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RESEARCH MEMORANDUM

INVESTIGATION OF TURBINES SUITABLE FOR USE IN A TURBOJET ENGINE WITH HIGH COMPRESSOR PRESSURE RATIO AND LOW COMPRESSOR-TIP SPEED

II - VELOCITY-DIAGRAM STUDY OF TURBINE FOR ENGINE OPERATION WITH CONSTANT EXHAUST-NOZZLE AREA

By Elmer H. Davison and Robert E. English

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Cleveland, Ohio

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS
WASHINGTON
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SUMMARY

The range of application of two-stage turbines to driving single-spool compressors was studied by investigating whether or not a two-stage turbine with frontal area no larger than the compressor frontal area could be satisfactorily designed to drive a particular compressor under take-off conditions. A previous investigation has shown that a turbine designed to drive a particular compressor under take-off conditions would also satisfactorily drive that compressor for cruising at altitude, maximum thrust at altitude, and engine acceleration at 80 percent of rated equivalent speed provided that the engine is operated with constant exhaust-nozzle area and variable rotative speed.

The low blade-tip speed, high work, and high air flow per unit of frontal area of this compressor make critical the problem of designing a two-stage turbine to drive this compressor. In this study of velocity diagrams for such a turbine, a simplified, approximate method of analysis was evolved which fairly accurately predicts design-flow conditions. From this analysis, the following conclusion was drawn:

For the purpose of driving a single-spool compressor as part of a turbojet engine, a two-stage turbine can be satisfactorily designed within the following limits:

(1) Turbine frontal area $\leq$ compressor frontal area

(2) Relative entrance Mach number to any blade row ≤ 0.82

(3) Turning by any blade row $\leq 113^{\circ}$

(4) No static-pressure rise across any blade row

(5) Exit tangential velocity $\leq$ 59 feet per second
provided that the following conditions are satisfied:

[redacted]

(1) The engine is operated with constant exhaust-nozzle area.

(2) The compressor has characteristics within the following range:

(a) Equivalent air flow per unit frontal area $\leq$ 25.8 pounds per square foot-second

(b) Equivalent tip speed $\geq$ 892 feet per second 2516

(c) Equivalent work input $\leq$ 131 Btu per pound

INTRODUCTION

Design of a turbine for a range of turbojet-engine operation rather than for a single operating condition requires that the turbine-design requirements for the various conditions of operation be considered. In reference 1, the design requirements are determined for turbines to drive a particular single-spool, high-pressure ratio, low-blade-tip-speed compressor under the following conditions:

(1) Take-off

(2) Maximum thrust at altitude

(3) Altitude cruising

(a) With maximum-thrust exhaust-nozzle area

(b) At rated rotative speed

(4) Engine acceleration at 80 percent design equivalent rotative speed

In reference 1, the engine operating conditions for take-off are considered to be: compressor total-pressure ratio, 8.75; turbine-inlet temperature, 2160° R; and rated rotative speed; at rated speed and a total-pressure ratio of 8.75, this compressor has an equivalent tip speed of 892 feet per second, an equivalent weight flow of 158 pounds per second, and a frontal area of 881 square inches.

The three compressor parameters that determine turbine-design requirements are mass flow per unit frontal area, work, and blade-tip speed. Increasing compressor work, increasing mass flow per unit compressor frontal area, or decreasing compressor-blade-tip speed make the turbine-design requirements more critical if an attempt is made to stay within a given number of turbine stages, a given turbine frontal area, and pre-established turbine aerodynamic limits. The characteristics of several engines currently being developed were investigated; the requirements imposed on a turbine to drive the compressor for this investigation constituted the most severe two-stage turbine-design problem because of the high mass flow per unit frontal area, the high work, and the low blade-tip speed of the compressor.

The high work output required of a turbine to drive this compressor precluded the possibility of achieving a single-stage turbine design within conventional aerodynamic limits and it appeared questionable whether or not even a two-stage turbine could be designed within conventional aerodynamic limits to satisfy the turbine-design requirements. Although by comparison with a two-stage turbine it would be relatively easy to obtain a three-stage turbine design within conventional aerodynamic limits to satisfy the turbine-design requirements, a three-stage turbine would, in general, have the disadvantages of being larger, heavier, and more complex.

For engine operation with constant exhaust-nozzle area, reference 1 shows that the turbine-design requirements are nearly identical for take-off, maximum thrust at altitude, and cruising at altitude. For engine operation at rated rotative speed, reference 1 shows that the minimum permissible exit annular area is greater than for operation with constant exhaust-nozzle area; this increase in the exit annular area makes more critical the problem of designing a turbine for satisfactory operation for take-off, maximum thrust at altitude, and cruising at altitude. A turbine which satisfies the requirements of satisfactory operation either with constant exhaust-nozzle area or at constant rotative speed is shown in reference 1 to also satisfy the requirements for acceleration at 80 percent of rated equivalent speed. The turbine-design problem may therefore be divided into two phases, one for each type of cruising operation, which correspond to the following two modes of engine operation for take-off, cruise, and maximum thrust at altitude: (1) engine operation with constant exhaust-nozzle area and variable rotative speed, and (2) engine operation at design rotative speed with variable exhaust-nozzle area.

The range of application of two-stage turbines to driving single-spool compressors is studied by investigating whether or not a two-stage turbine may be satisfactorily designed to drive a particular compressor over a range of engine operation for which the exhaust-nozzle area is constant. The possibility of designing a two-stage turbine for engine operation at rated rotative speed over the same range of conditions is a problem which remains to be studied.

Because a great number of turbine designs may be considered for driving this compressor, a simple method is desirable for scanning the most promising range of design. A series of turbine-design charts was therefore devised to provide the simple method. These charts were based on a one-dimensional analysis of flow within turbines and were applied to provide an approximation of the velocity diagrams at the hub, the radial station at which flow conditions are most critical.

For one possible design condition selected from these design charts, the velocity diagrams were more accurately estimated by considering radial variations in flow rather than just hub conditions. Simplified radial equilibrium was assumed (see reference 2) and a free-vortex distribution of tangential velocity was employed. 2516

This investigation was conducted at the NACA Lewis laboratory.

SYMBOLS

The following symbols are used in this report:

A frontal area, sq ft

a velocity of sound, ft/sec

a_{cr} critical velocity $\sqrt{\frac{2\gamma}{\gamma+1} gRT'}$, ft/sec

E work output, Btu/lb

g gravitational constant, 32.17 ft/sec^2

J mechanical equivalent of heat, 778.2 ft-lb/Btu

p absolute pressure, lb/sq ft

R gas constant, ft-lb/(lb)($^{\circ}$R)

T absolute temperature, $^{\circ}$R

U blade velocity, ft/sec

V absolute velocity of gas, ft/sec

W relative velocity of gas, ft/sec

w weight-flow rate of gas, lb/sec

α flow angle of absolute velocity measured from axial direction, deg

β flow angle of relative velocity measured from axial direction, deg

γ ratio of specific heats, 1.32

Δ prefix to indicate change

η adiabatic stage efficiency

θ angle in axial plane between inner shroud and axis of turbine (referred to herein as cone angle), deg

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ρ gas density, lb/cu ft

Subscripts:

1 entrance to first stator

2 entrance to first rotor

3 entrance to second stator

4 entrance to second rotor

5 exit of second rotor

T tip

u tangential component

x axial component

Superscript:

' stagnation or total state

GENERAL DESIGN CONSIDERATIONS

Design Conditions

According to the turbine-design requirements in reference 1, a turbine designed to drive the compressor under engine take-off conditions will satisfy the design requirements for the remaining three conditions for constant exhaust-nozzle operation. For this reason and, because it is conventional design practice to design for the take-off condition, only the design conditions imposed by the take-off operation of the engine are considered. For this type of engine operation, the following design conditions must be fulfilled by the turbine.

Work output, Btu/lb	131
Air flow per unit compressor frontal area, lb/sq ft-sec	25.8
Compressor-blade-tip speed, ft/sec	892
Turbine-inlet temperature, °R	2,160
Turbine-inlet pressure, lb/sq ft	17,600

For take-off, the minimum permissible exit annular area is 383 square inches (reference 1). Use of an area of 383 square inches would result in operation at limiting loading for a large part of the engine operating range. In order to provide additional latitude of turbine operation, the exit annular area was therefore arbitrarily increased by approximately 6 percent to 405 square inches. 2516

A turbine larger in diameter than the compressor is undesirable because the resulting increase in frontal area of the engine would reduce the advantage of the high mass flow per unit frontal area of the compressor. In general, the design problem of producing a given amount of work from a turbine stage within certain design limits is easier if the turbine-blade speed is high rather than low. If the rotative speed of the turbine is fixed, as in this case, high blade speeds can be obtained only by using large turbine diameters. For these reasons, the tip diameter of every turbine blade row was made equal to the tip diameter of the compressor. If the mass flow of the compressor is assumed to be equal to the mass flow of the turbine, the result of this selection of turbine diameters is that the turbine-blade-tip speed and the gas flow per unit of turbine frontal area are equal to the corresponding values for the compressor.

Design Limits

The following design limits and restriction on the velocity diagrams were chosen:

(1) Limits

(a) The relative Mach number at the entrance to any blade row should not exceed 0.85.

(b) The amount of turning required of any blade row should not exceed 120°; this applies to the velocity relative to each blade row.

(c) The static pressure should not rise across any blade row.

(2) Restriction

(a) The tangential component of absolute velocity of the gas leaving the last rotor-blade row should be as near zero as possible.

Because the understanding of losses and flow in turbines is incomplete, these limits are empirical and, consequently, somewhat arbitrary. If different magnitudes were assigned to these limits, the possible range of turbine design would be changed. Such a change would alter the numerical magnitudes in the computed results but the method of analysis of the design problem would be the same. The selected magnitudes of these limits are thought to be typical of those in current use.

The restriction that the exit tangential velocity should approach zero implies that no means is considered to be available for recovering the kinetic energy in this component of velocity. The use of exit straightening vanes as in a compressor is, of course, a possible means of recovering this energy. If, however, the exit tangential velocity can can be reduced in design to a sufficiently small value, satisfactory performance will very likely result without recovery of this energy. In a design of this type, for which the turbine operating conditions for cruising are almost identical with the conditions for take-off, considerable emphasis should be placed on keeping low the energy loss associated with the exit tangential velocity because such a loss would not only impair the thrust for take-off but would also result in high specific fuel consumption for cruise.

Design Problem

The turbine velocity diagrams depend in large part upon the division of work between the stages and on the axial variation in annular area. In the first stage the potential work capacity is greater than that of the second stage for two reasons: (1) Equal Mach numbers in the two stages produce higher velocities in the first stage because of the higher local sonic speeds. (2) High exit tangential velocity can be used in the first stage provided that the limits on entrance Mach number and static-pressure change are not exceeded in the second stator. The work output will therefore ordinarily be divided so that more work will be produced by the first stage than by the second. For a given exit annular area, decreasing the blade height at the turbine entrance results in increased Mach numbers and decreased amounts of turning. The design problem therefore becomes a matter of seeking a compatible combination of work division and axial variation in annular area that will result in low exit tangential velocity and flow conditions in the turbine which are within the design limits on Mach number, turning, and static-pressure change.

METHOD OF ANALYSIS

In order to solve the design problem, a means must be evolved to quantitatively relate the following factors:

(1) Total work output

(2) Mass flow

(3) Blade speed

(4) Inlet pressure and temperature

(5) Division of work between stages

(6) Axial variation in annular area 2516

(7) Exit tangential velocity

(8) Mach numbers

(9) Static-pressure changes

(10) Amounts of turning

A simplified method of analysis which relates these factors is outlined in the following paragraphs.

The nomenclature used in this analysis is shown in figure 1.

Assumed Conditions

The analysis will be confined to the hub section of the annulus, and the following conditions are assumed to prevail for purposes of this analysis:

(1) The specific mass flow ρV_x at the hub is the average for the annulus.

(2) The work output at the hub is the average for the annulus.

(3) The stage internal efficiency is 0.85 with all the loss occurring in the rotor.

For the temperature range of this analysis, a value of the ratio of specific heats γ equal to 1.32 is appropriate.

Primary Charts

In order to scan the most promising range of turbine design, a series of charts was constructed on the basis of a one-dimensional analysis of flow through the turbine. A method of turbine-velocity-diagram analysis by means of chart construction is presented in reference 3. Although the method of reference 3 is well suited to the analysis of a wide range of turbine-design conditions, the restrictions imposed on the turbine-design problem to be analyzed herein make a different type of chart more desirable. In the present case the turbine mass flow per unit of turbine frontal area and the turbine-tip speed are constant for the range of the analysis; for this reason, a new type of chart was evolved, the derivation of which is presented in the appendix.

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A particular division of work between the turbine stages must be assumed for the construction of each set of these charts. Four charts constitute a set, one each for the entrance and exit of each rotor-blade row. For this analysis, work divisions of 65/35 (65 percent of the total work of 131 Btu per pound is produced by the first stage and 35 percent, by the second), 70/30, and 75/25 were considered. One such set of charts is presented in figure 2 for a work division of 70/30. These charts present flow conditions at the hub of each rotor-blade row.

Use of Primary Charts

Each chart consists of a plot of the work parameter $UV_u/a_{cr,2}^2$ against the blade-speed parameter $U/a_{cr,2}$. On each chart, lines are plotted for constant values of other parameters of interest. These are generally absolute and relative flow directions α and β, and absolute and relative Mach numbers V/a and W/a. For station 5, the exit from the last rotor-blade row, lines are plotted for constant values of relative Mach number $(W/a)_5$, tangential-velocity parameter $(V_u/a_{cr})_5$, and axial-velocity parameter $(V_x/a_{cr})_5$.

The stage work E of the first stage, for instance, can be stated

$$E_{2-3} = \frac{U_2V_{u,2} - U_3V_{u,3}}{gJ}$$

This expression can be restated

$$\frac{gJE_{2-3}}{a_{cr,2}^2} = \frac{U_2V_{u,2}}{a_{cr,2}^2} - \frac{U_3V_{u,3}}{a_{cr,2}^2}$$

Each of the factors on the right side of this equation is an ordinate in figure 2(a) or 2(b); figures 2(a) and 2(b) are used by superimposing the two charts. The ordinates are offset by such an amount that the difference of the work parameters $UV_u/a_{cr,2}{}^2$ equals the required value of stage-work parameter $gJE_{2-3}/a_{cr,2}{}^2$. The lateral or horizontal orientation of the two charts is a matter of choice; each orientation corresponds to a definite difference in blade speed and thus a certain change in blade height. For a particular lateral orientation, each point on figure 2(a), for example, corresponds to a particular set of flow conditions at station 2 and the coinciding point on figure 2(b) corresponds to a particular set of flow conditions at station 3. Thus, the relation between the flow conditions at stations 2 and 3 is apparent, 2516 and the flow conditions at one station need not be optimized without regard for the other but rather compatible conditions can be selected.

In order to change the axial variation in annular area, a corresponding pair of charts need only be shifted laterally with respect to each other. A new set of charts must be constructed if the work division between the stages is to be altered. New charts for only stations 3 and 4 need be constructed because, with only minor inaccuracies, one set of charts for stations 2 and 5 can be used for a range of work division.

The following example will illustrate the use of the primary charts. Consider the flow conditions in the second stage for a 70/30 work division. The value of stage-work parameter $gJE_{4-5}/a_{cr,2}{}^2$ is 0.233 for this case. The relative Mach number at the rotor entrance $(W/a)_4$ is to be found for the following conditions:

(1) A blade-speed parameter $U_5/a_{cr,2}$ at the rotor exit of 0.30

(2) An annular-area increase corresponding to a change in the blade-speed parameter $U/a_{cr,2}$ of -0.02; that is, $U_4/a_{cr,2} = 0.32$

(3) An exit tangential velocity $V_{u,5}$ of zero

For these conditions, the work parameter at the turbine exit $U_5V_{u,5}/a_{cr,2}{}^2$ is zero and it is seen from figure 2(d) that the relative Mach number at the turbine exit $(W/a)_5$ is 0.58. At the rotor entrance the work parameter $U_4V_{u,4}/a_{cr,2}{}^2$ must be 0.233 in order that the stage will produce design work. Under this condition, the entrance Mach number relative to the last rotor $(W/a)_4$ is 0.73 (fig. 2(c)). This decrease in relative Mach number from 0.73 at the entrance to the blade row to 0.58 at the exit of the blade row produces a static-pressure rise within the rotor, a condition outside the design limits.

Auxiliary Charts

If it is desired to design a turbine for some given exit annular area, the problem of selecting a suitable design from figures 2(a) to 2(d) can be simplified by making cross plots of these figures as shown in figures 3 to 6.

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Figures 3 to 6 were constructed in the following manner: Figures 6(a), 6(b), and 6(c) were obtained by cross-plotting the values obtained from figure 2(d) by moving along a vertical line at the value of the blade-speed parameter at the turbine exit $U_5/a_{cr,2}$ required by the fixed exit area. For the turbine design considered, which has an exit annular area of 405 square inches, the blade-speed parameter $U_5/a_{cr,2}$ has a value of 0.319. Figure 6(d) was obtained from figures 6(b) and 6(c) and the following trigonometric relation of the velocity triangle:

$$\beta_5 = \tan^{-1}\left[\frac{\left(\frac{V_u}{a_{cr}}\right)_5 - \left(\frac{U}{a_{cr}}\right)_5}{\left(\frac{V_x}{a_{cr}}\right)_5}\right]$$

where from the definition of critical velocity a_{cr}:

$$\left(\frac{U}{a_{cr}}\right)_5 = \frac{U_5}{a_{cr,2}}\sqrt{\frac{T_2'}{T_5'}}$$

In order to obtain figure 5 it was necessary to assume different values for the annular area divergence between station 4 at the entrance to the last rotor and station 5 at the exit of the last rotor. When an axial variation of annular area is assumed with the exit area known, the blade-speed parameter $U_4/a_{cr,2}$ is fixed. With the change in work parameter $\Delta\left(UV_u/a_{cr,2}^2\right)$ known from the required work for the stage, it is possible to find values of the work parameter at the rotor entrance $(UV_u)_4/a_{cr,2}^2$ that correspond to values of the work parameter at the rotor exit $(UV_u)_5/a_{cr,2}^2$. These two conditions are used to obtain, from figure 2(c), the cross plots (figs. 5(a), 5(b), and 5(c)) of the parameters relative entrance Mach number $(W/a)_4$, relative entrance angle of flow β_4, and absolute entrance angle of flow α_4 against the work parameter at the rotor exit $(UV_u)_5/a_{cr,2}^2$ for lines of constant difference in annular area between stations 4 and 5 (constant values of blade-speed parameter $U_4/a_{cr,2}$). The plot of absolute entrance Mach number $(V/a)_4$ against work parameter at the rotor exit $(UV_u)_5/a_{cr,2}^2$ (fig. 5(d)) was obtained from figures 5(a) to 5(c) and the following trigonometric relation of the velocity triangle:

$$\left(\frac{V}{a}\right)_4 = \left(\frac{W}{a}\right)_4 \frac{\cos\ \beta_4}{\cos\ \alpha_4}$$

In a similar manner the cross plots shown in figures 3 and 4 were obtained from figures 2(a) and 2(b). 2516

These charts make it possible to scan the available velocity diagrams for a given work division, exit annular area, and various axial variations in annular area.

RESULTS

In tables I and II are shown the effects of work division and cone angle on the design of a turbine to drive the compressor of reference 1 over a range of flight conditions for which the exhaust-nozzle area of the engine remains constant. In order to simplify the following discussion, the term cone angle (see fig. 1) is used to denote axial variation in annular area. All the turbine designs shown in tables I and II are for an exit annular area of 405 square inches.

Effect of Work Division

The effects of three work divisions on the hub conditions of a turbine for a cone angle of 13.9° are shown in table I. As a basis for comparison, constant static pressure (impulse) across the hub of the last rotor and a single cone angle were assigned for the three work divisions. Table I shows that for the particular cone angle, increasing the amount of work done in the first stage has the following effects:

(1) The entrance Mach numbers to the first rotor and the second stator are increased.

(2) The turning in every blade row is decreased.

(3) The static-pressure drop in the second stator, as indicated by the difference between the entrance and exit absolute Mach numbers, is decreased.

(4) The exit tangential velocity $V_{u,5}$ is decreased.

Of the three work divisions, only the 75/25 division is outside the design limits. For the 75/25 division, (1) the entrance relative Mach number to the first rotor $(W/a)_2$ of 0.95 exceeds the limit of 0.85 placed on entrance Mach number, (2) the entrance Mach number to the second stator $(V/a)_3$ of 0.95 also exceeds this limit, and (3) the Mach number variation in the second stator from the entrance value $(V/a)_3$ of 0.95 to the exit value $(V/a)_4$ of 0.89 results in a static-pressure rise across the stator, a condition outside the design limits.

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For the 65/35 work division, the entrance Mach numbers, the amounts of turning, and the static-pressure changes appear to be more conservative than for the 70/30 work division. On the other hand, for the 70/30 and 65/35 work divisions, the exit tangential velocities are -96 and -206 feet per second, respectively. Because the kinetic energy associated with these velocities does not exceed 0.85 Btu per pound, both amounts of loss are small in comparison with the over-all work output (131 Btu/lb) of the turbine. Under conditions other than those analyzed, these losses may become larger. In addition, the occurrence of high exit tangential velocity (200 ft/sec or greater) may result in appreciable loss in the exit ducting or afterburner.

Effect of Variation in Cone Angle

The effect of changing the cone angle for a 70/30 work division is shown in table II. Each of the first three columns has a different uniform value of cone angle from turbine entrance to exit (constant cone angle) and column 4 is for a nonuniform value of cone angle from turbine entrance to exit (varying cone angle). The basis of comparison for all four columns is that all four designs have constant static pressure (impulse) across the hub of the last rotor.

Constant cone angle. - The first three columns of table II show that increasing the cone angle has the following effects:

(1) The inlet Mach numbers to the blade rows are increased.

(2) The amount of turning in each blade row is decreased.

(3) The static pressure drops, as indicated by the changes in Mach number, across the first rotor and the second stator are decreased.

(4) The exit tangential velocity $V_{u,5}$ is increased.

Columns 1 and 2 show that neither design has a marked advantage over the other. The design in column 2 has less turning in the first rotor than the design in column 1 but the inlet Mach number to the second stator is increased and the static-pressure drop across this stator is decreased. The difference in exit tangential velocity is very small.

Although turning is reduced appreciably by the cone angle shown in column 3, the inlet Mach number limit is exceeded for both the first rotor and the second stator. In addition, the amount of velocity increase is reduced appreciably in the second stator. The exit tangential velocity is also increased slightly over that in the designs shown in columns 1 and 2.

Varying cone angle. - In column 4, a design having an axial variation in cone angle is shown; the change in the blade-speed parameter $\Delta U/a_{cr,2}$ through the first rotor is twice the change in blade-speed parameter $\Delta U/a_{cr,2}$ through either of the succeeding two blade rows. This axial variation in cone angle allows the best features of the designs in columns 1 and 2 to be combined. In comparing column 4 with columns 1 and 2, it is seen that the entrance Mach numbers are lower, the static-pressure drop across the blade rows is greater, the exit tangential-velocity is as low or lower, and the amount of turning in the blade rows is only slightly greater than the lowest values of either column 1 or 2; these changes in flow conditions are all small.

An axial variation in cone angle can be accomplished by a smooth continuous curving inner-shroud profile or by a sharp corner if the inner-shroud profile is composed of straight-line segments. If a sharp corner were employed, the flow in the vicinity of the sharp corner could not be accurately forecast during the course of design. If a smooth continuous curving inner-shroud profile were used, the manufacturing complexity would be increased. There is apparently no distinct advantage in varying the cone angle.

Effects of Radial Variations on Velocity Diagrams

Velocity diagrams for three radial stations were calculated for a 70/30 work division with a constant cone angle the same as that in column 2 of table II. The following conditions were assumed for this calculation:

(1) Free-vortex distribution of the tangential velocity at each station

(2) Simple radial equilibrium at each station (see reference 2)

(3) Stage internal efficiency η of 0.85

(4) Ratio of loss (entropy rise) in the stator to loss in the rotor of 0.5

As in the chart analysis, the value of the ratio of specific heats γ was assumed to be 1.32. The velocity diagrams and the turbine configuration for which they were calculated are shown in figures 7 and 8, respectively. From figure 7, it can be seen that the hub conditions are slightly more conservative than those shown in column 2 of table II. The Mach number at the entrance to each blade row is slightly lower but the turning in each blade row is essentially the same. Furthermore, there is a small velocity increase in the last rotor and the exit tangential velocity has been changed from -96 to -59 feet per second. The 2516 simplified method of analysis fairly accurately predicted the design-flow conditions at the hub. For this particular set of calculations, the Mach numbers differed by 0.03 or less and the flow angles and turning differed by 3° or less.

The velocity increase and, therefore, static-pressure drop across the second stator remains about constant from the hub to the tip, whereas it increases from hub to tip for the two rotors (fig. 8). A decrease in the static-pressure drop across the blade profile increases the tendency toward flow separation and its attendant undesirable effects. If the static pressure were to rise across a rotor this rise would probably be confined to a small region near the hub since the pressure gradients become more favorable as the tip is approached. However, in the second stator, the pressure drop across the blade row does not become more favorable as the tip is approached; therefore, if the static pressure were to rise, the rise would not be confined to the hub but would extend over the entire blade height. For this reason, it is desirable in selecting a first approximation from the design charts to pick a design with a velocity increase across the second stator.

DISCUSSION

The results of the analysis, as so far presented, are specific rather than general because they are related to a particular compressor considered as part of an engine. The value of these results can be increased if, from them, general conclusions can be drawn concerning the entire class of single-spool compressors. In order for the general significance of these results to be apparent, the turbine-design requirements for driving this compressor over a range of engine operation with constant exhaust-nozzle area must be compared with the turbine-design requirements of other single-spool compressors. In the following discussion, the turbine-inlet temperature for take-off is presumed to be near 2160° R because a large change in temperature will alter the turbine-design requirements.

For the purpose of driving a single-spool compressor as part of a turbojet engine, a two-stage turbine can be satisfactorily designed within the following limits:

(1) Turbine frontal area $\leq$ compressor frontal area

(2) Relative entrance Mach number to any blade row $\leq$ 0.82

(3) Turning by any blade row $\leq 113^{\circ}$

(4) No static-pressure rise across any blade row

(5) Exit tangential velocity $\leq$ 59 feet per second

provided that the following conditions are satisfied: 2516

(1) The engine is operated with constant exhaust-nozzle area.

(2) The compressor has characteristics within the following range:

(a) Equivalent air flow per unit frontal area $\leq$ 25.8 pounds per square foot-second

(b) Equivalent tip speed $\geq$ 892 feet per second

(c) Equivalent work input $\leq$ 131 Btu per pound

If the turbine design requirements are made more critical by selecting other compressor characteristics or another range of flight conditions, it will very likely become necessary to depart from present turbine-design practice in order to obtain two-stage turbines with low exit tangential velocity and frontal area no larger than the compressor.

SUMMARY OF RESULTS

In order to study the range of application of two-stage turbines to drive single-spool compressors, an investigation was conducted to determine whether or not satisfactory velocity diagrams could be obtained for a turbine to drive a particular single-spool compressor over a range of engine operation with constant exhaust-nozzle area. The characteristics of several engines currently being developed were investigated; the requirements imposed on a turbine to drive the compressor chosen for this investigation constituted the most severe two-stage turbine-design problem because of the high mass flow per unit frontal area, the high work, and the low blade-tip speed of the compressor.

A simplified method of analysis, which fairly accurately predicted the design-flow conditions, was evolved for this study.

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For the compressor investigated, the following results regarding the turbine were obtained:

1. On the basis of the velocity-diagram study, it appears possible to design a two-stage turbine to drive the single-spool compressor. However, it is necessary to design very close to present turbine aerodynamic limits if the tangential velocity at the turbine exit is to be low. A set of velocity diagrams was determined within the following limits: Relative entrance Mach number, 0.82; turning by any blade row, 113°; no static-pressure rise; exit tangential velocity, -59 feet per second.

2. For most conservative design, the work output of the first turbine stage should be about twice that of the second stage.

3. With a given cone angle (axial variation of annular area) and a given exit annular area, the effects of increasing the work output of the first turbine stage were to increase the blade-row inlet Mach numbers, to reduce the amounts of turning, to decrease the static-pressure drops, and to decrease the exit tangential velocity.

4. For a given exit annular area and a given work division between the stages, the effects of increasing the cone angle (linear rate of change of the inner-shroud radius in the axial direction) were to increase the blade-row inlet Mach numbers, to reduce the amount of turning, to decrease the static-pressure drops, and to increase the exit tangential velocity.

5. A slight improvement in the velocity diagrams can be obtained by use of varying cone angle (nonlinear rate of change of the inner-shroud radius in the axial direction) at the expense of mechanical complexity.

CONCLUSION

For the purpose of driving a single-spool compressor as part of a turbojet engine, a two-stage turbine can be satisfactorily designed within the following limits:

(1) Turbine frontal area $\leq$ compressor frontal area

(2) Relative entrance Mach number to any blade row ≤ 0.82

(3) Turning by any blade row $\leq 113^{\circ}$

(4) No static-pressure rise across any blade row

(5) Exit tangential velocity ≤ 59 feet per second

provided that the following conditions are satisfied:

(1) The engine is operated with constant exhaust-nozzle area.

(2) The compressor has characteristics within the following range:

(a) Equivalent air flow per unit frontal area $\leq$ 25.8 pounds per square foot-second

(b) Equivalent tip speed $\geq$ 892 feet per second

(c) Equivalent work input $\leq$ 131 Btu per pound

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National Advisory Committee for Aeronautics
Cleveland, Ohio

APPENDIX - METHOD OF CONSTRUCTION OF PRIMARY CHARTS

The primary charts have as coordinates the work parameter $UV_u/a_{cr,2}^2$ and the blade-speed parameter $U/a_{cr,2}$. Lines of constant values of other parameters of interest are plotted on these coordinates. Two such charts are required for each row of rotor blades, one for the rotor entrance and one for the rotor exit. These charts are particularly suitable for velocity-diagram analysis of turbine stages having fixed values of blade-tip speed and equivalent mass flow per unit of turbine frontal area.

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In order to construct these charts, the following factors must be assumed or assigned:

(1) Ratio of specific heats, γ

(2) Work output per stage, E

(3) Weight flow per unit of frontal area, w/A

(4) Gas constant, R

(5) Stage internal efficiency, η

(6) Inlet total pressure, P'_1

(7) Inlet total temperature, T'_1

(8) Blade-tip speed, U_T

(9) Division of loss between stator and rotor of each stage

(10) That the specific mass flow ρV_x at the hub is the average for the annulus

(11) That the work output at the hub is the average for the annulus

From these factors, the values of critical velocity a_{cr} and total density ρ' can be computed for each axial station by means of standard thermodynamic procedures.

All of the symbols used in this appendix refer to conditions at the hub of the annulus with the single exception of U_T, the blade-tip speed.

For each chart a range of values of work parameter $UV_u/a_{cr,2}^2$ and blade-speed parameter $U/a_{cr,2}$ must be assigned. The specific-mass-flow parameter $\rho V_x/\rho' a_{cr}$ can then be determined from

$$\frac{\rho V_x}{\rho' a_{cr}} = \frac{w}{A\rho' a_{cr}}\left[1 - \left(\frac{U}{U_T}\right)^2\right]^{-1} \tag{A1}$$

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From the definition of critical velocity a_{cr}, that is

$$a_{cr} \equiv \sqrt{\frac{2\gamma}{\gamma+1}\, gRT'} \tag{A2}$$

the tangential-velocity parameter V_u/a_{cr} can be determined.

$$\frac{V_u}{a_{cr}} = \frac{UV_u/a_{cr,2}^{\;2}}{U/a_{cr,2}}\,\frac{a_{cr,2}}{a_{cr}} \tag{A3}$$

The values of specific-mass-flow parameter $\rho V_x/\rho' a_{cr}$ and tangential-velocity parameter V_u/a_{cr} can be used in conjunction with the flow chart in figure 3 of reference 3 to determine the axial-velocity parameter V_x/a_{cr}.

Even though the flow chart in reference 3 was constructed for a value of γ of 1.30, the chart is applicable for a range of values of γ. The results are not sensitive to ordinary variations in γ unless the axial velocity approaches sonic speed.

From the geometry of the vector diagrams (see fig. 1),

$$\frac{V}{a_{cr}} = \sqrt{\left(\frac{V_x}{a_{cr}}\right)^2 + \left(\frac{V_u}{a_{cr}}\right)^2} \tag{A4}$$

$$\frac{W}{a_{cr}} = \sqrt{\left(\frac{V_x}{a_{cr}}\right)^2 + \left(\frac{V_u}{a_{cr}} - \frac{U}{a_{cr}}\right)^2} \tag{A5}$$

where

$$\frac{U}{a_{cr}} = \frac{U}{a_{cr,2}}\,\frac{a_{cr,2}}{a_{cr}}$$

The Mach numbers relative to the stator V/a and relative to the rotor W/a can then be determined.

$$\frac{V}{a} = \frac{V}{a_{cr}}\left[\frac{\gamma+1}{2} - \frac{\gamma-1}{2}\left(\frac{V}{a_{cr}}\right)^2\right]^{-\frac{1}{2}} \tag{A6}$$

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$$\frac{W}{a} = \frac{W}{a_{cr}}\left[\frac{\gamma+1}{2} - \frac{\gamma-1}{2}\left(\frac{V}{a_{cr}}\right)^2\right]^{-\frac{1}{2}} \tag{A7}$$

From the velocity triangles

$$\alpha = \tan^{-1}\frac{V_u}{V_x} \tag{A8}$$

$$\beta = \tan^{-1}\frac{V_u-U}{V_x} \tag{A9}$$

The computed values of, for instance, W/a, α, and β can then be plotted against the work parameter $UV_u/a_{cr,2}^2$ with lines of constant blade-speed parameter $U/a_{cr,2}$. These results can then be cross-plotted to produce the primary charts as in figure 2.

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1. English, Robert E., Silvern, David H., and Davison, Elmer H.: Investigation of Turbines Suitable for Use in a Turbojet Engine with High Compressor Pressure Ratio and Low Compressor-Tip Speed. I - Turbine-Design Requirements for Several Engine Operating Conditions. NACA RM E52A16, 1952.

2. Wu, Chung-Hua, and Wolfenstein, Lincoln: Application of Radial-Equilibrium Condition to Axial-Flow Compressor and Turbine Design. NACA Rep. 955, 1950. (Supersedes NACA TN 1795.)

3. Alpert, Sumner, and Litrenta, Rose M.: Construction and Use of Charts in Design Studies of Gas Turbines. NACA TN 2402, 1951.

TABLE I - EFFECT OF WORK DIVISION ON TURBINE HUB CONDITIONS FOR

13.9° CONE ANGLE AND 405-SQUARE-INCH EXIT AREA

		Work division			
		65/35	70/30	75/25	
Cone angle	θ	13.9°	13.9°	13.9°	
Blade-speed parameter, station 2	$\frac{U_2}{a_{cr,2}}$	.359	.359	.359	
Blade-speed parameter, station 3	$\frac{U_3}{a_{cr,2}}$	.345	.345	.345	
Blade-speed parameter, station 4	$\frac{U_4}{a_{cr,2}}$	.332	.332	.332	
Blade-speed parameter, station 5	$\frac{U_5}{a_{cr,2}}$	.319	.319	.319	
Exit absolute Mach number	$\left(\frac{V}{a}\right)_2$	1.119	1.130	1.255	First stator
Exit absolute flow angle	α_2	65.3°	65.2°	63.8°	
Entrance relative Mach number	$\left(\frac{W}{a}\right)_2$	0.800	0.820	0.950	First rotor
Exit relative Mach number	$\left(\frac{W}{a}\right)_3$	.975	1.123	1.224	
Entrance relative flow angle	β_2	54.2°	54.7°	54.1°	
Exit relative flow angle	β_3	-57.0°	-55.5°	-53.0°	
Turning within first rotor	$\beta_2-\beta_3$	111.2°	110.2°	107.1°	
Entrance absolute Mach number	$\left(\frac{V}{a}\right)_3$	0.699	0.850	0.949	Second stator
Exit absolute Mach number	$\left(\frac{V}{a}\right)_4$	.993	.935	.887	
Entrance absolute flow angle	α_3	-40.2°	-41.8°	-39.0°	
Exit absolute flow angle	α_4	52.0°	49.2°	45.9°	
Turning within second stator	$\alpha_4-\alpha_3$	92.2°	91.0°	84.9°	
Entrance relative Mach number	$\left(\frac{W}{a}\right)_4$	0.742	0.701	0.679	Second rotor
Exit relative Mach number	$\left(\frac{W}{a}\right)_5$	.742	.701	.679	
Entrance relative flow angle	β_4	34.9°	30.3°	24.0°	
Exit relative flow angle	β_5	-37.5°	-34.0°	-30.0°	
Turning within second rotor	$\beta_4-\beta_5$	72.4°	64.3°	54.0°	
Tangential component of absolute-velocity parameter, turbine exit	$\left(\frac{V_u}{a_{cr}}\right)_5$	-.113	-.053	.005	
Axial component of absolute-velocity parameter, turbine exit	$\left(\frac{V_x}{a_{cr}}\right)_5$	.618	.612	.611	

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TABLE II - EFFECT OF CONE ANGLE (AXIAL VARIATION OF ANNULAR AREA) ON TURBINE HUB CONDITIONS FOR 70/30 WORK DIVISION AND 405-SQUARE-INCH EXIT AREA

		1	2	3	4	
Cone angle	θ	10.6°	13.9°	17.3°	Varying	
Blade-speed parameter, station 2	$\frac{U_2}{a_{cr,2}}$	.349	.359	.369	0.359	
Blade-speed parameter, station 3	$\frac{U_3}{a_{cr,2}}$	.339	.345	.353	.339	
Blade-speed parameter, station 4	$\frac{U_4}{a_{cr,2}}$	.329	.332	.336	.329	
Blade-speed parameter, station 5	$\frac{U_5}{a_{cr,2}}$	.319	.319	.319	.319	
Exit absolute Mach number	$\left(\frac{V}{a}\right)_2$	1.134	1.130	1.203	1.127	First stator
Exit absolute flow angle	α_2	68.0°	65.2°	60.7°	65.2°	
Entrance relative Mach number	$\left(\frac{W}{a}\right)_2$	0.820	0.820	0.890	0.815	First rotor
Exit Relative Mach number	$\left(\frac{W}{a}\right)_3$	1.115	1.123	1.140	1.106	
Entrance relative flow angle	β_2	58.7°	54.7°	48.3°	54.5°	
Exit relative flow angle	β_3	-58.3°	-55.5°	-50.0°	-58.3°	
Turning within first rotor	$\beta_2-\beta_3$	117.0°	110.2°	98.3°	112.8°	
Entrance absolute Mach number	$\left(\frac{V}{a}\right)_3$	0.824	0.850	0.890	0.816	Second stator
Exit absolute Mach number	$\left(\frac{V}{a}\right)_4$	.939	.935	.935	.939	
Entrance absolute flow angle	α_3	-44.7°	-41.8°	-35.5°	-44.5°	
Exit absolute flow angle	α_4	50.9°	49.2°	47.5°	50.9°	
Turning within second stator	$\alpha_4-\alpha_3$	95.6°	91.0°	83.0°	95.4°	
Entrance relative Mach number	$\left(\frac{W}{a}\right)_4$	0.701	0.701	0.713	0.701	Second rotor
Exit relative Mach number	$\left(\frac{W}{a}\right)_5$	.701	.701	.713	.701	
Entrance relative flow angle	β_4	32.1°	30.3°	27.8°	32.1°	
Exit relative flow angle	β_5	-33.2°	-34.0°	-34.9°	-33.2°	
Turning within second rotor	$\beta_4-\beta_5$	65.3°	64.3°	62.7°	65.3°	
Tangential component of absolute-velocity parameter, turbine exit	$\left(\frac{V_u}{a_{cr}}\right)_5$	-.043	-.053	-.065	-.043	
Axial component of absolute-velocity parameter, turbine exit	$\left(\frac{V_x}{a_{cr}}\right)_5$	.612	.612	.613	.612	

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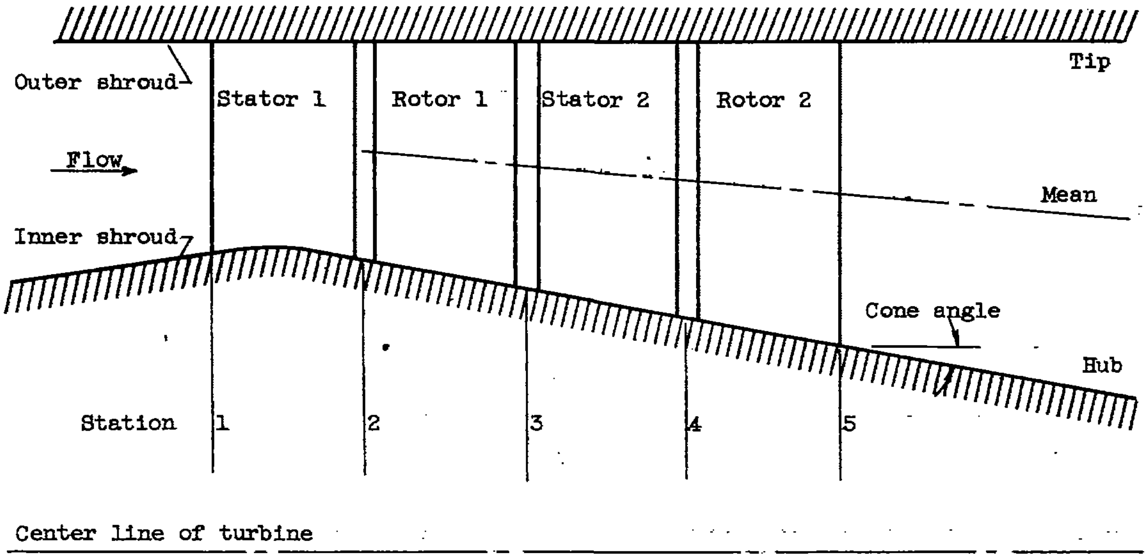
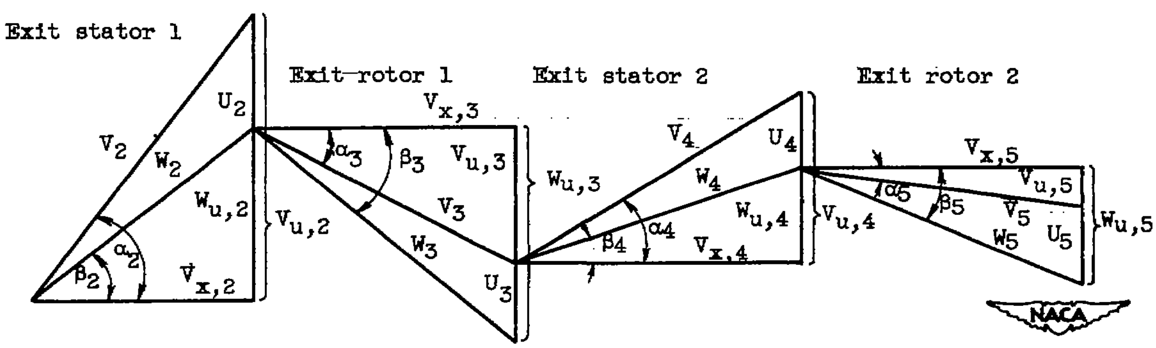


(a) Turbine nomenclature.



(b) Velocity-diagram nomenclature. V_u positive when in same direction as U; V_u negative when in direction opposite of U.

Figure 1. - Turbine and velocity-diagram nomenclature.

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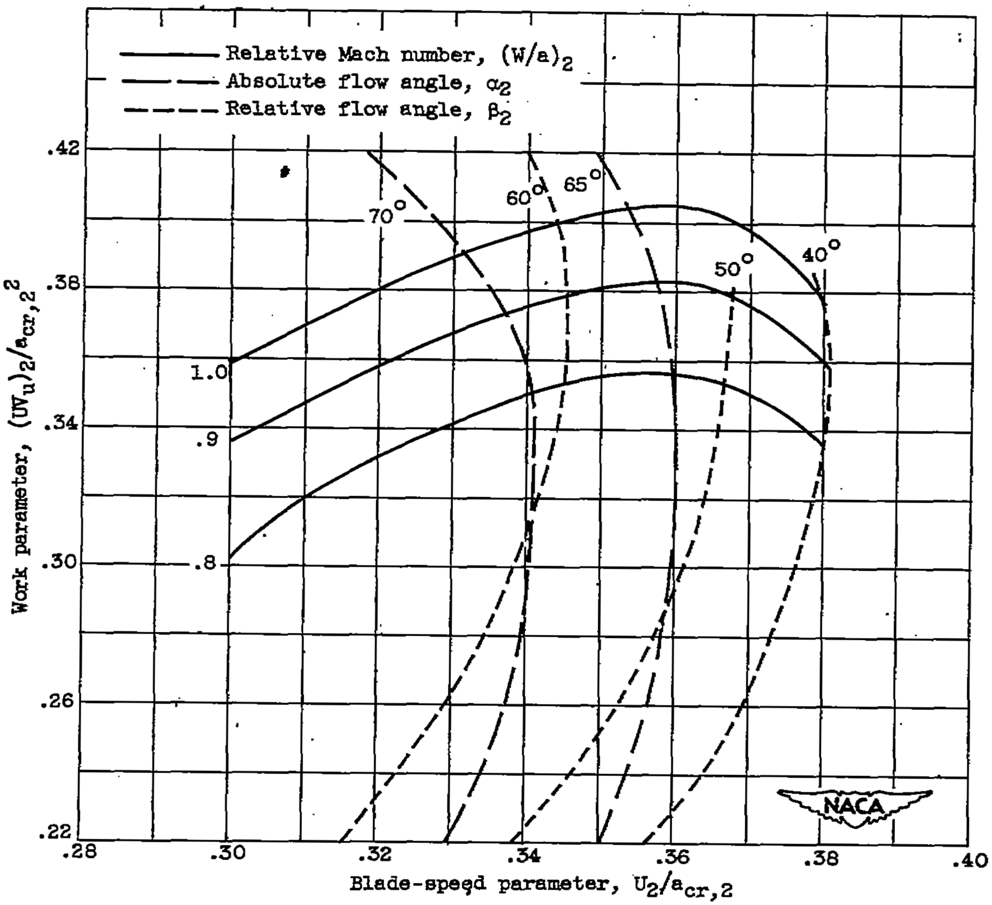

(a) At hub of station 2 for 70/30 work division.

Figure 2. - Primary turbine-design chart for analysis of flow conditions.

[redacted]

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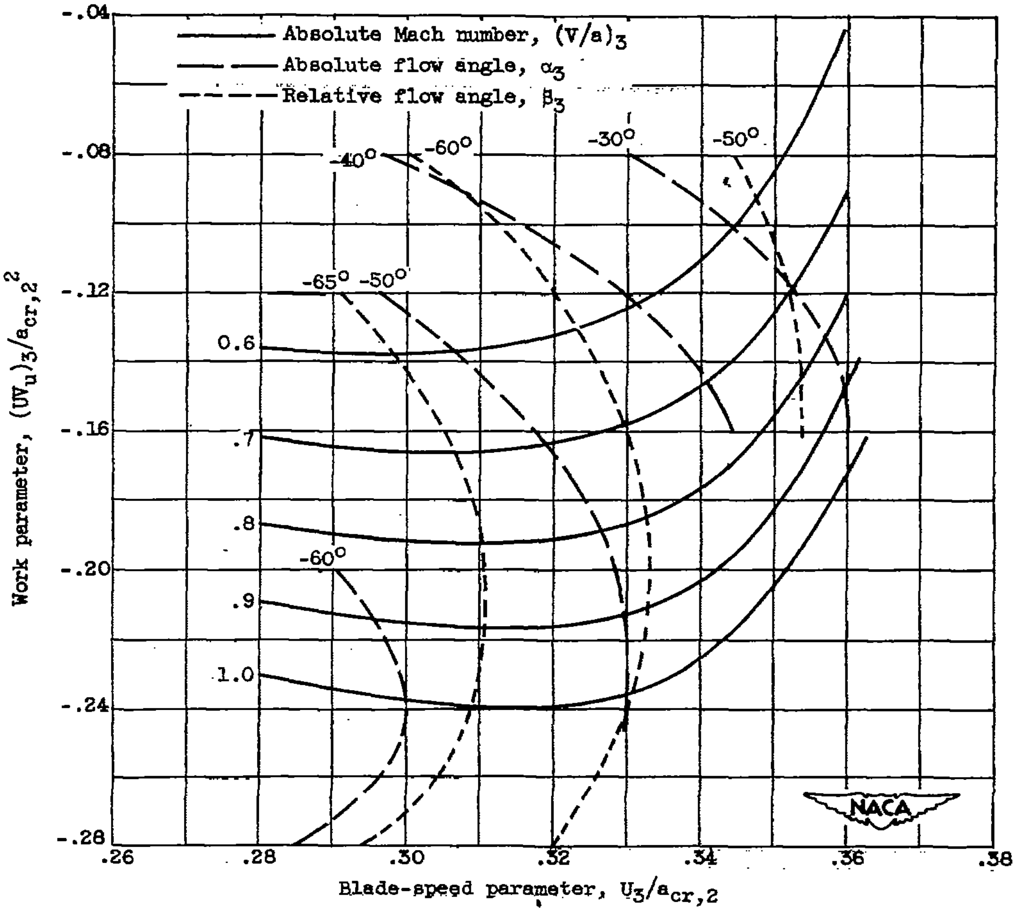

(b) At hub of station 3 for 70/30 work division.

Figure 2. - Continued. Primary turbine-design chart for analysis of flow conditions.

[redacted]

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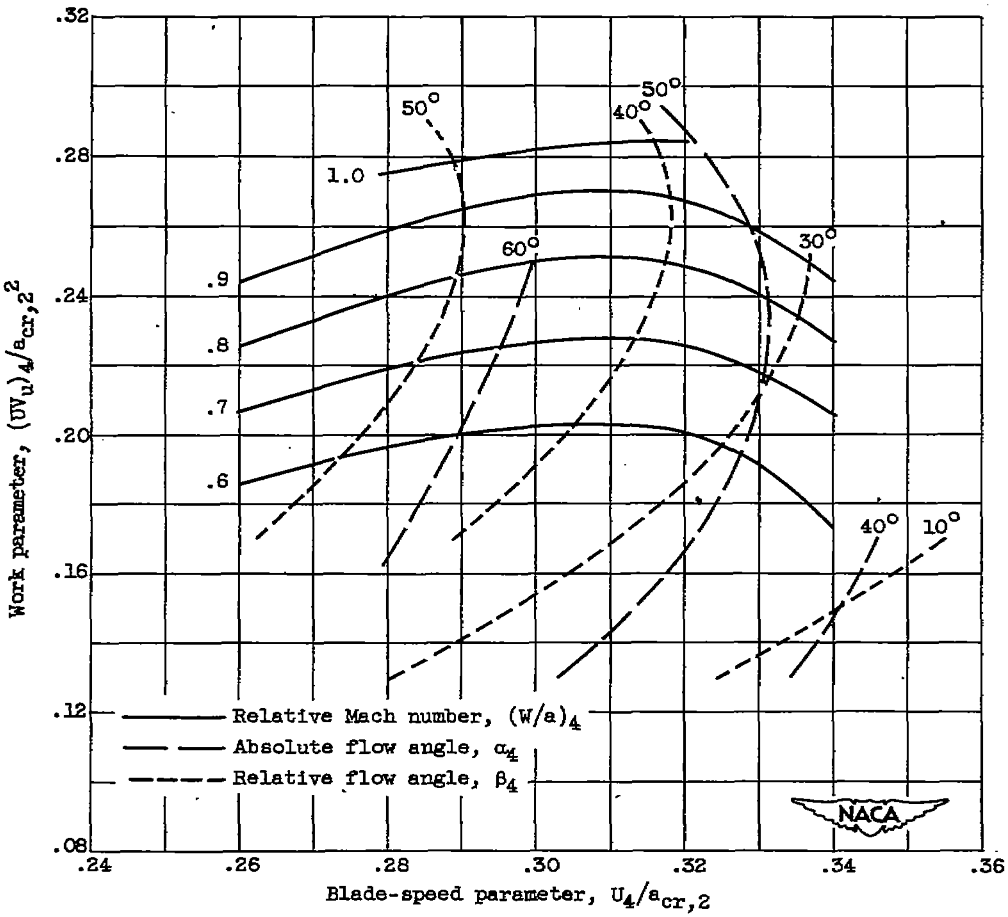

(c) At hub of station 4 for 70/30 work division.

Figure 2. - Continued. Primary turbine-design chart for analysis of flow conditions.

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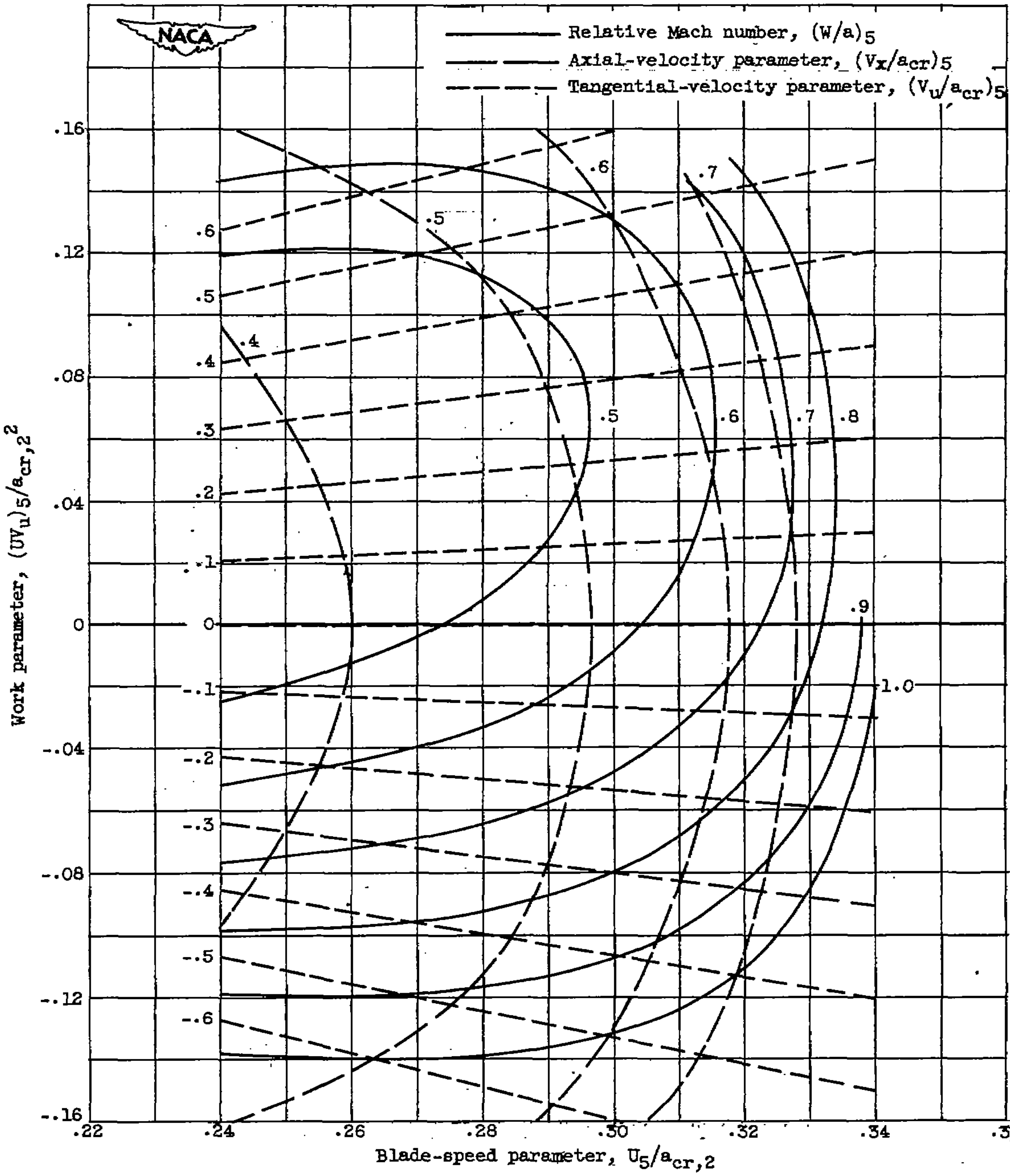

(d) At hub of station 5 for 70/30 work division.

Figure 2. - Concluded. Primary turbine-design chart for analysis of flow conditions.

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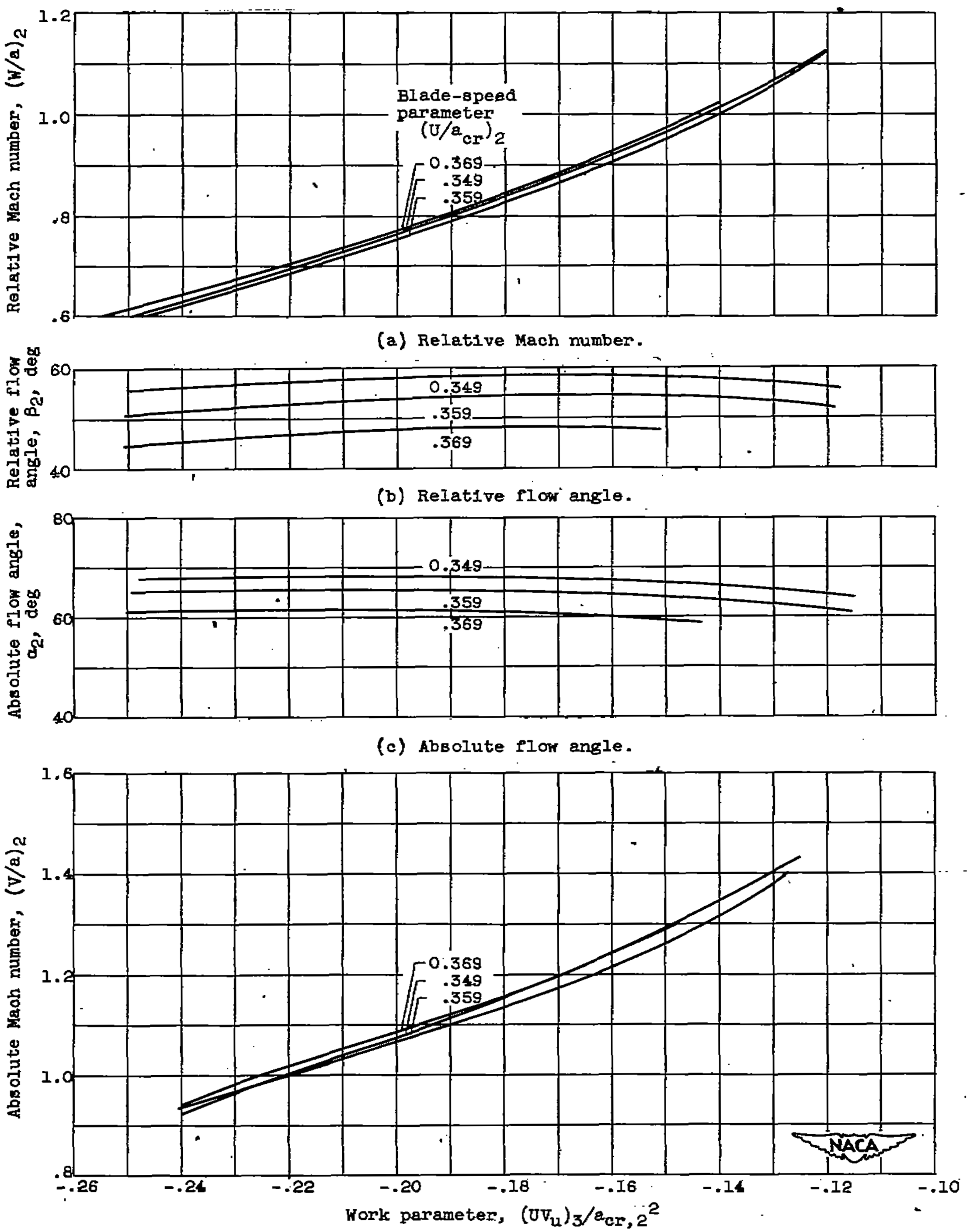

(a) Relative Mach number.

(b) Relative flow angle.

(c) Absolute flow angle.

(d) Absolute Mach number.

Figure 3. - Auxiliary turbine-design chart for analysis of flow conditions at hub of station 2 for 70/30 work division and an exit annular area of 405 square inches.

[redacted]

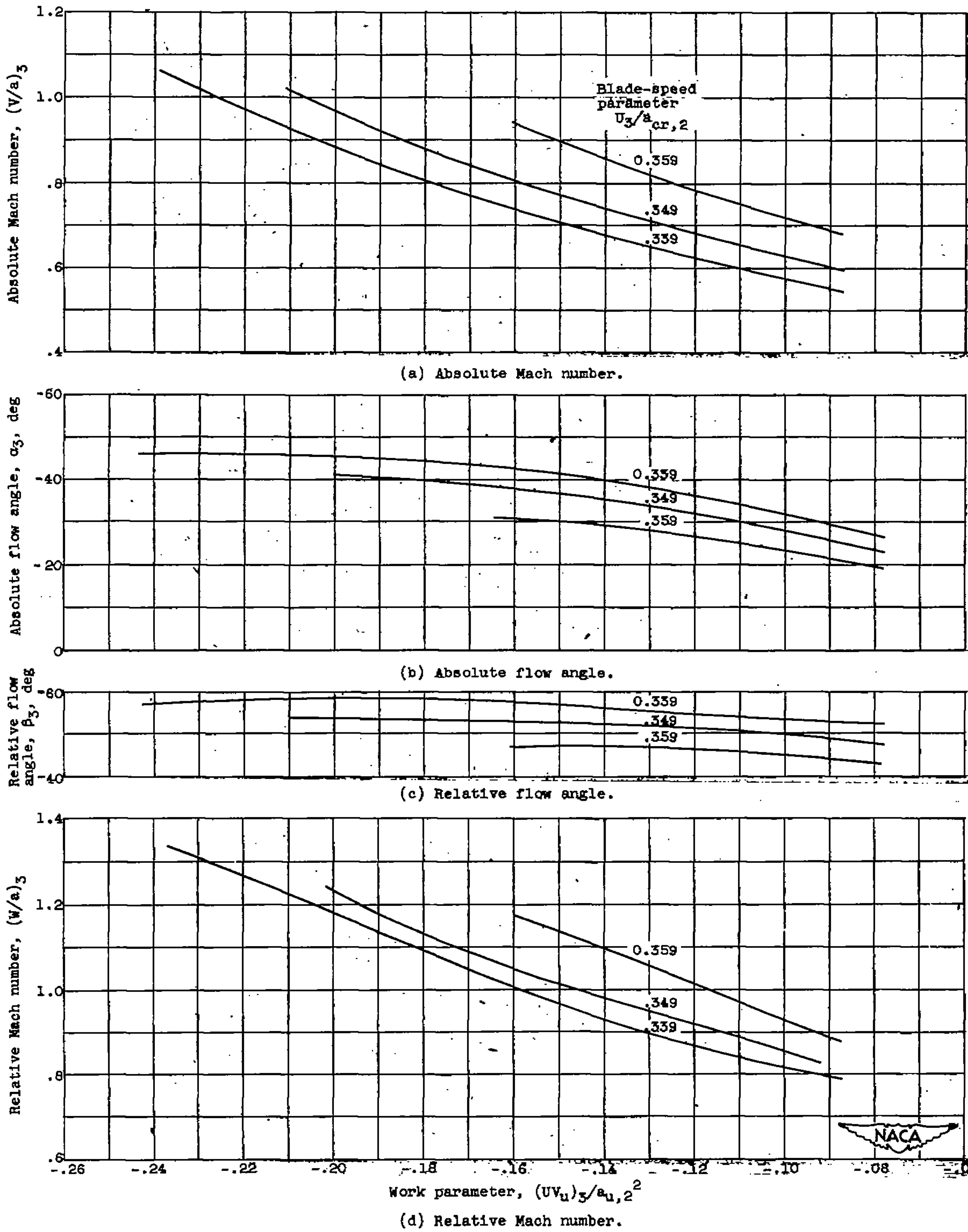

Figure 4. - Auxiliary turbine-design chart for analysis of flow conditions at hub of station 3 for 70/30 work division and an exit annular area of 405 square inches.

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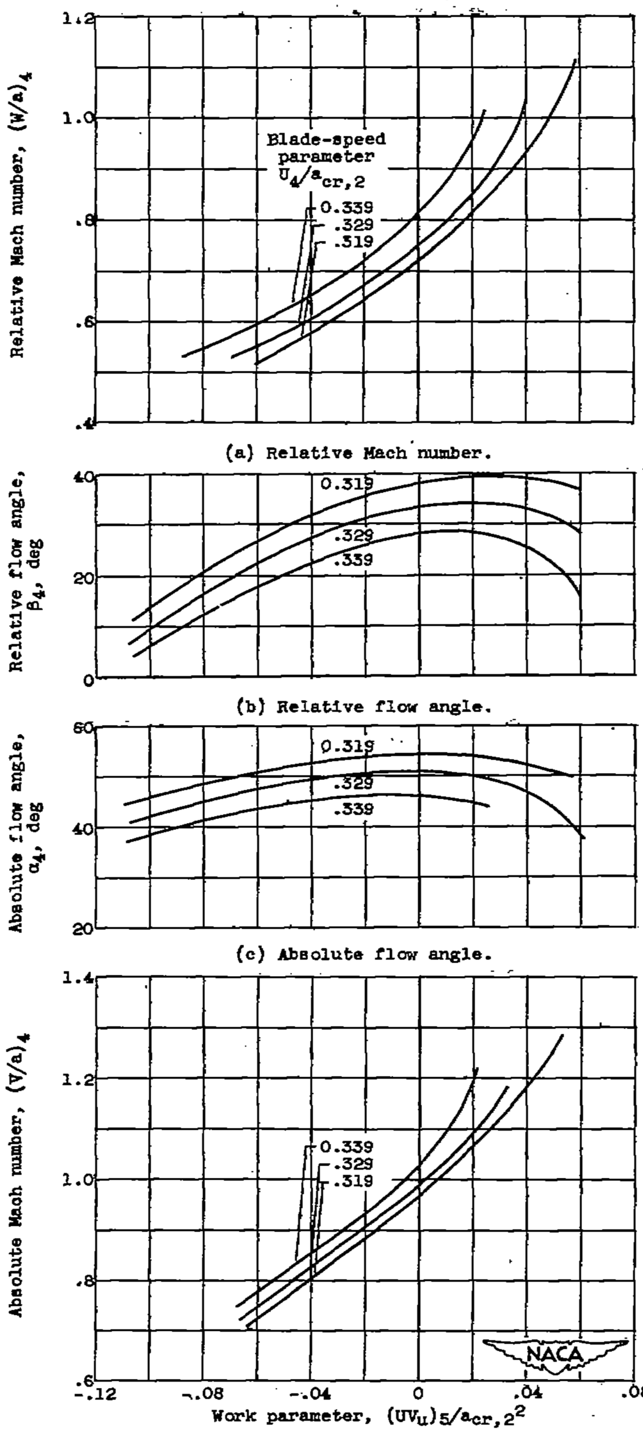

(a) Relative Mach number.

(b) Relative flow angle.

(c) Absolute flow angle.

(d) Absolute Mach number.

Figure 5. - Auxiliary turbine-design chart for analysis of flow conditions at hub of station 4 for 70/30 work division and an exit annular area of 405 square inches.

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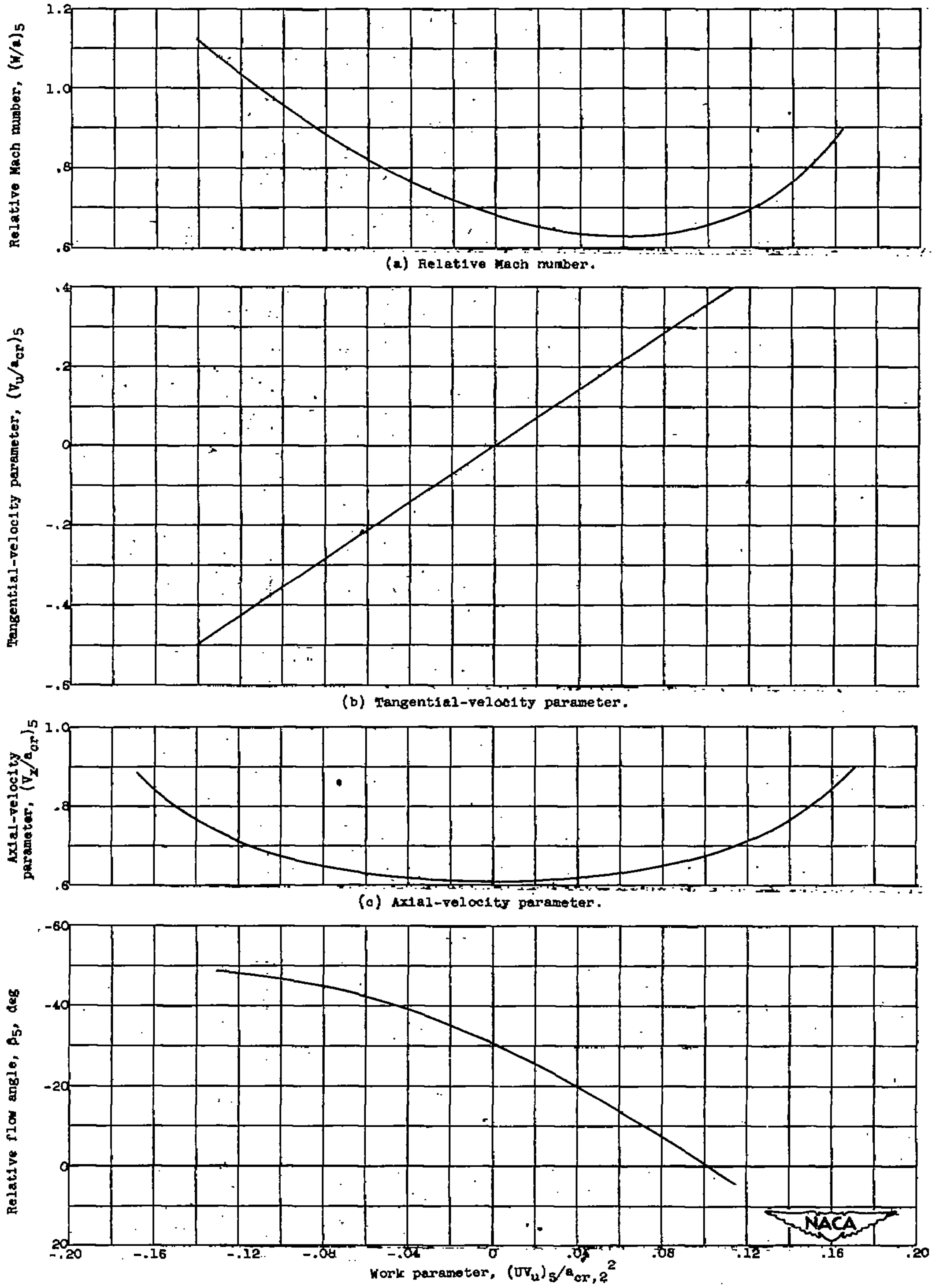
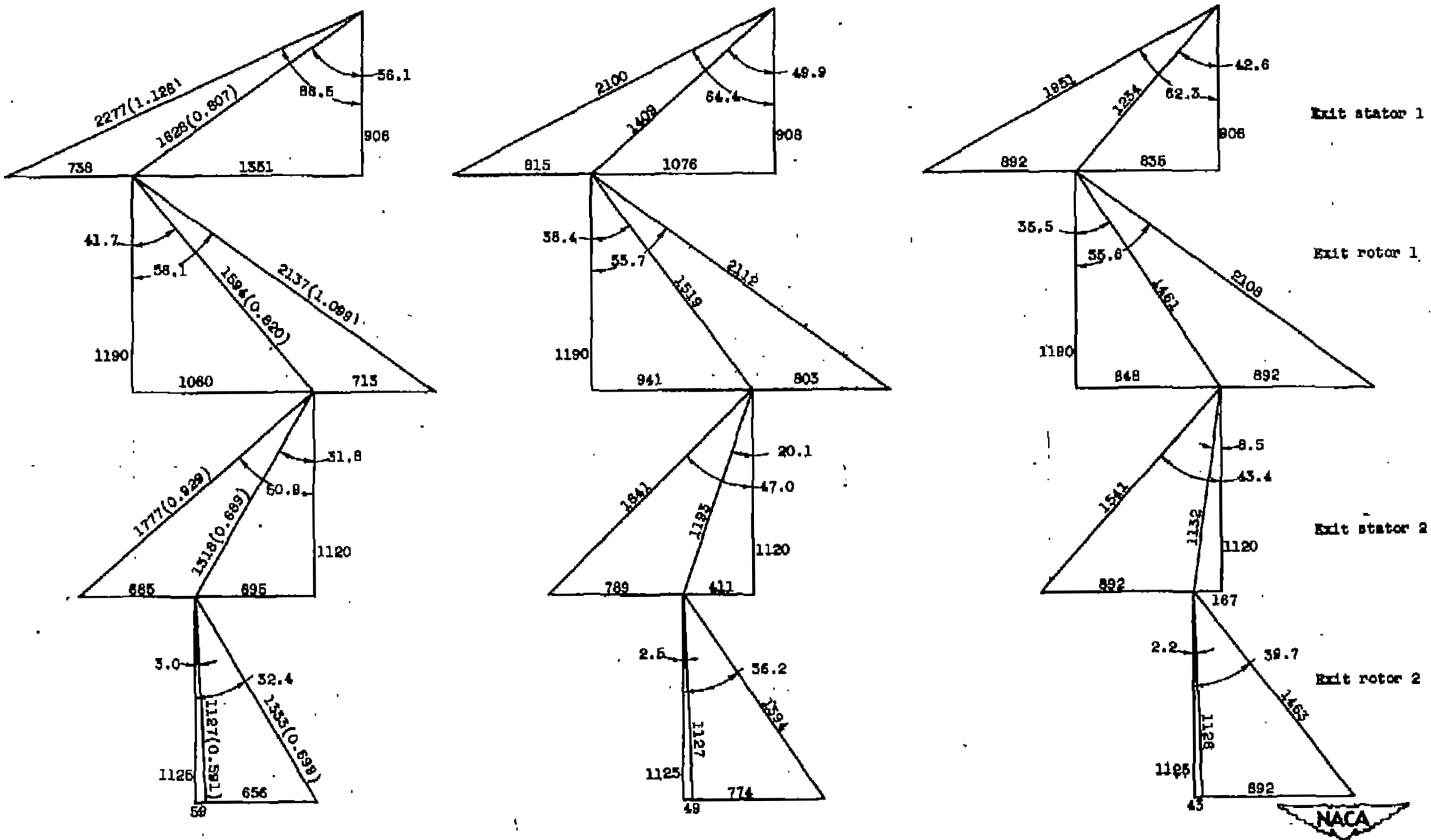


(d) Relative flow angle.

Figure 8. - Auxiliary turbine-design chart for analysis of flow conditions at hub of station 5 for 70/30 work division and an exit annular area of 405 square inches. Blade-speed parameter, 0.319.



(a) Hub. (b) Mean. (c) Tip.

Figure 7. - Velocity diagrams with assumed radial variations in flow for turbine having 70/30 work division and exit annular area of 405 square inches. Numbers in parentheses are Mach numbers based on local velocity of sound; velocities are in feet per second; angles are in degrees.

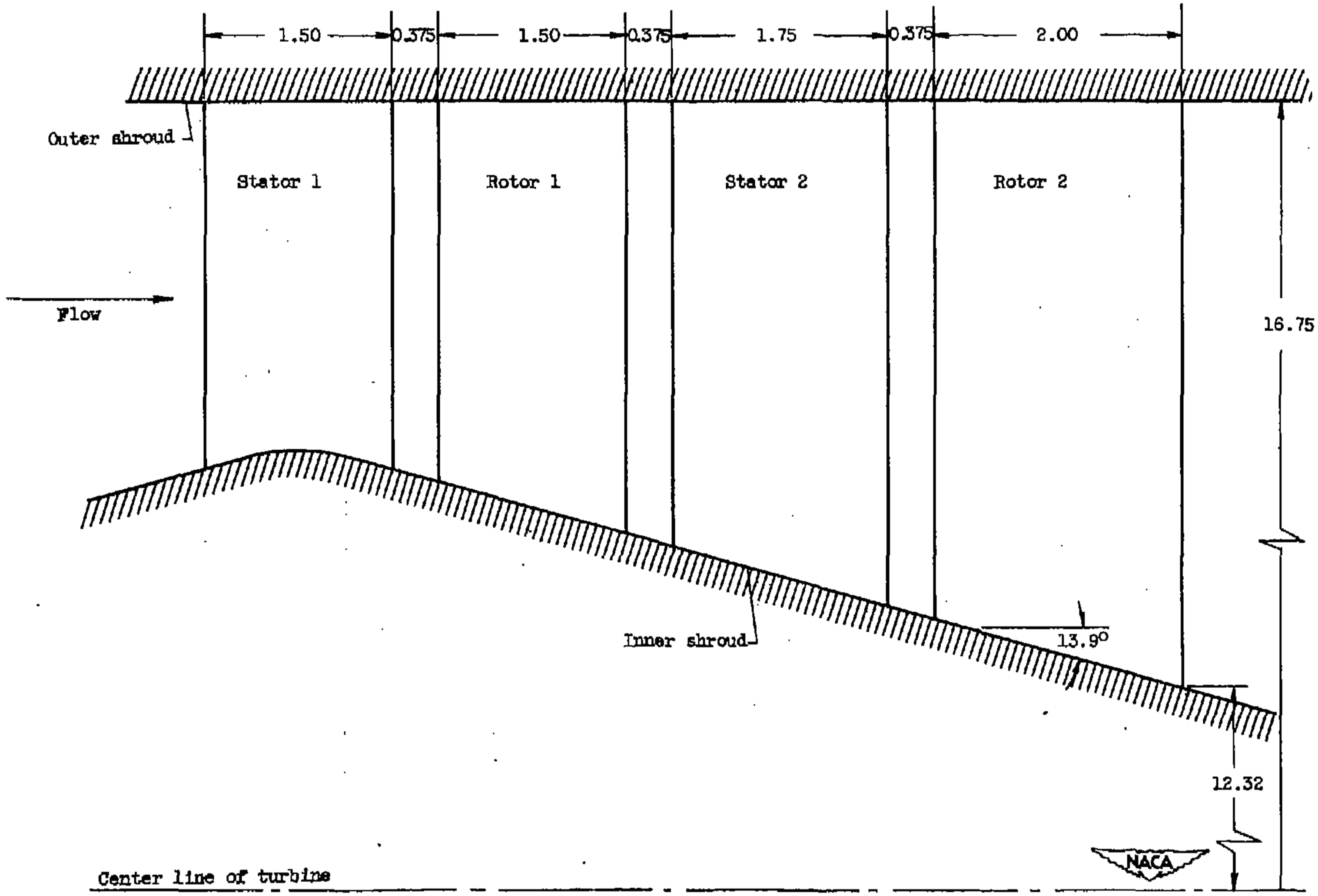


Figure 8. - Turbine configuration for which velocity diagrams with assumed radial variations in flow were calculated. (All dimensions in inches except where noted.)

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