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RESEARCH MEMORANDUM

A THEORETICAL INVESTIGATION OF THE INFLUENCE OF AUTOPILOT NATURAL FREQUENCY UPON THE DYNAMIC PERFORMANCE CHARACTERISTICS OF A SUPERSONIC CANARD MISSILE CONFIGURATION WITH A PITCH-ATTITUDE CONTROL SYSTEM

By Anthony L. Passera

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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A THEORETICAL INVESTIGATION OF THE INFLUENCE OF AUTOPILOT NATURAL FREQUENCY UPON THE DYNAMIC PERFORMANCE CHARACTERISTICS OF A SUPERSONIC CANARD MISSILE CONFIGURATION WITH A PITCH-ATTITUDE CONTROL SYSTEM

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SUMMARY

A theoretical investigation was made to determine the effects of autopilot natural frequency upon the dynamic performance characteristics of an attitude-controlled supersonic missile configuration with rate damping for a Mach number and altitude range. The autopilots considered were single-degree-of-freedom systems with a fixed damping ratio and various natural frequencies. The airframe was a supersonic canard missile configuration with a rate gyro-servo to give the required rate damping.

The adjustable gains of the autopilot and rate gyro-servo were set for each autopilot at one flight condition and held constant for other flight conditions. Transient-response curves of pitch angle, control-surface deflection, and normal acceleration in response to a unit step input signal were found for three supersonic Mach numbers and two altitudes.

Upon reviewing these transient responses, it was concluded that as the autopilot natural frequency increased, the response time and rise time decreased and keeping the autopilot natural frequency as high as possible therefore is advantageous; however, servo energy requirements along with diminishing improvement for the high-natural-frequency autopilots supports the use of a low-natural-frequency autopilot. This investigation was aimed at obtaining a compromise between these two conflicting ideas. The effect of Mach number and altitude changes upon transient characteristics is also presented.

INTRODUCTION

The general research program of automatic stabilization at the Pilotless Aircraft Research Division of the Langley Aeronautical Laboratory is concerned with the dynamic performance characteristics of an automatically controlled supersonic missile configuration. As a result of the analysis presented in reference 1, the dynamic performance characteristics were shown to be improved by the addition of rate damping to a small-static-margin canard airframe. In this reference, attitude feedback was obtained by the use of a perfect proportional autopilot. The study herein considers how the addition of dynamics to the autopilot in the form of a second-order characteristic equation with various natural frequencies affects the missile performance characteristics. In this paper the rate gyro-servo was represented by an experimental frequency response obtained at the Langley Laboratory.

The results of this theoretical investigation are presented in the form of pitch-angle, control-surface-deflection, and normal-acceleration transient responses for several flight conditions and autopilot natural frequencies in response to a unit input command signal.

SYMBOLS

K	gain constant for airframe
K_A	gain constant for autopilot
K_r	gain constant for rate gyro-servo
δ_r	canard deflection angle due to rate gyro-servo, degrees
δ_A	canard deflection angle due to autopilot, degrees
δ	total canard deflection
θ_i	input-pitch-angle command signal measured from some reference or uncaged autopilot gyro position, degrees
θ_o	output pitch angle measured from same reference as θ_i, degrees
s	Laplace transform variable corresponding to the differential operator, $\frac{d}{dt}$

ω_{n_1} undamped natural frequency of airframe, radians per second

ω_n undamped natural frequency of autopilot, radians per second

ζ_1 damping ratio of airframe

ζ damping ratio of autopilot

$\bar{c}$ mean aerodynamic chord of wing, feet

n normal acceleration, g units

τ time constant of a linear factor

DESCRIPTION OF THE AIRCRAFT AND CONTROL SYSTEM

The missile considered in this paper is the symmetrical cruciform configuration shown in figure 1. A flight test of this configuration is reported in reference 2. The wings and canard fins are of delta design with the leading edges swept back 60° and have modified-double-wedge cross sections. The fuselage fineness ratio is 16. The canard fins provide the required longitudinal control, while the auxiliary damping is provided through these same canard fins by the action of a rate gyro-servo.

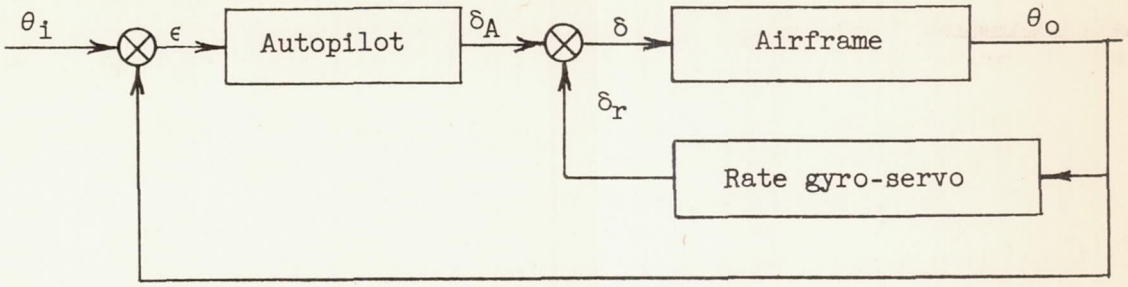
The rate gyro and servo combination used to give the additional damping to the dynamics of the airframe by providing a control-surface deflection proportional to the rate of pitch $\dot{\theta}_o$ is illustrated in figure 2. The valve controlling the flow of air to the servo is linked directly to the gyro gimbal. This gimbal, in turn, has its motion damped by two dashpots linked in parallel. The transient response to a step $\dot{\theta}_o$ of the rate gyro-servo was obtained experimentally at the Langley Laboratory by causing a step deflection of the rate-sensitive gyro gimbal. Figure 3 shows the transient response obtained and the associated frequency response determined by Fourier series.

Four attitude-sensitive autopilots are considered in the analysis approximated by the transfer function

$$\frac{\delta_A}{\epsilon}(s) = \frac{K_A \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

with the following constant coefficients: ω_n = 30, 50, 70, and 140 radians per second and ζ = 0.5. This transfer function has proven from experience

to be a good approximation of a gyro and servo combination and might well be the form that specifications would take in autopilot design.



The following is a description of the block diagram of the rate-damped missile and autopilot. An input signal or command θ_i calls for a change in pitch angle from some reference or uncaged position of the autopilot gyro. The error signal ϵ that causes the autopilot to respond is

$$\epsilon(s) = \theta_i(s) - \theta_o(s)$$

The autopilot responds to this signal and produces an output that satisfies the transfer function

$$\frac{\delta_A}{\epsilon}(s) = \frac{K_A\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

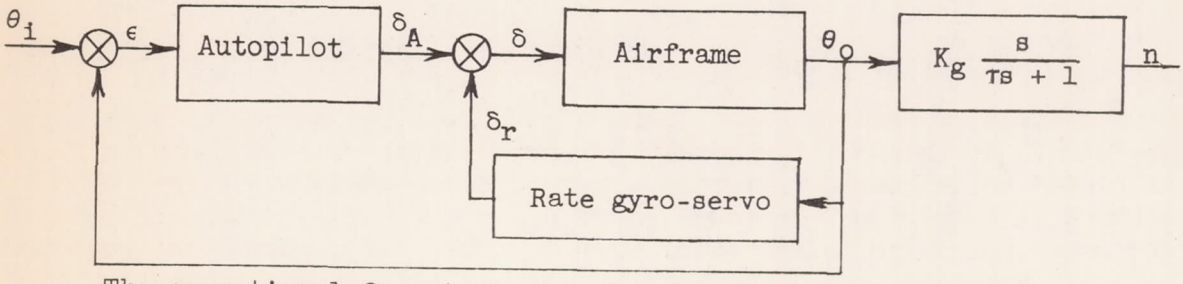
The rate gyro-servo produces a control-surface deflection δ_r in response to the signal $\dot{\theta}_o$. The transfer function for the rate gyro is not available in analytical form, but for this paper an experimentally determined transient response was available. Considering no change in lift due to control-surface deflection, this control-surface deflection $\delta = \delta_A - \delta_r$ causes the airframe to respond and a change in pitch angle θ_o is produced according to the transfer function

$$\frac{\theta_o}{\delta}(s) = \frac{K(\tau s + 1)}{s\left(s^2 + 2\zeta_1\omega_{n_1} s + \omega_{n_1}^2\right)}$$

This transfer function is obtained from the linear differential equations of motion with constant coefficients by assuming two degrees

of freedom longitudinally and disturbance from level flight. The Laplace transformation is applied to these equations with all initial conditions equal to zero, and then the equation is solved for θ_o/δ. The values of the constant coefficients in the airframe transfer function were determined by using the longitudinal stability derivatives given in reference 3. The resulting values of the coefficients are presented in table I.

The normal-acceleration transient response to a unit step input n(t) was obtained by cascading another transfer function with the original pitch-angle block diagram.



The operational form indicated for determining n is given by

$$n(s) = \frac{K_g s}{\tau s + 1}\,\theta_o(s)$$

where K_g is a constant.

Finally the control-surface-deflection transient response to a unit step input was obtained from the $\frac{\delta}{\theta_i}(s)$ response where

$$\frac{\delta}{\theta_i}(s) = \frac{\theta_o}{\theta_i}(s)\,\frac{\delta}{\theta_o}(s)$$

ANALYSIS PROCEDURE

The purpose of this investigation is to find the effects of the natural frequency of autopilots on the performance characteristics of an attitude-controlled canard missile configuration with rate damping over a Mach number and altitude range. The method and procedure of obtaining the over-all θ_o/θ_i frequency response by closing the two

loops of the block diagram is in accordance with linear servomechanism theory found in reference 4.

The adjustments of the gains K_r and K_A were obtained graphically. The rate gyro-servo and autopilot gain-constant adjustments were made to determine some of the best responses for the missile with a particular autopilot. The frequency responses of the missile and the rate gyro were plotted and the product, $\frac{\delta_r}{\delta} = \frac{\theta_o}{\delta}\frac{\delta_r}{\theta_o}$, taken by adding the log modulus and the phase angles on the graphs of log modulus plotted against log frequency and angle plotted against log frequency. This product was then plotted on the open-loop, rectangular coordinates of the plot of log modulus against angle and the closed-loop frequency response δ_r/δ_A was obtained by reading the coordinates of the superimposed closed-loop contours. At this point, the gain constant of the rate gyro can be increased or decreased by merely translating the open-loop curve vertically to a higher or lower position, respectively. Then the following operation is necessary to obtain the θ_o/δ_A frequency response:

$$\frac{\theta_o}{\delta_A} = \frac{\delta_r}{\delta_A}\frac{\theta_o}{\delta_r}$$

The autopilot transfer function is added to this response on the graphs of log modulus plotted against log frequency and angle plotted against log frequency to yield the over-all open-loop response $\frac{\theta_o}{\epsilon}$.

At this point, any variation of the rate-gyro gain constant alters the shape of the open-loop frequency-response curves $\frac{\theta_o}{\epsilon}$ and a family of curves of each missile and autopilot combination is produced for several values of the rate-gyro gain constant (see reference 1). This family of curves is examined, and a curve whose modulus resembles the shape of the closed-loop zero-decibel contour on the plot of log modulus against angle and has a high resonant frequency (if the curve has a peak) is chosen as that which would yield one of the best transient responses for the missile with that particular autopilot. Then, the autopilot

gain is adjusted to position the open-loop curve so it falls somewhere along the zero-decibel contour or between the zero- and 2.3-decibel contours, depending upon the shape of the curve. (See reference 4 for the significance of the gain adjustments to position the open-loop curve θ_o/ϵ tangent to the 2.3-decibel closed-loop contour.) This autopilot gain adjustment usually causes a resonant peak in the amplitude-ratio response. When this gain adjustment is made, the over-all closed-loop response θ_o/θ_i is obtained by reading the coordinates on the superimposed closed-loop contours. This is the final step in finding the pitch-angle response to a sinusoidal signal θ_i. When the response of the system to sinusoidal signals is known, the pitch-angle transient response to a square-wave input can be obtained by the method of superposition.

Since it is not possible by merely examining slightly different frequency responses to choose the one which will result in the best transient characteristics, it is necessary to obtain and examine the transient responses for several adjustments of K_r and K_A by the method described in the previous paragraph, and the combination of K_r and K_A which yields the best transient characteristics is selected. This method of adjusting the system gains does not necessarily give the optimum transient response but it is believed to give one which is nearly optimum. The gains were thus adjusted for each autopilot for $M = 1.6$ and an altitude of 4,000 feet.

Holding these gains fixed, the pitch-angle transient responses for other Mach numbers and altitudes were obtained by the procedure previously mentioned after making the required changes in the constants of the airframe transfer function.

The procedure for obtaining control-surface deflection and normal-acceleration transient responses to a step input signal is the same except for previously mentioned changes in block diagram.

The transient responses were obtained by the use of an electromechanical Fourier synthesizer at the Langley Laboratory. This machine adds a finite number of terms of a Fourier series. (See reference 5.) Since the frequency response of the system including the airframe, rate gyro-servo, and autopilot is available, the systems response to a square-wave input is determined by the method of superposition. As explained in reference 5, the output produced is

$$\frac{2}{\pi} \sum_{n=1,3\ldots} \left[\frac{(\text{Amplitude ratio})_{n\omega_1}}{n}\right] \sin\left[n\omega_1 t + (\text{Phase angle})_{n\omega_1}\right]$$

One requirement is that it is necessary to have the period of the fundamental frequency ω_1 large enough so all the transient motion will have essentially died out by the end of each half-cycle. Twelve odd harmonics usually give a good approximation for the response of the system to a square-wave input. The missile and autopilot are, in effect, low-pass filters so any high-frequency harmonics would be greatly attenuated relative to the fundamental and will thereby contribute little to the transient response at the output.

RESULTS AND DISCUSSION

This analysis was conducted to determine the effects of the natural frequency of autopilots on the performance characteristics of an attitude-controlled supersonic canard missile configuration with rate damping. Nearly optimum pitch-angle transient responses of the rate-damped missile and autopilot were obtained for $M = 1.6$, an altitude of 4,000 feet, and a static margin of $0.094\bar{c}$ with suitable adjustments of the rate gyro-servo and autopilot gain constants. The gain constants used with these autopilots are presented in table II. With these gain constants fixed, pitch-angle transient responses for flight conditions, $M = 1.2$ and $M = 2.0$ at an altitude of 4,000 feet and $M = 1.6$ at an altitude of 30,000 feet, were obtained to determine the effects on the system due to changes in Mach number and altitude. The choice of a small static margin is based upon the analysis presented in reference 1. Control-surface-deflection transient responses were found for flight conditions, $M = 1.2$, $M = 1.6$, and $M = 2.0$ at 4,000 feet, and all flight conditions considered for the missile with the autopilot whose natural frequency is 50 radians per second. Normal-acceleration transient responses were also obtained for the autopilot natural frequency of 50 radians per second for the same flight conditions and for all other autopilots at $M = 2.0$ and 4,000 feet.

In studying the results presented herein, there are transient characteristics used in this discussion that need to be defined: amplitude of the initial overshoot, rise time, and response time. The amplitude of the initial overshoot is the magnitude of the first peak above and measured from the steady-state value. The rise time is the time for the output θ_o to initially reach the steady-state value. The response time is the time required for the output to reach and remain within ± 5 percent of the steady-state or final value. These pitch-angle transient characteristics are illustrated in figure 4. Since the output of most physical systems can at best only follow the input with some small error, the best approximation of a desirable transient response is the one that has small amplitude for the initial overshoot, short rise time, and short response time. In other words, desirable transient-response characteristics

are those that reduce the transient error; however, consideration of structural and control-surface-deflection limitations may put some restrictions on these transient characteristics. Also, a missile and autopilot system may have transient characteristics that are desirable for one flight condition, but changes in Mach number or altitude may cause a radical change in the amplitude of the initial overshoot, response time, and rise time. Another system may have transient characteristics that yield a slow response or one with appreciable transient error, and yet, changes in flight conditions may not have much effect on these transient characteristics. It may become necessary, depending upon the application, to sacrifice desirable transient characteristics for poorer transient characteristics that are more consistent over a Mach number and altitude range.

Other considerations are that for a physical system there may be some limitation of the control-surface deflection either due to stops built into the control system, limit on the length of the servo stroke, or limit on aerodynamic control effectiveness. In order to produce the required pitch-angle transient response, the input to the servo may call for large oscillatory displacements through the combined outputs of the rate gyro and autopilot causing the servo to produce the maximum possible deflection and hold it until the input to the servo calls for a reduction in the servo displacement. The linear analysis for the system may also call for a rate of servo displacement that is beyond the physical limits of a particular valve and servo combination. This power limitation which, for example, might be due to some restriction in the time rate of volume flow for oil under a given pressure was not considered in the analysis. Precautions should be taken to prevent such nonlinear behavior in the missile control system. If, however, such behavior does exist, consideration should be given to determine to what extent the linear method of analysis is valid. Another consideration is that a desirable output transient response may require large total servo piston travel necessitating a large amount of stored potential energy for a given step input signal in the form of a stored volume of oil under pressure.

Pitch-Angle Transient Responses

Pitch-angle transient responses to a unit step input signal are presented in figures 5 to 8. These results are summarized in figures 9 and 10.

Figure 9 shows that in general increasing the natural frequency of the autopilot for all Mach numbers and altitudes considered causes the rise time and the response time to decrease and the initial overshoot to remain essentially the same.

The results of the pitch-angle transient responses indicate that it is desirable to include as high a natural-frequency autopilot as is available since transient characteristics tend to improve as the autopilot natural frequency increases; however, a consideration of the improvement quantitatively along with the economics and stored-energy requirements involved in the design and construction of higher-natural-frequency autopilots may favor the use of the lowest-natural-frequency autopilot which yields satisfactory transient characteristics.

For the pitch-angle transient responses, figure 9 shows that the greatest improvement in the transient characteristics occurs for the system with an autopilot natural frequency between 30 and 70 radians per second. For the system with the autopilot natural frequency greater than 70 radians per second the improvement is not so pronounced; however, at these frequencies the cost in design and construction of such an autopilot begins to increase appreciably. From the standpoint of economy, the possible added improvement of using an autopilot natural frequency greater than 70 might be outweighed by the increased cost.

The total transient error was integrated with a planimeter over the pitch-angle transient response from zero to the time required for the output to reach and remain within 5 percent of the steady-state value. This total transient error is a method of evaluating the combined effects of transient characteristics and is indicative of how well the output follows the input. This value of $\int|\epsilon(t)|\,dt$ for a nearly optimum system should be kept to a minimum if the system is free of noise; however, no attempt was made to minimize the value of this integral. This value was only used to illustrate the relative merits of different autopilot natural frequencies for the method of system adjustment used herein. Figure 10 shows that the system with an autopilot natural frequency greater than 70 radians per second does not substantially decrease the total transient error; therefore the argument for not increasing the autopilot natural frequency much above 70 radians per second is strengthened.

δ and n Transient Responses

In order to present a more complete analysis of the rate-damped missile and autopilot system, control-surface-deflection and normal-acceleration transient responses to a unit step input signal θ_i are presented in figures 11 to 16. Since there are physical limitations on structural loads and control-surface deflections, these transients are useful in determining what maximum values to expect for any input step signal. Also the δ transient responses indicate what total servo energy is required in response to the step θ_i.

Control-surface-deflection responses were obtained for the missile and autopilot combination with autopilot natural frequencies of 30, 50, 70, and 140 radians per second and for an altitude of 4,000 feet at all Mach numbers considered. Responses were also obtained for the autopilot natural frequency of 50 radians per second at $M = 1.6$ and 30,000 feet.

Reviewing the δ transients presented, some general conclusions are reached. For the missile and all autopilots considered, the amplitude of the maximum control-surface deflection, in general, decreases with increases in Mach number for an altitude of 4,000 feet. For flight conditions at 4,000 feet and all Mach numbers considered, increasing the ω_n of the autopilot increases the amplitude of the maximum overshoot. The maximum control-surface deflection for a step input signal θ_i increases with an increase in altitude for the autopilot with $\omega_n = 50$ at $M = 1.6$.

Figure 17 presents the total δ travel in response to a step input signal computed from the δ transient responses. Since the total servo displacement is proportional to the δ travel, this figure illustrates that as the autopilot natural frequency increases, more and more stored energy is required. The space and weight limitations for the airframe servos and associated gear make it a requirement to keep the autopilot natural frequency somewhere near the lowest value that yields satisfactory transient characteristics. Normal-acceleration transient responses were also obtained for the missile having an autopilot natural frequency of 50 radians per second for all Mach numbers and altitude ranges considered, and for the missile combined with the four autopilots considered at the highest Mach number. The highest Mach number was chosen since this would most likely yield the greatest number of g's for a given altitude, which would set a physical limit on the input step signal.

The normal-acceleration transient responses shown for $M = 2.0$ illustrate that as ω_n increases, the maximum normal acceleration increases. For $\omega_n = 50$ and $M = 1.6$, increasing the altitude decreases the maximum normal acceleration per degree of input θ_i. Finally, for the autopilot $\omega_n = 50$ at 4,000 feet increasing the Mach number increases the maximum overshoot for the n transient response.

CONCLUDING REMARKS

As a result of this theoretical investigation of the influence of autopilot natural frequency on the performance of a canard missile configuration with a pitch-attitude control system, the following conclusions are reached.

For all autopilots considered, increasing the autopilot natural frequency caused the rise time and response time to decrease while the initial overshoot remained essentially the same.

The improvement in the pitch-angle transient characteristics of the system with increasing autopilot natural frequency was greater for changes in natural frequency from 30 to 70 radians per second with smaller improvement for natural frequency greater than 70 radians per second for all flight conditions considered.

For all flight conditions considered, the required stored energy for a hydraulic servo in response to a given step input increased as the autopilot natural frequency increased.

The data obtained from investigations of this type may be used by the system designer in conjunction with space, weight, and economic considerations to determine the most practical automatic pilot specifications. It may be that for the desired application the additional cost and servo energy required might prohibit designing an autopilot with a natural frequency greater than 70 radians per second in view of the small improvement in system response obtained by using a higher natural frequency. For other configurations and control systems, a similar investigation would be required to obtain the data needed for selecting an autopilot compromise.

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TABLE I

AIRFRAME TRANSFER FUNCTION COEFFICIENTS FOR VARIOUS VALUES OF MACH NUMBER AND ALTITUDE

[Static margin, $0.094\bar{c}$ at $M = 1.6$;

$$\frac{\theta_o}{\delta}(s) = \frac{K(\tau s + 1)}{s(s^2 + 2\zeta_1 \omega_{n_1} s + \omega_{n_1}{}^2)}]$$

Mach number	Altitude (ft)	K	τ	ζ_1	ω_{n_1}
1.6	4,000	1800	0.268	0.26	13.8
1.6	30,000	241	.687	.17	8.0
1.2	4,000	1240	.287	.21	13.5
2.0	4,000	3250	.213	.37	11.8

TABLE II

AUTOPILOT AND RATE-GYRO GAIN CONSTANTS TABULATED AGAINST AUTOPILOT NATURAL FREQUENCY

[Adjusted for M = 1.6 and altitude of 4000 ft]

ω_n	K_r	K_A
140	0.08	2.32
70	.13	2.43
50	.16	2.82
30	.10	1.26

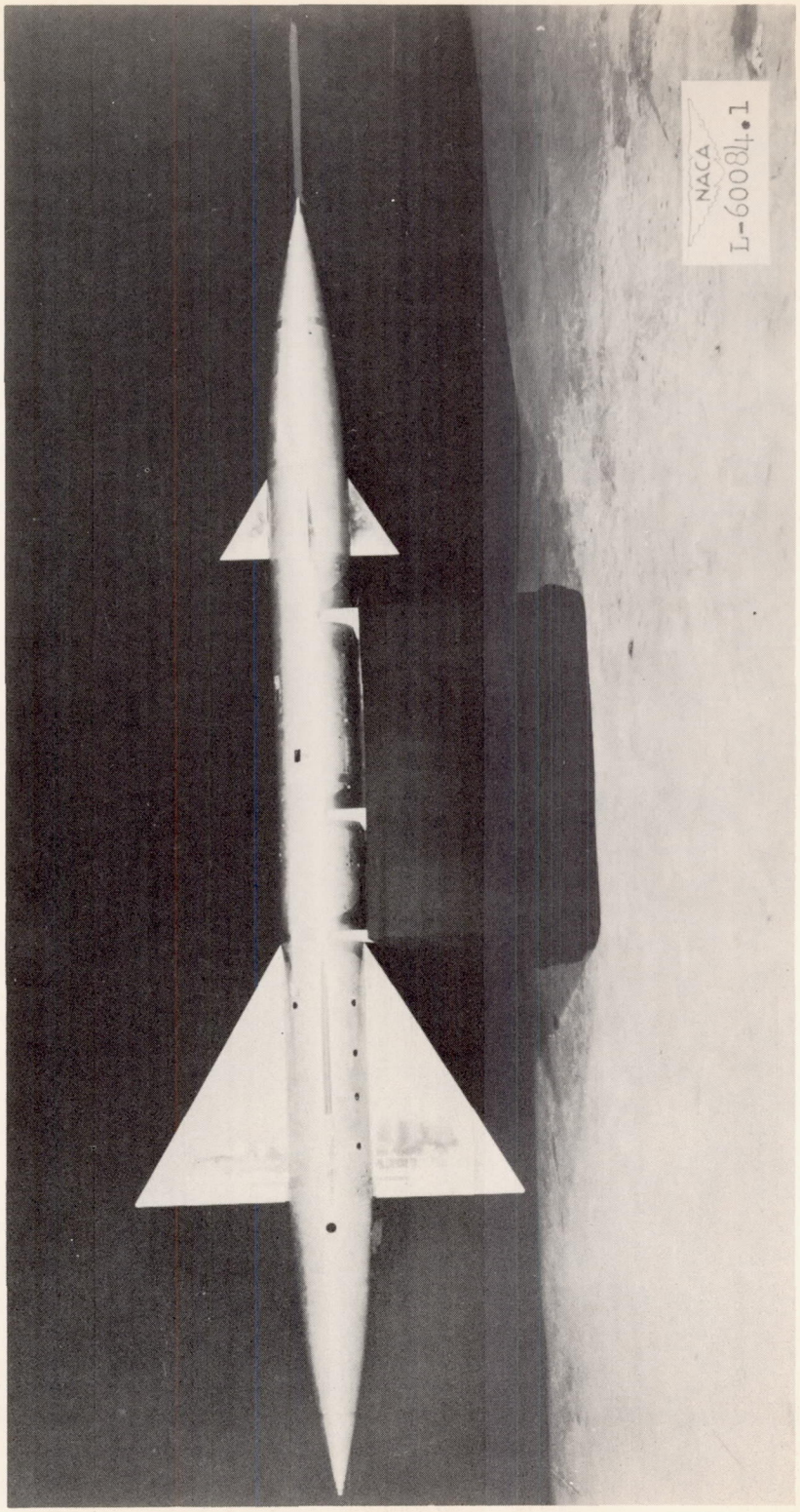


Figure 1.- Photograph of the missile configuration.

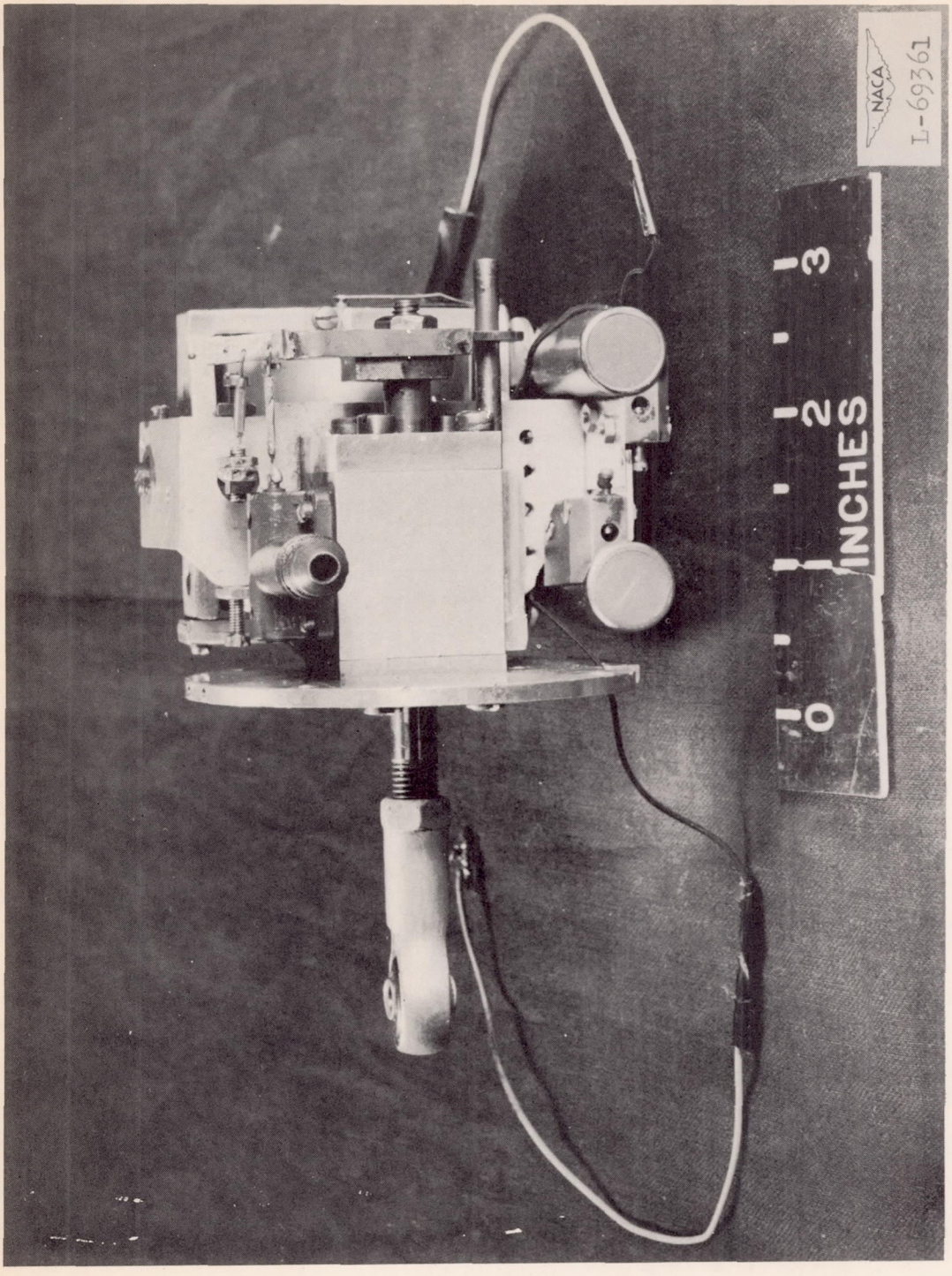


Figure 2.- Photograph of the rate gyro-servo.

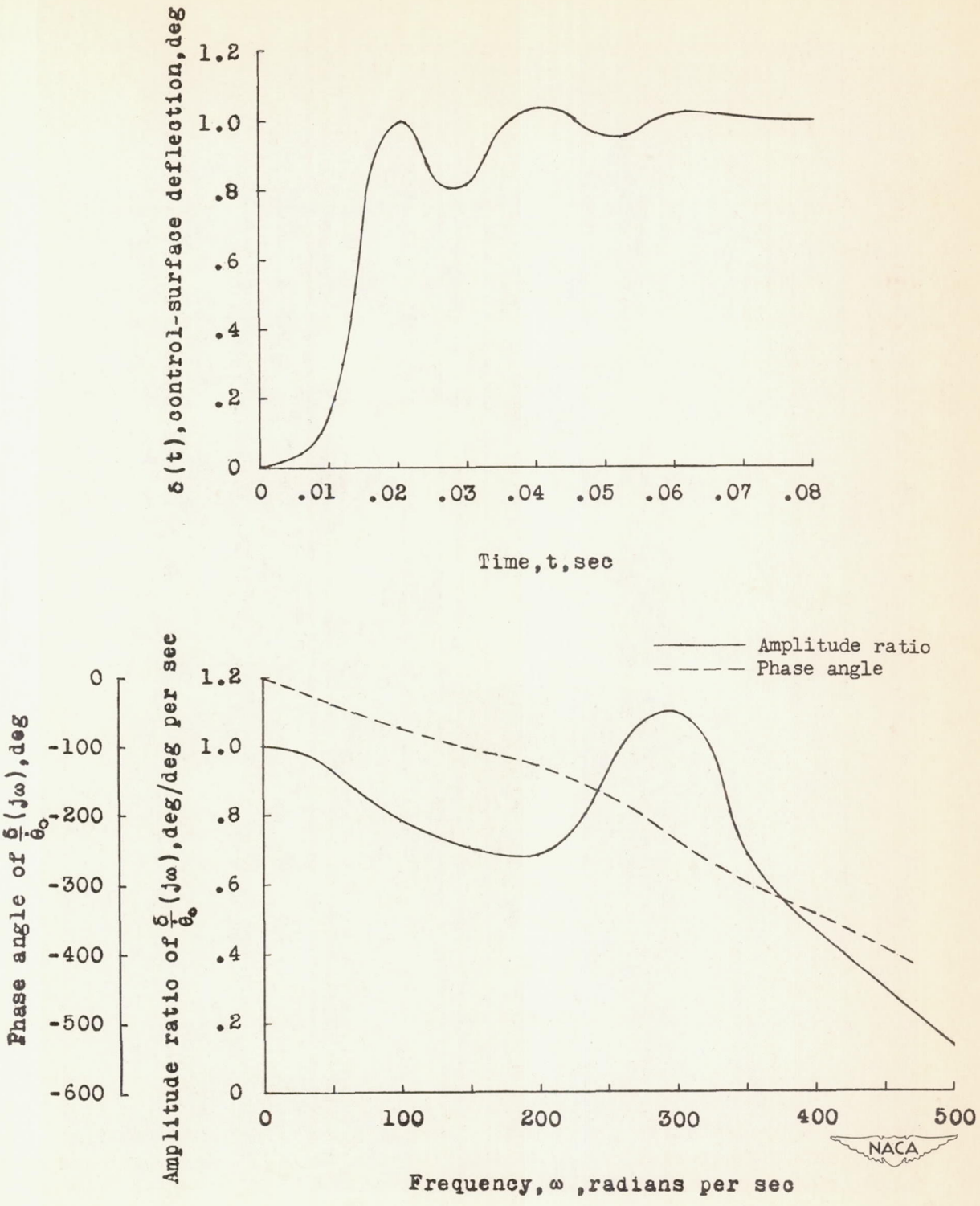
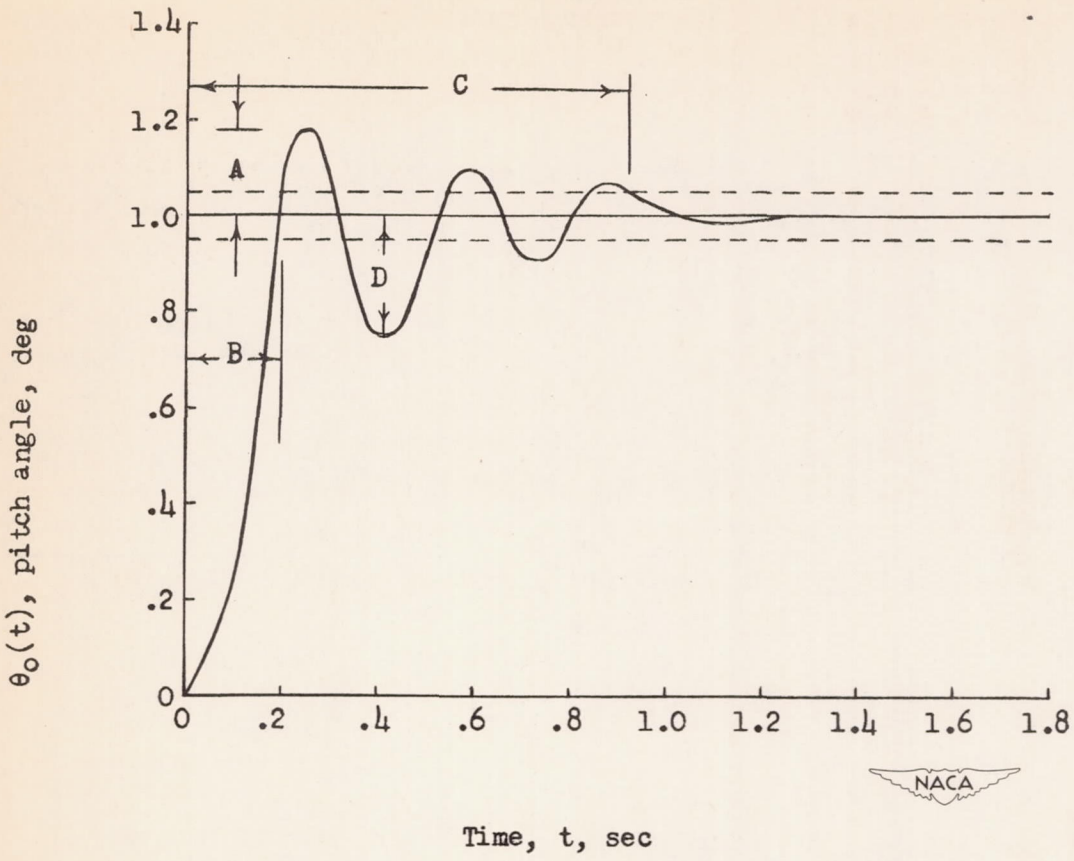


Figure 3.- Experimental transient response of the rate gyro and servo combination to a unit step, $\dot{\theta}_o$, and the frequency response determined from this transient by Fourier series.



A - Amplitude of the initial overshoot

B - Rise time

C - Response time

D - Transient error

Figure 4.- Representative pitch-angle transient response illustrating transient characteristics: amplitude of the initial overshoot, response time, rise time, and transient error.

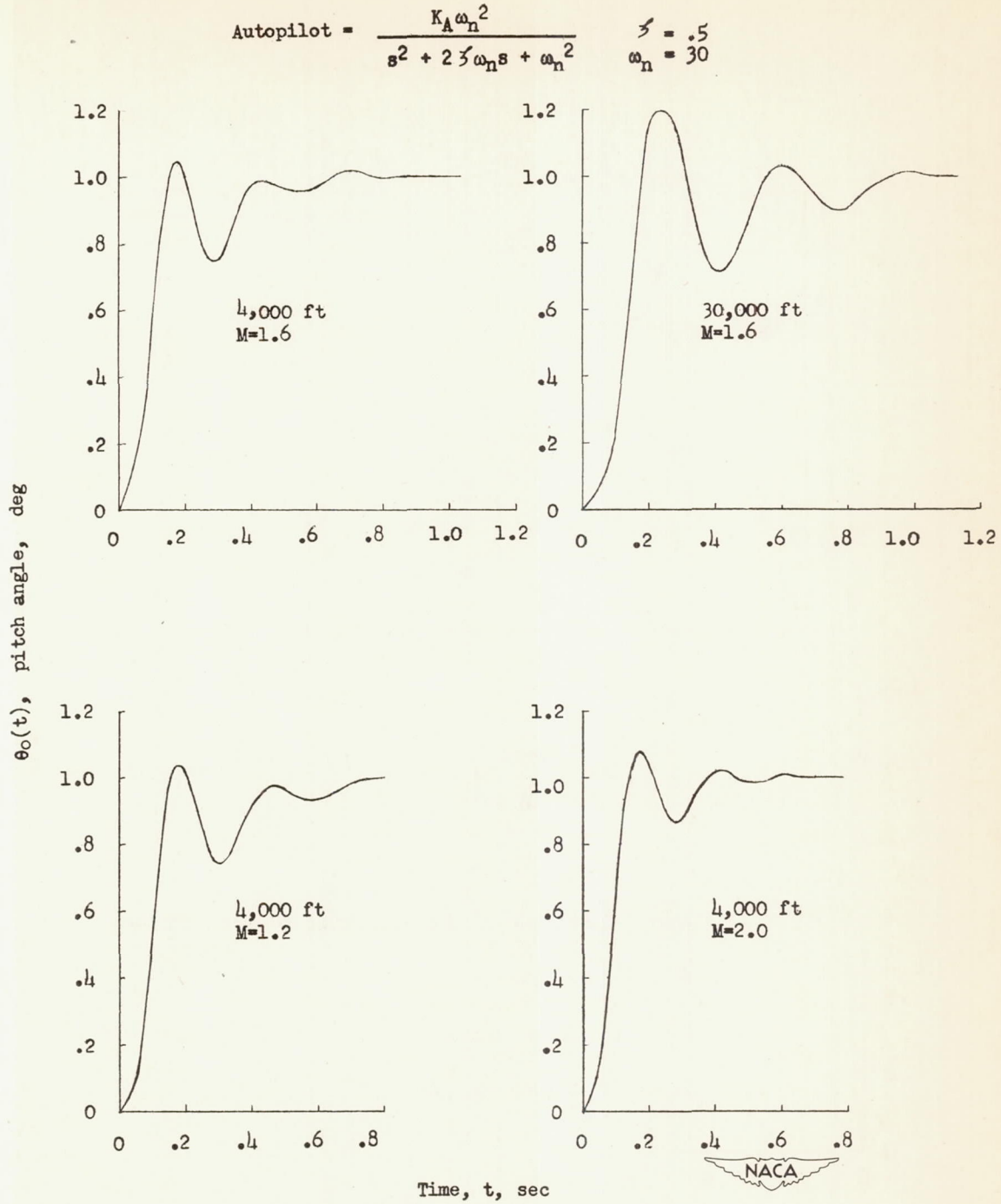


Figure 5.- Longitudinal transient responses $\theta_o(t)$ of the missile to a unit step input signal calling for a change in attitude of 1°. Static margin, 0.094$\bar{c}$ at M = 1.6; K_r = 0.10; K_A = 1.26.

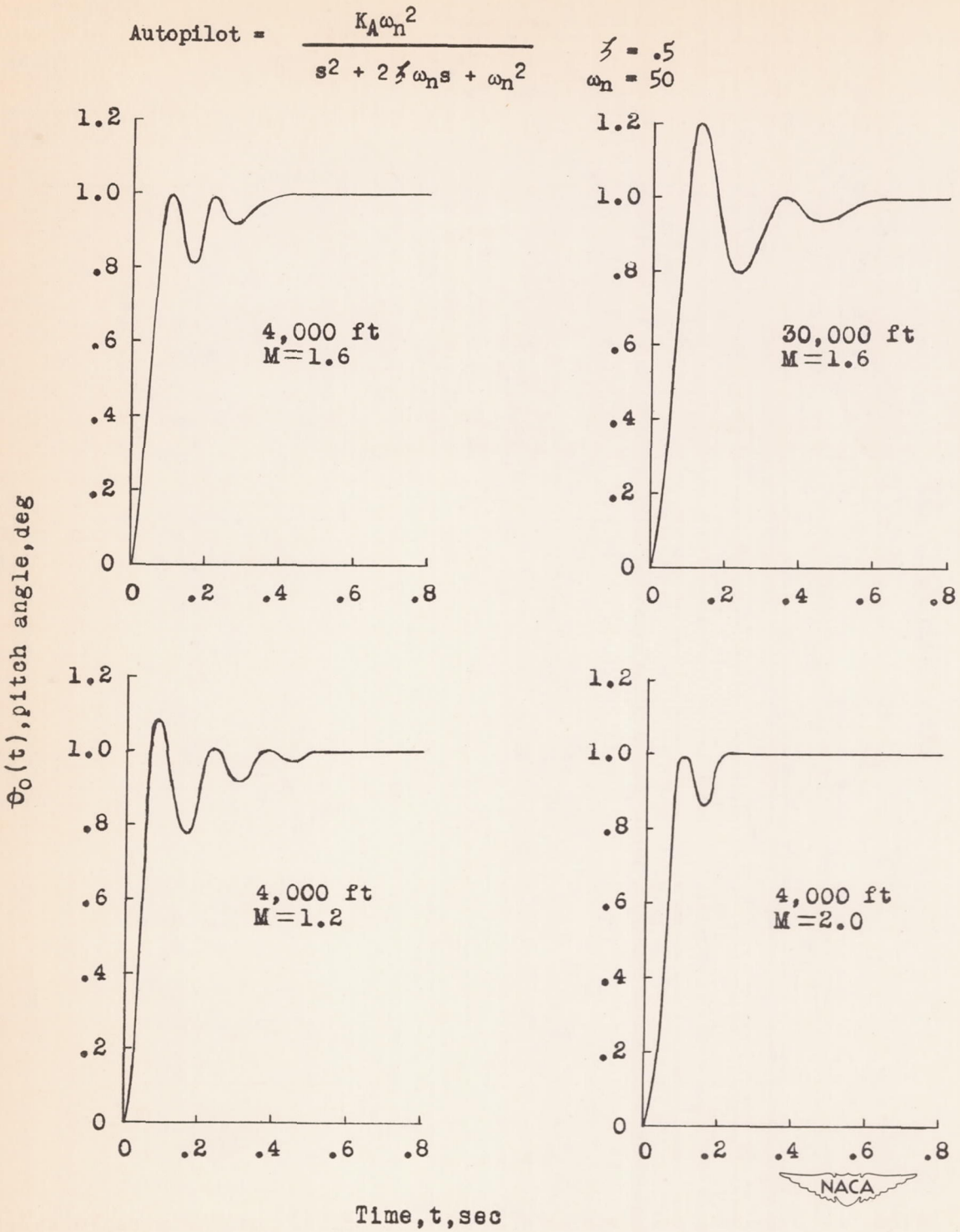


Figure 6.- Longitudinal transient responses $\theta_o(t)$ of the missile to a unit step input signal calling for a change in attitude of 1^o. Static margin, $0.094\bar{c}$ at M = 1.6; K_r = 0.16; K_A = 2.82.

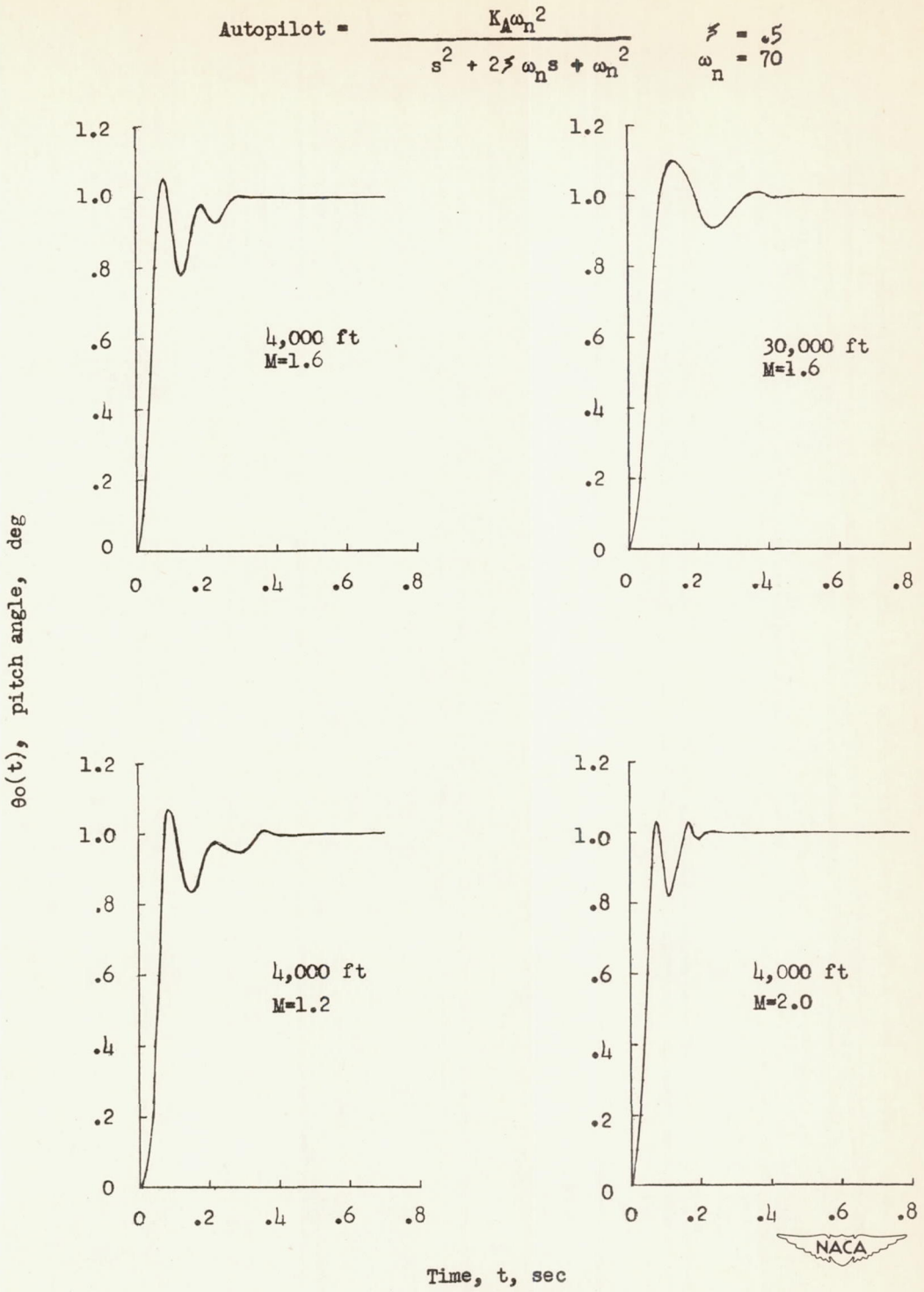


Figure 7.- Longitudinal transient responses $\theta_o(t)$ of the missile to a unit step input signal calling for a change in attitude of 1^o. Static margin, $0.094\bar{c}$ at M = 1.6; K_r = 0.13; K_A = 2.43.

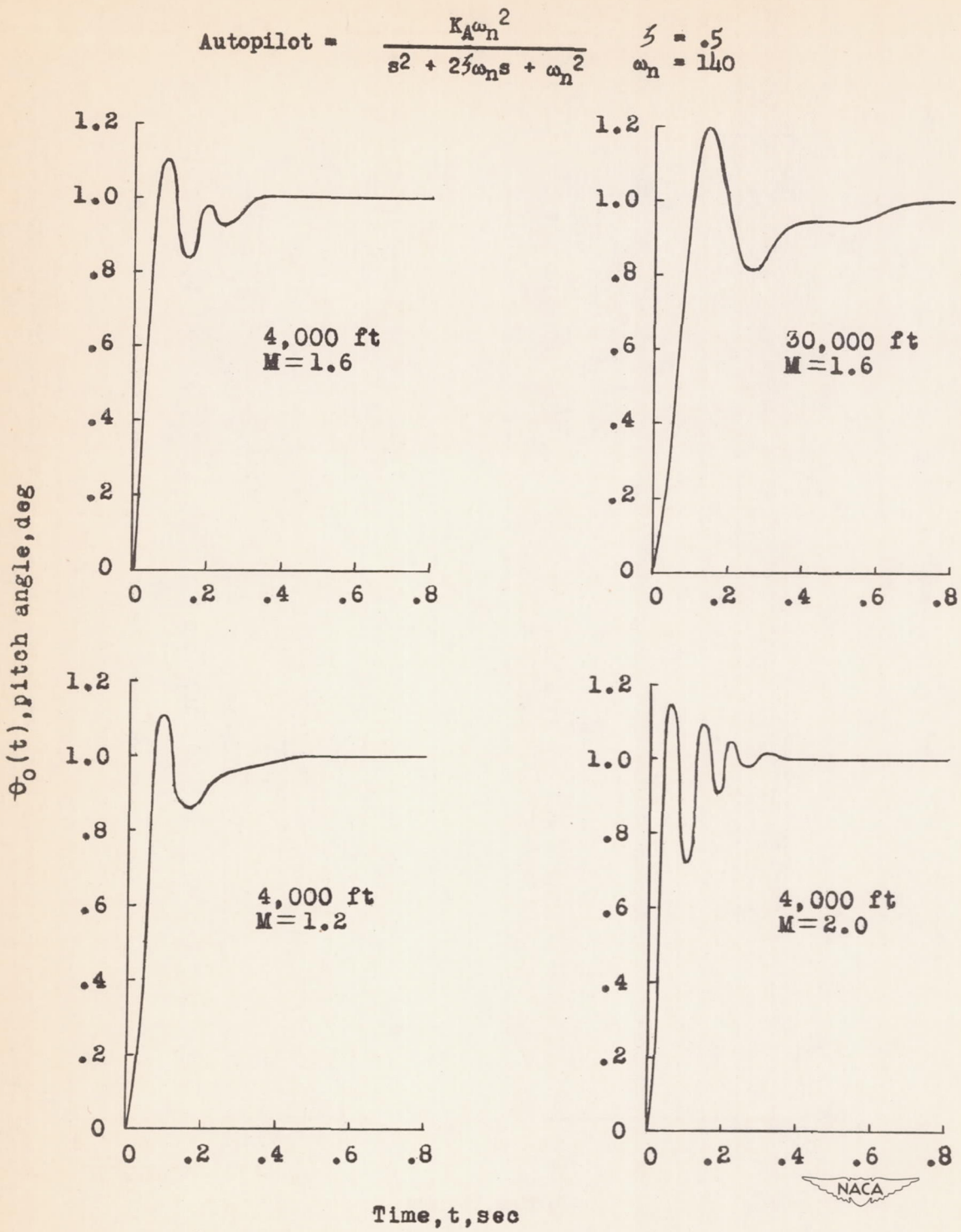


Figure 8.- Longitudinal transient responses $\theta_o(t)$ of the missile to a unit step input signal calling for a change in attitude of 1°. Static margin, $0.094\bar{c}$ at M = 1.6; K_r = 0.08; K_A = 2.32.

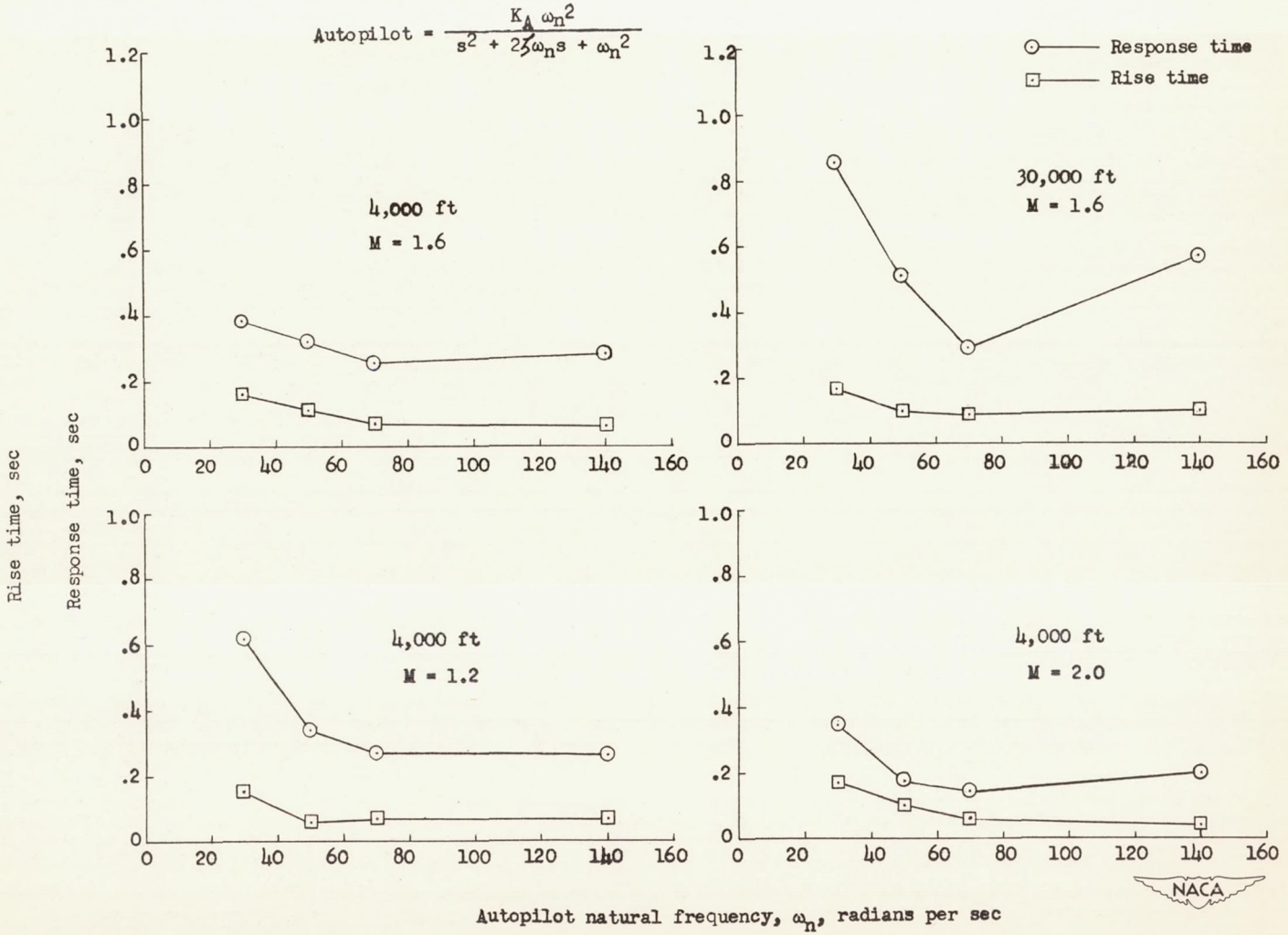


Figure 9.- Pitch-angle transient characteristics, response time, and rise time plotted against autopilot natural frequency. Static margin, $0.094\bar{c}$ at M = 1.6; ζ of autopilot, 0.5.

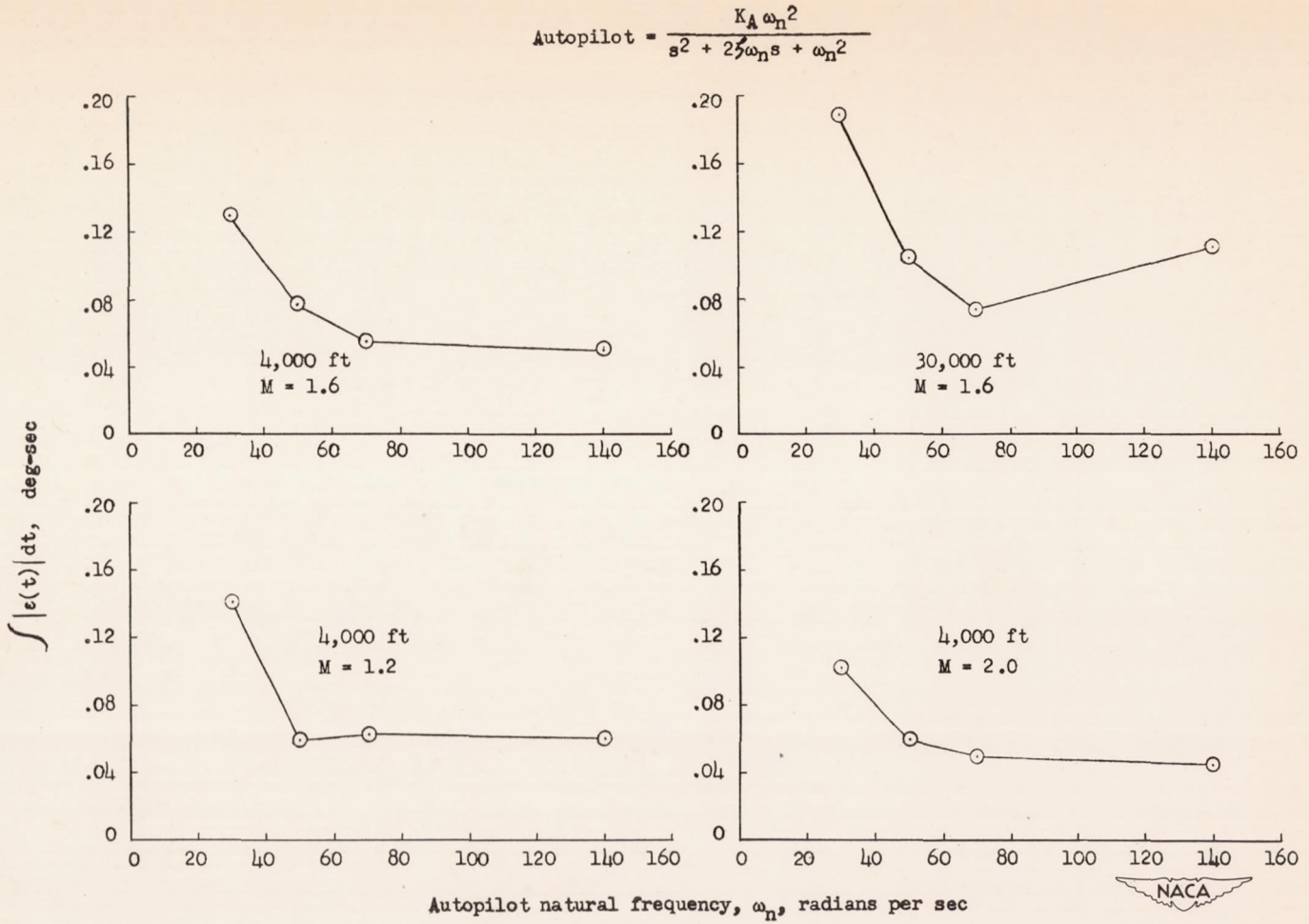


Figure 10.- Plot of $\int |\epsilon(t)| dt$ for the rate-damped airframe and autopilot combination against autopilot natural frequency in response to a step θ_i. Static margin, $0.094\bar{c}$ at $M = 1.6$; ζ of autopilot, 0.5.

$$\text{Autopilot} = \frac{K_A \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \zeta = .5 \quad \omega_n = 30$$

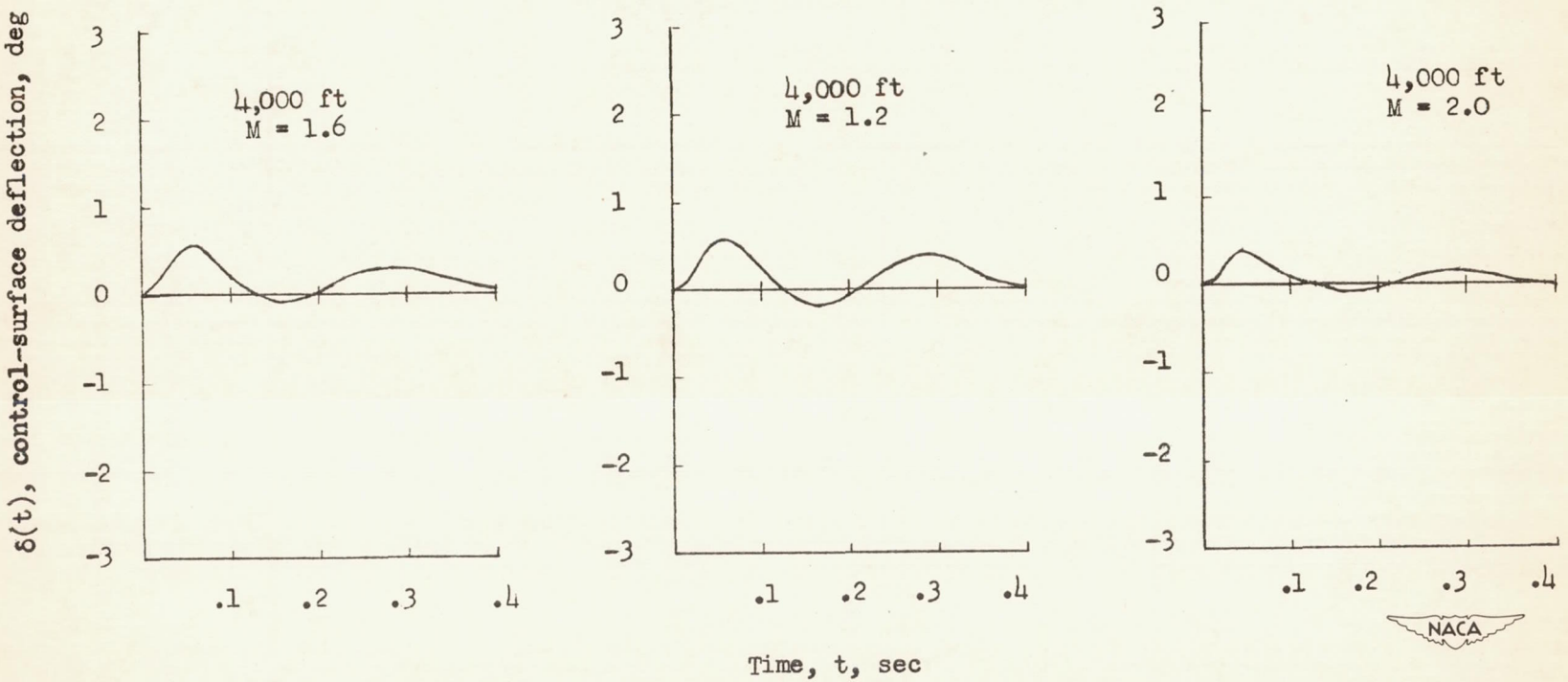


Figure 11.- Longitudinal transient responses $\delta(t)$ of the missile to a unit step input signal calling for a change in attitude of 1°. Static margin, $0.094\bar{c}$ at $M = 1.6$; $K_r = 0.10$; $K_A = 1.26$.

$$\text{Autopilot} = \frac{K_A \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \zeta = .5 \quad \omega_n = 50$$

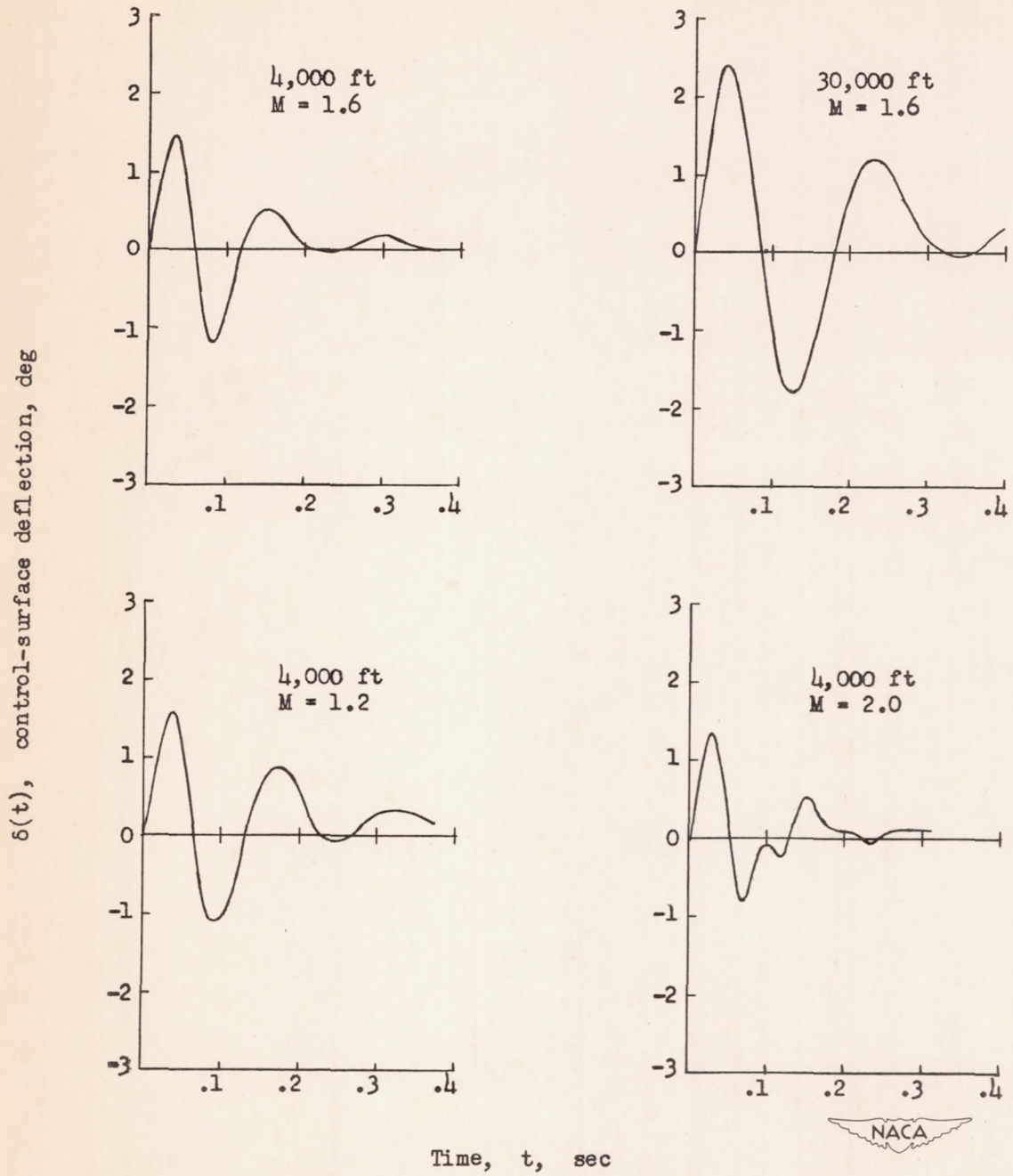


Figure 12.- Longitudinal transient responses $\delta(t)$ of the missile to a unit step input signal calling for a change in attitude of 1°. Static margin, $0.094\bar{c}$ at $M = 1.6$; $K_r = 0.16$; $K_A = 2.82$.

$$\text{Autopilot} = \frac{K_A \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \begin{matrix} \zeta = .5 \\ \omega_n = 70 \end{matrix}$$

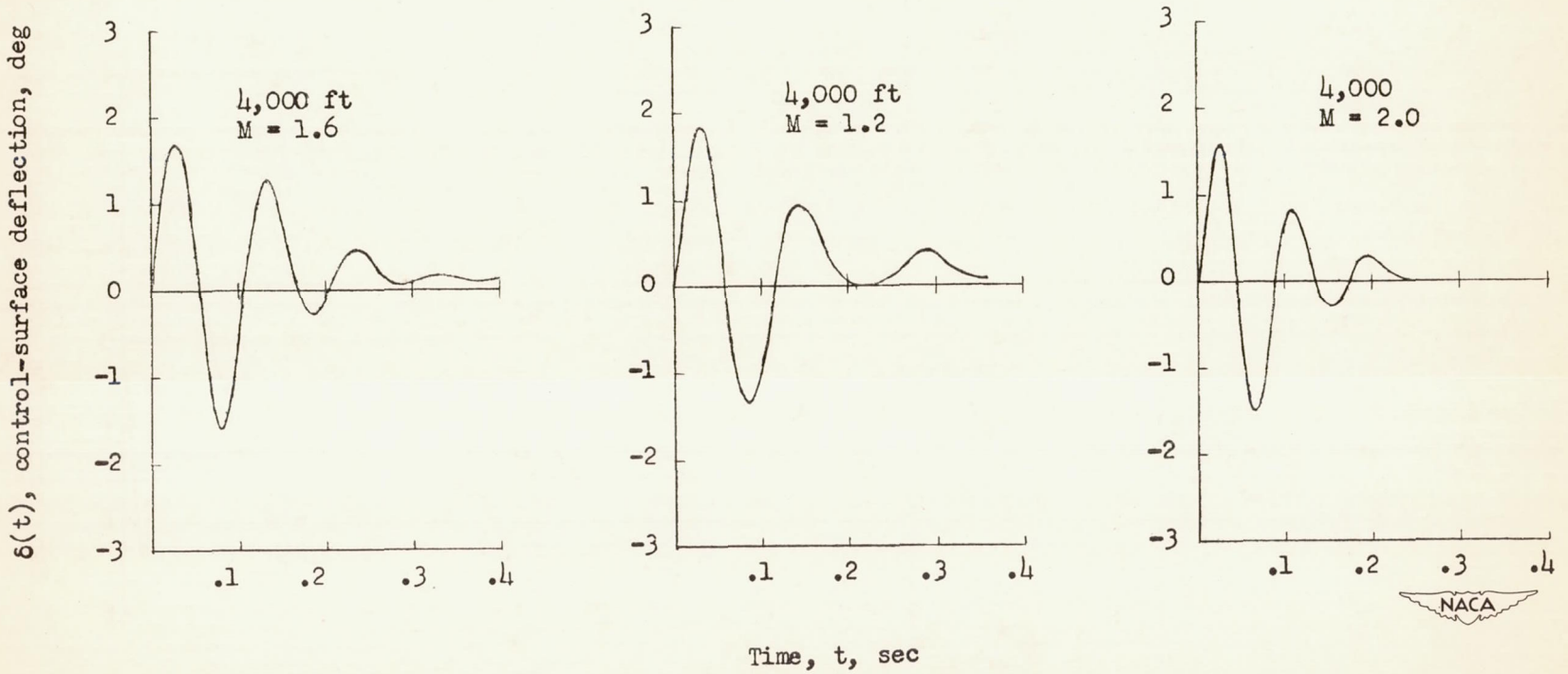


Figure 13.- Longitudinal transient responses $\delta(t)$ of the missile to a unit step input signal calling for a change in attitude of 1°. Static margin, $0.094\bar{c}$ at $M = 1.6$; $K_r = 0.13$; $K_A = 2.43$.

$$\text{Autopilot} = \frac{K_A \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \begin{matrix} \zeta = .5 \\ \omega_n = 140 \end{matrix}$$

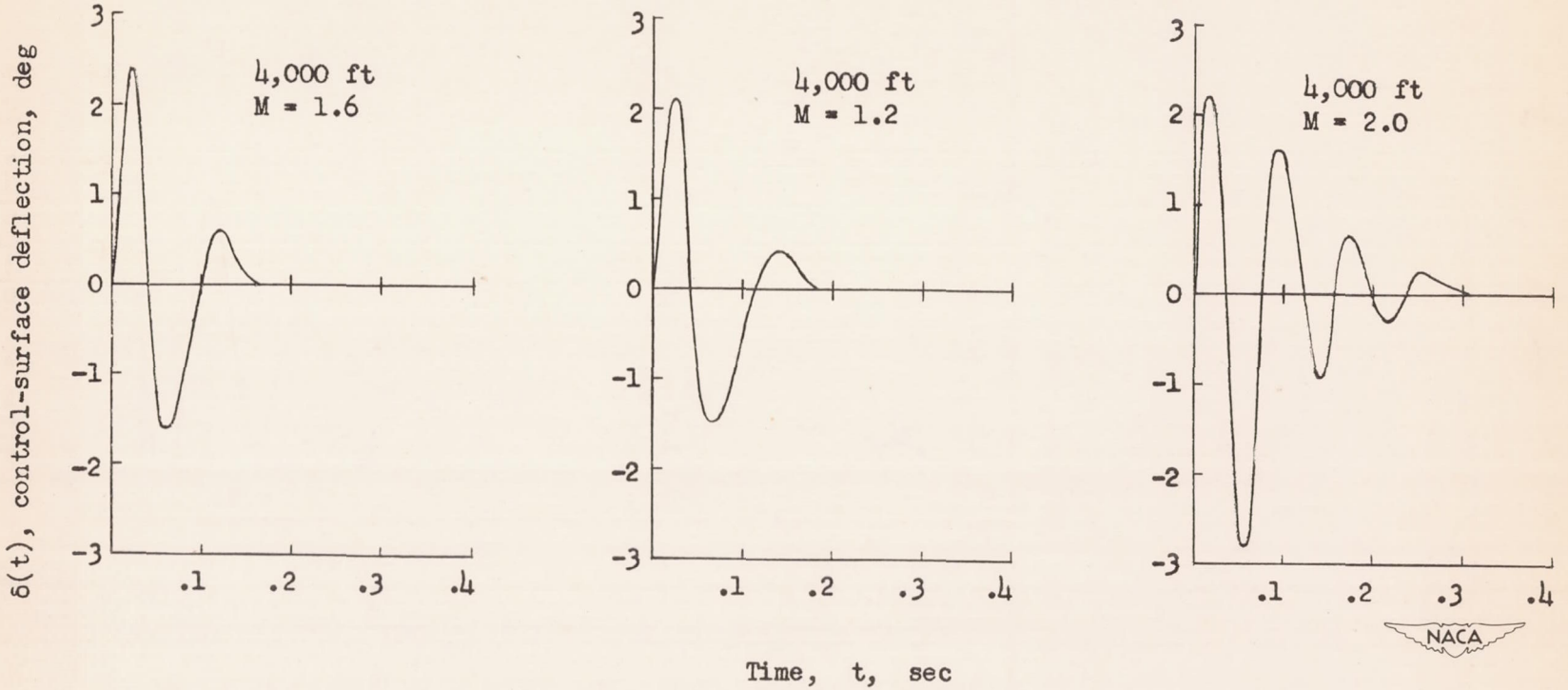


Figure 14.- Longitudinal transient responses $\delta(t)$ of the missile to a unit step input signal calling for a change in attitude of 1°. Static margin, $0.094\bar{c}$ at $M = 1.6$; $K_r = 0.08$; $K_A = 2.32$.

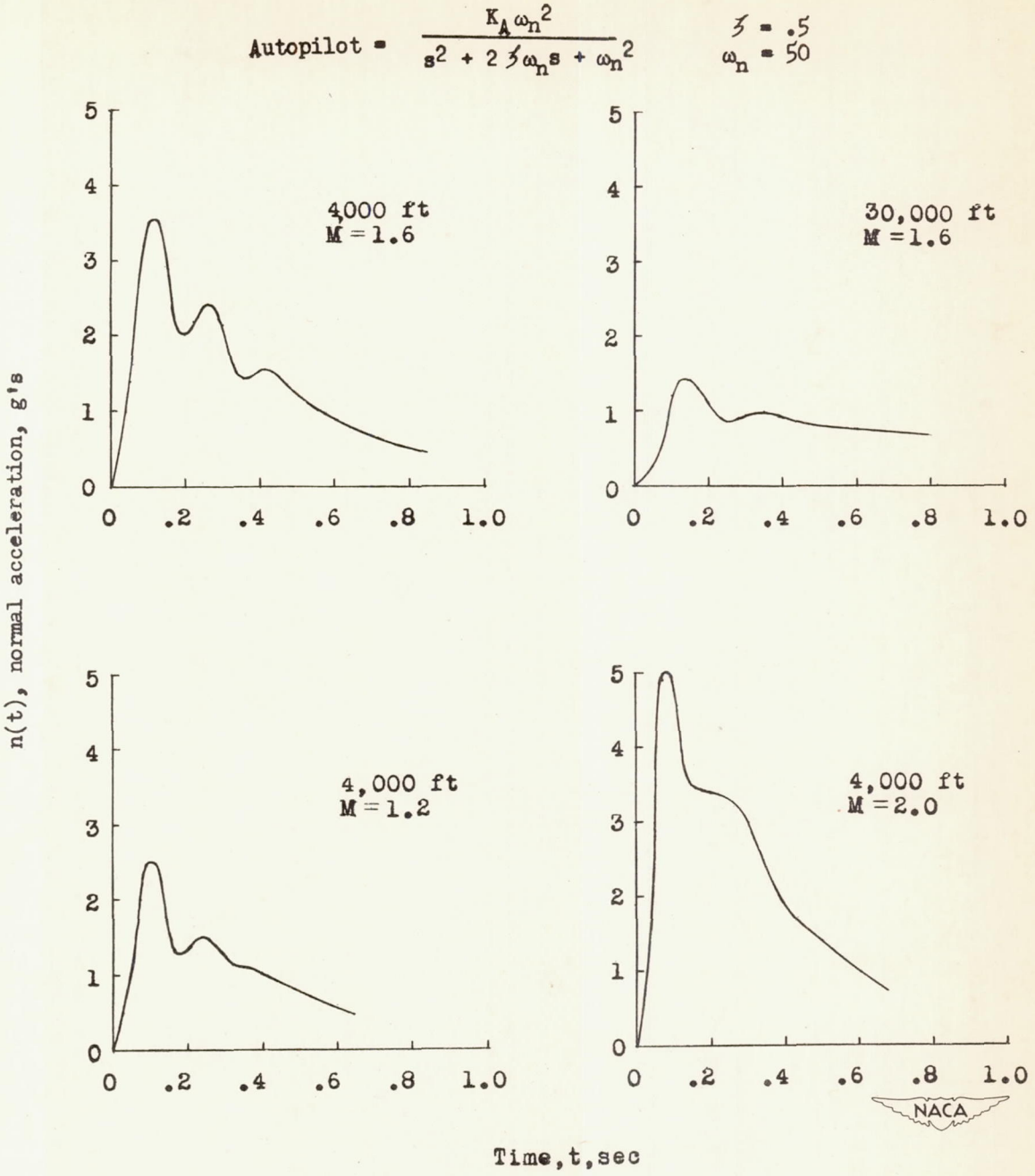


Figure 15.- Longitudinal transient responses for normal acceleration n(t) of the missile to a unit step input signal calling for a change in attitude of 1°. Static margin, 0.094$\bar{c}$ at M = 1.6; K_r = 0.16; K_A = 2.82.

$$\text{Autopilot} = \frac{K_A \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

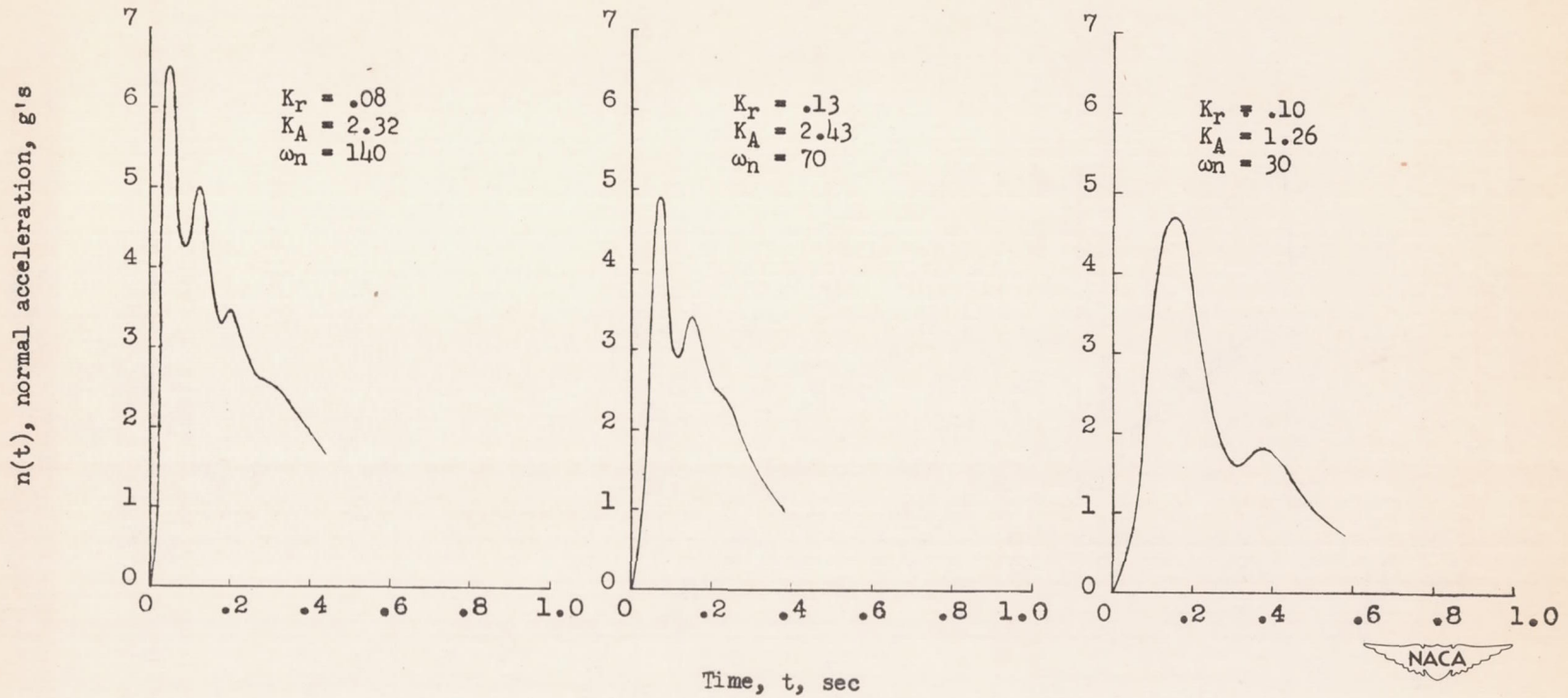


Figure 16.- Longitudinal transient responses for normal acceleration n(t) of the missile to a unit step input signal calling for a change in attitude of 1°. M = 2.0; static margin, 0.094$\bar{c}$ at M = 1.6; altitude, 4,000 feet; ζ of autopilot, 0.5.

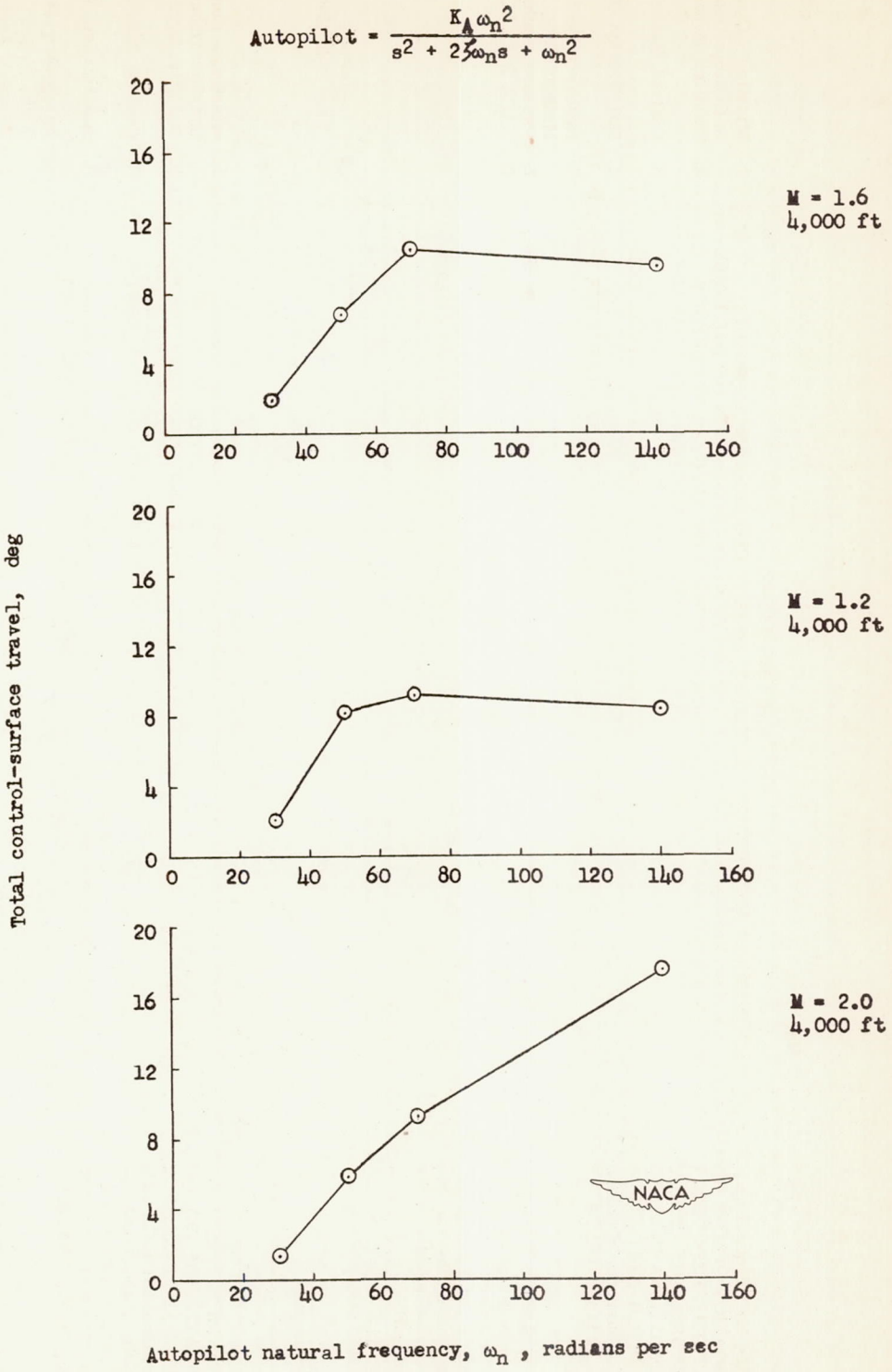


Figure 17.- Plot of total δ travel against autopilot natural frequency in response to a step input θ_i. Static margin, 0.094$\bar{c}$ at M = 1.6; ζ of autopilot, 0.5.