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RESEARCH MEMORANDUM

INVESTIGATION OF TURBINES SUITABLE FOR USE IN A TURBOJET ENGINE WITH HIGH COMPRESSOR PRESSURE RATIO AND LOW COMPRESSOR-TIP SPEED

III - VELOCITY-DIAGRAM STUDY OF TWO-STAGE AND DOWNSTREAM-STATOR TURBINES FOR ENGINE OPERATION AT CONSTANT ROTATIVE SPEED

By Robert E. English and Elmer H. Davison

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NATIONAL ADVISORY COMMITTEE
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WASHINGTON
November 21, 1952

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SUMMARY

The application of turbines to driving single-spool compressors was studied by investigating whether or not satisfactory velocity diagrams can be obtained for two-stage turbines either having or not having a downstream stator if these turbines are required to drive a particular single-spool compressor over a range of engine operation at rated rotative speed and to have a tip diameter no greater than the tip diameter of the compressor. The relatively low blade-tip speed, high work, and high air flow per unit frontal area of this compressor make critical the problem of designing such a two-stage turbine to drive this compressor.

A simplified method of analysis was developed in order to relate the turbine operations for take-off and cruise, both at rated rotative speed. In this way, the turbine velocity diagrams selected considered both take-off and cruising operation. From this analysis, the following conclusions were drawn:

A two-stage turbine which has a downstream stator and is required to drive a single-spool compressor as part of a turbojet engine operated at rated rotative speed for both take-off and cruise can be satisfactorily designed within the limits

(1) Turbine tip diameter $\leq$ compressor tip diameter.

(2) Relative entrance Mach number to any blade row ≤ 0.79.

(3) Turning by any blade row $\leq 123^{\circ}$.

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[redacted]

(4) Deceleration across only the downstream-stator-blade row.

provided that the following conditions are satisfied:

(1) Engine cruises with thrust at least as great as 0.64 maximum thrust at altitude.

(2) The compressor characteristics for take-off are within the following range:

(a) Equivalent air flow per unit frontal area $\leq$ 25.8 pounds per second-square foot.

(b) Equivalent blade-tip speed $\geq$ 892 feet per second.

(c) Equivalent work input $\leq$ 131 Btu per pound.

Because the turbine-design requirements for take-off are essentially the same as the requirements for cruising with the take-off value of exhaust-nozzle area, such a turbine design is also satisfactory for engine operation with constant exhaust-nozzle area.

When a single-spool compressor which has more severe turbine-design requirements is used, a satisfactory two-stage turbine cannot be designed within the given limits if the engine is to be operated at rated rotative speed for thrusts as low as 0.64 maximum thrust at altitude.

A downstream stator can be employed in turbine design to significantly improve internal flow conditions, to increase the margin to limiting loading, and to permit the use of considerable amounts of tangential velocity at the exit from the last rotor-blade row without the flow conditions at the entrance to the downstream stator exceeding the limits of modern compressor design. Even at constant rotative speed, engine thrust can be varied considerably without exceeding design limits on the downstream stator.

INTRODUCTION

The design of a turbine for a range of turbojet engine operation rather than for just a single operating condition requires that the turbine-design requirements for the various conditions of operation be considered. In reference 1, the design requirements are determined for turbines to drive a particular single-spool, high-pressure-ratio, low-blade-tip speed compressor under the following conditions:

(1) Take-off

(2) Maximum thrust at altitude

(3) Altitude cruising

  (a) With maximum-thrust exhaust-nozzle area

  (b) At rated rotative speed

(4) Engine acceleration at 80 percent equivalent rated speed

In reference 1, the engine operating conditions for take-off are a compressor total-pressure ratio of 8.75, a turbine-inlet temperature of 2160° R, and rated rotative speed. The compressor frontal area is 881 square inches. At rated rotative speed and a total-pressure ratio of 8.75, this compressor has an equivalent tip speed of 892 feet per second and an equivalent weight flow of 158 pounds per second.

For a given ratio of turbine to compressor frontal area, the three compressor parameters that determine the turbine-design requirements are mass flow per unit frontal area, work, and blade-tip speed. Increasing compressor work, increasing mass flow per unit compressor frontal area, or decreasing compressor blade-tip speed makes the turbine-design requirements more critical if an attempt is made to stay within a given number of turbine stages, a given turbine frontal area, and preestablished turbine aerodynamic limits. The characteristics of several engines currently being developed were investigated; the requirements imposed on a turbine to drive the compressor chosen for this investigation constituted the most severe problem in the design of two-stage turbines having a tip diameter no greater than that of the compressor because of the compressor's relatively high mass flow per unit frontal area, high work, and low blade-tip speed. If a two-stage turbine can be designed to drive this compressor satisfactorily, two-stage turbines can then be designed to drive all compressors presenting less severe turbine requirements.

The high work output required of a turbine to drive this compressor precluded the possibility of achieving a single-stage turbine design within conventional aerodynamic limits, and whether or not even a two-stage turbine could be designed within conventional aerodynamic limits to satisfy the turbine-design requirements appeared questionable. Although by comparison with a two-stage turbine a three-stage turbine which satisfies the turbine-design requirements would be relatively easy to design within conventional aerodynamic limits, a three-stage turbine would, in general, have the disadvantages of being larger, heavier, and more complex.

For engine operation with constant exhaust-nozzle area and variable rotative speed, reference 1 shows that the turbine-design requirements are nearly identical for take-off, maximum thrust at altitude, and cruise at altitude. Reference 1 shows that for engine operation at rated rotative speed and variable exhaust-nozzle area, the miminum permissible exit annular area is greater than that for operation with constant exhaust-nozzle area; this increase in the exit annular area makes more critical the problem of designing a turbine for satisfactory operation for take-off, maximum thrust at altitude, and cruise at altitude. A turbine which satisfies the requirements of operation either with constant exhaust-nozzle area or at constant rotative speed is shown in reference 1 also to satisfy the requirements for acceleration at 80 percent rated equivalent rotative speed. The turbine-design problem may therefore be divided into two phases, one for each type of engine operation for cruise. These phases correspond to the following two modes of engine operation for take-off, cruise, and maximum thrust at altitude: engine operation with constant exhaust-nozzle area and variable rotative speed, and engine operation at design rotative speed with variable exhaust-nozzle area.

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The problem of turbine design for engine operation with constant exhaust-nozzle area and variable rotative speed is analyzed in reference 2. Reference 2 indicates that a satisfactory two-stage turbine for driving this particular compressor may be designed within conventional aerodynamic design limits and be no larger in diameter than the compressor, but such a turbine design will be close to conventional design limits.

The application of turbines to driving single-spool compressors was investigated at the NACA Lewis laboratory by determining whether or not a two-stage turbine may be designed to drive this particular compressor satisfactorily over a range of engine operation for which the rotative speed is constant and the exhaust-nozzle area is varied. The results of reference 2 indicate that for this service a two-stage turbine design within conventional aerodynamic limits will probably be a marginal design. This investigation therefore utilized a downstream stator (or set of straightening vanes) in the design of such a turbine in addition to conventional two-stage designs for this application. This investigation of a downstream stator for this application has two purposes: to determine the extent to which the critical flow conditions within the turbine may be relieved by using downstream stators, and to determine the range of flow conditions that will exist at the entrance to the downstream stator between the take-off and cruising conditions, both at rated engine speed. A conventional two-stage turbine (not having a downstream stator) and a two-stage turbine having a downstream stator are hereinafter referred to as "two-stage turbine" and "downstream-stator turbine," respectively.

Because the design requirements for a turbine to drive this compressor under take-off and maximum-thrust-at-altitude conditions are shown in reference 1 to be essentially the same, a turbine design satisfactory for take-off is herein considered satisfactory for maximum thrust at altitude. On the other hand, a turbine for driving this compressor at rated engine speed for both take-off and cruise must operate under two widely differing conditions. The velocity-diagram study presented herein therefore examines two conditions of operation - take-off and cruise. In order to consider both these operations, a velocity-diagram study was made of the turbine-hub conditions for take-off; this velocity-diagram study was based on the one-dimensional analysis presented in reference 2. An extension of the analysis in reference 3 was then employed to relate each possible take-off design to its anticipated cruising performance. The method employed for this study may be summarized as follows: Possible designs for take-off were determined, and the cruising performance of these configurations was then predicted. These possible designs were then compared in order that the best of them could be selected for presentation.

By means of these approximate analyses, two possible designs were selected - one for a two-stage turbine and one for a downstream-stator turbine. For each of these possible designs, more accurate velocity diagrams were determined for take-off operation by considering radial variations in flow conditions rather than just hub conditions. Simplified radial equilibrium was assumed (reference 4), and a free-vortex distribution of tangential velocity was employed. For the downstream-stator design, a one-dimensional calculation was employed in order to estimate the flow conditions at the entrance to and the exit from each blade row under cruising operation.

The nomenclature used in this analysis is given in figure 1. The symbols used are defined in appendix A.

GENERAL DESIGN CONSIDERATIONS

Design Conditions

It is stated in reference 1 that for take-off the minimum permissible exit annular area is 383 square inches, and for cruising at rated engine speed it is 460 square inches. Exit annular areas somewhat larger than either of these were considered for this investigation. The annular area at the exit of the last rotor-blade row was 480 square inches for the two-stage design and 550 square inches for the downstream-stator design.

The turbine frontal area, in addition to exit annular area, was selected. A turbine larger in diameter than the compressor is undesirable because the resulting increase in frontal area of the engine would reduce the advantage of the high mass flow per unit frontal area of this compressor. Even though the design problem of producing a given amount of work from a turbine stage within certain design limits is easier to solve if the turbine-blade speed is high rather than low, higher blade speeds are obtainable in this case only by increasing the turbine diameter because the rotative speed of the turbine is fixed. For these reasons, the tip diameter of each turbine blade row was selected equal to the tip diameter of the compressor. If the mass flow of the compressor is assumed equal to the mass flow of the turbine, the result of this selection of turbine diameters is that the turbine-blade-tip speed and gas flow per unit of turbine frontal area are equal to the corresponding values for the compressor.

Take-off. - For engine operation during take-off, the following conditions must be fulfilled by the turbine:

Work output, Btu/lb	131
Total-pressure ratio	3.42
Air flow per unit frontal area, lb/(sec)(sq ft)	25.8
Blade-tip speed, ft/sec	892
Inlet temperature, °R	2160
Inlet pressure, lb/sq ft	17,600

Cruise. - The conditions for cruising operation are: flight altitude, 35,000 feet; flight Mach number, 0.75; engine thrust, 0.64 of maximum thrust for these flight conditions. For engine operation during cruise, the following conditions must be fulfilled by the turbine:

Work output, Btu/lb	115
Total-pressure ratio	4.24
Air flow per unit frontal area, lb/(sec)(sq ft)	10.2
Blade-tip speed, ft/sec	892
Inlet temperature, °R	1620
Inlet pressure, lb/sq ft	6000

Design Limits

The following turbine parameters are herein considered to limit the range of turbine design: relative Mach number at the entrance to each blade row, velocity change across each blade row (alternatively, amount of reaction), turning angle of each blade row, limiting loading, and absolute tangential velocity at the exit from the last rotor-blade row. In applying design limits, the designer should consider the operating conditions under which these limits will be approached. Reference 3 shows that for turbines having successive choked blade rows the most severe Mach numbers, velocity changes, and amounts of turning are encountered at low equivalent blade speeds. As was true for the turbine design in reference 2, a turbine to drive this compressor under the conditions specified herein will very likely have choking or nearly choking conditions in the first three blade rows at low equivalent blade speeds. For engine operation at constant rotative speed, the low values of equivalent blade speed are encountered under conditions resulting in high turbine-inlet temperature, and in particular under take-off rather than cruising conditions. The flow conditions in the first rotor and second stator will thus more closely approach the design limits under take-off rather than cruising conditions.

For take-off, the last row of rotor blades will very likely be limited by velocity change rather than by entrance Mach number just as was the turbine design in reference 2. The increasing turbine total-pressure ratio from take-off to cruise (from 3.42 to 4.24, reference 1) will result in an increase in the acceleration across the last rotor-blade row. The flow conditions which are critical during take-off thus differ from those which are critical during cruise. The following design limits were therefore applied in velocity-diagram computations for the take-off conditions:

(1) The relative Mach number at the entrance to any blade row should not exceed 0.85.

(2) The amount of turning required of any blade row should not exceed 120°; this applies to the velocity relative to each blade row.

(3) The velocity should not decrease across any blade row except the downstream stator.

For engine operation at rated rotative speed, the design limits which are approached during cruise are associated with large values of tangential velocity at the exit from the last rotor-blade row. The magnitude of the tangential component of the relative velocity at the exit from the last rotor-blade row will not exceed the value at the loading limit investigated in reference 5, and this condition establishes a design limit for cruise.

In addition, for a two-stage turbine which is to be operated without a downstream stator, the tangential component of absolute velocity of the gas leaving the last rotor-blade row should be as near zero as possible. For a downstream-stator turbine, on the other hand, the magnitude of the tangential velocity at the exit from the last rotor-blade row need not be made small; instead, conditions should be selected which will keep low the magnitude of the Mach number at the entrance to the downstream stator.

Design Problem

As in reference 2, the independent variables in the design study are work division between the turbine stages and axial variation in annular area or cone angle. Because reference 2 shows the effects of varying the cone angle from turbine entrance to exit to be very small, only constant values of cone angle are considered herein. The design problem is therefore a matter of seeking a combination of cone angle and work division for take-off such that for take-off operation, the design limits on Mach number, turning, and velocity change are not exceeded; and for operation of the same configuration under cruising conditions, neither is the loading limit exceeded in the last rotor nor (a) for a two-stage turbine without a downstream stator, is the exit tangential velocity objectionably large, nor (b) for a downstream-stator turbine, is a limiting value of stator-entrance Mach number exceeded.

METHOD OF ANALYSIS

General Method

The simplified method of velocity diagram analysis in reference 2 was employed to analyze the turbine internal flow conditions for take-off. A simplified method of analysis of cruising operation was used to relate the flow conditions at the exit from the last rotor-blade row during take-off and the flow conditions at this station during cruise. For any given design for take-off, the flow conditions at the exit from the last rotor-blade row can thus be determined and, in turn, the anticipated cruising performance can be evaluated.

Simplified Analysis of Cruising Operation

By means of the analysis presented in appendix B, charts like those in figure 2 were prepared for several values of exit annular area. The dashed lines show for the mean radius the relation between two factors: (1) the relative tangential velocity parameter at the exit from the last rotor-blade row which must be obtained under cruising conditions if the turbine is to produce enough work to drive the compressor under cruising conditions, and (2) the absolute tangential velocity parameter at the exit from the last rotor-blade row which may prevail under take-off conditions. The dot-dashed line represents the maximum value of relative tangential velocity parameter obtainable within the loading limit in reference 5. Points to the right of the intersection of these two lines represent possible design conditions and those to the left represent design conditions which, without departure from convention, are impossible. The distance by which the dot-dashed line is above the dashed line is indicative of the margin between design operation for cruise and operation at limiting loading. A solid line has been included to show the effect of increasing the required work output of the turbine by 4.5 percent. Because engine components are subject to some deterioration in performance during their useful life, these lines show the importance of designing for a condition somewhat removed from the condition of limiting loading.

Figures 3 and 4 present additional flow conditions at the exit from the last rotor-blade row for both take-off and cruising conditions. As before, the abscissa is the take-off values of absolute tangential velocity parameter at the exit from the last rotor-blade row. The Mach numbers at the exit from the second rotor-blade row for both take-off and cruise are shown. For the exit annular area of 550 square inches, the flow directions are shown as well. For the exit annular area of 480 square inches, the leaving loss, equal to the kinetic energy contained in the tangential velocity at the exit from the last rotor-blade row, is shown as a fraction of the ideal turbine work.

Plots such as these were used to relate the take-off and cruising operations in order that a design might be obtained which would operate within the design limits for both operating conditions.

Interblade-Row Calculation

For the downstream-stator design, interblade-row conditions for cruise were computed for the design configuration. For these calculations, the exit flow direction relative to each blade row at the mean radius was assumed to be constant from take-off to cruise. The equivalent mass flow at the turbine exit which results in production of the required turbine work was determined. In order to determine approximate hub conditions from these mean-radius calculations, the flow was assumed to vary radially according to the free-vortex, simplified-radial-equilibrium relations.

RESULTS

For each of the two types of turbine design, namely, two-stage and downstream-stator, the results are presented for single values of work division between stages, cone angle of the inner shroud, and exit annular area. The effects of work division and cone angle were investigated in reference 2 and are therefore not presented herein. Selection of the exit annular areas will be discussed.

For each type of turbine design, a possible design condition was selected which is a good compromise of factors affecting the velocity diagrams. This selection was based on the results of simplified velocity-diagram analyses similar to those presented in reference 2. The velocity diagrams presented are based on this selected design condition and include the radial variations in flow resulting from the assumption of simplified radial equilibrium and free-vortex flow; these diagrams are for take-off operation.

Two-Stage Turbine Design

Take-off. - The velocity diagrams in figure 5 are for a two-stage turbine in operation during take-off having an exit annular area of 480 square inches, a 70/30 work division, and a cone angle of 20.6°. The assumed turbine configuration is presented in figure 6. Even though the velocities for take-off operation are within the specified design limits, every blade row must operate very near a design limit. The acceleration across a rotor-blade row increases considerably from hub to tip. On the other hand, the acceleration across the second stator-blade row is nearly constant from hub to tip, having the smallest value at the tip.

The design limit of no deceleration across a blade row results from experience in design and operation of rotor-blade rows near this limit. Because of the difference between the radial distributions of acceleration, operation of a stator-blade row near the design limit of no deceleration may be less efficient than operation of a rotor-blade row near this limit. Even though the design limits were not exceeded for this design, operation of the second stator-blade row may therefore be critical.

By changing the work division so that less work is produced by the first stage, the flow conditions in the second stator-blade row may be made more conservative. In order to remain within the design limit of no deceleration in the second stage, this change in work division will require that the tangential velocity at the exit from the second rotor-blade row be increased. The leaving loss $V_{u,5}^2/2gJ$ will thereby be increased.

Cruise. - The relation between turbine exit tangential velocity for take-off and the turbine exit flow conditions for cruise are shown in figures 2(a) and 3. For the velocity diagram presented in figure 5 for take-off operation, the value of exit tangential velocity parameter $(V_u/a_{cr})_5$ at the mean radius is -0.065. For this take-off value of exit tangential velocity parameter, figures 2(a) and 3 show that for cruising (1) the flow conditions at the rotor exit are very near the condition of limiting loading and (2) the kinetic energy associated with the exit tangential velocity is over 2 percent of the ideal turbine work output. Because this kinetic energy is herein assumed to be the loss associated with the exit tangential velocity, this loss reduces the turbine efficiency by 0.02 for cruising. This turbine design is thus marginal for both take-off and cruise.

When the velocity diagram for take-off is adjusted by employing an increased amount of exit tangential velocity (larger negative value), the effect on the turbine exit flow conditions is as shown in figures 2(a) and 3. Increasing the exit tangential velocity for take-off by 55 feet per second corresponds to a change of -0.03 in the tangential velocity parameter $(V_u/a_{cr})_5$, that is, from -0.065 to -0.095. For cruise under this condition, the loading limit would be reached in the last rotor-blade row and the leaving loss would be increased to nearly 4 percent of the ideal turbine work. Improvement in the flow conditions within the second stator-blade row during take-off operation would thus be at the expense of impaired cruising performance.

Exit annular area. - Although results of the analysis are presented for only a single value of exit annular area (480 sq in.), several values were considered. From figure 2(a), it is apparent that an exit annular area of 480 square inches results in operation at limiting loading. For this reason, a smaller exit annular area could not be employed without exceeding the loading limit for cruise. On the contrary, a greater annular area is required if the compressor work for cruise rises above 115 Btu per pound (fig. 2(a)). An exit annular area of 480 square inches is thus about the smallest that can be employed.

Increasing the exit annular area above 480 square inches will reduce the severity of the problem of limiting loading but will aggravate other problems. A comparison between this two-stage design and the design presented in reference 2 (which has an exit annular area of 405 sq in.) shows that increasing the exit annular area from 405 to 480 square inches has made more critical the problem of designing for satisfactory turbine operation under take-off conditions. A further increase in exit annular area will result in an even more critical two-stage design. Thus, in order to alleviate the critical nature of this design, some additional feature (such as a downstream stator or an increased tip diameter) should be incorporated into the design.

Downstream-Stator Turbine Design

Take-off. - The velocity diagrams in figure 7 are for a downstream-stator turbine in operation under take-off conditions having an exit annular area of 550 square inches, a 67/33 work division, and a cone angle of 27.6°. The assumed turbine geometry is shown in figure 8. The flow conditions are within the specified design limits with the single exception of the amount of turning at the hub of the first rotor-blade row. The maximum Mach number at the entrance to any blade row is 0.79.

The amount of turning in the first rotor-blade row could be reduced from 123° to the limiting value of 120° by increasing the cone angle by a small amount and so adjusting the work division that the work output of the first stage is slightly decreased. Because of the arbitrary nature of the design limits, this change was not made.

The differences of greatest significance between the two-stage and downstream-stator designs are the improved acceleration in the second stator-blade row, the increased amount of exit tangential velocity, and the increased margin between the required cruising output of the second rotor-blade row and the value at limiting loading. A comparison of figures 5 and 7 shows that at the tip of the second stator-blade row the velocity increase rose from a value of 2.3 to 9.4 percent. The increased values of exit tangential velocity are tolerable because of the anticipated pressure recovery in the downstream stator.

Limiting loading. - The increased margin between the required cruising output and limiting loading is apparent in figure 2. For the downstream-stator design, the take-off value of the exit tangential velocity parameter $(V_u/a_{cr})_5$ at the mean radius is -0.15. Figure 2(b) shows that for cruise the turbine pressure ratio may be increased in order to increase the turbine work output by 4.5 percent without exceeding the loading limit, an improvement over the two-stage design.

Stator-entrance conditions. - One factor which significantly affects the practicability of using a downstream stator is the range of flow conditions which will prevail at the stator entrance. If the stator-entrance Mach number for cruise is too high or the range of stator-entrance flow direction between take-off and cruise is too great, a downstream stator will not efficiently recover the kinetic energy contained in the exit tangential velocity. Between take-off and cruise, the range of stator-entrance flow conditions at the mean radius for a take-off value of exit tangential velocity parameter of -0.15 is shown in figure 4 to be the following:

Variable	Take-off	Cruise
Mach number, $(V/a)_5$	0.42	0.61
Flow angle, α_5, deg	-20	-28

For this range of stator-entrance conditions, the experimental cascade data of reference 6 indicate that the kinetic energy contained in the exit tangential velocity can be efficiently recovered for both take-off and cruise by using an NACA 65-(12)10 airfoil section with a solidity of 1.

These results indicate that a downstream stator can be employed in design to significantly improve the internal flow conditions for take-off, to increase the margin to limiting loading, and to permit the use of considerable amounts of exit tangential velocity with good efficiency. Even at constant rotative speed, engine thrust can be varied considerably without exceeding design limits on the downstream stator.

Exit annular area. - The exit annular area of 550 square inches was chosen somewhat arbitrarily. An examination of figures 2(a) and 3 indicates that even with a downstream stator an area of 480 square inches is not satisfactory for the following reasons: (1) Within the loading limit, only a small amount of exit tangential velocity can be employed, thereby limiting the usefulness of a downstream stator. (2) For significant amounts of exit tangential velocity, the stator-entrance Mach number is prohibitively large. Even with the addition of a downstream stator, such a design would remain marginal.

An exit annular area of 550 square inches was therefore selected in order to overcome the shortcomings of an area of 480 square inches. This value probably does not represent an optimum. Within the limits of this analysis, it is found that any area near 550 square inches will probably be satisfactory.

The use of a large exit annular area has some advantages not brought out by this analysis. For the turbine geometry shown in figure 8, the axial Mach number at the exit from the third stator under take-off conditions is only 0.34. Because this Mach number is low, the diffusion required at the entrance to an afterburner is reduced considerably below that required with a smaller annular area. Another advantage of the large annular area is the increased rate of engine acceleration which is possible.

Interblade-row velocities for cruise. - The approximate hub conditions determined during the interblade-row calculation are presented in table I. The conditions at the entrances to and exits from the first rotor-blade row and second stator-blade row are more conservative under the cruising rather than the take-off condition for the following reasons: The entrance Mach number is lower, the acceleration is greater, and the amount of turning is decreased. Although the entrance Mach number in the second rotor-blade row is greater for cruise than for take-off, the flow conditions for cruise are more conservative because the entrance Mach number is still so low (0.64) that it is not critical, the acceleration is increased, and the amount of turning is essentially unchanged. For the downstream stator-blade row, the more severe conditions are encountered during cruise, as shown by the simplified cruise analysis; the stator-entrance Mach number increased from 0.44 to 0.63 and the flow angle, from 26 to 32°. Considered as a whole, the flow conditions for cruise are thus at least as favorable as those for take-off.

Although the flow angle at the mean radius and the exit from each blade row for cruise was assumed to be equal to the angle for take-off, the exit flow angles for the hub radius in table I vary slightly for the second rotor-blade row. This variation results from the assumed radial variations and is a measure of the approximate nature of this assumption.

DISCUSSION

A comparison of the results of reference 2 with those of this analysis indicates the relative difficulty of designing for either of two modes of engine operation: (1) constant exhaust-nozzle area and variable rotative speed, or (2) constant rotative speed and variable exhaust-nozzle area. The increase in difficulty of designing a turbine for constant-speed operation results largely from the range of conditions over which the turbine must operate between take-off and cruise; whereas for engine operation with constant exhaust-nozzle area, the turbine operating conditions are essentially unchanging between take-off and cruise.

The results of reference 1 show that the over-all blade-jet speed ratio for cruise at rated speed is slightly higher (and thus more conservative) than for take-off. Despite the high value of blade-jet speed ratio for cruise at rated rotative speed, the tangential velocity at the exit from the last rotor-blade row of the designs investigated was higher for cruise than for take-off; for the two-stage design, this introduced an undesirable leaving loss. On the other hand, the flow conditions in the first rotor-blade row and second stator-blade row were not up to the design limits for the cruising condition. If a turbine were to be designed for the cruising condition alone, the first stage could produce a greater share of the work without exceeding the design limits, thereby reducing the leaving loss. A cursory study of designing for cruise alone indicated that for this condition a satisfactory two-stage design could be obtained having zero exit whirl. A satisfactory two-stage design for take-off operation is presented in reference 2. Despite the fact that a satisfactory two-stage design can be obtained for either take-off or cruise at rated engine speed, a satisfactory two-stage design cannot be obtained for operation under both the cruise and take-off conditions. For engine operation at constant speed, the required range of operation must thus be considered in turbine design.

The results of the analysis, as so far presented, are specific rather than general because they are related to a particular compressor considered as part of a turbojet engine. The value of these results can be increased if, from them, general conclusions can be drawn concerning the entire class of single-spool compressors. In order for the general significance of these results to be apparent, the characteristics of this particular compressor must be related to the characteristics of other single-spool compressors. In the following discussion, the turbine-inlet temperature for take-off is presumed to be near 2160° R because a large change in temperature will alter the turbine-design requirements.

For the purpose of driving a single-spool compressor as part of a turbojet engine to be operated at rated rotative speed for both take-off and cruise, a satisfactory downstream-stator turbine can be designed within the limits

(1) Turbine tip diameter $\leq$ compressor tip diameter.

(2) Relative entrance Mach number to any blade row ≤ 0.79.

(3) Turning by any blade row $\leq 123^{\circ}$.

(4) Deceleration across only the downstream-stator-blade row.

provided that the following conditions are satisfied:

(1) Engine cruises with thrust at least as great as 0.64 maximum thrust at altitude.

(2) The compressor characteristics for take-off are within the following range:

(a) Equivalent air flow per unit frontal area ≤ 25.8 pounds per second-square foot.

(b) Equivalent blade-tip speed ≥ 892 feet per second.

(c) Equivalent work input ≤ 131 Btu per pound.

Because the turbine-design requirements for take-off are essentially the same as the requirements for cruise with the take-off value of exhaust-nozzle area (reference 1), such a turbine design is also satisfactory for engine operation with constant exhaust-nozzle area.

When a single-spool compressor which has more severe turbine-design requirements than these is used, a satisfactory two-stage turbine cannot be designed within the given limits if the engine is to be operated at rated rotative speed for thrusts as low as 0.64 maximum thrust at altitude.

SUMMARY OF RESULTS

In order to study the application of turbines to driving single-spool compressors, an investigation was conducted to determine whether or not satisfactory velocity diagrams could be obtained for two-stage turbines either having or not having a downstream stator if these turbines are required to drive a particular single-spool compressor over a range of engine operation at rated rotative speed. The characteristics of several engines currently being developed were investigated; the requirements imposed on a turbine to drive the compressor chosen for this investigation constituted the most severe problem in the design of two-stage turbines having a tip diameter no greater than that of the compressor because of the relatively high mass flow per unit frontal area, high work, and low blade-tip speed of this compressor.

A simplified method of analysis was developed in order to relate the turbine operations for cruise and take-off, both at rated rotative speed. In this way, turbine velocity diagrams were selected by considering both take-off and cruising operation.

For this particular compressor, the following results were obtained regarding the turbine:

1. A two-stage design not having a downstream stator was at best a marginal design. For take-off operation, the velocity diagrams closely approached the design limits. For cruise, the leaving loss was objectionably large and the margin below limiting loading was scant.

2. A two-stage design having a downstream stator and an exit annular area of 550 square inches was obtained within the following limits: relative entrance Mach number, 0.79; turning by any blade row, 123°; no deceleration except within the downstream stator-blade row. The cone angle of the inner shroud was 27.6°. The work division between the stages was 67/33.

3. For the hub of the downstream stator-blade row, the range of entrance conditions between take-off and cruise was as follows: entrance Mach number, 0.44 to 0.63; entrance flow angle, 26° to 32°.

CONCLUSIONS

1. For the purpose of driving a single-spool compressor as part of a turbojet engine to be operated at rated rotative speed for both take-off and cruise, a satisfactory two-stage turbine having a downstream stator can be designed within the limits

(1) Turbine tip diameter $\leq$ compressor tip diameter.

(2) Relative entrance Mach number to any blade row ≤ 0.79.

(3) Turning by any blade row $\leq 123^{\circ}$.

(4) Deceleration across only the downstream stator-blade row.

provided that the following conditions are satisfied:

(1) Engine cruises with thrust at least as great as 0.64 maximum thrust at altitude.

(2) The compressor characteristics for take-off are within the following range:

(a) Equivalent air flow per unit frontal area ≤ 25.8 pounds per second-square foot.

(b) Equivalent blade-tip speed ≥ 892 feet per second.

(c) Equivalent work input ≤ 131 Btu per pound.

Because the turbine-design requirements for take-off are essentially the same as the requirements for cruise with the take-off value of exhaust-nozzle area, such a turbine design is also satisfactory for engine operation with constant exhaust-nozzle area.

2. When a single-spool compressor which has more severe turbine-design requirements than these is to be driven, a satisfactory two-stage turbine cannot be designed within the given limits if the engine is to be operated at rated rotative speed for thrusts as low as 0.64 maximum thrust at altitude.

3. A downstream stator can be employed in turbine design to significantly improve internal flow conditions, to increase the margin to limiting loading, and to permit the use of considerable amounts of tangential velocity at the exit from the last rotor-blade row without the flow conditions at the entrance to the downstream stator exceeding the limits of modern compressor design. Even at constant rotative speed, engine thrust can be varied considerably without exceeding design limits on the downstream stator.

4. Even if turbines can be designed for satisfactory operation under each of two operating conditions, satisfactory performance of a single design under both operating conditions is not guaranteed.

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APPENDIX A

SYMBOLS

The following symbols are used in this report:

A	annular area, sq ft
a	velocity of sound, ft/sec
a_{cr}	critical velocity, $\sqrt{\frac{2\gamma}{\gamma+1} gRT'}$, ft/sec
B	trailing-edge blockage factor
E	turbine work output, Btu/lb
g	standard acceleration due to gravity, 32.17 ft/sec^2
J	mechanical equivalent of heat, 778.3 ft-lb/Btu
n	polytropic exponent
p	pressure, lb/sq ft
R	gas constant, ft-lb/(lb)($^{\circ}$R)
r	radius, ft
T	temperature, $^{\circ}$R
U	blade speed, ft/sec
V	absolute velocity, ft/sec
W	relative velocity, ft/sec
w	weight flow, lb/sec
α	absolute flow angle measured from axial direction, deg
β	relative flow angle measured from axial direction, deg
γ	ratio of specific heats
$\Delta\eta$	leaving loss

ρ	density, lb/cu ft

Subscripts:

h	hub
R	upstream of rotor trailing edge
r	rotor throat
s	stator throat
T	tip
u	tangential component
x	axial component
1	entrance to first stator
2	entrance to first rotor
3	entrance to second stator
4	entrance to second rotor
5	exit of second rotor

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Superscripts:

$'$	stagnation or total state relative to stator
$''$	stagnation or total state relative to rotor

APPENDIX B

METHOD OF ANALYSIS OF CHOKED-FLOW TURBINES

For a turbine which does not choke the last rotor during take-off operation, the analysis in reference 3 was extended to calculate the cruising performance of turbines designed for take-off operation. In this analysis the first stator is assumed to choke at take-off and both the first stator and the last rotor are assumed to choke at cruise.

Take-Off

From the take-off conditions the throat area of the first stator, the throat area of the last rotor, and the blade exit angle for the last rotor can be calculated for a given exit annular area A_5 and an assigned value of the tangential velocity parameter $(V_u/a_{cr})_5$. In order for these values to be calculated, the following factors must be assumed or assigned: ratio of specific heats γ, work output E, weight flow w, internal efficiency η, inlet total pressure p_1', inlet total temperature T_1', exit annular area A_5, percent of trailing-edge blockage B, blade-tip speed U_T, and the hub-tip radius ratio at the turbine exit $(r_h/r_T)_5$; the work output at the mean radius is the average for the annulus, and the specific-mass-flow parameter $(\rho V_x/\rho' a_{cr})$ at the mean radius is the average for the annulus; the flow loss from the throat to the exit of the last rotor-blade row and from the entrance to the throat of the first stator-blade row (assumed isentropic herein). From these factors, the values of critical velocity a_{cr} and total density ρ' can be computed at the turbine exit (station 5) and the first stator exit (station 2) by means of standard thermodynamic procedures.

The symbols used in the following analysis refer to conditions at the mean radius of the annulus with the exceptions of blade-tip speed U_T, hub radius r_h, and tip radius r_T.

If a trailing-edge blockage B is assumed, the specific-mass-flow parameter upstream of the trailing edge (based on the annular area within the blade row) can be determined from

$$\left(\frac{\rho V_x}{\rho' a_{cr}}\right)_{5,R} = \frac{w}{\rho_5' a_{cr,5} B A_5} \tag{1}$$

The values of the specific-mass-flow parameter $(\rho V_x/\rho' a_{cr})_{5,R}$ and the assigned value of the tangential velocity parameter $(V_u/a_{cr})_5$ can be used in conjunction with the flow chart in figure 3 of reference 7 to determine the axial velocity parameter $(V_x/a_{cr})_{5,R}$. The tangential velocity parameter (V_u/a_{cr}) is assumed to remain constant between the rotor throat and downstream of the rotor. With this assumption $\left(\frac{V_u}{a_{cr}}\right)_5$, $\left(\frac{V_u}{a_{cr}}\right)_{5,R}$, and $\left(\frac{V_u}{a_{cr}}\right)_{5,r}$ are all equal.

Even though the flow chart in reference 7 was constructed for a value of γ of 1.30, the chart is applicable for a range of values of γ. The results are not sensitive to ordinary variations in γ unless the axial velocity approaches sonic speed.

If the gas density between the throat and the trailing edge of the last rotor is assumed constant, the throat area of the rotor $A_{5,r}$ can be calculated from the geometry of the vector diagrams (see fig. 1).

$$A_{5,r} = \frac{BA_5\left(\frac{V_x}{a_{cr}}\right)_{5,R}}{\sqrt{\left[\left(\frac{V_u}{a_{cr}}\right)_5 - \left(\frac{U}{a_{cr}}\right)_5\right]^2 + \left(\frac{V_x}{a_{cr}}\right)^2_{5,R}}} \qquad (2)$$

where from the geometry of the turbine,

$$U = \frac{1}{2} U_T\left[1 + \left(\frac{r_h}{r_T}\right)_5\right] \qquad (3)$$

The exit blade angle $\beta_{5,R}$ can also be calculated from the geometry of the vector diagrams.

$$\beta_{5,R} = \tan^{-1}\frac{\left(\frac{V_u}{a_{cr}}\right)_5 - \left(\frac{U}{a_{cr}}\right)_5}{\left(\frac{V_x}{a_{cr}}\right)_{5,R}} \qquad (4)$$

(The positive V_u direction is in the direction of blade motion.)

Since the specific-mass-flow parameter by definition takes on the value at the choked condition

$$\left(\frac{\rho V}{\rho' a_{cr}}\right) = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \quad \text{(one-dimensional flow)}$$

(5)

the throat area of the choked first stator can be determined from

$$A_{2,s} = \frac{w}{\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \rho_2{}' a_{cr,2}} \tag{6}$$

The polytropic exponent $n/(n-1)$ relating the ratio of total pressure across the turbine to the ratio of total temperature across the turbine is assumed the same for take-off and cruise and can be determined from the take-off conditions by the thermodynamic relation

$$\frac{n}{n-1} = \frac{\ln\left(\frac{p_1{}'}{p_5{}'}\right)}{\ln\left(\frac{T_1{}'}{T_5{}'}\right)} \tag{7}$$

Cruise

For this cruise analysis, the following factors must be assumed or assigned at the cruise condition: work output E, polytropic exponent $n/(n-1)$, turbine inlet total temperature $T_1{}'$, blade-tip speed U_T, and flow loss from the throat to the exit of the last rotor-blade row and from the entrance to the throat of the first stator-blade row (assumed isentropic herein).

If the first stator and the last rotor are assumed choked, continuity of mass flow requires that

$$\frac{p_1{}' A_{2,s}}{\sqrt{T_1{}'}} = \frac{p_{5,r}{}'' A_{5,r}}{\sqrt{T_{5,r}{}''}} \tag{8}$$

which can be written

$$\frac{A_{5,r}}{A_{2,s}} = \frac{p_1{}'}{p_5{}'} \sqrt{\frac{T_5{}'}{T_1{}'}} \frac{p_5{}'}{p_{5,r}{}''} \sqrt{\frac{T_{5,r}{}''}{T_5{}'}} \tag{8a}$$

and upon introducing the thermodynamic relation

$$\left(\frac{p_1{}'}{p_5{}'}\right) = \left(\frac{T_1{}'}{T_5{}'}\right)^{\frac{n}{n-1}} \tag{9}$$

and, by definition of the stagnation state,

$$\left(\frac{p_5'}{p_{5,r}''}\right) = \left(\frac{T_5'}{T_{5,r}''}\right)^{\frac{\gamma}{\gamma-1}} \tag{10}$$

equation (8a) becomes

$$\frac{A_{5,r}}{A_{2,s}} = \left(\frac{T_1'}{T_5'}\right)^{\frac{n+1}{2(n-1)}} \left(\frac{T_5'}{T_{5,r}''}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{8b}$$

The area ratio $A_{5,r}/A_{2,s}$ is known from the take-off calculations and the temperature ratio T_1'/T_5' can be calculated from the assigned work output E and inlet total temperature T_1' by standard thermodynamic procedures. Thus, from equation (8b) the temperature ratio $T_5'/T_{5,r}''$ required at cruise can be determined.

Modifying equation (12) of reference 3 gives,

$$-\left(\frac{W_u}{a_{cr}''}\right)_5 = \frac{1}{2}\sqrt{\frac{T_5'}{T_{5,r}''}}\left[\left(\frac{U}{a_{cr}'}\right)_5 + \left(\frac{a_{cr}'}{U}\right)_5 \frac{\gamma+1}{\gamma-1}\left(\frac{T_{5,r}''}{T_5'} - 1\right)\right] \tag{11}$$

For the assigned value of blade-speed parameter $(U/a_{cr}')_5$, equation (11) specifies the magnitude of relative tangential velocity parameter $(W_u/a_{cr}'')_5$, which is required if the turbine is to produce the required work under cruising conditions.

The maximum obtainable $(W_u/a_{cr}'')_5$ for the $\beta_{5,R}$ calculated at take-off can be determined from figure 3 in reference 5. The blade angle plotted in figure 3 of reference 5 is measured from the tangential direction and is therefore the complement of the blade angle $\beta_{5,R}$ used in this analysis. If the maximum obtainable $(W_u/a_{cr}'')_5$ is greater than the required $(W_u/a_{cr}'')_5$, the turbine is capable of cruising the engine at the blade speed assigned (rated blade speed in this report); if not, the reverse is true.

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TABLE I - INTERBLADE-ROW CONDITIONS AT HUB RADIUS FOR TAKE-OFF AND CRUISE AT RATED ROTATIVE SPEED FOR DOWNSTREAM-STATOR DESIGN

Blade row	Variable	Take-off	Cruise
First stator	Exit absolute Mach number, $(V/a)_2$	1.12	1.04
	Exit absolute flow angle, α_2	69°	69°
First rotor	Entrance relative Mach number, $(W/a)_2$	0.79	0.68
	Exit relative Mach number, $(W/a)_3$	.96	.95
	Entrance relative flow angle, β_2	60°	57°
	Exit relative flow angle, β_3	-63°	-63°
	Turning within first rotor, $\beta_2-\beta_3$	123°	120°
Second stator	Entrance absolute Mach number, $(V/a)_3$	0.67	0.63
	Exit absolute Mach number, $(V/a)_4$	.85	.95
	Entrance absolute flow angle, α_3	-50°	-46°
	Exit absolute flow angle, α_4	62°	62°
	Turning within second stator, $\alpha_4-\alpha_3$	112°	108°
Second rotor	Entrance relative Mach number, $(W/a)_4$	0.59	0.64
	Exit relative Mach number, $(W/a)_5$	.62	.86
	Entrance relative flow angle, β_4	48°	47.0°
	Exit relative flow angle, β_5	-50°	-52°
	Turning within second rotor, $\beta_4-\beta_5$	98°	99°
Downstream stator	Entrance absolute Mach number, $(V/a)_5$	0.44	0.63
	Entrance absolute flow angle, α_5	-26°	-32°

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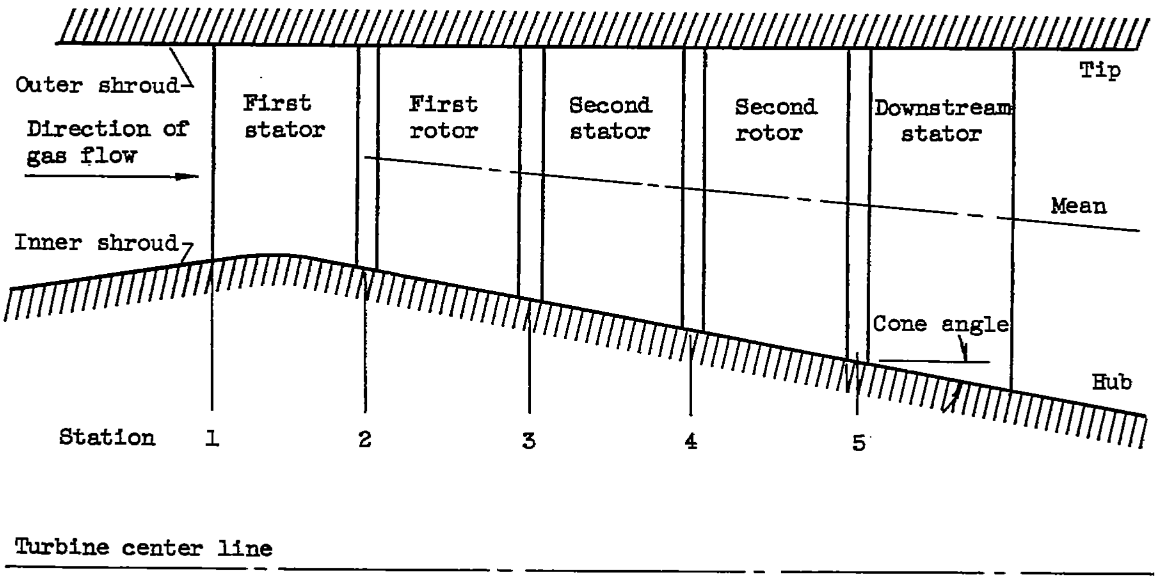
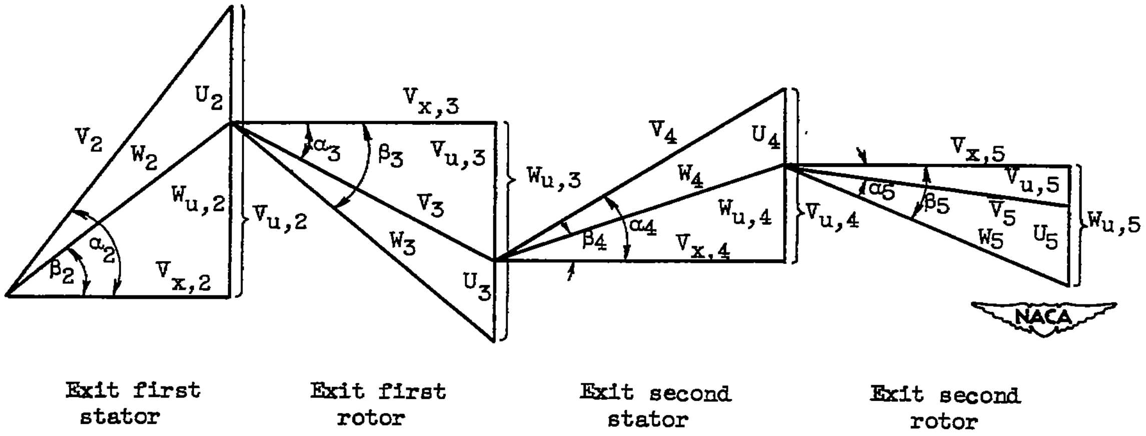
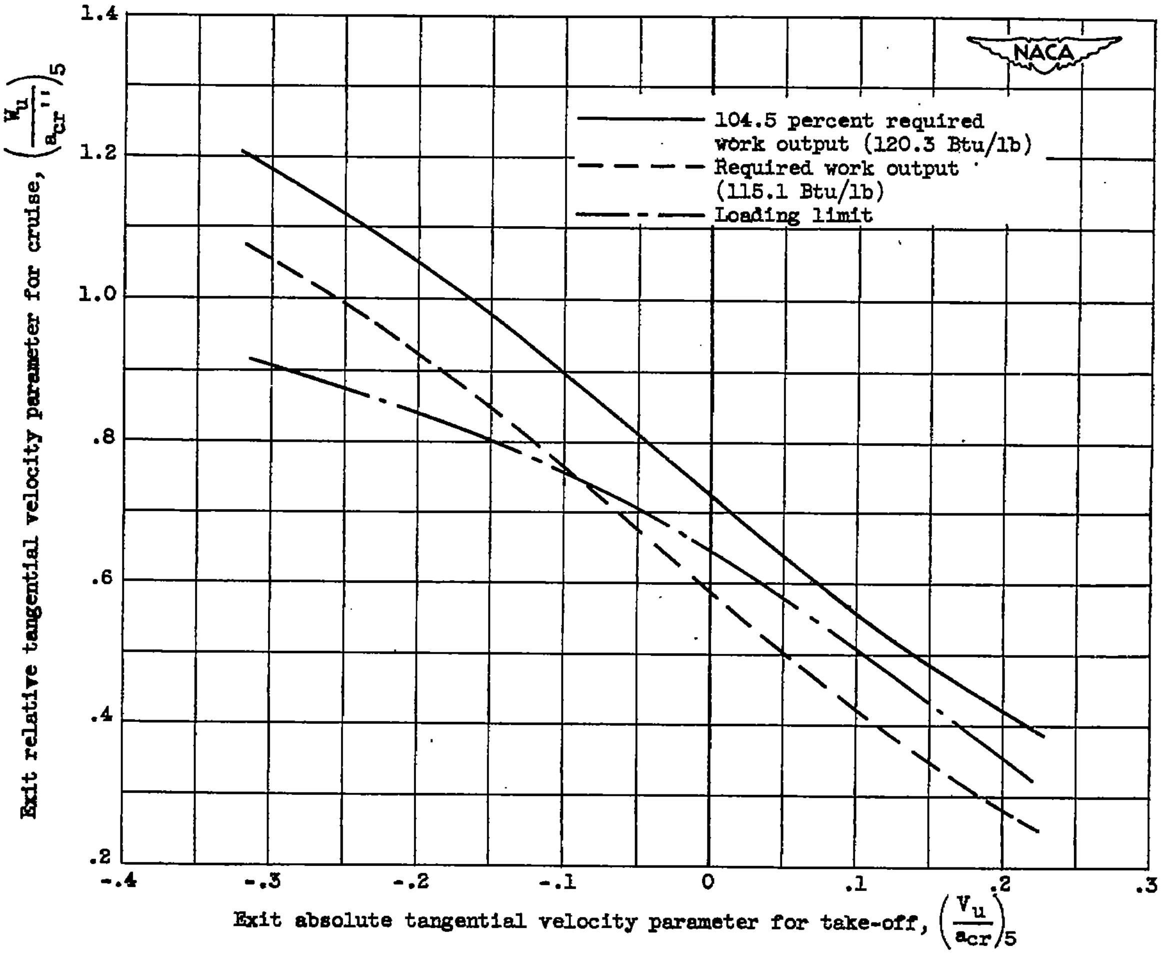


(a) Turbine nomenclature.



(b) Velocity-diagram nomenclature.

Figure 1. - Turbine and velocity-diagram nomenclature. V_u positive when in same direction as U; V_u negative when in direction opposite to U.



(a) Exit annular area of 480 square inches.

Figure 2. - Comparison of required and limiting exit relative tangential velocity parameter $\left(\frac{W_u}{a_{cr}''}\right)_5$ at cruise.

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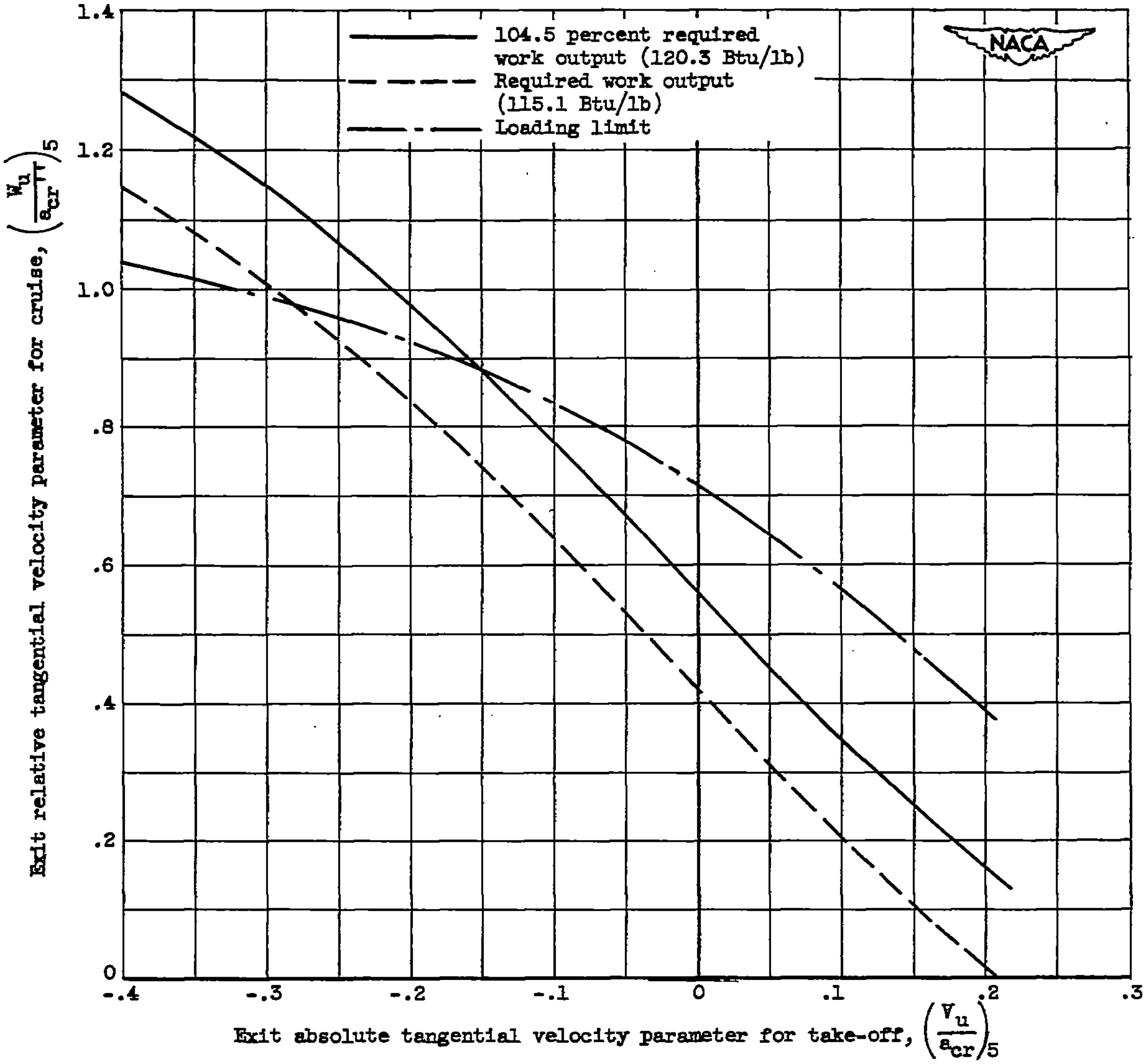

(b) Exit annular area of 550 square inches.

Figure 2. - Concluded. Comparison of required and limiting exit relative tangential velocity parameter $\left(\frac{W_u}{a_{cr}''}\right)_5$ at cruise.

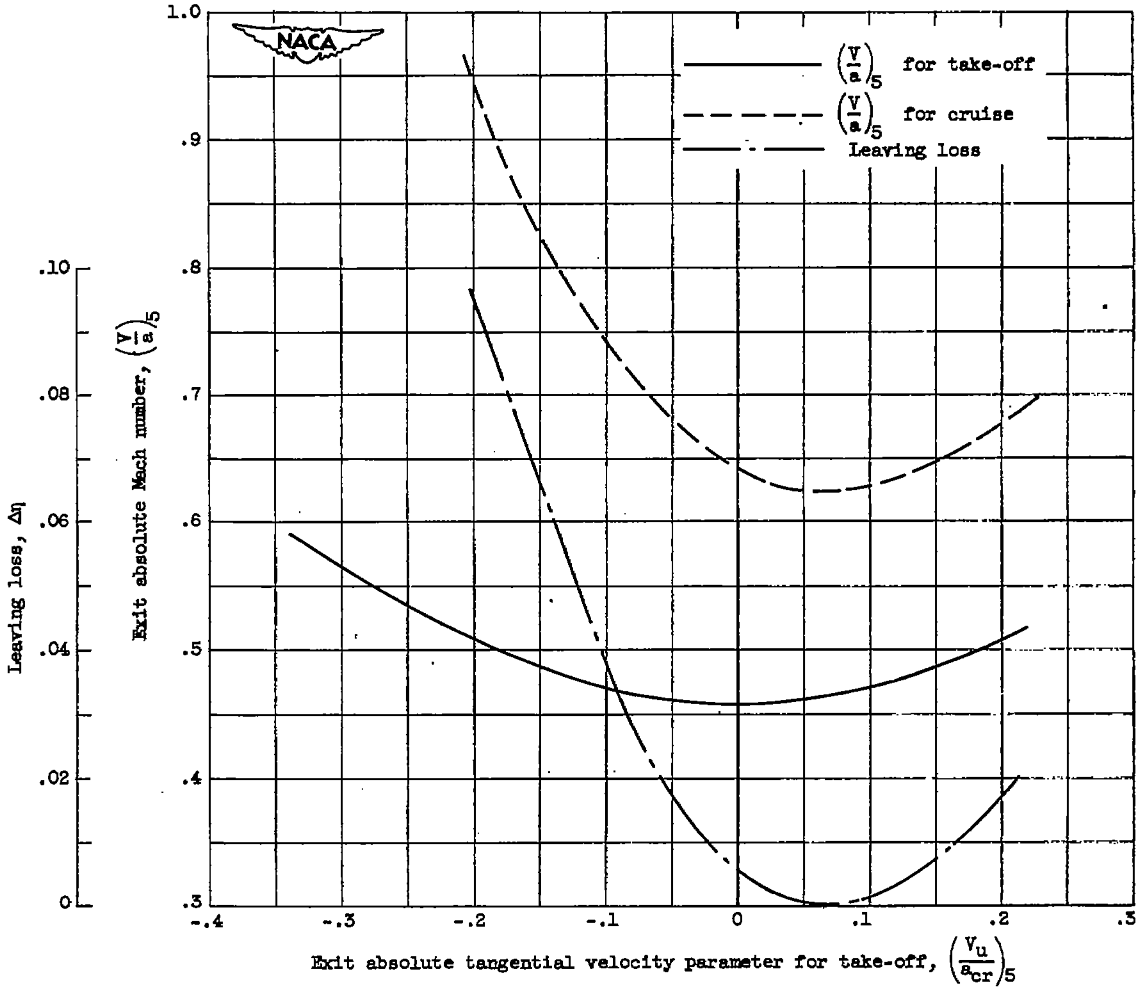

Figure 3. - Change in exit Mach number between take-off and cruise and leaving loss at cruise for exit annular area of 480 square inches.

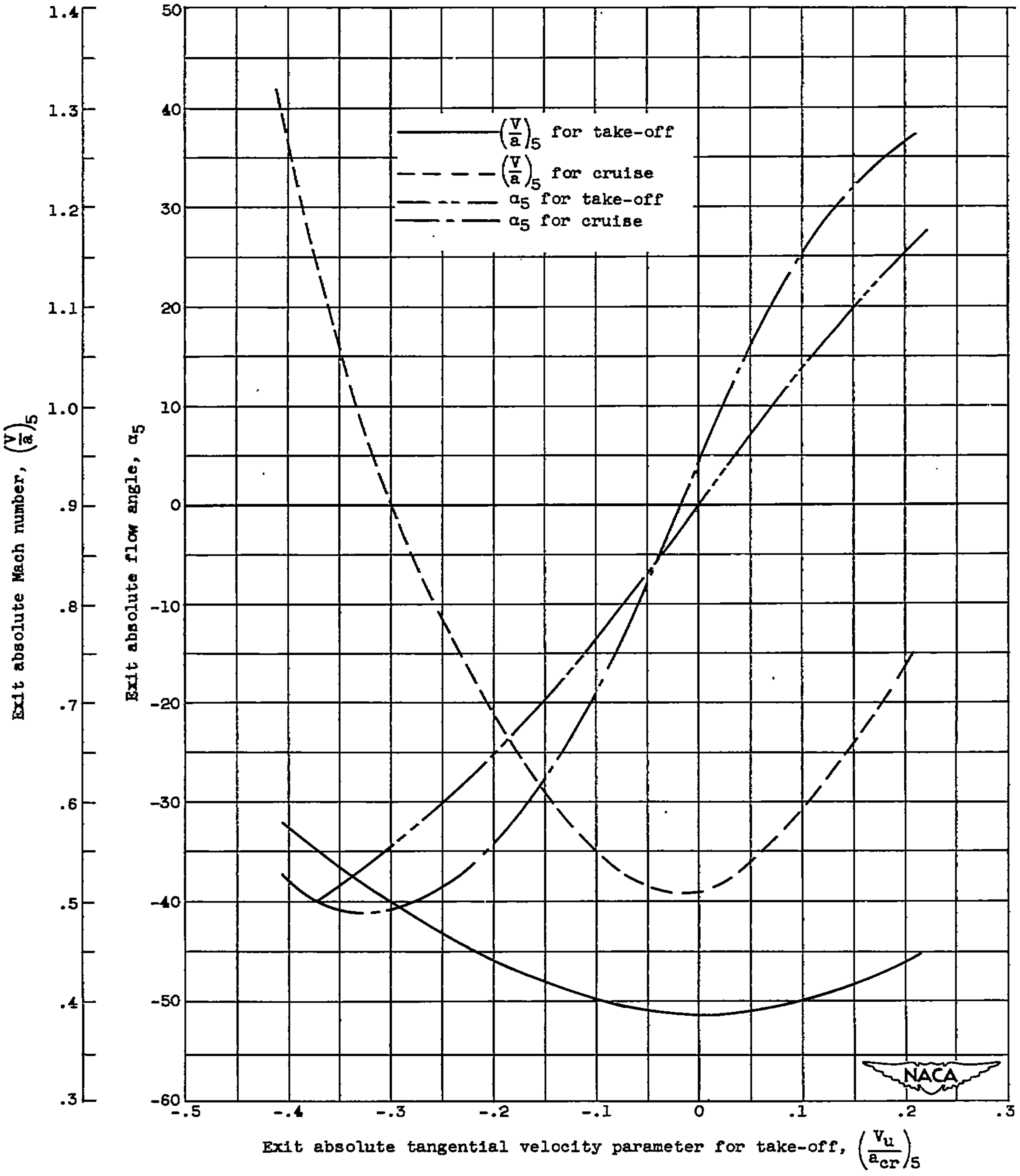

Figure 4. - Change in exit absolute Mach number and flow direction between take-off and cruise for exit annular area of 550 square inches.

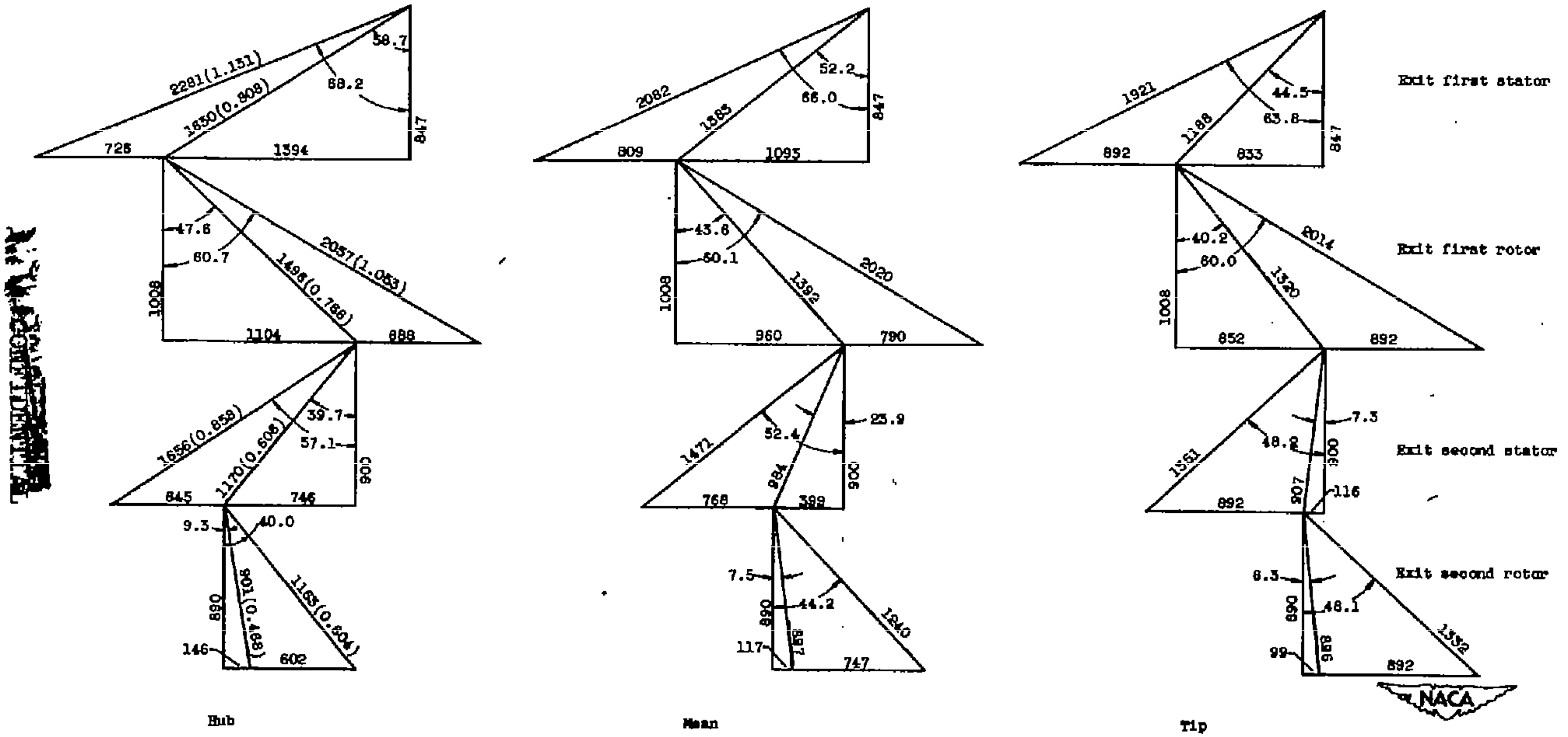

Figure 5. - Take-off velocity diagrams with assumed radial variations in flow for two-stage turbine having 70/30 work division, exit annular area of 480 square inches, and cone angle of 20.6°. Numbers in parentheses are Mach numbers based on local velocity of sound; velocities are in feet per second; angles are in degrees.

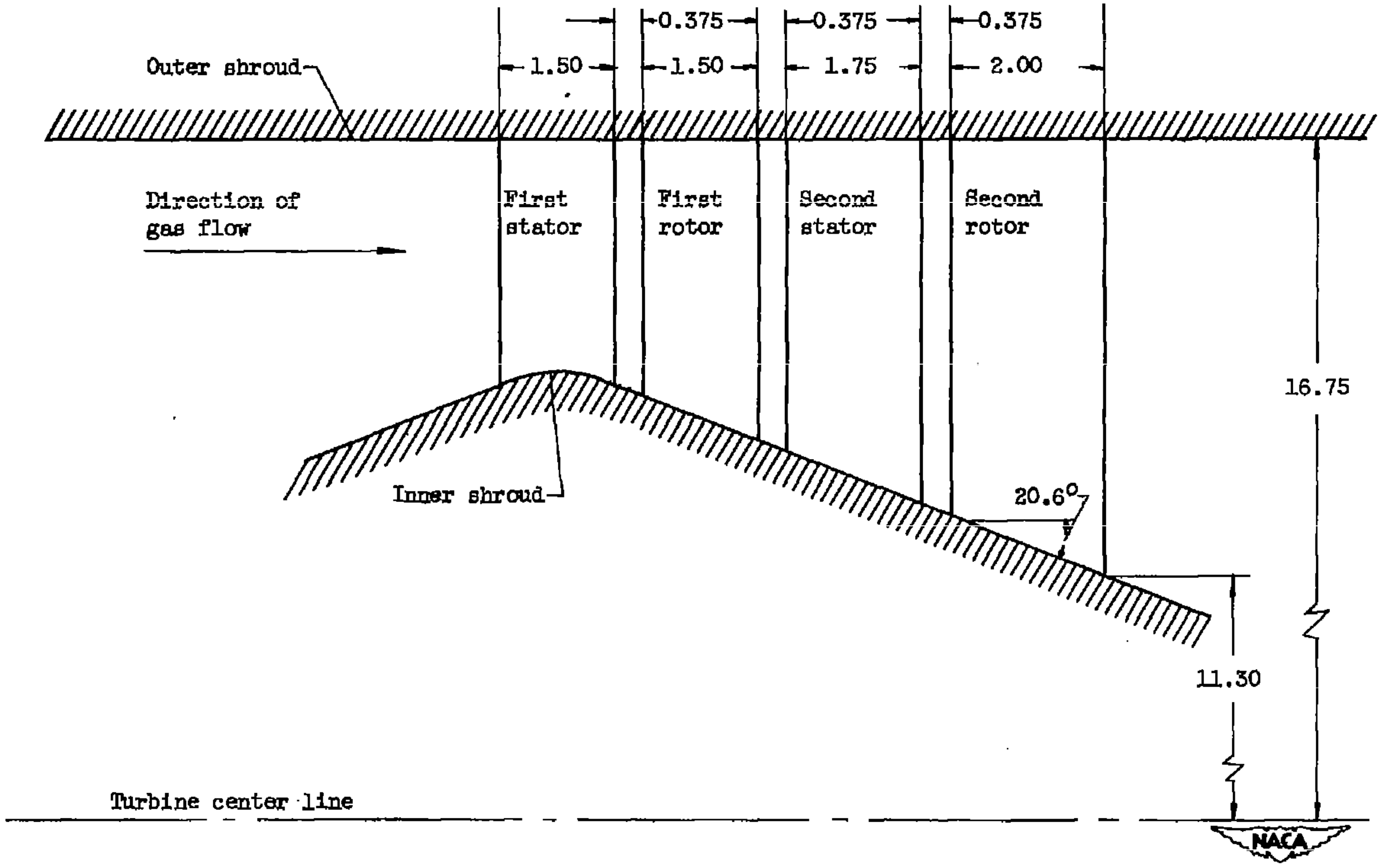

Figure 6. - Two-stage turbine configuration for which velocity diagrams are calculated. Dimensions are in inches.

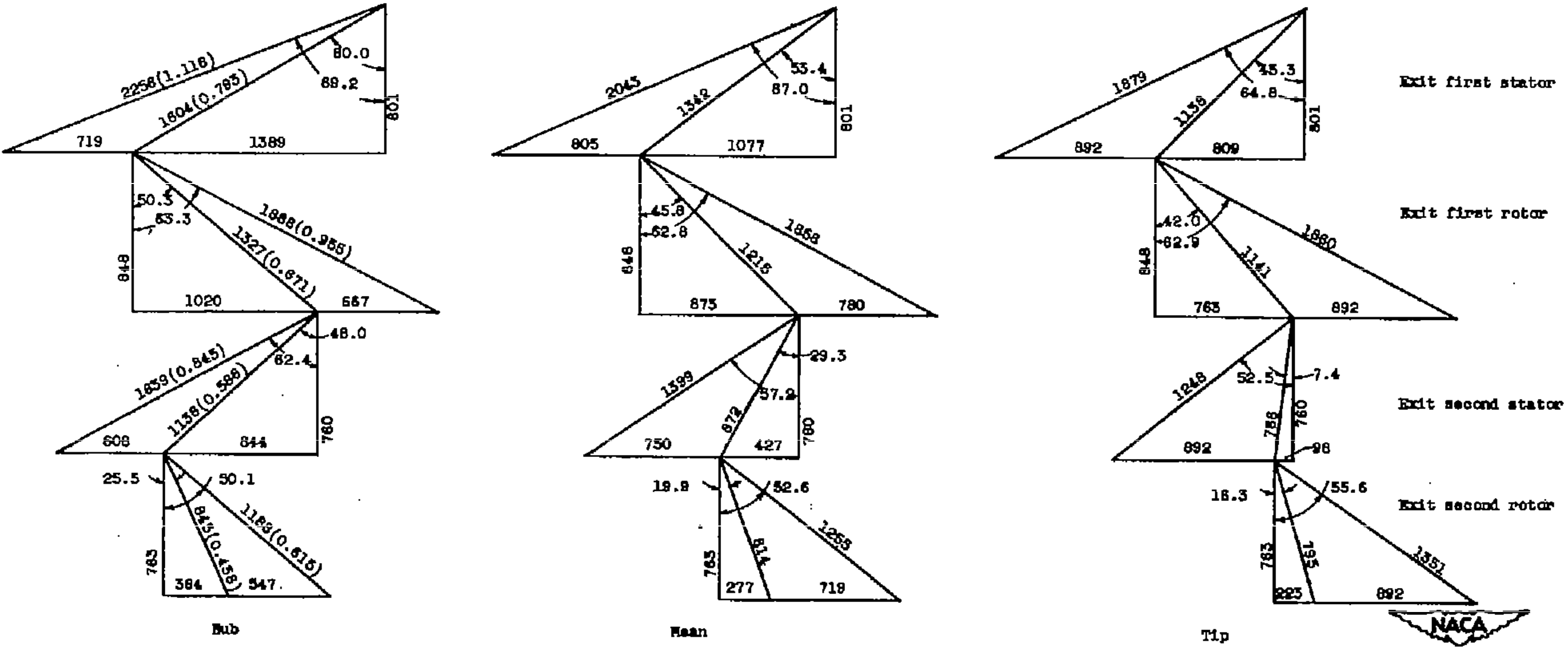

Figure 7. - Take-off velocity diagrams with assumed radial variations in flow for downstream-stator turbine having 67/33 work division, exit annular area of 550 square inches, and cone angle of 27.5°. Numbers in parentheses are Mach numbers based on local velocity of sound; velocities are in feet per second; angles are in degrees.

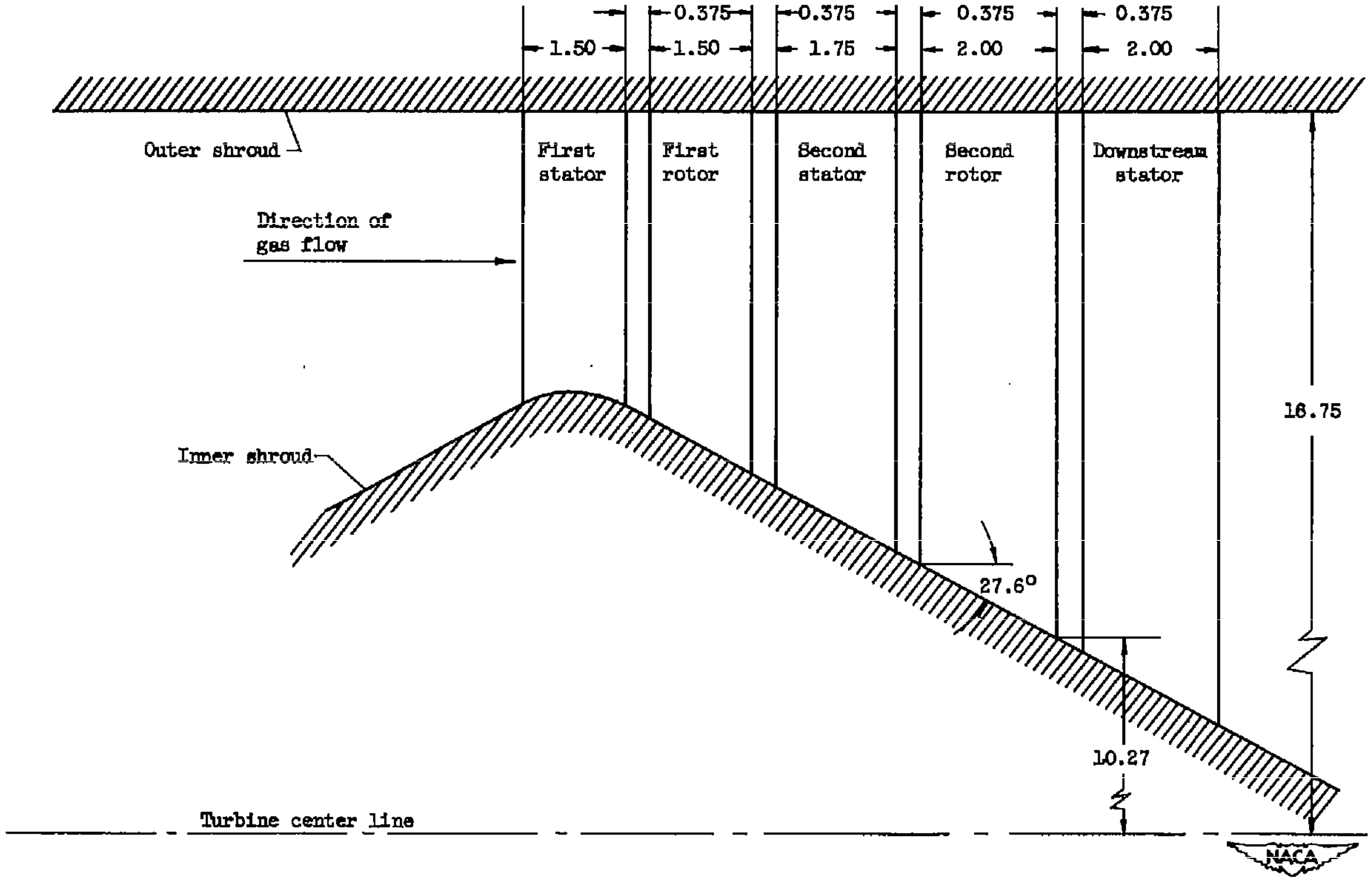

Figure 8. - Turbine configuration with downstream stator for which velocity diagrams are calculated. Dimensions are in inches.

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