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RESEARCH MEMORANDUM

LONGITUDINAL FREQUENCY-RESPONSE CHARACTERISTICS OF THE DOUGLAS D-558-I AIRPLANE AS DETERMINED FROM EXPERIMENTAL TRANSIENT-RESPONSE HISTORIES TO A MACH NUMBER OF 0.90

By Ellwyn E. Angle and Euclid C. Holleman

Langley Aeronautical Laboratory
Langley Field, Va.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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LONGITUDINAL FREQUENCY-RESPONSE CHARACTERISTICS OF THE DOUGLAS D-558-I AIRPLANE AS DETERMINED FROM EXPERIMENTAL TRANSIENT-RESPONSE HISTORIES TO A MACH NUMBER OF 0.90

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SUMMARY

During flight tests of the Douglas D-558-I research airplane, several transient oscillations of the airplane in response to elevator pulses were obtained over a Mach number range of 0.52 to 0.90 at altitudes between 30,000 and 37,000 feet. Through an application of the Fourier transform to the experimental input and output functions, the longitudinal frequency response of the airplane was determined as a function of Mach number. A comparison of the response data estimated from wind-tunnel data with the experimental results showed good agreement. It was found that the maximum response amplitude was a minimum at a Mach number of 0.88. At lower Mach numbers (0.52 to 0.66) the effects of lift coefficient on frequency response were indicated.

INTRODUCTION

In the analysis and synthesis of automatic stabilized dynamical systems such as the airplane and autopilot combination, there are two important elements: (1) the transfer function of the airplane and (2) the transfer function of the autopilot.

There is a definite need for airplane frequency-response data at high subsonic, transonic, and supersonic speeds to aid in the design of automatic-control equipment and to aid in the study of the dynamic stability of the transfer function of the airplane. In satisfying this need it is more feasible to obtain frequency-response data from transient oscillations in response to pulse disturbances than from the response to sinusoidal inputs of various frequency magnitudes.

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The purpose of this paper is to present the longitudinal frequency-response characteristics determined from a Fourier analysis of transient oscillations of the Douglas D-558-I research airplane. The data presented herein were obtained over a Mach number range from 0.52 to 0.90 at altitudes between 30,000 and 37,000 feet.

SYMBOLS

θ	angle of pitch, degrees or radians
$\dot{\theta}$	rate of change of pitch angle, radians per second
δ_e	control deflection, degrees
ϕ	phase angle between $\dot{\theta}$ and δ_e, degrees
α	angle of attack, degrees
i_t	stabilizer incidence, degrees
ω	exciting frequency, radians per second
ω_{R_A}	undamped natural frequency of airplane, radians per second
ω_O	damped natural frequency, radians per second
ξ_A	damping ratio of airplane $\left(\frac{\text{Amount of damping}}{\text{Critical damping}}\right)$
S	manipulation variable of Laplace transform
D	differential operator per second (d/dt)
t	time, seconds
C_O,C_1	disturbance-function parameters of transfer function
M	Mach number
n	normal acceleration, g units
h_p	pressure altitude, feet
C_{N_A}	normal-force coefficient of airplane
$C_{L_{\delta_e}}$	rate of change of lift coefficient with elevator deflection, per degree

$C_{m_{\delta_e}}$ rate of change of pitching-moment coefficient with elevator deflection, per degree

C_{L_α} rate of change of lift with angle of attack, per degree

C_{m_α} rate of change of pitching-moment coefficient with angle of attack, per degree

$C_{m_{D\alpha}}$ rate of change of pitching-moment coefficient with angular velocity of angle of attack, per degree per second

$C_{m_{D\theta}}$ rate of change of pitching-moment coefficient with pitching velocity, per degree per second

$R_{\dot{\theta}}, I_{\dot{\theta}}$ real and imaginary parts of output function, respectively

$R_{\delta_e}, I_{\delta_e}$ real and imaginary parts of input function, respectively

Subscript:

max maximum condition

EQUIPMENT AND PROCEDURE

Airplane and Instrumentation

The test airplane, D-558-I, is a high-speed research airplane of conventional design with an unswept wing and tail. The physical characteristics are presented in table I and a sketch of the airplane is shown as figure 1.

All quantities necessary for the analysis of the airplane motion were recorded by standard NACA recording instruments and were synchronized by a common timer. The quantities recorded were: normal acceleration, airspeed, altitude, pitching velocity, elevator position, and stabilizer position. Accuracies of the recorded quantities are indicated by the following instrument accuracies:

Pitching velocity, $\dot{\theta}$, radians per second	±0.005
Elevator position, δ_e, degrees	±0.1
Mach number, M	±0.01
Normal acceleration, n, g units	±0.05

Method of Obtaining Data

During flights of the D-558-I airplane, several stick-fixed longitudinal responses to elevator pulses were obtained. The responses were recorded during stabilized 1 g flight over a Mach number range of 0.52 to 0.90 at altitudes between 30,000 and 37,000 feet. The input function consisted of a pulse of the elevator of about 2° in magnitude and a duration of from 0.5 to 1.0 second.

The pulse resulted in an initial airplane oscillation of approximately $\pm 1/2$ g to ± 1 g and a pitching velocity of ± 0.1 radian per second. A restricting device was used on the elevator control to insure the return of the control to approximately its original position following the disturbance and to aid in maintaining fixed-elevator condition during the subsidence of the oscillation. All other controls, ailerons, rudder, and stabilizer, were fixed during the maneuvers. The stabilizer was fixed at -2.00°, airplane nose down, throughout the tests. The elevator position was recorded at four stations along the span external of the fuselage. In order to compensate for elevator twist, which amounted to about 0.4°, an average of the four control positions was used as the input function.

METHOD OF ANALYSIS

The analysis of the transient-response flight data is based on the assumptions that the equations of motion are linear differential equations with constant coefficients and that forward velocity and altitude are constant. The equations of motion are as follows:

$$C_{L_{\delta_e}}\delta_e + C_{L_\alpha}\alpha = 2\tau D(\alpha - \theta) \tag{1}$$

$$C_{m_{\delta_e}}\delta_e + C_{m_\alpha}\alpha + C_{m_{D\alpha}} D\alpha + C_{m_{D\theta}} D\theta = \frac{\tau c}{2V} K_y{}^2 D^2\theta \tag{2}$$

where

$$\tau = \frac{m}{\rho S V}$$

$$K_y{}^2 = \frac{4I_y}{mc^2}$$

m	mass of airplane, slugs
S	wing area, square feet
V	forward velocity, feet per second
ρ	air density, slugs per cubic foot
I_y	moment of inertia in pitch, slug-feet^2
c	mean aerodynamic chord, feet

Time histories of pitching velocity and elevator position provide the data from which the transfer functions of the system are determined by means of the Fourier transform. The use of this transform is briefly described herein but a more complete explanation may be found in references 1 to 3.

The Fourier transform, as applied in harmonic analysis, is

$$F(i\omega) = \int_{-\infty}^{\infty} F(t)e^{-i\omega t}\,dt \tag{3}$$

By transferring the experimental functions, input and output, so that the initial and final conditions are equal to zero, the transform may be simplified to

$$F(i\omega) = \int_{0}^{T} F(t)e^{-i\omega t}\,dt \tag{4}$$

where the quantity $F(t)$ reaches a steady-state condition at time $t = T$ identical to the initial condition at the time $t = 0$. If, however, $F(t)$ assumes some steady-state condition other than that equivalent to the initial condition, the transform is modified to

$$F(i\omega) = \int_{0}^{T} F(t)e^{-i\omega t}\,dt + \int_{T}^{\infty} F(t)e^{-i\omega t}\,dt \tag{5}$$

for which the second term on the right-hand side becomes finite as

$$\int_{T}^{\infty} F(t)e^{-i\omega t}\,dt = \frac{F(T)}{i\omega}\,e^{-i\omega T} \tag{6}$$

Evaluation of the integral equation (5) yields a set of real and imaginary parts each for the input and output as a function of consecutive integral multiples of the fundamental frequency ω_0. The integrals in the real and imaginary parts of equation (5) are obtained by numerical integration (Simpson's One-Third Rule) of the respective parts as

$$\left.\begin{aligned}
R_{\dot{\theta}} &= \int_0^T \dot{\theta}(t)\ \cos \omega t\ dt - \frac{\dot{\theta}(T)}{\omega}\ \sin \omega T \\
I_{\dot{\theta}} &= -\int_0^T \dot{\theta}(t)\ \sin \omega t\ dt - \frac{\dot{\theta}(T)}{\omega}\ \cos \omega T \\
R_{\delta_e} &= \int_0^T \delta_e(t)\ \cos \omega t\ dt - \frac{\delta_e(T)}{\omega}\ \sin \omega T \\
I_{\delta_e} &= -\int_0^T \delta_e(t)\ \sin \omega t\ dt - \frac{\delta_e(T)}{\omega}\ \cos \omega T
\end{aligned}\right\} \tag{7}$$

including the corrections for a finite steady-state condition.

It is possible to combine these quantities directly into a frequency-response expression as

$$\frac{\dot{\theta}}{\delta_e} = \frac{R_{\dot{\theta}} + iI_{\dot{\theta}}}{R_{\delta_e} + iI_{\delta_e}} = \left|\frac{\dot{\theta}}{\delta_e}\right| e^{i\phi} \tag{8}$$

The results determined by equations (7) and (8) are exact within the limitations of numerical integration accuracy.

The frequency response can be determined in another way from transient-response data by assuming a form for the transfer function as

$$\frac{\dot{\theta}}{\delta_e} = \frac{C_0 + C_1 S}{\left(S^2 + {\omega_{R_A}}^2\right) + 2\xi_A \omega_{R_A} S} \tag{9}$$

derived from the assumed two-degree-of-freedom system described by equations (1) and (2). These results, however, can only be classed as faired data. Application of the incomplete Fourier transform to equation (9) yields:

$$\left[(i\omega)^2 + 2\xi_A\omega_{R_A} i\omega + \omega_{R_A}^2\right]\int_0^T \dot{\theta}(t)e^{-i\omega t}\,dt + e^{i\omega T}\left[D\dot{\theta}(T) + 2\xi_A\omega_{R_A}\dot{\theta}(T) + i\omega\dot{\theta}(T)\right] - \left[D\dot{\theta}(0) + 2\xi_A\omega_{R_A}\dot{\theta}(0) + i\omega\dot{\theta}(0)\right] = (C_0 + C_1 i\omega)\int_0^T \delta_e(t)e^{i\omega t}\,dt + C_1\delta_e(T)e^{-\omega T} - C_1\delta_e(0) \tag{10}$$

from which a real and an imaginary equation can be set up. By using the method of least squares and a matrix solution, the transfer-function coefficients C_0, C_1, $2\xi_A\omega_{R_A}$, and $\omega_{R_A}^2$ are determined from the solution of the simultaneous equations derived from the number of frequencies desired to give a frequency spectrum. (Least-squares method is described in reference 4.) The transfer function for the system is then calculated for the desired exciting frequencies by substituting the transfer-function coefficients back into equation (9).

RESULTS AND DISCUSSION

Presented as figures 2(a) to 2(d) are four representative time histories of the quantities and the type of transient oscillations used in the determination of the transfer functions. The numerical transformation of time histories such as these from the time plane to the frequency plane yields the frequency-response data shown as figures 3(a) to 3(l) which show response amplitude and phase angle plotted against frequency.

The points in the figures represent the experimental frequency response determined by equations (7) and (8). The same real and imaginary parts, equation (7) without end correction, are used in conjunction with equation (10) and the least-squares method of curve fitting to give the variations represented by the continuous lines and presented as faired data. These results show the variation of the frequency-spectrum characteristics at constant Mach numbers over a Mach number range of 0.52 to 0.90. It can be seen that the maximum response amplitude and the natural frequency take on a definite variation with Mach number. This variation is demonstrated in figure 4 in which these quantities are plotted against Mach number. The maximum amplitude decreases from 9.3 at a Mach number of 0.52 to a minimum of 6.4 at a Mach number of 0.65, increases to 9.9 at a Mach number of 0.75, then decreases to a minimum value of 2.7 at a Mach number of 0.88 after which the curve again increases to a value of 7.1 at a Mach number of 0.9. Over the same Mach number range the natural frequency decreases from a value of 4.00 at a Mach number of 0.52 to 1.7 at a Mach number of 0.75 and then increases to a value of 5.2 at a Mach number of 0.90.

Between Mach numbers of 0.60 and 0.90 the maximum response amplitude and natural frequency derived from wind-tunnel data (reference 5) are compared with the experimental results determined from Fourier analysis. The agreement is considered good from Mach number 0.69 to Mach number 0.90. The decrease in the natural frequency might possibly be attributed to a lift-coefficient effect since the lift coefficient decreases from 0.8 at a Mach number of 0.52 to 0.4 at a Mach number of 0.75.

CONCLUDING REMARKS

An application of the Fourier transform has been used to obtain the frequency-response characteristics of the Douglas D-558-I airplane from experimental transient oscillations in response to a pulse elevator input over a Mach number range of 0.52 to 0.90 at altitudes between 30,000 and 37,000 feet. The experimental frequency-response data were obtained by numerically applying the Fourier transform to the transient response obtained. A comparison of the response data estimated from wind-tunnel data with the experimental results showed good agreement. It was found that the maximum response amplitude had a minimum value at a Mach number of 0.88. At the lower Mach numbers (0.52 to 0.66) the effects of lift coefficient on the frequency response were indicated.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va.

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1. Greenberg, Harry: A Survey of Methods for Determining Stability Parameters on an Airplane from Dynamic Flight Measurements. NACA TN 2340, 1951.

2. Brown, Gordon S., and Campbell, Donald P.: Principles of Servomechanisms. John Wiley & Sons, Inc., 1948, pp. 195-235.

3. Swanson, Robert S., and Mastrocola, N.: Preliminary Report on Methods for Determining Dynamic Stability Parameters from Free-Flight Tests. Tech. Rep. No. 58, U. S. Naval Air Missile Test Center (Pt. Mugu, Calif.), Sept. 5, 1949. 5375.4 / 31

4. Shinbrot, Marvin: A Least Squares Curve Fitting Method with Applications to the Calculation of Stability Coefficients from Transient-Response Data. NACA TN 2341, 1951.

5. Wright, John B.: High-Speed Wind-Tunnel Tests of a $\frac{1}{16}$-Scale Model of the D-558 Research Airplane. Longitudinal Stability and Control of the D-558-1. NACA RM L8C23, 1948.

TABLE I

PHYSICAL CHARACTERISTICS OF DOUGLAS D-558-I AIRPLANE

Airplane:	
Mass, slugs	315
Moment of inertia in pitch, slug-ft^2	16,750
Fuselage:	
Length, ft	35.04
Wing:	
Span, ft	25.0
Area, sq ft	150.7
Taper ratio	0.54
Aspect ratio	4.17
Mean aerodynamic chord, ft	6.21
Horizontal tail:	
Span, ft	12.25
Area, sq ft	35.98
Taper ratio	0.55
Aspect ratio	4.17
Tail length, from 0.25 chord line of wing to hinge line, ft	16.34

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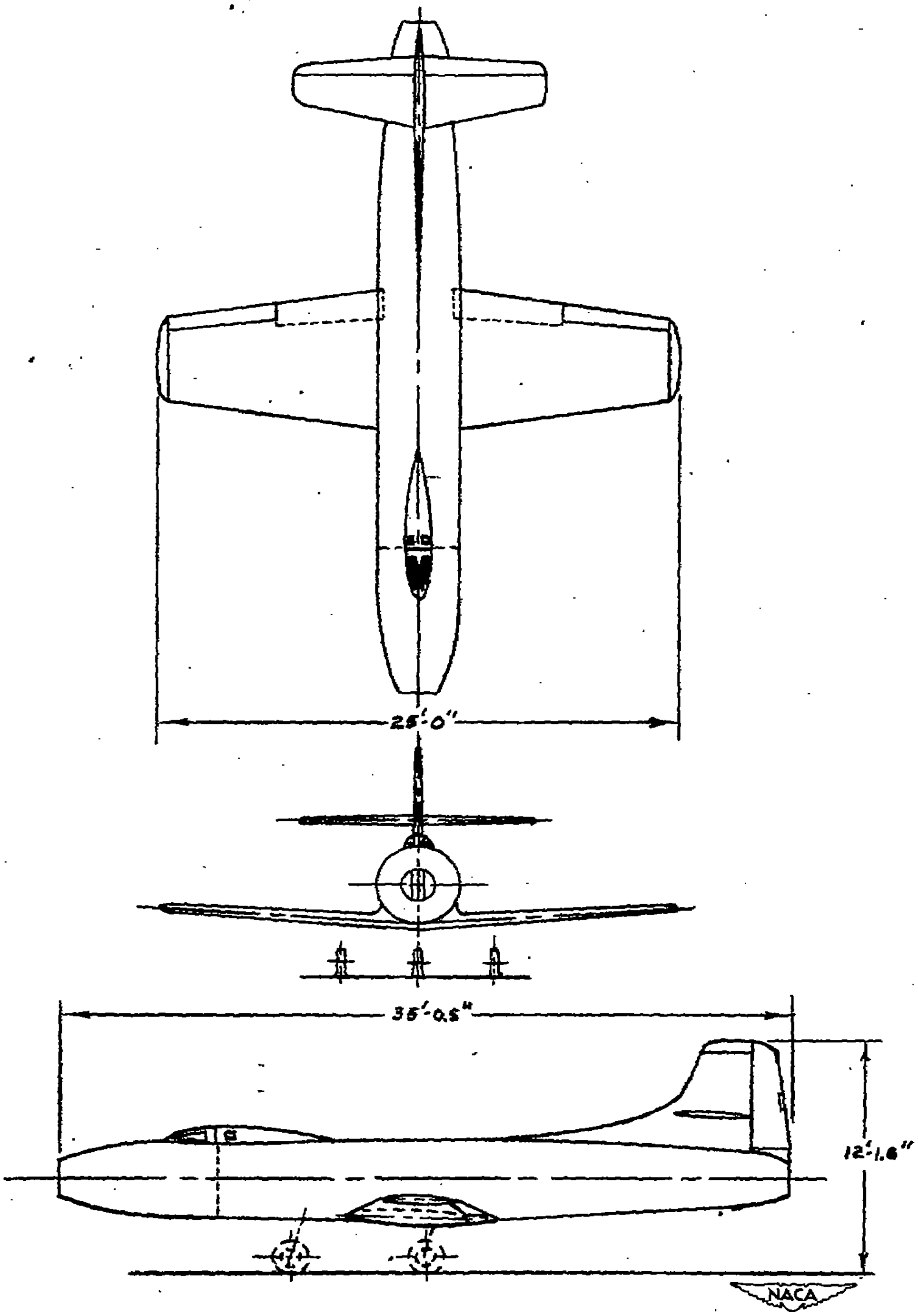
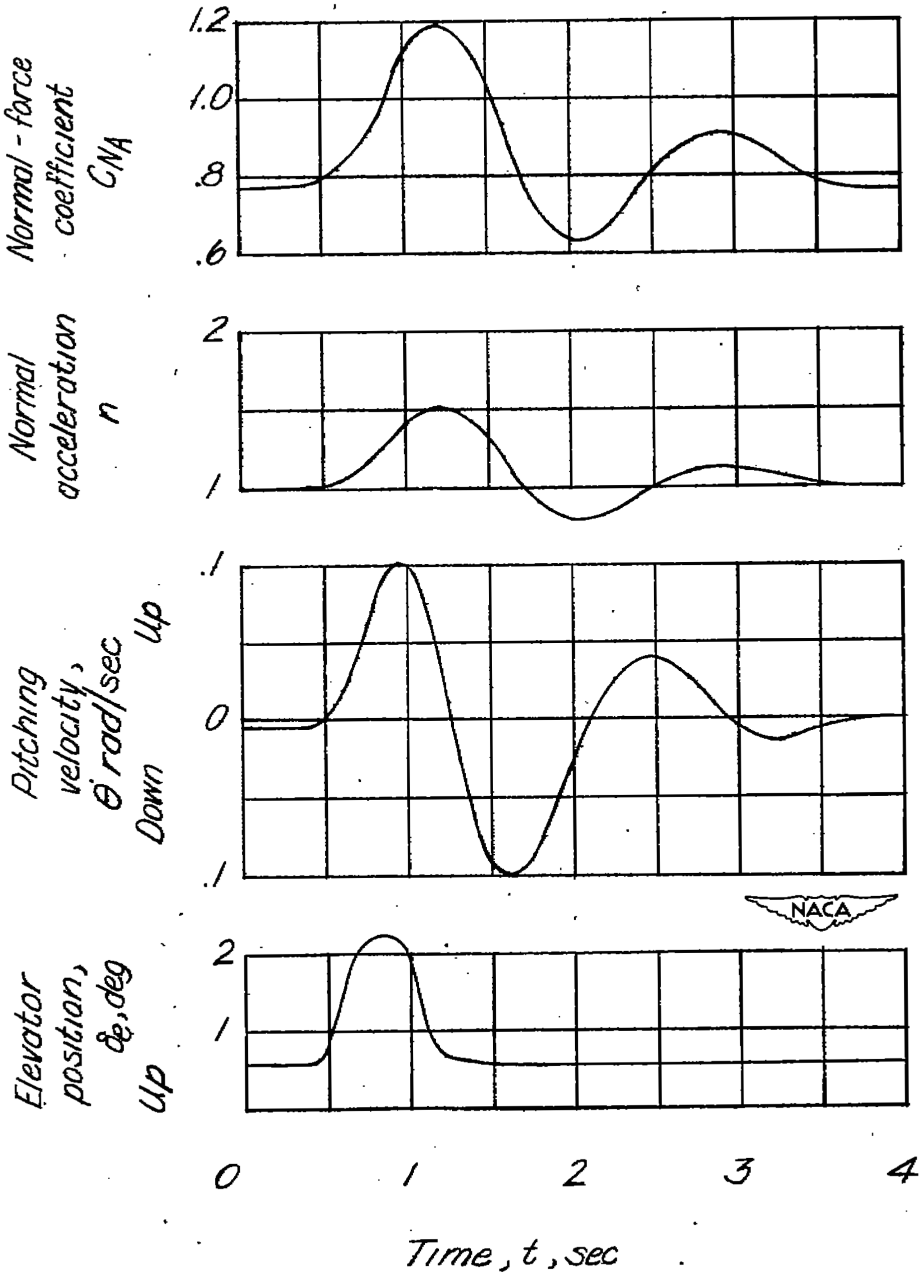
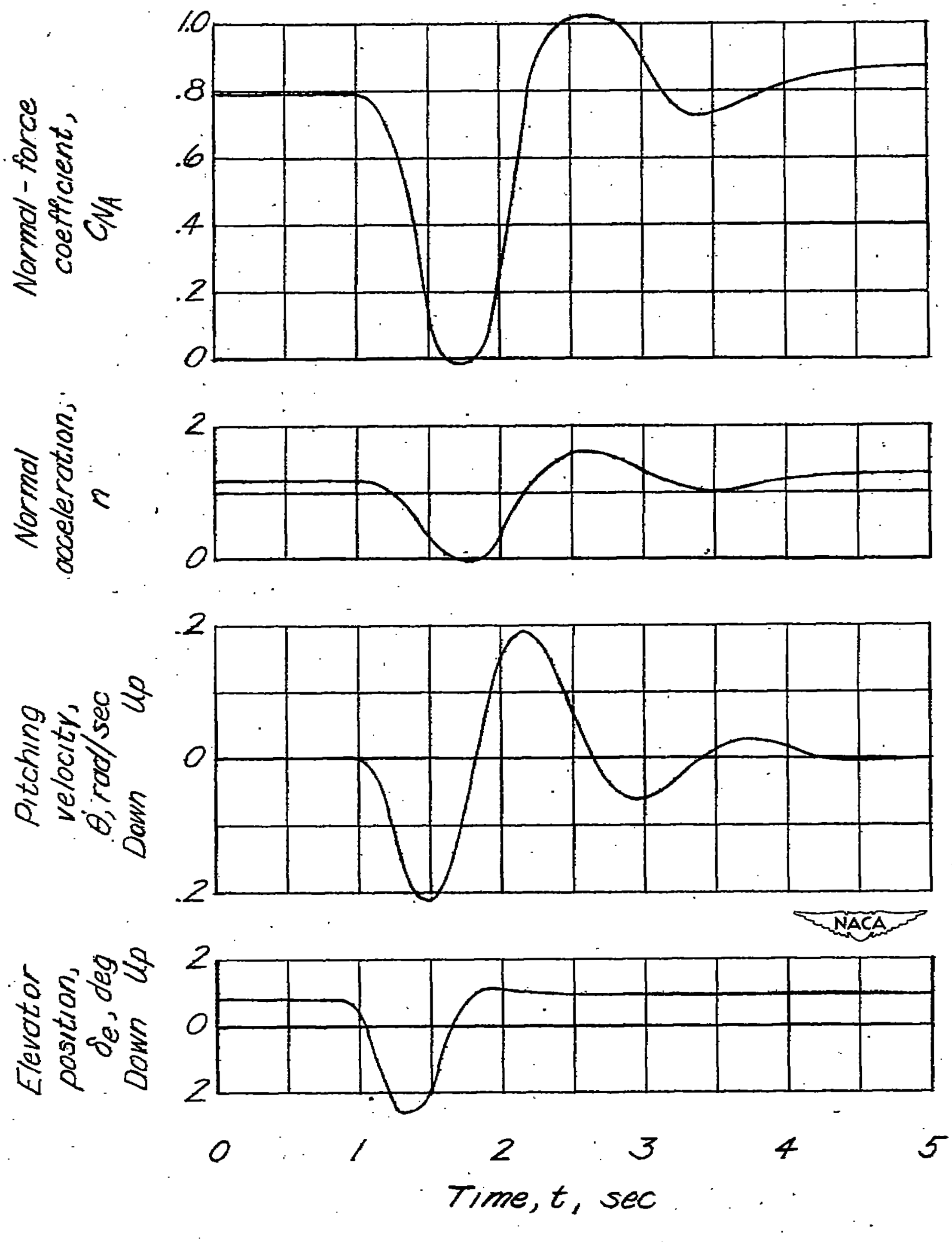


Figure 1.- Three-view drawing of the Douglas D-558-I airplane.



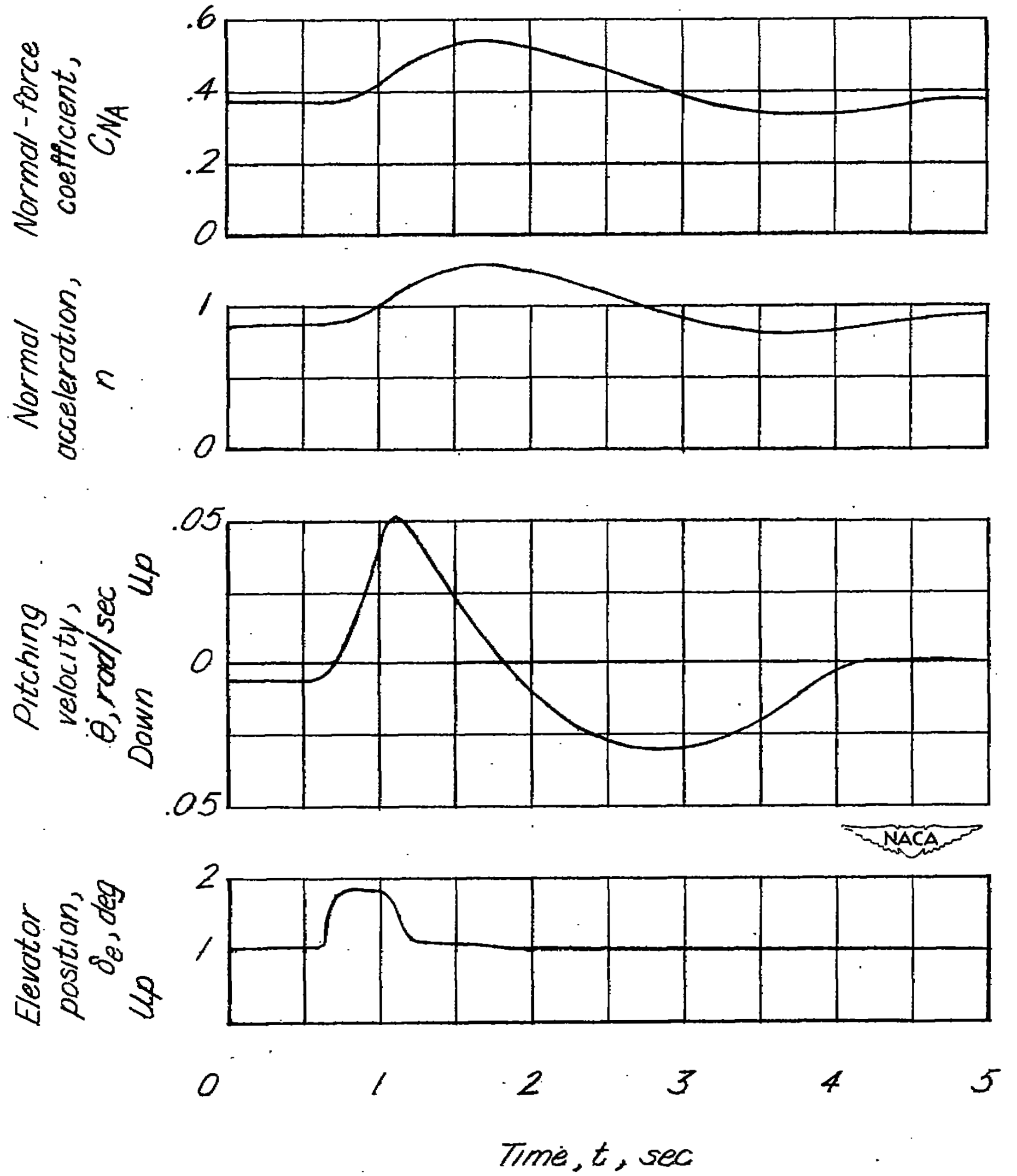
(a) M = 0.52; h_p = 37,400 feet.

Figure 2.- Time histories of typical airplane response to an elevator pulse. $i_t = -2.00^{\circ}$.



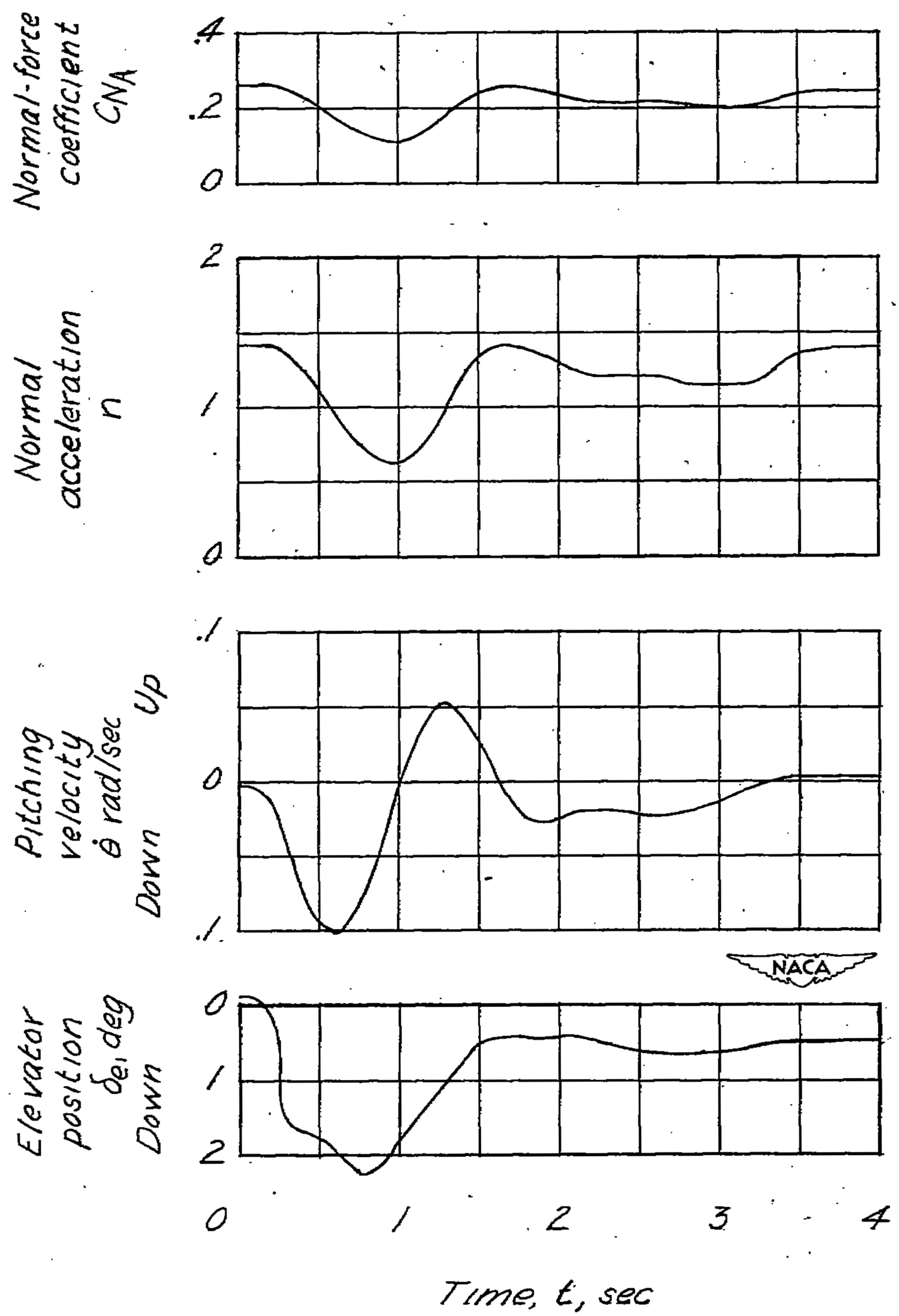
(b) M = 0.55; h_p = 35,800 feet.

Figure 2.- Continued.



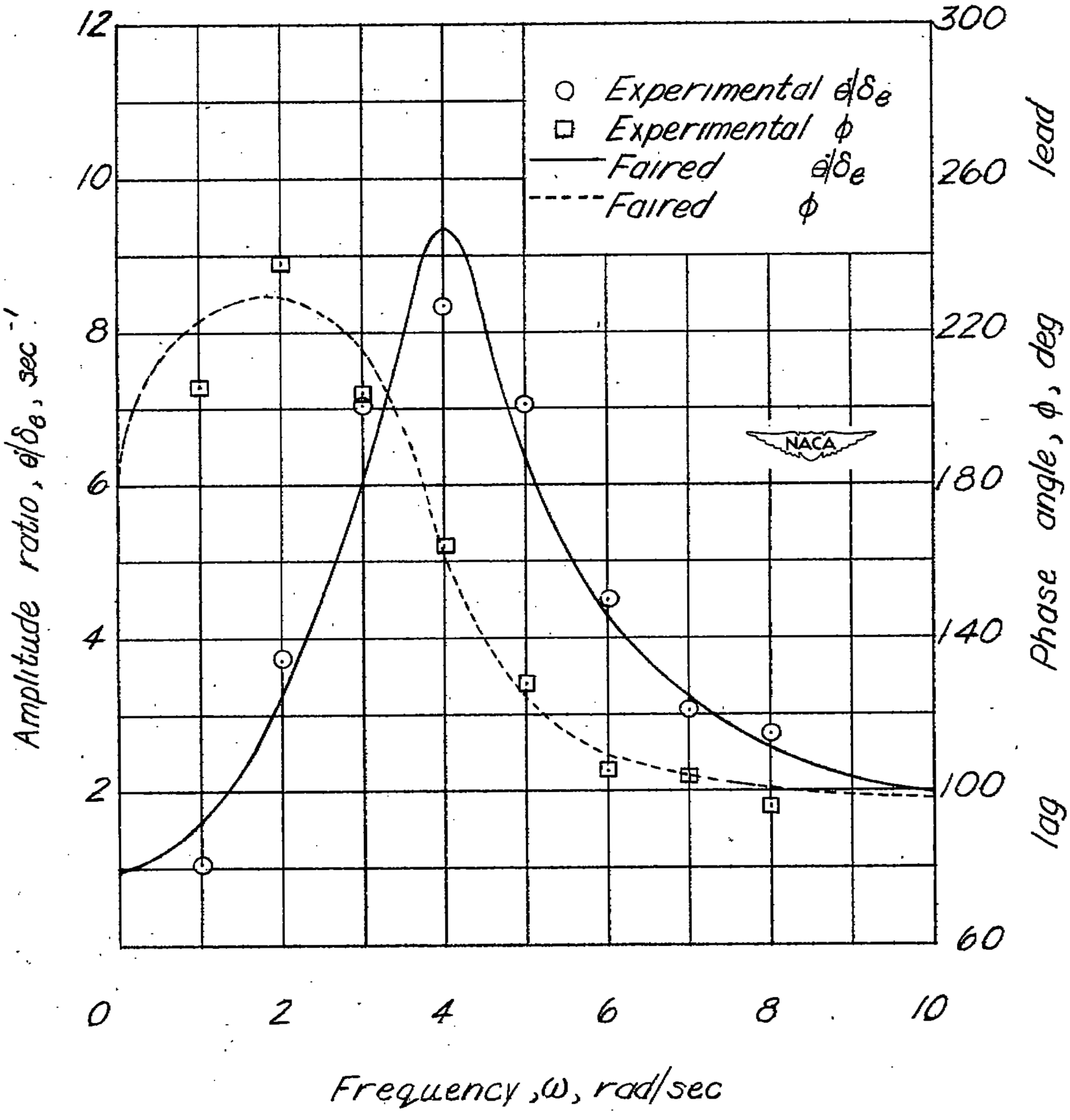
(c) $M \approx 0.75$; $h_p = 38{,}000$ feet.

Figure 2.- Continued.



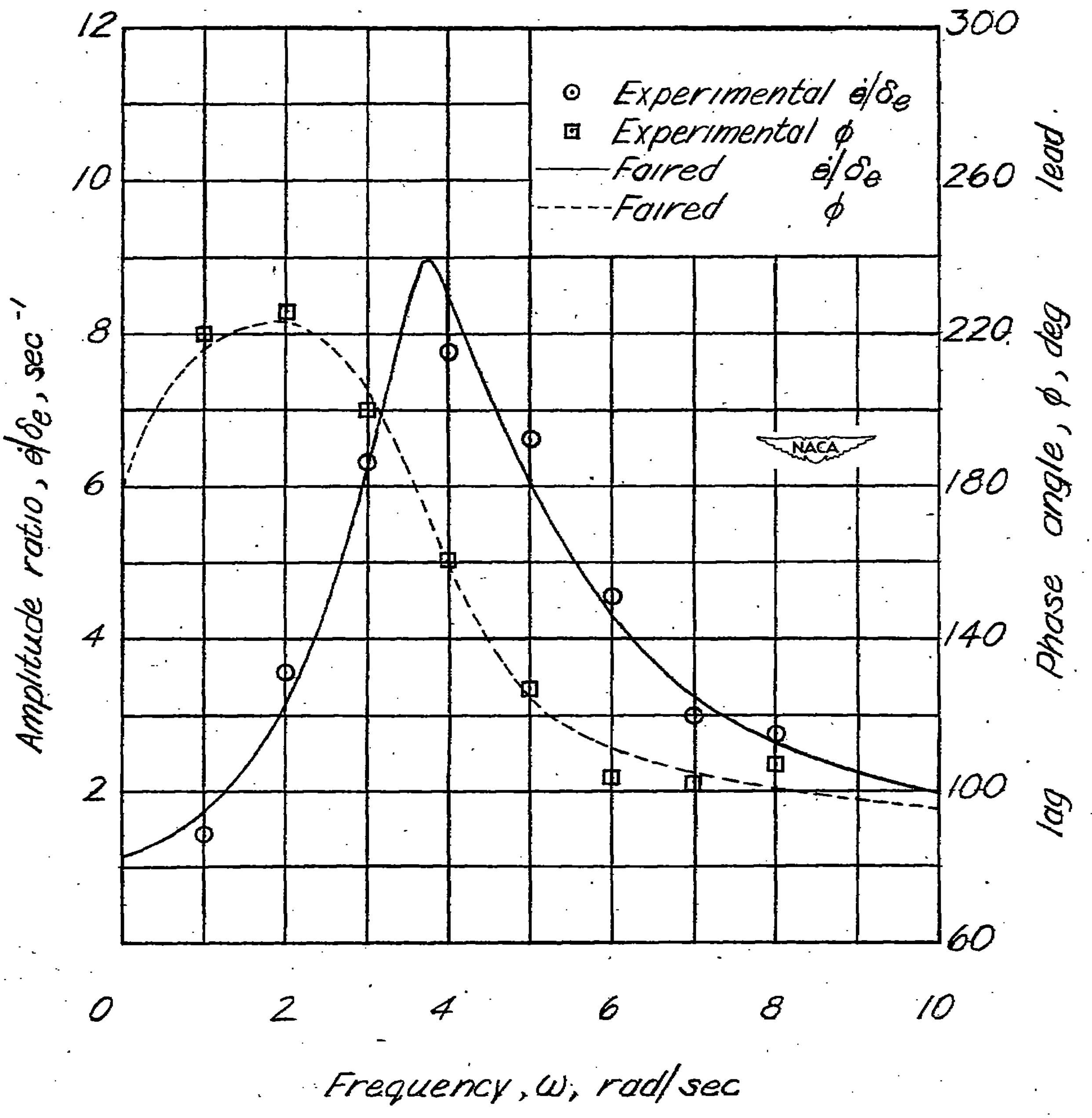
(d) M = 0.90; h_p = 30,000 feet.

Figure 2.- Concluded.



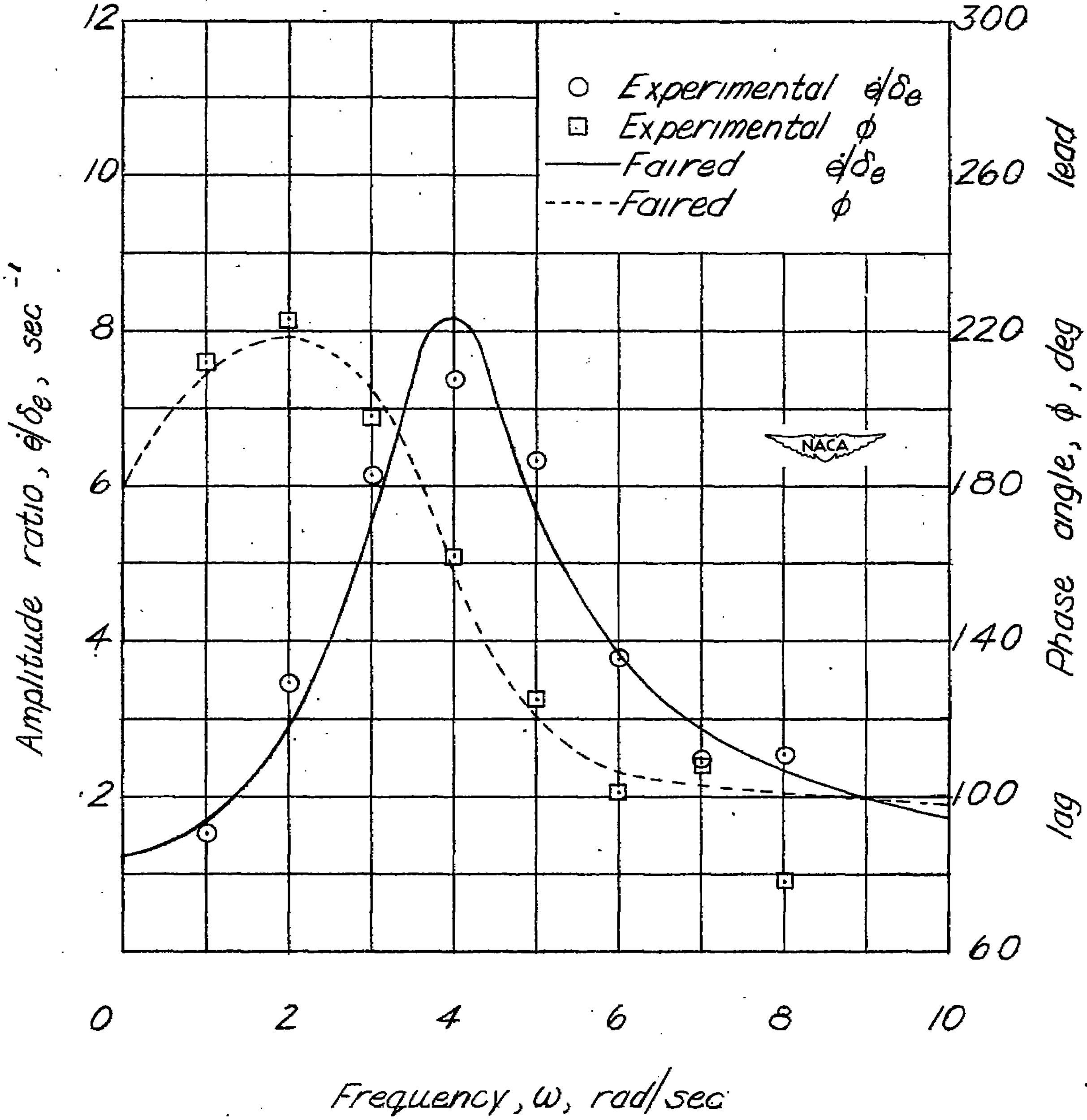
(a) M = 0.52; h_p = 37,400 feet.

Figure 3.- Frequency response of the Douglas D-558-I airplane as determined from pitching velocity response to an elevator pulse.



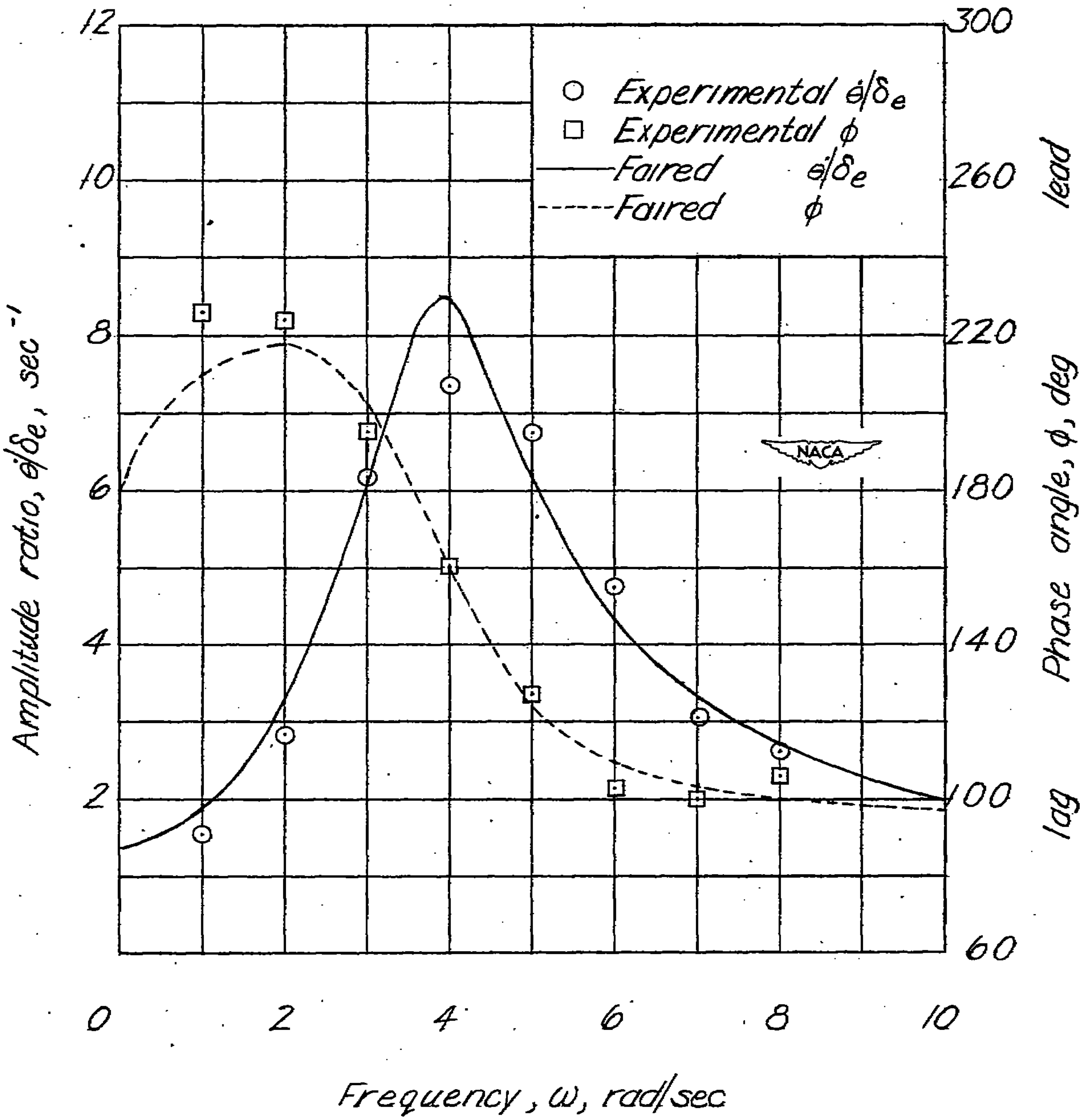
(b) M = 0.54; h_p = 36,000 feet.

Figure 3.- Continued.



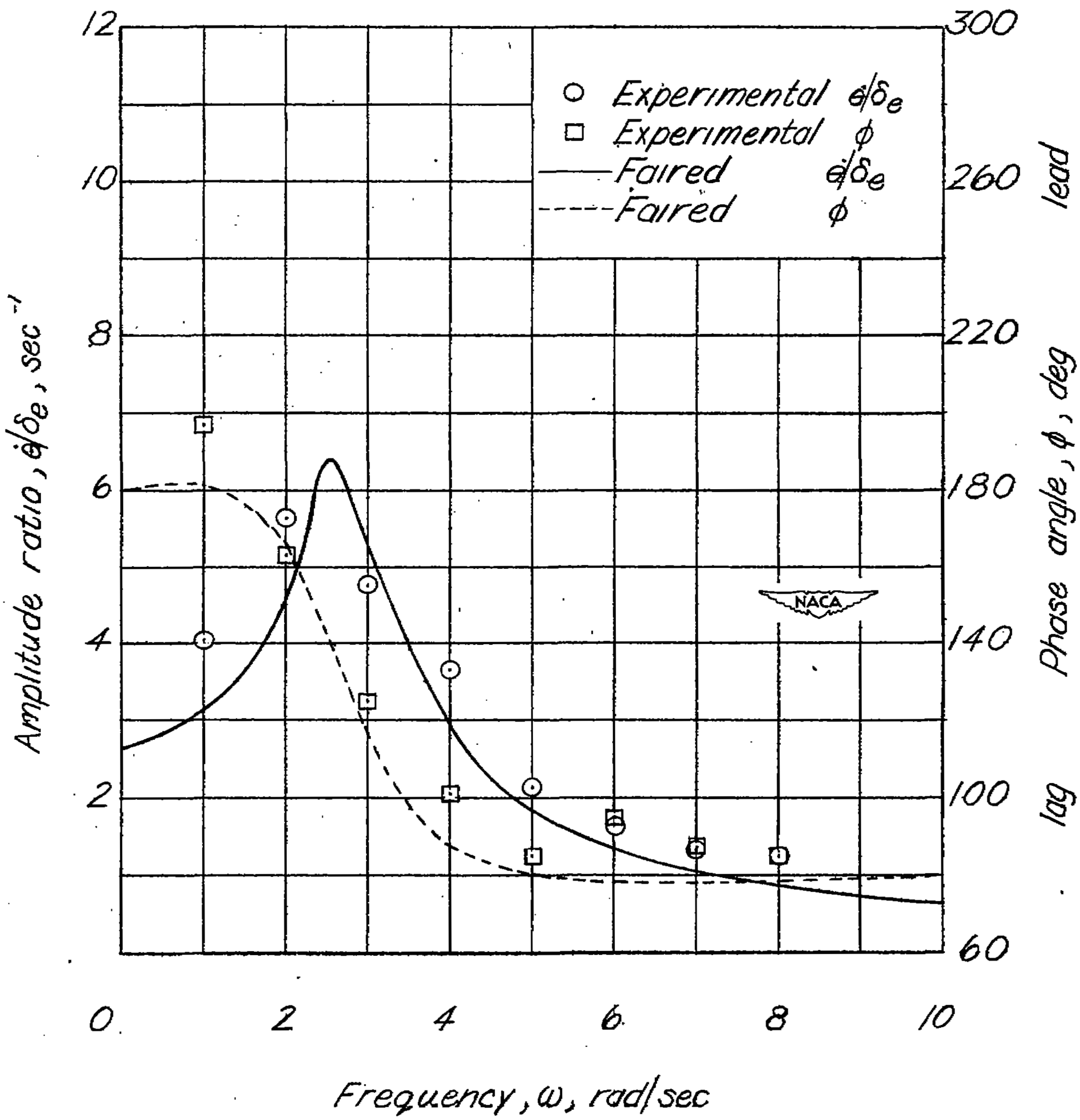
(c) M = 0.55; h_p = 36,400 feet.

Figure 3.- Continued.



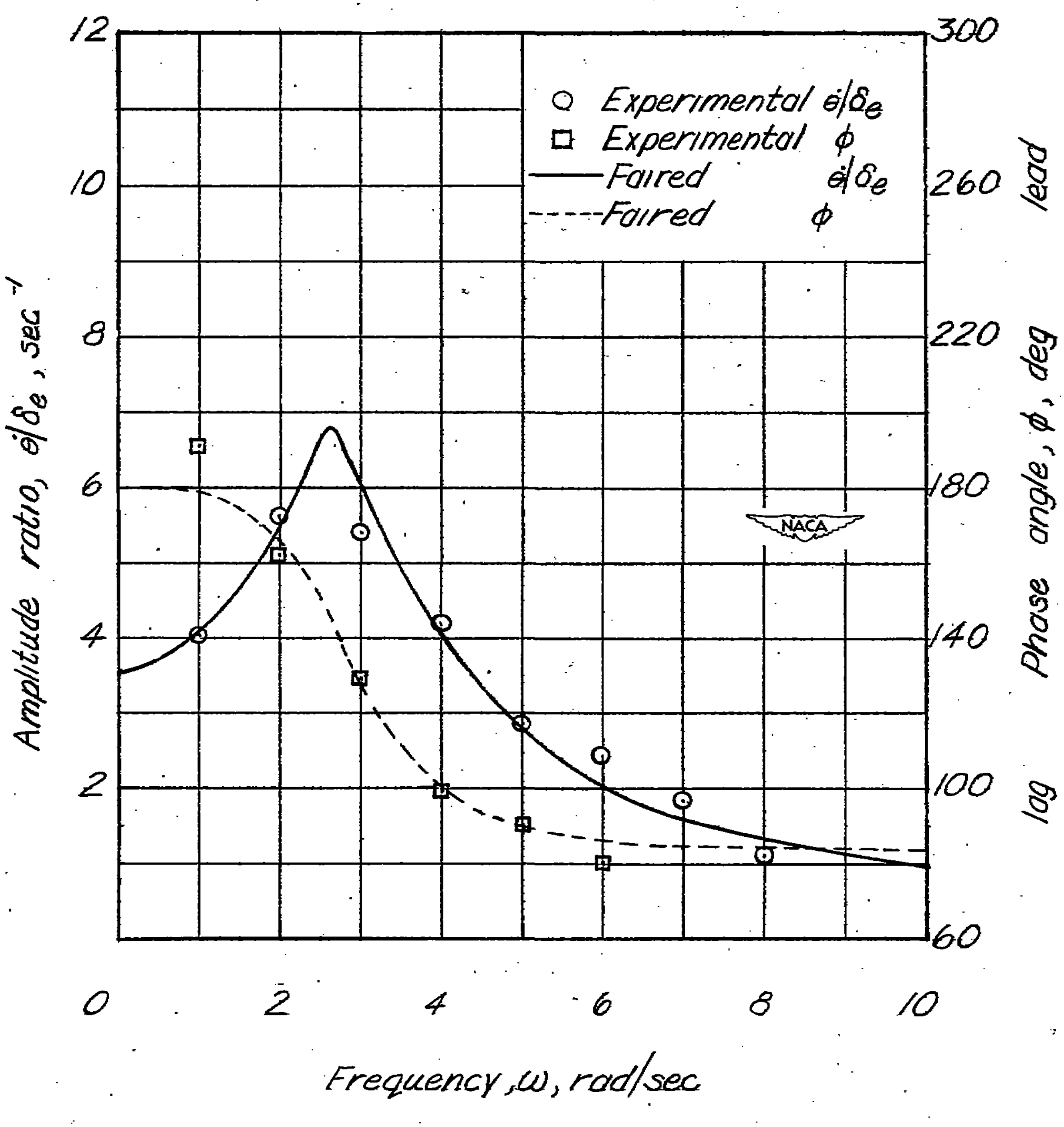
(d) $M = 0.55$; $h_p = 35{,}800$ feet.

Figure 3.- Continued.



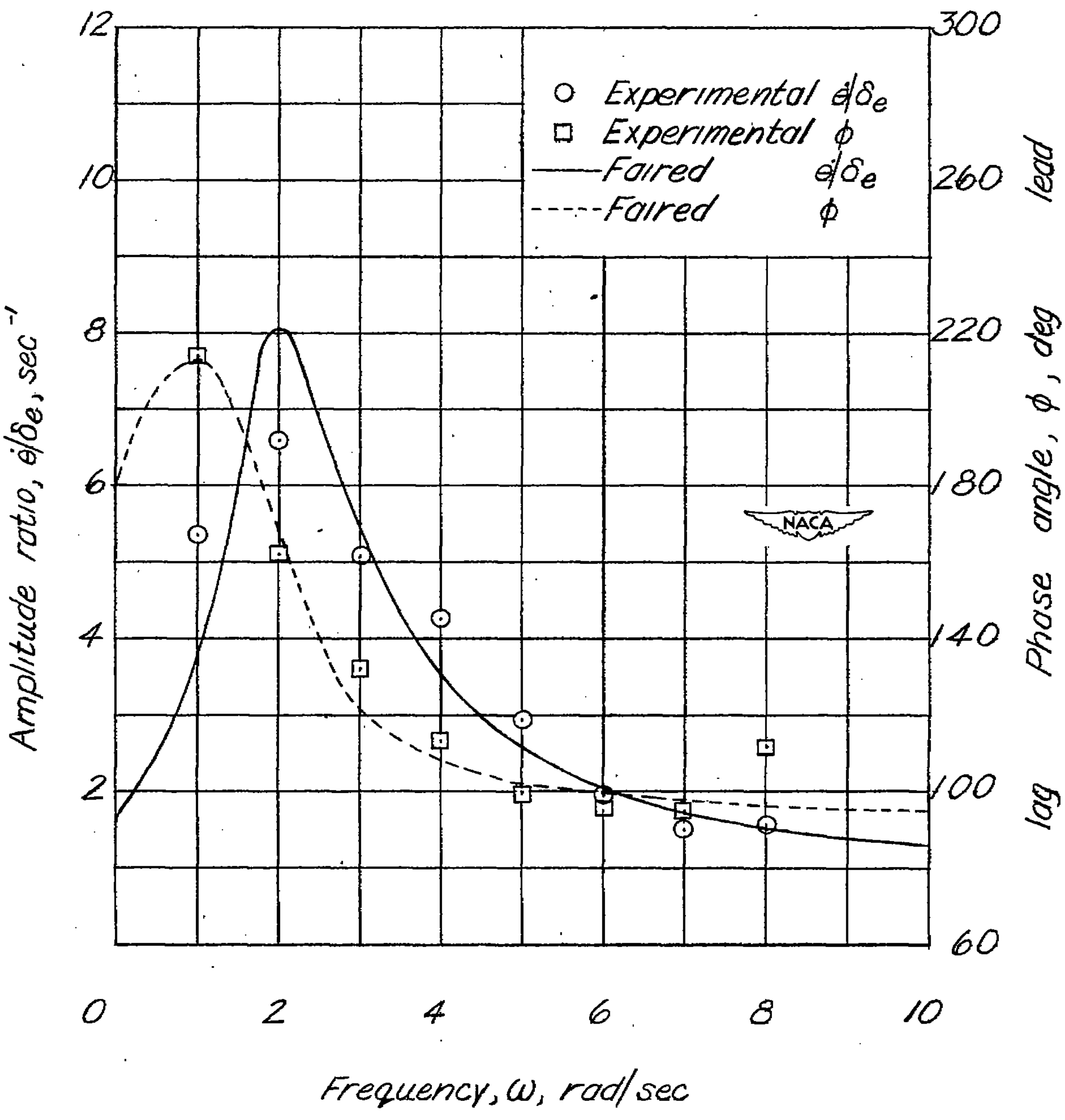
(e) M = 0.65; h_p = 40,200 feet.

Figure 3.- Continued.



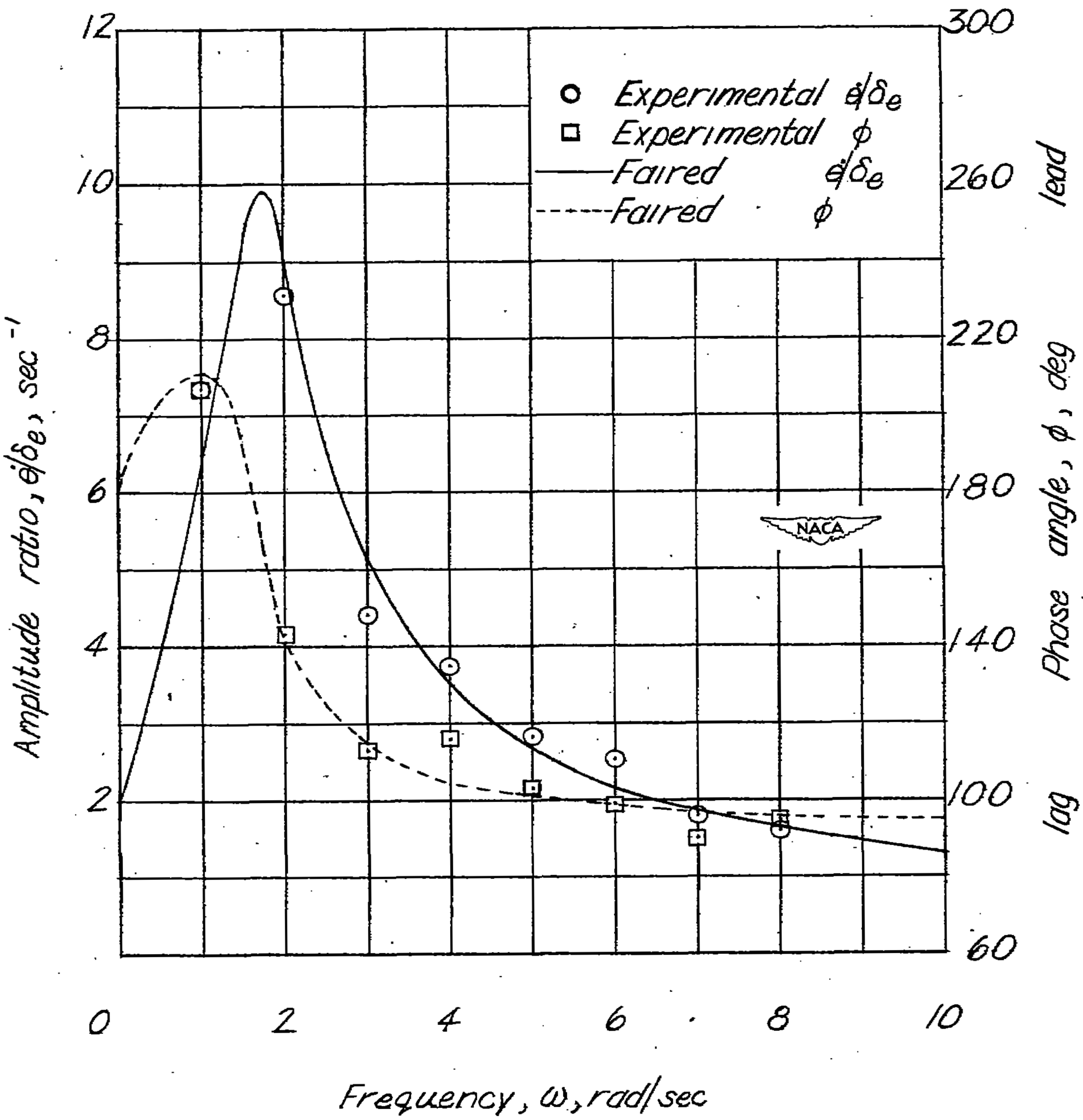
(f) M = 0.66; h_p = 39,200 feet.

Figure 3.- Continued.



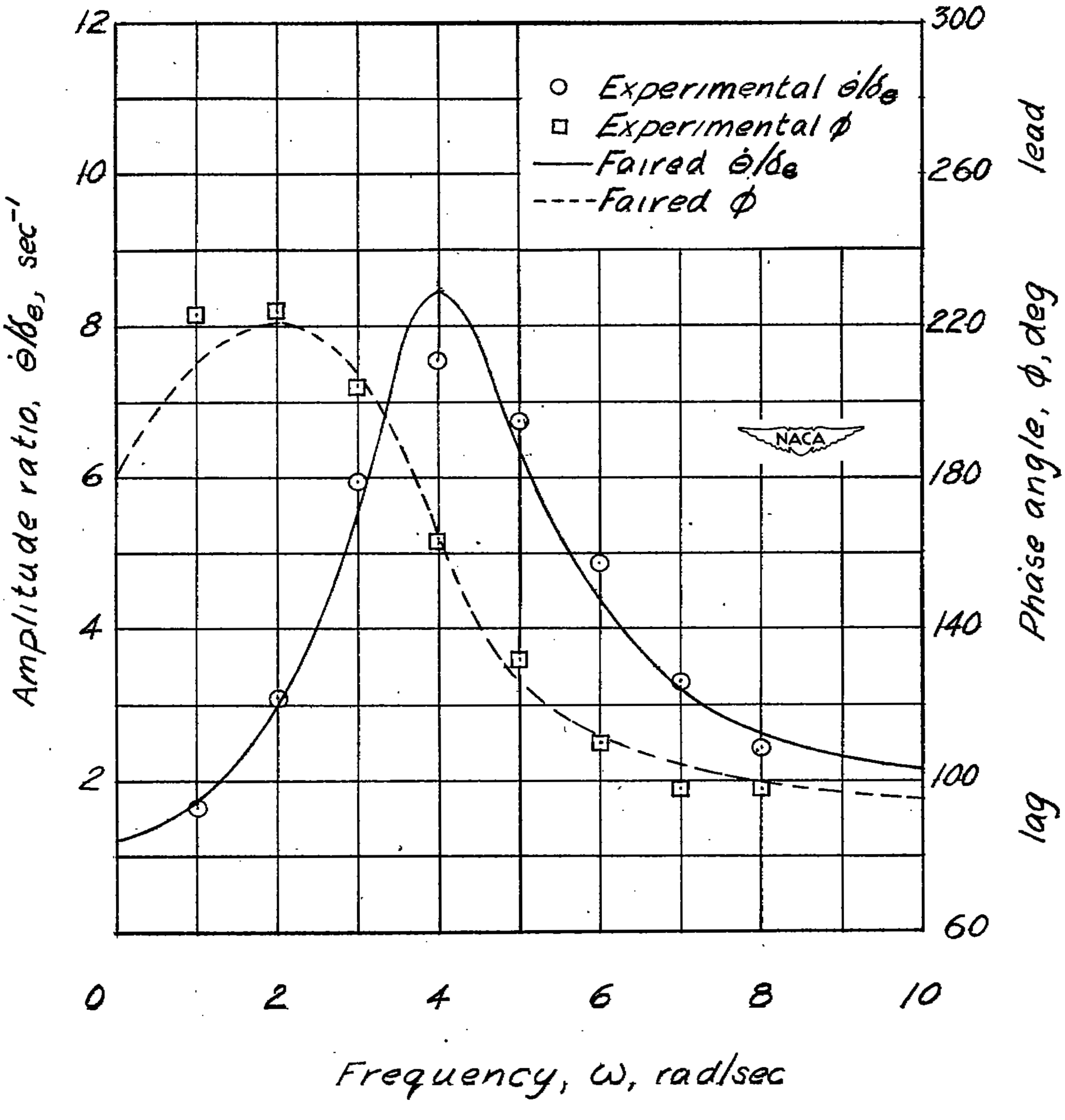
(g) M = 0.69; h_p = 38,000 feet.

Figure 3.- Continued.



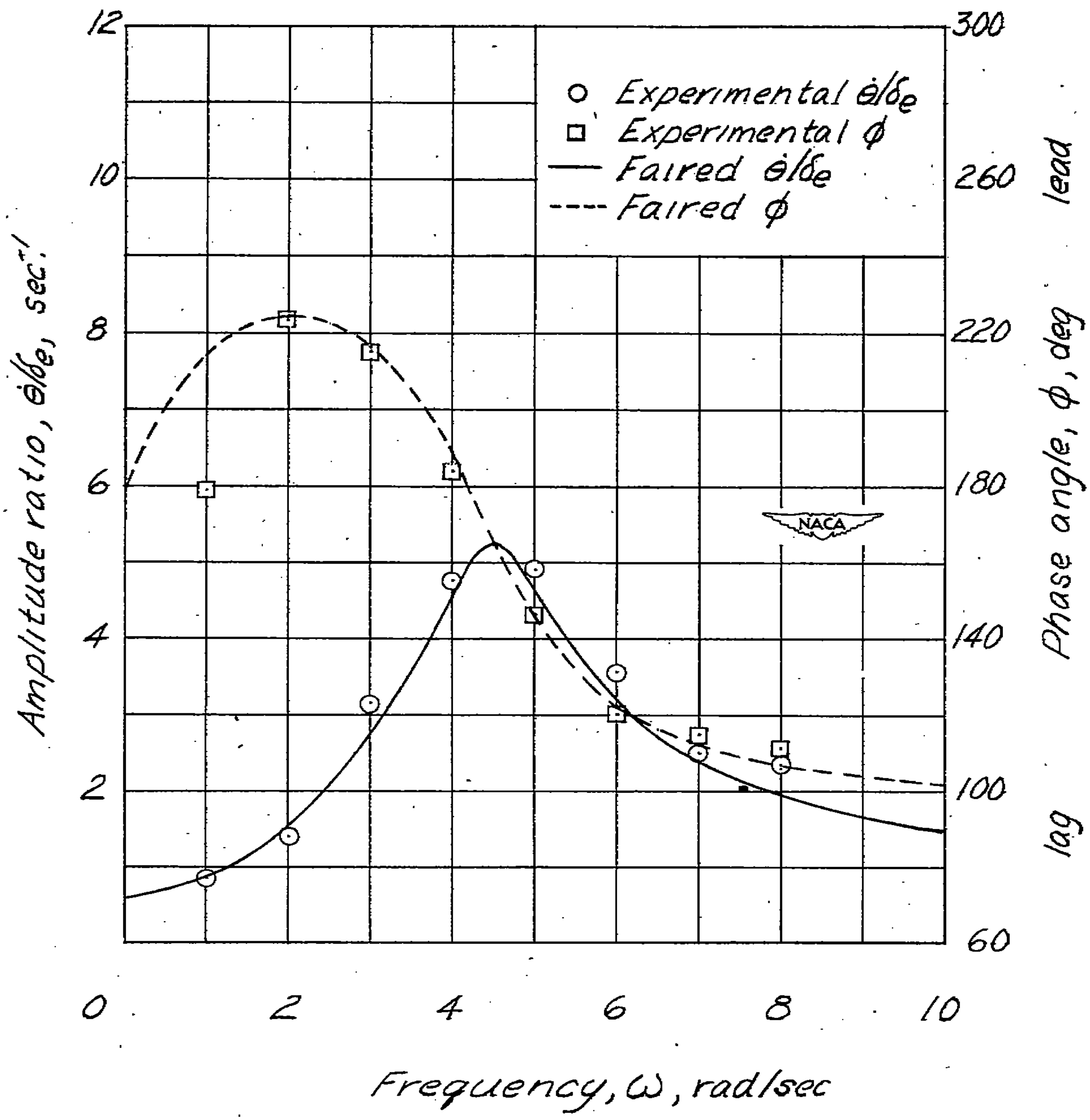
(h) M = 0.75; h_p = 38,000 feet.

Figure 3.- Continued.



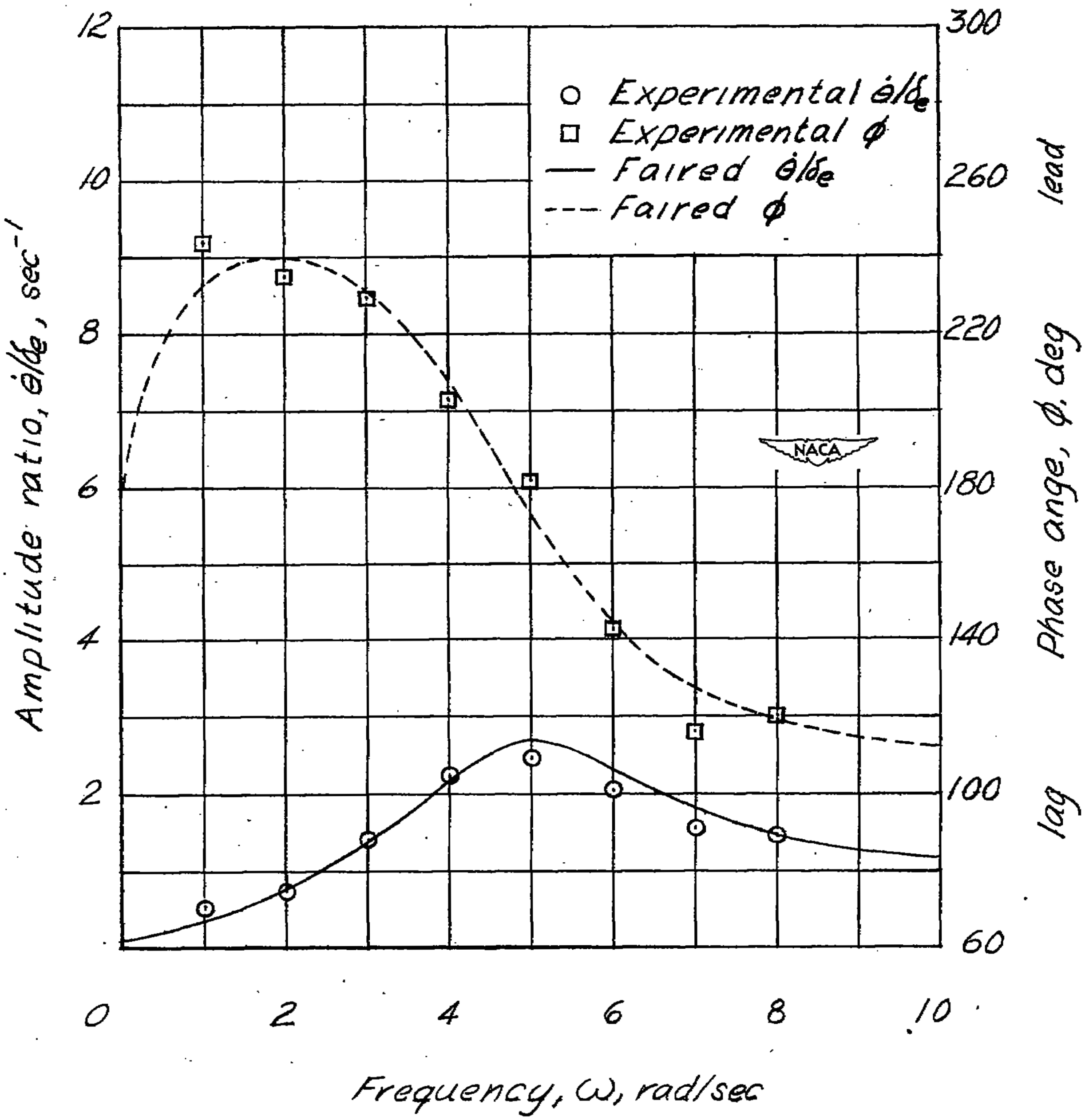
(i) M = 0.82; h_p = 34,000 feet.

Figure 3.- Continued.



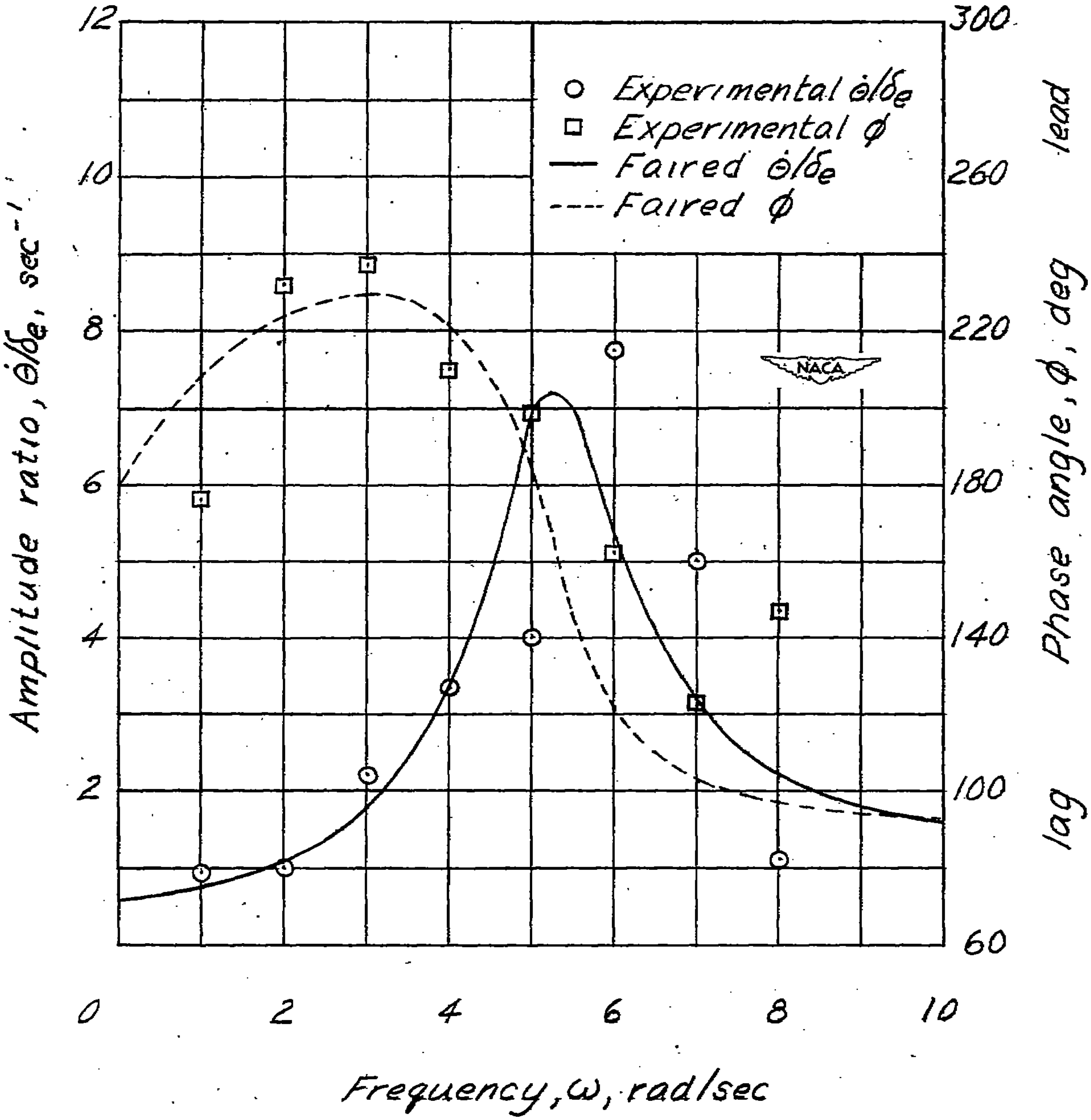
(j) M = 0.86; h_p = 33,000 feet.

Figure 3.- Continued.



(k) M = 0.88; h_p = 33,400 feet.

Figure 3.- Continued.



(l) M = 0.90; h_p = 30,000 feet.

Figure 3.- Concluded.

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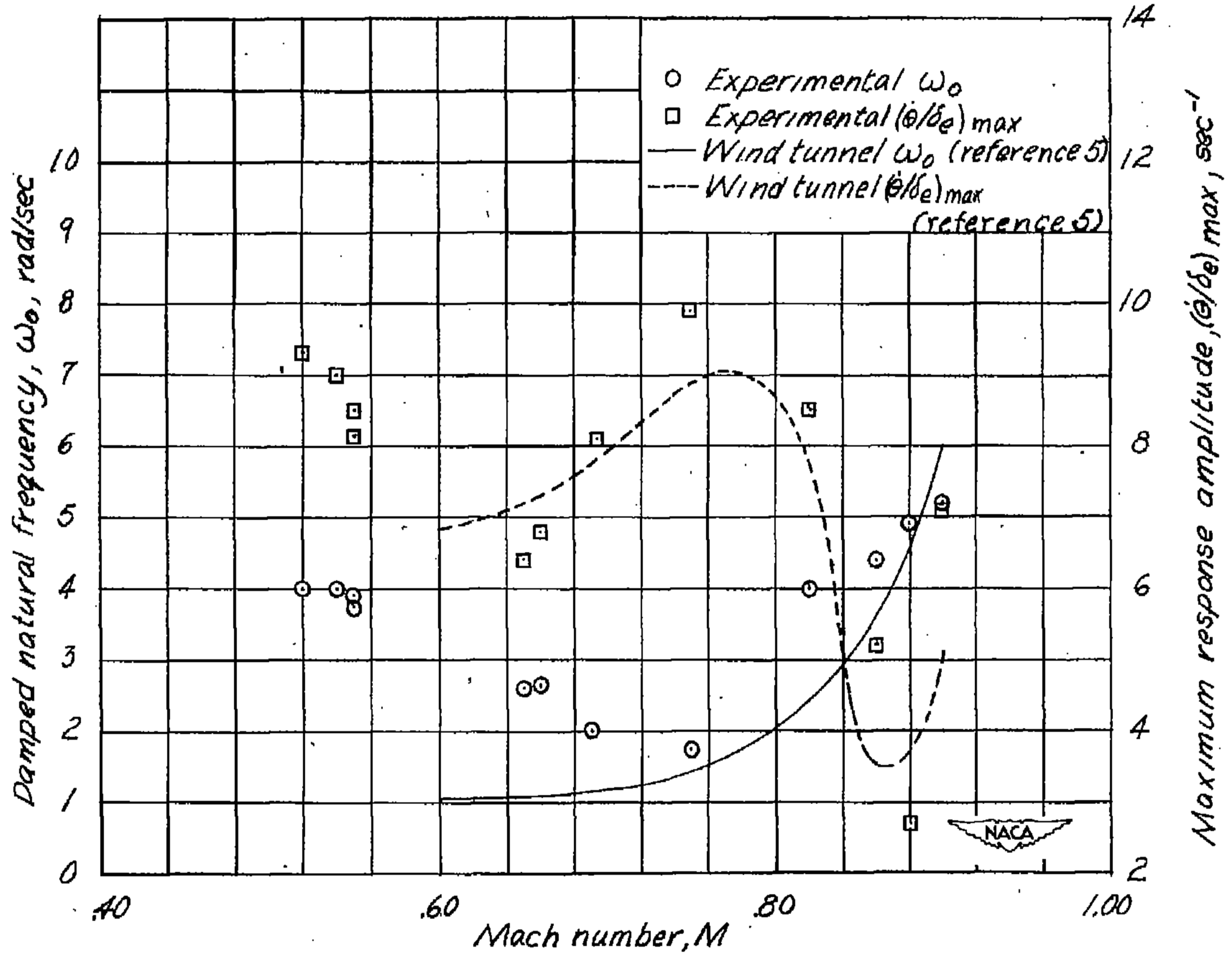

Figure 4.- Comparison of wind tunnel and experimental maximum response amplitude and damped natural frequency as a function of Mach number.

SECURITY INFORMATION

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