and porosity. Only an approximate solution of the differential equations is possible for this case (reference 17). The local behavior of the boundary-layer thickness, the wall temperature, and the heat-transfer coefficient is different from the case previously discussed. As in the other case, the boundary-layer thickness starts with the value zero at the leading edge of the plate but it increases linearly in the downstream direction for great values of x . The local heat-transfer coefficient starts with the value infinity at the leading edge and approaches asymptotically the value zero with increasing x . The wall temperature is equal to the gas temperature at the leading edge and approaches the cooling-air temperature asymptotically with increasing x . It is interesting to note that the behavior of these values is

different when the air is sucked into the plate instead of blown from it. In this case the boundary-layer thickness, the heat-transfer coefficient, and the wall temperature reach a constant value some distance from the leading edge.

Conditions near the leading edge of a turbine blade where the boundary layer is expected to be laminar differ from those on a flat plate by the fact that the pressure and, therefore, the velocity V_g outside the boundary layer are not constant but vary along the blade surface. Exact calculations are possible only for certain types of this velocity variation. Solutions were presented for constant property values, small Mach numbers, and for the case that V_g varies proportional to x (references 15 and 16). The solutions can be extended to include the effect of the variation of property values and a variation of the gas velocity V_g proportional to x^m . The resulting formula for the heat-transfer coefficient will have the same form as equation (11). The factor F will then also depend on the ratio of the gas temperature T_g to the wall temperature $T_{w,g}$ and on the exponent m . This exponent is related to the local pressure gradient dp/dx by the equation

$$\frac{dp}{dx} = \frac{m\rho_g V_g^2}{x}$$

Normally, the variation of the velocity V_g proportional to x^m does not correspond too well to the velocity variation encountered on turbine blades. For such a variation only approximate calculations of the heat-transfer coefficient are possible. A method using the integrated momentum and heat-flow equations of the boundary layer, similar to the procedure proposed by von Kármán (reference 18), seems to be best suited for this purpose.

Heat Transfer in Transpiration-Cooled Turbulent Boundary Layer

No exact calculations are possible for the turbulent boundary layer with or without transpiration cooling. Experimental information obtained on transpiration-cooled turbulent boundary layers is also very meager. The present knowledge of the heat transfer in the transpiration-cooled turbulent boundary layer is therefore much less reliable than that of the laminar boundary layer. A first, approximate theory for the turbulent case was presented by Rannie (reference 19). The essential features of this theory may be explained with the help of figure 5. The turbulent fluctuations within the boundary layer on the gas side of the wall decrease in a direction towards the wall. On the wall

itself they must be zero. Very little detailed information exists on this decrease of turbulence. However, to simplify the real conditions for heat-transfer investigations the boundary layer can be divided into two parts, a very thin sublayer near the wall (indicated by δ_L in fig. 5) within which the flow is assumed to be laminar, and a turbulent part where the heat transfer by turbulent mixing is assumed to exceed by far the heat transfer by conduction. This idealization, which gave useful results for the boundary layer on a solid wall, is used by Rannie in his theory. In addition he assumes that the transpiration-cooling process does not alter the value of δ_L or the conditions in the turbulent part of the boundary layer; only the laminar sublayer is influenced. The temperature drop through the laminar sublayer on a solid wall is practically linear in accordance with equation (2). For the porous wall it is non-linear and can be described by equation (6). In this way a formula was derived in reference 19 for the heat-transfer coefficient and by use of equation (10) the wall temperature could also be calculated. Comparisons of this formula with experimental data obtained in the Jet Propulsion Laboratory, California Institute of Technology showed fair agreement (reference 19). The formula was simplified by Friedman (reference 20) in restricting it to fluids with a Prandtl number near one. The formula for the wall temperature $T_{w,g}$ derived by him is

$$\frac{T_{w,g} - T_c}{T_g - T_c} = \frac{r}{e^{r\phi} + r - 1} \quad (12)$$

where r is the ratio of the velocity parallel to the surface at the border between the laminar sublayer and the turbulent part of the boundary layer (at the distance δ_L in fig. 5) to the stream velocity V_g outside the boundary layer. The value ϕ is given by

$$\phi = \frac{\rho_c c_p v_c}{H'} \quad (13)$$

with ρ_c , the density; c_p , the specific heat; v_c , the velocity of the cooling air; and H' , the heat-transfer coefficient that would be present on a solid surface under the same outside flow conditions.

The formula was used by Friedman to study heat transfer in a tube, but it can be applied to the flat plate as well. Reference 21 gives the equation for heat transfer over a flat plate with turbulent flow as

$$Nu = 0.0296 (Re)^{0.8} (Pr)^{1/3} \quad (14)$$

If both sides of equation (14) are divided by $(Re)(Pr)$, the equation takes the form

$$\frac{Nu}{(Re)(Pr)} = St = \frac{H'}{\rho_g c_p V_g} = \frac{0.0296}{(Re)^{1/5} (Pr)^{2/3}} \quad (15)$$

where the Reynolds number is based on the length x . Substituting the value of H' from equation (15) into equation (13) results in

$$\varphi = \frac{(Re)^{1/5} (Pr)^{2/3}}{0.0296} \frac{\rho_c v_c}{\rho_g V_g} \quad (16)$$

The value φ therefore depends mainly on the ratio of density times velocity of the coolant to density times velocity of the outside gas flow and, to a smaller degree, on the Reynolds number for the outside flow and on the Prandtl number. The value 0.5 was used by Friedman for the velocity ratio r . For the flat plate it is better to use the relation in reference 22

$$r = \frac{2.11}{(Re)^{0.1}} \quad (17)$$

Friedman also compared equation (12) with test data and found good agreement. For the specific values $Re = 10^5$ and $Pr = 0.7$, the temperature ratio given by equation (12) is plotted in figure 9 as a function of the mass-flow ratio. The temperature ratio calculated from equations (10) and (11) for the laminar boundary layer on a flat plate is also included. The figure reveals that the amount of coolant flow necessary to obtain the same wall temperature is very much lower for the laminar boundary layer than for the turbulent one. This fact means that very big gains can be obtained if the laminar portion of the boundary layer on the surface of the turbine blade can be increased.

The conditions on a turbine blade in the region of the turbulent boundary layer differ again from the conditions on a flat plate by the fact that the velocity varies along the blade surface. Practically no information on the influence of this velocity variation is available, neither on the solid surface nor on a transpiration-cooled wall. It seems, however, that the influence of a pressure gradient is less on a turbulent boundary layer than on a laminar one. Nothing is known of the heat transfer in the separated region that often exists on the downstream part of the suction surface of the blade.

Heat Transfer in Film Cooling

It was pointed out that in film cooling the cooling-air film is diffused by turbulent mixing with the hot gases and, in this way, is gradually destroyed on its downstream path after leaving the slot. When the wall surface does not receive heat by radiation or by conduction into or from the interior of the wall, it assumes a temperature that is called the adiabatic wall temperature.

More information on the value of this adiabatic wall temperature $T_{w,ad}$ may be obtained from an experimental investigation by Wieghardt (reference 6), which is concerned primarily with the de-icing of airplane wings. He therefore investigated a hot-air jet blown into a cold air stream by a slot in a flat plate. As a first approximation, however, his results should be applicable to the film-cooling process as well. He derived the expression

$$\frac{T_g - T_{w,ad}}{T_g - T_c} = 21.8 \left(\frac{\frac{s}{x} \frac{\rho_c V_c}{\rho_g V_g}} \right)^{0.8} \quad (18)$$

in which x is the distance in downstream direction from the slot and s , the slot width. The formula is applicable for x/s values greater than 100 and for a mass-flow ratio smaller than or equal to one. It gives a decreasing adiabatic wall temperature at a certain distance x with increasing mass-flow ratio. More research is required for x/s values smaller than 100, because this range will find most application in gas-turbine blades. The experiments (reference 6) showed that, for mass-flow ratios greater than one, the adiabatic wall temperature increased again with increasing mass-flow ratio. A value of one for the mass-flow ratio therefore gives the best cooling effectiveness.

When there is heat flow through the wall by internal conduction from a hotter location in the wall, or when heat is radiated from the gas to the wall surface, a finite heat transfer by convection from the wall to the air film has to balance this heat exchange and the temperature profile on a normal to the wall must have a finite gradient at the wall surface. A heat-transfer coefficient H describing this heat flow dQ has to be defined by the equation

$$dQ = HdA (T_e - T_{w,g}) \quad (19)$$

in analogy to the corresponding equation for the heat transfer in high-velocity flow. The effective temperature T_e has to be chosen in

such a way that the temperature difference $T_e - T_{w,g}$ is zero where the heat flow dQ is zero. Only this difference avoids a change of the heat-transfer coefficient to values infinite and zero as the temperature difference is varied. The exact evaluation of T_e is difficult owing to wall temperature gradients in the direction of flow, which influence the air-film temperature profile normal to the wall surface. The temperature T_e affecting heat transfer will therefore probably be slightly different from the adiabatic wall temperature $T_{w,ad}$ in equation (18). No information is presently available on the magnitude of the heat-transfer coefficients in film cooling or on an exact method of evaluating T_e in equation (19).

The information obtained by Wieghardt's investigations may be used to compare the effectiveness of film and transpiration cooling for the case when the heat flow dQ in equation (19) is zero. If a wall is assumed to be cooled by a continuous succession of slots having the distance x from each other, equation (18) gives the highest values for the effective wall temperature. The amount of coolant flow per length x of the surface is $s\rho_c V_c$. In a transpiration-cooled wall of the same dimensions, the coolant flow through the length x is $x\rho_g V_g$. In order to compare both walls at the same amount of coolant flow per unit area, the coolant velocity v_c on the transpiration-cooled wall has to be compared with a coolant velocity $\frac{s}{x} V_c$ on the film-cooled wall. Therefore the expression

$$\frac{s}{x} \frac{\rho_c V_c}{\rho_g V_g}$$

in equation (18) is equivalent to the expression

$$\frac{\rho_c v_c}{\rho_g V_g}$$

in equation (16). In this way the curve for film cooling is inserted in figure 9. It can be seen that the transpiration cooling is more efficient than the film cooling. In this comparison it must be kept in mind that equation (18) describing the temperatures obtained in film cooling is valid only for a ratio x/s greater than 100 and that the slot width is of the same order of magnitude as the boundary-layer thickness.

FLOW THROUGH POROUS WALLS

Required Flow Distribution

The information in the previous section can be used to determine the wall-temperature distribution around the surface of a transpiration-cooled porous blade when the velocity of the cooling air v_c is prescribed, or it can be used to determine the distribution of the cooling-air velocity around the blade surface that is necessary to keep the wall temperature uniform. A first approximate answer to this question may be obtained with the equations presented and the accuracy of such a determination can be increased when more information on the heat-transfer coefficients under the circumstances prevailing on blade surfaces becomes available.

The local distribution of the cooling-air velocity v_c necessary to keep the surface temperature of a plane wall at a constant value is shown in figure 10. The necessary coolant velocity begins with an infinite value at the leading edge and decreases rapidly in the region of the laminar boundary layer with increasing distance from the leading edge. It rises again when the boundary layer becomes turbulent in accordance with figure 9 and decreases in the turbulent region. This decrease, however, is much slower than the decrease in the laminar portion.

Basically, the conditions around a turbine blade for constant surface temperature will be similar. They differ mostly in the region near the stagnation point where the finite thickness of the blade causes the cooling-air velocity to start out with a finite value. The required velocity v_c decreases from this value along both the suction and pressure side of the surface to the transition point, where a rise in the necessary velocity v_c has to be anticipated followed by a more gradual decrease of its value in the downstream direction. The leading edge region where high coolant velocities v_c are required probably extends a greater distance downstream on a blade than on a flat plate.

The problem that has to be solved in the design of a turbine blade is providing the necessary distribution of the cooling air by proper choice of the local porosity of the blade wall and its thickness. Probably the variation of v_c determined theoretically along both sides of the blade can be approximated well enough by a constant value; however, the high v_c values necessary near the leading edge have to be provided by some means. Figure 11 shows the temperature distribution that was measured on a porous blade in a static cascade at the

NACA Lewis laboratory. The manufacturer tried to maintain a uniform porosity around the blade perimeter. It can be seen that the temperatures are very low on both sides of the blade. For comparison, blade temperatures are also given which were measured in a static cascade on two blades cooled internally with air; namely, a hollow blade and a blade where the internal area was increased by brazing 10 tubes into the hollow blade shell. It can be seen that the temperatures obtained in the central part of the porous transpiration-cooled blade are extremely low. Even with liquid cooling such low temperatures have not been obtained at comparable coolant flows.

The reason for the high leading-edge temperatures is apparent from a comparison of the blade cross section in figure 11 and the velocity distribution necessary to maintain a constant wall temperature as shown in figure 10. Figure 11 shows that the wall thickness near the leading edge was appreciably higher than at the sides of the blades. Owing to a high pressure drop through the thicker wall, the velocity was therefore small in the region where, according to figure 10, a high value of this velocity is necessary to maintain a constant wall temperature. Near the trailing edge, no especially high values of the coolant velocity are necessary according to figure 10; however, the coolant velocity in this region in the turbine blade was extremely low because of the long path length of the coolant through the material in this region. This fact is evidenced by a measured local coolant-flow distribution shown in figure 12. Special attention must therefore be directed to the leading-edge and the trailing-edge regions of such a transpiration-cooled turbine blade. Further investigations are necessary to determine whether a local change in the porosity can make the temperature sufficiently uniform around the periphery of the blade or whether other special means have to be used for cooling the leading- and trailing-edge regions.

Another difficulty is present in turbine blades, which may be explained with the help of figure 13. This figure shows the outside pressure distribution around the turbine blade. The cooling-air pressure inside the turbine blade is practically constant because of the low cooling-air velocities. This constant value is also indicated in the upper part of the figure. It can be seen that the pressure difference available to force the cooling air through the porous wall differs along the blade surface. Near the leading edge where high coolant velocities v_c are required, the pressure difference is smallest. The higher pressure difference and higher coolant flow on the suction side of the blade is responsible for the lower temperature

seen in figure 11 on this side of the blade. The differences in the available pressure difference can be decreased by using a wall with low porosity, which makes the required inside pressure high. A high coolant pressure, however, should be avoided because this pressure may not be available in the compressor of the gas-turbine engine. It would be sufficient to provide a high coolant pressure only for the region near the stagnation point and possibly near the trailing edge. Another possibility is to let the variation of the local porosity take care of the differences in the available pressure as well as the differences in the necessary coolant flow.

Porosity and Permeability

The manufacturer of porous material is concerned with the fabrication of porous structures with a certain predetermined and reproducible porosity. On the other hand, the considerations in the previous paragraphs determine what the permeability of a wall should be. The porosity is defined as the value one minus the ratio of the density of the porous material to the density of a solid piece made of the same material. The permeability expresses the capacity of a porous material to pass liquids or air when pressure differences are present. In order to prescribe the required porosity to the manufacturer of transpiration-cooled walls, the law relating the porosity and the mass flow through a porous wall must be known. For low velocities Darcy's law

$$\frac{\Delta p}{\tau} = C_3 \frac{\mu_c v_c}{d} \quad (20)$$

gives the relation between the pressure difference Δp acting on the two surfaces of a plain porous wall with the thickness τ , the viscosity μ_c , the velocity v_c of the coolant flowing through the porous wall, and the average width d of the pores. This law holds for Reynolds numbers smaller than one when the Reynolds number is based on the dimension d . It gives a linear relationship between pressure drop and velocity analogous to laminar flow in a tube. For large Reynolds numbers another law holds

$$\frac{\Delta p}{\tau} = C_4 \frac{\rho_c v_c^2}{d} \quad (21)$$

according to which the pressure drop is proportional to the density ρ_c of the coolant and the square of the velocity v_c . Usually, there

is a large range of Reynolds numbers within which the law gradually changes from the linear to the quadratic relationship. Usually, this is the range that is of interest for the applications considered. In addition the constants in both equations depend on the structure of the porous wall and cannot be predicted exactly (reference 23). The relation between the pressure drop and the velocity, therefore, has to be experimentally determined for each material. When the density of the coolant varies considerably on its way through the porous wall, it is more expedient to calculate with the mass flow $\rho_c v_c$ per unit area. It was shown in references 23 and 24 that, when the change in density is caused by pressure variation, the two foregoing equations become relations between the differences Δp^2 of the square of the pressure and the mass flow rate $\rho_c v_c$.

In turbine blades the surfaces cannot always be considered as plane walls in calculating the mass flow through the walls, especially near the leading and the trailing edge of the blade. The mass flow through curved walls may be determined by methods similar to the ones used to study the heat flow by conduction through curved walls. In the range of low Reynolds numbers where the relations between the pressure square and the mass flow rate is linear, there is a complete analogy between both physical processes. The lines of constant temperature then correspond to the lines of constant pressure and the direction of the heat flow on any place within the wall coincides with the direction of the mass flow. In the range where the relation between pressure-square values and mass-flow rate is nonlinear this analogy ceases to exist. In this case an adaptation of the relaxation method to the nonlinear character of the equations may be used to determine local flow rates through a porous wall.

SUMMARY OF RESULTS

The results of a survey on transpiration and film cooling have shown that:

1. Transpiration cooling is an extremely effective means of cooling objects in high-temperature, high-velocity gas streams such as gas-turbine blades; however, special care will be required to provide sufficient cooling at the leading and trailing edges.

2. The laminar boundary layer on most turbine blades is apparently limited to the portion of the blade near the leading edge. Because transpiration cooling is much more effective for laminar boundary layers than for turbulent layers, means of extending the laminar layer require investigation.

3. Fabrication techniques must be developed for porous blades to provide high strength with controlled, variable permeability.

4. Film cooling is also an effective method for the cooling of turbine blades, although it is less effective than transpiration cooling.

5. Although considerable research has been conducted on transpiration and film cooling, additional research is required for a more thorough understanding of the heat transfer under the special conditions prevailing for turbine blades.

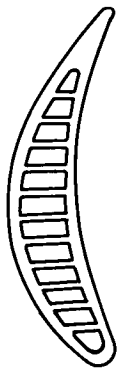
Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio.

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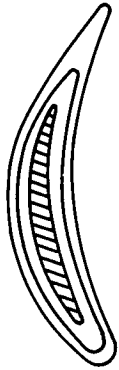
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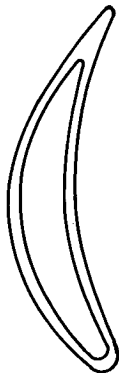
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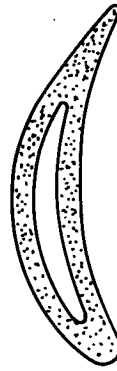


Hollow

(a) Convection-cooled blades.



(b) Film-cooled blade.



(c) Porous blade.

Figure 1. - Air-cooled turbine blades.

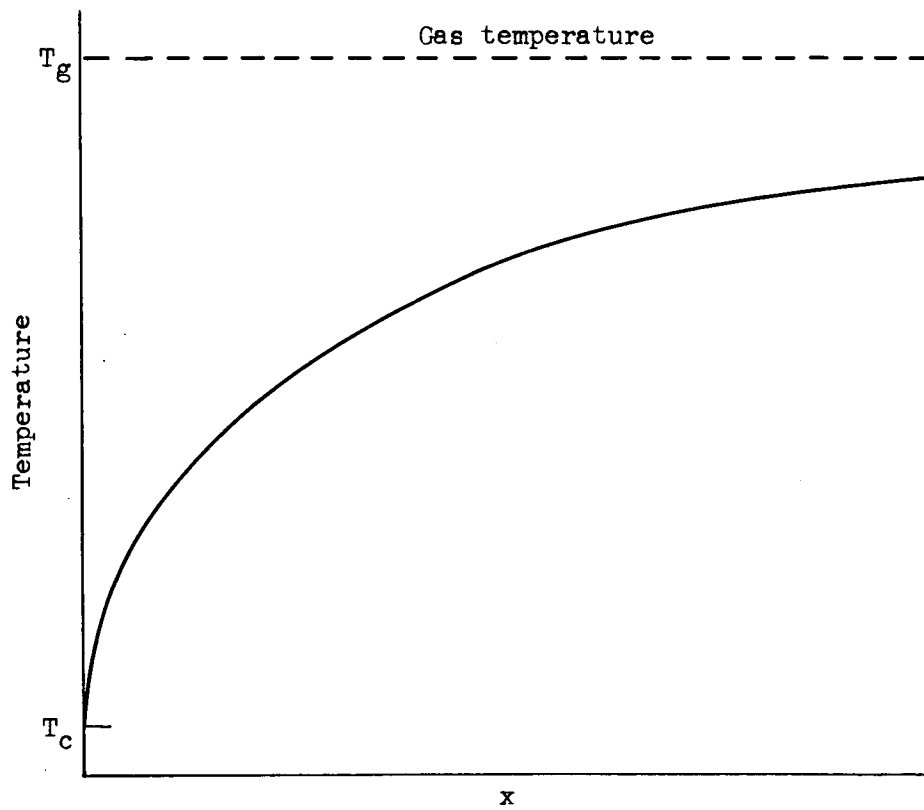
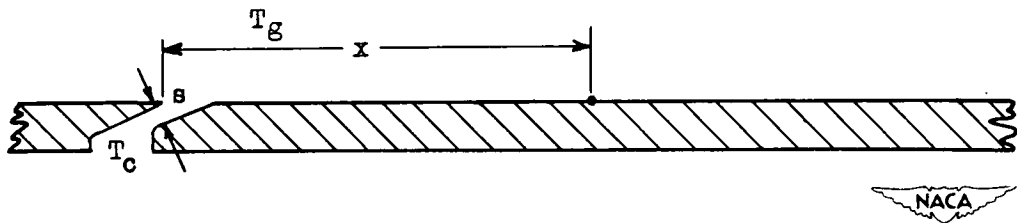


Figure 2. - Temperature distribution along film-cooled plate.

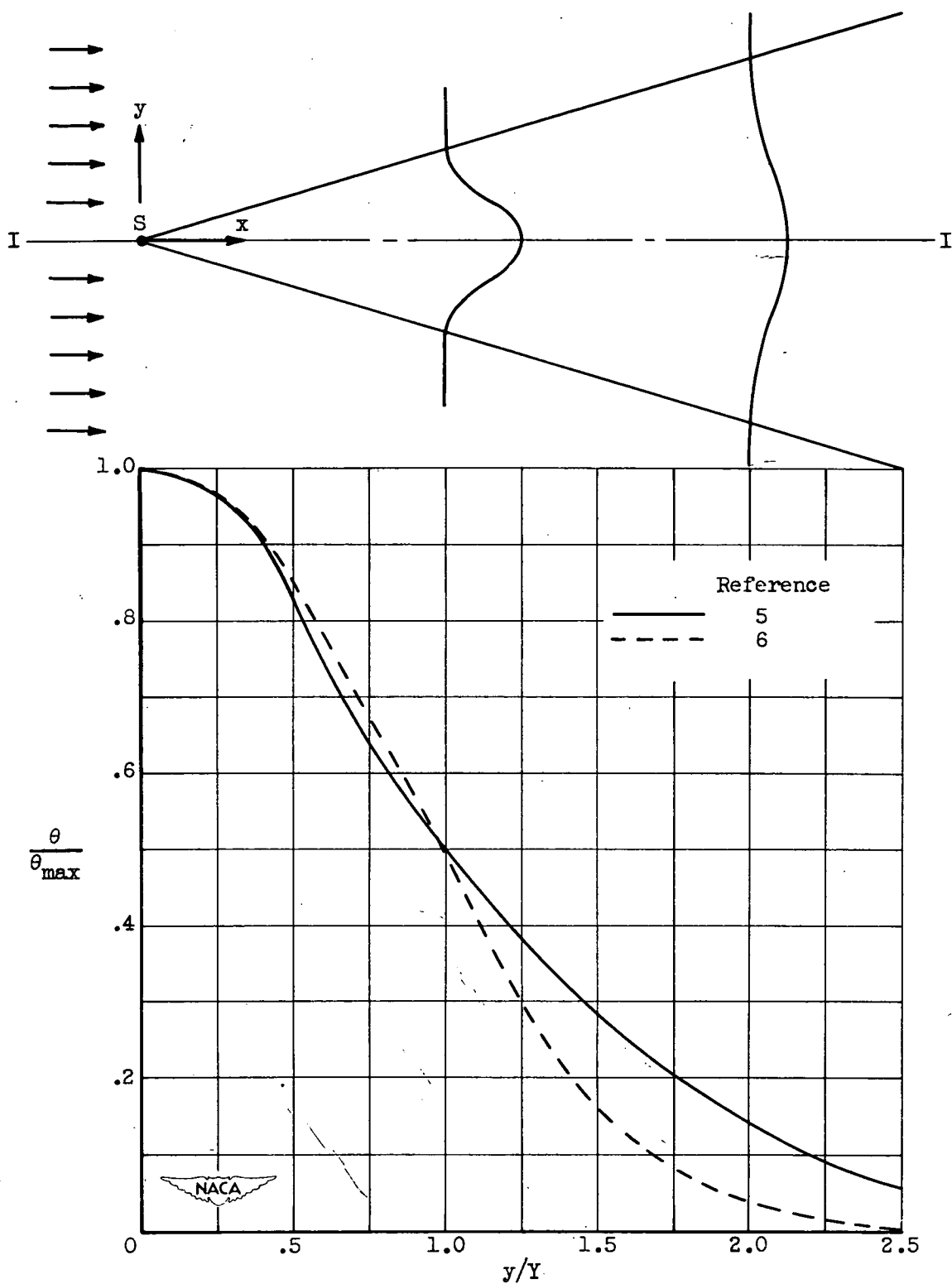


Figure 3. - Temperature profile downstream of line source of heat.

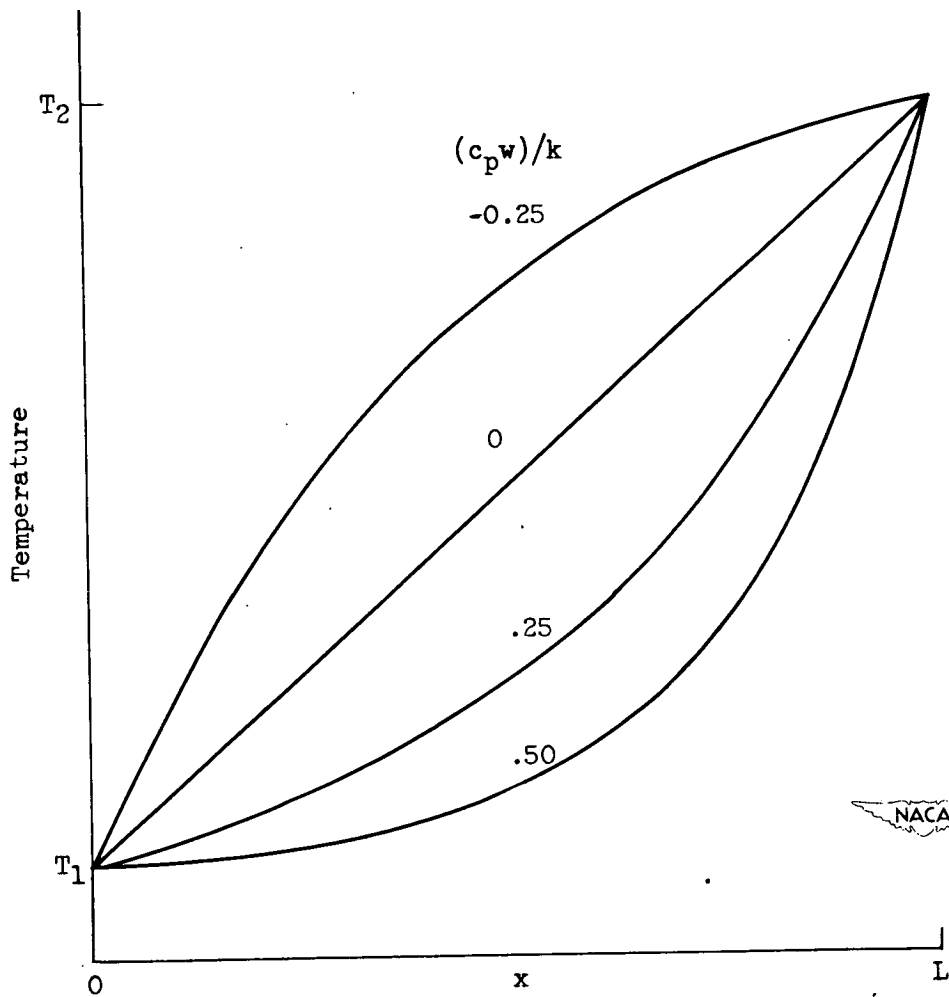
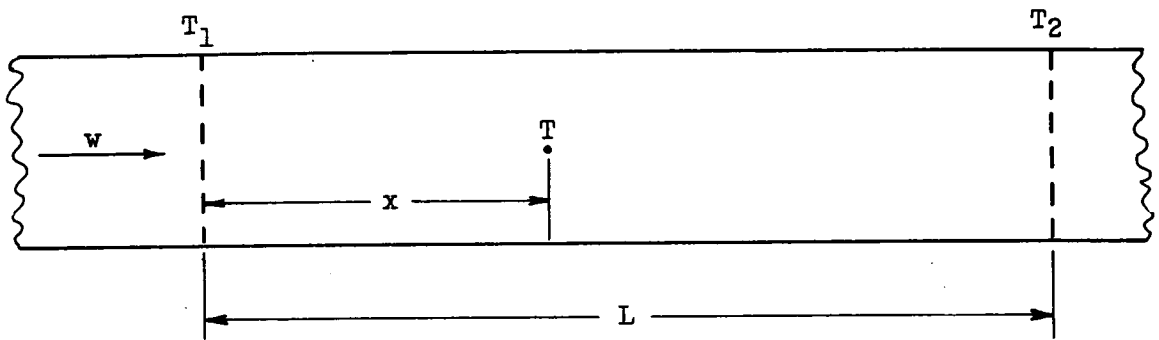
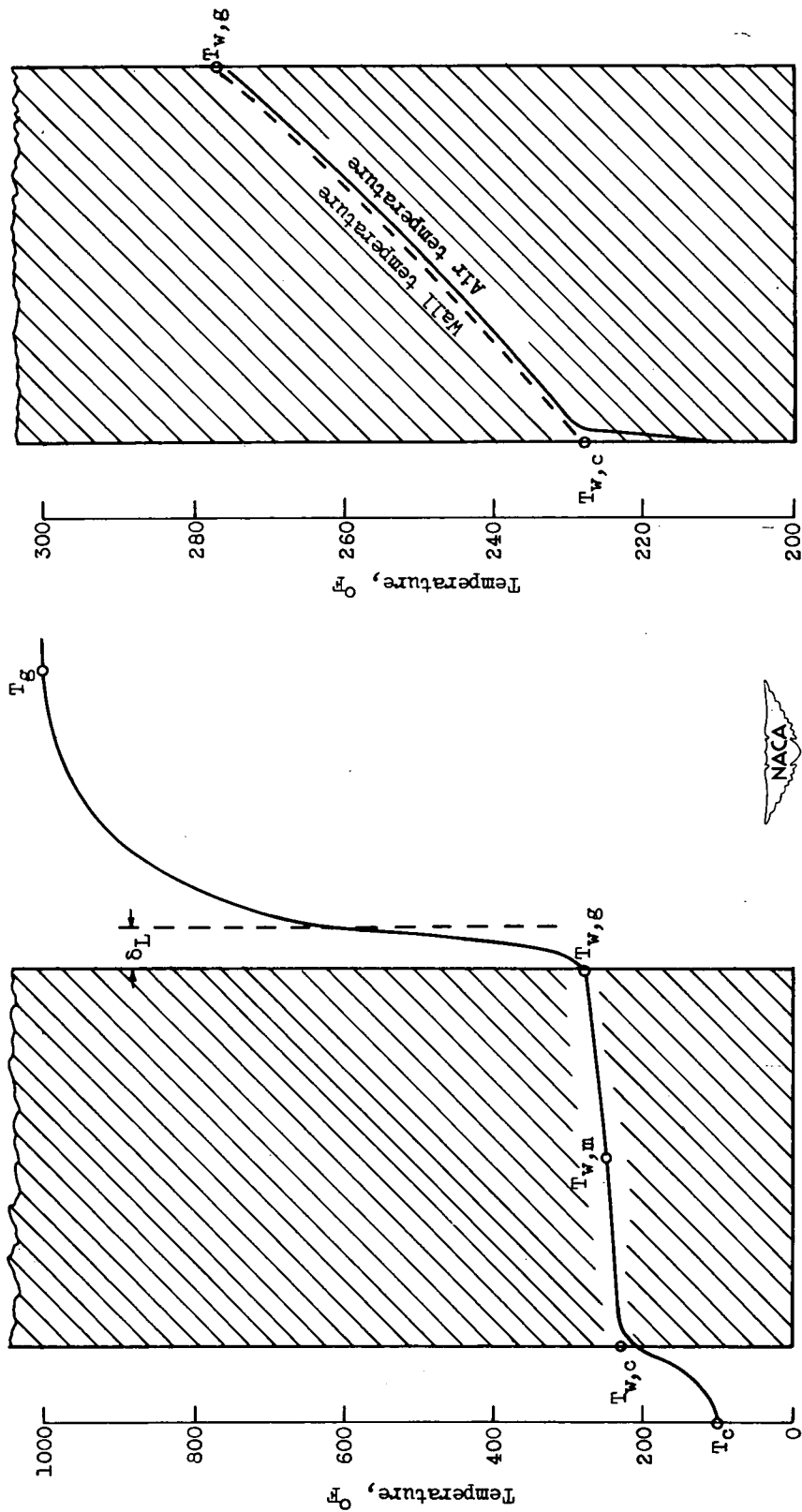


Figure 4. - Effect of air-flow rate on temperature distribution between two locations having prescribed temperatures.



(a) Temperature variation between coolant and gas.

(b) Temperatures in porous wall.

Figure 5. - Temperature variation through porous wall and adjacent boundary layers.

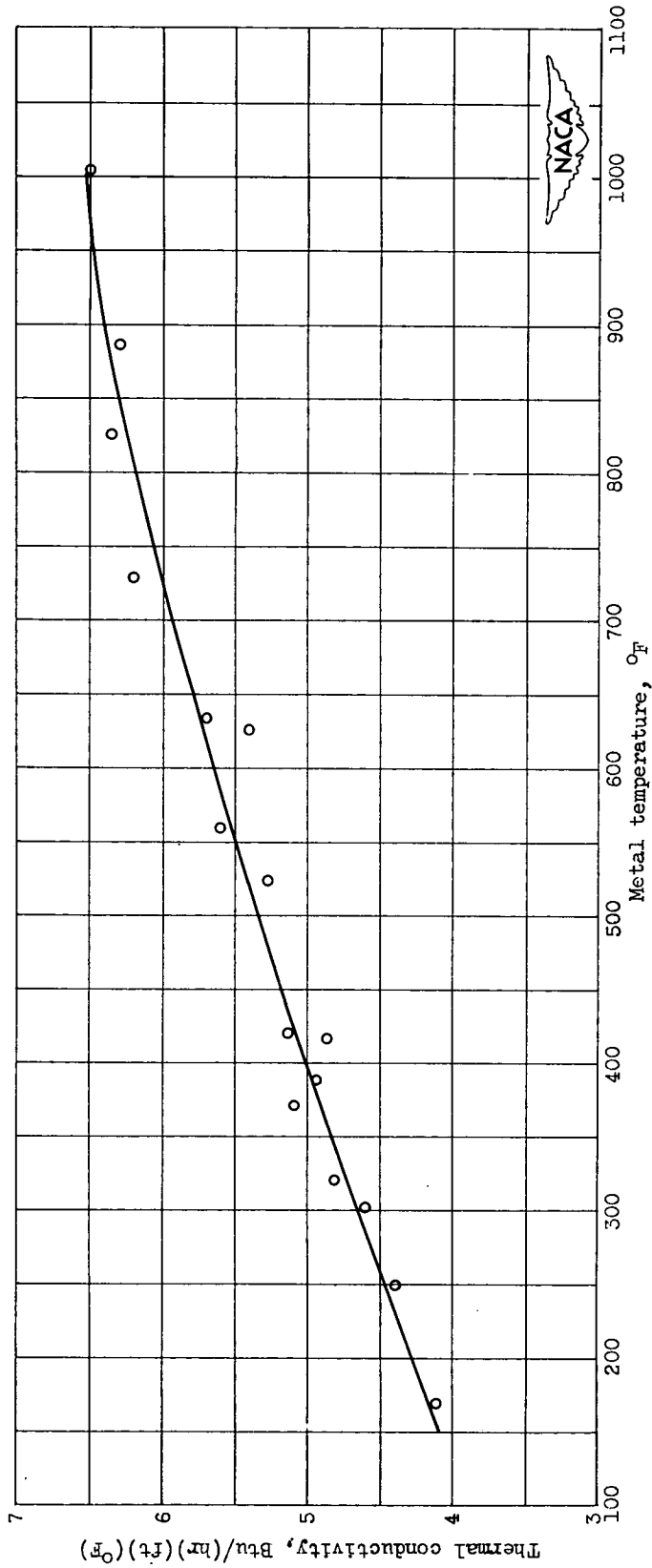


Figure 6. - Thermal conductivity of sintered AISI 301 stainless steel with 25-percent porosity. (NACA calibration.)

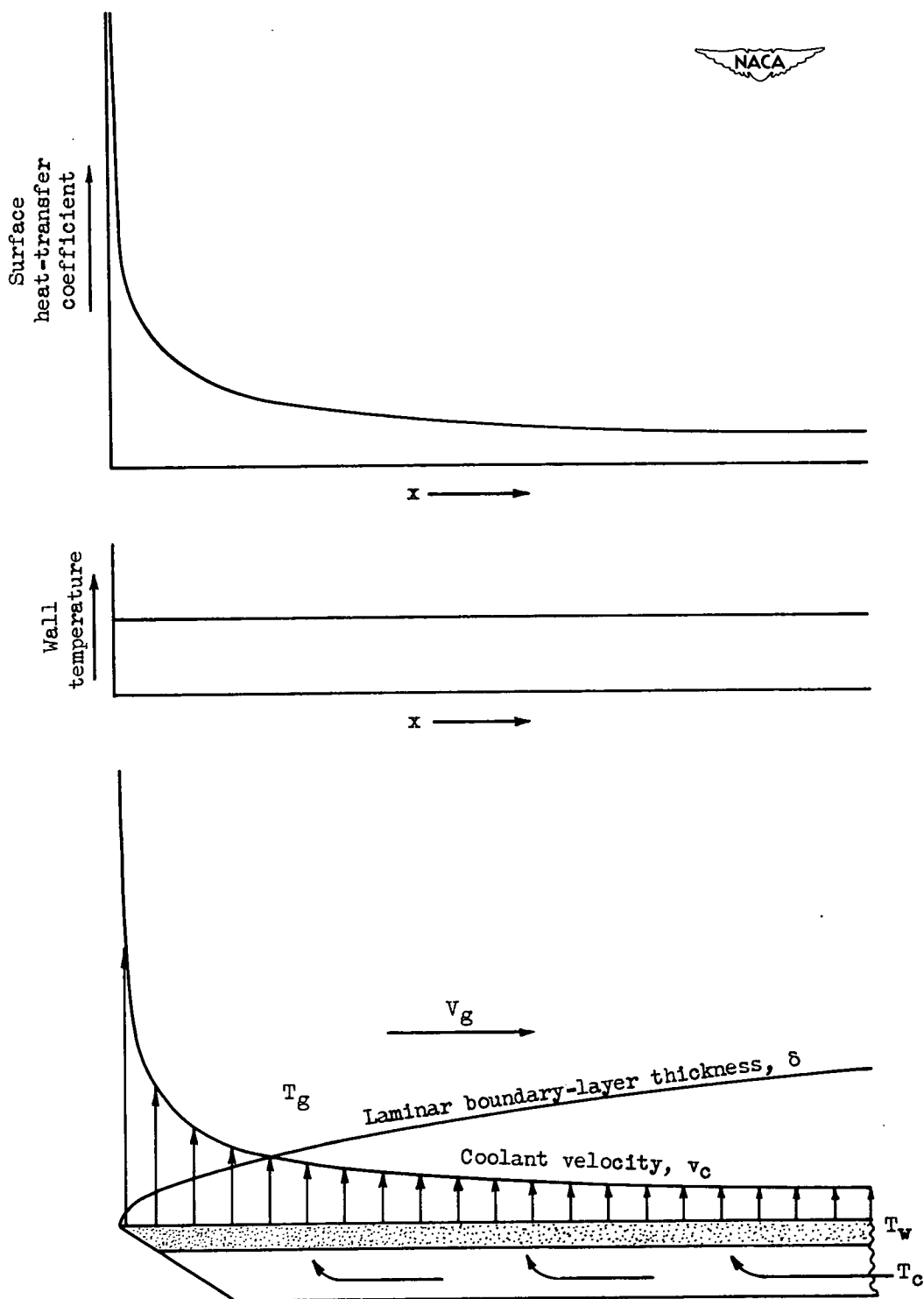


Figure 7. - Coolant velocity, heat-transfer coefficient, and boundary-layer thickness for transpiration-cooled plate at constant temperature with laminar flow.

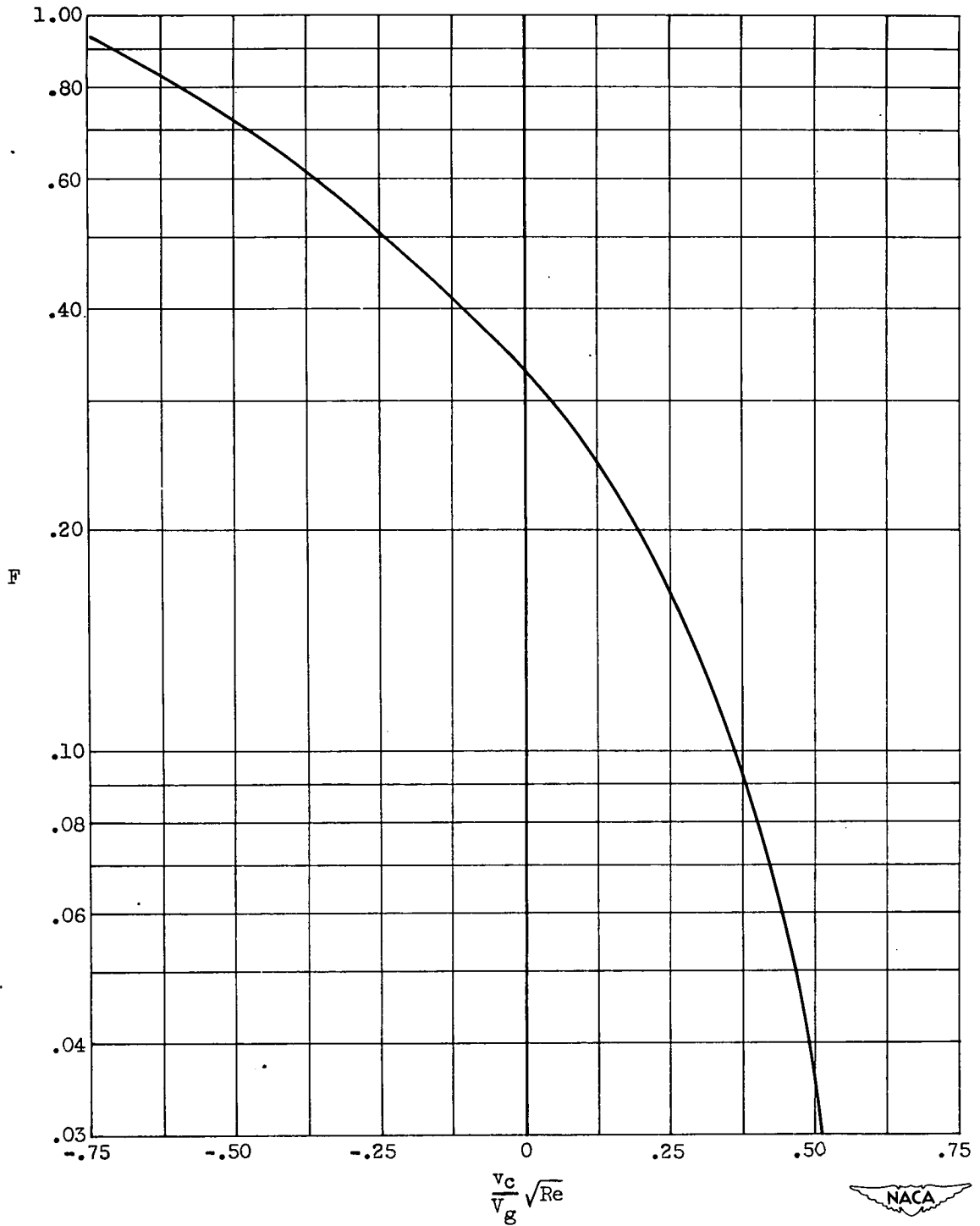


Figure 8. - Factor F in laminar-flow Nusselt equation $Nu = F\sqrt{Re}$ for transpiration cooling.

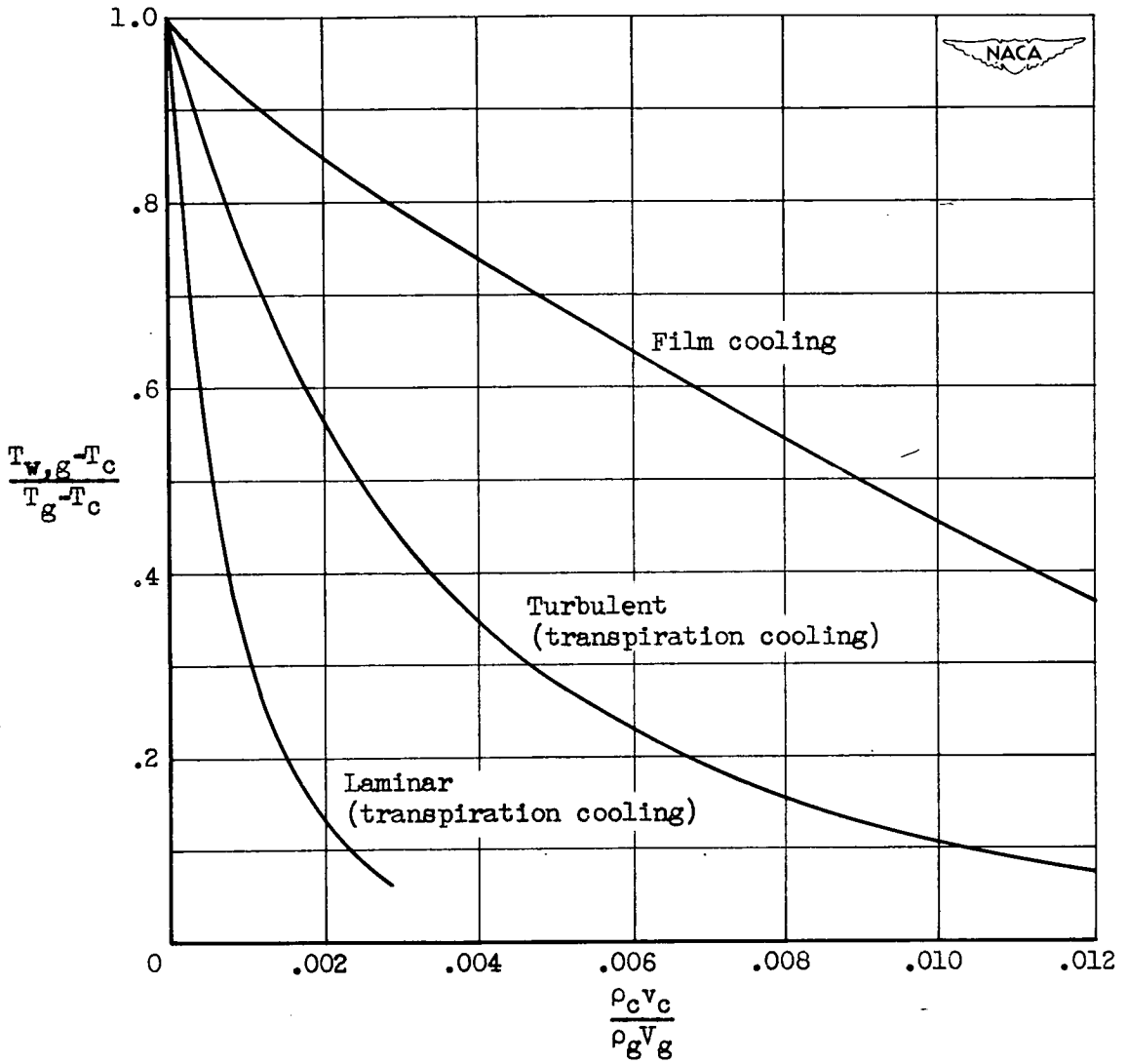


Figure 9. - Effect of mass-flow ratio on wall temperature for transpiration and film cooling. $Re = 10^5$.

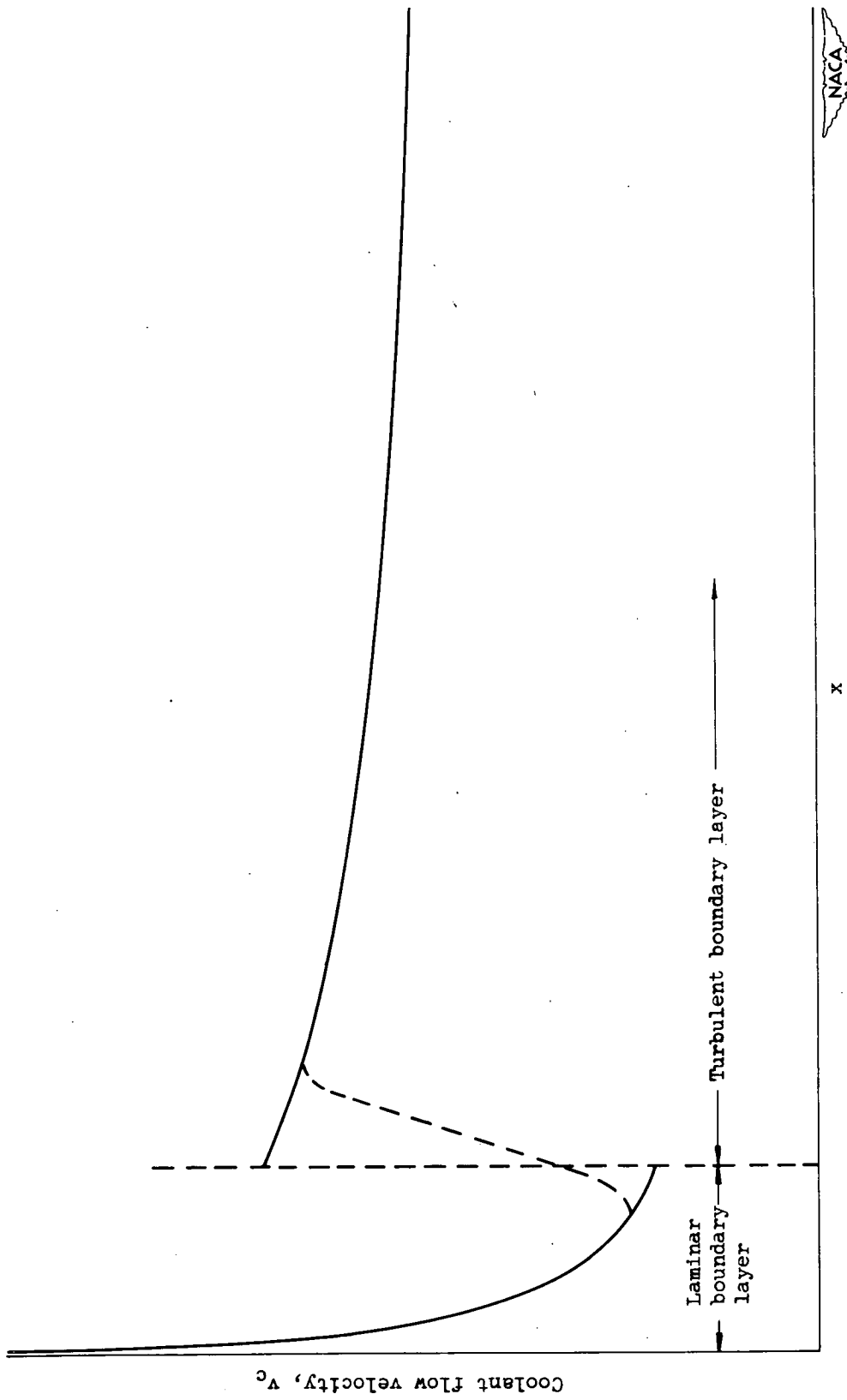


Figure 10. - Coolant-flow velocity requirements for constant-temperature transpiration-cooled plate.

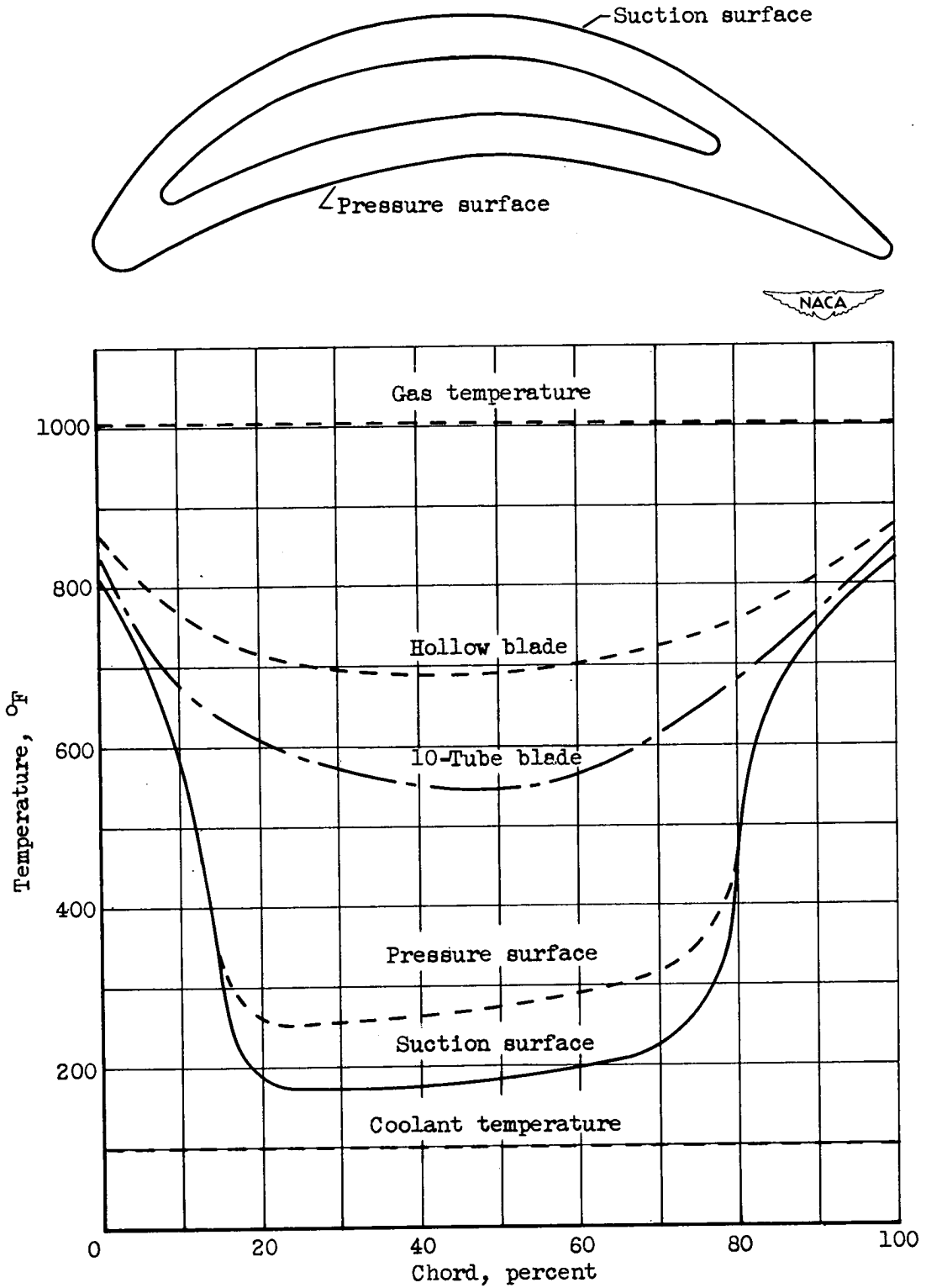


Figure 11. - Porous-blade temperature distribution for $w_c/w_g = 0.026$.

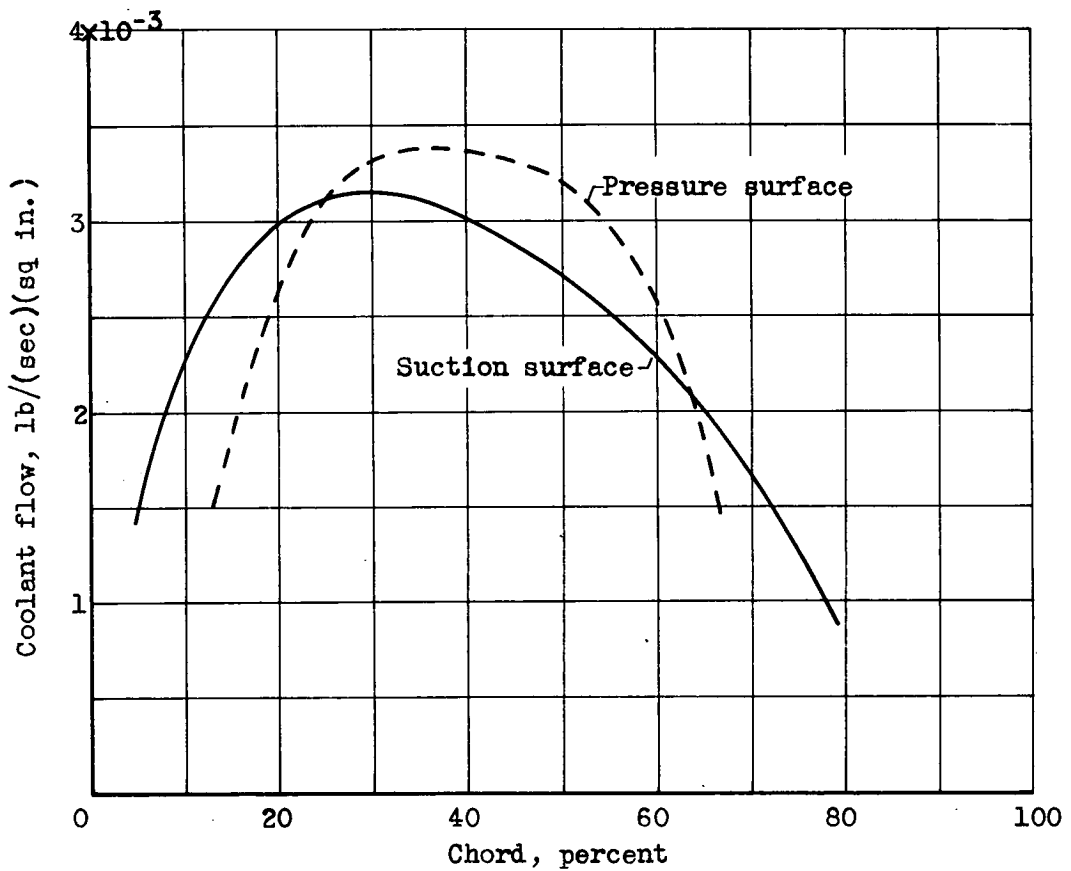
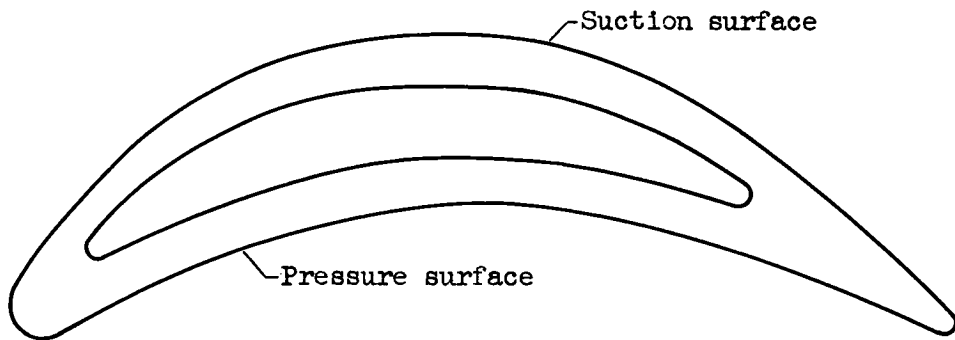


Figure 12. - Permeability variation around porous blade for $\rho \Delta P = 49.5 \text{ lb}^2/\text{ft}^5$.

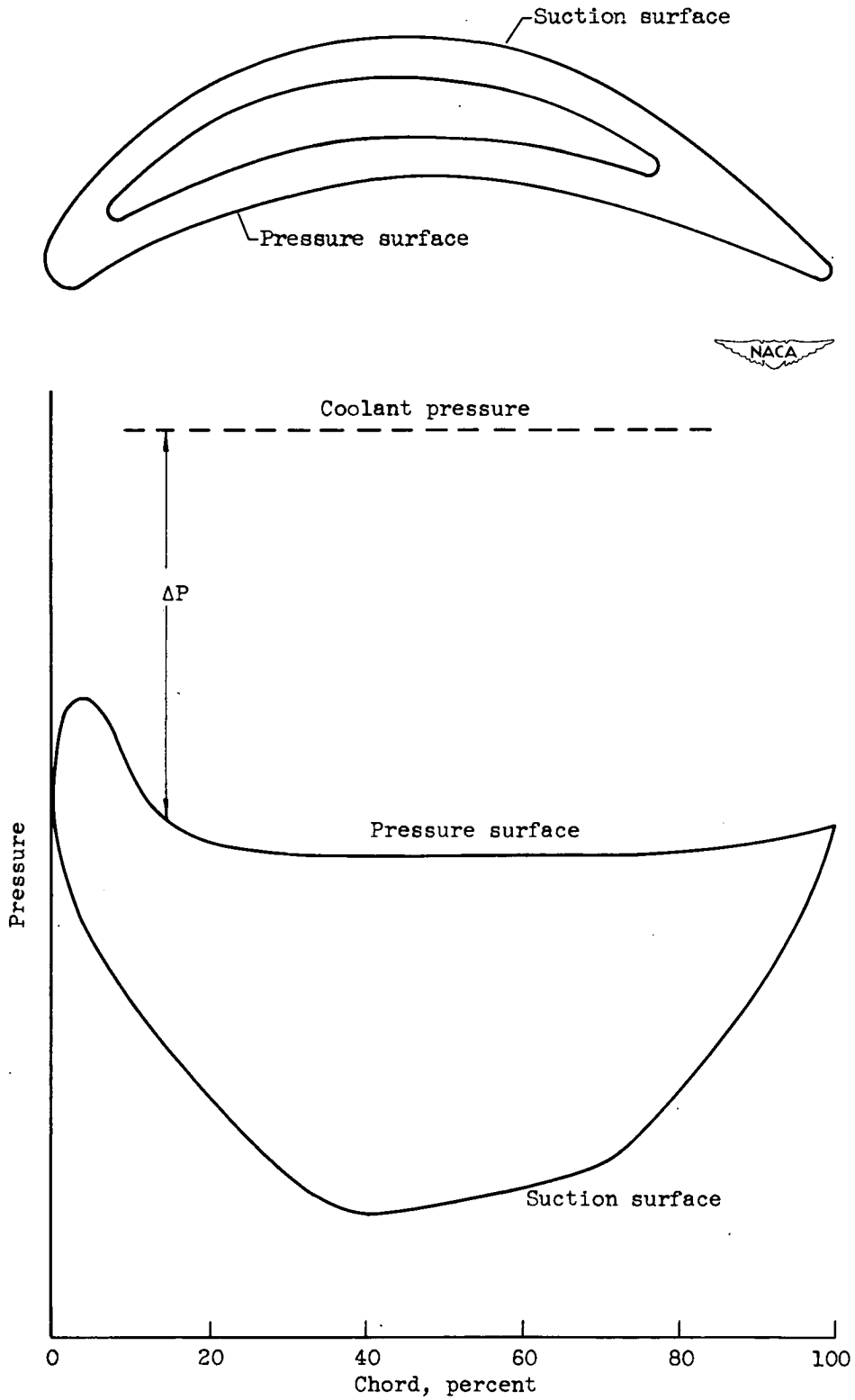


Figure 13. - Pressure distribution around turbine blade.