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RESEARCH MEMORANDUM

LUMPED REFLECTOR PARAMETERS FOR TWO-GROUP REACTOR CALCULATIONS

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SUMMARY

Equations are developed to show that, for one-dimensional two-group neutron-distribution calculations, the properties of a neutron-reflector region can be combined in three parameters that effectively represent the action of the reflector. The use of these parameters and boundary conditions based on them for single and multizone reflectors is considered and from the results of computations, the following applications of the parameters are shown:

(1) Use of the parameters in connection with reactor core properties for comparison of reflector configurations

(2) Direct use of parameters as circuit constants in electrical analog

(3) Use of boundary conditions based on reflector parameters to obtain numerical solutions of two-group equations in a neutron-producing region.

INTRODUCTION

The close analogy between the physical concepts and equations for the diffusion of neutrons in a reactor and the flow of current in an electrical transmission line suggests the application of already developed electrical engineering techniques for handling problems connected with reactor design. The use of lumped electrical circuit elements to approximate a continuous transmission line has been successfully employed to simulate the diffusion of neutrons by electrical analog simulators (reference 1). In the analogy, the neutron fluxes and the diffusion currents are simulated, respectively, by the electrical potentials and the currents of the transmission line. The following analysis and examples carried out at the NACA Lewis laboratory describe an additional transmission-line concept that can be applied to the one-dimensional solution of the neutron-group equations.

The neutron diffusion, absorption, and moderating properties of the material in a passive reactor zone of finite extent give rise to a particular relation between neutron fluxes and diffusion current at the interface between that medium and an adjacent reactor zone. The diffusion of neutrons into the passive zone can be treated in identical manner to the flow of current into an analogous electrical transmission line connected to an unspecified sending circuit. The transmission line is ordinarily characterized by a definite length, terminating impedance, characteristic impedance, and attenuation constant. In order to specify its reaction on the sending circuit, the line is replaced by an equivalent sending-end impedance that behaves like the line with respect to its reaction on any other electric circuit. The neutron reflector, when described by a similar set of properties, can be treated in the same analytical manner as the transmission line. In the two-group approximation the only processes occurring in a passive reactor zone are the reflection of fast and thermal neutrons and the moderation and reflection of fast neutrons as thermal neutrons. A minimum of three reflector parameters will, therefore, describe the action of the passive zone. The use of transmission-line-like properties gives this set of three parameters and introduces the following advantages and simplifications.

(1) Reflectors or general passive zones with structures based on usual engineering requirements can be directly compared by a consideration of these three parameters for each in connection with the requirements imposed by the reactor core. (A simplified example presented for water and beryllium reflectors demonstrates the comparison of reflector parameters.)

(2) The definition of a neutron-diffusion impedance for a reflector region is immediately applicable to the nuclear-reactor simulator. Here, the reflector network is actually replaced by a single resistor or combined resistor and current source. The simplification leads to more rapid operation of the simulator and permits the use of network sections usually assigned to the reflector to give a more detailed measure of the neutron distribution in the active region of the reactor.

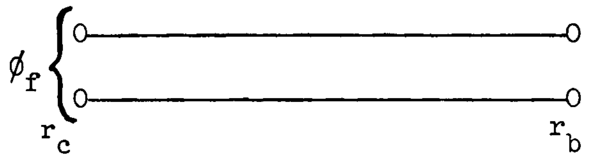
(3) The effective replacement of a reflector by suitable properties applied at the core-reflector interface suggests the use of these properties in numerical calculations to define new boundary conditions applied only to the neutron flux in the reactor core (appendix A). This simplified boundary-condition concept for a simply reflected reactor leads to a set of two boundary equations in place of the usual four equations. The 2×2 criticality determinant derived from the boundary equations is, however, of approximately the same complexity as the ordinary criticality determinant reduced in order to a 2×2 determinant. The simplified boundary conditions are especially useful when applied to a multiregion reflector (appendix B). There is then no increase in the complexity of the criticality determinant.

ANALYSIS

Neutron-diffusion properties. - The definition of characteristic diffusion properties for a neutron-reflector material is based on the neutron-balance equations applied to the reflector region of a nuclear-reactor segment with its center at the origin of the variable r. Consider first the equation for the fast-neutron group:

$$\omega r^m \frac{\lambda_{tr,f}}{3}\left(\frac{d^2\phi_f}{dr^2} + \frac{m}{r}\frac{d\phi_f}{dr}\right) - \omega r^m \Sigma_{a,f}\,\phi_f - \omega r^m \frac{\lambda_{tr,f}}{3} B^2\,\phi_f = 0 \qquad (1)$$

The constant ω determines the size of the reactor segment considered; m is an integer, 1, 2, or 0, respectively, depending on whether the reactor geometry is cylindrical, spherical, or rectangular. The factor ωr^m then serves to make equation (1) a neutron-conservation equation applicable to the entire reflector segment and permits a comparision with the analogous electrical-transmission-line equation with a direct numerical equivalence of terms. The nuclear constants are defined in the list of symbols (appendix C), and the buckling constant B applies only to reactor configurations having finite linear dimensions in other than the r-direction. Equation (1) is similar in form to the equation for the potential on the pictured nonuniform electric transmission line with distributed resistance and leakage.



The line of length (r_b-r_c) has a series resistance per unit length $R_{0,f}/r^m$ and a shunt conductance per unit length $r^m G_{0,f}$. A potential ϕ_f is applied at the sending end of the line at r_c. The terminating impedance on the line is zero or infinite, respectively, for the requirements of $\phi_f(r_b) = 0$ or of $\partial\phi_f(r_b)/\partial r = 0$. These requirements correspond to those for a finite neutron reflector bounded by (1) a nonreflecting region, or (2) a perfect reflector or identical image reflector region and source of neutrons. Equation (1) transforms to the transmission-line equation by the definition of the following quantities:

$$R_{0,f} = \frac{3}{\omega\lambda_{tr,f}} \qquad (2a)$$

$$G_{0,f} = \omega\Sigma_{a,f} + \frac{\omega\lambda_{tr,f}\,B^2}{3} \qquad (2b)$$

The transformed equation (1) is then

$$\frac{r^m}{R_{0,f}} \frac{d^2\phi_f}{dr^2} + \frac{m}{r}\frac{d\phi_f}{dr} - r^m G_{0,f}\,\phi_f = 0 \tag{3a}$$

or its equivalent

$$\frac{d^2\phi_f}{dr^2} + \frac{m}{r}\frac{d\phi_f}{dr} - G_{0,f}\,R_{0,f}\,\phi_f = 0 \tag{3b}$$

Two combinations of the quantities defined in equations (2) appear in equation (3b) and its solution. Therefore let

$$Z_{0,f}^{\;2} = \frac{R_{0,f}}{G_{0,f}} \tag{4a}$$

$$\alpha_f^{\;2} = R_{0,f}\,G_{0,f} \tag{4b}$$

Because of their transmission-line analogs, these quantities are defined as a fast-neutron characteristic impedance $Z_{0,f}$ and a fast-neutron attenuation constant α_f.

The neutron-balance equation for the thermal group corresponding to equation (1) is

$$\omega r^m \frac{\lambda_{tr,th}}{3}\left(\frac{d^2\phi_{th}}{dr^2} + \frac{m}{r}\frac{d\phi_{th}}{dr}\right) - \omega r^m \Sigma_{a,th}\,\phi_{th} - \omega r^m \frac{\lambda_{tr,th}}{3} B^2\,\phi_{th}$$

$$= -\,\omega r^m\,p_{th}\,\Sigma_{af}\,\phi_f \tag{5}$$

This equation also transforms into a transmission-line equation that is equivalent term-by-term with the thermal-group equation.

$$\frac{r^m}{R_{0,th}}\left(\frac{d^2\phi_{th}}{dr^2} + \frac{m}{r}\frac{d\phi_{th}}{dr}\right) - r^m G_{0,th}\,\phi_{th} = -\,r^m g_{th,f}\,\phi_f \tag{6a}$$

or

$$\frac{d^2\phi_{th}}{dr^2} + \frac{m}{r}\frac{d\phi_{th}}{dr} - G_{0,th}\,R_{0,th}\,\phi_{th} = -\,R_{0,th}\,g_{th,f}\,\phi_f \tag{6b}$$

with the substitution of the following quantities:

$$R_{0,th} = \frac{3}{\omega\lambda_{tr,th}} \tag{7a}$$

$$G_{0,th} = \omega\,\Sigma_{a,th} + \frac{\omega\lambda_{tr,th}\ B^2}{3} \tag{7b}$$

$$g_{th,f} = \omega p_{th}\,\Sigma_{a,f} \tag{7c}$$

The thermal-neutron source indicated on the right side of equation (6a) is analogous to a current introduced per unit length in an electrical transmission line. It is therefore helpful to define further nuclear parameters appearing in equations (6) and in their solution that correspond to transmission-line parameters. Let

$$Z_{0,th}{}^2 = \frac{R_{0,th}}{G_{0,th}} \tag{8a}$$

$$\alpha_{th}{}^2 = R_{0,th}\,G_{0,th} \tag{8b}$$

These thermal parameters correspond to the fast-neutron parameters defined in equations (4).

Reflector solutions. - Equations (3b) and (6b) with $m = 0, 1,$ or 2 are to be solved satisfying either of the following two boundary conditions depending on the desired reactor configuration:

(1) The flux vanishes at the outer boundary of the reflector $(\phi(r_b) = 0)$

(2) The gradient of the flux is zero at the origin $(\phi'(0) = 0)$

The general solution of equation (3b) satisfying the appropriate boundary conditions is

$$\phi_f(r) = C_f\,\Phi(\alpha_f r) \tag{9}$$

where C_f is a constant determined from the known flux at the reflector-core interface r_c, and $\Phi(\alpha_f r)$ is a generalized function the specific value of which is given in table I.

The net neutron diffusion current $I_f(r)$ in the r-direction at any point in the reflector is

$$I_f(r) = -\frac{r^m}{R_{0,f}}\frac{d\phi_f(r)}{dr} = -r^m\frac{C_f}{Z_{0,f}}\Phi'(\alpha_f r)$$

where

$$\Phi'(\alpha r) \equiv \frac{d}{d(\alpha r)}\Phi(\alpha r)$$

The ratio $\dfrac{\phi_f(r)}{I_f(r)}$ is determined entirely by reflector properties and can be defined as the fast-neutron impedance of the reflector $Z_f(r)$:

$$Z_f(r) = -\frac{Z_{0,f}}{r^m}\frac{\Phi(\alpha_f r)}{\Phi'(\alpha_f r)} \tag{10}$$

Specific values of $Z_f(r)$ appear in table I.

Then

$$I_f(r_c) = \frac{\phi_f(r_c)}{Z_f(r_c)} \tag{11}$$

defines a new boundary condition at $r = r_c$ for the neutron flux in the adjacent region.

The solution of equation (6b),

$$\frac{d^2\phi_{th}(r)}{dr^2} + \frac{m}{r}\frac{d\phi_{th}(r)}{dr} - \alpha_{th}^2\,\phi_{th}(r) = -R_{0,th}\,g_{th,f}\,C_f\Phi(\alpha_f r)$$

for the thermal neutron flux is

$$\phi_{th}(r) = C_{th}\,\Phi(\alpha_{th} r) - \frac{g_{th,f}\,R_{0,th}\,C_f\Phi(\alpha_f r)}{\alpha_f^2 - \alpha_{th}^2} \tag{12}$$

where $\Phi(\alpha_{th} r)$ is the generalized homogeneous solution of (6b) given in table I for the reflectors considered.

The net thermal diffusion current $I_{th}(r)$ in the r-direction is

$$I_{th}(r) = -\frac{r^m}{R_{0,th}} \frac{d\phi_{th}(r)}{dr}$$

or

$$I_{th}(r) = -r^m \frac{C_{th}}{Z_{0,th}} \Phi'(\alpha_{th}r) + \frac{r^m g_{th,f} C_f \alpha_f}{\alpha_f^2 - \alpha_{th}^2} \Phi'(\alpha_f r) \tag{13}$$

Equation (13) can be put into a more convenient form with the definition of a thermal-neutron impedance $Z_{th}(r)$:

$$Z_{th}(r) = -\frac{Z_{0,th}}{r^m} \frac{\Phi(\alpha_{th}r)}{\Phi'(\alpha_{th}r)} \tag{14}$$

which corresponds to $Z_f(r)$ defined in equation (10).

With the use of this thermal-neutron impedance, equation (13) becomes

$$I_{th}(r) = \frac{\phi_{th}(r)}{Z_{th}(r)} + \frac{R_{0,th}\, g_{th,f}\, C_f}{\alpha_f^2 - \alpha_{th}^2} \left[\frac{\Phi(\alpha_f r)}{Z_{th}(r)} + r^m \frac{\alpha_f}{R_{0,th}} \Phi'(\alpha_f r)\right]$$

or equivalently at the reflector boundary r_c,

$$I_{th}(r_c) = \frac{\phi_{th}(r_c)}{Z_{th}(r_c)} - P_{th,f}(r_c)\, I_f(r_c) \tag{15a}$$

or

$$I_{th}(r_c) = \frac{\phi_{th}(r_c)}{Z_{th}(r_c)} - \frac{P_{th,f}(r_c)}{Z_f(r_c)} \phi_f(r_c) \tag{15b}$$

where

$$P_{th,f}(r_c) = \left[\frac{R_{0,f}}{R_{0,th}} - \frac{Z_f(r_c)}{Z_{th}(r_c)}\right]\left(\frac{g_{th,f}\, R_{0,th}}{\alpha_f^2 - \alpha_{th}^2}\right) \tag{16}$$

Equations (11) and (15) state that the net fast-neutron and thermal-neutron diffusion currents into the reflector are linear functions of the fast- and thermal-neutron fluxes at the reflector boundary.

Equations (11) and (15b) can be used as a new set of boundary conditions for the two-group equations of the core.

Two-group criticality equations. - The fast- and thermal-neutron diffusion currents are given by equations (11) and (15b) for a particular reactor segment. Division of equations (11) and (15b) by ωr^m results in two equations giving the neutron diffusion currents per unit area. These equations applied at the core-reflector interface as boundary conditions to the general solution of the two-group core diffusion equations (appendix A)

$$\phi_f(r) = A\,\Theta_1(\beta_1 r) + D\,\Theta_2(\beta_2 r) \tag{A7}$$

$$\phi_{th}(r) = A\,S_1\,\Theta_1(\beta_1 r) + D\,S_2\,\Theta_2(\beta_2 r) \tag{A8}$$

lead to the criticality determinant

$$\begin{vmatrix} \left[-\frac{\lambda_{tr,f}}{3}\frac{\partial\Theta_1(\beta_1 r)}{\partial r} - \frac{\Theta_1(\beta_1 r)}{\omega r^m Z_f(r)}\right]_{r=r_c} & \left[-\frac{\lambda_{tr,f}}{3}\frac{\partial\Theta_2(\beta_2 r)}{\partial r} - \frac{\Theta_2(\beta_2 r)}{\omega r^m Z_f(r)}\right]_{r=r_c} \\ \left[\begin{matrix}-\frac{\lambda_{tr,th}}{3}S_1\frac{\partial\Theta_1(\beta_1 r)}{\partial r} \\ -\left(\frac{S_1}{\omega r^m Z_{th}(r)} - \frac{P_{th}(r)}{\omega r^m Z_f(r)}\right)\Theta_1(\beta_1 r)\end{matrix}\right]_{r=r_c} & \left[\begin{matrix}-\frac{\lambda_{tr,th}}{3}S_2\frac{\partial\Theta_2(\beta_2 r)}{\partial r} \\ -\left(\frac{S_2}{\omega r^m Z_{th}(r)} - \frac{P_{th}(r)}{\omega r^m Z_f(r)}\right)\Theta_2(\beta_2 r)\end{matrix}\right]_{r=r_c} \end{vmatrix} = 0 \tag{17}$$

The solution of this determinant can be obtained by varying any of several variables: (1) the reflector thickness in $Z(r)$, (2) the radius of the core, or (3) the fissionable-material concentration in S and the β.

The ratio of the parameters in equations (A7) and (A8) is

$$\frac{D}{A} = -\frac{\left[\frac{\lambda_{tr,f}}{3}\frac{\partial\Theta_1(\beta_1 r)}{\partial r} + \frac{\Theta_1(\beta_1 r)}{\omega r^m Z_f(r)}\right]}{\left[\frac{\lambda_{tr,f}}{3}\frac{\partial\Theta_2(\beta_2 r)}{\partial r} + \frac{\Theta_2(\beta_2 r)}{\omega r^m Z_f(r)}\right]} \tag{18}$$

This ratio leads to knowledge of the spatial variation of $\phi_f(r)$ and $\phi_{th}(r)$ in the core.

APPLICATION OF REFLECTOR PARAMETERS

Comparison of reflector properties. - A group of reflectors satisfactory from an engineering and nuclear standpoint can be further compared by a knowledge of $P_{th,f}$, Z_f, and Z_{th}, which give a description of the over-all behavior of the reflector. These quantities are particularly useful for a comparison of reflector properties in the case of a multireflector reactor where the problem is difficult to solve by ordinary two-group methods.

The following properties of $Z_f(r_b-r_c)$, $Z_{th}(r_b-r_c)$, and $P_{th,f}(r_b-r_c)$ facilitate a choice of reflector for a particular core structure.

(1) A low $Z_f(r_b-r_c)$ will depress the fast flux in the core leading to a large neutron current into the reflector region, whereas a high $Z_f(r_b-r_c)$ tends to confine the neutrons to the core where they will slow down.

(2) A large $Z_{th}(r_b-r_c)$ tends to prevent the diffusion, and hence the loss, of thermal neutrons from the core.

(3) The quantity $P_{th,f}(r_b-r_c)$ is a measure of the moderating power of a reflector, a large value of $P_{th,f}(r_b-r_c)$ indicating considerable slowing down, resulting in more thermal neutrons returning to the core.

A demonstration of these properties for simple water and beryllium reflectors of various thicknesses is given in figures 1 and 2; spherical geometry and a core radius of 30 centimeters are assumed. A water reflector thickness of 10 centimeters gives values of Z_f and Z_{th} 95.1 and 99 percent of the respective saturation values while the corresponding values for 20 centimeters of beryllium are 98.1 percent and 89.2 percent. The saturation value of $P_{th,f}$ for water, 0.286, means that not more than 28.6 percent of the fast neutrons can slow down to return to the core. The graphs therefore indicate that little is gained by choosing a water reflector thickness of greater than 10 centimeters or a beryllium thickness of greater than 20 centimeters for this particular reactor configuration. Although the beryllium reflector has a lower value of Z_{th} than the water reflector, the beryllium reflector would be more desirable on the basis of its higher value of Z_f and $P_{th,f}$ providing reflector thickness is no problem.

Simulator operation using reflector parameters. - A typical reactor criticality determination was performed on the NACA nuclear reactor simulator for a simply reflected cylindrical reactor in order to establish a procedure for the direct use of reflector parameters. For the fast-neutron group, the reflector was replaced by a fast-neutron resistance equal to $Z_f(r_c)$ that terminated the electric network for the active zone. Measurements made on the network then included that of the current through this terminating resistance. The thermal-neutron network for the reactor core was then terminated by a resistance equal to $Z_{th}(r_c)$, the thermal-neutron diffusion impedance. In addition, a current $P_{th,f}$ multiplied by that measured through the reflector fast-neutron resistance was introduced at the terminal point of the core network.

The over-all reflector parameters give the following advantages in simulator operation:

(1) The network sections ordinarily assigned to the reflector can be used to give more detailed information of the neutron distribution in the reactor core.

(2) The reduction in time required to set one resistor value and one current value in contrast to a large number of adjustments required for a reflector network greatly simplified the operation of the simulator. This feature is especially useful for reactor calculations involving the adjustment of core parameters by iterative processes.

(3) Various reflectors can be employed in calculations by the simple readjustment of one resistor value for each neutron group.

(4) If a detailed variation in flux for a multizone reflector is required, the analytical expression with boundary values measured on the simulator gives more complete information than can be found with most practical reflector networks.

Numerical calculation example. - In order to determine computational time, a numerical calculation based on the criticality determinant (equation (17)) was made for a spherical reactor having a single reflector and compared with the computational time involved in solving the two-group core-reflector equations.

Equation (17) has in it several possible variables: (1) the radius of the core r_c in the parameters Z_f, Z_{th}, $P_{th,f}$, Θ; (2) the reflector structure in the parameters Z_f, Z_{th}, $P_{th,f}$; and (3) the fissionable material concentration in the parameters S, β, and Θ.

Any of these quantities, for example, the fissionable material concentration, may be used as a variable; however, r_c was made to vary as with the usual computational method in order to obtain a direct comparison. Because of the dependence of Z and $P_{th,f}$ on r_c, the 2×2 determinant is approximately equal in complexity to the reduced form of the 4×4 determinant and it was found that there is no loss or gain in computational ease or time between the two iterative methods.

For a single core having internal and external passive zones, the concept of reflector impedances and a $P_{th,f}(r)$ leads to a 4×4 determinant; and there is no increase in the order of the determinant due to the addition of more passive zones. The conventional solution to the same problem requires an 8×8 determinant with 4 additional rows and columns for each additional passive zone. Thus, a considerable saving in computational effort and time should result from the use of the reflector impedance method over the conventional method of solution of this particular problem.

CONCLUSIONS

A neutron-reflector region can be effectively represented by only three parameters for two-group calculations in one dimension. For the fast-neutron group a single quantity, the fast-neutron impedance, is necessary, and for the thermal group, a thermal-neutron impedance and current parameter are required. The simplified boundary conditions dependent on these parameters have the following advantages:

1. Consideration of the three reflector parameters in connection with a proposed core structure permits a direct comparison of the effectiveness of reflectors that is especially useful for multizone reflectors.

2. Reflector parameters are applicable as actual circuit elements on an electrical network simulator for the group diffusion model.

3. Boundary conditions based on the reflector parameters can be applied to flux calculations in neutron-producing regions without further consideration of the neutron flux in the reflector regions.

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APPENDIX A

DERIVATION OF CRITICALITY EQUATIONS FOR REACTOR-PLUS-REFLECTOR CONFIGURATION BASED ON TWO-GROUP THEORY AND TRANSMISSION-LINE THEORY

The results of the work on the finite transmission line can be applied to the two-group equations for a reactor-plus-reflector configuration. Essentially, the reflector equations and the four core-reflector interface boundary conditions are replaced by two boundary conditions; this, in effect, reduces the ordinary 4×4 determinant solution of the diffusion equations to a 2×2 determinant solution.

The one-dimensional two-group equations for a reactor core are (reference 2)

$$\frac{\lambda_{tr,f}}{3} \nabla_r^2 \phi_f(r) - \frac{\lambda_{tr,f}}{3} B^2 \phi_f(r) - \Sigma_{a,f} \phi_f(r) + k_f(1-p_{th}) \Sigma_{a,f} \phi_f(r) + k_{th} \Sigma_{a,th} \phi_{th}(r) = 0 \quad \text{(A1)}$$

and

$$\frac{\lambda_{tr,th}}{3} \nabla_r^2 \phi_{th}(r) - \frac{\lambda_{tr,th}}{3} B^2 \phi_{th}(r) - \Sigma_{a,th} \phi_{th}(r) + p_{th} \Sigma_{a,f} \phi_f(r) = 0 \quad \text{(A2)}$$

where B^2 represents the buckling constant for directions normal to r; it applies only to the solutions involving rectangular and cylindrical geometry and does not apply to a spherical geometry.

The two boundary conditions replacing the two reflector equations and the four core-reflector interface boundary conditions are

$$-\frac{\lambda_{tr,f}}{3} \frac{d\phi_f(r)}{dr} = \frac{\phi_f(r)}{\omega r^m Z_f(r)} \quad \text{(A3)}$$

and

$$-\frac{\lambda_{tr,th}}{3} \frac{d\phi_{th}(r)}{dr} = \frac{\phi_{th}(r)}{\omega r^m Z_{th}(r)} - \frac{P_{th,f}(r)}{\omega r^m Z_f} \phi_f(r) \quad \text{(A4)}$$

Equations (A1) and (A2) must satisfy the following relation:

$$\nabla_r^2 \phi_f(r) = \beta^2 \phi_f(r) \tag{A5}$$

$$\nabla_r^2 \phi_{th}(r) = \beta^2 \phi_{th}(r) \tag{A6}$$

Substitution of equations (A6) and (A5) in equations (A1) and (A2) will give two equations in β^2, the buckling constant (reference 2). This constant β^2 may have either of two values: one value positive $\left[\beta_1^2\right]$ and one value negative $\left[\beta_2^2\right]$.

The general solution of equations (A5) and (A6) will be given by

$$\phi_f(r) = A\,\Theta_1(\beta_1 r) + D\,\Theta_2(\beta_2 r) \tag{A7}$$

$$\phi_{th}(r) = A\,S_1\Theta_1(\beta_1 r) + D\,S_2\,\Theta_2(\beta_2 r) \tag{A8}$$

where $\Theta_1(\beta_1 r)$ and $\Theta_2(\beta_2 r)$ represent the value of the solution for $\phi_f(r)$ and $\phi_{th}(r)$ corresponding to the two values β_1^2 and β_2^2, respectively. The constants S_1 and S_2 represent the ratios of Θ_{th}/Θ_f, the components of the fluxes for the values of β_1^2 and β_2^2, respectively. The values of S_1 and S_2 are

$$S_1 = -\frac{\lambda_{tr,f}}{3k_{th}\,\Sigma_{a,th}}\left(\beta_1^2 - B^2\right) + \frac{\Sigma_{a,f}}{k_{th}\,\Sigma_{a,th}}\left[1 - k_f(1 - p_{th})\right] \tag{A9}$$

$$S_2 = -\frac{\lambda_{tr,f}}{3k_{th}\,\Sigma_{a,th}}\left(\beta_2^2 - B^2\right) + \frac{\Sigma_{a,f}}{k_{th}\,\Sigma_{a,th}}\left[1 - k_f(1 - p_{th})\right] \tag{A10}$$

The values for $\Theta_1(\beta_1 r)$ and $\Theta_2(\beta_2 r)$ are listed in the following table for $m = 0, 1, 2$:

	∇_r^2	$\Theta_1(\beta_1 r);\ \beta_1^2 > 0$	$\Theta_2(\beta_2 r);\ \beta_2^2 < 0$
m = 0	$\frac{d^2}{dr^2}$	$\cosh\ (\beta_1 r)$	$\cos\ (\beta_2 r)$
m = 1	$\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}$	$I_0\ (\beta_1 r)$	$J_0\ (\beta_2 r)$
m = 2	$\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}$	$\sinh\ (\beta_1 r)/r$	$\sin\ (\beta_2 r)/r$

Substitution of the boundary conditions ((A3), (A4)) into equations (A7) and (A8) at $r = r_c$ will give

$$-\frac{\lambda_{tr,f}}{3}\left[A\frac{\partial\ \Theta_1(\beta_1 r)}{\partial r} + D\frac{\partial\ \Theta_2(\beta_2 r)}{\partial r}\right] = \frac{1}{\omega r^m Z_f(r)}\left[A\ \Theta_1(\beta_1 r) + D\ \Theta_2(\beta_2 r)\right] \tag{A11}$$

$$-\frac{\lambda_{tr,th}}{3}\left[A\ S_1\frac{\partial\ \Theta_1(\beta_1 r)}{\partial r} + D\ S_2\frac{\partial\ \Theta_2(\beta_2 r)}{\partial r}\right] = \frac{1}{\omega r^m Z_{th}(r)}\left[A\ S_1\ \Theta_1(\beta_1 r)\right.$$
$$\left. + D\ S_2\ \Theta_2(\beta_2 r)\right] - \frac{P_{th,f}(r)}{\omega r^m Z_f(r)}\left[A\ \Theta_1(\beta_1 r) + D\ \Theta_2(\beta_2 r)\right] \tag{A12}$$

Simplification of equation (A11) and (A12) results in two equations in the parameters A and D:

$$\left[-\frac{\lambda_{tr,f}}{3}\frac{\partial\ \Theta_1(\beta_1 r)}{\partial r} - \frac{\Theta_1(\beta_1 r)}{\omega r^m Z_f(r)}\right]A + \left[-\frac{\lambda_{tr,f}}{3}\frac{\partial\ \Theta_2(\beta_2 r)}{\partial r} - \frac{\Theta_2(\beta_2 r)}{\omega r^m Z_f(r)}\right]D = 0 \tag{A13}$$

$$\left[-\frac{\lambda_{tr,th}}{3} S_1 \frac{\partial\, \Theta_1(\beta_1 r)}{\partial r} - \left(\frac{S_1}{\omega r^m Z_{th}(r)} - \frac{P_{th,f}(r)}{\omega r^m Z_f(r)}\right)\Theta_1(\beta_1 r)\right] A$$

$$+ \left[-\frac{\lambda_{tr,th}}{3} S_2 \frac{\partial\, \Theta_2(\beta_2 r)}{\partial r} - \left(\frac{S_2}{\omega r^m Z_{th}(r)} - \frac{P_{th,f}(r)}{\omega r^m Z_f(r)}\right)\Theta_2(\beta_2 r)\right] D = 0 \tag{A14}$$

There will be a nontrivial solution to equations (A13) and (A14) only if the determinant of the coefficients vanishes. Thus, the general criticality equation is

$$\left[-\frac{\lambda_{tr,f}}{3}\frac{\partial\, \Theta_1(\beta_1 r)}{\partial r} - \frac{\Theta_1(\beta_1 r)}{\omega r^m Z_f(r)}\right]\left[-\frac{\lambda_{tr,th}}{3} S_2 \frac{\partial\, \Theta_2(\beta_2 r)}{\partial r} - \left(\frac{S_2}{\omega r^m Z_{th}(r)} - \frac{P_{th,f}(r)}{\omega r^m Z_f(r)}\right)\Theta_2(\beta_2 r)\right]$$

$$+\left[\frac{\lambda_{tr,f}}{3}\frac{\partial\, \Theta_2(\beta_2 r)}{\partial r} + \frac{\Theta_2(\beta_2 r)}{\omega r^m Z_f(r)}\right]\left[-\frac{\lambda_{tr,th}}{3} S_1 \frac{\partial\, \Theta_1(\beta_1 r)}{\partial r} - \left(\frac{S_1}{\omega r^m Z_{th}(r)} - \frac{P_{th,f}(r)}{\omega r^m Z_f(r)}\right)\Theta_1(\beta_1 r)\right] = 0 \tag{A15}$$

Depending upon the type of problem under consideration, the reflector thickness, the radius of the core r_c, or the fissionable material concentration may be varied to achieve criticality.

The ratio of the two constants in equations (A7) and (A8) is

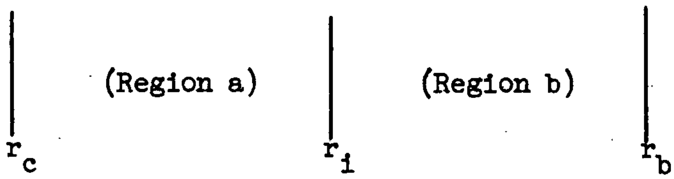
$$\frac{D}{A} = -\frac{\left[\frac{\lambda_{tr,f}}{3}\frac{\partial\, \Theta_1(\beta_1 r)}{\partial r} + \frac{\Theta_1(\beta_1 r)}{\omega r^m Z_f(r)}\right]}{\left[\frac{\lambda_{tr,f}}{3}\frac{\partial\, \Theta_2(\beta_2 r)}{\partial r} + \frac{\Theta_2(\beta_2 r)}{\omega r^m Z_f(r)}\right]} \tag{A16}$$

If the ratio D/A is known, the spatial variation of flux in the core may be plotted as a function of the distance r. The value of the flux in the core will then be known to an arbitrary constant. With the values of the fast- and thermal-neutron fluxes known at the core-reflector interface, the reflector fluxes may be calculated by using equations (9) and (12) with the constants adjusted to give the proper flux values at the core-reflector interface.

APPENDIX B

TWO-GROUP PARAMETERS FOR MULTIPLE PASSIVE ZONES

The reflector consisting of two regions as shown schematically in the following figure can be represented by three parameters similar to those for a simple one-region reflector:



Region b, extending from r_i to the boundary r_b, is immediately replaced by a set of parameters as described in the body of the report or, if it is a multizone region, by a set previously determined by the following process. These parameters establish all necessary boundary conditions for the region-a neutron flux solutions $(r_c \leq r \leq r_i)$ and lead to reflector parameters that describe the effect of the entire reflector combination.

Fast-group parameters. - The general one-dimensional solution for the fast-neutron flux in region a has the form

$$\phi_f = A_f\left[\psi_1(\alpha_f r) + D_f\ \psi_2(\alpha_f r)\right] \tag{B1}$$

where A_f and D_f are constants to be determined by the applied boundary conditions, and the functions $\psi_1(\alpha_f r)$ and $\psi_2(\alpha_f r)$ are listed in the accompanying table for three coordinate systems:

Coordinate system	$\psi_1(\alpha r)$	$\psi_2(\alpha r)$
Rectangular	$\sinh\ (\alpha r)$	$\cosh\ (\alpha r)$
Cylindrical	$K_0\ (\alpha r)$	$I_0\ (\alpha r)$
Spherical	$\dfrac{\sinh\ (\alpha r)}{r}$	$\dfrac{\cosh\ (\alpha r)}{r}$

At r_i, the boundary of region a, the neutron current is

$$I_f(r_i) = -\frac{r_i^{\ m}}{R_{0,f}}\frac{d\phi_f}{dr}\bigg|_{r_i} = -\frac{r_i^{\ m}\ A_f}{Z_{0,f}}\left[\psi_1'(\alpha_f r_i) + D_f\ \psi_2'(\alpha_f r_i)\right] \tag{B2}$$

This current is related to the flux at the boundary r_i by the impedance $Z_{f,b}$

$$Z_{f,b} = \frac{\phi_f(r_i)}{I_f(r_i)} = -\frac{Z_{0,f}}{r_i^m}\left[\frac{\psi_1(\alpha_f r_i) + D_f\ \psi_2(\alpha_f r_i)}{\psi_1'(\alpha_f r_i) + D_f\ \psi_2'(\alpha_f r_i)}\right] \tag{B3}$$

Boundary condition (B3) permits evaluation of the constant D_f

$$D_f = -\left[\frac{Z_{0,f}\ \psi_1(\alpha_f r_i) + r_i^m\ Z_{f,b}\ \psi_1'(\alpha_f r_i)}{Z_{0,f}\ \psi_2(\alpha_f r_i) + r_i^m\ Z_{f,b}\ \psi_2'(\alpha_f r_i)}\right] \tag{B4}$$

The fast-group flux and current solutions obtained with D_f permit the definition of a fast-neutron impedance for the reflector combination according to

$$Z_{f,c} = \frac{\phi_f(r_c)}{I_f(r_c)}$$

The value of this impedance is

$$Z_{f,c} = -\frac{Z_{0,f}}{r_c^m}\left[\frac{\psi_1(\alpha_f r_c) + D_f\ \psi_2(\alpha_f r_c)}{\psi_1'(\alpha_f r_c) + D_f\ \psi_2'(\alpha_f r_c)}\right] \tag{B5}$$

<u>Thermal-group parameters</u>. - The thermal-group equation for region a is

$$\nabla^2\ \phi_{th} - \alpha_{th}^2\ \phi_{th} = -R_{0,th}\ g_{th,f}\ A_f\left[\psi_1(\alpha_f r) + D_f\ \psi_2(\alpha_f r)\right] \tag{B6}$$

The general solution of equation (B6) is separated into two parts consisting of a complementary function

$$\phi_{th,h} = C_{th}\left[\psi_1(\alpha_{th} r) + \delta_{th}\ \psi_2(\alpha_{th} r)\right] \tag{B7}$$

and a part made up of the particular integral and an arbitrary multiple of the complementary function

$$\phi_{th,p} = A_{th}\left[\psi_1(\alpha_{th} r) + D_{th}\ \psi_2(\alpha_{th} r)\right] - \frac{R_{0,th}\ g_{th,f}\ A_f}{\alpha_f^2 - \alpha_{th}^2}\left[\psi_1(\alpha_f r) + D_f\ \psi_2(\alpha_f r)\right] \tag{B8}$$

where C_{th}, δ_{th}, A_{th}, and D_{th} are constants. This separation of the solution leads to equations that simplify the definition of parameters for the double reflector.

The boundary conditions for the two parts of the thermal-group solution are

$$\phi_{th,h}(r_c) = \phi_{th}(r_c) \tag{B9a}$$

$$I_{th,h}(r_i) = -\frac{r_i^m}{R_{0,th}} \frac{d\phi_{th,h}}{dr}\bigg|_{r_i} = \frac{\phi_{th,h}(r_i)}{Z_{th,b}} \tag{B9b}$$

$$\phi_{th,p}(r_c) = 0 \tag{B10a}$$

$$I_{th,p}(r_i) = -\frac{r_i^m}{R_{0,th}} \frac{d\phi_{th,p}}{dr}\bigg|_{r_i} = \frac{\phi_{th,p}(r_i)}{Z_{th,b}} - P_{th,f,b}\, I_f(r_i) \tag{B10b}$$

Evaluation of boundary condition (B9b) gives

$$Z_{th,b} = -\frac{Z_{0,th}}{r_i^m}\left[\frac{\psi_1(\alpha_{th}r_i) + \delta_{th}\,\psi_2(\alpha_{th}r_i)}{\psi_1'(\alpha_{th}r_i) + \delta_{th}\,\psi_2'(\alpha_{th}r_i)}\right]$$

from which the constant δ_{th} is evaluated

$$\delta_{th} = -\left[\frac{Z_{0,th}\,\psi_1(\alpha_{th}r_i) + r_i^m\, Z_{th,b}\,\psi_1'(\alpha_{th}r_i)}{Z_{0,th}\,\psi_2(\alpha_{th}r_i) + r_i^m\, Z_{th,b}\,\psi_2'(\alpha_{th}r_i)}\right] \tag{B11}$$

The thermal-neutron diffusion impedance for the combined reflector is defined in terms of the homogeneous part of the flux solution in region a. The current of thermal neutrons into the reflector

$$I_{th}(r_c) = I_{th,h}(r_c) + I_{th,p}(r_c)$$

is written

$$I_{th}(r_c) = \frac{\phi_{th}(r_c)}{Z_{th,c}} + I_{th,p}(r_c) \tag{B12}$$

The homogeneous part of equation (B12) is then solved for $Z_{th,c}$ with condition (B9a)

$$Z_{th,c} = \frac{\phi_{th}(r_c)}{I_{th,h}(r_c)} = -\frac{Z_{0,th}}{r_c^{m}}\left[\frac{\psi_1(\alpha_{th}r_c) + \delta_{th}\ \psi_2(\alpha_{th}r_c)}{\psi_1'(\alpha_{th}r_c) + \delta_{th}\ \psi_2'\ (\alpha_{th}r_c)}\right] \tag{B13}$$

The undetermined constants in the particular flux solution (B8) are fixed by the boundary conditions (B10). Equation (B10a) gives

$$A_{th} = \frac{R_{0,th}\ g_{th,f}\ \phi_f(r_c)}{(\alpha_f^2-\alpha_{th}^2)\left[\psi_1(\alpha_{th}r_c) + D_{th}\ \psi_2(\alpha_{th}r_c)\right]} \tag{B14}$$

This value of A_{th} permits the evaluation of boundary condition (B10b), giving

$$-h\ \phi_f(r_c)\left[\frac{\psi_1'(\alpha_{th}r_i) + D_{th}\ \psi_2'(\alpha_{th}r_i)}{\psi_1(\alpha_{th}r_c) + D_{th}\ \psi_2(\alpha_{th}r_c)}\right] + n\ \phi_c(r_c)$$

$$= l\ \phi_f(r_c)\left[\frac{\psi_1(\alpha_{th}r_i) + D_{th}\ \psi_2(\alpha_{th}r_i)}{\psi_1(\alpha_{th}r_c) + D_{th}\ \psi_2(\alpha_{th}r_c)}\right] \tag{B15}$$

where

$$h = \frac{r_i^{m}\ R_{0,th}\ g_{th,f}}{Z_{0,th}\ (\alpha_f^2-\alpha_{th}^2)} \tag{B16a}$$

$$l = \frac{R_{0,th}\ g_{th,f}}{Z_{th,b}(\alpha_f^2-\alpha_{th}^2)} \tag{B16b}$$

and

$$n = \left[\frac{R_{0,th}\ g_{th,f}}{Z_{th,b}(\alpha_f^2-\alpha_{th}^2)} - \frac{g_{th,f}\ R_{0,f}}{Z_{f,b}(\alpha_f^2-\alpha_{th}^2)} + \frac{P_{th,f,b}}{Z_{f,b}}\right]\left[\frac{\psi_1(\alpha_f r_i) + D_f\ \psi_2(\alpha_f r_i)}{\psi_1(\alpha_f r_c) + D_f\ \psi_2(\alpha_f r_c)}\right]$$

(B16c)

The constant D_{th} then is obtained directly from equation (B15):

$$D_{th} = \frac{l\ \psi_1(\alpha_{th}r_i) - n\ \psi_1(\alpha_{th}r_c) + h\ \psi_1'(\alpha_{th}r_i)}{n\ \psi_2(\alpha_{th}r_c) - l\ \psi_2(\alpha_{th}r_i) - h\ \psi_2'(\alpha_{th}r_i)} \tag{B17}$$

This permits a complete evaluation of $I_{th,p}(r_c)$:

$$I_{th,p}(r_c) = -\left\{\frac{h\ r_c^{\ m}}{r_i^{\ m}}\left[\frac{\psi_1'(\alpha_{th}r_c) + D_{th}\ \psi_2'(\alpha_{th}r_c)}{\psi_1(\alpha_{th}r_c) + D_{th}\ \psi_2(\alpha_{th}r_c)}\right] + \frac{R_{0,f}\ g_{th,f}}{Z_{f,c}(\alpha_f^{\ 2}-\alpha_{th}^{\ 2})}\right\} \phi_f(r_c) \tag{B18}$$

In the form employed for the single reflector region this is equivalent to

$$I_{th,p}(r_c) = - \frac{P_{th,f,c}}{Z_{f,c}} \phi_f(r_c) \tag{B19}$$

The three constants $Z_{th,c}$, $Z_{f,c}$, and $P_{th,f,c}$ appearing in equations (B5), (B13), and (B19), respectively, then serve to represent all properties of the double reflector for two-group calculations.

APPENDIX C

SYMBOLS

The following symbols are used in this report:

A,C,D,δ	undetermined constants
B^2	buckling constant for directions normal to r (applies to solutions involving rectangular or cylindrical geometry)
G_0	neutron conductance defined by equation (2b)
$g_{th,f}$	neutron transconductance defined by equation (7c)
$I(r)$	neutron diffusion current at point r
k	neutron multiplication constant
m	0, 1, 2
$P_{th,f}$	reflection parameter defined by equation (16)
p_{th}	neutron resonance escape probability
R_0	neutron diffusion resistance defined by equation (2a)
r	distance variable
r_b	position of reflector outer boundary
r_c	position of core-reflector interface
r_i	position of boundary between two reflector regions
S	ratio of flux components Θ_{th}/Θ_f
Z	reflector impedance
Z_0	characteristic impedance of reflector material
α	attenuation constant of reflector material
β^2	buckling constant for core (r-direction)

ζ $K_0(\alpha r_b)/I_0(\alpha r_b)$

Θ general function

λ_{tr} neutron transport mean free path

Σ_a macroscopic neutron absorption cross section (includes slowing in fast group)

Φ general function

$\phi(r)$ neutron flux at point r

ψ general function

ω reactor segment size factor

∇_r^2 $\dfrac{d^2}{dr^2} + \dfrac{m}{r}\dfrac{d}{dr}$

Subscripts:

b parameter for region b

c parameter for combined reflector

f fast neutron group

h indicates solution of homogeneous equation

p indicates particular solution

th thermal neutron group

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2. Soodak, Harry, and Campbell, Edward C.: Elementary Pile Theory. John Wiley and Sons, Inc., 1950.

TABLE I - SOLUTIONS OF TRANSMISSION-LINE EQUATIONS

	$\Phi(\alpha r)$	$\Phi'(\alpha r) \equiv \frac{d}{d(\alpha r)} \Phi(\alpha r)$	$Z(r)$
$\underline{m = 0}$			
$\phi(r_b) = 0$	$2e^{-\alpha r_b} \sinh \alpha(r_b - r)$	$-2e^{-\alpha r_b} \cosh \alpha(r_b - r)$	$Z_0 \tanh \alpha(r_b - r)$
$\phi'(0) = 0$	$2 \cosh \alpha r$	$2 \sinh \alpha r$	$-Z_0 \coth \alpha r$
$\underline{m = 1}$			
$\phi(r_b) = 0$	$K_0(\alpha r) - \zeta I_0(\alpha r)$	$-K_1(\alpha r) - \zeta I_1(\alpha r)$	$\frac{Z_0}{r} \frac{K_0(\alpha r) - \zeta I_0(\alpha r)}{K_1(\alpha r) + \zeta I_1(\alpha r)}$
$\phi'(0) = 0$	$I_0(\alpha r)$	$I_1(\alpha r)$	$-\frac{Z_0}{r} \frac{I_0(\alpha r)}{I_1(\alpha r)}$
$\underline{m = 2}$			
$\phi(r_b) = 0$	$2e^{-\alpha r_b} \frac{\sinh \alpha(r_b - r)}{r}$	$-2e^{-\alpha r_b} \frac{\sinh \alpha(r_b - r)}{\alpha r^2}$ $-2e^{-\alpha r_b} \frac{\cosh \alpha(r_b - r)}{r}$	$\frac{Z_0 \alpha}{r} \frac{1}{(1 + r \alpha \coth \alpha(r_b - r))}$
$\phi'(0) = 0$	$\frac{\sinh \alpha r}{r}$	$-\frac{1}{\alpha r^2} \sinh \alpha r + \frac{\cosh \alpha r}{r}$	$\frac{Z_0 \alpha}{r} \frac{1}{(1 - r\alpha \coth \alpha r)}$

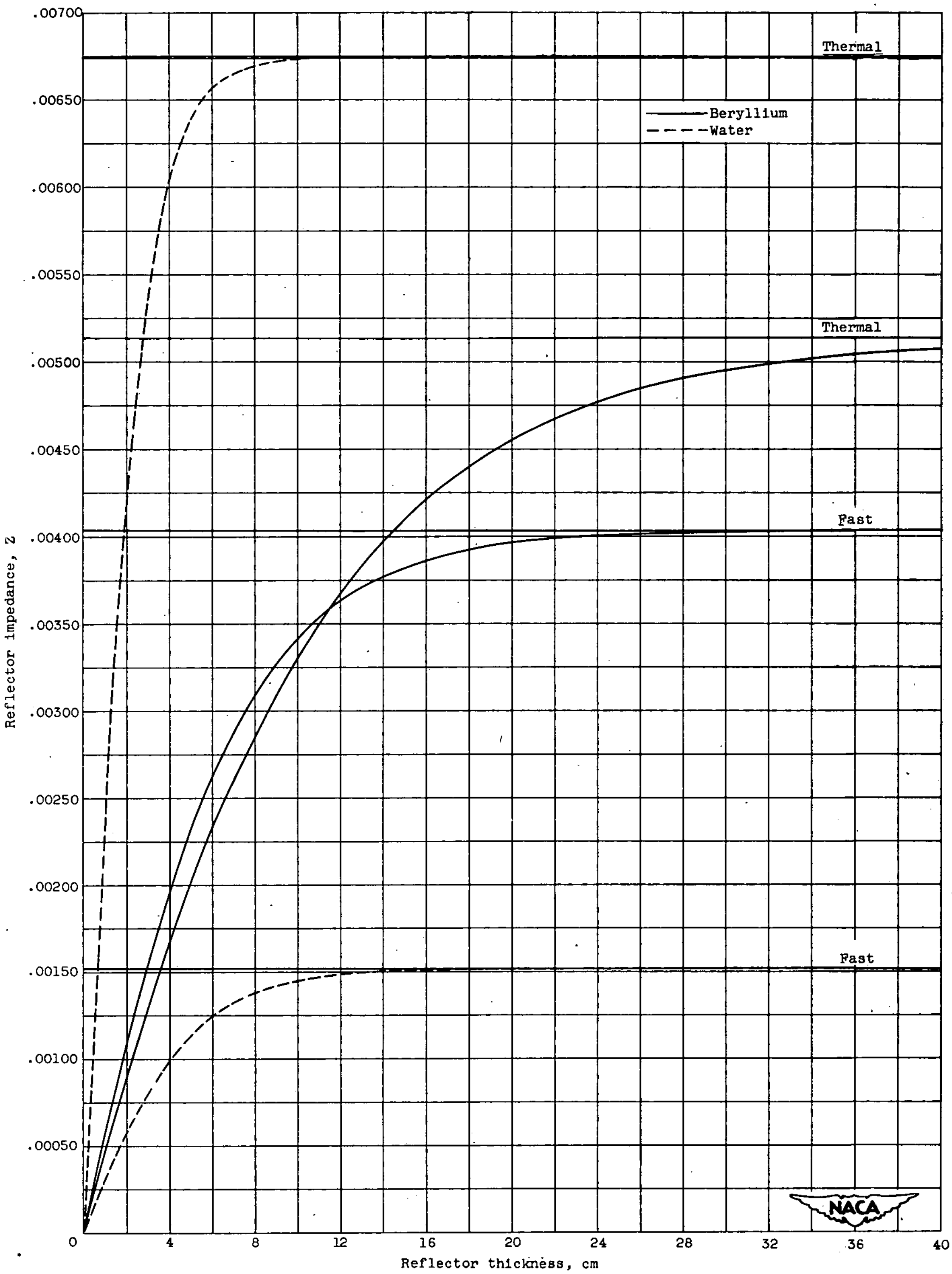

Figure 1. - Fast and thermal impedances for beryllium and water reflectors.

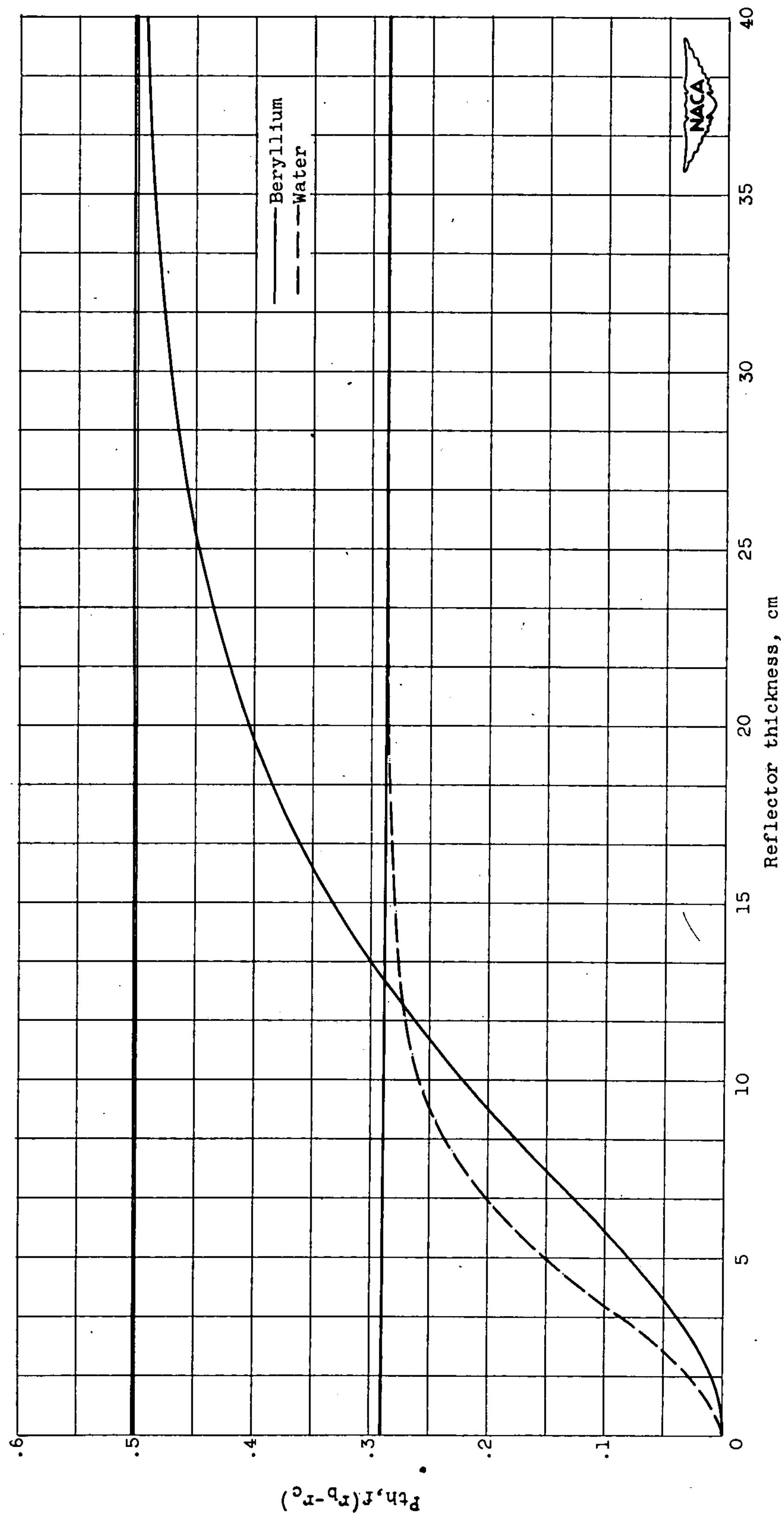

Figure 2. - Reflection parameter for beryllium and water reflectors.